Constrained Inefficiency over the Life-cycle

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February 14, 2018

Abstract

We quantitatively explore how saving and human capital investment are pushed away from efficient levels, measured in constrained optimum, through one’s course of life. In this paper, we compare realistically calibrated competitive equilibrium profiles of saving and time investment on human capital with constrained efficient profiles. We find that top 10% income earners would save 256% more at age 25 in the constrained optimum than in the competitive equilibrium and would invest almost no time on human capital. On the contrary, the bottom 10% of households would save 79% less at age 25 but would start saving more than competitive equilibrium after age 43. They would also spend 24% of their time on human capital investment at age 25, instead of 10% in the competitive equilibrium. We extend this framework to welfare implications of student debt burden associated with the attainment of higher education over the life-cycle.

Keywords: Constrained efficiency, life-cycle, incomplete insurance, uninsurable idiosyncratic shocks, saving, human capital investment

JEL Codes: D15, D52, D62, H23, J24

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1 Introduction

How much are we away from efficient levels of saving and investment on human capital over the course of life? In this paper, we answer this question by comparing realistically calibrated competitive equilibrium age profiles to constrained efficient profiles. We adopt the notion of constrained efficiency—which is recently studied by Dávila et al. (2012) in a standard incomplete market model with uninsurable idiosyncratic income shocks—as a baseline optimal level of profiles. In this efficient allocation, a planner improves on market allocations by internalizing the effect on prices without completing the market and without changing the household’s budget constraints. In other words, households depart from their self-interested optimization by internalizing the effect on market prices.\(^1\)

In an incomplete market with idiosyncratic income shocks, constrained inefficiency (sometimes known as “pecuniary externality”) arises from two sources: i) incomplete insurance and ii) wealth and income inequality. The constrained inefficiency may vary by age in a life-cycle model in three ways. First, since the future income risks that individuals face vary by age, incomplete insurance induces age-varying precautionary saving. For example, older cohorts nearing retirement only have to insure against limited years of future income risks, as compared to younger cohorts. Therefore, their precautionary saving could be smaller than younger cohorts. Second, due to the increasing profiles of income and consumption over the life-cycle, a large fraction of younger cohorts tend to be categorized as consumption-poor within a cross-sectional consumption distribution of all age cohorts. Therefore, a unit increase in the consumption of younger cohorts can contribute relatively more to overall welfare improvement than the same unit increase in the consumption of older cohorts. Finally, labor supply/human capital investment of young cohorts reacts more elastically to wage changes than that of older cohorts, which also differentiate welfare effects of price change across age. (Peterman, 2016).

We quantify the inefficiency of saving and investment on human capital over the life-cycle. For this analysis, we first calibrate an overlapping-generations model with endogenous human capital accumulation where households face idiosyncratic human capital shocks, a borrowing constraint, a progressive tax on labor income, proportional taxes on asset returns and consumption, and social security payments. We numerically solve the model and match model-simulated aggregate moments as well as household’s income and consumption profiles to their data counterparts. Then, we compute constrained efficient profiles and compare to the competitive equilibrium profiles.

Our results show that median households of all ages would save 34–88% more than competitive equilibrium saving, while they would reduce time investment on human capital by 62–90% before retirement in the constrained optimum. The marginal benefit of less (more) time investment on human capital (labor supply) today would be higher labor income today at a given wage and current human capital, whereas the lower future labor income due to a lower accumulation of human capital would be the marginal cost. Therefore, the efficient allocations redistribute labor income from old to young and wealth from young to old. On the other hand, the increase in wage

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\(^1\)Constrained optimum can be found when a planner maximizes a utilitarian objective assigning equal weights to all households. Nevertheless, the constrained optimum may not be Pareto efficient.
due to higher aggregate capital would partially offset the low accumulation of human capital of older cohorts.

In a life-cycle model, the constrained inefficient allocations can be improved on by the redistribution of income and wealth not only across one’s own life, but also across individuals of the same age cohort. To observe distributional effects within each age cohort, we also examine the constrained efficient profiles of top and bottom 10% income earners. The top 10% income earners would save up to 256% more at age 25 and would spend almost no time on human capital investment. The lack of investment on human capital is mainly driven by diminishing returns to human capital and a substitution effect from the rise of wage. Since the top 10% of households have endowed high human capital, they have less incentive to invest on human capital. In addition, the low accumulation of human capital can be offset by the rise of wage. On the contrary, the bottom 10% of households would save 79% less at age 25 but starts saving more than competitive equilibrium saving after age 43. On the other hand, the time investment on human capital would be 24% of their time at age 25 in constrained optimum, instead of 10% in the competitive equilibrium. Although more time invested on human capital reduces current and future labor income, an increase in wage and an accumulation of human capital offsets the lifetime income loss. Overall, the bottom 10% would have higher labor income and consumption. Since utilitarian social welfare assigns equal weights to all individuals, and a welfare increase of the consumption-poor dominates a welfare decline of the consumption-rich, constrained efficient allocations are welfare-improving.

We further extend this analysis to a current issue on student debt burden associated with the attainment of higher education. Although student loans (either private or federal) allow young cohorts to attain higher education, the debt burden put young people in a tight financial position (i.e. student debt crisis). With student debt, young people are unable to purchase a house and are unable to save for precautionary purposes. In other words, student loans allow young people to upgrade their human capital through higher education at a cost of student debt burden after graduation, which inhibits their saving for future income risks. Using this life-cycle framework, we can provide welfare implications of student debt burden over the life-cycle.

This paper sheds light on constrained efficiency where a planner can directly affect the household’s saving and investment on human capital—as opposed to a planner controlling the market prices as in optimal taxation policy—keeping household budget constraints and market structures unchanged. Two closely related studies are Dávila et al. (2012) and Park (2017)’s steady state analyses of constrained efficiency. We apply the notion of constrained efficiency to an overlapping generations framework. Our life-cycle analysis not only yields consistent results as those in the steady state analysis, but also introduces an additional channel for welfare improvement. With a calibrated Aiyagari model, Dávila et al. (2012) show that welfare improves by increasing aggregate capital which eventually makes the consumption-poor better off while making the consumption-rich worse off. Park (2017) extends this analysis to endogenous human capital model in an incomplete market with exogenous labor income shocks. She examines the two sources of constrained inefficiency arising from monetary investment on human capital. In the steady state analysis, the constrained inefficient allocations can be improved on by the redistribution of in-
come and wealth across households (consumption-rich to consumption-poor) and across states of world. One contribution of this paper to the literature is that the constrained inefficiency over the life-cycle can differ by age and can be further improved on if one redistributes income and wealth within one’s life. In other words, we quantitatively investigate an additional inefficiency and redistribution channel arising from the life-cycle property.

In an overlapping-generations model with uninsurable idiosyncratic income shocks, however, much attention has been drawn to optimal tax policy where a planner can only impose taxes on prices with a balance in government budget constraints. Among many others, for instance, Peterman (2016) and Costa and Santos (2015) quantitatively analyze optimal labor and capital taxes in a life-cycle model with endogenous human capital investment and idiosyncratic labor income shocks. Heathcote et al. (2017a,b) analytically study optimal progressivity of income taxes varying by age. Conesa and Krueger (2006) and Conesa et al. (2009) quantitatively compute the optimal income taxes in an overlapping-generations model without human capital investment, while Krueger and Ludwig (2013, 2016) examine education policy as an alternative to progressive taxation.

The rest of the paper is organized as follows. Section 2 introduces a life-cycle human capital model to characterize the competitive equilibrium profiles. Then, the model is numerically solved and calibrated in section 3. In section 4, we compare the competitive equilibrium profiles to constrained efficient profiles to examine how far our saving and human capital investment are away from efficient levels. In section 5, we extend our model to incorporate student debts to endogenously determine the initial human capital; and finally conclusion follows in section 6.

2 The Benchmark Model

In this benchmark model, we apply Conesa et al. (2009)’s tax structures to Huggett et al. (2011)’s endogenous life-cycle human capital model.

Demographics
The economy is populated by \( J + 1 \) overlapping generations. In each period, a continuum of new age cohort is born, whose population grows at a constant rate \( n \). Defining a fraction of age \( j \) households in the population as \( \mu_j \), the demographics satisfies \( \mu_j = \mu_{j-1}/(1+n) \) for all \( j = 1, \ldots, J \) and \( \sum_{j=0}^{J} \mu_j = 1 \).

Preference and per-period decisions
Households live until age \( J < \infty \) and retires at \( J_R \leq J \).\(^2\) Households are \textit{ex ante} heterogeneous in their initial human capital \( (h_0) \), initial wealth \( (a_0) \), and learning ability \( (\theta_0) \). Households are employed throughout their life until retirement. When a household is at work at age \( j \), she chooses how much to consume \( (c_j) \) and save \( (a_{j+1}) \). One unit of time endowment is allocated into human

\(^2\)In this life-cycle model, we assume that there is no fertility choice and family structure. Therefore, an individual represents a household and hence two terms are used interchangeably.
capital investment \((s_j \in [0, 1])\) and labor supply \((1 - s_j)\) as in Ben-Porath (1967). There is a borrowing limit of \(a \geq 0\). A household maximizes time-separable expected lifetime utility over consumption: \(E \sum_{t=0}^{J} \beta^t u(c_j)\).

**Labor income risks**

A household encounters labor income risks attributable to the idiosyncratic shocks \((\varepsilon_j)\) to human capital in each age. The human capital shocks affect the stock of human capital accumulated in the previous period. The shock is uninsurable due to the absence of full insurance contracts. The shocks are distributed as iid normally distributed with mean zero and variance of \(\sigma^2_{\varepsilon}\) across individuals, \(\varepsilon_j \sim N(0, \sigma^2_{\varepsilon})\).

**Human capital accumulation**

Human capital accumulation follows a Ben-Porath type of technology, which is mainly governed by the human capital depreciation \((\delta_h)\), a learning ability \((\theta_0)\), the current stock of human capital \((h_j)\), and time investment on the accumulation:

\[
h_{j+1} = \exp(\varepsilon_{j+1})H(h_j, s_j; \theta_0)
\]

where the accumulation of human capital before the shock arrival is defined as \(H(h_j, s_j; \theta_0) := (1 - \delta_h)h_j + \theta_0(h_js_j)^\gamma\). We assume that skills represented by the stock of human capital are general and labor market is competitive.

**Taxes and transfers**

Government receives tax income by levying a flat-rate consumption tax \((\tau_c)\), a capital income tax \((\tau_k)\), a social security tax \((\tau_{ss})\), and a potentially progressive labor income tax \((T(y))\). After retirement, households receive social security payments \((y_{ss})\).

**Household budget constraint**

Given the features of the model, a household budget constraint for an age cohort \(j\) at each point in time is:

\[
\begin{align*}
(1 + \tau_c)c_j + a_{j+1} &= \tilde{y}_j - T(\tilde{y}_j) + (1 + r(1 - \tau_k))a_j, & \text{for } j \leq J_R \\
(1 + \tau_c)c_j + a_{j+1} &= y_{ss} + (1 + r(1 - \tau_k))a_j, & \text{for } j > J_R.
\end{align*}
\]

where \(r\) is a rate of return on saving and \(\tilde{y}_j\) is taxable labor income. The taxable labor income is the pre-tax labor income minus the employer’s portion of social security tax up to a wage base limit, \(\tilde{y}\).

\[
\tilde{y}_j = \begin{cases} 
wh_j(1 - s_j) - 0.5\tau_{ss}\min(wh_j(1 - s_j), \tilde{y}) & \text{if } j \leq J_R \\
0 & \text{if } j > J_R.
\end{cases}
\]

where \(w\) is a real wage on effective labor.

**Production technology**

The production in this economy takes CRS production technology.

\[
Y = AK^\alpha L^{1-\alpha}
\]

That is, labor supply and human capital investment are rival and the cost of on-the-job training is borne by workers.
where $K$ is an aggregate capital stock, $L$ is an aggregate effective labor, and $Z$ is Hick-neutral technical change. We assume that there is no aggregate shock for simplicity and hence $A$ is constant over time.

**Initial Distribution**
We assume that all the age cohorts start with no wealth ($a_0 = 0$). A joint distribution of individual’s initial human capital and learning ability is bivariate log-normally distributed, where the log of the random variable follows 2-dimensional mean vector, $\mathbf{M} = \left( \begin{array}{c} m_h \\ m_\theta \end{array} \right)$, and 2 × 2 covariance matrix, $\Sigma = \left( \begin{array}{cc} \sigma^2_h & \lambda \theta \sigma_h \sigma_\theta \\ \lambda \theta \sigma_h \sigma_\theta & \sigma^2_\theta \end{array} \right)$.

**Aggregation**
At each point in time, households at age $j$ are characterized by an individual state $x_j = (a_j, h_j; \theta_0)$ where $a_j \in A$ is an amount of saving/borrowing, $h_j \in H$ is the stock of human capital, and $\theta_0 \in \Theta_0$ is an innate (learning) ability, where the sets of control variables are convex.

Define a state space as $X = A \times H \times \Theta_0$. Then, for age-$j$ households at a particular point in time, we define a joint probability measure $\Psi_j$ over the probability space $(X, B(X), \Psi_j)$, where $B(X)$ is the Borel $\sigma$-algebra on $X$. The distribution of age-$j$ households at each point in time is denoted as $\Psi_j(x_j)$ for $x_j \in B(X)$. The transition function of individual states in each age $j$ given the current state $x_j$ is characterized as $Q_j(x_j, B) = \Pr(x_j + 1 \in B | x_j)$ for all $B \in B(X)$. Then, the transition of household distribution of each state is characterized as $\Psi_{j+1}(B) = \int_X Q_j(x_j, B) d\Psi_j(x_j)$ for all $B \in B(X)$.

In equilibrium, goods, labor, and capital markets clear at the competitive prices at each point in time:

\[ K = \sum_{j=1}^{J} \mu_j \int_{X} a_j(x_j) d\Psi_j(x_j) \quad (2) \]
\[ L = \sum_{j=1}^{J} \mu_j \int_{X} h_j(x_j)(1 - s_j(x_j)) d\Psi_j(x_j) \quad (3) \]
\[ \sum_{j=0}^{J} \mu_j \int_{X} c_j(x_j) d\Psi_j(x_j) + (1 + n)K' + G = Y + (1 - \delta)K \quad (4) \]

where $a_j(x_j)$, $h_j(x_j)$, $s_j(x_j)$, and $c_j(x_j)$ are policy functions of saving, human capital stock, human capital investment, and consumption, respectively, in each age $j$.

**Government budget balance**
The government budget balances when

\[ G = \sum_{j=0}^{J} \int_{X} (r c_j(x_j) + r\tau h_j a_j(x_j) + T(\tilde{y}_j(x_j))) d\Psi_j(x_j), \quad (5) \]

\[ 4 \text{Note that the current stock of human capital } h \in H \text{ encompasses the current realization of human capital shock } \varepsilon \in \mathcal{E}; \text{ therefore, the current human capital stock and physical assets are enough to characterize the current state of a household.} \]
and the social security policies satisfy

$$
\sum_{j=0}^{J_R} \int_{x} \tau_{ss} \min\{wh_j(x_j)(1-s_j(x_j)), \bar{y}\} d\Psi_j(x_j) = \sum_{j=J_R+1}^{J} \int_{x} y_{ss} d\Psi_j(x_j). \quad (6)
$$

Then we define the benchmark competitive equilibrium.

*Definition 2.1.* A competitive equilibrium with tax distortions is a collection of policy functions \(\{a_j(x_j), h_j(x_j), c_j(x_j), s_j(x_j)\}_{j=0}^{J_R}\), value functions \(\{V_j(x_j)\}_{j=0}^{J_R}\), \(\{W_j(a_j)\}_{j=J_R+1}^{J}\), factor prices \(\{r, w\}\), aggregate input factors \(\{K, L\}\), and the distribution of individual states across age \(\{\Psi_j(x_j)\}_{j=1}^{J}\), given the social security payments, \(y_{ss}\), government spending \(G\), a set of taxes \(\{\tau_c, \tau_k, \tau_{ss}\}\), and the initial distribution \(\{M, \Sigma\}\), such that

1. Given factor prices, the household policy functions solve the recursive household problem, for \(j \leq J_R\),

$$
V_j(a_j, h_j; \theta_0) = \max_{c_j, a_{j+1}, s_j, h_{j+1}} u(c_j) + \beta \mathbb{E} V_{j+1}(a_{j+1}, h_{j+1}; \theta_0) \quad (7)
$$

subject to

$$
(1 + \tau_c) c_j + a_{j+1} = \bar{y}_j - T(\bar{y}_j) + (1 + r(1 - \tau_k)) a_j \quad (8)
$$

$$
h_{j+1} = \exp(\varepsilon_{j+1}) H(h_j, s_j; \theta_0) \quad (9)
$$

$$
\bar{y}_j = wh_j(1 - s_j) - 0.5 \tau_{ss} \min\{wh_j(1 - s_j), \bar{y}\} \quad (10)
$$

$$
a_j \geq -a \quad (11)
$$

2. And for \(j > J_R\)

$$
W_j(a_j) = \max_{c_j, a_{j+1}} u(c_j) + \beta W_{j+1}(a_{j+1}) \quad (12)
$$

subject to

$$
(1 + \tau_c) c_j + a_{j+1} = y_{ss} + (1 + r(1 - \tau_k)) a_j \quad (13)
$$

$$
a_j \geq -a \quad (14)
$$

3. Competitive factor prices equal marginal product of input factor in the production technology: \(w = (1 - \alpha) \frac{Y}{L}\) and \(r = \alpha \frac{Y}{K} - \delta\),

4. Markets clear as in equations (2)-(4)

5. Government budgets balance as in equations (5)-(6)

6. and distributions of households in each state and age \(\Psi_{j+1}(B) = \int_{x} Q_j(x_j, B) d\Psi_j(x_j)\) for all \(B \in \mathbb{B}(\mathcal{X})\) and \(j < J\) are consistent with household’s policy functions.
3 Quantitative Analysis

3.1 Functional Forms and Calibration

In this section, we briefly discuss the calibration of model parameters. The calibration of tax parameters mostly follows Conesa et al. (2009) and the rest of model parameters follow Huggett et al. (2011). Table 2 shows the overview of the predetermined and calibrated parameters.

Demographics We set \( J = 58 \) and \( J_R = 40 \) where we assume that individuals enter the labor market at age 23, retire at age 62, and live until age 80.\(^5\) The population growth \( n \) is 0.011 from the BLS estimate.

Production technology The average capital income share, \( \alpha \), is set to be 0.322 from NIPA data. Since we disregard aggregate effects in the model, TFP \( (A) \) is set to normalize the return to human capital \( w = (1 - \alpha)A^{\frac{1-\alpha}{1-\alpha}} \), given that the capital-output ratio \( K/Y \) to be 2.947. The benchmark elasticity of human capital production, \( \gamma \), is set to be 0.7 as estimated in the literature. We need a sensitivity check for this parameter.

Depreciation Targeting annual average rate of return to capital as 4.2\% and the capital-output ratio \( K/Y \) as 2.947, the depreciation of capital \( \delta \) is set to be 0.067 from the equilibrium condition \( r + \delta = F_k(K, N) \).

Preferences We assume isoelastic utility function: \( u(c) = \frac{c^{1-\psi}}{1-\psi} \). The coefficient of relative risk aversion \( \psi \) is unidentifiable from the model. We set \( \psi = 2.0 \) as in the literature. The discount factor \( \beta \) is determined from the rate of return on physical capital by setting annual 4.2\%.

Initial heterogeneity As we assume that there is no wealth at the begining \( (a_0 = 0) \), the parameters for the initial heterogeneity, \( (m_h, m_\theta, \sigma_h^2, \sigma_\theta^2, \lambda_{h\theta}) \), are identified by targeting data moments of log wage profile. According to Huggett et al. (2011), a heterogeneity in learning ability captures increasing variance of log incomes over the life-cycle, and hence, in the absence of heterogeneity in learning ability, they show that the variance becomes quite flat over the ages. Therefore, we target initial mean log wage to identify average initial human capital \( (m_h) \); the variance of initial wage for the variance of initial human capital \( (\sigma_h^2) \); average wage growth of first 10 years for the initial distribution of learning ability \( (m_\theta) \); and the first 10 years of wage growth.

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\(^5\)Since there are two peaks of retirement ages, 62 and 65, we choose the former. The choice is solely due to the sample size. We eliminate individuals with their annual working hours below 520 and above 5200 hours and hourly nominal wage less than the half of that year’s minimum wage. Hence, the data samples do not explicitly exclude individuals switching from full-time to part-time or to unemployed. Also retired, students, housewife, and permanently disabled individuals are dropped.
variance for the variance of initial learning ability ($\sigma^2_\theta$). Further, the wage decline in old age in the data identifies the depreciation of human capital ($\delta_h$) as the model implies that $s_j \approx 0$ in old age. Finally, the correlation between initial human capital and the learning ability, ($\lambda_{h\theta}$), can be identified from the variance of wages, $\{\text{var}(\tilde{w}_j)\}_{j=0}^{JR}$.

**Government Policies**  In the US as of 2017, the social security taxes is 12.4% total and wage base limit is $127,200. Therefore, we set the payroll tax rate $\tau_{ss}$ to be 12.4% and the maximum labor income $\bar{y}$ to be 2.5 time the average income, as in Conesa et al. (2009). The social security benefits ($y_{ss}$) are determined by the social security budget balance. The consumption tax rate ($\tau_c$) takes 6% from Mendoza et al. (1994) and and capital income tax rate ($\tau_k$) takes 40% from Domeij and Heathcote (2004). For the labor income tax, we implement a nonlinear tax function estimated by Gouveia and Strauss (1994):

$$T(y) = \tau_0(y - (y^{-\tau_1} + \tau_2)^{-1/\tau_1})$$

This functional form allows a progressivity in the tax system, where it becomes a flat rate tax as $\tau_1 \to 0$. We set the tax parameters at $\tau_0 = 0.258$ and $\tau_1 = 0.768$ and determine $\tau_2$ by the government budget balance.

**Human Capital Shocks**  The source of labor income risks in the model attributes to human capital shocks, where the shocks have long-lasting effects on the labor income. We estimate variances of human capital shocks $\sigma^2_\varepsilon$ by matching age-dependent variance and persistence of wage dynamics estimated from PSID data covering 1968-1997. We eliminate PSID samples after 1997 due to the biannually conducted survey data. We keep male heads of the household aged between 23-62. We drop self-employed and keep individuals (household) with at least 3 years of consecutive income observations. We also eliminate individuals with their annual working hours below 520 and above 5200 hours and hourly nominal wage less than the half of that year’s minimum wage. Retired, students, housewife, and permanently disabled individuals are dropped.\(^6\)

In the model, individual’s log wage is defined as

$$\tilde{w}_j^i = \tilde{w} + \tilde{h}_j^i$$

where $w$ is a rental rate of human capital and $h_j$ is the stock of human capital at age $j$.\(^7\)  Human

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\(^6\)Details about data statistics and sample selection can be found in Appendix A.

\(^7\)This specification of wage implies that human capital investment occurs “off” the job. Hence firm does not change the wage rate depending on the amount of time the worker invest on human capital. An alternative is to define $w_j = wh_j(1 - s_j)$. In this case, human capital investment happens “on” the job that firm offers a training to the workers and pays less during the training periods. Even with the alternative specification of wage, the estimation procedure that we define shortly does not change and hence either specification does not eventually affect the estimation results.
capital stocks in logs at age $j$ is determined by the human capital accumulation equation:

$$
\hat{h}^i_j = \hat{h}^i_{j-1} + q(h_{j-1}, s_{j-1}) - \delta_h + \varepsilon^i_j
$$

$$
= \hat{h}^i_0 + \sum_{k=1}^j q(h_{k-1}, s_{k-1}) - j\delta_h + \sum_{k=1}^j \varepsilon^i_k.
$$

That is, the human capital stocks at age $j$ is determined by three components: an initial human capital $h_0$, an endogenously driven human capital growth $\sum_{k=1}^j q(h_{k-1}, s_{k-1}) - j\delta_h$ and accumulated human capital shocks $\sum_{k=1}^j \varepsilon^i_k$. We assume that the human capital shocks $\varepsilon^i_j$ are normally distributed with mean zero and variance $\sigma^2_{\varepsilon,j}$. To identify the distribution of human capital shocks in the model, the data counterpart of wage process in log is defined as

$$
\tilde{w}^i_j = \alpha^i + f(X^i_{j,t}, \theta_t) + z^i_j + u^i_j
$$

$$
z^i_j = z^i_{j-1} + \eta^i_j
$$

where $\alpha^i$ is a fixed effect; $f(X^i_{j,t}, \theta_t)$ is an observable component of wage growth, which captures changes in returns to education over time; $z^i_j$ is a persistent shock to the wage; $u^i_j$ is a measurement error and transitory shocks; and $\eta^i_j$ is an innovation to the persistent shock. The innovations $u^i_j$ and $\eta^i_j$ are normally distributed with mean zero and age-dependent variances $\sigma^2_{u,j}$ and $\sigma^2_{\eta,j}$, respectively. This specification of wage process is standard in the literature, which is also consistent with the income process in the model. Therefore, we estimate the variance of human capital shocks across age ($\sigma^2_{\varepsilon,j}$) by matching the estimated variances of persistent shocks ($\sigma^2_{\eta,j}$) from the data.

### 3.2 Competitive Equilibrium Profiles

We use EGM algorithm to solve for competitive equilibrium profiles. The household problem at age $j$ in a recursive form is:

$$
V_j(x_j) = \max u((\bar{y}_j - T(\bar{y}_j) + (1 + r(1 - \tau_k))a_j - a_{j+1})/(1 + \tau_c))
$$

$$
+ \beta \mathbb{E} [V_{j+1}(a_{j+1}, \exp(\varepsilon_{j+1})H(h_j, s_j; \theta_0); \theta_0)|a_j, h_j]
$$

(16)
Then, FOCs and Envelope conditions of the household problem for \( j \leq J_R \) are:

\[
\begin{align*}
\frac{u'(c_j)}{1 + \tau_c} &= \beta \mathbb{E} V_j^{a+1} \\
V_j^a &= \frac{u'(c_j)}{1 + \tau_c} (1 + r(1 - \tau_k)) \\
&\quad + \frac{u'(c_j)}{1 + \tau_c} w h_j (1 - 0.5 \tau_{ss} y_j \leq \bar{y}) (1 - T'(\bar{y}_j)) = \beta \mathbb{E} V_{j+1}^h \exp(\varepsilon_{j+1}) \frac{\partial H}{\partial s_j} \\
\partial H \bigg|_{\partial h_j} &= \gamma \theta_0 (h_j s_j)^{-1} h_j \\
V_j^h &= \frac{u'(c_j)}{1 + \tau_c} w (1 - s_j) (1 - 0.5 \tau_{ss} y_j \leq \bar{y}) (1 - T'(\bar{y}_j)) + \beta \mathbb{E} V_{j+1}^h \exp(\varepsilon_{j+1}) \frac{\partial H}{\partial h_j} \\
\partial H \bigg|_{\partial h_j} &= (1 - \delta_h) + \gamma \theta_0 (h_j s_j)^{-1} s_j \\
(1 + \tau_c) c_j + a_{j+1} &= \bar{y}_j - T(\bar{y}_j) + (1 + r(1 - \tau_k)) a_j \\
h_{j+1} &= \exp(\varepsilon_{j+1}) H(h_j, s_j; \theta_0) \\
\end{align*}
\]

where \( V_j^a \) and \( V_j^h \) indicate partial derivatives of value function with respect to \( a_j \) and \( h_j \). Combining equations (19) and (21), the envelope condition for \( h_j \) becomes

\[
V_j^h = \frac{u'(c_j)}{1 + \tau_c} w (1 - T'(\bar{y}_j)) (1 - 0.5 \tau_{ss} y_j \leq \bar{y}) \left[ 1 + \frac{1 - \delta_h}{\gamma \theta_0 (h_j s_j)^{-1}} \right]
\]

Similarly, FOCs and envelope conditions of the household problem for \( j > J_R \) are:

\[
\begin{align*}
&u'(c_j) = (1 + \tau_c) \lambda_j \\
&\beta W_{j+1}^a = \lambda_j \\
&(1 + \tau_c) c_j + a_{j+1} = y_{ss} + (1 + r(1 - \tau_k)) a_j \\
&W_j^a = \lambda_j (1 + r(1 - \tau_k)) \\
\end{align*}
\]

We apply endogenous grid method (EGM) algorithm together with exogenous grid methods (EXGM) (see Carroll (2005), Barillas and Fernández-Villaverde (2007), Ludwig and Schöhn (2016)). Based on the FOCs and envelope conditions, EGM algorithm for a working-age household proceeds in the following way:

**Step 1.** Define grid points \((a_{j+1}, h_j, \theta_0) \in \tilde{A} \otimes \tilde{H} \otimes \tilde{\Theta}_0 \) for \( j \leq J_R \) and \( a_{j+1} \in \tilde{A} \) for \( j > J_R \), where a set \( \tilde{X} \) denotes a set of \( N_X \) discretized points of a convex set \( X \), \( \tilde{X} = \{ x_{\min} = x_0, x_1, \ldots, x_{N_x-1}, x_{N_x} = x_{\max} \} \).

**Step 2.** At \( j = J \), for any given \( a_j \), we have \( a_{j+1} = 0 \). Therefore, we obtain

\[
\begin{align*}
c_j^* &= (y_{ss} + (1 + r(1 - \tau_k)) a_j)/(1 + \tau_c) \\
W_j^a &= u'(c_j^*) (1 + r(1 - \tau_k))/(1 + \tau_c)
\end{align*}
\]
Step 3. At $J_R - 1 \leq j < J$, we find $(\hat{c}_j, \hat{a}_j)$ for each grid point $a_{j+1} \in \bar{A}$:

$$u'(\hat{c}_j) = (1 + \tau_k)\beta W_{j+1}a$$
$$\hat{a}_j = ((1 + \tau_k)\hat{c}_j + a_{j+1} - y_{ss}) / R(1 - \tau_k),$$

and using a 3-tuple $(\hat{c}_j, \hat{a}_j, a_{j+1})$, interpolate policy functions $g_j^a(a_j) = c_j^*$ and $g_j^a(a_j) = a_j^{*+1}$ for each endogenous grid point, $a_j \in \bar{A}$. Finally, if $a_j^{*+1} \geq -a$, proceed to $j - 1$ problem. Otherwise, find $a_j^{*+1}$ and $c_j^*$ for the particular states $a_j \in \bar{A} \setminus \bar{A}(a_j; a_j^{*+1} \geq -a)$ using a nonlinear solution method.\(^8\)

Step 4. At $j < J_R - 1$, for each grid point $(a_{j+1}, h_j, \theta_0) \in \tilde{X}$ with $V_{j+1}^a$ and $V_{j+1}^h$, we obtain $\hat{c}_j$ from equation (17). Taking the expectation over the human capital shocks $\varepsilon_{j+1}$ on equation (18), we can find $\hat{s}_j$. Then, we obtain $\hat{a}_j$ from equation (19). Using a 6-tuple $(\hat{c}_j, \hat{a}_j, \hat{s}_j, a_{j+1}, h_j, \theta_0)$, interpolate policy functions $g_j^a(a_j, h_j, \theta_0) = c_j^*$, $g_j^a(a_j, h_j, \theta_0) = s_j^*$ and $g_j^a(a_j, h_j, \theta_0) = a_j^{*+1}$ for each endogenous grid point, $(a_j, h_j, \theta_0) \in \tilde{X}$. Finally, if $a_j^{*+1} \geq -a$, proceed to $j - 1$ problem. Otherwise, find $c_j^*$, $s_j^*$ and $a_j^{*+1}$ for the particular states $(a_j, h_j, \theta_0) \in \tilde{X} \setminus \bar{X}(a_j, h_j, \theta_0; a_j^{*+1} \geq -a)$ using a nonlinear solution method. Given the solutions $(c_j^*, s_j^*, a_j^{*+1})$, we compute envelope conditions $V_{j+1}^a$ and $V_{j+1}^h$ from equations (20) and (21) for each $(a_j, h_j, \theta_0) \in \tilde{X}$.

### 3.3 Targeting Data Moments

We calibrate model parameters by matching simulated moments to targeting data moments. There are 10 calibrating parameters:

$$(\beta, \delta_h, m_h, m_\theta, \sigma_h^2, \sigma_\theta^2, \lambda_{h\theta}, \tau_2, y_{ss}, \sigma_\varepsilon^2).$$

Now, we choose XX data moments to be targeted:

$$(K/Y, \{E(w_j)\}_{j=0}^{J_R}, \{Cov(w_i, w_j)\}_{j=0}^{J_R-1}, \{E(c_j)\}_{j=0}^{J_R}, \{V(c_j)\}_{j=0}^{J_R})$$

\(^8\)The hybrid use of endogenous and exogenous grid methods are computationally more efficient than the use of only exogenous grid methods so long as very few states are binding with the borrowing constraint.

\(^9\)From $j = J_R - 1$ to $j = J_R$, there is a transformation of state space from three dimensions to one dimension. That is, the household problem at $j = J_R - 1$ becomes

$$V_j(a_j, h_j; \theta_0) = \max_{c_j, a_{j+1}, s_j} u(c_j) + \beta W_{j+1}(a_{j+1})$$

s.t. $$(1 + \tau_k)c_j + a_{j+1} = \bar{y}_j - T(\bar{y}_j) + (1 + r(1 - \tau_k))a_j$$
$$\bar{y}_j = wh_j(1 - s_j) - 0.5\tau_{ss} \min(wh_j(1 - s_j), \bar{y})$$
$$a_j \geq -a.$$
Also each competitive equilibrium clears all the markets and satisfies government budget balances.

3.4 Constrained Efficient Profiles

The constrained optimum can be found when a planner improves on market allocations by internalizing the effect on prices without completing the market and without changing household’s budget constraints. In other words, households depart from their self-interested optimization by internalizing the effect on market prices. To solve for the efficient allocations, a planner maximizes a utilitarian objective assigning equal weights to all households with initial assets \((a_0, h_0; \theta)\):

\[
\sum_{j=1}^{J_R} \mu_j \int_{x_j} V_j(x_j) d\Psi_j + \sum_{j=J_R+1}^{J} \mu_j \int_{x_j} W_j(a_j) d\Psi_j
\]

\[= \max \sum_{j=1}^{J_R} \mu_j \int_{x_j} u((\tilde{y}_j - T(\tilde{y}_j) + F_k(1 - \tau_k)a_j - a_{j+1})/(1 + \tau_c))d\Psi_j \]

\[+ \beta \sum_{j=1}^{J_R} \mu_j \int_{x_j} \mathbb{E}[V_{j+1}(a_{j+1}, \exp(\varepsilon_{j+1})H(h_j, s_j; \theta_0); \theta_0)|x_j] d\Psi_j \]

\[+ \sum_{j=J_R+1}^{J} \mu_j \int_{x_j} u((y_{ss} + F_k(1 - \tau_k)a_j - a_{j+1})/(1 + \tau_c))d\Psi_j \]

\[+ \beta \sum_{j=J_R+1}^{J} \mu_j \int_{x_j} W_{j+1}(a_{j+1})d\Psi_j \]

\[\Psi_{j+1} = G_j(\Psi_j) \]
The first-order conditions of planner’s problem with respect to $a_j$ are

$$
\frac{u'(c_j)}{1 + \tau_c} = \beta E V^a_{j+1}
$$

$$
V^a_j = \frac{u'(c_j)}{1 + \tau_c} F_k(1 - \tau_k) + \Delta_k
$$

$$
\Delta_k = \sum_{j=1}^J \mu_j \int \frac{u'(c_j)}{1 + \tau_c} \left[ F_{kk}(1 - \tau_k) a_j + \frac{d\tilde{y}_j}{da_j} (1 - T'(\tilde{y}_j)) \right] d\Psi_j
$$

$$
= \sum_{j=1}^J \mu_j \int \frac{u'(c_j)}{1 + \tau_c} F_{kk}(1 - \tau_k) a_j d\Psi_j
$$

$$
+ \sum_{j=1}^{J_R} \mu_j \int \frac{u'(c_j)}{1 + \tau_c} F_{h_k} h_j (1 - s_j)(1 - 0.5 \tau_{\bar{y}_j} y_j) (1 - T'(\bar{y}_j)) d\Psi_j
$$

$$
= \sum_{j=1}^J \mu_j \int \frac{u'(c_j)}{1 + \tau_c} F_{kk} K \frac{(1 - \tau_k) a_j}{K} d\Psi_j
$$

$$
- \sum_{j=1}^{J_R} \mu_j \int \frac{u'(c_j)}{1 + \tau_c} F_{kk} K \frac{h_j (1 - s_j)}{L} (1 - 0.5 \tau_{\bar{y}_j} y_j) (1 - T'(\bar{y}_j)) d\Psi_j
$$

Since individuals after retirement do not face any idiosyncratic income risks, her saving must be optimal at given prices.

The first-order conditions of planner’s problem with respect to $s_j$ and an envelope condition with respect to $h$ are

$$
\frac{u'(c_j)}{1 + \tau_c} F_h h_j (1 - T'(\bar{y}_j)) (1 - 0.5 \tau_{\bar{y}_j} y_j) + h_j \Delta_h = \beta E V^h_{j+1} \exp(\varepsilon_{j+1}) \frac{\partial H_j}{\partial s_j}
$$

$$
\Delta_h = \sum_{j=1}^{J_R} \mu_j \int \frac{u'(c_j)}{1 + \tau_c} F_{kh} K \left[ \frac{(1 - \tau_k) a_i}{K} - \frac{h_i (1 - s_i)}{L} (1 - 0.5 \tau_{\bar{y}_j} y_j) (1 - T'(\bar{y}_j)) \right] d\Psi_j
$$

$$
V^h_j = \frac{u'(c_j)}{1 + \tau_c} \left[ \frac{d\tilde{y}_j}{dh_j} (1 - T'(\tilde{y}_j)) + \frac{\partial F_k}{\partial h_j} (1 - \tau_k) a_j \right] + \beta E V^h_{j+1} \exp(\varepsilon_{j+1}) \frac{\partial H_j}{\partial h_j}
$$

$$
= \frac{u'(c_j)}{1 + \tau_c} F_h (1 - s_j)(1 - T'(\bar{y}_j)) (1 - 0.5 \tau_{\bar{y}_j} y_j) + (1 - s_j) \Delta_h + \beta E V^h_{j+1} \exp(\varepsilon_{j+1}) \frac{\partial H_j}{\partial h_j}
$$
As we combine the FOC and EC

\[
V_j^h = \frac{u'(c_j)}{1 + \tau_c} F_h(1 - T'(y_j))(1 - 0.5\tau_{ss}y_j \leq \bar{y}) \left[ 1 + \frac{1 - \delta_h}{\gamma \theta_0(h_j s_j)^{\gamma-1}} \right] + (1 - s_j) \Delta_h + h_j \Delta_h \frac{\partial H}{\partial s_j} \left( \frac{\partial H}{\partial s_j} \right)^{-1}
\]

\[
= \frac{u'(c_j)}{1 + \tau_c} F_h(1 - T'(y_j))(1 - 0.5\tau_{ss}y_j \leq \bar{y}) + \Delta_h \left[ 1 + \frac{1 - \delta_h}{\gamma \theta_0(h_j s_j)^{\gamma-1}} \right]
\]

(39)

Then, the following algorithm computes the constrained efficiency.

Step 1. Guess an initial distribution \( \hat{\Psi} = \{ \hat{\Psi}_1, \hat{\Psi}_2, \ldots, \hat{\Psi}_J \} \). A good initial guess could be a competitive equilibrium distribution.

Step 2. Compute the value functions and policy functions as in competitive equilibrium and simulate the distribution \( \tilde{\Psi} \).

Step 3. If \( \| \hat{\Psi} - \tilde{\Psi} \| < \delta_{\Psi} \), then stop. Otherwise, repeat the process with the new distribution \( \tilde{\Psi} \).

4 Quantitative Results

In this section, we first discuss the competitive equilibrium profiles of mean households. Then, we compare aggregate moments of competitive equilibrium, constrained efficiency, and full insurance. Our main results are the comparison of competitive equilibrium profiles with constrained efficient profiles. Hence, we first present median households’ constrained efficient profiles; and then we show the top and bottom 10% income earners’ constrained efficient profiles.

Competitive Equilibrium Profiles  The model parameters in the life-cycle human capital model are calibrated to match average hourly wage and nondurable consumption profiles in data. Figure 1 shows the data match of the simulated moments. Both lifetime hourly wage and consumption in the data take a hump-shape, where the profiles reach the peak at around age 45-50. Even though the simulated income (measured in real wage times the current human capital, \( w_h \)) and consumption profiles reach their peak a little later at age 50-55, the model captures the hump-shape profiles of hourly wage and consumption. Assuming zero asset holdings at age 23 in the model, there is a sharp and monotone increase in asset holdings until the retirement age. The time investment on human capital accumulation at age 23 starts with 12.4% of total hours and it gradually declines until it reaches zero at the retirement age.

The variances of each age cohort also match the data moments well. The hourly wage dispersion in the data increases from 0.217 at age 23 to 0.412 at age 62, while the model simulated variance of wage increases from 0.174 at age 23 to 0.388 at age 62. The variance of consumption remains quite flat with wide confidence intervals at level 0.16, while the simulated variance rises from 0.162
at age 23 to 0.256 at age 62. The variance of asset holdings drastically increases after age 50 and then plummets after retirement. The variance of time investment reaches its peak at age 38 and gradually declines. These benchmark profiles are considered as constrained inefficient profiles of average households in the U.S. at each age of their life, where their decisions are supposedly distorted by income taxes and incomplete insurance induces inefficient accumulation of assets and human capital. We characterize the degree of inefficiency embedded in these profiles by studying constrained efficient profiles.

**Aggregate Variables** Table 3 compares the aggregate variables of three economies with tax distortions: competitive equilibrium, constrained optimum, and full insurance. As is consistent with the results in Dávila et al. (2012), the capital in the competitive equilibrium is underaccumulated relative to the capital in the constrained optimum. More specifically, the capital-output ratio would be 35% higher, while capital-labor ratio would be 55% higher in the constrained optimum. Because of the underaccumulation of capital in the competitive equilibrium, the interest rate would be 66% lower and the wage would be 15% higher in the constrained optimum. These effects on the aggregate variables can also be inferred from the calculated capital and labor wedge in the constrained optimum. The sign of the capital wedge that corrects constrained inefficiency in saving is positive, while the sign of labor wedge that corrects inefficiency in effective labor supply is negative. Therefore, households would accumulate more assets and reduce time investment on human capital in the constrained optimum. Also note that the aggregate effects of constrained inefficiency are smaller than those in Dávila et al. (2012) or Park (2017). This is mainly due to the tax distortions present in the model. Further, we compute the allocations in full insurance economy for our comparison. When the uninsurable income risks are removed in the full insurance economy, the source of inefficiency that induces precautionary saving would vanish and hence there would be less accumulation of capital (10% lower capital-labor ratio) with 20% higher interest rate.

**Median Household’s Profiles** The primary question was: how much are we away from efficient levels of saving and investment on human capital over the course of life? We now answer this question by comparing the competitive equilibrium life-cycle profiles to constrained efficient profiles. Figure 3 plots median household’s life-cycle profiles of labor income, consumption, asset accumulation, and time investment on human capital. Competitive equilibrium profiles for median households are not too different from those for mean households in Figure 1. The life-cycle profile on asset accumulation in panel (a) shows that the constrained efficient saving would be 34–88% 10Kaplan (2012) shows that consumption profiles are sensitive to the sample selection criteria and the choice of equivalence scales. Our simulated mean and variance profiles of consumption in fact match quite closely to raw data consumption profiles in Kaplan (2012).

11Full insurance economy is computed by taking $\sigma_\varepsilon \to 0$ of the competitive equilibrium economy. Given the values of exogenous parameters in the competitive equilibrium allocations, we re-compute market clearing prices. The allocations in the full insurance economy is the first-best allocation with the tax distortions. All the taxes in the competitive equilibrium are kept unchanged because any efficient allocations could also be distorted by the taxes.
higher until age 50 than the competitive equilibrium saving. More specifically, it would be 85% higher at age 30, 65% higher at age 40, and 34% higher at age 50. Prior to the retirement, the constrained efficient asset accumulation slows down and its peak at the retirement is 5% lower than the peak of the competitive equilibrium. This implies that the median households would redistribute their wealth from young to old for an efficient allocation.

On the contrary, time investment on human capital in panel (b) drops by 62–90% before the retirement. In the constrained efficient allocations, median households at age 23 would spend 4.8% of their time for investment, while it is 12.7% in the competitive equilibrium allocations. Since human capital is a risky asset, increasing risk-free assets and reducing risky assets would provide more insurance to the households. The decline of time investment on human capital implies an increase in labor supply which would eventually yield 6% higher labor income and 6.5% higher consumption at age 23 (panels (c) and (d)). Nevertheless, the lower human capital accumulation gradually lowers the wage profile. An increase in asset accumulation would raise the wage by 15% (see Table 3), which would partially offset labor income loss of older cohorts in the constrained optimum.

For the comparison, we also plot full insurance profiles along with the other profiles. With the full insurance, human capital is no longer a risky asset and hence precautionary motive would vanish. This yields 15–25% lower asset accumulation over the life-cycle in panel (a). Surprisingly, however, the time investment on human capital in full insurance economy would also be 65% lower at age 23. This is mostly due to the income effect of labor supply. A decrease in capital accumulation lowers wage by 3.4%. Since the wage decline is for all age—which is considered to be a decline of permanent wage—the income effect on labor supply dominates the substitution effect. Overall, labor income profile and consumption profile in a full insurance economy are much smoother than the two other profiles over the life-cycle.

**Distributional Effects**  In a life-cycle model, a welfare-improving is not only through redistribution across one’s life, but also across individuals of the same age cohort. To observe distributional effects within each age cohort, we study the constrained efficient profiles of top and bottom 10% households in Figure 4.\(^\text{12}\) The top 10% income earners would save 14–267% until retirement (panel (a)), while they would invest almost no time on human capital accumulation (panel (b)). No investment on human capital is mainly driven by diminishing returns to human capital—since top 10% income earners are born with high initial human capital—and a income effect from the rise of permanent wage. This low accumulation of human capital would yield 4.4% higher labor income at age 23 with a gradual decline thereafter (panel (c)). Also, their consumption is constantly lower than the competitive equilibrium profiles as shown in panel (d).

As opposed to the top 10%, bottom 10% income earners would save 84% less at age 23 but starts saving more than competitive equilibrium saving after age 43. There is clearly an income–class turnaround at around age 43 that makes them save more just as median income

\(^{12}\) We classify the top and bottom 10% income earners by their initial human capital. Therefore, they may not stay in top or bottom 10% throughout their life.
earners. This turnaround comes from 25% of their time investment on human capital at age 23 in the constrained optimum, instead of 10% in the competitive equilibrium. Although more time investment on human capital reduces current and future labor income, an increase in wage and an accumulation of human capital offset the lifetime income loss. Also, a high human capital accumulation allow them to jump up to median or even top income classes. Thus, the bottom 10% would have higher labor income and consumption overall. Since utilitarian social welfare assigns equal weights to all individuals, and a welfare increase of consumption-poor dominates a welfare decline of consumption-rich, constrained efficient allocations are welfare-improving.

We perceive that in the constrained optimum labor income and consumption of top 10% income earners are relatively lower than their competitive equilibrium levels. On the contrary, labor income and consumption of bottom 10% would be higher than in the competitive equilibrium. Therefore, the variance of labor income and consumption in the constrained optimum economy would be substantially lower than the variance in the competitive equilibrium and would no longer be monotonically increasing over the life-cycle (panel (e) and (f)).

5 Extensions

This constrained efficiency analysis in a life-cycle model allows us to extend this framework to a variety of current issues. For example, we can provide a welfare implication of student debt burden for current young cohorts. Although student loans (either private or federal) allow young cohorts to complete higher education, the debt burden put young people in a tight financial position (aka. student debt crisis). With the student debt to be paid off, young people have to refrain from purchasing an own house and are not able to save much for precautionary purposes. In other words, young people can move up to a relatively higher income class with student debts at a cost of not being able to save for future income risks.

[TBC]

6 Conclusion

In this paper, we quantify the degree of inefficiency in saving and human capital investment of each age over the life-cycle. Since the future income risks that individuals face vary by age, the incomplete insurance induces age-varying precautionary saving. We analyze how much saving and human capital investment are driven by this age-varying inefficiency and how much the efficient saving and human capital investment ought to be. For this analysis, we adopt a notion of constrained efficiency where a planner can internalize the effect on market prices (interest rate and wage) without controlling household budgets and without providing extra insurance contracts. We examine that the constrained inefficiency over the life-cycle can differ by age and can be further improved on by the redistribution of income and wealth not only across individuals and states.
of the world but also within one’s life. The main contribution of this study is that we explicitly present an additional inefficiency and redistribution channel arising from the life-cycle property.

Our quantitative life-cycle analysis show that median households’ saving until retirement is underaccumulated by the constrained inefficiency, while time investment on human capital is over-invested by the inefficiency at all age until retirement. When we examine the constrained optimum profiles by income groups, top 10% income earners would save more and invest almost no time on human capital in the constrained optimum. On the contrary, bottom 10% income earners would dissave until age 43 and would save more thereafter, while they would invest more time on human capital accumulation in early age of their life.

The life-cycle analysis of constrained efficiency with endogenous human capital accumulation can be extended to welfare implications of current issues. For example, one issue with the recent student debt crisis in the U.S. is that young people are able to upgrade their human capital by attaining higher education at a cost of student debt burden after graduation, which inhibits their saving for future income risks. Thus, welfare implications of student debt burden associated with endogenously determined initial human capital can be explored in this framework. More applications to current issues will be our future research.

7 Bibliography

References


A Appendix

A Data

For income and employment data, we use family-individual longitudinal data set from Panel Study of Income Dynamics (PSID) covering 1980-1997. Since CEX interview survey starts from 1980, we select the identical PSID sample periods for a consistency. Also the PSID interview has been conducted biannually since 1997. The estimates of income process and our results do not change much by extending the sample periods.

For consumption data, we use Consumer Expenditure Survey (CEX) data from 1980-1997 to compute moments of consumption profile. We use a cleaned data set provided by Krueger and Perri (2006). The consumption expenditure data highly depends on the number of family members and children. Since the model does not characterize the proliferation of generations, we apply an equivalence scale used in Deaton and Paxson (1994) to the consumption data, which assigns 1 for each adult and 0.5 for each child.\textsuperscript{13}

B Sample selection criteria

For PSID, we drop SEO and Latino samples that are added to the survey since 1990. For both PSID and CEX, we keep male heads of the household aged between 23-62. We drop self-employed and kept individuals (household) with at least 4 years of consecutive income observations. We also eliminate individuals with their annual working hours below 520 and above 5200 hours and hourly nominal wage less than the half of that year’s minimum wage. Retired, students, housewife, and permanently disabled individuals are dropped. We deflate the income by Consumer Price Index (2005=100 based) to transform nominal labor earnings into real. We compute hourly wages by dividing annual labor income by annual work hours. This sampling criteria apply to both PSID and CEX data sets. In addition, we choose CEX samples with 4 consecutive observations. Data description is presented in Table 1.

\textsuperscript{13}A comparison of different equivalence scales are reported in Kaplan (2012).
Table 1: Data Description

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</tr>
<tr>
<td>57</td>
<td>22.09</td>
<td>17.70</td>
</tr>
<tr>
<td>58</td>
<td>21.50</td>
<td>17.12</td>
</tr>
<tr>
<td>59</td>
<td>21.32</td>
<td>17.08</td>
</tr>
<tr>
<td>60</td>
<td>20.00</td>
<td>17.06</td>
</tr>
<tr>
<td>61</td>
<td>20.68</td>
<td>17.69</td>
</tr>
<tr>
<td>62</td>
<td>21.94</td>
<td>17.02</td>
</tr>
</tbody>
</table>

Total 18.45 15.70 0.60 37,821 1203.44 1047.50 0.49 97,732

Note: The unit of mean and median is in dollar. Standard deviation is a logged value.
## Table 2: Calibrated and Predetermined Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>0.011</td>
<td>from BLS data</td>
</tr>
<tr>
<td>$J_R$</td>
<td>40</td>
<td>working age from 23 to 62</td>
</tr>
<tr>
<td>$J$</td>
<td>58</td>
<td>age up to 80</td>
</tr>
<tr>
<td><strong>Preferences and Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.042</td>
<td>annual net interest rate at 4.2%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9695*</td>
<td>average capital-output ratio $K/Y = 2.947$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.322</td>
<td>average capital income share</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.067</td>
<td>$\alpha \frac{Y}{K} - r$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>[0.7, 0.9]</td>
<td>[0.7 as a benchmark from Huggett et al. (2011)]</td>
</tr>
<tr>
<td>$\psi$</td>
<td>[0.5, 2.5]</td>
<td>2 as a benchmark from Huggett et al. (2011)</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.011*</td>
<td>labor income decline in old age</td>
</tr>
<tr>
<td>$a$</td>
<td>0</td>
<td>zero borrowing limit</td>
</tr>
<tr>
<td><strong>Initial conditions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_h$</td>
<td>2.652*</td>
<td>average initial wage at age 23</td>
</tr>
<tr>
<td>$m_\theta$</td>
<td>-1.453*</td>
<td>average wage growth</td>
</tr>
<tr>
<td>$\sigma^2_{h}$</td>
<td>0.415*</td>
<td>initial variance of wage profile</td>
</tr>
<tr>
<td>$\sigma^2_{\theta}$</td>
<td>0.144*</td>
<td>variance growth of wage profile</td>
</tr>
<tr>
<td>$\lambda_{h\theta}$</td>
<td>0.059*</td>
<td>the covariance of initial wage and initial wage growth</td>
</tr>
<tr>
<td><strong>Shock process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.0809*</td>
<td>variance of iid human capital shocks</td>
</tr>
<tr>
<td><strong>Tax system</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>0.124</td>
<td>Data</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>2.5×$E(y)$</td>
<td>Data</td>
</tr>
<tr>
<td>$y_{ss}$</td>
<td>5.193*</td>
<td>social security budget balance</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>0.06</td>
<td>Mendoza et al. (1994)</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.40</td>
<td>Domeij and Heathcote (2004)</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>0.258</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>0.768</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>0.004*</td>
<td>government budget balance</td>
</tr>
</tbody>
</table>

Notes: * on Value column indicates calibrated parameter values. Otherwise, parameter values are predetermined.
Figure 1: Data vs. Model: Mean Profiles

(a) Hourly Wage

(b) Consumption

(c) Assets

(d) Human Capital Investment

Notes:
Figure 2: Data vs. Model: Variance Profiles

(a) Hourly Wage

(b) Consumption

(c) Assets

(d) Human Capital Investment

Notes:
| Table 3: Aggregate Variables of Competitive Equilibrium and Constrained Optimum |
|------------------|------------------|------------------|
|                   | Competitive Equilibrium | Constrained Optimum | Full Insurance |
| Aggregate Output (Y) | 17.262           | 16.748           | 13.956          |
| Aggregate Assets (K)  | 50.316           | 65.707           | 37.784          |
| Aggregate Labor (L)   | 13.428           | 11.314           | 11.244          |
| Aggregate Consumption (C) | 12.154       | 11.019           | 10.208          |
| Capital-Output Ratio (K/Y)  | 2.915           | 3.923           | 2.707          |
| Capital-Labor Ratio (K/L)    | 3.747           | 5.807           | 3.360          |
| Interest Rate (r)       | 4.32%            | 1.48%            | 5.17%          |
| Real Wage (w)           | 0.872            | 1.0036           | 0.842          |
| Capital Wedge (Δk)      | -                | 0.000246         | -              |
| Labor Wedge (Δh)        | -                | -0.001686        | -              |
Figure 3: Constrained Efficient Profiles: Median Households

(a) Assets

(b) Human Capital Investment

(c) Labor Income

(d) Consumption

Notes:
Figure 4: Distributional Effects: Top 10% vs. Bottom 10% Income Earners

(a) Assets

(b) Human Capital Investment

(c) Labor Income

(d) Consumption

(e) Variance of Labor Income

(f) Variance of Consumption

Notes: