Demographics, monetary policy and the zero lower bound *

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This draft: February 2018

Abstract

The recent literature shows that demographic trends may affect the natural rate of interest (NRI), which is one of the key parameters affecting stabilization policies implemented by central banks. However, little is known about the quantitative impact of these processes on monetary policy, especially in the European context, despite persistently low fertility rates and an ongoing increase in longevity in many euro area economies. In this paper we develop a New Keynesian life-cycle model, and use it to assess the importance of population ageing for monetary policy. The model is fitted to euro area data and successfully matches the age profiles of consumption-savings decisions made by European households. It implies that demographic trends have contributed significantly to the decline in the NRI, lowering it by 2 percentage points between 1980 and 2030. Despite being spread over a long time, the impact of ageing on the NRI may lead to a sizable and persistent deflationary bias if the monetary authority fails to account for this slow moving process in real time. We also show that, with the current level of the inflation target, demographic trends have already exacerbated the risk of hitting the lower bound (ZLB) and that the pressure is expected to continue. Delays in updating the NRI estimates by the central bank elevate the ZLB risk even further.

JEL: E43, E52

Keywords: ageing, monetary policy, zero lower bound, life-cycle models

*We would like to thank Jacek Suda for his advice on modelling learning, and Janusz Jablonowski for sharing his estimates of the life-cycle profiles from the HFCS. Comments received at the Chief Economists Meeting at the Bank of England, NBP-NBU Conference in Kyiv, Dynare conference in Tokyo and seminars at Deutsche Bundesbank, Lietuvos Bankas and Narodowy Bank Polski are gratefully acknowledged as well. The views expressed herein are those of the authors and not necessarily those of Narodowy Bank Polski.

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1 Introduction

Many economies, developed and developing alike, experience (or will soon begin to experience) a substantial demographic transition. Increasing longevity and sub-replacement fertility rates translate into ageing of societies, with the speed of this process varying between countries. Ageing affects aggregate output, pension system sustainability, structure and volume of fiscal expenditures, housing markets, and many other issues.

These developments have also become of interest to central bankers and monetary economists. A decreasing population growth rate can translate into a drop in the natural rate of interest (NRI) through higher capital per worker. Increasing longevity lengthens the planning horizon of households, inducing them to save more, and thus exerting a further downward pressure on the NRI. Longer living households, dependent largely on accumulated wealth and asset income during their retirement, may prefer lower and more stable inflation rates, influencing politicians’ and central bankers’ preferences towards the inflation-output tradeoff. Demographics can lead to changes in monetary transmission via its impact on asset distribution. Last but not least, for given inflation targets, lower NRIs translate into lower nominal interest rates, leaving less space for conventional monetary policy during slowdowns in economic activity and thus increasing the risk of hitting the zero lower bound (ZLB) constraint.

In spite of the topic’s importance the impact of demographics on monetary policy has been tackled only in a limited number of papers to date. Kara and von Thadden (2016) calibrate a Blanchard-Yaari overlapping generations model to the euro area and project a decrease of the natural interest rate for the euro area by 0.9 percentage points between 2008 and 2030. They conclude that such adjustments are not important within the horizon that is relevant for monetary policy and do not call for an adjustment in its conduct, consistent with the outlook of some central bankers, e.g. Bean (2004). Carvalho et al. (2016) calibrate a similar model to the average of several developed countries and simulate a more significant decline of the equilibrium interest rate (1.5 percentage points between 1990 and 2014). In contrast to Kara and von Thadden (2016), they conclude that low and declining real interest rates carry important challenges for the monetary authorities. However, in both papers the claims about monetary policy implications are qualitative rather than quantitative.

Another strand of literature signalizes that population ageing may affect the monetary policy environment and effectiveness. Gagnon et al. (2016) argue that the demography of the US is virtually the sole culprit of the recent permanent decline in real GDP growth, rate of aggregate investment and safe asset yields, suggesting that this situation is the “new normal”. Wong (2016) uses household-level data to show that older people are less responsive to interest rate shocks, and that the credit channel loses importance as societies age. Imam (2015) argues that the effectiveness of monetary policy transmission weakens in older societies, with
exception of the wealth and expectation channels which gain in importance. This analysis is based on a dynamic panel model and shows that indeed demographic change is associated with decreased monetary policy effectiveness. Bullard et al. (2012), Vlandas (2016) and Juselius and Takats (2015) analyse the impact of demography on social preferences and, as a result, on the inflation rate targeted by central banks.

Our paper concentrates on the following questions: (i) What is the quantitative impact of ageing on the economy, and in particular on the NRI and potential output? (ii) Should monetary authorities be concerned about the scale and speed of the decline in these variables? In particular, what happens if the central bank observes them only with a lag? (iii) How do the demographic processes affect the probability of hitting the ZLB on the nominal interest rates? We concentrate on the euro area. As evidenced by Figure 1, the euro area is currently undergoing a rapid drop in the number of people entering the working-age period and future fertility rates are projected to remain persistently low. Moreover, mortality rates are consistently falling and the probability of reaching the retirement age is expected to increase from 83% in 1980 to almost 95% around 2080. These two forces reinforce each other in leading to a rapid increase in the old-age dependency ratio, which is projected to reach 70% by 2080. Similar processes, though less severe in their scale, affect the entire European Union.

We offer a number of contributions to the current state of knowledge. First, while most of the literature models ageing by employing simplifying Blanchard-Yaari type assumptions, we opt for modelling the demographic structure in full detail. In this respect, our model draws on the full-scale life-cycle framework pioneered by Auerbach and Kotlikoff (1987). Importantly, those studies employ the Blanchard (1985) assumption of constant mortality risk (while retired). Heijdra and Romp (2008) show that embedding a realistic mortality risk within the Blanchard-Yaari framework has a major impact on the behavior of the model economy, including the shape of impulse response functions to shocks, although their approach is tractable only in the case of a small open economy. We improve on both strands of the literature, modeling a realistic mortality profile also in the case of a closed economy. Moreover, since our focus is on monetary policy implications of ageing, we include the key real and nominal rigidities identified in the New Keynesian literature. This richer structure is key to deliver reliable quantitative answers, also to questions that have not been addressed in the literature before. In particular (this being our second contribution), to our knowledge, we are the first to quantitatively relate demographics to the ZLB problem and consider the impact of imperfect central bank knowledge in this context. Third, we show quantitatively, that in spite of its glacial speed, the demographic transition can have a substantial impact on monetary policy - in particular it can be responsible for a deflationary bias (whose estimates we deliver).

We offer the following answers to the topics enumerated above. First, we find that the
impact of ageing on the economy can be substantial from the monetary policy perspective. In particular, given the currently available demographic projections, the equilibrium interest rate in the euro area is projected to decline by almost 2 percentage points between 1980 and 2030. The growth rate of potential output declines by 0.6 percentage points over the same period. Second, observing the declining equilibrium variables by the central bank in real time makes the process neutral for price stability. However, if the bank learns about the impact of demographic processes on the NRI and potential output only slowly over time, a prolonged period of below-target inflation follows. This deflationary bias may be sizable - over 0.3 percentage points for several years. Third, according to our simulations, demographic developments should be expected to increase the ZLB probability as well. When equilibrium variables are observed the annual probability of hitting the ZLB increases from below 1% in the 1980s to over 4% in 2030. It must be noted that these numbers accumulate to a dramatic increase of the chance of hitting the ZLB in longer horizons. The probability rises from less than 4% for the whole 1980s decade to over 40% for the 2020s decade. Under the learning scenario the risk increases even more - the annual probabilities rise to 4-5% already in 2020.

The paper is organized as follows. The next section exposits the model used in our analysis. Section 3 documents the construction of demographic input data and our calibration strategy. In Section 4 we discuss our simulation results, in particular pertaining to the natural interest rate, the zero lower bound and the role of central bank learning. Section 5 concludes.

2 Model

We construct a New Keynesian model with overlapping generations as well as real and nominal frictions to analyze the implications of demographic transition for monetary policy. The model economy is populated by households facing age- and time-dependent mortality risk, three types of firms, and a monetary policy authority.

2.1 Households

2.1.1 Optimization problem

Each household consists of a single agent who appears in our model at age 20 and is assigned age index \( j = 1 \). Agents can live up to 99 years \((j = J = 80)\), at each year subject to age- and time-dependent mortality risk \( \omega_{j,t} \). Hence, at each time period the model economy is populated by 80 cohorts of overlapping generations, with the size of cohort \( j \) denoted by \( N_{j,t} \).

A \( j \)-aged household maximizes its expected remaining lifetime utility that depends on
consumption $c_{j,t}$ and hours worked $h_{j,t}$ according to

$$U_{j,t} = E_t \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \exp \left( \varepsilon_t^u \right) \left( \ln c_{j+i,t+i} - \phi_{j+i} \frac{h_{j+i,t+i}}{1 + \varphi} \right)$$  \hspace{1cm} (1)$$

where $\beta$ denotes the discount factor, the ratio $N_{j+i,t+i}/N_{j,t}$ represents the probability of surviving for at least $i$ more years, $\varepsilon_t^u$ is a preference shock, $\phi_j$ is the age-dependent labor disutility parameter and $\varphi$ is the inverse of the Frisch elasticity of labor supply.

All households face the following budget constraint

$$P_t c_{j,t} + P_t a_{j+1,t+1} = W_t z_j h_{j,t} + R^a_t P_{t-1} a_{j,t} + P_t b_{eq_t}$$  \hspace{1cm} (2)$$

where $P_t$ denotes the aggregate price level, $a_{j,t}$ stands for the beginning-of-period $t$ real stock of assets that are managed by investment funds and yield the gross nominal rate of return $R^a_t$, $W_t$ is the nominal wage per effective hour, while $z_j$ represents age-specific labor productivity. Our model features exogenous retirement upon reaching age 64 ($J = JR = 45$) and hence we set $z_j = 0$ for $j \geq JR$. Finally, since most agents die before reaching their maximum age, they leave unintentional bequests, which are redistributed equally across all living agents in the form of lump-sum transfers $b_{eq_t}$.

2.1.2 Demography and aggregation

Demographic processes are governed by changes in the size of initial young cohorts $N_{1,t}$ and mortality risk $\omega_{j,t}$, both of which are assumed to be exogenous. The total number of agents alive $N_t$ and the population growth rate $n_{t+1}$ are given by

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad 1 + n_{t+1} = \frac{N_{t+1}}{N_t}$$  \hspace{1cm} (3)$$

where the number of agents evolves according to

$$N_{j+1,t+1} = (1 - \omega_{j,t}) N_{j,t}$$  \hspace{1cm} (4)$$

To better capture the impact of expected demographic changes, we allow population growth in the steady state to differ from zero. As then the number of agents within each cohort becomes nonstationary, it is useful to define the size of cohorts relative to that of the youngest one

$$N^{rel}_{j,t} = \frac{N_{j,t}}{N_{1,t}}$$  \hspace{1cm} (5)$$
and the growth rate of initial young $n_{1,t+1}$

$$1 + n_{1,t+1} = \frac{N_{1,t+1}}{N_{1,t}}$$

(6)

This allows us to rewrite equations (3) and (4) in relative terms

$$N_{rel}^t = \sum_{j=1}^{J} N_{rel}^{j,t}$$

and

$$1 + n_{t+1} = \frac{N_{rel}^{t+1}}{N_{rel}^t} (1 + n_{1,t+1})$$

(7)

Then the allocations of all households can be aggregated to the following per capita variables

$$c_t = \sum_{j=1}^{J} \frac{N_{rel}^{j,t}}{N_{rel}^t} c_{j,t}$$

(9)

$$h_t = \sum_{j=1}^{J} \frac{N_{rel}^{j,t}}{N_{rel}^t} z_{j,t} h_{j,t}$$

(10)

$$a_{t+1} = \sum_{j=1}^{J} \frac{N_{rel}^{j,t}}{N_{rel}^{t+1}} a_{j+1,t+1} (1 + n_{1,t+1})$$

(11)

$$P_{t,beq} = \sum_{j=1}^{J} \frac{N_{rel}^{j,t-1} - N_{rel}^{j,t} (1 + n_{1,t})}{N_{rel}^t (1 + n_{1,t})} R_{t} P_{t-1} a_{j,t}$$

(12)

### 2.2 Firms

There are three types of firms in the economy – investment funds, final goods producers and intermediate goods producers. The first two groups are perfectly competitive while intermediate goods producers operate in a monopolistically competitive environment. All firms are risk-neutral, i.e. they maximize the expected present value of future profits, discounting them using nominal interest rates.

#### 2.2.1 Investment funds

Investment funds use households’ savings to buy and manage a portfolio of assets, transferring every period the earned gross return back to households. The portfolio consists of physical capital, bonds and shares of intermediate goods producing firms. Investment funds maximize
the expected present value of future gross returns

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} R_s \right)^{-1} \left[ \sum_{s=0}^{\infty} \left[ R_{t+1}^k + (1 - \delta) Q_{t+1} \right] K_{t+1} + R_t P_t B_{t+1} \right] + \int_0^{N_{t+1}} \left[ P_{t+1} F_{t+1} (i) + (N_{t+2}/N_{t+1}) P_{t+1}^d (i) \right] D_{t+1} (i) \, di \tag{13}
\]

where \( R_t \) denotes the gross nominal interest rates on bonds \( B_{t+1} \), \( R_{t+1}^k \) is the nominal rental rate on capital while \( Q_{t+1} \) is the nominal price of a unit of capital \( K_{t+1} \), which depreciates at rate \( \delta \). The mass of intermediate goods firms is tied to the size of population and at the end of time period \( t \) it is equal to \( N_{t+1} \). At the end of period \( t + 1 \) a randomly selected fraction \( (N_{t+2}/N_{t+1}) - 1 \) of firms generates spin-offs (or dies in case of negative population growth). The spin-offs are identical clones of their parents. \( D_{t+1} (i) \) stands for the number of shares issued by intermediate goods producing firm \( i \) that are traded at the end of period \( t + 1 \) at price \( P_{t+1}^d (i) \) and throughout the period \( t + 1 \) yield dividends \( F_{t+1} (i) \).

The balance sheet of investment funds can be written as

\[
P_t A_{t+1} = Q_t (1 - \delta) K_t + P_t I_t + P_t B_{t+1} + \int_0^{N_{t+1}} P_t^d (i) D_{t+1} (i) \, di \tag{14}
\]

where investment \( I_t \) is used to produce capital according to the law of motion

\[
K_{t+1} = (1 - \delta) K_t + \exp (\varepsilon_t^i) \left[ 1 - S_k \left( \frac{I_t}{I_{t-1}} \right) \right] I_t
\]

where \( \varepsilon_t^i \) is a shock to efficiency of investment, subject to adjustment costs of the following form

\[
S_k \left( \frac{I_t}{I_{t-1}} \right) = \frac{S_1}{2} \left( \frac{I_t}{I_{t-1}} - (1 + n_t) \right)^2 \tag{15}
\]

Since we assume that all revenue from asset management is transferred back to households, the ex-post rate of return on assets is given by

\[
R_t^a P_{t-1} A_t = \left[ R_t^k + (1 - \delta) Q_t \right] K_t + R_{t-1} P_{t-1} B_t + \int_0^{N_t} \left[ P_t F_t (i) + (N_{t+1}/N_t) P_t^d (i) \right] D_t (i) \, di \tag{16}
\]

\[\textsuperscript{1}\text{This particular form ensures that adjustment costs are zero in the steady state even if the population size is not stationary.}\]
2.2.2 Final goods producers

Final goods producers purchase varieties of intermediate goods and produce a homogenous final good according to the following CES aggregator

\[
y_t = \left[ \frac{1}{N_t} \int_0^{N_t} y_t(i) \frac{1}{\mu_t} \, di \right]^\mu_t
\]

(17)

where \( y_t \) is per capita final goods output, \( y_t(i) \) are purchases of differentiated intermediate goods and \( \mu_t = \exp(\varepsilon_t^\mu) \mu \) is the stochastic gross markup, with steady state value \( \mu \geq 1 \), that depends on the elasticity of substitution between intermediates. The associated aggregate price index is given by

\[
P_t = \left[ \frac{1}{N_t} \int_0^{N_t} P_t(i) \frac{1}{\mu_t} \, di \right]^{1-\mu_t}
\]

(18)

The demand for a given intermediate good can be then expressed as

\[
y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\frac{\mu_t}{1-\mu_t}} y_t
\]

(19)

2.2.3 Intermediate goods producers

Intermediate goods producers hire capital and labor and produce differentiated output according to the Cobb-Douglas production function

\[
y_t(i) = \exp(\varepsilon_t^z) k_t(i)^\alpha h_t(i)^{1-\alpha}
\]

(20)

where \( \varepsilon_t^z \) is a productivity shock. They face demand schedule given by equation [19] and set their prices taking into account the Calvo friction, with \( \theta \) representing the probability of not receiving the reoptimization signal, in which case prices are fully indexed to steady state inflation.

2.3 Monetary authority

The monetary authority sets the nominal interest rate according to a Taylor-like rule that takes into account the zero lower bound constraint

\[
R_t = \begin{cases} 
R_t^{cb} & \text{if } R_t^{cb} > 1 \\
1 & \text{if } R_t^{cb} \leq 1 
\end{cases}
\]

(21)

where

\[
R_t^{cb} = R_t^{Rt-1} \left[ \frac{\bar{R}_t^e}{\pi_t} \gamma^e \left( \frac{y_t}{\bar{y}_t} \right)^\gamma^y \right]^{1-\gamma_R} \exp(\varepsilon_t^R)
\]

(22)
In the above equation, $\pi_t \equiv P_t / P_{t-1}$ is the gross rate of inflation, $\pi$ is the inflation target and $\varepsilon^R_t$ is a monetary policy shock. The coefficients $\gamma_R$, $\gamma_\pi$ and $\gamma_y$ control, respectively, the degree of interest rate smoothing, response to deviations of inflation from the target and response to deviation of output growth from its potential.

The variables $\tilde{R}_t^e$ and $\tilde{y}_t^e$ describe the central bank perceptions of the natural nominal interest rate $\tilde{R}_t \equiv \pi \tilde{r}_t$ and natural output $\tilde{y}_t$, respectively, where $\tilde{r}_t$ denotes the natural real interest rate. These natural quantities are defined as hypothetical values of the relevant variables that would be observed under fully flexible prices (i.e. $\theta = 0$) and absent stochastic shocks, but with demographic changes taken into account. Unless indicated otherwise, the perceptions of the monetary authority are assumed to be consistent with current economic developments, i.e. $\tilde{R}_t^e = \tilde{R}_t$ and $\tilde{y}_t^e = \tilde{y}_t$. Alternatively, we assume that these perceptions are linked to the actual values with a constant gain learning process as in Evans and Honkapohja (2001)

\begin{align*}
\tilde{R}_t^e &= \tilde{R}_{t-1}^e + \lambda (\pi \tilde{r}_{t-1} - \tilde{R}_{t-1}^e) \quad (23) \\
\tilde{y}_t^e &= \tilde{y}_{t-1}^e + \lambda (\tilde{y}_{t-1} - \tilde{y}_{t-1}^e) \quad (24)
\end{align*}

so that the central bank observes the true natural interest rate and output only with a lag, and updates their current guess with a fraction $\lambda$ of the previous forecast errors. This way of formulating the feedback rule ensures the long-run consistency of the equilibrium with central bank targets, but also allows us to model imperfect knowledge of the monetary authority.

### 2.4 Market clearing conditions

The model is closed with a standard set of market clearing conditions. Equilibrium on the final goods market implies

\[ y_t = c_t + i_t \quad (25) \]

The market clearing conditions for capital can be written as

\[ \frac{1}{N_t} \int_0^{N_t} k_t (i) = k_t \quad (26) \]

while that for labor is

\[ \frac{1}{N_t} \int_0^{N_t} h_t (i) = h_t \quad (27) \]

\footnote{The resulting estimate of the unobserved quantity is equivalent to an exponentially weighted average of its all past values.}
This allows us to write the aggregate production function as

\[ y_t \Delta_t = \exp (\varepsilon_{zt}^t) \kappa_t^{\alpha} \theta_t^{1-\alpha} \]  

(28)

where \( \Delta_t \equiv \frac{1}{N_t} \int_0^{N_t} \left( \frac{P_t(i)}{P_t} \right)^{\mu_t/(1-\mu_t)} d_i \) measures the price dispersion across intermediate goods.

Since bonds are traded only between (identical) investment funds, we have

\[ B_t = 0 \]  

(29)

Without loss of generality, the number of shares issued by each intermediate goods producing firms can be normalized to unity at all times, which gives

\[ D_{t+1}(i) = D_t(i) = 1 \]  

(30)

### 2.5 Exogenous shocks

The model economy is driven by the following exogenous variables. Demographic processes are characterized by the the growth rate of initial young \( n_{1,t} \) and age-specific mortality risk \( \omega_{j,t} \), all of which are treated as deterministic, i.e. known to all optimizing agents. Additionally, the economy is hit by stochastic shocks to productivity \( \varepsilon_{zt}^t \), household preferences \( \varepsilon_u^t \), investment specific technology \( \varepsilon_i^t \), monetary policy \( \varepsilon_R^t \), as well as markup shocks \( \varepsilon_{\mu}^t \).

### 3 Calibration

The model is calibrated for the euro area, using annual time frequency. In this section we first explain how we parametrize the life cycle characteristics, then present the assumptions regarding demographic trends, and next describe the chosen values for other structural parameters as well as properties of shocks driving the stochastic version of the model.

#### 3.1 Life-cycle characteristics

Our model allows two exogenous household characteristics to vary with age, namely labor productivity \( z_j \) and weight on labor in utility \( \phi_j \), \( j = 1, ..., JR - 1 \). We calibrate the age profiles associated with these parameters by relying on the second wave of the Household Finance and Consumption Survey (HFCS), conducted in 18 euro area countries between 2012 and 2014. The life-cycle characteristics are extracted at a household level, and the age of the household is determined by the age of the household head. We use data on labor income, including wage employment and self-employment, and hours worked, defined as time spent working at the main job. Since our model does not explicitly account for changes in
the household composition or the family size, the extracted age profiles of labor income and hours worked are next divided by the square root of the number of household members, which is one of the equivalence scales used while working with household level data, see Fernandez-Villaverde and Krueger (2007) and OECD (2008). Thus obtained profiles are next smoothed using fourth-order polynomials, which are also used for extrapolation over the age groups 20-24. We approximate the life-cycle pattern of productivity $z_j$ by dividing the age profile of income by that for hours worked. The age-specific weights on labor disutility $\phi_j$ are chosen such that in the initial steady state the model-implied hours worked match exactly the obtained empirical age profile for this variable.

Figure 2 presents the matched age profiles for productivity and hours worked. The former follows the well-documented pattern, increasing up to late middle age of a household head, and then declining. It might be interesting to note that this profile peaks about later than that estimated for the US by Gourinchas and Parker (2002) or Fernandez-Villaverde and Krueger (2007), which may be explained by the fact that our data measure consumption of households expecting longer lifetimes. As regards hours worked, they are almost flat, with some increase at young age and drop close to retirement. In the figure we also show how our model matches two other important lifecycle profiles that we do not target, namely consumption and net assets. Again, as an empirical benchmark we use the HFCS data described above, with assets defined as net wealth excluding public and occupational pensions, and consumption approximated by spending on food (at home and outside) and utilities. As before, we use fourth-order polynomials to smooth the profiles and extrapolate them for age groups 20-24 and 67-70. Given the simple structure of our model, the profiles are matched remarkably well. In particular, we capture the timing of peaks in consumption and asset accumulation, the latter expressed relative to consumption, and the slopes are not very different from their empirical counterparts as well, especially for assets.

### 3.2 Demographic trends

Our demographic scenarios use the past and projected estimates of the fertility and mortality rates for the European Union. We base on the historical data provided by Eurostat and the Europop2013 projection, which encompasses years 2013-2080. Population data and age-specific death rates prior to 2015 come from the demo_pjan and demo_m lifetable series, respectively. For years where mortality rates are not documented for the oldest cohorts, we employ exponential extrapolation. The future mortality rates are taken from the main projection scenario proj_13naasmr, while population projections use the no-migration variant.

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3The lifecycle consumption pattern in Gourinchas and Parker (2002), based on the CEX survey in years 1980-1993 peaks at around age 45, and in Fernandez-Villaverde and Krueger (2007), based on the CEX survey in years 1980-2001 peaks at around age 50. This already provides some evidence that due to increasing longevity the lifecycle consumption peak occurs later in life.
proj_13npzms for internal consistency.

This data is available for individual EU member states (28 countries), and we use the historical and projected cohort sizes to aggregate them over those countries for which the projected no-migration variant estimates are available. The historical mortality rates are available as of 2002. For years 1986-2001, we use the French estimates, as the EU-28 and France had similar mortality rates in the 2000s. For the population data, we use directly the cohort sizes for years 2001-2014. For years 1995-2000, we rescale proportionately the cohort sizes reported for EU-27 (Croatia is not included). Population prior to 1995 is constructed using French mortality rates.

To ensure that the model predictions for the years that we focus on, i.e. 1980 and after, are not contaminated by frontloading effects, we start our deterministic simulations in the year 1900. To that end, we construct artificial population data for the years 1900-1994 and backcast the sizes of historical 20-year old cohorts while holding the historical mortality rates at the earliest available date (1986 for France) to accurately match the existing population structure from 1990 onwards. This is important as the consequences of the WW2 and post-war demographic booms are still visible in the age pyramids.

Finally, the mortality rates were smoothed using the Hodrick-Prescott filter, with smoothing parameter 6.25 to avoid jumps in the demographic input data produced by data revisions and splicing historical data and projections. As for the growth rate of the youngest cohort, we apply a much larger degree of smoothing (10,000) as we want it to capture secular trends in fertility rates rather than reflect the post-war baby booms and its echos. The resulting population pyramids for years 1995 and 2015 (corresponding to the first year in which we explicitly observe the population structure and the last year of available historical data) can be seen in Figure 3, which confirms that we are able to capture accurately the underlying trends even after significant smoothing. At the end of our projection horizon, we assume that mortality rates stabilize while the rate of change of 20-year olds stays at the level projected for 2080.

### 3.3 Other parameters

Our calibration of the remaining model parameters is based on the previous literature, complemented with econometric estimates performed outside of the model and a moment matching exercise. The chosen values of the structural parameters are reported in Table 1, while Table 2 reports the properties of shocks that we use in stochastic simulations.

The discount factor is calibrated at 1.001 to match the average real interest rate of 1.2% observed in the euro area during the years 1999-2008. This is the longest time span during which the Eurozone interest rates can be considered close to their equilibrium values. In this period inflation was relatively stable, and after 2008 the euro area faced a prolonged
crisis that pushed the interest rates down for cyclical rather than structural reasons. Our calibration of the Frisch elasticity of labor supply of 0.25 is consistent with estimates from the microeconomic literature (see e.g. [Peterman, 2016]). Physical capital is assumed to depreciate at a standard rate of 10% annually. The capital elasticity of output is calibrated at 0.25, which ensures that the investment rate is close to the average values observed in the Eurozone. This parametrization, together with a markup of 4%, implies that the labor income share is about 72%. Since our model does not feature wage rigidities, we calibrate the Calvo probability at a somewhat high value of 0.73, or 0.925 in quarterly frequency.

The parametrization of the monetary policy feedback rule, including the standard deviation of monetary shocks, is based on econometric estimation of a log-linearized version of the monetary policy feedback rule (22), using euro area data over the period 1980-2012 from the AWM database, converted to annual frequency. We cut the sample at 2012 as this was the last year during which the ECB’s monetary policy was not constrained by the zero lower bound. Prior to estimation, all series are detrended using the Hodrick-Prescott filter, with the smoothing parameter set to 100, which is a conventional value used for annual data. Subtracting trends from the data is aimed to capture the time variation in the natural interest rate, potential output and inflation target.

We consider three variants of the speed at which the central bank learns about changes in the natural interest rate and potential output. Our benchmark is 1, which implies perfectly knowing these two latent variables in real time. Whenever we allow for imperfect learning, we either follow the empirical literature documenting the observed speed of learning (Branch and Evans, 2006; Malmendier and Nagel, 2016; Milani, 2011), which suggests annual learning rates of approximately 8%, or make use of popular estimates of the NRI. In the latter case, we compare two types of NRI estimates, i.e. smoothed and filtered, both based on the methodology of Holston et al. (2016). The filtered estimates use only past data available at a given point in time to estimate the NRI. The smoothed estimates, on the contrary, are based on the entire data sample. As such, these two measures can be seen as proxies for the real-time and “true” estimates of the NRI, respectively. Next, we estimate a regression following formula (23), using the filtered series for the perceived and the smoothed series for the true NRI. The estimated learning speed is approximately 20%, which is our alternative estimate for $\lambda$.

Apart from the demographic transition scenario, driven by the purely deterministic evolution of the fertility and mortality rates, the model is also used in a stochastic context. The five stochastic shocks affect: productivity, intertemporal preferences, investment-specific technological progress, firms’ monopolistic power and monetary policy. All of them follow

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4Clearly, this way of reasoning is not appropriate for the last part of the sample, as the smoothed estimates converge, by construction, to the filtered ones. For this reason, we decided to drop the last 10 years of observations from further analysis. As a consequence, $\lambda$ is estimated on the sample 1980-2005.
first-order autoregressive processes, except for the monetary shock that we assume to be white noise, and whose volatility is estimated outside of the model together with the remaining parameters describing the monetary policy rule. The inertia and standard deviations of the remaining shocks, as well as the investment adjustment cost function curvature, are determined in a moment matching exercise, in which we use detrended data on euro area real GDP, real private consumption, real investment, HICP inflation and the short-term nominal interest rate. More specifically, we minimize the distance between the model-based standard deviations, first-order autocorrelation and correlation with output and their respective data counterparts. All model-based moments are calculated using the first-order approximation of the policy functions around the point defined as the mean of the state variables in our demographic transition scenario over the period 1999-2008, which is the period during which we match the level of the real interest rate. We use the same weights for all matched moments, except the volatility and inertia of the nominal interest rate, for which we assign much higher weights so that the fit is exact. It is important to achieve a perfect match for this variable as one of our goals is to evaluate the impact of demographic transition on the probability of hitting the ZLB. As can be seen from Table 3 the achieved fit is very good also for other moments.

4 Effects of demographic change

In this section we seek to answer several important questions about the consequences of the demographic transition for monetary policy. As it is well know from the literature, population ageing can have a sizeable impact on savings and the equilibrium interest rate. We start by documenting this effect. Then we move to analysing the consequences for monetary policy: we compare the impact of the demographic change on inflation, under various assumptions about the speed with which the central bank notices the decline of the NRI an potential output growth rate. Third, we asses how much the declining NRI raises the probability of hitting the zero lower bound on the nominal interest rates. This is done both for the case when the central bank observes the declining NRI and potential output in real time and when it gradually learns about their changes.

4.1 Impact of the demographic transition

We begin with describing the impact that the demographic transition exerts on main macroeconomic variables. To this end we run a deterministic simulation, assuming that the demographic processes described in Section 3 are known to all agents.

Figure 4 presents the main results. The upper-left panel shows the dependency ratio. The economy faces a sharp increase of this ratio, resulting from lower birth rates and higher life
expectancy. Two main factors are at play. First, due to a longer expected time in retirement, workers increase savings. Second, low birth rates result in a declining population and, thus result in higher per capita asset holdings. Both factors operate in the same direction - they lower the real interest rate. The decline is substantial, though spread over time. Between 1980 and 2030 the interest rate declines by almost 2 percentage points. Other developments worth mentioning are declining labor supply, increasing real wage (due to the lower supply) and, as a consequence, a higher capital-labor ratio chosen by firms. With less labor the economy produces less goods, so that ultimately GDP per capita and capital per capita are lower after the transition is over. The growth rate of potential output declines as well and bottoms out in the 2030s 0.6 percentage points below its 1980s level.

It is interesting to see the relative role of the two factors - fertility and longevity. Figure 6 decomposes the impact of demographic developments on the natural rate of interest. Clearly, both matter, with increased life expectancy being slightly more important - this factor is responsible for 50-60% of the NRI decline, depending on the considered period.

We also find it interesting to compare our simulated path with a popular NRI estimate used in the literature. Figure 5 plots our NRI together with the smoothed and filtered estimates of Holston et al. (2016) refered to in Section 3.3 (the available sample is 1972-2015). While (not surprisingly) the exact numbers differ, it is striking that our projected decline of the NRI is much in line with the downward trend in the econometric estimates. This is even more evident if we ignore the post 2008 data, which most probably has been driven by cyclical factors related to the financial crisis. Between 1972 and 2008 our NRI declines by 1 percentage point and the Holston et al. (2016) estimates by 1.1-1.5 percentage points. Hence, it seems that over this period demographic processes have been responsible for most of the downward trend in the natural rate.

## 4.2 Consequences for monetary policy

We see two potential consequences of the demographic transition on monetary policy. First, potential misperceptions of the NRI or of potential output could bias the monetary policy stance. In the past equilibrium interest rates have been frequently assumed constant (or at least stationary). For instance, until approximately 2000 many economists and central bankers placed NRIs in the US, UK or euro area in the range of 2-3% (Laubach and Williams, 2003). With relatively rare exceptions, Taylor rules that were calibrated or estimated for these (and several other developed) countries assumed a constant intercept, and hence implied a constant NRI (see e.g. Taylor (1993); Smets and Wouters (2003) for standard rules and Orphanides and Williams (2002); Trehan and Wu (2007) for exceptions with time-varying NRI). In such an environment possible misperceptions of the NRI result only in short-run over- or underrestrictiveness of monetary policy.
Things change when the natural rate is declining for long periods. If the central bank observes the evolution of the natural rate in real time then the impact of a declining NRI on inflation can be neutralized by an appropriate policy rate adjustment (leaving the ZLB problem aside for a moment). However, it seems likely that the central bank might notice the declining NRI only with a lag, and hence overestimate it during the transition. As a consequence, the monetary policy stance may be unintentionally too contractionary for a long time. A similar (though of opposite consequences) problem applies to potential output. Its declining growth rate could be noticed with a lag, leading the central bank to perceive output growth as being below potential. As a consequence the bank could attempt to conduct too expansionary policy. In what follows we take a deeper look at both of these issues.

Second, a lower natural rate decreases, ceteris paribus, the average nominal central bank interest rate, and as a consequence, raises the probability of hitting the zero lower bound. In what follows, we analyze these two potential consequences in detail.

4.2.1 Speed of learning and inflation bias

As was explained in Section 2, our monetary policy rule was designed to account for declining NRI and potential output growth. In the baseline calibration we assume that the central bank observes both in real time. Now we compare this result to the case when the monetary authority follows a learning process, described by equations (23) and (24), so that every period the central bank’s guesses of these two unobservable variables is being updated for a fraction $\lambda$ of the last period forecast error. The resulting expected NRI and potential output growth rate are equivalent to exponentially weighted averages of all their past values.

Figure 7 documents our findings by comparing the inflation rate under the baseline and learning assumptions. As expected, in the baseline scenario inflation is constant at the central bank target. Monetary authorities adjust the interest rate one-for-one with the NRI and so neutralize the impact of a declining natural rate on the economy. However, if the central bank fails to timely account for the demographic trends, monetary policy ceses to be neutral. As already mentioned two forces are at play - the declining NRI makes policy overly restrictive while the declining potential makes it overly expansionary.

Figure 7 shows that the NRI effect dominates - policy becomes clearly too contractionary and results in a deflationary bias. Even though the NRI drops on an average rate of only 0.04 percentage points per year, the permanent bias in monetary policy has a substantial impact on inflation. In particular, inflation remains permanently below the target, the gap in the analyzed period reaches 0.35 percentage points on maximum (in the 2020s). As the natural rate stabilizes after 2030, the deviations in its perception stop being permanently biased and inflation slowly returns towards the target.
4.2.2 Probability of hitting the ZLB

During the recent decade, the interest rates in many countries have hit the zero lower bound (ZLB). Something that had looked like a textbook curiosity has become a part of central bank reality. While the affected banks managed to elaborate alternative tools that allowed (at least partially) to overcome the consequences posed by the constraint (e.g. quantitative easing), it seems that they still prefer to use the short-term interest rate as the main policy instrument. Keeping this in mind we decided to check whether the lower NRI raises substantially the probability of hitting the ZLB. If this is the case, central banks might have to consider increasing their inflation targets to compensate for the declining NRI.

We proceed as follows. We run stochastic simulations with productivity, time preference, risk premium and foreign shocks calibrated as described in Section 3 for 100,000 periods. This exercise is repeated in the vicinity of every point on our deterministic path using first-order Taylor approximation to the model equilibrium conditions. The simulations are done twice. First, we assume that the central bank knows the NRI and potential output. Second and third, we check what happens if the bank learns about these unobservable variables as described in Section 4.2.1 using two alternative parametrizations for the speed of learning. Since learning can result in a long period when the economy faces persistently low inflation, we expect the probability of hitting the ZLB to be higher at some points. Then, we approximate the probability of hitting the ZLB by calculating the frequency of periods during which the nominal interest rate is constrained by this limit.

Our findings are summarized in Figure 8. According to the baseline scenario, the annual probability of hitting the ZLB in the 1980s has been relatively low (below 1%). However, as the equilibrium real rate was declining, the probability was increasing as well, reaching about 2% in 2010, and it is projected to exceed 4% by 2030. While the annual probabilities do not seem very high, one should note what they imply over longer horizons. For instance, the chance of hitting the ZLB during the whole 2020s decade increases to 42.2% from 3.9% in the 1980s. The results from the learning scenario are even more alarming. Not only does the annual probability increase to much higher levels, but even the numbers for the contemporaneous times are quite high, especially if the speed of learning is slow. In 2020 the probability of meeting the ZLB exceeds 4% under \( \lambda = 0.2 \) and rises above 5% if \( \lambda = 0.08 \).

These results show that a slowly, but permanently declining equilibrium interest rate, especially if not properly accounted for, can result in a serious deterioration of monetary policy quality. This finding stipulates our main conclusion for monetary policy. Even if the

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5Regarding the perceived NRI, the central bank is assumed not to learn the equilibrium rate or potential output growth rate during the stochastic simulation. We assume both perceived latent variables to be constant during each stochastic simulation at the respective level from the deterministic scenario.

6The simulations are performed using Dynare OBC (Holden, 2016).

7We do not discuss in detail the adverse consequences of hitting the ZLB, this has been done in many studies, see e.g. Gust et al. (2012); Ireland (2011); Neri and Notarpietro (2014).
central bank is aware of the consequences presented in this paper, the demographic transition is an issue. However, if the bank fails to timely account for the declining NRI, monetary policy becomes too contractionary, generates a deflationary bias and further increases the risk of ending up in a liquidity trap.

5 Robustness

We conduct two experiments to assess the robustness of our findings. First, we investigate how the presence of a pension system affects our results. Second, we analyze how important it is to account for the age-specific productivity profile.

5.1 Impact of pension systems

We augment our baseline model with a streamlined pay-as-you-go pension system. Working households pay proportional a social security contribution \( \tau_p \) levied on labor income. Retired households receive pension benefits, calculated as the product of the replacement rate \( \varrho \) and the wage net of social security contribution. The pension system is assumed to be balanced at all times, satisfying

\[
\tau_p t w_t \sum_{j=1}^{JR-1} N_{j,t} z_j h_{j,t} = pen_t \sum_{j=JR}^{J} N_{j,t} \tag{31}
\]

where \( pen_t \), equal across all retired households, is given by

\[
pen_t = \varrho_t (1 - \tau^p_t) w_t \tag{32}
\]

Accordingly, the budget constraint of the households is modified as follows, while the rest of the model remains unchanged

\[
P_t c_{j,t} + P_t a_{j+1,t+1} = (1 - \tau^p_t) W_t z_j h_{j,t} + R^a_t P_{t-1} a_{j,t} + P_t beq_t + 1_{j \geq JR} P_t pen_t \tag{33}
\]

Due to dynamically changing demographics, the pension system can remain balanced only if either the contribution rate, or the replacement rate (or both) change over time\(^8\). Since we do not intend to take a stance regarding the future approaches to balancing the pension systems, we analyze two edge-case scenarios. In the first, the contribution rate is held constant at \( \tau^p = 0.2 \), while the replacement rate decreases. This puts additional downward pressure on the NRI, as the expectation of lower future pensions induces households to save more privately. In the second scenario we keep the replacement rate unchanged at \( \varrho = 0.2 \)

\(^8\)Here we consciously ignore the possibility that the imbalance can be filled with either adjusting the public debt or by changing other taxes.
and as a result the contribution rate increases over time. Here the decrease in the NRI is mitigated, as the increasing old-age dependency ratio translates to higher contribution rates, reducing disposable income and in turn asset accumulation.

The consequences of those two scenarios for the NRI are depicted in Figure 9. The constant contribution rate scenario generates a similar decrease in the NRI as in the baseline scenario up to year 2015, and then reaches a plateau at the level of 1.4 percentage points lower than in 1980. The replacement rate scenario implies an even stronger decreases in the NRI relative to the baseline, of the magnitude of up to 2.5 percentage points. Most notably, prior to 2015 the baseline scenario predicts a smaller decrease in the NRI than in either of the two pension scenarios, while after 2015 the baseline scenario rests firmly between the two edge-case scenarios and in this sense provides a reasonable guess on the future developments.

5.2 Importance of age-dependent productivity

As a second robustness check, we investigate the importance of accounting for the age-dependent productivity profile for our results. As due to the aging processes the age distribution of households in the economy changes over time, this introduces also shifts in the average labor productivity. To assess the quantitative impact of this effect, we construct a scenario where the productivity is age-independent, while maintaining the same profile of hours worked and effective aggregate labor input in the initial steady state. Additionally, we also analyze a scenario where the initial steady state profile of hours worked is perfectly flat, while productivity remains age-dependent. Both scenarios require an appropriate adjusting of the age-dependent labor disutility parameters, but the rest of the model is unchanged.

The results of these scenarios are presented on Figure 10. The constant productivity scenario generates almost the same decline in NRI as the baseline up to year 2015 and afterwards the decline is by approx. 0.1 percentage points lower than in the baseline. The mechanism behind this difference stems from a shift in the distribution of a lifecycle income from the “thrifty” middle-aged to the “profligate” young. In the constant hours worked scenario the NRI behaves virtually identically to the baseline scenario. Since the lifecycle profile of hours worked is relatively flat already in the baseline, this should not come off as a surprise.

6 Conclusions

How does the demographic transition, resulting from lower fertility rates and higher life expectancy, affect monetary policy? While the question has already been tackled in the

Both scenarios generate effective net pension replacement rates (ratio of net pension to pre-retirement net labor income) of approx. 80% in year 1980.
literature, and there is agreement that equilibrium interest rates will be affected, many issues remain unclear. First, whether the impact is large or small. Second, what happens if the central bank learns about the declining equilibrium interest rate only with a lag. Third, whether a similar role can be played by the declining growth rate of potential output? Fourth, how does the transition affect the probability of hitting the zero lower bound on interest rates?

Regarding the first problem, we are confident that our modeling approach, based on an OLG framework carefully calibrated to projected birth and mortality rates is able to deliver more precise simulations than the earlier studies. We show that the effects can be substantial. In particular, between 1980 and 2030, the equilibrium interest rate in the euro area declines by almost 2 percentage points and the growth rate of potential output falls by 0.6 percentage points.

In principle this decline should not pose a problem for monetary authorities - they should simply adjust interest rates to follow the declining natural rate and potential output. Two issues emerge, however. First, both variables are not directly observable, and it seems possible that monetary policy learns about their decline only with a lag. Second, a lower natural rate implies, ceteris paribus, a higher probability of hitting the zero lower bound.

We show that both problems are acute. Learning about the declining natural rate and potential output growth rate can result in a prolonged period of below-target inflation. While the two mismeasurements work in opposite directions, the NRI effect proves much stronger. Our simulations show a deflationary bias during the whole transition process of up to 0.35 percentage points. As expected, the annual probability of hitting the ZLB also increases, from a benign 0-1% in the 1980s, to over 4% in 2030 and over 6% after the transition is over. Taking additionally learning into account makes the problem much more pronounced. In this case the probability of hitting the ZLB grows to 4-6% already in 2020. It must be noted that such annual probabilities accumulate to much higher numbers over longer horizons. For instance, even without learning, the probability of hitting the ZLB during the 2020s decade exceeds 40%.

All in all, our main conclusion is that in spite of being spread over a long period, the demographic transition can have a significant impact on the conduct of monetary policy. In particular, if the central bank fails to timely account for the declining NRI policy may become too contractionary. Moreover, for the given inflation target, the demographic transition implies a dramatic increase of the probability of hitting the zero lower bound. This risk materializes already contemporaneously.
References


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Vlandas, Tim (2016) ‘The impact of the elderly on inflation rates in developed countries.’ LEQS - ‘LSE Europe in Question’ Discussion Paper Series 107, European Institute, LSE

# Tables and figures

Table 1: Calibrated structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.001</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\phi^{-1}$</td>
<td>0.25</td>
<td>Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>Capital depreciation rate</td>
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<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>Capital share in output</td>
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<td>$S_1$</td>
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<tr>
<td>$\mu$</td>
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<td>Product markup</td>
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<td>$\theta$</td>
<td>0.73</td>
<td>Calvo probability</td>
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<td>$\pi_{ss}$</td>
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<td>Inflation target</td>
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<td>$\gamma_R$</td>
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<td>$\gamma_\pi$</td>
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<td>Reaction to inflation</td>
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<td>$\gamma_y$</td>
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<td>Reaction to GDP growth</td>
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<tr>
<td>$\lambda$</td>
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<td>Learning parameter values</td>
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Table 2: Calibrated stochastic shocks

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<tr>
<th>Parameter</th>
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<th>Description</th>
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<tr>
<td>$\rho_z$</td>
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<td>Inertia of productivity shocks</td>
</tr>
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<td>$\rho_u$</td>
<td>0.75</td>
<td>Inertia of preference shocks</td>
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<tr>
<td>$\rho_i$</td>
<td>0.73</td>
<td>Inertia of investment specific shocks</td>
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<tr>
<td>$\rho_p$</td>
<td>0.82</td>
<td>Inertia of price markup shocks</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.005</td>
<td>Standard dev. of innovations to productivity shocks</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.029</td>
<td>Standard dev. of innovations to preference shocks</td>
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<tr>
<td>$\sigma_i$</td>
<td>0.033</td>
<td>Standard dev. of innovations to investment specific shocks</td>
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<tr>
<td>$\sigma_p$</td>
<td>0.019</td>
<td>Standard dev. of innovations to price markup shocks</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.009</td>
<td>Standard dev. of monetary shocks</td>
</tr>
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</table>

Table 3: Matched data moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard dev.</th>
<th>Autocorrelation</th>
<th>Corr. with GDP</th>
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<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
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<tr>
<td>GDP</td>
<td>1.56</td>
<td>1.65</td>
<td>0.79</td>
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<tr>
<td>Consumption</td>
<td>1.62</td>
<td>1.56</td>
<td>0.83</td>
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<tr>
<td>Investment</td>
<td>4.48</td>
<td>4.47</td>
<td>0.95</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.98</td>
<td>0.76</td>
<td>0.42</td>
</tr>
<tr>
<td>Interest rate</td>
<td>1.49</td>
<td>1.49</td>
<td>0.64</td>
</tr>
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</table>
Figure 1: Demographics

Figure 2: Lifecycle profiles
Figure 3: Population pyramids
Figure 4: Impact of demographic transition on the euro area

Note: Capital per hour worked, hours per capita, capital per capita, the real hourly wage and GDP per capita have been normalized to unity in 1980.

Figure 5: The natural rate of interest: comparison with Holston et al. (2016) estimates
Figure 6: Decomposition of demographic forces affecting the NRI

![Graph showing the decomposition of demographic forces affecting the NRI.](image1)

Figure 7: Inflation bias under learning about NRI

![Graph showing inflation bias under learning about NRI.](image2)

Note: solid line - baseline model; dashed and dotted lines - models with NRI learning.
Figure 8: Probability of hitting the ZLB

Figure 9: Robustness: alternative pension systems
Figure 10: Robustness: alternative assumptions regarding hours worked and productivity profiles
A Appendix

A.1 Complete list of model equations

Households

\[ c_{j,t} + a_{j+1,t+1} = w_t z_j h_{j,t} + \frac{R^o_t}{\pi_t} a_{j,t} + beq_t \]  
(A.1)

\[ a_{0,t} = 0 \]  
(A.2)

\[ a_{j,t} = 0 \]  
(A.3)

\[
1 = \beta (1 - \omega_{j,t}) \mathbb{E}_t \left[ \frac{c_{j,t}}{c_{j+1,t+1}} \frac{R^o_t}{\pi_t} \right] \]  
(A.4)

\[
h_{j,t} = \left( \frac{w_t z_j}{\phi_j c_{j,t}} \right)^{1/\varphi} \]  
(A.5)

Demography

\[ N_{1,t}^{rel} = 1 \]  
(A.6)

\[ N_{j+1,t+1}^{rel} = \frac{(1 - \omega_{j,t}) N_{j,t}^{rel}}{1 + n_{1,t+1}} \]  
(A.7)

\[ N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \]  
(A.8)

\[ 1 + n_{t+1} = \frac{N_{t+1}^{rel}}{N_t^{rel}} (1 + n_{1,t+1}) \]  
(A.9)

Aggregation

\[ c_t = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \]  
(A.10)

\[ h_t = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \]  
(A.11)

\[ a_{t+1} = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} a_{j+1,t+1}}{N_{t+1}^{rel}} \]  
(A.12)

\[
beq_t = \frac{\sum_{j=1}^{J} \left[ N_{j-1,t-1}^{rel} - N_{j,t}^{rel} (1 + n_{1,t}) \right] \left( R^o_t / \pi_t \right) a_{j,t}}{N_t^{rel} (1 + n_{1,t})} \]  
(A.13)
Financial intermediary

\[(1 + n_{t+1}) k_{t+1} = (1 - \delta) k_t + \exp (\varepsilon_t) \left[ 1 - \frac{S_1}{2} (1 + n_t) \left( \frac{i_t}{i_{t-1}} - 1 \right) \right] i_t \]  
(A.14)

\[(1 + n_{t+1}) a_{t+1} = q_t (1 - \delta) k_t + i_t + p_t^d \]  
(A.15)

\[\frac{R_t}{\pi_t} a_t = (r_t^k + (1 - \delta) q_t) k_t + (p_t^d + f_t) \]  
(A.16)

\[R_t q_t = \mathbb{E}_t \left[ (r_t^k + (1 - \delta) q_{t+1}) \pi_{t+1} \right] \]  
(A.17)

\[R_t p_t^d = \mathbb{E}_t \left[ (1 + n_{t+1}) (p_{t+1}^d + f_{t+1}) \pi_{t+1} \right] \]  
(A.18)

\[1 = q_t \exp (\varepsilon_t) \left[ 1 - \frac{S_1}{2} (1 + n_t)^2 \left( \frac{i_t}{i_{t-1}} - 1 \right) \right] - S_1 (1 + n_t) \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \]  
(A.19)

Intermediate goods producers

\[\frac{r_t^k}{w_t} = \frac{\alpha}{1 - \alpha} \frac{h_t}{k_t} \]  
(A.20)

\[m_{ct} = \frac{1}{\exp (\varepsilon_t^i)} \left( \frac{r_t^k}{\alpha} \right)^\alpha \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha} \]  
(A.21)

\[\tilde{p}_t = \mu \frac{\Omega_t}{\Upsilon_t} \]  
(A.22)

\[\Omega_t = m_{ct} y_t + \theta \mathbb{E}_t \left[ \frac{\pi_{t+1}}{R_t} \left( \frac{\pi_{t+1}^\zeta}{\pi_t^\zeta} \right)^{\frac{1}{\tau}} \Omega_{t+1} \right] \]  
(A.23)

\[\Upsilon_t = \exp (\varepsilon_t^\mu) \tau y_t + \theta \mathbb{E}_t \left[ \frac{\pi_{t+1}}{R_t} \left( \frac{\pi_{t+1}^\zeta}{\pi_t^\zeta} \right)^{\frac{1}{\tau}} \Upsilon_{t+1} \right] \]  
(A.24)

Inflation and price dispersion dynamics

\[\pi_t^\zeta = \pi_{ss} \]  
(A.25)

\[1 = \theta \left( \frac{\pi_t^\zeta}{\pi_t} \right)^{\frac{1}{1-\mu}} + (1 - \theta) (\tilde{p}_t)^{\frac{1}{1-\mu}} \]  
(A.26)

\[\Delta_t = (1 - \theta) (\tilde{p}_t)^{\frac{1}{1-\mu}} + \theta \Delta_{t-1} \left( \frac{\pi_t^\zeta}{\pi_t} \right)^{\frac{1}{1-\mu}} \]  
(A.27)
Monetary policy

\[ R_t = \begin{cases} R_t^{cb} & \text{if } R_t^{cb} > 1 \\ 1 & \text{if } R_t^{cb} \leq 1 \end{cases} \]  

(A.28)

\[ R_t^{cb} = R_t^{\gamma R} \left[ \bar{R}_t^e \left( \frac{\pi_t}{\pi} \right)^{\gamma_e} \left( \frac{y_t/y_{t-1}}{\bar{y}_t^e/\bar{y}_{t-1}^e} \right)^{\gamma_y} \right]^{1-\gamma_R} \exp (\varepsilon_t^R) \]  

(A.29)

Alternative assumptions about perceived variables

\[ \bar{R}_t^e = \bar{R}_t \quad \text{or in learning case} \]  

(A.30)

\[ \bar{y}_t^e = \bar{y}_t \quad \text{or in learning case} \]  

(A.31)

Market clearing

\[ y_t \Delta_t = \exp (\varepsilon_t^x) k_t^\alpha h_t^{1-\alpha} \]  

(A.32)

\[ y_t = c_t + i_t \]  

(A.33)

\[ f_t = y_t - w_t h_t - r_t^k k_t \]  

(A.34)