Real Interest Rates and Productivity in Small Open Economies*

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Abstract

In emerging market economies (EMEs), capital inflows are associated to productivity booms. However, the experience of advanced small open economies (AEs), like the ones of the Euro Area periphery, points to the opposite, i.e., capital inflows lead to lower productivity, possibly due to capital misallocation. We measure capital flow shocks as (exogenous) variations in (world) real interest rates. We show that, in the data, the misallocation narrative fits the evidence only for AEs: lower real interest rates lead to lower productivity in AEs, whereas the opposite holds for EMEs. We build a business cycle model with firms’ heterogeneity, financial imperfections and endogenous productivity. The model combines a misallocation effect, stemming from capital inflows, with an original sin effect, whereby capital inflows, via a real exchange rate appreciation, affect the borrowing ability of the incumbent, marginally more productive firms. The estimation of the model reveals that low (high) trade elasticity and high (low) firm’s dispersion in EMEs (AEs) are crucial ingredients to account for the different effects of capital inflows. The relative balance of these characteristics is able to simultaneously rationalize the evidence in both EMEs and AEs.

Keywords: world interest rates, financial frictions, firms’ heterogeneity, small open economies.

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1 Introduction

In emerging market economies (EMEs) capital inflows typically lead to output and asset price booms, appreciating real exchange rates, and excessive credit growth (Blanchard et al. 2016).\footnote{The latter is often considered as one of the best predictor of financial crisis (Gourinchas and Obstfeld 2012; Schularick and Taylor 2012).} Capital inflows, however, are not only a story of emerging markets. With the onset of the euro, large capital inflows in the European periphery have been associated to current account imbalances, loss of competitiveness, and a slowdown in productivity. The dismal performance of productivity in the euro periphery, in particular, has ignited a wider debate on the alleged \textit{misallocation} effects of capital (in)flows (Rey 2013; Gopinath et al. 2015).

In this paper we study the effects of capital inflows on business cycles, in both EMEs and advanced economies (AEs). In particular, and in light of the recent “misallocation debate”, we focus our attention on the effects of capital inflows on aggregate \textit{productivity}.

In our analysis, capital inflow “shocks” are measured as exogenous variations in (world) real interest rates. This is not the only way to measure capital inflows shocks. But it has the advantage of speaking to two sets of issues. First, the recent heated debate on the effects of ultra-easy monetary policy in the advanced economies for capital flow spillovers in emerging markets (Rey 2013; Miranda-Agrippino and Rey 2015). Second, a previous literature investigating the role of real interest rates fluctuations for EMEs business cycles (Neumeyer and Perri 2005; Uribe and Yue 2006). Noticeably, that literature has never investigated the \textit{causal} effect of real interest rates variations on productivity.

The cyclical properties of real interest rates and productivity differ sharply across EMEs and AEs. Figure 1 and 2 display the cross-correlation function of the real interest rate with (de-trended) GDP (top panel) and (de-trended) total factor productivity (bottom panel) respectively, for a sample of AEs and EMEs.\footnote{The real interest rate for each country is constructed as the sum of the US real interest rate and of a spread measure computed from the EMBI+ dataset. See Section 2 for more details. Concerning the cyclical correlation of the real interest rate with GDP in EMEs, this figure updates Neumeyer and Perri (2005) to the 1994Q1-2016Q3 period. Interestingly, cross correlations computed in the more recent time frame are higher, both for EMEs and AEs, than the one computed in Neumeyer and Perri (2005), where the sample ends in 2002Q2.} In EMEs, the real interest rate is countercyclical, and negatively correlated with productivity. Conversely, in AEs, real interest rates are procyclical, and positively correlated with productivity. Relatedly, a well-known business cycle literature (Neumeyer and Perri 2005; Uribe and Yue 2006) argues that, in the data,
real interest rate shocks account for a significant fraction of output volatility in EMEs, but for a negligible one in AEs.

The evidence reported in Figure 1 and 2 is unconditional and does not establish any causal link. We therefore first provide (VAR-based) evidence that the effects of real interest rate shocks on productivity are starkly different in EMEs and AEs (exemplified by the euro periphery). We show that a (suitably identified) positive innovation to the real interest rate causes (on average) a fall in productivity in EMEs, while the opposite holds for the euro-periphery countries (i.e., a real interest rate shock causes a rise in productivity). In other words, we show that the “misallocation narrative” describes well the experience of the euro area periphery countries (in that case, lower real interest rates, with the onset of the euro, associated to lower productivity), but the same narrative is at odds with the evidence for EMEs.

The empirical difference across EMEs and AEs poses a theoretical challenge. We therefore build a unified theoretical framework which can rationalize the evidence on the link between real interest rates and productivity for both groups of small open economies. We proceed in two steps. We first build a model of a small open economy which extends the standard RBC model (e.g., Mendoza 1991) to allow for two main features: (i) financial

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Figure 1: Cross-correlation between the real interest rate \((t+j)\) and log GDP\((t)\). The sample period is 1994Q1-2016Q3 for EMEs, and 1996Q1-2007Q4 for EA periphery countries. GDP series are detrended using the Hodrick-Prescott filter with smoothing parameter 1600. For a detailed description of the data refer to Appendix A1 and Appendix A2.
imperfections; and (ii) firms’ heterogeneity in productivity. Noticeably, the combination of these two features, and in contrast to a standard RBC model, makes total factor productivity endogenous. We label the latter the misallocation model.

In principle, an environment with imperfect financial markets and heterogeneous firms (leading to misallocation of production) would seem more genuinely suited to account for business cycle fluctuations in EMEs rather than in AEs (Restuccia and Rogerson 2017). The misallocation model, however, generates a puzzle. Relative to a standard RBC setup, this model leads to two main findings: first, an exogenous rise (fall) in the real interest rate leads to a rise (fall) in productivity; second, misallocation leads to a dampening of the effects of real interest rate shocks on output. These results are at odds with the evidence in Figure 1. Also they contradict the overwhelming evidence whereby output volatility is significantly larger in EMEs than in AEs, and real interest rate shocks explain a large fraction of output volatility in EMEs.

The puzzle stemming from the misallocation model can be explained as follows. Consider, for instance, an exogenous rise in the (world) real interest rate. At the margin, and in the presence of borrowing frictions, this makes the opportunity cost of producing (i.e., the marginal benefit of saving) higher for less productive firms, inducing the latter to exit the market, thereby driving up average productivity. The endogenous positive effect on

Figure 2: Cross-correlation between the real interest rate \((t+j)\) and log TFP\((t)\). The sample period is 1994Q1-2016Q3 for EMEs, and 1996Q1-2007Q4 for EA periphery countries. TFP series are detrended using the Hodrick-Prescott filter with smoothing parameter 1600. For a detailed description of the data refer to Appendix A1 and Appendix A2.
productivity dampens the standard contractionary effect of higher real interest rates on output stemming from intertemporal substitution. Furthermore, the dampening effect on output is increasing in the dispersion of new entrants in the production sector. Therefore, and somewhat paradoxically, a model characterized by financial frictions and misallocation of production seems better suited to account for business cycle dynamics in AEs than in EMEs.

We then modify the misallocation model to allow for an additional feature that typically characterizes financial markets in EMEs: the widespread inability of those countries to borrow in their own currency. We label this the misallocation cum original sin model. We show that this model, in line with the EMEs narrative, can generate both amplification of output fluctuations and a negative (positive) effect of higher (lower) real interest rates on productivity. The condition that allows to obtain the latter results is that periods of higher (lower) real interest rates be also periods of tightening (loosening) financial conditions. The introduction of an original sin channel allows to make the latter effect endogenous: higher (lower) real interest rates, in fact, lead to a depreciation (appreciation) of the real exchange rate - as typically witnessed during capital outflow (inflow) episodes in EMEs. If domestic firms can mostly borrow in foreign currency, the real depreciation (appreciation) lowers (boosts) their collateral values and their ability to borrow. The most productive firms, which are ex-ante the constrained ones, contract (expand) their borrowing, and therefore production, at the margin, leading to a decrease (increase) in average productivity. In turn, this generates a positive wedge between the marginal product of capital and the safe real interest rate, thereby amplifying the effect on aggregate output.

**Related literature.** Mendoza (1991) and Correia et al. (1995) show that interest rate fluctuations account only for a small fraction of business cycle fluctuations in a standard RBC small open economy model. Neumeyer and Perri (2005) find that the importance of interest rate shocks can be restored by augmenting a real business cycle model with a working capital constraint, zero wealth elasticity of labor supply and correlated movements of productivity and country risk (the latter being a component of the interest rate). In line with this finding, Neumeyer and Perri (2005) show that an (exogenous) negative correlation between interest rates and (temporary) productivity shocks allows to better match the business cycle moments of EMEs. Uribe and Yue (2006) show that this approach might overestimate the role of world interest rate shocks as it doesn’t account for the endogenous movements of
domestic rates to domestic macroeconomic conditions. Other papers investigating the role of real interest rates for emerging market business cycles are García-Cicco et al. (2010) and Akinci (2013). All these previous papers treat aggregate productivity in the standard way, i.e., like an exogenous stochastic process. The main difference of our paper is that we model productivity as endogenous. In this vein, we take a route similar to Pratap and Urrutia (2012), who concentrate on endogenous falls in productivity during EMEs financial crises, focusing on a systematic relationship between capital flows, misallocation and productivity movements.

2 Empirical analysis

The goal of this section is to investigate the role of real interest rates on productivity and economic activity in small open economies. Moving from the unconditional evidence presented in Figure 1, we now aim at estimating the causal relationship of suitably identified real interest rate shocks on the economy, differentiating between emerging and advanced economies. We do it by combining impulse responses from country-specific Structural Vector Autoregressions (henceforth SVARs) with recursive identification, using the stochastic pooling Bayesian approach introduced in Canova and Pappa (2007). This allows us to report a single measure of location and a 68 percent credibility set differentiated for EMEs and AEs, using all the relevant cross-sectional information.

We use quarterly data over the period 1994:1 to 2016:3. Four EMEs (Argentina, Brazil, Korea and Mexico) and four AEs (Ireland, Italy, Portugal and Spain) are included in the analysis. For EMEs, the selection and the length of the sample is driven by data availability, mostly constrained by the lack of reliable data on employment, hours worked and investment. The latter are in fact necessary for the construction of a measure of quarterly TFP. For AEs, the choice of the four Euro Area periphery countries is driven by the consideration that, especially in the time period of convergence towards the adoption of the euro, these countries experienced large and supposedly exogenous variations in the real interest rate. We start by describing the methodology used for the construction of the quarterly TFP measures. Next, we define our measure of the real interest rate and we finally set-up the empirical model used for the structural analysis.
Measuring TFP  We construct a non-adjusted quarterly measure of Total Factor Productivity (TFP henceforth) for four EMEs (Argentina, Brazil, Korea and Mexico) and four euro-periphery countries (Ireland, Italy, Portugal and Spain). As in Fernald (2014) we assume that total output is produced employing the capital stock ($K_t$) and labor ($L_t$) through a Cobb-Douglas production function:

$$Y_t = (TFP_t)K_t^aL_t^{1-a}.$$  

This implies that both capital and labor have a constant contribution to total production over time. This simplifies our analysis as we can measure TFP movements (aka, the Solow residual) as the change in total output unexplained by variation in capital and/or labor. While total output is proxied by aggregate GDP, it becomes important to correctly measure the capital stock and labor.

As for capital, we apply the perpetual inventory method (henceforth PIM, Fernald (2014, Bergeaud et al. (2016)) and construct an end-of-the-period measure starting from data on physical investment. We assume that investment is undertaken in one flow at the middle of the quarter, implying partial depreciation during the same quarter. The PIM capital accumulation equation reads:

$$K_{t+1}^j = (1 - \delta^j_q)K_t^j + I_{t+1}^j \sqrt{1 - \delta^j} \cdot j = E, B$$

(1) where investment is separated in two categories $j = E, B$, which capture the different longevity of capital, and where $\delta^j_q$ denotes the quarterly depreciation rate of capital of type $j$. The first category, $j = B$, captures the slowly depreciating capital with a rate of annual depreciation of $(\delta^B_q)^4 = 2.5$ percent, and is defined as buildings (Dwellings, Cultivated Biological Resources and Other Buildings and Structure); the second category, labeled equipment ($j = E$), captures the capital with quick turnover, with a yearly 10 percent depreciation rate (Intellectual Property Products, Machinery and Equipment and WPN Systems). One final assumption is needed to initialize the capital series. We assume that the growth rate of capital between the initial and the first period is equal to the average GDP growth rate. This implies that $\frac{1}{n} \sum_{t=0}^{n-1} \frac{Y_{t+1} - Y_t}{Y_t} = \frac{K_t - K_0}{K_0} = -\delta^j + \sqrt{(1 - \delta^j) \frac{I_t}{K_0^j}}$, allowing us to compute the initial value $K_0^j$. Given $\delta^j$, and applying (1), one can then recover the sequence for $K_t^j$, and compute the series for aggregate capital as $K_t = \sum_{j=E,B} K_t^j$ for all $t$.

As for the labor input, we proceed as follows. The total amount of labor used in production is computed multiplying data on hours worked with those on employment. Quarterly
data on employment are not always directly available for EMEs and are, when necessary, reconstructed using Census data. Appendix A provides a detailed description of the data and the methodology used country by country.

The resulting TFP measure has two well known limits. First, it has to be interpreted as an aggregate measure of productivity and not as the correct aggregate measure of technology (see Kimball et al. 2006; Basu et al. 2012). Second, our measure does not account for changes in factor utilization (Fernald 2014), failing to account for the intensive margin, due, for example, to modifications of hours in the workweek or of labor effort. However, we claim that this measure of aggregate productivity is still informative and gives us a statistical object which we will be able to meaningfully relate to our model.

**Real interest rates** The real interest rate we want to measure is the expected quarterly real rate at which households and firms in the economy can borrow or lend domestically and internationally. Aside from the fragmentation of financial markets and the co-existence of different nominal rates in the economy, the largest difficulty in defining a real interest rate is the measurement of domestic expected inflation. While for AEs past inflation can be used to form quarterly reliable expectations, in EMEs the high volatility of inflation often generates implausible movements in (ex-post) real interest rates.

For EMEs we follow Neumeyer and Perri (2005) and Uribe and Yue (2006), and compute the real interest rate in a typical economy as the sum of the U.S. risk free rate (measured as the 90-day U.S. Treasury Bill rate) and a measure of the country’s interest rate premium reported by the JP Morgan Emerging Market Bond Global spread Index (henceforth EMBI+ global spread). The EMBI global spread is a quarterly bond spread index of foreign denominated (US dollar) fixed income debt instruments which is collected by JP Morgan. To the nominal interest rate we subtract expected US inflation, computed as the three-period moving average of the current deflator inflation. Hence the real interest rate for the typical EME is constructed as:

\[
RR^j_t = (R^USt - \pi^USt_t) + \Delta^EMBI_t, \ j \in EM
\]

where \(R^USt\) is the 90-day U.S. treasury bill rate, \(\pi^USt\) is expected inflation in the US, and \(\Delta^EMBI_t\) is the EMBI+ spread. For a typical euro-periphery economy (\(j \in AE\)) we compute the real interest rate as:
\[ RR_t^j = R_t^{j,IB} - E\pi_t^j, \quad j \in AE \]

where \( R_t^{j,IB} \) is the 90-day nominal interbank rate in country \( j \), and \( E\pi_t^{AE} \) is expected inflation. Details on the construction of our data set are available in Appendix A1 and A2.

### 2.1 SVARs

Our empirical model takes the typical form:

\[ A_0 Y_t = A_1 Y_{t-1} + ... A_p Y_{t-p} + \varepsilon_t \tag{2} \]

where \( Y_t \) is a \( n \times 1 \) vector, \( A_0, A_1, ..., A_p \) are \( n \times n \) matrices of structural coefficients, and \( \varepsilon_t \) is a \( n \times 1 \) vector of random disturbances with mean zero and identity variance-covariance matrix \( \Sigma_\varepsilon \). The vector \( Y_t \) comprises \( n = 5 \) variables: total factor productivity (TFP), real GDP, net exports as a ratio to GDP (\( NX_t \)), the real effective exchange rate (REER\( _t \)), and the real interest rate (RR\( _t \)):

\[
Y_t = \begin{bmatrix}
TFP_t \\
GDP_t \\
NX_t \\
REER_t \\
RR_t
\end{bmatrix}
\]

\( TFP_t, GDP_t \) are first expressed in logs and then with \( NX_t \) in levels are HP-filtered. \( REER_t \) is expressed in logs. The number of lags is set to 2, to preserve enough degrees of freedom.

We assume that \( A_0 \) is a lower triangular matrix and that the real interest rate is ordered last in \( Y_t \). These assumptions, which imply that TFP reacts to the shock hitting the real interest rate, \( \varepsilon_t^{RR} \) only with a lag, allow us to identify innovations in the real interest rate which are orthogonal to domestic economic conditions, summarized by \( (n-1) \times 1 \) sub-vector of domestic variables \( Y_t^{d} \equiv (TFP_t, GDP_t, NX_t, REER_t) \).\(^3\) Consider a typical EMEs. The real interest rate \( RR_t \) is the sum of two components: the first is the US real interest rate, which is a proxy for the world real interest rate, and is therefore strictly exogenous from the

\(^3\)A possibly problematic assumption concerns the relative ordering of REER\( _t \) and RR\( _t \). Our baseline specification states that the real exchange rate is ordered in position \( n-1 \), implying that the real exchange rate does not react on impact to innovations in the real interest rate. We have experimented with an alternative ordering in which REER\( _t \) is ordered in position \( n \) and RR\( _t \) is ordered in position \( n-1 \). Our results are generally robust.
viewpoint of the EM small open economy; the second component, however, is the EMBI+ spread, whose variations are endogenous to the domestic economic conditions captured by $Y_t^d$. Hence ordering $RR_t$ last allows to identify those components of the innovations to the spread $\Delta_t^{EMBI}$ which are orthogonal to the domestic business cycle. Premultiplying both sides of (2) by $A_0^{-1}$ our model assumes the reduced form structure:

$$Y_t = C_1 Y_{t-1} + ... + C_p Y_{t-p} + u_t$$  \hspace{1cm} (3)

where $C_i \equiv A_0^{-1} A_i$, $u_t \equiv A_0^{-1} \varepsilon_t$ and $Var(u_t) = \Sigma_u = A_0^{-1} I (A_0^{-1})'$. It is then straightforward to compute $A_0^{-1}$ as the Choleski factor of the matrix $\Sigma_u$. In the figures below, however, we normalize the size of the shock to the real interest rate $\varepsilon_t^{RR}$ to 1.

**Stochastic pooling** Following Canova and Pappa (2007), we pool the impulse responses of the different countries. We assume that each country-specific impulse response of variable $r$ to $\varepsilon_t^{RR}$ has the prior distribution:

$$\alpha^r_{t,h} = \mu^r_h + v^r_{t,h} \text{ where } v^r_{t,h} \sim N(0, \tau^r_h)$$

where $h$ is the impulse response horizon, $h = 0, 1, ..., H$ and $\iota \in N$ is the country identifier ($\alpha^r_{t,10}$ is therefore the impulse response of variable $r$, for country $\iota$, 10 periods after the shock).

We choose a diffuse prior for $\mu^r_h$, so that the average impulse responses are essentially driven by the data. We assume $\tau^r_h = \delta_r / h$, where $\delta_r$ takes into account the observed dispersion of the impulse responses for variable $r$ across countries.$^4$

Under a Normal-Wishart prior for each country-specific VAR, the posterior for $\mu^r_h$ is

$$\mu^r_h | \tau^r_h, \hat{\Sigma}_u \sim N(\tilde{\mu}^r_h, \tilde{V}^r_{\mu,h})$$

where $\tilde{\mu}^r_h = \hat{V}^r_{\mu,h} \sum_{\iota=0}^N (\hat{V}^r_{\alpha^r_{\iota,h},h} + \tau^r_h)^{-1} \hat{\alpha}^r_{\iota,h}, \hat{V}^r_{\mu,h} = (\sum_{\iota=0}^N (\hat{V}^r_{\alpha^r_{\iota,h},h} + \tau^r_h)^{-1}^{-1} \text{ and } \hat{\Sigma}_u \text{ is the estimated variance-covariance matrix of the reduced form residuals } u_t \text{ in the VAR for country } \iota, \hat{\alpha}^r_{\iota,h} \text{ is the country } \iota \text{-specific OLS estimator of } \alpha^r_{\iota,h} \text{ and } \hat{V}^r_{\alpha^r_{\iota,h}} \text{ its variance. The intuition behind this approach is that impulse responses are weighted by their precision. More precise impulse responses are weighted more than those estimated with less precision.}$

$^4$Namely, it is computed by averaging the cross-sectional variance of the impulse responses across horizons.
Results  Figure 3 depicts (weighted) impulse-responses of selected variables to a one-standard error innovation in the real interest rate for EMEs, whereas Figure 4 reports the same responses for the Euro Area periphery countries. Three main results are worth emphasizing.

First, in EMEs, an exogenous innovation in the real interest rate induces a contraction in both GDP and TFP, a rise in net exports and a real exchange rate depreciation. This picture is consistent with the typical narrative of capital outflow episodes. In the EA periphery, a rise in the real interest rate causes a similar effect on net exports and the real exchange rate; but, remarkably, the effect on GDP and TFP is the opposite relative to EMEs: both GDP and TFP rise in response to a real interest rate innovation. Interestingly the two results above are consistent with the unconditional evidence reported in Figure 1. Third, and conditional on a real interest rate innovation, net exports are countercyclical in EMEs, whereas they are procyclical in AEs. Below we build a theoretical model that is able to simultaneously account for these three main results.

While the results for EMEs are reported for a time sample extending to 2016:3, the ones for the EA periphery countries (Figure 4) are based on a sample that excluded the period comprising the euro-zone sovereign debt crisis. Figure (5) displays the results for the EA periphery countries extending the sample beyond 2007 and until 2016:3. The figure shows that our key result remains unchanged: a rise in the real interest rate generates a rise in TFP, although the effect on GDP and net exports looses statistical significance. The latter result is somewhat in line with previous evidence pointing out the weak relevance of real interest rate shocks for the volatility of GDP in advanced economies.

3 Theoretical model

In this section we develop a theoretical framework in order to rationalize the different effects of real interest shocks on productivity in the two groups of countries. Our model builds on a series of theoretical contributions emphasizing the role of firms’ heterogeneity and financial frictions - such as, e.g., Reis (2013), Liu and Wang (2014), Moll (2014) and Buera and Moll (2015). Our contribution is to extend (elements of) these setups to a dynamic small open economy environment featuring balance sheet effects of real exchange fluctuations.

Consider a small open economy populated by two types of agents: (i) a family of (a large number of) firms, labeled entrepreneur; (ii) a representative worker. Only the entrepreneur is
Figure 3: Impulse responses to a one standard deviation innovation to the real interest rate ($RR_t$). Sample of pooled countries: Argentina, Brazil, Korea and Mexico. Sample period 1994:1 - 2016:3. REER = Foreign/Domestic, therefore a rise is a real depreciation.
Figure 4: Impulse responses to a one standard deviation innovation to the real interest rate ($RR_t$). Sample of pooled countries: Ireland, Italy, Portugal and Spain. Sample period: 1996:1 - 2007:4. REER = Foreign/Domestic, therefore a rise is a real depreciation.
Figure 5: Extended sample period. Impulse responses to a one standard deviation innovation to the real interest rate ($RR_t$). Sample of pooled countries: Ireland, Italy, Portugal and Spain. Sample period: 1996q1 - 2016q3. REER = Foreign/Domestic, therefore a rise is a real depreciation.
allowed to save. The entrepreneur consumes/saves the income returned by the firms. Firms belonging to the family are allowed to borrow and lend to each other at the (exogenous) world interest rate $r^*_t$. The worker supplies homogeneous labor to the firms and consumes her labor income. Domestic agents consume both a domestically produced good and an imported good.

**Relative prices** Let the domestic CPI index be denoted by

$$P_t = \left[ \gamma P_{H,t}^{1-\theta} + (1-\gamma)P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of the domestic and foreign good respectively, $\gamma$ is the share of the domestically produced good in the consumption basket, and $\theta > 0$ is the elasticity of substitution between the domestic and the foreign good. Let $\epsilon_t$ be the CPI-based real exchange rate:

$$\epsilon_t = \frac{P_t^*}{P_t} = \frac{P_{F,t}}{P_{H,t}}$$

where $P_t^*$ is the foreign CPI (expressed in units of domestic currency). The second equality follows from the law of one price and from assuming that the weight of domestically produced goods in the consumption basket of the rest of the world is infinitesimally small.

In units of CPI, the price of the domestic good therefore reads:

$$q_t = \frac{P_{H,t}}{P_t} = \left[ \frac{1 - (1 - \gamma)\epsilon_t^{1-\theta}}{\gamma} \right]^{\frac{1}{1-\theta}} = q(\epsilon_t)$$

with $q'(\epsilon_t) < 0$. Hence a real (CPI) depreciation, i.e., a rise in $\epsilon_t$, causes a fall in the relative price of the domestic good $q_t$, with an elasticity $(1 - \gamma)/\gamma$, which is increasing in the share of imported goods (or degree of openness).\(^5\)

### 3.1 Entrepreneur

The agent named Entrepreneur, like a family construct, holds a large number of firms, each indexed by $i$. Each firm $i$ produces a homogenous good via a constant-return to scale

\(^5\)To see this, notice that a log-linear approximation of (6) around a steady state with $\varepsilon = q = 1$ yields: $\hat{q}_t = -\frac{1}{\gamma} \hat{\epsilon}_t$, where a hat denotes percentage deviations from the steady state. Alternatively, one can define the terms of trade $\tau_t = P_{F,t}/P_{H,t}$ as the relative price of the imported good. The relationship between the terms of trade and the real exchange rate then reads: $\tau_t = \tau(\epsilon_t) = \epsilon_t/q(\epsilon_t)$, with $\tau'(\epsilon_t) > 0$. 

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production function, but is heterogeneous in its own productivity. The production function of a generic firm \( i \) is:

\[
y_{i;t} = A_t (z_{i,t-1} k_{i,t-1})^{\alpha} l_{i,t}^{1-\alpha}, \quad \alpha \in [0, 1]
\]  

(7)

where \( y_{i;t} \) is output of firm \( i \), \( A_t \) is a common productivity shifter, \( z_{i,t-1} \) is firm \( i \)'s own productivity, and \( l_{i,t} \) is labor hired from the workers at the wage \( w_t \). Firm \( i \)'s productivity is drawn from a continuous distribution \( \Psi(z) \):

\[
z \sim \Psi(z)
\]  

(8)

with \( \psi(z) \) being the marginal density function.

Each firm \( i \) draws its own productivity before the end of each period and before making its borrowing/lending decision. Hence \( z_{i,t-1} \) denotes time \( t \) productivity of firm \( i \) drawn before the end of period \( t - 1 \). Firm \( i \) starts period \( t \) with an equal share of net wealth distributed by the Entrepreneur in period \( t - 1 \), and before period-\( t \) idiosyncratic productivity (drawn at time \( t - 1 \)) was known. In addition, each firm \( i \) owns an amount of capital borrowed from (or lent to) other firms. We assume that borrowing and lending is denominated only in units of the foreign good. This assumption will be critical for some of our results.

**Timing**  
The timing of events is illustrated in Figure 6. Let \( S_{i,t} \) denote the state vector of firm \( i \) at the beginning of time \( t \):

\[
S_{i,t} = (\pi_{t-1}, z_{i,t-1}, d_{i,t-1}, A_t, r^*_{t-1}),
\]

where \( \pi_{t-1} \) is net worth, expressed in domestic CPI units, and uniformly distributed by the Entrepreneur across firms in period \( t - 1 \); \( d_{i,t-1} \) is outstanding borrowing (or lending), expressed in foreign consumption units; and \( r^*_{t-1} \) is the gross real interest rate (between \( t - 1 \) and \( t \)) expressed in units of foreign goods.

The capital stock available to firm \( i \) at the beginning of time \( t \) therefore is equal to:

\[
k_{i,t-1} = \pi_{t-1} + \epsilon_{t-1} d_{i,t-1}
\]  

(9)

Equation (9) states that, conditional on production, firm \( i \) faces an external finance problem, i.e., the same firm needs to acquire external funds beyond its net worth in order to finance the purchase of physical capital.
1. At the beginning of time $t$ aggregate uncertainty $A_t$ is resolved.

2. Given $S_{i,t}$, each firm $i$ chooses the optimal quantity of labor $l_{i,t}$ in order to produce output $y_{i,t}$ using (7). After production, and after paying interest on its outstanding debt, each firm $i$ returns both the inherited wealth and the capital previously borrowed from other firms to the Entrepreneur. Profits $\Gamma_{i,t}$ are realized for all $i$'s and returned to the Entrepreneur (as dividends).

3. Given the return on the rented capital plus the received dividends, the Entrepreneur chooses consumption $C^e_t$ and savings in new aggregate wealth $N_t$.

4. Realized aggregate wealth $N_t$ is distributed in equal shares $\pi_t$ to all firms, before the realization of idiosyncratic productivity.

5. Before the end of period $t$, and before its borrowing/lending decision is made, each firm $i$ draws its period $t+1$ idiosyncratic productivity $z_{i,t}$, which is i.i.d. across firms and time. The realized difference in productivity generates a motive for borrowing and lending across firms.

6. After observing $z_{i,t}$, although not aggregate productivity $A_{t+1}$ yet, firm $i$ chooses new borrowing from (or lending to) other firms, $d_{i,t}$, and maximizes the expected discounted value of next period profits.

7. At beginning of time $t + 1$ aggregate uncertainty $A_{t+1}$ is resolved and firms that have available capital optimally chose the level of labor and produce.

**Borrowing frictions and original sin** Conditional on production, new borrowing in period $t$, $d_{i,t}$, is limited by the value of collateral:

$$d_{i,t} \leq \frac{X \cdot k_{i,t}}{\epsilon_t}$$

(10)
where \( \chi \) is an exogenous and constant loan-to-value ratio. Notice that fluctuations in the real exchange rate affect the value of collateral. In particular, a real appreciation (i.e., a fall in \( \epsilon_i \)) boosts, ceteris paribus, firm \( i \)'s ability to borrow. We will show below that this feature - which we label, in line with a large literature, "original sin" - is particularly important to allow the model to account for the effects of real interest rate shocks on productivity (and the business cycle in general) in EMEs.

3.1.1 Individual firm’s problem

Next we formally study the problem of each individual firm \( i \) owned by the Entrepreneur. Let firm \( i \)'s real profits in period \( t \) (expressed in domestic CPI units) be given by

\[
\Gamma_{i,t} = q_t y_{i,t} - w_t l_{i,t} - (1 + r_{t-1}^*) \epsilon_t d_{i,t-1} + (1 - \delta) k_{i,t-1} - \pi_{t-1}
\]

where \( q_t y_{i,t} \) is firm's \( i \) output expressed in units of domestic CPI, \( w_t l_{i,t} \) is the real cost of labor, \( r_{t-1}^* \) is the exogenous one-period real interest rate on (foreign good denominated) debt, \( (1 - \delta) k_{i,t-1} \) is undepreciated capital, and \( \pi_{t-1} \) is outstanding net worth at the beginning of time \( t \).

Let \( M_{t,t+j} \) be the Entrepreneur’s stochastic discount factor, which is common across firms. Each firm \( i \) chooses labor demand \( l_{i,t} \), borrowing \( d_{i,t} \) and holdings of physical capital \( k_{i,t} \) in order to solve:

\[
\max_{\{l_{i,t}, k_{i,t}, d_{i,t}\}} \sum_{s=0}^{\infty} \mathbb{E}_t M_{t,t+j} \Gamma_{i,t+s}
\]

subject to (7), (9) and (10).

The problem of firm \( i \) can be split into a static optimal labor choice and an intertemporal choice. As in Angeletos and Calvet (2006) and Angeletos (2006), since labor \( l_{i,t} \) affects only time \( t \) profits and is chosen after the state \( S_{i,t} \) has been observed, the optimal \( l_{i,t} \) maximizes \( \Gamma_{i,t} \) state by state. Given that the constant-return nature of production, this implies that optimal labor demand is linear in capital. Formally:

\footnote{A constraint of this type can be due, as in Kiyotaki and Moore (1997), to the limited ability of the borrower to commit to repay its debt. Anticipating this, a given lender will require collateral at the time of the loan contract.}
\[ l_{i,t} = l(A_t, w_t, z_{i,t-1}) \cdot k_{i,t-1} \]  \hfill (12)

where
\[
l(A_t, w_t, z_{i,t-1}) \equiv \max_{l_{i,t}} \{ q_t y_{i,t} - w_t l_{i,t} \} = \left( \frac{w_t}{1 - \alpha} \right)^{-\frac{1}{\alpha}} (A_t q_t)^{\frac{1}{\alpha}} z_{i,t-1}.
\]  \hfill (13)

In the intertemporal stage, and conditional on (12), firm \( i \) chooses capital and debt after receiving net wealth from the family \( \pi_t \) and after drawing next period idiosyncratic productivity \( z_{i,t} \).

Let the gross real interest rate (between \( t + s - 1 \) and \( t + s \)) expressed in units of domestic CPI be denoted by:
\[ \mathcal{R}_{t+s-1} \equiv (1 + r_{t+s}^t) \frac{\epsilon_{t+s}}{\epsilon_{t+s-1}}. \]  \hfill (14)

Substituting \( l_{i,t} \) from (12) and for \( d_{i,t} \) from (9), we can write the firm’s maximization problem as a function only of the choice of capital:
\[
\max_{\{k_{i,t}\}} \sum_{s=0}^{\infty} \mathbb{E}_t \mathcal{M}_{t,t+s} \left\{ \left[ \alpha (q_{t+s} A_{t+s}) \frac{w_{t+s}}{1 - \alpha} \right]^{-\frac{1}{\alpha}} z_{i,t+s-1} + 1 - \delta \right\} k_{i,t+s-1} - \mathcal{R}_{t+s-1} k_{i,t+s-1} + (\mathcal{R}_{t+s-1} - 1) \pi_{t+s-1}
\]  \hfill (15)

subject to
\[ k_{i,t+s} \leq \lambda \pi_{t+s}, \]  \hfill (16)

where \( \lambda \equiv 1/(1 - \chi) \). Notice that equation (16) is a leverage constraint on the net wealth equally distributed to each firm \( i \) by the Entrepreneur.

**Optimality conditions** Let \( \nu_t \) be the period-\( t \) multiplier on constraint (16). The period-\( t \) first-order optimality conditions for firm \( i \) read:
\[
\nu_t > 0 : \quad k_{i,t} = \lambda \pi_t \]  \hfill (17)

\[
\nu_t = 0 : \quad \mathbb{E}_t \mathcal{M}_{t,t+1} \left[ (q_{t+1} A_{t+1}) \frac{w_{t+1}}{1 - \alpha} \right]^{-\frac{1}{\alpha}} \alpha z_{i,t} + (1 - \delta - \mathcal{R}_t) \]  \hfill (18)
Since $\mathcal{M}_{t,t+1}$ is common across firms it is possible to show that there exists a value of firm $i$'s productivity $z_t$, common to all firms $i$, which satisfies:

$$z_t = \frac{\mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \left[ R_t - 1 + \delta \right] \right\}}{\mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \left[ \alpha (q_{t+1} A_{t+1})^{\frac{1}{2}} \left( \frac{m_{t+1}}{1-\alpha} \right)^{-\frac{1-\alpha}{\alpha}} \right] \right\}} \equiv z (R_t^*)$$ \quad (19)

such that:

$$k_{i,t} = \begin{cases} \lambda \bar{m}_t & \text{if } z_{i,t} > z_t \\ \in (0, \lambda \bar{m}_t] & \text{if } z_{i,t} = z_t \text{ and } \epsilon_t d_{i,t} = \begin{cases} (\lambda - 1) \bar{m}_t & \text{if } z_{i,t} > z_t \\ (\bar{m}_t, (\lambda - 1) \bar{m}_t] & \text{if } z_{i,t} = z_t \\ -\bar{m}_t & \text{if } z_{i,t} < z_t \end{cases} \\ 0 \end{cases}$$ \quad (20)

**Remarks** Two observations are in order concerning equations (19) and (20). First, and conditional on $z_{i,t} > z_t$, the choices of both capital and debt are linear in net worth, and are equal across firms. In particular, each firm $i$ whose productivity draw exceeds the threshold borrows up to the maximum limit. This is an implication of the constant-return production function, coupled with the assumption that the productivity draw is iid across firms. Conversely, if $z_{i,t} < z_t$, i.e., the productivity draw is below the threshold, the firm does not purchase capital and simply decides to lend its net worth $n_t$ to the more productive firms. Second, at the optimum, and for any given sequence $\epsilon_t$ of the real exchange rate, the threshold productivity $z_t$ is increasing in the real interest rate:

$$\frac{\partial z_t}{\partial r^*_t} = \frac{\partial z (\cdot)}{\partial r^*_t} > 0$$ \quad (21)

The intuition for this result is as follows. The marginal firm is indifferent between entry (and produce) and stay idle and lend its capital to the more productive firms. An exogenous rise in the real interest rate $r^*_t$ makes the opportunity cost of production or, equivalently, the marginal return on saving, higher for the marginal firm. The latter, therefore, finds it optimal to exit the market and act as an unproductive lender. This "cleansing" effect raises the productivity threshold, because it now requires, in equilibrium, a higher productivity draw in order to make it profitable for the marginal firm to enter and become productive.

Notice, however, that (21) describes only a partial equilibrium effect. In general equilibrium, variations in the real interest rate affect the real exchange rate $\epsilon_t$, and in turn the collateral value in equation (10). A rise in the real interest rate (for instance) induces a capital outflow and a depreciation of the real exchange rate, which in turn tightens the borrowing constraint for the incumbent firms. Ceteris paribus, the marginally productive
firm will then be induced to enter the market, thereby lowering average productivity. This effect can potentially overturn the positive effect on average productivity stemming from the higher return on saving inducing the marginally less productive firm to exit the market.

### 3.1.2 Aggregation

Before moving to the specification of the Entrepreneur’s problem, we need to aggregate across individual firms. This is useful, in particular, to derive our measures of aggregate and average productivity, which evolve endogenously in our setting. To begin with, aggregate net worth reads:

\[ N_t = \int_0^1 \pi_t di = \pi_t \int_0^\infty \psi(z)dz = \pi_t \]

for all \( i \in [0,1] \). Since, from (17), \( k_{i,t} = 0 \) if \( z_{i,t} < z_t \) and \( k_{i,t} = \lambda \pi_{i,t} \) otherwise, aggregate capital can be written:

\[ K_t = \int k_t(i)di \]

\[ = \lambda \pi_t \int_{z_t}^\infty \psi(z)dz \]

\[ = \lambda N_t[1 - \Psi(z_t)] \]

Hence aggregate capital depends on aggregate net worth \( N_t \) and on the fraction of firms \([1 - \Psi(z_t)]\) which are productive. The latter, in turn, is a decreasing function of the productivity threshold \( z_t \).

Similarly, aggregate debt can be expressed, in units of domestic CPI, as:

\[ \epsilon_t D_t = \int_0^1 \epsilon_t d_{i,t} di \]

\[ = -\pi_t \int_0^{z_t} \psi(z)dz + [\lambda - 1]\pi_t \int_{z_t}^\infty \psi(z)dz \]

\[ = \pi_t \Psi(z_t) + [\lambda - 1]\pi_t[1 - \Psi(z_t)] \]

\[ = N_t[\lambda(1 - \Psi(z_t)) - 1] \]

Notice that, in equilibrium, and due to the valuation mismatch between the firm’s liability side (denominated in units of foreign goods) and the asset side (denominated in units of the domestic good) movements in the real exchange rate \( \epsilon_t \) drive a wedge between aggregate debt and aggregate net worth.
Aggregate labor can be written as:
\[
L_t = \int_0^1 L_{i,t} \, d\bar{i} 
\]
\[
= \left[ \frac{w_t}{1 - \alpha} \right]^{1 - \frac{1}{\alpha}} [q_t A_t]^{\frac{1}{\alpha}} \lambda n_{t-1} \cdot Z_t \tag{25}
\]

where \( Z_t \equiv \int_{Z_{t-1}}^{\infty} z \psi(z) \, dz \) is aggregate productivity.

Then using (22) we obtain:
\[
L_t = \left[ \frac{w_t}{1 - \alpha} \right]^{1 - \frac{1}{\alpha}} [q_t A_t]^{\frac{1}{\alpha}} K_{t-1} Z_t, 
\]

where
\[
Z_t \equiv \frac{Z_t}{1 - \Psi(z_{t-1})} = \int_{Z_{t-1}}^{\infty} z \psi(z) \, dz 
\]
\[
\tag{26}
\]
is average productivity, i.e., aggregate productivity divided by the number of productive firms.

Aggregate home goods production can be written:
\[
Y_t = \int_0^1 y_t(i) \, d\bar{i} 
\]
\[
= \left[ \frac{1 - \alpha}{q_t} A_t^{\frac{1}{\alpha}} \left( \frac{w_t}{1 - \alpha} \right)^{-\frac{1 - \alpha}{\alpha}} \right] \int_0^1 z_{i,t-1} k_{i,t-1} \, d\bar{i} 
\]
\[
= q_t^{1 - \alpha} A_t^{\frac{1}{\alpha}} \left( \frac{w_t}{1 - \alpha} \right)^{-\frac{1 - \alpha}{\alpha}} \lambda n_{t-1} \int_{Z_{t-1}}^{\infty} z \psi(z) \, dz 
\]

Substituting (22) and (25) yields the following relationship between aggregate output and aggregate labor and capital:
\[
Y_t = A_t (Z_t K_{t-1})^\alpha L_t^{1 - \alpha} \tag{28}
\]

In equilibrium, aggregate output depends (positively) on both the exogenous productivity index \( A_t \) and on the endogenous measure of average productivity \( Z_t \).

**Aggregate profits and wealth** Finally, it is useful to derive an expression for the evolution of aggregate profits. Aggregating across firms we can write:
\[
\Gamma_t = \int_0^1 \Gamma_{i,t} di = \left[ \alpha (q_t A_t)^{\frac{1}{\alpha}} \left( \frac{w_t}{1-\alpha} \right)^{-\frac{1-\alpha}{\alpha}} \right] \int_0^1 z_{i,t-1} k_{i,t-1} di + [1 - \delta - \mathcal{R}_{t-1}] \int_0^1 k_{i,t-1} di + [\mathcal{R}_{t-1} - 1] \int_0^1 \pi_{i,t-1} di
\]

which can be simply rewritten, as a function of aggregate capital, as:

\[
\Gamma_t = (\Pi_t - \mathcal{R}_{t-1} + 1 - \delta) K_{t-1} + (\mathcal{R}_{t-1} - 1) N_{t-1}
\]

where \( \Pi_t \equiv \alpha (q_t A_t)^{\frac{1}{\beta}} \left( \frac{w_t}{1-\alpha} \right)^{-\frac{1-\alpha}{\alpha}} \mathcal{Z}_t \).

It is also useful to notice that aggregate profits can be written, as a function of aggregate wealth, as:

\[
\Gamma_t = \{ (\Pi_t - \mathcal{R}_{t-1} + 1 - \delta) [1 - \Psi (\mathcal{Z}_{t-1})] \lambda + (\mathcal{R}_{t-1} - 1) \} N_{t-1}
\]

### 3.2 Family

The wealth and the aggregate profits of the individual firms are returned to the entrepreneur. The family, as a standalone agent, maximizes the present discounted value of utility, which depends on a composite consumption index of domestic and foreign goods:

\[
C_{t}^e = \left[ \gamma^{\frac{\theta}{\rho + 1}} C_{H,t}^{\frac{\rho + 1}{\theta}} + (1 - \gamma)^{\frac{\theta}{\rho + 1}} C_{F,t}^{\frac{\rho + 1}{\theta}} \right]^\frac{\theta}{\rho - \theta}
\]

where both \( \gamma \) and \( \theta \) have been defined above. Notice that \( \gamma \) is also a measure of home bias in consumption.

The family’s flow of funds constraint reads:

\[
C_{t}^e + N_t = \Gamma_t + N_{t-1}
\]

Combining (32) with (30) yields:

\[
C_{t}^e + N_t = (\Pi_t - \mathcal{R}_{t-1} + 1 - \delta) \{ 1 - \Psi (\mathcal{Z}_{t-1}) \} \lambda + \mathcal{R}_{t-1} N_{t-1}
\]

The problem of the family is the one of choosing allocations for \( \{ C_t, N_t, C_{H,t}, C_{F,t} \} \) in order to solve:
max \{c_t, n_t, c_{H,t}, c_{F,t}\} \frac{\mathbb{E}_t}{\beta_t} \sum_{s=0}^{\infty} \chi_{t+s} \ln C_{t+s}^c \quad \text{subject to} \quad (31), (33).

In the above expression we have that $\chi_{t+s} = \beta_{t+s-1} \chi_{t+s-1}$ $\forall s \geq 0$, and $\beta_{t+s-1} \equiv \frac{1}{1+\psi^\beta(\log C_{t+s-1}^c - \chi^\beta)}$. Notice that we have assumed that the family becomes more impatient when average consumption, $C_t^c$, increases. The resulting equilibrium conditions read:

$$\frac{1}{C_t^c} = \beta_t \mathbb{E}_t \frac{1}{C_{t+1}^c} \left\{ \left[ \frac{\alpha q_{t+1} Y_{t+1}}{K_t} + (1-\delta) \right] \frac{K_t}{N_t} + R_t \left[ 1 - \frac{K_t}{N_t} \right] \right\}$$

$$C_{H,t} = \gamma q_t^{-\theta} C_t^c \quad (34)$$

$$C_{F,t} = (1-\gamma) \epsilon_t^{-\theta} C_t^c \quad (35)$$

$$\quad (36)$$

where we have used the fact that $\Pi_{t+1} = \alpha q_{t+1} \frac{Y_{t+1}}{K_t}$ and $\frac{K_t}{N_t} = \lambda[1 - \Psi(\zeta)]$. Note that, since $q_t = q(\epsilon_t)$, the relative demand for the domestic good, $C_{H,t}/C_{F,t}$, is an increasing function of the real exchange rate $\epsilon_t$: a real depreciation raises the relative demand for the domestic good, with elasticity $\theta > 0$.

Equation (34) is an intertemporal condition equating the family’s marginal utility of consumption to the family’s marginal utility of saving. Equations (35) and (36) describe the optimal allocation of any given composite consumption basket into domestic and imported goods.

### 3.3 Worker

The representative worker derives income only from labor. His problem is the one to maximize the following utility function:

\[ \max \{c_t, n_t, c_{H,t}, c_{F,t}\} \frac{\mathbb{E}_t}{\beta_t} \sum_{s=0}^{\infty} \chi_{t+s} \ln C_{t+s}^c \quad \text{subject to} \quad (31), (33). \]
subject to

$$C_t^w = w_t L_t,$$

where $C_t^w$, $L_t$ and $w_t$ denote, respectively, worker’s consumption, hours worked and the real wage expressed in units of CPI, $\sigma$ is the intertemporal elasticity of substitution, $\phi$ is the inverse of the Frisch elasticity and $\psi^L$ is a labor supply preference parameter. Notice that, for simplicity and without loss of generality, the worker does not have access to financial markets.

The first order condition of the worker’s problem is:

$$\psi^L L_t^{\phi} = w_t$$

### 3.4 Equilibrium

For given processes $\{r_t^*, A_t\}$ a rational expectations equilibrium is a set of endogenous variables $\{\Pi_t, C_t^e, C_t^w, Y_t, N_t, K_t, D_t, \epsilon_t, L_t, q_t, z_t, w_t, R_t\}$ solving the set of equilibrium conditions which, for convenience, are described in detail below.

Let aggregate domestic absorption be given by:

$$C_t \equiv C_t^e + C_t^w + K_t - (1 - \delta) K_{t-1}$$

Market clearing for Home goods then requires:

$$Y_t = \gamma q_t^{-\theta} C_t + X^*(Y_t^*, \epsilon_t)$$

where

$$X_t^* \equiv X^*(Y_t^*, \epsilon_t) = (1 - \gamma) \left( \frac{\epsilon_t}{q(\epsilon_t)} \right)^\theta Y_t^*$$

is foreign demand for the domestic good (or, simply, exports). Notice that $\partial X_t^*/\partial \epsilon_t > 0$, with $\theta > 0$ being the elasticity of exports to the real exchange rate.
The optimality conditions of the family’s problem comprise two equations. The first describes the evolution of net aggregate wealth:

\[ C_t^e + N_t = \left( \Pi_t - (1 + r_t^{*}) \frac{\epsilon_t}{\epsilon_{t-1}} + 1 - \delta \right) \left[ 1 - \Psi(z_{t-1}) \right] \lambda + (1 + r_{t-1}^{*}) \frac{\epsilon_{t-1}}{\epsilon_{t-1}} N_{t-1} \]

The second equation describes intertemporal optimization:

\[ \frac{1}{C_t^e} = \beta_t \mathbb{E}_t \frac{1}{C_{t+1}^e} \left\{ \left[ \alpha \frac{q_{t+1} Y_{t+1}}{K_t} + 1 - \delta - (1 + r_t^{*}) \frac{\epsilon_{t+1}}{\epsilon_t} \right] \frac{K_t}{N_t} + (1 + r_t^{*}) \frac{\epsilon_{t+1}}{\epsilon_t} \right\}, \]

where the expression for aggregate profits \( \Pi_t \) is

\[ \Pi_t = \alpha (q_t A_t)^{\frac{1}{\alpha}} \left( \frac{w_t}{1 - \alpha} \right)^{-\frac{1-\alpha}{\alpha}} \int_{z_{t-1}}^{z_{t+1}} z \psi(z) dz \left[ 1 - \Psi(z_{t-1}) \right] \]

The aggregate condition describing the optimal allocation of net wealth into capital reads:

\[ K_t = \lambda N_t [1 - \Psi(z_t)], \]

whereas the one that describes the optimal allocation of net wealth into debt is:

\[ D_t = \frac{N_t [\lambda (1 - \Psi(z_t)) - 1]}{\epsilon_t} \]

Aggregate labor demand and threshold productivity are respectively given by

\[ L_t = \left[ \frac{w_t}{1 - \alpha} \right]^{\frac{1}{\alpha}} \left( q_t A_t \right)^{\frac{1}{\alpha}} K_{t-1} \frac{\int_{z_{t-1}}^{z_{t+1}} z \psi(z) dz}{1 - \Psi(z_t)} \]

\[ z_t = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \left[ (1 + r_t^{*}) \frac{\epsilon_t}{\epsilon_{t-1}} - 1 + \delta \right] \right\} \]

\[ \bar{z}_t = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \left[ \alpha q_{t+1} A_{t+1}^{\frac{1}{\alpha}} (\frac{\epsilon_{t+1}}{1-\alpha})^{-\frac{1-\alpha}{\alpha}} \right] \right\} \]

In equilibrium, the relationship between aggregate output and average productivity is given by:

\[ Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \frac{\int_{z_{t-1}}^{z_{t+1}} z \psi(z) dz}{1 - \Psi(z_{t-1})} \]

Finally, the worker’s optimality conditions comprise a budget constraint and an optimal labor supply choice, respectively given by:

25
\[ C_t^w = w_t L_t \]
\[ \psi_t L_t^\phi = w_t \]

To complete the description of the equilibrium it is useful to recall that the expression for the price of the domestic good in units of the CPI, \( q_t \), and for the CPI-based real interest rate \( R_t \) are given respectively by (6) and (14).

**Net exports** Let net exports \( NX_t \), expressed in units of domestic goods, be given by
\[ NX_t = X^*(Y_t^*, \epsilon_t) - \frac{\epsilon_t}{q_t} C_{F,t} \]
where \( C_{F,t} \) is absorption of imported (both consumption and investment) goods, given by
\[ C_{F,t} = (1 - \gamma) \epsilon_t^{-\theta} C_t \]

Using (39) we can write
\[
NX_t = \left( Y_t - \gamma q_t^{-\theta} C_t \right) - \left( 1 - \gamma \right) \frac{\epsilon_t^{1-\theta}}{q_t} C_t
\]
\[
= Y_t - \left[ q_t^{-\theta} \left( \gamma + (1 - \gamma) \left( \frac{\epsilon_t}{q_t} \right)^{1-\gamma} \right) \right] C_t
\]
\[
= Y_t - \frac{C_t}{q_t}
\]
where the last step follows from (6).

**4 Calibration**

In this section we describe the calibration of the model. We assume a mean-preserving Pareto distribution for new productivity draws. Let
\[
\Psi(z) = \left\{ \begin{array}{ll}
1 - \left( \frac{z}{z_m} \right)^\gamma & \text{if } z \geq z_m \\
1 & \text{if } z < z_m
\end{array} \right. \quad (40)
\]
and

\[ \psi(z) = \begin{cases} \frac{\eta z_m}{z} & \text{if } z \geq z_m \\ 0 & \text{if } z < z_m \end{cases} \tag{41} \]

be respectively the cumulative and the density function, where \( \eta > 1 \) is the shape parameter.

We normalize the mean of the distribution to 1 by setting the Pareto scale parameter \( z_m = \frac{\eta - 1}{\eta} \), allowing us later to compare distributions with different degrees of heterogeneity. We set the baseline value of the shape parameter \( \eta = 3 \), although we show robustness exercises below.

We employ the following calibration for the structural parameters. The time unit is a quarter. We set the capital share \( \alpha = 0.32 \), the capital depreciation rate \( \delta = 0.025 \) (per quarter), and the inverse Frisch elasticity \( \phi = 1.5 \). The value of the maximum leverage ratio \( \chi \) is set equal to 2/3, which implies \( \lambda = 3 \). As for consumption preferences, we set the share of domestic goods \( \gamma \), which is also an index of home bias in consumption, equal to 0.8, and a baseline value of the trade elasticity of substitution \( \theta = 1 \). It is well known, both in the international trade and in the macroeconomic literature, that there exists considerable uncertainty concerning the value of the trade elasticity of substitution. As suggested by Corsetti et al. (2008) empirical estimates for the value of \( \theta \) based on aggregate time series range between 0.1 and 2. Using a moment estimation strategy, and conditional on a share of distribution costs equal to 50 percent, Corsetti et al. (2008) estimate a value of the trade elasticity of substitution equal to 0.425, which is close to the low end of the spectrum.\(^8\) A low value of the trade elasticity of substitution is critical to generate a sufficiently high volatility in the real exchange rate. In our context this is important to control the strength of the balance sheet effect of exchange rate fluctuations, acting via the borrowing constraint (10). It will however be crucial to experiment with alternative values for this parameter.

Finally, we assume that the (world) gross real interest rate follows an exogenous AR(1) stochastic process:

\[ \log(1 + r^*_t) = \rho^* \log(1 + r^*_{t-1}) + \varepsilon^*_t. \tag{42} \]

We fit the above AR(1) process (augmented by a constant) with quarterly US data from 1993:1 to 2007:4. The time series for the US real interest rate is constructed as in our previous

\(^8\)If we let \( s_d \) be the share of distribution costs, the price elasticity of tradable goods is equal to \( \theta(1 - s_d) \). Corsetti et al. (2008) estimate a value of \( \theta = 0.85 \), and calibrate the share of distribution costs equal to 1/2, based on the evidence in Burstein et al. (2003). The resulting value for the price elasticity of tradables is therefore 0.85/2 = 0.425.
empirical section. \footnote{Estimates are similar if we include the Great Recession period.} Our estimates (with standard errors in parenthesis) yield $\hat{\rho}^* = 0.96_{(27.09)}$ and $\hat{\sigma}_x^* = 0.44$.

## 5 Financial frictions and (mis)allocation

We start by studying the following experiment: how does the presence of financial frictions and firms’ heterogeneity affect the transmission of real interest rate shocks? The natural benchmark to answer this question is a standard small open economy real business cycle (RBC) model as, e.g., in Mendoza (1991).

Figure 7 displays impulse responses of selected variables to a one standard deviation (44 bps) exogenous increase in the real interest rate $r^*_t$. Broadly speaking this corresponds to a capital outflow shock. We focus on two alternative economies. The first (labeled RBC Model) is a standard RBC economy with perfect financial markets and a representative firm.\footnote{As a baseline we use a standard Mendoza (1991) small open economy real business cycle model simply modified to account for the separation between workers and entrepreneurs, as in our model with financial frictions.} The second (labeled financial frictions) is our model economy with heterogenous firms and borrowing constraints. To illustrate our argument, we assume that the latter is a one-good only economy. This allows us to abstract from any valuation effect on borrowing stemming from real exchange rate movements.

In both economies, a rise in the real interest rate causes a contraction in output, consumption and investment. What is noteworthy, however, is that the response of output in the economy with financial frictions is significantly dampened relative to the one of the baseline RBC economy. In other words, the introduction of financial frictions causes an attenuation effect of real interest rate shocks. The reason for the attenuation effect is simple, and lies in the behavior of aggregate TFP. Notice that in the baseline RBC economy TFP is exogenous, and constant. In the economy with financial frictions, TFP is endogenous and is driven by the reallocation of capital across firms with heterogenous productivity. However, in response to a rise in the real interest rate, capital reallocation drives productivity up, thereby dampening the contraction of output.

The intuition for why TFP rises in response to a rise in the real interest rate works as follows. After idiosyncratic productivity is drawn, and given the assumption of constant returns to scale in production, the decision of firms whether or not to produce depends
Figure 7: Theoretical impulse responses to a one standard deviation rise in the real interest rate: baseline RBC model (solid) vs one-good model with firms’ heterogeneity and financial frictions (dashed). All variables expressed in percent deviations from steady state.
linearly on capital. Therefore, whenever its productivity draw ensures that the return on capital is above its marginal cost, an individual firm $i$ will decide to employ capital up to the maximum allowed by the borrowing constraint. The latter is given by the outside option of lending capital to "more lucky" firms, i.e., those firms whose productivity draw is above the cutoff level $z_t$. That cutoff, as shown in equation (19), is also a function of the real interest rate. For a marginally (un)productive firm, a rise in the real interest rate increases the return from "remaining idle", i.e., not producing, and simply renting capital to the more productive firms. Put differently, a higher real interest rate makes the opportunity cost of entry higher. The exit of the marginally (un)productive firm induces a (mis)allocation effect: as a result, average productivity rises.

In short, the rise in the real interest rate induces, via a "cleansing-type effect", an upward movement in average productivity, which dampens the contractionary effect on output induced by the fall in consumption and investment. The conclusion is that the model is inconsistent with the following twofold evidence for EMEs: (i) real interest rate innovations explain a significant portion of aggregate fluctuations; and (ii) the conditional correlation between aggregate productivity and real interest rates is negative.

The above result is surprising on two different grounds. First, it suggests that a model augmented with firms' heterogeneity and financial frictions is better able to account, at least qualitatively, for the effects of real interest rate shocks on productivity in AEs rather than EMEs. However, the presence of financial frictions is typically supposed to be a feature that, more genuinely, characterizes the structure of an emerging market economy as opposed to an advanced economy. Second, it generally contradicts the widely held belief, in the business cycle literature, that the presence of financial frictions amplifies aggregate fluctuations, consistent with the overwhelming evidence that the volatility of output is significantly higher in EMEs relative to AEs.

**The role of heterogeneity** The counteracting force stemming from the endogenous movement in productivity is quantitatively relevant only if firms entering are enough to significantly affect average productivity. This implies that what matters for the elasticity of aggregate output to a real interest rate shock is the degree of heterogeneity across firms. If firms' heterogeneity is large, a rise in the real interest rate induces a sufficiently large fraction of firms to exit the market, and therefore a possibly large reallocation effect.

The degree of heterogeneity, i.e., the dispersion of firms' productivity, is determined by
the shape parameter $\eta$ of the Pareto distribution summarized by (40) and (41). Figure 8 displays the effect of varying the shape parameter $\eta$ on the response of output to an exogenous increase in the real interest rate. The lower is $\eta$, i.e., the larger the heterogeneity across firms, the less pronounced the response of output. Conversely, by reducing heterogeneity to a single concentrated firm ($\eta \to \infty$), one can reproduce the same effect on output that would prevail in the baseline RBC model with a representative firm.

6 Original sin

Our model so far (featuring heterogenous firms and financial frictions) seems better able to account for the role of real interest rate shocks in AEs rather than EMEs. However, another feature that characterizes many EMEs is the widespread inability to borrow in domestic currency. As traditionally done in the literature, we label this as the "original sin" effect.

A necessary condition for this effect to be at work is that the economy features both domestic and imported goods, thereby causing relative price (i.e., real exchange rate) movement.
ments. In turn, since borrowing is expressed in units of foreign goods, relative price movements affect the ability to borrow of productive, yet constrained, firms. In particular, a depreciation (appreciation) of the real exchange rate in response to a rise (fall) in the real interest rate can, ceteris paribus, tighten (relax) the financial constraint for those firms. In this vein, the original sin effect - which affects already productive yet constrained firms - interacts with the misallocation effect in driving the response of average productivity to real interest rate shocks.

Figure 9 displays the effects of selected variables to a 50bps rise in the real interest rate for alternative values of $\epsilon$, the elasticity of substitution between domestic and foreign goods. This parameter typically controls the strength of the expenditure switching effect, and therefore the elasticity of the relative price of domestic goods to real interest rate innovations. The results are reported for two cases corresponding to alternative values of the trade elasticity of substitution: $\theta = 0.3$ and $\theta = 1$.

As already hinted above, there is a vast literature in international (macro)economics investigating the empirically plausible value of the trade elasticity of substitution. Estimates based on higher frequency (quarterly or monthly) data in quantitative DSGE models typically report values below unity. A stream of the international trade literature, however, looks at the effects of variations in the relative price of exported goods over longer time periods, and estimates values of the trade elasticity between 1 and 2. Given that our model is calibrated on quarterly data a value of $\theta$ below 1 seems the natural benchmark. Notice also that, once we account for the fact that our model does not feature distribution costs, the "low" elasticity case of $\theta = 0.3$ is in line with the empirical estimates reported in Corsetti et al. (2008). A relatively low value of the elasticity of substitution could also be justified on the grounds that our model does not feature a distinction between a traded and a non-traded good sector. In addition, it would seem more natural that a low elasticity of substitution between domestically produced and imported goods be a feature of an emerging-market, rather than advanced, small open economy.

With all these considerations in mind, notice, first, that a rise in the real interest rate generates a depreciation of the real exchange, and to a larger extent the lower is the elasticity $\epsilon$, i.e., the lower the degree of substitutability between domestic and foreign goods. The key result is that for a sufficiently low value of the elasticity of substitution the model is able

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to generate a positive conditional comovement between output and productivity, exactly in line with the empirical evidence for EMEs.

As suggested above, the key element behind the positive conditional comovement between output and average TFP is the presence of an "original sin" effect. This effect is induced (in this case) by a depreciation of the real exchange rate, which lowers the value of collateral for the incumbent firms, thereby tightening their borrowing constraint. At the margin, a tightening of the credit constraint induces the more productive firms (those for which the return on capital is higher than the return on savings) to reduce their borrowing from the less productive firms, for which lending becomes less convenient than producing. The entry of less productive firms reduces the productivity of the marginal incumbent firm thereby causing a fall in the average productivity of the active firms in the economy. The resulting fall in average productivity (for a sufficiently low value of $\epsilon$) exacerbates the contractionary effect of the increase in the real interest rate, as shown by the larger contraction in output. This result suggests that an original sin effect (working through firms’ balance sheet), combined with the presence of firms’ heterogeneity and financial frictions, can help to account for the relatively larger importance of real interest rate shocks in explaining EMEs’ business cycles.

**Robustness** Figure 10 displays the effect of varying the trade elasticity $\theta$ and the degree of home bias $\gamma$ on the impact response of a few selected variables to a rise in the real interest rate. A negative response of average productivity requires both a sufficiently low elasticity of substitution and a sufficiently high degree of home bias. The reason is that for relatively lower values of $\theta$ and higher values of $\gamma$ the impact response of the real exchange rate becomes larger (a larger depreciation in this case), thereby amplifying the negative balance sheet effect on incumbent firms. Interestingly, the higher the degree of home bias $\gamma$, the larger the range of values of the trade elasticity (extending also above 1) for which the response of average productivity to a rise in the real interest rate remains negative. This suggests that additional "trade frictions" such as non-tradability and/or deviations from the law of one price (due e.g., to distribution costs), which would contribute to lowering the price elasticity of tradables, would in turn magnify the equilibrium response of the real exchange rate and, potentially, the negative response of average productivity to a capital outflow shock. All these features would help bringing the model further in line with our established empirical evidence.
Figure 9: Theoretical impulse responses to a one standard deviation rise in the real interest rate. Model with two goods and original sin effect.
Figure 10: Impact effect of a rise in the real interest rate as a function of the trade elasticity $\theta$ and of the degree of home bias $\gamma$. 
7 Empirical fit

Despite its simplicity, we show in this section that the model is able to fit well some relevant features of the data. We estimate key structural parameters of the model for EMEs as well as of the model for AEs. For EMEs, we estimate the more general version of our two-good model featuring both the misallocation channel (i.e., firms’ heterogeneity coupled with financial frictions) and the original sin channel (i.e., foreign currency borrowing, whereby fluctuations in the real exchange rate affect the ability to borrow). For the AEs, we estimate the model featuring the misallocation channel only (i.e., a two-good economy where borrowing is only in domestic currency).

Some structural parameters are calibrated and some others are estimated using a minimum distance estimator. Let \( \zeta \) be the vector of parameters to be estimated. We estimate \( \zeta \) by minimizing the distance between the empirical impulse responses obtained in Section 2 and the model-implied theoretical impulse responses. Denote by \( \hat{\Psi} \) the vector in which the estimated impulse responses to be matched are stacked in column and denote by \( \Psi(\zeta) \) the corresponding stacked DSGE-based impulse responses, evaluated at \( \zeta \). Our estimator for \( \zeta \) is:

\[
\hat{\zeta} = \arg \min_{\zeta} (\hat{\Psi} - \Psi(\zeta))'V^{-1}(\hat{\Psi} - \Psi(\zeta))
\]

The weighting matrix \( V \) is a diagonal matrix with the variances of the marginal distributions of \( \hat{\Psi} \) on the main diagonal. Actually, we are considering \( \hat{\Psi} \) as the "data" and estimate \( \hat{\zeta} \) as those parameters that make the structural impulse responses \( \Psi(\zeta) \) to lie as close as possible to \( \hat{\Psi} \).

The comovement between the real interest rate and TFP is the key moment that differentiates the conditional dynamics in the EMEs as opposed to the AEs (it is negative in our sample of EMEs and it is positive in our sample of AEs). In light of this, in our estimation, we match two impulse responses to a real interest rate shock: the response of TFP and the response of the real interest rate. As both in the DSGE model and in the VAR TFP does not respond on impact to a shock to the real interest rate, we match the impulse response of TFP at horizons 2 to 4. For the response of the interest rate, we normalize the size of the shock to one and match the impulse responses at horizons 2 to 4. As a result, for each model, the vector \( \hat{\Psi} - \Psi(\zeta) \) is a \( 1 \times (3 \cdot 2) \) vector.

Relative to the setup presented in the above sections, we specify a more general model for the real interest rate process. We assume that the world real interest rate \( r^* \) follows an
AR(2) process of the form:

$$\log(1 + r_t) = \rho_1^* \log(1 + r_{t-1}^*) + \rho_2^* \log(1 + r_{t-2}^*) + \epsilon_t^*$$

The vector $\zeta$ of structural parameters to be estimated is:

$$\zeta = [\theta, \eta, \rho_1^*, \rho_2^*]$$

where $\theta$ is the trade elasticity and $\eta$ is the Pareto distribution parameter. As illustrated in figures 8 and 10, the values of these two parameters are critical in shaping the effects of real interest rate shocks on productivity.

Figures 11 and 12 show the empirical impulse responses, respectively for the EMEs and AEs. The solid line is the impulse response of the VAR, surrounded by the credible bands (dashed, thin lines). Dashed thick lines are the impulse responses from the DSGE under $\hat{\zeta}$. The model for the EMEs (top) matches the data extremely well and the model for AE (bottom) is able to match the sign and the size of the impulse response functions. It is interesting to note that the models are able to match the negative (for the EME) and positive (for AE) response of TFP to a positive real interest rate shock.

The estimated values of the critical parameters are reported in Table 1 below\textsuperscript{14}.

\textsuperscript{14}To compute standard errors we follow the procedure outlined in Altig et al. (2011).
Table 1. Estimated parameter values

<table>
<thead>
<tr>
<th></th>
<th>Trade elasticity $\theta$</th>
<th>Pareto distribution $\eta$</th>
<th>$\rho_1^2$</th>
<th>$\rho_2^2$</th>
</tr>
</thead>
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<tr>
<td>EMEs</td>
<td>0.353</td>
<td>1.046</td>
<td>0.821</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>(0.0259)</td>
<td>(0.0417)</td>
<td>(0.1274)</td>
<td>(0.1360)</td>
</tr>
<tr>
<td>AEs</td>
<td>0.430</td>
<td>6.021</td>
<td>1.078</td>
<td>-0.130</td>
</tr>
<tr>
<td></td>
<td>(0.0497)</td>
<td>(0.0297)</td>
<td>(0.0086)</td>
<td>(0.0496)</td>
</tr>
</tbody>
</table>

There are two main findings. First, the estimated value of the trade elasticity of substitution $\theta$ is low, and clearly below 1, for both sets of economies. Second, the value of parameter $\eta$ (which shapes the Pareto distribution for new productivity draws) changes considerably between different sets of countries (and therefore models). Recall that the shape parameter $\eta$ controls the degree of heterogeneity, i.e., the dispersion of firms’ productivity. The lower $\eta$, the larger the heterogeneity across firms. Our estimates indicate that firms’ (productivity) dispersion is therefore larger for EMEs relative to AEs. Interestingly, this result is line with existing cross-country empirical evidence on market concentration. Koren and Tenreyro (2007) show that the degree of sectoral concentration declines with development at early stages and increases at later stages; Imbs and Wacziarg (2003) finds that sectoral concentration follows a U shape pattern as a function of the degree of development, pointing to a degree of firms’ (or sectors’) productivity dispersion being larger in EMEs.
relative to AEs.

8 Conclusions

In emerging market economies (EMEs), capital inflows are associated to productivity booms, while the opposite is true for advanced small open economies (AEs), like the ones of the Euro periphery. VAR-based evidence shows that, conditional on suitably identified real interest rate innovations, aggregate TFP and output fall in EMEs, whereas they rise in AEs. We have built a general equilibrium small open economy model simultaneously able to account for both facts. The key element of our model is twofold: misallocation of capital across heterogeneous firms, due to financial frictions, and the widespread "original sin" phenomenon whereby EMEs cannot borrow in domestic currency. The relative balance of these two effects can rationalize the evidence in both groups of countries.
References


