What Determines the Neutral Rate of Interest in an Emerging Economy?

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Abstract  

Recent evidence shows that, in the last 25 years, the natural (or neutral) rate of interest of some advanced and emerging economies has followed a common downward trend, which suggests that there might be international factors behind the determination of equilibrium real interest rates worldwide. In this paper, we estimate the natural rate of interest for Mexico, a prototype emerging economy, and measure the importance of domestic and external factors in its determination. We explore different methodologies and propose a model-averaging measure for the short and medium term, and for the long term too. Among relevant external factors, we find that foreign aggregate shocks, the instrumentation of monetary policy in advanced economies, and the gradual reduction in the global long-term real interest rate may explain recent trends in the Mexican natural rate of interest.  

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1 Introduction

A central bank may influence aggregate demand, credit supply, and inflation expectations by targeting the overnight interbank nominal interest rate (the conventional monetary-policy instrument when the economy is away from the effective lower bound). In this context, the natural rate of interest is a relevant concept for central banks because it helps them determine a neutral stance of monetary policy towards economic activity and inflation. The natural rate of interest (hereafter, natural rate or $r^\star$) can be defined as the level of the short-run real interest rate that is consistent with output near its potential, and stable inflation near its target (see Laubach and Williams, 2003). If we add a medium-term measure of inflation expectations to $r^\star$, we get the level of the policy interest rate at which monetary policy “would be neither expansionary nor contractionary if the economy were operating near its potential” (see Yellen, 2015).\footnote{In contrast, the stance of monetary policy would be contractionary (expansionary), putting downward (upward) pressure to economic activity and prices, if the short-run real interest rate would lie above (below) the natural rate.} For this reason, $r^\star$ is also known as the neutral rate of interest.\footnote{In this paper, we use indistinctively the names of natural rate and neutral rate when we refer to $r^\star$.}

Despite its importance, the natural rate is an elusive reference indicator for monetary policy because: (i) it is not observable and must be inferred from quantitative methods that are subject to a relevant statistical uncertainty; and (ii) it may vary over time due to changes in economic factors, either structural (i.e., potential growth, demographics, market structures) or transitory (i.e., macroeconomic shocks). It is important to notice that these factors are outside the control of the central bank, so $r^\star$ is not a policy choice. Changes in structural factors, which normally happen at low frequencies, govern the longer-run trend of $r^\star$ (see Calstrom and Stehulak, 2015), while macro shocks, which occur at higher frequencies (like those that hit aggregate demand), may temporarily deviate $r^\star$ from its longer-run value (see Cúrdia, Ferrero, Ng and Tambalotti, 2015).

Lately, several studies have estimated recent trends for $r^\star$ in different economies, as well as the longer-run level at which it may converge in the absence of further shocks to these economies. This renewed interest on $r^\star$ may be explained by the low levels of policy interest rates observed worldwide after the 2008 global financial crisis, the slow recovery that followed and uncertain outlook for potential growth going forward, as well as the gradual reduction in the global long-term real interest rate observed in the last three decades (see Brainard, 2015; Rachel and Smith, 2015). The evidence gathered so far for different countries is striking: in almost all studies $r^\star$ has followed a downward trend that started well before the global financial crisis (although it strengthen during the crisis, see Section 3 for further details). It is not exactly clear why this result seems to appear over and over again in different countries. However, Holston, Laubach and Williams (2017) have...
signaled that global factors could be important drivers in shaping the trends of potential growth and natural rates, at least for a group of 4 advanced economies.\(^3\)

The aim of this paper is to analyse the relative importance of domestic and foreign factors in the determination of the natural rate in an emerging market economy. We focus on Mexico to conduct our study, from the period 2001:M1 to 2016:M9. Mexico is a prototype emerging, open-market economy, with an important volume of international trade and subject to portfolio capital flows that may affect the equilibrium real interest rates of the country. The techniques used for Mexico can serve for other countries alike, where the workable sample period is shorter than for typical studies that focus on advanced economies.\(^4\) We first present an analysis of the recent evolution of \(r^*\) in Mexico for the short- and medium-run, and discuss the longer-run level at which \(r^*\) may converge in the absence of further shocks to the economy. We then explore different domestic and foreign factors that may be behind the dynamics of \(r^*\) in the short and medium run, as well as its trend in the long run.

Due to the uncertainty associated with the measurement of \(r^*\), we consider 6 different methods to achieve a robust estimation of this variable in the short and medium run. To assess the longer-term level for \(r^*\), we resort to an heuristic analysis of structural factors, and to 3 different quantitative methods that aim to filter out very persistent transitory factors. Among the methodologies used for the short and medium run, we consider from simple to standard and sophisticated techniques, including averages and filters, a simple Taylor rule estimated recursively, the affine financial model of Adrian, Crump and Moench (2013), an adaptation of the Laubach and Williams (2003) model for a small open economy (SOE), and a BVAR with time-varying intercepts (or TVI-BVAR for short) for the U.S. and Mexico, which assumes that the U.S. dynamics are block exogenous to those of Mexico. For the longer-run quantitative analysis, we computed long-term averages of the equilibrium real interest derived from Lama (2011)’s RBC model for SOEs, a revisited Taylor rule estimated recursively with controls for important transitory factors, and averages of interest-rate swaps with long-term maturities.

Remarkably, all of these methods reach similar findings, either for the short- and medium-run analysis, as well as for the long-term assessment. Our results suggest that short- and medium-run \(r^*\) has followed a downward trend at least since 2001, fell sharply to record low levels in the

\(^3\) These economies are the U.S., Canada, the U.K., and the Euro Area. Holston et al. (2017) find a substantial amount of co-movement in potential growth and the natural rates of these economies over time.

\(^4\) In the case of Mexico, the adoption of inflation targeting in 2001, among other factors, propelled a drastic change in the time-series properties of inflation (see Chiquiar, Noriega and Ramos-Francia, 2010). This fact, along with a formal change in the monetary-policy instrument during the 2000s, make difficult to combine different monetary-policy regimes using a single estimation method (Banco de México gradually changed its targeting instrument from the monetary base to the short-run nominal interest rate during that decade).
aftermath of the global financial crisis, and has recovered only recently. In turn, for the longer-term level of the natural rate, the evidence suggests that $r^*$ may converge to a somewhat lower value than the one that prevailed at the beginning of the 2000s, i.e., falling from around 3 percent in real terms to around 2.5 percent. It is noteworthy that the uncertainty surrounding these estimates is quite important, and thus punctual results must be taken with caution. As mentioned above, similar evidence has been gathered for the U.S. and other economies for both short-run $r^*$ and its longer-term convergence level.

We then gauge the relative importance of domestic and foreign factors as drivers of the natural rate in Mexico. For the short- and medium-run analysis, we argue that both domestic and foreign transitory factors pushed short-run $r^*$ downwards in the aftermath of the global financial crisis. This is the case because the sharp drop in $r^*$ after 2008 cannot be explained by the dynamics of estimated potential growth. Among domestic transitory factors, we point out to persistent conditions of economic slack that hit the Mexican economy since the financial crisis. Among the foreign transitory factors, we point out to two. The first one concerns the dynamics of short-run $r^*$ in the U.S., and the second one is the implementation of unconventional monetary policies (UMPs) by the U.S. Federal Reserve (or Fed). Concerning the first one, the evidence suggests that a medium-run measure of $r^*$ in Mexico seems to be co-integrated with medium-run measure of $r^*$ in the U.S. The latter implies that persistent transitory factors affecting $r^*$ in the U.S. may influence as well $r^*$ in Mexico through open-economy and business-cycle channels. Regarding the importance of UMPs by the Fed, the exercises with the estimated Taylor rules are quite illustrative. In a first stage, we recursively estimate a simple Taylor rule where the short-run nominal interest rate reacts to inflation and output deviations from their long-run levels, and the rule’s intercept corresponds to the estimate of $r^*$. The results of this exercise, included in the short- and medium-run analysis section, show important time variation in the estimate of $r^*$; this estimate gradually decreases as we add observations to the estimation. In contrast, when we include an indicator of the Fed’s UMP in the Taylor rule, an exercise presented in the long-term analysis section, the estimate of $r^*$ remains relatively steady after 2008. An interpretation of this result is that the abundant liquidity in financial markets prompted by the implementation of UMPs by the Fed spilled over to Mexican assets as well. In other words, capital inflows in the recovery period could have persistently shift outwards the supply of loanable funds in Mexico, thus reducing short-run $r^*$ for a sizable amount of time.

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5Potential growth is estimated in the Laubach-and-Williams type model and the TVI-VAR. Although this variable exhibits important statistical uncertainty, its trend remains relatively steady through out the sample period.

6It is important to notice that Mexico and the U.S. share an important degree of business-cycle synchronization. The latter can be partially rationalized because aggregate shocks in the U.S. seem to explain a large proportion of output fluctuations in Mexico. See Carrillo, Elizondo and Hernández-Román (2017).
To explain the slight fall in the longer-term level of $r^*$ in the 15 year window of our study, we point out that both domestic and foreign factors might be important here too. Among those factors, we document changes in demographics and the downward trend observed in the global and Mexican long-term real interest rates. However, we find it more difficult to obtain quantitative evidence that this low-frequency changes are co-integrated with global factors.

The remainder of the paper is organized as follows. Section 2 discusses the effects of structural and transitory factors on $r^*$ using a simple general-equilibrium model. Although we do not use this model in the quantitative analysis, it helps to fix ideas on the differentiation between the dynamics of short-run $r^*$ and its longer-term convergence level. In Section 3 we provide a selected revision of the literature and extract common evidence on the estimated trend of natural rates around the world. In Section 4 we present the evidence for Mexico, and in Section 5 we discuss in more detail the drivers of the trends found for $r^*$ in this country. The final section concludes.

2 A Primer on the Natural Rate and its Determinants

The natural rate can be defined as the level of the short-run real interest rate that is consistent with economic activity near its potential or efficient level, and inflation near its long-run objective. In other words, the natural rate is the level of the short-run real interest rate that maintains the output gap closed, in an stable inflation environment. Such a rate may vary over time due to changes in two types of factors: structural and transitory. The former may change at very low frequencies through time (e.g., demographics, markets structure), while the latter relates to transitory macroeconomic shocks, which happen at a higher frequency but are expected to disappear over time. In order to fix ideas, we illustrate the effect of both structural and transitory factors on $r^*$ with the help of a simple RBC model. Later, we bring these ideas to a more general environment.

Assume a representative agent chooses consumption $C_t$, savings $B_t$, labor supply $n_t$, investment $I_t$, capital purchases $K_t$, and production $y_t$ in order to maximize her expected welfare. The agent faces different constraint and exogenous processes, such as a budget constraint, production technologies for output and capital, a resource constraint, a labor-neutral deterministic trend $A_t$, and a transitory stochastic disturbance $\varepsilon_t$ on savings returns. The problem of the representative agent is thus

$$\max_{C_{t+j}, B_{t+j}, n_{t+j}, I_{t+j}, K_{t+j}, y_{t+j}} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( \log C_{t+j} - \chi \frac{n_{t+j}^{1+\omega}}{1+\omega} \right) \right\},$$

7Like other emerging economies, the Mexican long-term real interest rate gradually decreased from the beginning of the 2000s until the “taper tantrum” episode in 2013 (see Rachel and Smith, 2015, for similar evidence for a group of EMEs).
subject to

\[ C_{t+j} + I_{t+j} + \frac{B_{t+j}}{(1 + r_{t+j}) \exp (\varepsilon_{t+j})} \leq Y_{t+j} + B_{t+j-1}, \]
\[ K_{t+j} \leq (1 - \delta) K_{t+j-1} + I_t - \frac{\vartheta}{2} \left( \frac{I_{t+j}}{K_{t+j-1}} - (\delta + \gamma) \right)^2 K_{t+j-1}, \]
\[ Y_{t+j} = K_{t+j-1}^\alpha (A_{t+j} n_{t+j})^{1-\alpha} \]
\[ Y_{t+j} = C_{t+j} + I_{t+j}, \]

with \( A_t = A_{t-1} \exp (\gamma) \), and \( \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_{\varepsilon,t} \) for all \( t \). Variable \( r_t \) is the real interest rate, \( \eta_{\varepsilon,t} \) is a white noise random innovation, \( \beta < 1 \) is the subjective discount factor, \( \omega > 0 \) is the inverse of the Frisch elasticity of labor supply, \( \chi \) is a normalizing constant, \( \gamma \) is the rate of potential economic growth, \( \vartheta > 0 \) measures the intensity of capital adjustment costs and ensures that the relative price of capital varies in the equilibrium dynamics, and \( 0 < \rho_\varepsilon < 1 \) ensures that the stochastic shock is transitory. Notice that this economy contains a unit root due to deterministic growth. The equilibrium dynamics of the detrended economy can be summarized by the following two relations (where small-case letters denote growth-detrended variables, such as \( x_t = \frac{X_t}{A_t} \) for \( X_t \in \{ C_t, I_t, Y_t, K_t \} \)):

\[ 1 + r_t = E_t \left\{ \frac{c_{t+1}}{c_t} \right\} \frac{\exp (\gamma)}{\beta} \frac{1}{\exp (\varepsilon_t)}, \]
\[ 1 + r_t = E_t \left\{ \frac{1}{q_t \exp (\varepsilon_t)} \left[ \alpha \frac{y_{t+1}}{k_{t+1}} + q_{t+1} \left( 1 - \delta - \text{adj}_{t+1} \right) \right] \right\}, \]

where \( q_t = \left[ 1 - \vartheta \left( \frac{h_{t+1}}{K_{t+1}} - (\delta + \gamma) \right) \right]^{-1} \) is the relative price of capital and \( \text{adj}_{t} \) is a proportion of last-period capital that is lost due to adjustment costs when producing new capital.\(^8\) The first expression is the Euler equation for consumption and savings. It determines the supply of loanable funds in the economy. Since \( c_t = y_t - i_t \), it follows that, everything else held constat, the supply of loanable funds is upward sloping in the \((i_t, r_t)\)-space. The second expression results from combining the FOC for capital and investment, and it denotes aggregate investment demand or the economy’s demand of loanable funds. Replacing the price of capital in that equation, it can be shown that the demand of loanable funds is downward sloping in the \((i_t, r_t)\)-space.

To illustrate the effects of structural factors on \( r^* \), let us assume that the stochastic shock \( \varepsilon_t \) is equal to zero at all times. From the supply of loanable funds, it can be seen that the real natural rate in the long run, denoted by \( \bar{r}^* \), is determined by \( \gamma \) and \( \beta \), such that

\[ 1 + \bar{r}^* = \frac{\exp (\gamma)}{\beta}. \]

\(^8\) It can be shown that \( \text{adj}_{t} = \vartheta \left( \frac{h_t}{K_{t+1}} - (\delta + \gamma) \right) \left[ \frac{1}{2} \left( \frac{h_t}{K_{t+1}} - (\delta + \gamma) \right) - \frac{1}{\kappa_{t+1}} \right]. \)
A higher potential economic growth $\gamma$ or a more impatient agent (i.e., a lower $\beta$), will tend to rise the level at which $r^*$ will converge in the long run (i.e., in the absence of shocks). This is the case because in both scenarios the agent is willing to cut savings to increase consumption, so the supply of loanable funds shifts leftwards.\(^9\) Deep determinants that can change potential growth are related to the trend of total factor productivity, labor force growth, or even institutional market arrangements that promote or impede productivity. On the other hand, a change in demographics composition or a democratization of sophisticated financial instruments may change households preferences towards savings (for instance, a higher proportion of the working-age population or more affordable plans for retirement savings tend to increase the national saving rate; see Rachel and Smith, 2015). These factors that are typically mentioned in heuristic analyses that aim to predict a long-run level of the natural rate (see Carlstrom and Stehulak, 2015).

In order to understand the effect of transitory factors on $r^*$, suppose there is a positive innovation for $\varepsilon_t$, so the returns of savings suddenly increase. Let us focus on the supply of loanable funds. After a shock in $\varepsilon_t$, there are two alternatives to satisfy this equation. In the first one, the real interest rate does not change and current consumption falls, deviating from its long-run equilibrium level. In the second alternative, consumption stays put while the real interest rate drops in the same proportion as the shock increases. This latter scenario defines the real natural rate in the short-run, or $r^*_t$, since a sufficient drop in the real rate ensures that consumption does not deviate from its long-run equilibrium level. In this example, the relationship between short-run $r^*$ and its long-run level is given by\(^{10}\)

$$1 + r^*_t = \frac{E_t \left\{ \frac{c^*_{t+1}}{c^*_t} \right\} \exp(\gamma) \frac{1}{\beta} \exp(\varepsilon_t)}{\exp(\varepsilon_t)},$$

where $c^*_t$ is the detrended-steady-state value for consumption (notice that $c^*_t = c^*$ for all $t$). Figure 1 displays a graphical representation of short-run $r^*$ and its long-run convergence level through time. In the picture, we assume low frequency changes for potential growth and time preferences, so $\bar{r}^*_t$ depends on $\gamma_t$ and $\beta_t$. In turn, $\varepsilon_t$ displays temporary fluctuations at a higher frequency than $\gamma_t$ and $\beta_t$.

\(^9\)Changes in other deep parameters, like $\alpha$ or $\delta$, have an impact on the long-run level of total savings, investment and output, but leave the natural rate unchanged. The reason is that changes in these parameters shift both the supply and demand of loanable funds in a way that they eventually balance their effect on the equilibrium real interest rate.

\(^{10}\)It is noteworthy that changes in $r^*_t$ in response to demand-side shocks will generally leave real activity unchanged. However, supply-side shocks, such as those that change productivity, will tend to affect both the real interest rate and the efficient level of output. In such a case, $r^*_t$ is the level of the real interest rate consistent with efficient output, which is obtained when prices are flexible.
The distinction between the short-run dynamics and the longer-run trend of $r^*$ is not only important for academic forums, but also for policymakers when they instrument monetary policy, as the following quote from the Fed’s October 2015 FOMC Minutes show:

Estimates derived using a variety of empirical models of the U.S. economy and a range of econometric techniques indicated that short-run $r^*$ fell sharply with the onset of the 2008–09 financial crisis and recession, quite likely to negative levels. Short-run $r^*$ was estimated to have recovered only partially and to be close to zero currently [...] A number of participants indicated that they expected short-run $r^*$ to rise as the economic expansion continued, but probably only gradually. Moreover, it was noted that [...] the longer-run normal level to which the nominal federal funds rate might be expected to converge in the absence of further shocks to the economy –that is, the level that would be consistent, in the long run, with maximum employment and 2 percent inflation– would likely be lower than was the case in previous decades.

2.1 Open-Economy Determinants of the Natural Rate

The previous model is useful to illustrate in simple terms how structural and transitory factors affect $r^*$, in the short and long run. However, we need to incorporate further dimensions to understand how broader type of determinants influence the natural rate. For small open economies and emerging markets (EMEs) in general, an important dimension is the international capital market, which is governed by capital flows across borders. A relevant condition to determine the direction of these flows (in the absence of capital controls) is the real interest rate parity condition, which
reads

\[ r_t = r_t^w + E_t \Delta rer_{t+1} + \varphi_t, \]

where \( r_t^w \) represents the world real interest rate, \( \Delta rer_t \) is the percent change of the real exchange rate, and \( \varphi_t \) is the country-risk premium. A persistently low \( r_t^w \) may increase the demand for domestic assets, which in turn will shift outwards the supply of loanable funds. Consequently, as capital inflows continue, domestic \( r^* \) may exhibit a persistent downward pressure coupling the trend of \( r_t^w \). Persistent trends in capital flows influence \( r^* \) over time not only through the supply of loanable funds, but also through its effects on economic activity. Fed’s Governor Brainard described clearly these open-economy determinants for the U.S. economy in a 2015 speech:11

The broad-based reduction in interest rates in the rest of the world, by increasing demand for U.S. assets, puts upward pressure on the dollar, which in turn implies downward pressure on the U.S. neutral rate. One way to think about the spillover from abroad is how much adjustment in the federal funds rate might be necessary to insulate domestic employment from an appreciation in the dollar that is expected to persist.

2.2 Challenges in the Use of the Natural Rate as a Guide for Monetary Policy

So far, we have assumed that prices are flexible and so the economy can always attain an efficient level of output. However, the presence of nominal rigidities are an obstacle for the short-run real interest rate \( r_t = i_t - E_t \pi_{t+1} \) to reach its natural level \( r_t^* \), where \( i_t \) is the short-run nominal interest rate and \( E_t \pi_{t+1} \) is expected inflation. An important lesson derived from the literature on optimal monetary policy is that, when nominal rigidities are present, the central bank may influence (or directly set) \( R_t \) to emulate changes in \( r_t^* \) in order to minimize inefficient fluctuations in output (see Woodford, 2003). This is the case because prices adjust slowly due to nominal rigidities, and so the central bank may improve welfare by influencing \( r_t \) in the right direction using its instruments.

However, an important challenge to use \( r_t^* \) as a guide for the implementation of monetary policy is that it is not observable and, as argued above, it is also time-varying. \( r_t^* \) is not observable because it corresponds to the level of the real interest that would prevail if prices and wages were flexible, an unrealistic environment.12 The fact that \( r_t^* \) is affected by changes in structural and transitory factors make more challenging the inference of \( r^* \) in different periods of time. Indeed,

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12 In this context, price flexibility should be understood as a characteristic of relative prices to always reflect efficient resource allocation signals, and adjust accordingly when shocks occur. As many micro-data studies show, like that of Nakamura and Steinsson (2010), prices vary at different frequencies, depending on sector, industry, competition, and many other considerations that may differ from only efficient allocation signals.
there exists an important amount of uncertainty around econometric measures of $r^*_t$, and one must be cautious in interpreting punctual estimates of this variable as its undeniable actual level for the current period. A risk-averse approach would be to use different types of methods to infer a robust trend of $r^*_t$ through time. We follow this strategy for the estimates of $r^*$ for Mexico.

In the next section, we discuss some of the evidence on perceived changes in $r^*$ for advanced and emergent economies in late years. Afterwards, we present the estimations for Mexico.

3 Recent Evidence on Natural Rates in AEs and EMEs

This section briefly surveys some of the recent evidence related with estimated trends of $r^*$ in advanced and emerging market economies. The main message of this brief survey is that almost all the studies capture a downward trend in $r^*$ that started around the 90s, and that sharpened after the 2008 global financial crisis.

3.1 Advanced Economies (AEs)

Recently, the process of monetary policy normalization in the U.S. has fueled a renewed interest on the level at which the federal funds rate is expected to converge in the long term. More broadly, the discussion focuses on the trajectory followed by $r^*$ in the short and medium term in recent years, and in the longer-term level at which $r^*$ might converge in the absence of further shocks in the economy. For the U.S., Yellen (2015) presents a set of estimates of short-run $r^*$ obtained from New-Keynesian DSGE models, and shows that this variable fell sharply towards negative levels in the onset of the 2008 global financial crisis and reached zero by the end of 2015. These models interpret the reduction in short-run $r^*$ as a response to persistent macro shocks to aggregate demand, such as tighter financing conditions and lesser access to credit, de-leveraging by households, lower global growth, and greater uncertainty. DSGE models have advantages and shortcomings when measuring $r^*$. Among the first, these models are able to pin down changes in short-run $r^*$ to particular transitory shocks that affect the economy. An obvious limitation is that these interpretations are model-dependent. A more important shortcoming is that current and affordable solution methods for medium- and large-scale DSGE models, based on perturbation

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14 The estimates of these DSGE models assume the existence of nominal rigidities and other frictions to capture transitory macroeconomic shocks. To estimate short-run $r^*$, the models compute the real interest rate that would prevail if prices and wages were flexible. Therefore, the estimated short-run $r^*$ in this type of models is a counterfactual measure, not observable, and highly volatile since it is subject to a wide set of transitory shocks.
methods, are ill suited for capturing low-frequency movements in structural factors that affect the long-term level of $r^*$. More flexible methodologies, like state-space models with a time-varying structure designed to capture low-frequency changes in $r^*$, provide medium-term estimates of $r^*$.\footnote{These models assume that high-frequency transitory shocks have dissipated, and condition $r^*$ on structural factors perceived in the measurement period.} For the case of the U.S., Laubach and Williams (2016) and Johanssen and Mertens (2016) estimate a clear downward trend for $r^*$ that started at least since the 80s, but it deepened since the financial crisis. The drawback of this type of state-space models models is that, because of their flexibility, they cannot distinguish precisely which part of the changes in the medium-term trend of $r^*$ are due to changes in structural factors and which other to very persistent transitory shocks, like those related to the financial crisis (this fact will become clearer when we formally introduce a SOE version of the Laubach and Williams model and the TVI-BVAR in the analysis for Mexico). We thus consider the evidence provided by these models as valid trends for the medium term, but not so accurate measures for long-run convergence levels of $r^*$.

The evidence of a downward trend of $r^*$ is not exclusive to the U.S. Holston et al. (2017), using a version of the Laubach and Williams model, find evidence that $r^*$ and potential growth in Canada, the Euro Area and the U.K. have followed a downward trend for several decades. Additionally, they find that these estimates and those for the U.S. have a considerable co-movement over time. Thus, the results in in this paper suggest that there exists an important role for global factors that may explain the trends of $r^*$ and potential growth in these economies. Similarly, Bouis, Rawdanowicz, Renne, Watanabe and Christensen (2013) find that for seven OCDE economies (the U.S., Japan, the Euro Area, the U.K., Canada, Sweden and Switzerland) $r^*$ has generally fallen since 1980.\footnote{Sweden and Switzerland are exceptions, as their estimates of $r^*$ have remained stable, and relatively high, since the financial crisis, see Bouis et al. (2013). Other results for the case of Sweden can be seen in Armelius, Bonomolo, Lindskog, Radahl, Strid and Walentin (2015)} They argue that the fall of $r^*$ is likely the result of a lower potential growth. In addition, they mention that, according to OECD projections, $r^*$ may converge to a lower level than before the global financial crisis. For Japan, Fujiwara, Iwasaki, Muto, Nishizaki and Sudo (2016) show that $r^*$ has followed a downward trend since the nineties and relate this trend to a slowdown in potential growth.\footnote{More evidence of $r^*$ for US, Euro Area and Japan, and some inflation targeting countries (Australia, Canada, New Zealand, Norway, UK, Sweden and Chile) is shown in Schmidt-Hebbel and Walsh (2009). Although they do not find clear evidence of a downward trend in $r^*$ in all cases, but they show that neutral rates of this economies, with the exception of Japan, are highly correlated.} Similarly, the European Central Bank (2004) find that $r^*$ in the Euro Area has decreased since the middle nineteens, and argue that this trajectory may reflect the slowdown in productivity and population growth in the region, as well as the reduction of...
inflationary risk premia. Likewise, Bernhardsen and Gerdrup (2007) find that $r^*$ has fallen since at least 1990 in Norway, and explain that one of the reasons is partly a lower inflationary risk premia, since inflation and its expectations stabilized towards low levels. For New Zealand, Basdevant, Björksten and Karagedikli (2004), using different models, find evidence that suggest a downward trend in $r^*$ since 1992, while Björksten and Karagedikli (2003) conclude that the reduction in $r^*$ can be partly explained by a worldwide decline in natural rates, and by local factors. Finally, Richardson and Williams (2015) also present evidence that $r^*$ has fallen in New Zealand since the global financial crisis. Other group of studies, based on either structural or reduced-form models find similar evidence.\footnote{For example, Goldby, Laureys and Reinold (2015), using a DSGE model, show that short-run $r^*$ in the U.K. fell sharply after the financial crisis towards negative levels, and start recovering after 2012. According to this model, the greater part of this fall in $r^*$ was due to a domestic risk-premium shock, although global shocks also pulled down $r^*$, which may capture the effects of a global slowdown. Other studies for different AEs are Cuaresma, Gnan and Ritzberger-Gruenwald (2004), Mesonnier and Renne (2007), Manrique and Marqués (2004), Garnier and Wilhelmsen (2009), Benati and Vitale (2007), Kirker (2008), Berger and Kempa (2014), and Fries, Mouabbi, Msonnier and Renne (2016), among others.}

### 3.2 Emerging Market Economies (EMEs)

The evidence for EMEs is not very different from this for AEs. Trends of natural rates in EMEs have also declined. In particular, Magud and Tsounta (2012) using different methodologies document some stylized facts for $r^*$ in ten Latin American countries:\footnote{The countries included are Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Guatemala, Mexico, Paraguay, Peru and Uruguay. The methodologies used by the authors include: HP filter, implicit common stochastic trend using short- and long-term interest rates, dynamic Taylor rule, expected-inflation augmented Taylor rule, Laubach and Williams model, consumption-smoothing models and uncovered interest parity (UIP) condition. Their sample was from 2000 to 2012.} (i) $r^*$ tended to be lower in countries with stronger fundamentals; (ii) wider ranges in their $r^*$ estimates were associated with weaker monetary frameworks and higher inflation risk premiums, although the dispersion can be caused by little information data; and (iii) $r^*$ showed a downward trend in the last decade, with exception of Brazil and Uruguay. The authors argue that the behaviour of this trend is possibly due to stronger economic fundamentals in the region, as well as easing global financial conditions that increased available savings in the region.

Similar evidence for EMEs about a downward trend of $r^*$ was found by Perrelli and Roache (2014).\footnote{The countries considered in this analysis are: Brazil, Chile, China, Colombia, Czech Rep., Egypt, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey and Uruguay. The authors use statistical filters to document the decline of $r^*$. The sample used was from 2002 to 2011.} In particular, within the period 2010-2013, the economies that more decreased their $r^*$ were Brazil and Turkey (fell down around 500 bases points and 800 bases points, respectively). Additionally, they also found evidence that these trends were affected by domestic and foreign fac-
tors. With regard to the first are included saving and investment demand. The second one includes the world natural rate level, which represents common factors such as excess global savings and unconventional monetary policies in U.S. and Europe.\textsuperscript{21}

In individual analysis Fuentes and Gredig (2008), González, Ocampo, Pérez and Rodríguez (2012) and Perrelli and Roache (2014) document the particular cases of \( r^* \) in Chile, Colombia and Brazil, respectively. In each case, authors used a battery of models to determine the level of \( r^* \).\textsuperscript{22}

In the case of Chile, from 1980 to 2007 all models gave similar results, the \( r^* \) dynamics followed a downward trend. The \( r^* \) in Colombia suggested a significant variability. In particular, from 1980 to 1994 the trend of \( r^* \) declined significatively, after that it reverted its trend, reaching high levels in 1999. Finally, in the period from 2000 to 2009 this trend fell down again. In Brazilian’s case, from 2006 to may of 2013 \( r^* \) declined, after that period it reverted its trajectory. Additionally, the decline in long-term \( r^* \) in Brazil is explained by domestic and external factors. Within the first one includes: lower public debt, reduced sovereign risk, and increased supply of saving. The external factor was characterized by lower global interest rate, which contributed with 200 bases point to the decline of \( r^* \) in the period from 2010 to 2013.

\section{Evidence for Mexico}

To cope with the uncertainty associated to the estimation of the natural rate and obtain robust results for Mexico, we consider different methodologies to study the evolution of \( r^*_t \) in the short, medium and long run. It is noteworthy that the degree of uncertainty surrounding the estimates of \( r^*_t \) depends on how well can these methodologies differentiate between transitory factors and changes in structural factors. In Section 4.1 we present methods that are better suited for the short and medium run, while in Section 4.2 we provide some evidence indicating a likely value for long-run \( r^*_t \), given current trends in structural factors.

For all of the estimates below, we use the ex-ante short-term real interest rate, measured as the overnight interbank nominal interest rate less the one-year ahead expectations of headline inflation, which we extract from a survey of private sector professional forecasters, gathered by Banco de

\textsuperscript{21}Further, with principal components analysis, they found that two common factors explain about 45\% of the common variations in real rates of the analyzed countries. The first one represents the common trend and the second one is the common cycle.

\textsuperscript{22}The models used in these papers can be classified in three categories: (i) pure economic theory (traditional consumption model and uncovered parity interest rate condition); (ii) \( r^* \) implicit in financial instruments (forward rates, state-space models with common stochastic rate in short-run and long-run interest rates, yield curve models); and (iii) estimated \( r^* \) from statistical models using macroeconomic data (filters, the Laubach and Williams model, and other semi-structural models).
México. For multivariate models, we use other macro variables and information from financial markets. We discuss these variables in detail when we present each method. The period of study spans, in general, from January 2000 to September 2016.

4.1 The Neutral Rate in the Short and Medium Run

In this section, we present from the simpler univariate methods to more complex multivariate systems that aim to extract a trend from the observed short-term real interest rate. We present evidence from averages and univariate filters, a standard Taylor rule estimated recursively, an affine term structure model, an adaptation of the Laubach and Williams (2003)’s model for a small open economy, and a BVAR for the U.S. and Mexico with time-varying intercepts. The point estimates from these models are shown in Figure 2. Despite their differences, all methodologies suggest an important reduction of $r^\star$ in the short and medium run in the aftermath of the 2008 global financial crisis, and a moderate increase since 2014.

![Figure 2: Summary of Results for Short- and Medium-Run $r^\star$](image)

Source: Own estimates with data from INEGI, Banco de México, and the FRED Database from the St. Louis Fed.

The advantage of the first methods listed in Figure 2 is their simplicity, while for the latter ones it is their richer economic environment. A common shortcoming in all of these methods is that

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23See the Encuesta sobre las expectativas de los especialistas en economía del sector privado by Banco de México (http://www.banxico.org.mx/informacion-para-la-prensa/comunicados/resultados-de-encuestas/expectativas-de-los-especialistas/index.html).
they have problems to disentangle changes in structural factors (like those affecting demographics or trend productivity) from persistent shocks (like those that affected aggregate demand in the aftermath of the global financial crisis). For this reason, we consider all the results presented in this section as useful guides to assess the path of $r^*$ in the short and medium run. We leave for the next section a longer-run analysis of $r^*$, where we try to control for very persistent shocks that affected temporarily its trend. Next, we describe in detail the methods we displayed in Figure 2.

### 4.1 Averages and Trends

A simple indicator of $r^*_t$ in the medium run is the average of the ex-ante real interest rate during a full business cycle, which we define here as a completed downturn and upturn of output with respect to its longer-term level. In our sample we have one full business cycle from 2001 to 2008, while we assume that the second one is still ongoing, starting from 2009 up to present. These cycles are shown in the left picture of Figure 3, denoted with a different color in the shadowed area are between the output gap series and the $x$-axis. The picture also depicts the ex-ante real interest rate at a quarterly frequency in our sample. According to this methodology, $r^*_t$ decreased substantially in the current economic cycle, falling from 3.33% to 0.45%.

As an alternative, we use univariate statistical filters to disentangle the trend and cycle of the ex-ante real rate. The trend can be thus interpreted as an approximate measure of $r^*_t$ in the medium term. To this end, we use the Christiano-Fitzgerald filter and the Hodrick-Prescott filter with an end-point correction. The right picture of Figure 3 suggests a downward trend in the ex-ante real rate for two periods in the sample. The first one goes from the beginning of the 2000s up to 2004, and the second one goes from 2008 up to 2014, whereby there seems to be a reversion.

Albeit their simplicity, these measures tell us nothing about the drivers behind the suggested fall in $r^*$.

### 4.1.2 Standard Taylor Rule

Next, we assess the behavior of $r^*$ by approximating the reaction function of the central bank in response to deviations of inflation and output from desired levels. To this end, we propose the following Taylor rule with interest-rate smoothing:

$$i_t = (1 - \rho)[\bar{r}^* + \bar{\pi} + \delta(\pi_t - \bar{\pi}) + \theta \hat{y}_t] + \rho i_{t-1} + \varepsilon_t,$$

(1)

---

24 We could assume as well that the second cycle ended in 2014, and the qualitative results from this approach would not change.

25 For the Christiano and Fitzgerald (2003)’s filter, we use an asymmetric band-pass filter, isolating the cyclical components between 2 and 96 months (which is the usual belief in the length of business cycle). In the case of Hodrick-Prescott filter, we use a smoothing parameter of 14,400 and the end-point correction as suggested by St-Amant and van Norden (1997).
where $i_t$ is the overnight interbank nominal interest rate, $\pi_t$ is inflation, $\hat{y}_t$ is the output gap, and $\varepsilon_t$ captures any change in $i_t$ not explained by the rule. Additionally, as it is common in the literature, the rule includes a lag of the nominal interest rate to capture the fact that, in general, a central bank adjusts its policy rate gradually. Finally, the neutral rate $\bar{r}^*$ denotes the level of the real interest rate that should prevail when inflation equals the inflation target $\bar{\pi}$ and the output gap equals 0. Notice that we have added a bar to this definition of the neutral rate, since in principle $\bar{r}^*$ denotes a long-run equilibrium level. However, as we have noted above, the evidence suggests that the equilibrium real interest rate has been changing over time.

In order to capture changes in $\bar{r}^*$ over our sample period, we estimate equation (1) recursively at a monthly basis. In particular, we first estimate the rule from January 2000 to December 2002, and then we re-estimate it for every month in our sample by adding this month to the previous estimation, while leaving the initial period fixed.\textsuperscript{26} In the estimation, we use headline annual inflation, and approximate the output gap using Mexico’s Global Indicator of Economic Activity (or IGAE, by its Spanish acronym), published monthly by INEGI.\textsuperscript{27} The results from this exercise are shown in Figure 4, and are similar to the evidence from averages and filters, namely, that the neutral rate seems to have fallen from 2008 to 2014, and has recovered moderately ever since.

\begin{itemize}
  \item \textsuperscript{26}We estimate the rule through OLS.
  \item \textsuperscript{27}To compute a measure of economic slack from IGAE, we used its percent deviation from trend, which we estimated using the Hodrick-Prescott filter with end-point correction.
\end{itemize}
A question that remains unsolved from this exercise is why does $\bar{r}^*$ seem to change over our sample period, especially when it is supposed to be a long-term indicator of the neutral rate? The answer to this question may be given by omitted factors in equation (1), such as those that capture the extraordinary monetary policy accommodation that was observed worldwide, and in particular in the U.S., in the aftermath of the global financial crisis. We come back to this point in subsection 4.2.1, where we revisit this estimation of the Taylor rule for Mexico.

![Figure 4: Taylor Rule Intercept and Short-Run Real Interest Rate](image)

Note: The range corresponds to two standard deviations of the estimates. The actual rate is calculated as the difference between the nominal rate and the inflation target. Source: Own estimates made with data from INEGI and Banco de México.

### 4.1.3 Affine Term Structure Model

In this exercise, we estimate the affine term structure model of Adrian et al. (2013) with Mexican data. The model exploits no-arbitrage conditions in financial markets to compute an expected path of a $n$th-months-maturity bond’s nominal interest rate for an horizon of $k$ periods. In particular, the model is specified as an state-space system of the form:

\[
X_t = \mu + \phi X_{t-1} + \vartheta_{t+1} \tag{2}
\]

\[
i^{(n)}_t = A_n + B'_n X_t \tag{3}
\]

with $E(\vartheta_{t+1} \vartheta'_{t+1}) = \Sigma$, and where $X_t$ is the vector of state variables, $i^{(n)}_t$ is the nominal interest rate of a bond with maturity of $n$ months, $\vartheta_t$ are white-noise state innovations, $\phi$ and $B'_n$ are
coefficient matrices, and \( \mu \) and \( A_n' \) are coefficient vectors. We assume that vector \( X_t \) contains 5 factors related to the yield curve: (1) its level, (2) its slope, (3) its curvature, (4) the implied excess returns, and (5) the term premia. These state variables explain the dynamics of the term structure of interest rates. We estimate the aforementioned factors through principal components using data for government bonds yields at different maturities. The model parameters are then estimated with OLS using auxiliary regressions.\(^{28}\) Our estimate of \( r^* \) is computed through the expected average path of the short-run nominal interest rate (for 1-month bonds) for an horizon of 10 years or 120 months, minus the expectations of inflation for a similar horizon, such that

\[
r^*_t = \frac{1}{120} \sum_{k=1}^{120} E_t \left\{ i_{t+k}^{(1)} \right\} - \bar{\pi}_t^e,
\]

where \( \bar{\pi}_t^e \) denotes the long-term expectations of headline inflation, which are estimated using the affine model of Aguilar-Argaez, Elizondo and Roldán (2016). We use government bonds with zero-coupon yields at maturities 1,2,3,...,120 months. The estimate of \( r^*_t \) using this methodology thus reflect the average level of the short-run real interest over 120 months ahead.

Figure 5: Average Expectation of the Short-Run Real Interest Rate Implicit in Financial Instruments

Note: The implicit expectation estimate is made for the short-run nominal interest rate. To get the short-run real interest rate we consider the inflation expectations from Aguilar-Argaez et al. (2016). Source: Own calculations with data of PiP and Valmer.

\(^{28}\)More details of this methodology can be found in Adrian et al. (2013). In particular, the coefficients \( A_n \) and \( B_n \) are estimated recursively and depend on risk parameters. When these parameters equal zero, we obtain the risk-free coefficients \( A_n^{RF} \) and \( B_n^{RF} \). Using these coefficients, we can compute the average expectation at time \( t \) of short-term interest rates over the next \( k \) periods, since

\[
E_t[i_{t+1,t+k}^{(1)}] = -(1/n)(A_n^{RF} + B_n^{RF} X_t).
\]
Figure 5 displays the term-structure estimate of \( r^* \), which shows two behaviors. First, similar to the filters and the standard Taylor rule, the results of the affine model suggest that investors’ expectations of the short-run real interest rate declined sharply at the onset of the global financial crisis. And second, investors anticipated that this rate would remain low for a long period of time. Finally, again the trend of investors’ expectations shows a reversal in the middle of 2014.

4.1.4 Laubach and Williams Model Adapted for a SOE

Laubach and Williams (2003)’s model is perhaps the most popular framework to estimate \( r^* \) in the literature. In this exercise, we adapt this model for a small open economy. As the original setting, we include reduced-form representations of aggregate supply and demand (i.e., an IS curve and a Phillips curve), and a set of transition equations that drive the dynamics of the unobserved variables of the model, like the natural rate, or potential output. The model is thus represented by

**MEASUREMENT EQUATIONS**

\[
y_t - y^*_t = a_y (y_{t-1} - y^*_{t-1}) + a_r (r_{t-1} - r^*_{t-1}) + a_y \hat{y}_{US} + \sum_{\ell=1}^{4} a_q,\ell \hat{q}_{t} + \epsilon_{y,t} \quad (5)
\]

\[
\pi_t - \bar{\pi} = b_\pi (\pi_{t-1} - \bar{\pi}) + b_y \sum_{\ell=1}^{2} \left( y_{t-\ell} - y^*_{t-\ell} \right) + b_s (\Delta s_{t-1} + \pi_{US}^t) + \epsilon_{\pi,t} \quad (6)
\]

**TRANSITION EQUATIONS**

\[
r^*_t = \gamma_t + z_t \quad (7)
\]

\[
z_t = z_{t-1} + \epsilon_{z,t} \quad (8)
\]

\[
y^*_t = y^*_{t-1} + \gamma_{t-1} + \epsilon_{y^*,t} \quad (9)
\]

\[
\gamma_t = \gamma_{t-1} + \epsilon_{\gamma,t} \quad (10)
\]

where \( y_t \) is output, \( y^*_t \) is potential output, \( r_t \) is the short-run real interest rate, \( \hat{y}_{US}^t \) is a measure of the U.S. output gap, \( \hat{q}_t \) is the percent deviation of the real exchange rate from trend, \( \pi_t \) is inflation, \( \bar{\pi} \) is the central bank’s inflation target, \( \Delta s_t \) is the nominal exchange rate depreciation, \( \pi_{US}^t \) is the U.S. inflation rate. The IS and Phillips curves, expressed in equations 5 and 6, are affected by transitory shocks \( \epsilon_{y,t} \) and \( \epsilon_{\pi,t} \), which we assume are white noise. The neutral rate \( r^*_t \) is determined in equation 7, and is a function of potential output growth \( \gamma_t \) and a time-varying latent component \( z_t \), which captures model-omitted factors. Here, it is worth noticing that while \( \gamma_t \) is clearly a structural factor, \( z_t \) can contain both structural and transitory factors. Similar to Laubach and Williams, we assume that \( z_t \), and \( \gamma_t \) are random walks, while \( y^*_t \) is a random walk with drift. Similarly, we assume that the state innovations \( \epsilon_{z,t} \), \( \epsilon_{\gamma,t} \), and \( \epsilon_{y^*,t} \) are white noise.
The main difference between system 5-10 and Laubach and Williams’s original framework is that we incorporate small-open-economy considerations into the IS and Phillips curves. As such, the former includes a measure of U.S. output and the real exchange rate to take into account the dynamics of the trade balance on domestic demand. In the same vein, the Phillips curve takes on board that home inflation may be affected by the relative purchasing power parity condition, and so it includes the nominal depreciation and the inflation of U.S. prices.

We estimate the model’s parameters through maximum likelihood, using the Kalman filter with monthly data from 2000M1-2016M6.\(^{29}\) As a measure of Mexico’s output, we use the indicator of Elizondo (2012), who approximates monthly GDP from the IGAE index (see Section 4.1.2) using a mixed-frequencies Kalman filter method. For the inflation measures and the nominal depreciation, we use annual rates for headline CPI inflation for both Mexico and the U.S., and the peso-dollar nominal exchange rate. The short-run real interest rate is the same as in preceding sections. In turn, we use the U.S. industrial production index as a proxy for U.S. output, and the real exchange rate index between U.S. and Mexico, which is available from Banco de México’s website. We compute gaps from these variables using a Hodrick-Prescott filter with a \(\lambda\)-parameter equal to 14,400 (because of their monthly frequency).

The left panel of Figure 6 shows the \(r^*\) estimate from this methodology, along with the short-run real interest rate. Consistent with preceding exercises, the results from the Laubach and Williams model suggest that the neutral rate started to decline in the year 2006, and has not returned to pre-crisis levels ever since. The right panel of the Figure redraws the estimated \(r^*\) (blue area) together with a counterfactual of this estimate assuming that the latent factor \(z_t\) is equal to zero during all periods (red line). In other words, the picture shows the proportion of the neutral rate that is explained by estimated potential growth alone. As it can be seen, the two measures diverge increasingly after the global financial crisis, which suggests that model-omitted factors contained in \(z_t\), and no potential growth, can explain the sharp fall in the neutral rate after the crisis. We will come back to this point later on.

\(^{29}\)As it is well known, this estimation procedure suffers from the so-called “pile-up problem”, which bias the estimation of the variances of \(\epsilon_{\gamma,t}\) and \(\epsilon_{z,t}\) towards zero, i.e. \(\sigma_{\gamma}\) and \(\sigma_{z}\), respectively. For this reason, it is necessary to restrict and calibrate these parameters, so that \(\lambda_{\gamma} = \frac{\sigma_{\gamma}}{\sigma_{y\star}}\) and \(\lambda_{z} = \frac{\sigma_{z}}{\sigma_{y}}\). An estimation strategy is to set a grid with different combinations of \(\lambda_{\gamma}\) and \(\lambda_{z}\), and then pick the couple using a comparison criterion. One of these criteria is to select the couple that achieves the best dynamic adjustment for output and inflation, but the results from this exercise drew practically constant values for \(\gamma_t\), \(z_t\) and \(r^*_t\). These implausible results could be explain in part by the “pile-up” problem mentioned above. Therefore, we decided to choose the couple \((\lambda_{\gamma},\lambda_{z})\) that maximize a “concordance index,” which measures the percentage of periods that the model’s estimated output gap is of the same sign than the official output gap estimates by Banco de México (more details on this latter estimate can be found in Banco de México’s 2009Q2 Quarterly Report, pp. 69). Further information about the ML estimation procedure of these parameters can be found in Laubach and Williams (2003), Mesonnier and Renne (2007), Soto, Enciso, Arango, Corredor and Álvarez.
Figure 6: Short-Run Real Rate, Neutral Rate and Its Determinants in Laubach and Williams Model

Note: The confidence intervals in the left panel are of 90 percent significance. Source: Own estimates made with data from Banco de México.

4.1.5 BVAR with Time-Varying Intercepts and Block Exogeneity (TVI-BVAR)

For the final exercise of this section, we consider a joint vector autoregression model for Mexico and the U.S. The model takes into account the interactions between the two countries, while at the same time assumes that the U.S. aggregate dynamics are block exogenous to those of Mexico, which means that U.S. variables influence the Mexican ones, but not vice versa. This is indeed a small-open-economy assumption. The main difference with a typical VAR model is that we let its intercepts to vary with time, which allow us to capture low frequency changes in the trend of \( r_t^* \).

Let \( X_t = \begin{bmatrix} X_{f,t} \ X_{h,t} \end{bmatrix}' \) be the joint vector of foreign (U.S.) variables \( X_{f,t} \), and home (Mexican) variables \( X_{h,t} \), including the short-run real interest rate \( r_t \). The VAR model reads

\[
X_t = C_t + A_1 X_{t-1} + A_2 X_{t-2} + \xi_t,
\]

\[
C_t = C_{t-1} + \upsilon_t,
\]

where \( C_t \) is a vector of time-varying intercepts that follow random walk processes, \( A_\ell \) are conformable matrices of parameters with block exogeneity,\(^{30}\) and \( \xi_t \) and \( \upsilon_t \) are white-noise innova-


\(^{30}\)In particular, we assume that \( A_\ell = \begin{bmatrix} A_{f,\ell} & 0 \\ A_{h,\ell} & A_{h,\ell} \end{bmatrix} \), so that \( X_{f,t-k} \) explains part of the fluctuations of \( X_{h,t} \), but \( X_{h,t-k} \) do not explain those of \( X_{f,t} \).
tions. Notice that in the absence of shocks, the variables in the system converge to

\[ \hat{X}_t = (I - A_1 - A_2)^{-1} \times C_t. \]  (13)

We take the row of vector \( \hat{X}_t \) that corresponds to the short-run real interest rate as the BVAR estimate of the real neutral rate \( r_t^\star \). We consider the latter as a medium-run estimate, as vector \( C_t \) may be affected by both structural factors and very persistent transitory factors. The main difference between the BVAR estimate and that of the Laubach-and-Williams model (presented in the previous section) is that the former incorporates a wider set of foreign and home variables, within a more flexible structure, so the BVAR estimate carries on more information about the joint dynamics of foreign and home variables.

We estimate the above system through Bayesian techniques, where we use the Kalman filter to infer the path of the state vector \( C_t \). The vector of foreign variables \( X_{f,t} \) contains the U.S. PCE inflation rate, a monthly approximation of U.S. GDP growth, a shadow measure for the fed funds rate, the 10-year yield curve term premia as estimated by Kim and Wright (2005), and the VIX index to control for financial markets volatility. In turn, the vector of home variables \( X_{h,t} \) includes core inflation, a monthly approximation of GDP growth using the methodology proposed in Elizondo (2012), the short-term real interest rate, and the nominal peso-dollar exchange rate depreciation.

Figure 7 shows the results from this methodology. Similar to all previous results, the estimated \( r_t^\star \) declined around 2006, and reached its lowest value at the end of 2012. Since then, it has increased somewhat but it has not reached pre-crisis levels. In the left panel of the Figure we

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31 A caveat of this methodology is that the results may be sensitive to the selection of the priors. Since we have a hybrid BVAR, with one part of parameters time-invariant and the other time-varying, we use two different strategies to discipline our priors selection for each part. With respect to the invariant part, we use a Minnesota-style prior for \( A_\ell \), where we selected the hyperparameters that maximize the forecasting performance of the model, while we impose block exogeneity in the upper-right quadrant of these matrices (we opt for maximizing the forecasting performance rather than the marginal likelihood, as computing the latter turned challenging for the hybrid model). Further, we use a diffuse inverse Wishart prior for the variance-covariance matrix \( E\{\xi_t\xi_t'\} \). With respect to the time-varying part, we use a diffuse prior for the initial state vector \( C_0 \), and an adjusted inverse Wishart prior for \( E\{\nu_t\nu_t'\} \) so that the distance between the model’s estimated output gap and the output gap estimate by Banco de México was minimized (for details on this latter estimate, see Banco de México’s 2009Q2 Quarterly Report, pp. 69). Further details about the estimation of the TVI-BVAR are available upon request.

32 We approximate the U.S. GDP monthly series applying Denton’s method to the quarterly series.

33 In particular, we use the average of the measures proposed by Wu and Xia (2016) and Krippner (2015). These series take negative values during the fed funds rate’s zero lower bound period, from 2009M1 to 2015M12, and are equal to the fed funds rate outside that period. The negative values in the shadow rate aim to measure the degree of monetary policy accommodation of unconventional monetary policies by the Fed if they were translated into movements in the fed funds rate.

34 See also Section 4.1.4.

35 Given the wide set of home and foreign variables in the model, it soon became cumbersome to estimate it with fully-fledged time-varying parameters and stochastic volatility, like the models studied in Cogley and Sargent (2005) and Primiceri (2005).
have plotted the estimate of potential growth. As it can be seen, the fluctuations in the estimated potential growth do not fully explain the trend observed in the estimated neutral rate.

Figure 7: Short-Run Real Rate, Neutral Rate, and Trend Growth from the TVI-BVAR Model

Note: The confidence intervals are of 90 percent significance. Source: Own estimates made with data from Banco de México and the FRED database of the St. Louis Fed.

4.1.6 Summary

All methodologies show that the neutral rate $r^*$ estimated for short- and medium-term in Mexico fell down in global financial crisis, digits close to 3.4%, to around 1% in real terms for the periods indicated in Table 1. This means neutral rate in nominal terms decreased from 7.4% to 4.8%.\(^{36}\)

These results are consistent with similar estimates have been made for United States, which show a decline in $r^*$ during the global financial crisis of 2008 and their permanence in low levels since then. The foregoing is related with the deleveraging of houses and a weak economic activity leading to a lower demand for credit and, on the other hand, to more stringent conditions for the granting of credit that reduced the offer. It is noteworthy that this fall in aggregate demand had a greater impact on the neutral rate level in the short-term. With regard to the reduction of $r^*$ in Mexico, this can be attributed to transitory factors related to: (i) abundant monetary liquidity in the financial markets resulting from measures of non-conventional monetary policy in advanced economies, such as those carried out by the Federal Reserve; and (ii) the persistent slack conditions

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\(^{36}\)To calculate the nominal neutral rate, was added to the average of each methodology the average inflation expectations in the next 12 months, according to the Survey of Banco de México.
Table 1: Summary of Quantitative Results for the Short and Medium Run

<table>
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<td>4.2</td>
<td>1.2</td>
<td>8.2</td>
<td>5.0</td>
</tr>
<tr>
<td>Laubach and Williams model</td>
<td>2.7</td>
<td>0.9</td>
<td>6.7</td>
<td>4.7</td>
</tr>
<tr>
<td>TVI-BVAR model</td>
<td>3.7</td>
<td>1.2</td>
<td>7.7</td>
<td>5.0</td>
</tr>
<tr>
<td>Average</td>
<td>3.4</td>
<td>1.0</td>
<td>7.4</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Note: We compute the nominal neutral rate by adding to the real neutral rate the average for the indicated period of headline inflation expectations one-year ahead, which we extracted from the survey of professional forecasters gathered by Banco de México. According to this survey, for the period 2001Q1-2008Q4, the average inflation expectation was 4.01%, while for the period 2009Q1-2016Q3, the latter reached 3.83%.

have prevailed in the Mexican economy in recent years. When transitory factors that led to this scenario dissipate, it is expected that $r^*$ converges to its long-term level in absence of shocks. Thus, it is evident that there have been global and local factors affecting the dynamics of neutral rate in short- and medium-term.

The following subsection are considered three different quantitative methods to infer the level at which is expected to converge $r^*$ in the long-term, in the absence of further transitory shocks.

4.2 **Long-run $r^*$**

The long-term level of $r^*$ depends on structural factors relatively outside the scope of monetary policy, such as the potential growth (affected, between other things, by the demography and the total factor productivity trend), the preferences of savings and risk aversion of households and investors, among others. In the case of the United States and other advanced economies, recent studies find that both potential growth and the long term level of $r^*$ of these economies have shown a downward trend in the past 25 years. The foregoing suggests that global factors have played an important role in the determination of the potential growth and the neutral rate at the international level (see Holston *et al.*, 2017).

4.2.1 **Taylor Rule Revisited**

In the short- and medium-term analysis, it was noted that the monetary policy of the Fed, specifically expansionary non-conventional policies, could have affected the real neutral rate in the medium term. To control for this factor, is considered an alternative configuration of the Taylor rule, where it introduces an indicator of the effect of this unconventional policies of the Fed
as a regressor more in the estimation of the monetary rule presented earlier. Thus the modified Taylor rule is defined as

\[ i_t = (1 - \rho)\left[ r^* + \bar{\pi} + \gamma i_t^{US, shadow} + \beta (\pi_t - \bar{\pi}) + \theta \hat{y}_t \right] + \rho i_{t-1} + \varepsilon_t, \tag{14} \]

The variable \( i_t^{US, shadow} \) takes the value of zero when the Federal Funds rate is positive (until June 2009), and takes the “shadow” value calculated by Wu and Xia (2016) from July 2009 to December 2015.

Similar to Taylor rule showed in Figure 4, the modified Taylor rule also is estimated recursively since 2002.

\[ \text{Figure 8: Augmented Taylor Rule} \]

\[ \text{r}^* \text{ estimate in the long run} \]
\[ \text{Short-run real interest rate} \]

\[ \text{Note: The confidence intervals are of 90 percent significance. Source: Own estimates made with data from Banco de México.} \]

Figure 8 shows that if it is controlled for unconventional policies of the Fed, then neutral rate remains relatively constant from 2008, around 2.4% in real terms. This correspond to a level of 5.4% in nominal terms in the long-term, adding the inflation target of 3%.

37 This result is in line with some conclusions found in ?. Who argue that one of the reasons why models estimate a time-varying \( r^* \) is due to bias by omission of some variables or even bias by omitted equations.

38 This rate was updated by the Federal Reserve Bank of Atlanta until December 2015.
4.2.2 RBC Model for a SOE with Efficiency Wedges

An alternative to measure the long-term $r^*$ is through a real business cycles model (RBC model). In this model we follow to Lama (2011), who proposes a model of open economy with flexible prices to replicate the dynamics of product, consumption, investment and the worked hours.

This model uses adjustment wedges, which capture the effects of possible rigidities in the economy that may affect the evolution of macroeconomic variables and that the model does not explicitly incorporate. These wedges help the model to perfectly fit the observed dynamics in the data. Thus, we can estimate the equilibrium real interest rate (marginal productivity of capital) that is consistent with these fluctuations. Given that the wedges capture frictions affecting the economy, but they are do not model of structural shape, it is difficult to distinguish between changes in structural and transitory factors in short periods of time. However, we can filter the transitory factors of $r^*$ in the long-term if we calculate the average for long period of time.

Specifically we use four wedges, the first one is for total factor productivity (TFP), the second one affects the consumption-work decision, the third wedge affects the balance of the capital market and the last one affects the intertemporal transmission of sources.

Thus, the model is characterized by a representative household and enterprises. Thus, the first maximize his utility as

$$\max_{c_t,b_{t+1},l_t} E_0 \{ \sum_{t=0}^{\infty} N_t \beta^t u(c_t, l_t) \},$$

$$c_t + x_t + (1 + n)(1 + \gamma)b_{t+1} \leq (wedge_{l,t}) w_t l_t + (wedge_{k,t}) z_t k_t + (wedge_{b,t}) (1 + \tilde{r}_t)b_t + T_t$$

$$(1 + \tilde{r}) = (1 + r^w)(b_t/b^*)^\nu$$

$$(1 + n)k_{t+1} = (1 - \delta) K_t + x_t - f(x_t, k_t)$$

The enterprises maximize their utility in the following way

$$\max_{l_t,k_t} y_t = w_t l_t - r_t k_t$$

$$y_t = (wedge_{TFP,t}) F(k_t, (1 - \gamma)^tl_t)$$

With $N_t$ population, $c_t$ consumption, $l_t$ worked-hours, $x_t$ investment, $b_t$ international bonds, $k_t$ capital, $T_t$ net taxes lump-sum, $w_t$ real salaries, $z_t$ real rate of capital income, $\tilde{r}_t$ domestic real interest rate, $r^w$ international real interest rate, $p^k$ Q of Tobin, $\beta$ subjective discount rate, $n$ population growth rate, $\gamma$ productivity growth rate, $\delta$ depreciation rate of capital, $u(\cdot)$ home utility, $f(\cdot)$ investment adjustment costs, and $F(\cdot)$ production function.
In this small open economy, the equilibrium interest rate is determined through two optimality conditions, the decision of consumption/savings and the demand for investment, i.e.,

\[ u'(c_t, l_t) = \frac{\beta}{1 + \gamma} E_t[u'(c_{t+1}, l_{t+1})(\text{wedge}_{b,t})(1 + \tilde{r}_t)] \quad \text{and} \]

\[ E_t\{1 + r^{k}_{t+1}\} = \frac{E_t\{(\text{wedge}_{TFP,t})F'_k(\cdot) + (1 - \delta)p^k_{t+1} - \text{cost.adj}_t\}}{p^k_t} \]

Finally, the nominal neutral rate in long-term is approximated by

\[ \bar{i} = \bar{r} + \bar{\pi}, \quad \text{and} \quad \bar{r} = \frac{1}{T} \sum_{t=1}^{T} (\text{wedge}_{b,t})(1 + \tilde{r}_t). \quad (17) \]

Similarly to the result obtained from the estimation of the modified Taylor rule, we can observe in the Figure 9 that the average level of the real neutral rate \( r^* \) is 2.4%, it is equivalent to 5.4% in nominal term.

**Figure 9: SOE-RBC Model of Lama (2011)**

Source: Own estimates made with data from Banco de México and INEGI.

### 4.2.3 Financial Markets Information

Derivative instruments provide financing alternatives and flexible hedging. In the particular case of the interest rate swaps (swap of TIIE) to different maturities, counterparts exchanged between
them flows at a fixed rate per flows to a floating rate. One of the benefits of follow the implicit expectations of interest rates in these instruments is that they reflect very precisely the risk levels that market agents perceive.

It is noteworthy that this kind of instruments are very liquid. In particular, in June 2013, MexDer stressed that swaps of TIIE represented the largest OTC market of Mexico. Thus, the price of an interest rate swap is determined in the following way: the legs of the swap (fixed leg and variable leg) are associated with different schemes of recurring payment flows referenced to different interest rates. For that both sides are willing to hire the swap, the net present value of the flows of both legs must be equal. This is achieved through a fixed interest rate, with this rate the contract is agreed. The factors which are discounted flows of each leg depend on yield curve expectations in the future.

The fixed rate of this contract may be associated with the expected level of the TIIE for a determined period by the participants in the swap contract. To capture the market expectations on the level of long-term rate, we consider the average of agreed fixed rate in the contracts swaps of TIIE at 5 and 10 years, least 30 basis points, which is the historical difference between the TIIE and the one-day interbank funding rate.

Figure 10 shows the moving average to one-month of implicit reference interest rates that is traded on the derivatives market through contracts swaps of TIIE to 5 and 10 years. The dotted line corresponds to the average of these contracts. In particular, from December 2015 to September 2016, the average of the swaps contracts, less 30 bases points, suggests that the implicit reference interest rate will be located around 5.6%, where swaps to 5 years are located in 5.2% and those of 10 years in 5.9%. This result is again consistent with the previous methods.

4.2.4 Summary

Table 2 summarizes the results of the methodologies to estimate the long-term level of \( r^* \). The range for this rate, calculated on the base of average of minimum and maximum levels obtained in each method, this suggests that the long-term level of \( r^* \) is from 1.7% to 3.3% in real terms and from 4.7% and 6.3% in nominal terms, with media point in 2.5% and 5.5%, respectively. We add inflation target of 3% in nominal terms.

\[39\] In Mexico TIIE represents the interbank equilibrium interest rate.

\[40\] More detail about this can be found in the link: http://www.mexder.com.mx/
Figure 10: Implicit Policy Rate in interest-rate Swaps (for TIIE) at long-term Maturities

Note: The implicit policy rate in interest-rate swaps for TIIE is calculated as the difference between the swap rate minus 30 basis points, according to historical differences between the policy rate and TIIE. Source: Own computations with data from Bloomberg.

Table 2: Summary of Quantitative Results for the Long Run

<table>
<thead>
<tr>
<th>Methods</th>
<th>Real neutral rate, $r^*_t$</th>
<th>Nominal neutral rate, $r^*_t + \bar{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Central point</td>
<td>Range</td>
</tr>
<tr>
<td><strong>Augmented Taylor rule</strong></td>
<td>2.5</td>
<td>1.6 - 3.4</td>
</tr>
<tr>
<td><strong>RBC-SOE model</strong></td>
<td>2.4</td>
<td>1.3 - 3.6</td>
</tr>
<tr>
<td><strong>Swaps market</strong></td>
<td>2.6</td>
<td>2.2 - 2.9</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>2.5</td>
<td>1.7 - 3.3</td>
</tr>
</tbody>
</table>

Note: We compute the long-run nominal neutral rate by adding to the estimated long-run real neutral rate the long-run inflation target of Banco de México, which equals 3%.

5 Importance of Domestic and Foreign Factors

5.1 Short-run $r^*$

5.1.1 Domestic Factors

Between the transitory factors that have pushed downward to neutral rate are the persistent slack in Mexico that have prevailed since global financial crisis. In addition, as mentioned above, monetary liquidity in the capital market caused by the unconventional monetary policy measures in advanced economies, in particular in the U.S. The foregoing, was caused by the excess of desired savings
that exceeded the investment from 2003 to 2007 in the United States, what is known as “saving glut”. This in a certain way was also characteristic of Latin America, from 2004 to 2007 the EMBI indicator of country risk declined sharply from 1200 to 200 basis points, reaching its historical lows around 2006 (see Figure 11, left graph). In the case of Mexico, this indicator was in the range of 400 to 150 basis points from 2003 to 2007. This made to Mexico attractive place to invest. In addition, pension funds began to increase its savings significantly, from 5% to 15% as a proportion of GDP from 2004 to 2015 (see Figure 12). All these were reflected in lower interest rates of the entire curve, and in particular in the corresponding to 10-years, which showed a downward trend from 2003, reaching its minimum historic until before the ”taper tantrum” (see Figure 11, central graph), this was due to that term premia decreased too (see Figure 11, right graph). The same pattern showed both long-term interest rate ant term premium in the U.S. This may be evidence of the strong fall found in 2006 in the neutral rate estimated by Laubach and Williams and TVI-VAR methods, as well as its tendency after global financial crisis.

Figure 11: Financial Variables: Key Interest Rates

![Chart showing interest rates and EMBI](image)

**Note:** Note: the EMBI is measured as the difference in basis points with respect to U.S. treasury bonds yield. For the estimate of the 10-years term premium for Mexico, we follow Adrian et al. (2013). The estimate for U.S. follows as well Adrian et al. (2013), and is updated by the New York Fed. **Sources:** Banco de México, Valmer, PiP and Bloomberg.
5.1.2 Foreign Factors

The literature has documented some foreign factors that have affect the dynamics of $r^*_t$ in advance and emerging economies. In particular, in Hördahl, Sobrun and Turner (2016) find that there are two global factor that move interest rates: in the short-run this factor is Federal Funds rate, because this rate was which affected financial conditions in emerging market economies, and in the long-run one important factor that move interest rates is a world real interest rate. The last rate will be explain in detail in long-run analysis.

In our Laubach and Williams estimate we find that the “other factors” into $z_t$ explain the behaviour of neutral rate after global financial crisis, see Figure 6. In this sense, we can compare the index $z_t$ with factors that can affect the $r^*_t$ in Mexico, such as the monetary policy of the Fed (left side of Figure 13). Thus, there is a high correlation between the index $z_t$ and the target rate of Fed, especially when it includes the “shadow” measure of Wu and Xia (2016). The correlation persists even if it is considered the $r^*_t$ for U.S., calculated by Laubach and Williams (2016), and updated by the staff of the Fed (side right of the Figure 13).

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41The shadow rate of Wu and Xia (2016) translates the effect of the unconventional monetary policies of the Fed as a counterfactual negative target rate. While more negative is this rate more lax was the unconventional monetary policy of the Fed.
As in Laubach and Williams model, the results of the TVI-VAR suggest that the dynamics of potential growth, appears not to be the factor that explains the fall in the \( r^*_t \) (see Figure 7). In both methodologies the neutral rates begin to fall from 2006. Similarly, to Laubach and Williams model, we calculate the \( z_t^{VAR} = r^*_t - \gamma_t \) index implicit in TVI-VAR in order to identify other factors different to monetary policy of Fed, that may be related to the decline in the \( r^*_t \). Thus, there is a high correlation between the \( z_t \) index and the real long-term interest rate of Mexican bonds, see Figure 14. Both variables began to rise during the “ taper tantrum”.

To verify if those correlations are consistent, the following subsection analyses cointegration relationships between Mexican neutral rate and some of its determinants.

### 5.1.3 Cointegration Analysis and Discussion

As mentioned in Holston et al. (2017) the economic theory documents that movements in \( r^* \) must be correlated between countries. In this context, Rachel and Smith (2015) present a stylized scheme in which they argue that for a small open economy the real rate of equilibrium or neutral rate is probably influenced by global factors.

In this section, following to Holston et al. (2017) we perform a cointegration analysis, which suggests that, in the sample considered, \( r^* \) of Mexico is influenced by external or global factors.
Specifically, we run cointegration tests to discern whether there is a long-term relationship between the neutral rates of Mexico and U.S.. To do so, in the case of Mexico we use the average of the estimates from Laubach and Williams and TVI-VAR models, while for the U.S. we use the estimates of Laubach and Williams (2003).42

As a first approach, Figure 15 shows the relationship between the neutral rates and their components (the growth rate of potential output and the latent variable $z_t$) by means of dispersion graphs. It is easy to appreciate that the relationship between neutral rates is very strong. Apparently, this is mainly because latent variables ($z_t$) are even more correlated, while potential growth rates do not appear to have a relationship. In order to verify these relations slightly more formal way, cointegration tests are carried out following the methodology of Johansen (1988). Although by construction those variables are modeled as random walk, it is checked empirically through the unit root test of Augmented Dickey-Fuller that the series can be characterized as integrated processes of order one, I(1). It is noteworthy that, the results generated in this section should not

---

Note: The nominal interest rate of coupon bonds was obtained from PiP for the period 2003Q3 to 2015Q4. From 2002Q1 to 2003Q2, this rate was estimated from the 10-years zero-coupon bonds, obtained from Valmer. It should be noted that, for the sample 2003Q3-2015Q4, the correlation between the coupon rate and the zero coupon rate is 99.5 percent. To compute the long-term real interest, we used the long-term inflation expectations of Aguilar et al. (2016).

Source: Own calculations with data of PiP, Valmer and results from the TVI-BVAR.

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42 The updated data are available in http://www.frbsf.org/economic-research/files/LaubachWilliamsUpdatedEstimates.xlsx
be considered as conclusive, because the variables used are the product of econometric estimates so that the resulting statistical inference is not all valid. However, the results can be considered as one more piece of evidence in favor of the idea that there is a co-movement between neutral rates among economies.

Figure 15: Scatter plots of $r^*$ in Mexico and the U.S. and their Components

Source: Own calculations from the average of Laubach-and-Williams and the TVI-VAR models for the $r^*$ in Mexico, and with data from Laubach and Williams (2003) for the U.S., updated by the San Francisco Fed.

Table 3 presents the results of cointegration tests. As can be seen in the three cases ($r^*$, $z$ and $g$) it can be concluded that there is a long-term relationship between these variables of Mexico and U.S. In turn the cointegration relationship are reported in Table 4. Firstly, it should be noted that although the cointegration test suggests that potential growth rates are cointegrated the estimated long-term coefficient is very small (0.232), and if the statistical inference was valid this would be not significant. On the other hand, the results indicate an important relationship both in neutral rates and latent variables $z$s. Even the long-term effect of the American $z$ on that of Mexico is greater than American neutral rate on $r^*$ of Mexico.

These results as a whole suggest that neutral rate of Mexico is related to neutral external or global rate (measured by the $r^*$ in the U.S.), and this is mainly due to other factors beyond those that could link to growth rates of potential output. This evidence for an emerging economy such as the Mexican complements the findings of other studies such as Holston et al. (2017), who show evidence of co-movement in neutral rates for a group of advanced economies. In addition, it reinforces the argument that global factors are very important in determination of the neutral rate of economies.
Table 3: Cointegration Test

<table>
<thead>
<tr>
<th>Variables</th>
<th>Hypothetical # of Coint. Vectors</th>
<th>Trace</th>
<th>Max-Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Statistic</td>
<td>p-value</td>
</tr>
<tr>
<td>( r^* )</td>
<td>None</td>
<td>14.021</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>Al most 1</td>
<td>1.389</td>
<td>0.239</td>
</tr>
<tr>
<td>( z )</td>
<td>None</td>
<td>15.726</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>Al most 1</td>
<td>2.023</td>
<td>0.155</td>
</tr>
<tr>
<td>( g )</td>
<td>None</td>
<td>24.337</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Al most 1</td>
<td>0.006</td>
<td>0.937</td>
</tr>
</tbody>
</table>

In all cases the cointegration model is using unrestricted intercept and no trend. Estimations include three lags.

Table 4: Cointegration Relationships

<table>
<thead>
<tr>
<th>Cointegration Vector</th>
<th>( r^* - mx )</th>
<th>( r^* - us )</th>
<th>( z - mx )</th>
<th>( z - us )</th>
<th>( g - mx )</th>
<th>( g - us )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.000</td>
<td>-1.057</td>
<td>1.000</td>
<td>-1.503</td>
<td>1.000</td>
<td>-0.232</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.087)</td>
<td>(0.087)</td>
<td>(0.087)</td>
<td>(0.241)</td>
<td>(0.963)</td>
</tr>
<tr>
<td></td>
<td>[-13.444]</td>
<td>[-17.335]</td>
<td>[-17.335]</td>
<td>[-17.335]</td>
<td>[-0.963]</td>
<td>[-0.963]</td>
</tr>
<tr>
<td>Adjustment Velocity</td>
<td>-0.071</td>
<td>0.195</td>
<td>-0.041</td>
<td>0.284</td>
<td>-0.085</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.057)</td>
<td>(0.021)</td>
<td>(0.105)</td>
<td>(0.019)</td>
<td>(0.009)</td>
</tr>
<tr>
<td></td>
<td>[-3.423]</td>
<td>[3.420]</td>
<td>[-1.948]</td>
<td>[2.703]</td>
<td>[-4.411]</td>
<td>[2.601]</td>
</tr>
</tbody>
</table>

Standard errors are in parenthesis and \( t \)-statistics are in brackets.
In all cases the cointegration model is using unrestricted intercept and no trend. Estimations include three lags.
5.2 Long-run $r^*$ (Heuristic Analysis)

In the case of Mexico, a heuristic analysis suggests that long-term level of $r^*$ would be affected by factors that determine the level of potential growth, as well as the dynamics of the international capital market. By the side of the potential growth, this could be negatively affected by a lower growth of the population and the labor force. However, it is expected that the structural reforms carried out recently in the country may compensate the drop in the population growth rate. Thus, if the potential growth increases, so will have the long-run level of $r^*$ of the economy.

However, with regard to international capital market, it has been observed that the long-run global real interest rate has gradually decreased in the last 30 years in response to international determinants that, on the one hand, have increased the global desired savings and, on the other, have reduced the demand for global investment. It is also expected that this rate will be stabilized at around one percent for a long time. In this context, as well as other emerging economies, the long-run real interest rate of Mexico followed a similar trend to the global rate from 2002 to 2013, although from a higher level, see Figure 16. This trend can be explained by global capital flows that sought higher yields than those offered in their own markets, which exerted downward pressure on the long-term level of $r^*$ during the referred period. Thus, to the extent that capital flows are directed to domestic financial assets in search of higher yields, this could lead to the long-term level of $r^*$ will be lower than the level that prevailed before the global financial crisis, offsetting the effect of a possibly higher potential growth.

5.2.1 Domestic Factors

As mentioned previously, economic theory suggests that long-run neutral rate is determined by domestic structural factors, such as total factor productivity, population growth, and preferences of added savings.

According to those factors, there is pessimism about the future growth (increasing the demand for safe assets). The concerns on the decline in growth are related with lower population growth and decrease in total factor productivity (TFP). The foregoing is related to: i) limited growth of education, due to capital per worker has grown but the years of schooling can not grow forever; ii) inequality can contribute to low real rates increasing savings by people with high incomes; and iii) oversupply of government bonds will increase public debt.

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43The CONAPO estimate that from 2010 to 2030 the total population will grow more slowly, moving it from a rate of 1.3 % to 0.7%. For the population of between 16 to 65 years, its rate would rise from 1.8% to 0.6% in the same period.

44More details can be found in Rachel and Smith (2015).

45Starting this year, we can see a slight increase in long-term real interest rate, in particular the inflation-indexed rate to 10-years. However, this rate has not reached the levels observed before the financial crisis.
With regard to changes in the preferences of saving and investment (increasing savings and falling investment). By part of the greatest savings are related with the demographic structure, greater inequality in the countries, and enhanced savings on emerging economies. In the case of lower investment is related with lower marginal productivity of capital, less public investment, and an increase in the interest rate differential.

### 5.2.2 Foreign Factors

According to external structural factors that have affected the Mexican long-run neutral rate downward is the international capital market, whose price is the world real interest rate in the long-term. This is because has had a greater global savings.

In particular, the low interest rates that have been registered in the rest of the world involved an increase in the demand for American assets, which implied downward pressures on the nominal neutral rate of that country. Additionally, Rachel and Smith (2015) argue that structural factors related to the global savings and the demand for global investment could explain up to 300 basis points of the fall in the world real interest rate from 1985 to mid 2015, see Figure 16.

In this sense, the factors that shifted the global savings curve to the right according to Rachel and Smith (2015) are: an increase in the proportion of adults of working age, a propensity to save in the governments bonds of emerging economies, and a higher income inequality. Among the factors that have unmotivated global investment demand, these authors found: a significant fall in the relative price of capital, a lower public investment and an increase in the difference between the return rate of capital and the risk-free rate. With regard to the global growth rate, these authors argue that did not influence negatively to the world interest rate from 1980 to 2007. However, the expectation of a lower growth in the future from 2008, either by a lower growth in emerging countries or by the labor force in advanced countries, would have contributed to a fall of 100 bases points in the world real interest rate in recent years. In summary, the reasons could explain until 400 bases points of the fall in the world real interest rate in the past 35 years.

### 6 Concluding Remarks

In this paper, we summarize different estimations for the trend of the natural rate in Mexico in the short and medium run, and assess the longer-term level at which this variable may converge in the absence of shocks. We find that short-run $r^*$ has followed a downward trend since the beginning of the 2000s, fell sharply during the 2008 global financial crisis, reached record low levels in 2014, and has recovered just recently. We argue that potential growth in Mexico cannot explain the fall in short-run $r^*$ in the aftermath of the crisis; these dynamics could be better explained by transitory...
Figure 16: Global, U.S., and Mexico Long-Term Real Interest Rates

Note: The global real rate was extracted from King and Low (2014), which uses instruments that discount the inflation of the G7 countries except Italy. For the U.S. and Mexico, we use the rate of 10-year coupon bonds minus the long-term inflation expectations implicit in financial instruments. Source: Own calculations with data from King and Low (2014), the Federal Reserve, PiP, Valmer, and Aguilar et al. (2016).

We argue that lower growth in the labor force and a change in the demographic composition of the Mexican population, favoring a greater proportion of adults, along with the lower trend in the global and the Mexico’s long-term real interest rates, might have pressure downwards the longer-term level of \( r^* \). These conclusions rest however tentative, as they are drawn from an heuristic analysis. More quantitative approaches point out that the long-term level of \( r^* \) might be lower now of what it was in the 2000s. These results are consistent with the heuristic analysis.
all of the results shown here must be taken with caution. The latter, along with the difficulty to assess the phase of the business cycle that the economy undergoes in real time, implies that a central bank must search for a wide set of economic indicators in order to assess the stance of monetary policy necessary to achieve price stability in the middle run.
References


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