Abstract

We apply the weak axiom of revealed preferences (WARP) in the context of a two-period small open-economy model. According to WARP, certain changes in net exports and net foreign asset positions should be ruled out. For example, a country which has initially run a trade deficit, should remain doing so following a drop in the exogenous interest rate. Similarly, a country which has run a surplus should maintain it if the interest rate goes up. The argument also applies to shifts in the sign of net foreign asset position. It holds for both an endowment economy and a production economy. Using these restrictions, we test the model on an aggregate panel data for 22 developed economies. The results suggest that WARP violations tend to concentrate around the two oil crises, suggesting an important role of terms of trade and commodity shocks in small open economy models.

Keywords: trade balance, net foreign asset positions, revealed preferences, small open economy

JEL Classification: F32; D01

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1 Introduction

Since the early work of Sachs (1981) and Svensson and Razin (1983), it has become standard to view the current account and the international investment positions as outcomes of an inter-temporal choice between consumption and savings and the small open economy model has become the workhorse of open economy macroeconomics. Ever since, the basic model was extended in various directions to analyze phenomena such as sovereign default, exchange rate movements, financial crises, and global imbalances. The sign, direction and magnitude of international capital flows and stocks is at the core of each of these problems. For example, exchange rate and balance of payments crises, which have plagued many, mainly emerging, economies, can be described as sudden stops events (see, e.g., Dornbusch et al. (1995) and Calvo (1998)) during which foreign financing collapses and the economy witnesses an abrupt ”reversal” of its current account deficit (see, e.g., Edwards (2004)) into either a surplus or at least a drastically lower deficit. On the other hand, the persistence in the flows and global imbalances, mainly due to trade deficits in the US and surpluses in China, is curious not only because it epitomizes the Lucas paradox (see Lucas, 1990) but also because over time these imbalances accumulate. In the US case the persistent trade deficits have eroded the country’s net foreign wealth and turned it from a net foreign creditor to a net debtor. Such switch in sign has important consequences because the transversality condition imposes different restrictions on the expected future path of net exports for net debtors relative to net creditors.

This paper offers a novel perspective on the dynamics and the signs of the two international accounts variables: net exports and net foreign asset position. In particular, we apply the weak axiom of revealed preferences (henceforth WARP) in the context of a simple two-period endowment model of a small open economy. WARP implies that, following an exogenous change in the world interest rate, certain choices of intertemporal consumption bundles, i.e. combinations of current and future consumption, would violate the WARP. For instance, a country which initially ran a trade surplus, should remain in surplus when the interest rate it faces goes up. In a similar vein, a country running a deficit should not switch to surplus if, ceteribus paribus, the cost of borrowing falls. In other words, a country that initially revealed its preference for being a net borrower, should, assuming stable preferences, continue to prefer to borrow under even more generous conditions.

The argument survives a few modifications. Most importantly, one can apply it in the context of net foreign asset positions. For example, a country which was initially a net foreign debtor, should remain a debtor if the interest rate drops. Furthermore, one can also inspect the sign of the change in net exports (as opposed to the level). This requires a Slutsky decomposition to isolate the substitution effect. Finally, the argument is generalizable to a two-period production economy. In this case, however, due to the convexity of the production possibility frontier, not only does the WARP preclude the switches described above, but also requires some switches to occur, e.g. a mild surplus country has to switch to deficit following an interest rate drop.
If one takes the stability of preferences as an axiom, then the natural interpretation of any possible violations of the WARP is interpreted as a failure of the model or its parametrization. Hence, our exercise can be treated as a test of the simple small open economy model. The advantage of our approach, relative to standard statistical tests, is that since it highlights the particular data points at which WARP is violated, it may also help identifying the economic mechanisms that the benchmark model should be amended with to make it rationalizable by the data. For example, we find that WARP violations in net exports switches tend to be clustered around the times of the first and second oil crises. Several countries at the time, net energy importers, reversed into trade deficits following the sharp increase in global oil prices. Many of these switches were followed be opposite switches back to surplus as the shock got absorbed. The violation of WARP comes from the fact that these were also years in which the world interest rate was on the rise. Accordingly, despite not being hard evidence, the results suggest that including a terms of trade mechanism and shock may improve the fit of the model.

The paper is mostly related to the literature testing small open economy models, for example Shreffin and Woo (JIE, 1990), Glick and Rogoff (JME, 1995), Gruber (JME, 2004), Nason and Rogers (JIE, 2006). It also speaks to the vast literature on revealed preferences. Since the initial idea of Samuelson (1938), the revealed preferences theory grew substantially and has been applied in various contexts. For broader up-to-date reviews of this work the reader is referred to Crawford and De Rock (2014) and Chambers and Echenique (2016). Here instead we only discuss the strands of the literature which intersect with this paper to at least some degree.

A vast majority of the work using revealed preferences axioms has been applied to problems of *intra-temporal* choice in which the agents choose a basket of multiple goods within a single period. More recently, however, there have been applications of the revealed preference theory to *inter-temporal* problems in the sense that the axioms are applied to choices between goods in different points of time.\(^1\) The early work of Browning (1989) contains both approaches. Firstly, GARP tests are applied explicitly to check the stability of intra-temporal utility functions. They are applied to aggregate consumption time series, divided into a few classes of goods. In this step the time dimension kicks in only to the extent that it is a transmitter of new data points and of variability in observed intra-temporal choices. Secondly, the paper provides cyclical monotonicity conditions, which constitute a simple test for rational expectations hypothesis/ life-cycle model, which describes inter-temporal consumption choices. Cyclical monotonicity conditions are similar, but stronger than those for GARP. Importantly, in this approach choices are associated with consumption points in time rather than with vectors of consumption over time. Browning’s methodology is also used

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\(^1\)Implementability of this approach relies on the assumption of weakly separable time aggregator. A shown by Kubler (2004), for a general class Kreps-Porteus of non-time-separable utility functions the revealed preferences axiom does not impose any observable restrictions on the choices of households. Kubler (2003) made this point more general by introducing agents’ heterogeneity and making a distinction between what is observable on aggregate vs. individual level.
in Deaton (1987). Recently, Echenique et al. (2017) generalize this framework to exponential discounting. Observable implications of the discounted utility model is also studied by Dziewulski (2015), although using a different setting. Using Afriat (1967) theorem, Crawford (2010) derives testable conditions for a model with habits and applied it to a panel of micro data (see also Crawford and Polisson, 2014). Demuynck and Verriest (2013) use a similar approach to test a model with rational addiction. Finally, in a recent work, Adams et al. (2014) use a revealed preference argument to show that instabilities in preferences on a household level can be attributed to mild heterogeneity between its members.

Revealed preference analysis has also been used in macroeconomics. A common application has been to study aggregate consumption data divided into a few classes of goods (e.g. nondurables, durables and services or more detailed categories like food, clothing, transportation, entertainment, etc.). Early papers include Landsburg (1981) and Varian (1982). The general picture stemming from this literature is that preference violations are very rare. This has been largely attributed to little variation in relative prices combined with steady growth in absolute income, although not without objections (see Bronars, 1987). Following the GARP-based test of Varian (1983) the focus has also been on the closely related question of separability. For example, the evidence for separability of consumption categories appears problematic, as reported by Fleissig et al. (2000). The issue has become relevant in the context of modeling the demand for money. Early studies of Swofford and Whitney, 1986, 1987, 1988 as well as Patterson (1991) analyze separability of money in the utility function relative to goods and leisure whereas Belongia and Chalfant (1989), Belongia and Chrystal (1991) and Fisher and Fleissig (1997), among others, focus on the aggregation of money categories.\footnote{The issue of separability of money has grown into a separate strand of literature, a review of which is far beyond the scope of this paper. For an extensive overview of it the reader is instead referred to Binner et al. (2009) and references therein.}

Finally, Dowrick and Quiggin (1994, 1997) use revealed preference tests in the study of international living standards based on international (PPP-based) prices. Based on GARP tests, they argue that that the official ICP rankings are problematic and propose a new utility-consistent Ideal Afriat Index.

The rest of the paper is organized as follows. Section 2 discusses the simple model of intertemporal consumption choice for a small open economy through the lens of the revealed preference theory. Section 3 presents the data and discusses some basic empirical regularities related to switches in trade balance and net foreign asset positions. Section 4 lays out the testing strategy used in the paper as discusses the related technical issues. Section 6 presents the main results as well as some robustness checks. Section 7 concludes.

## 2 WARP and the open economy model

In this section we present an application of the revealed preferences theory in the context of a small open economy. We use a basic, two-period textbook model (see e.g. Obstfeld and Rogoff, 1996) in which net exports (and current account) reflects the country’s preferences over its intertemporal consumption structure. When
looked through the lens of the weak axiom of revealed preferences (WARP) the model has, under certain assumptions, testable implications. These are restrictions on the admissible switches in the signs of net exports and net foreign asset position. We start by presenting the basic argument. In Subsection 2 we discuss the conditions and assumptions on preferences under which the argument holds and when it breaks. This discussion will influence the design of our empirical test in Section 3. In Subsection 3 we also show that similar testable predictions arise in a production economy. Here, however, the picture is more complicated and the testable conditions are only necessary, but not sufficient for the model to be rationalizable by WARP.

2.1 Endowment economy

Consider an endowment economy inhabited by a representative agent who lives over two periods, where period 1 can be interpreted as the present and period 2 as the future. In both periods the agent receives an endowment, $Y_1$ and $Y_2$, respectively and, as of period 1, there is no uncertainty about period 2. The agent chooses a consumption bundle $(C_1, C_2)$ given a budget constraint

$$C_1 + \frac{C_2}{R_2} = Y_1 + \frac{Y_2}{R_2} \quad (2.1)$$

where $R_2$ denotes the present gross interest rate (i.e. from the present until the future period). The economy is assumed to be small in the sense that it takes the world interest rate $R^*$ as given. It is also open in the sense that access to international financial markets is free. Those assumptions imply that the agent can freely substitute between consumption over time at $R_2 = R^*$.

Suppose the agent chooses a bundle denoted as $B$ in Figure 1. Given $R_2$ the country chooses to run a trade deficit today, i.e. $C_1 > Y_2$. It has then to pay back in period 2 the debt acquired in 1 and to run a trade (and current account) surplus in the future, so that $C_2 < Y_2$. This necessitates a switch of the sign of the net exports (and current account) positions between periods 1 and 2.

![Figure 1: Intertemporal consumption choice in an endowment economy.](image-url)
Now, suppose the country faced a lower interest rate, \( R'_2 < R_2 \). Some new consumption baskets to the right of \( Y_1 \) would become affordable, whereas some to the left of \( Y_1 \) would become unaffordable. Note that the country would have to choose a consumption basket that lies to the right of \( Y_1 \), i.e. it would still run a trade deficit in the present. The new optimal point \( B' \) may be both to the right and to the left of point \( B \), so the deficit may both increase or decrease. However, the country may not switch into a trade surplus (indicated by \( B'' \)) because this would violate the WARP. All bundles to the left of \( Y_1 \) were considered inferior at \( R_2 \), and hence they were not chosen. So they cannot be chosen at \( R'_2 \) either. If there are some bundles to the left of \( Y_1 \) which would be preferred to point \( B \), they must be beyond the country’s budget constraint both under \( R_2 \) and \( R'_2 \).

To put things formally, suppose the bundle \( B = (C_1, C_2) \), indicating trade deficit in period 1, would have been chosen at prices \( p = (p_1, p_2) = \left(1, \frac{1}{R_2}\right) \) and \( B' = (C'_1, C'_2) \) (trade surplus in the present) would have been chosen at prices \( p' = (p'_1, p'_2) = \left(1, \frac{1}{R'_2}\right) \), where all prices are expressed in terms of present consumption units. Then, WARP is satisfied if

\[
pB' \leq pB \Rightarrow p'B > p'B'
\]

In our context, the implications are the following. Suppose a trade deficit in period 1 was preferred under higher relative price of present consumption \( p \) so that \( pB' \leq pB \). Suppose further, that a deficit was preferable also under \( p' \) so that \( p'B > p'B' \). Then, if under \( p' \) a surplus \( B'' \) was chosen despite the fact that a deficit was preferred, it must have been because a deficit was not affordable under \( p' \). Here, however, if trade deficit \( B \) (and hence net borrowing) is affordable in period 1 under higher interest rates, then it is even more so affordable under lower interest rates because present consumption is cheaper in the latter case. This implies that a surplus \( B'' \) couldn’t have been chosen under lower interest rates, if deficit was chosen already under higher interest rates. If it were, WARP would have been violated. A country which has initially chosen to run a deficit \( B \) has to remain doing so by choosing some \( B' \) if interest rates are lower.

Before moving to the limitations of this argument, two remarks are in order. First, and rather obviously, the same logic applies to countries running an initial trade (and current account) surplus. These countries, in turn, cannot switch to a deficit following an interest rate increase. If they revealed preferred to lend under some interest rate, then they also have to lend under an even higher rates. Secondly, the revealed preferences argument doesn’t provide any guidance on when will the country shift the trade balance sign. It only predicts when the country will not switch. Without additional assumptions on the country’s preferences, the model cannot say more about the direction of the reaction to an interest rate change. A borrowing country which faces a decrease in the interest rate may both decrease or increase its trade (and current account) deficit. Yet it has to remain a borrower.
2.2 Robustness and extensions

In this subsection we explore the role of some of the assumptions that we imposed in the model in Subsection 2.1. We start with the assumption that the present and futureendowments are constant. It is easy to show that any change in $Y_1$ or $Y_2$ invalidates the restrictions that WARP puts on the sign of the trade balance. To illustrate the problem, consider Figure 2. Initial endowment is given by point $A$. The corresponding initial interest rate is $R_2$. Suppose that the country chooses to run a deficit in the first period and surplus in the future, denoted again by point $B$. If the interest rate falls (total endowment is still constant), the country can not choose any point to the left of $A$, i.e. it can not switch to a surplus. Now, consider a simultaneous increase of endowment $Y_1$ to $Y'_1$ so that the total endowment is given by $A'$ and the budget constraint shifts outwards. Note that now the country may choose any point to the right of point $E$, for example point $B'$. Such choice would result in a switch from a deficit to surplus in period 1 (the point lies to the left of $Y'_1$) but it would be still consistent with WARP.

The fact that endowment changes invalidate the predictions of the WARP regarding switches in the trade balance sign makes the standard empirical tests for WARP impractical for our purpose. In traditional applications of revealed preference tests in the literature, it usually doesn’t matter if shifts of the budget constraint are due to movements in relative prices, absolute prices or in income. It is, however, crucial in the intertemporal context. In Section 4 we will discuss this problem further and present a strategy to isolate out these undesired changes.

The second tacit but important assumption made in the previous subsection was that the net foreign wealth inherited from the past is $B_0 = 0$. Again, it is highly unlikely to be satisfied in the data.\footnote{See Lane and Milesi-Ferretti (2007) for a detailed empirical analysis of net and gross foreign asset positions.} As with endowment changes, one has to take this fact into account in the empirical exercise when switches in the sign...
of trade balance are analyzed. But what happens if the $B_0 = 0$ assumption is relaxed? It turns out that the WARP reasoning still holds. However, it no longer restricts switches in the balance of trade, but switches in the sign of net foreign asset position. To see this, consider again the endowment economy. If $B_0 \neq 0$, the present disposable amount of own wealth is $B_0 R_1 + Y_1$. The additional future income is still $Y_2$. Suppose now that the small open economy initially chooses $C_1$ such that $R_1 B_0 + Y_1 > C_1$. Then, after an increase of $R_2$ it cannot switch to any $C_1'$ such that $R_1 B_0 + Y_1 < C_1'$. In other words, the term $R_1 B_0 + Y_1 - C_1'$ has to remain positive. However, note that

$$R_1 B_0 + Y_1 - C_1 > 0$$
$$B_0 + Y_1 - C_1 + r_1 B_0 > 0$$
$$B_0 + CA_1 > 0$$
$$B_1 > 0$$

Similarly, $R_1 B_0 + Y_1 - C_1' > 0$ translates to $B_1' > 0$. This means, that a country which initially chooses $B_1 > 0$, i.e. being a net foreign creditor at the end of the present period, has to remain a net creditor $B_1' > 0$ when the exogenous interest rate $R_2$ is higher. By analogy, a net foreign debtor has to remain a debtor if interest rates are lower. Whether violations of this WARP constraint occur in the data can be tested empirically using a similar methodology as the one applied to trade balance and will be presented in Subsection 4.2.

### 2.3 Production economy

The revealed preferences argument still holds if we extend the simple model into a production economy with investment. Consider a country with a production function $Y = F(K)$ satisfying $F'(K) > 0$ and $F''(K) < 0$. Then, assuming zero depreciation without loss of generality, the production possibility frontier is given by

$$C_2 = F [F(K_1) - C_1 + K_1] + F(K_1) - C_1 + K_1$$

where, as as of period 1, $K_1$ is predetermined and $K_2$ is chosen through $I_1$. Assuming again that the country is small and has free access to international financial markets, the levels of production and investment are determined solely by the condition $F'(K_2) = r_2$, as denoted by point $A$ in Figure 3.

Now consider a fall of the world interest rate to $r_2' < r_2$. The optimal investment condition requires a change in $I_1$ and therefore a shift in the intertemporal output schedule from point $A$ to point $A'$. Note that a country which was initially a net borrower (choosing a consumption bundle $C$, or any other to the right of $A$), cannot switch into a lender position, because this would require picking a consumption bundle on the $-(1 + r_2)$ curve and to the left of point $A'$. This would again violate the WARP. The country can only pick a bundle on $-(1 + r_2)$, but to the right of point $D$. Yet all those points denote a net borrower (current account deficit) position. This requirement stems from the concavity of the production function (and of the PPF.
line) and the linearity of the $-(1 + r_2)$ budget constraint line. Again, the country may increase or decrease its lending by running a smaller or larger current account deficit.

In fact, in a production economy some additional interesting switching implications arise relative to the endowment economy. Consider a country running initially a CA surplus and choosing a bundle on the line between points $A$ and $D$ (i.e. a relatively mild surplus). Then, after a drop of interest rates, the country is not allowed to pick any bundle to the left of point $D$. So it can neither remain in surplus (point to the left of $A'$) nor run a deficit of relatively limited size (the $A'$ to $D$ segment). Hence it actually has to switch its position from a (relatively moderate) surplus into a (sizable) deficit. However, such scenario does not translate into a general behavioral rule and is reported solely for completeness. It will therefore not be tested in the empirical part.

3 The data and some empirical regularities

We start the empirical part with an initial look at international data. Our dataset is a panel of national accounts series for 22 developed OECD countries. The data is of annual frequency and spans the years 1970-2016. We focus on developed countries primarily because of the limitations regarding the data on net foreign asset positions. This data is often shorter than that for national accounts and also patchy (see Lane and Milesi-Ferretti (2007) for details). The stringent restrictions of our model, such as free flow of capital and lack of risk may also be less unrealistic for our selection of countries. In principle, however, they may well contribute to any possible violations of the WARP and cause rejection of the model. ns as well as country

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4The countries studied are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK and the USA. Details on data sources are provided in Appendix ??.
risk premia. Both elements are assumed away in the model. Finally, we are not able to test countries that have not switched (unless production economy model is considered). Nevertheless, in principle the data can be extended to include a broader set of countries.

Figure 4 plots the number of switches in net exports in our panel, whereas Figure 5 shows similar switches in the net foreign asset positions. The first observation stemming from these figures is that the number of trade balance switches was relatively high in the first half of the sample, i.e. in the 1970s and 1980s. Switches "in minus", i.e. from surplus to deficit are particularly numerous in two years, 1974 and 1980. These are the years that mark the first and the second oil crisis, respectively. Many of these switches were then reversed by a return to surplus in the following years, as the economies adjusted to the oil price shocks. Some switches tend to be, nevertheless more long-lasting, especially those "in plus". In a few occasions, they were associated with sudden stop-like events (see, e.g., Dornbusch et al. (1995) and Calvo (1998)) and reversals of current account deficits (see, e.g., Edwards (2004)). This applies, for example, to Finland in 1992 or Iceland in 2009 and, more gradually, to the southern European countries following the sovereign debt crisis. A similarly dramatic switch was in Norway in 1978 following the start of North Sea oil extraction.

In a few cases, notably Italy and Sweden, the trade deficits around the first oil crisis turned the countries
into prolonged net debtor positions. In general, however, switches in the net foreign asset positions are much rarer events, if only due to the fact that net external position, being a stock variable, moves more slowly than net exports (a flow). They concentrate not only during the oil crises, but occurred quite frequently throughout the 1990s and just before the Global Financial Crisis, but without a clear dominance of either direction.

4 Testing method

4.1 Trade balance

In order to be able to empirically analyze the data and check for possible violations of the WARP, we need to put some more structure on the model, on top of that discussed in Subsection 2.1. These assumptions make the empirical exercise implementable and the model testable. They also make the model more stylized and therefore more likely to be rejected because ultimately any WARP violations found in the data are interpreted as an evidence for a failure of the model and its assumptions, rather than evidence for unstable preferences. They are, however, the building blocks of a standard textbook intertemporal choice model and
are widely used, especially in applied macroeconomic work. It is therefore of interest to put the simple model to a revealed preference test, especially since the testable predictions are simple and intuitively appealing. Our strategy is therefore very different than that usually pursued in the literature looking for violations of revealed preferences axioms in the data. A standard approach is to exploit the Afriat (1967) theorem. The advantage of this strategy is that one has to impose very little structure on the data. The cost is that many questions become untestable, especially in intertemporal context (see Kubler (2004)). Our approach allows to use use the revealed preferences paradigm in previously unexplored contexts at the price of imposing more structure.

For concreteness, we assume that the economy is inhabited by a representative agent. The country has free access to international financial markets at a given real and riskless gross interest rate $R$. The agent maximizes her lifetime utility $U(C_1, C_2)$ subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{R_2} = R_1 B_0 + Y_1 + \frac{Y_2}{R_2}$$  \hfill (4.1)

Utility function $U(C_1, C_2)$ is intertemporally additive, twice differentiable and strictly concave in both present and future consumption, the latter being discounted by the subjective discount factor $\beta < 1$:

$$U(C_1, C_2) = u(C_1) + \beta u(C_2), \quad u'(\cdot) > 0, \quad u''(\cdot) < 0 \hfill (4.2)$$

where the period-specific utility function $u(\cdot)$ is of the CRRA type $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$, and $\sigma$ is the risk aversion coefficient. Optimal intertemporal choice is described by the familiar Euler condition

$$u'(C_1) = \beta R_2 u'(C_2) \hfill (4.3)$$

To be able to bring the model to the data, we proceed as follows. In the first step, we eliminate $C_2$ from the optimality condition 4.3 using the budget constraint 4.1 and then solve for $C_1$. This yields the familiar Marshallian demand function that relates consumption to prices and income. Next, we take a first-order Taylor expansion of the demand function. In particular, we approximate it around the vector of variable values before any changes, i.e. around its former self. For example, $C_1$ denotes present consumption before, e.g., a change in the interest rate or endowments, whereas $C_1'$ is present consumption after the change, so that, e.g., $\hat{C_1} = \frac{C_1' - C_1}{C_1}$ denotes percentage deviation of present consumption from its former self.

In the last step, the Taylor-expanded demand function is combined with the Taylor-approximated version of the net exports definition $NX_1 = Y_1 - C_1$. By substituting out $\hat{C_1}$ and solving for $NX_1'$ we arrive at:

$$NX_1' = NX_1 - \frac{C_1 (B_0 R_1 R_2 + Y_1 R_2 - C_1 R_2 - C_2) R_2}{C_2 + C_1 R_2} \hat{R_2} - \frac{C_1 R_1 R_2}{C_2 + C_1 R_2} \hat{B_0} - \frac{C_1 B_0 R_1 R_2}{C_2 + C_1 R_2} \hat{R_1} + \frac{C_2 Y_1}{C_2 + C_1 R_2} \hat{Y_1} - \frac{C_1 Y_2}{C_2 + C_1 R_2} \hat{Y_2} \hfill (4.4)$$

where, as the only exception to the notation rule, $\hat{B_0} = B_0' - B_0$, due to the possibility of negative values. Equation 4.4 shows the optimal evolution of the level of net exports, which depends on changes in present and future endowments, interest rates and the net foreign asset position.
Let us now step back for a moment and revisit the comparative statics exercise studied in Subsection 2.1 through the lens of the parametric model summarized by equation 4.4. Back then, we considered a change in the current interest rate (i.e. \( \hat{R}_2 \neq 0 \)), treating endowments as constant parameters, i.e. we imposed \( \hat{Y}_1 = 0 \) and \( \hat{Y}_2 = 0 \). The initial net foreign asset position equal to zero (i.e. \( \hat{B}_0 = 0 \)) but it also remained zero following the interest rate change (i.e. \( \hat{B}_0 = 0 \)). Under those assumptions, equation 4.4 reduces to

\[
NX'_1 = NX_1 - C_1 \frac{Y_1 R_2 - C_1 R_2 - \frac{C_2}{\sigma}}{C_2 + C_1 R_2} \hat{R}_2
\]  

(4.5)

This equation captures the change in net exports through adjustments of \( C_1 \) due to a change in the present interest rate \( R_2 \). The change is a net outcome of the substitution, income and wealth effects.

Consider now the effects of a drop in the current interest rate (i.e. \( \hat{R}_2 < 0 \)) for country that initially ran a trade deficit, i.e. \( NX_1 = Y_1 - C_1 < 0 \). Following our theoretical argument, we should observe no switch in the net exports sign. A sufficient (but not necessary) condition for that is for \( C_1 \) to go up. This would be the case if the (positive) substitution and wealth effects jointly dominated the negative income effect. Making use of the intertemporal budget constraint 4.1, this sufficient condition becomes

\[
Y_2 > \sigma \frac{1}{\sigma} C_2
\]

(4.6)

Is this indeed always the case? Note that the assumption \( B_0 = 0 \) implies that a trade deficit today is equivalent to a trade surplus in the future, i.e. \( Y_2 \geq C_2 \). Given that \( \sigma > 0 \), it follows that 4.6 indeed remains negative under any parametrization because the negative income effect is always dominated and present consumption will always rise for a trade deficit country following an interest rate drop. In other words, 4.6 is slack given \( Y_2 \geq C_2 \).

Consider now a reverse scenario, in which the country starts with a trade surplus and the interest rate goes up. According to the theoretical argument, that country should sustain a positive trade balance in the present and, as a consequence, a deficit in the future, i.e. \( C_2 \geq Y_2 \). To preclude a switch from surplus to deficit, a sufficient condition is, again, that 4.6 be satisfied. Now, however, the condition is no longer slack In particular, it will be violated if future deficits are relatively large and/or if \( \sigma \) is sufficiently larger than 1. The intuition behind this is the following. The ultimate effect of interest rate increase will depend on the relative strength of the substitution, income and wealth effects. The substitution effect will tend to shift consumption from the present to the future, as higher \( R_2 \) means relatively more costly present consumption. The income effect works in the opposite direction. A higher interest rate means that future consumption expressed in today’s prices is cheaper, which, for any given intertemporal endowment, allows to consume more. However, the same increase of interest rates reduces the present value of intertemporal endowment. This negative wealth effect works towards a reduction in present consumption. The relative strength of the income effect grows with \( \sigma \), the degree of relative risk aversion. For \( \sigma > 1 \) the income effect dominates the substitution effect. On top of that, the fact that \( C_2 \geq Y_2 \) means that the income effect dominates the wealth effect. If large enough, it may then offset the combined substitution and wealth effects. In this case, 4.6 is violated.

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and consumption goes up following an interest rate increase for trade surplus countries. If large enough, such rise in consumption may generate a switch to trade deficit and hence a violation of WARP. Such violation of WARP is not implied by a violation of 4.6 given that this condition is only sufficient, but not necessary. However, since a violation of WARP implies that preferences cannot be represented by a utility function in the first place, this case may potentially lead to a curious theoretical paradox in the model.

In the data, the assumptions \( B_0 = 0, \dot{B}_0 = 0, \dot{Y}_1 = 0 \) and \( \dot{Y}_2 = 0 \) are practically never satisfied. Moreover, even in an iterated version of the model itself \( \dot{B}_0 = 0 \) has to be violated because if we iterate the model one period forward (so that \( B_1 = B_0' \)) the net foreign asset position is necessarily affected by a non-zero trade balance in the previous period, as trivially follows from the law of motion \( B_0' = B_0 R_1 + NX_1 \). As discussed in Subsection 2.2, all these changes invalidate the WARP restrictions put on net exports changes and make the commonly used nonparametric tests for WARP not applicable for our purpose. Equation 4.4, on the other hand, can be brought to the data and can be naturally thought of as a regression that allows to "control for" these undesired endowment and wealth changes in the data. In particular, it decomposes the total net exports dynamics into orthogonal changes due to interest rate movements as well as changes in net foreign wealth and endowment. When taken to the data, equation 4.4 tells if the concurrent changes in net foreign wealth and endowments are enough to explain a switch in the sign of net exports or not. Putting it differently, one can subtract the terms related to \( \dot{B}_0, \dot{Y}_1 \) and \( \dot{Y}_2 \) from both sides and ask what would the sign of trade balance be were these changes controlled for. If the sign is admissible given the initial trade balance \( NX_1 \) and the movement in interest \( \dot{R}_2 \), there is no violation. If the sign doesn’t match, we conclude that WARP was violated.

### 4.2 Net foreign asset position

As discussed in Subsection 2.2, the assumption of zero initial net foreign wealth \( B_0 = 0 \) can be relaxed. In this case, WARP imposes restrictions on switches in the net foreign wealth. To derive the testable equation, we start with the identity defining the evolution of the net foreign asset position:

\[
B_1 = R_1 B_0 + Y_1 - C_1
\]  
(4.7)

As before, the equation is log-linearized and \( \bar{C}_1 \) is eliminated using the linear version of the Marshallian demand function. The final equation is:

\[
B_1' = B_1 + \frac{B_0 R_1 C_2}{C_2 + C_1 R_2} \dot{R}_1 + \frac{R_1 C_2}{C_2 + C_1 R_2} \dot{B}_0 - \frac{C_1 (B_0 R_1 R_2 + Y_1 R_2 - C_1 R_2 - \frac{C_2}{\bar{\sigma}} \dot{R}_2}{C_2 + C_1 R_2} \dot{R}_2 + \frac{C_2 Y_1}{C_2 + C_1 R_2} \dot{Y}_1 - \frac{C_1 Y_2}{C_2 + C_1 R_2} \dot{Y}_2
\]  
(4.8)
5 Taking the model to the data

Let us now discuss the issue of taking equation 4.4 to the data. This poses an obvious challenge given that the theoretical model has, strictly speaking only three periods only: the initial, 0, present, 1, and future, 2. Accordingly, the multi-period time series has to be collapsed to three time periods. Period 1 in the model is associated with a single year in the data, for example year \( t = 2004 \). Variables indexed by 0 take the empirical value for the end of year \( t - 1 \). Variables indexed as 2 are less obvious and can be proxied in various ways. We exploit several options. The simplest is to assume that agents are “myopic” and ignore the future 2 and more years ahead. In this case, any \( Y_2 \) is associated with a single year \( t + 1 \) in the data, as it is the case with \( Y_1 \). A more reasonable approach is to think of \( Y_2 \) as the whole future income discounted as of period 2. Since the sample is finite, one can use observations for all available future years \( t + 1 \ldots T \) to year \( t + 1 \) and sum them up. This case constitute our benchmark results (to be reported). Finally, one can depart from the perfect foresight assumption and work with expectations regarding future output, which allows to set the forecast *ad infinitum*. At each point of time, total future endowment \( Y_2 \) is proxied by the discounted sum of the total forecasted future income (up to 1000 periods ahead), where the annual discount factor is equal to 0.96. Out-of-sample forecasted growth rates are obtained using an ARMA(1,1) model on the past growth rates. The benchmark proxy for the world interest rate is the 3 month U.S. T-bill rate. We also explore a weighted average of national interest rates in a robustness check (to be reported). We also modify the world rate by removing the rate for country tested from the world average.

Consider now studying an interest rate change. In the data this occurs as we move from one year to the next. Now, present variables after the change (with period 1 and prime) correspond to year \( t + 1 \) in the data, e.g., \( C'_1 = C_{2005} \). Period 2 corresponds to years \( t + 2 \) and later, whereas the initial period is end \( t \). The strategy is depicted in Figure 6:

![Figure 6: Taking the model to the data.](image)
6 Results

We start by reporting the first set of results in which future GDP is "myopic" and $Y_2$ reflects only next-year GDP. The model tested is the endowment economy with perfect foresight (with respect to the next year). Figure 7 reports switches from trade surplus to trade deficit. Cases marked with yellow denote unsusceptible switches, in which a country switched to deficit amid a global interest rate drop. Cases in orange are susceptible in the sense that the switch to deficit occurred despite a rise in the world interest rate. In these cases, however, the switch can be explained by the model and attributed to changes in endowment (current and future), the evolution of the net foreign asset position, and/or changes in the return on these net assets. Cases highlighted in red are considered WARP violations because the model cannot account for the switch despite taking into account all concurrent changes (in endowment, in returns and initial NFAP). Altogether there are 21 violations, including one case (highlighted in green) which is not robust to parameterization. That case is a violation only for $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 1.5$, but for higher degrees of risk aversion. All other 20 violations are robust to changes in $\sigma$, up to the analyzed limit of $\sigma_{\text{max}} = 8$.

The general message stemming from the figure is that most WARP-violating switches occurred in years 1973-74 and 1979-81, i.e. during the first and second oil crises. The second period is also associated with the Volcker disinflation and a steep liftoff of real interest rates in the US were a much more common phenomenon in the 1970s and 1980s. The only other two occasions in which violations occurred were the year 1988-89 and 2005. Both of these periods were characterized by US monetary policy tightening.

Figure 8 reports switches in the opposite direction, i.e. from trade deficit to surplus. A switch is susceptible is it occurs amid a drop in the world (US) interest rate. As the figure illustrates, violations in this direction have been much less numerous than in the case of switches into deficit. Only 8 instances are observed in the sample, 3 of them associated only with Australia. Many switches "in plus" followed the oil crises, as the economies adjusted to higher energy prices. However, these switches were, with at most 4 exceptions in agreement with the WARP.

Figures 10 and 9 report similar switches, but not in net foreign asset positions. Figure 10 documents switches from initial net creditors to net debtors. The picture is similar as with the case of countries switching from trade surplus to trade deficit. The only qualitative difference is that switches in NFAP are a rarer event than switches in the trade balance. As before, violations of WARP concentrate around the two oil crises. We also report violations in mid-1990s as well as mid-2000s. Italy and Netherlands are the only two countries for which 2 violations are reported. All results, except for Japan in 1974, are fully robust to our choice of the risk aversion parameter $\sigma$. The most frequently discussed switch of that type was the US in 1985 (see e.g. Gourinchas and Rey (2007)). This switch is highlighted in yellow. It is not meaningful to analyze whether it constituted a violation of WARP, because the interest rate exogeneity assumption in this case does not hold.

Finally, in Figure 9 we report switches of net creditors to net debtors. Here, it is much harder to find a strong regularity. Two violations occurred in mid-1970s and two in early 1990s. The only clustering of
violations can be observed in 2008-09, during the Global Financial Crisis. These violations are attributed to Denmark, Finland and Netherlands.

In sum, the initial results suggest that violations of WARP tend to cluster around times of economic turbulence: the two oil crises and the Global Financial Crisis. Some violations are also reported at times of US monetary policy contraction.

7 Concluding remarks

To be written...

References


Figure 7: Switches and WARP violations of countries running initially a trade surplus and switching to deficit.
Figure 8: Switches and WARP violations of countries running initially a trade deficit and switching to surplus.
Figure 9: Switches and WARP violations of countries, initial net creditors, switching to debtors.
Figure 10: Switches and WARP violations of countries, initial net debtors, switching to creditors.