Monetary Policy and the Term Structure of Interest Rates

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Abstract

We analyze the effects of monetary policy on the term structure of interest rates in a medium DSGE model including long-run nominal and real risk estimated using Bayesian methods. The estimated model replicates key stylized facts of the nominal and real term structures of interest rates and term premia as well as macroeconomic variables. We quantify the effects of unexpected changes in the monetary authority’s policy rate and stance on the short and long ends of the term structure and on agents’ precautionary motives that drive the pricing of nominal and real risk in the economy. In our application to forward guidance, we find that these precautionary motives lead the pricing of risk to dampen the effects of a preannounced expansionary monetary policy on the real economy.

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1 Introduction

What are the effects of monetary policy on the term structure of interest rates? The empirical literature has yet to reach a definitive conclusion on this question, with respect to not only the quantitative but also the qualitative effects. The differences in empirical results could be driven by different underlying samples or identification approaches, see Campbell, Fisher, Justiniano, and Melosi (2016) for a discussion of potential shortcomings of the different approaches in isolating the effects of monetary policy, and, furthermore, the “shocks” identified in the empirical literature are not always the empirical counterparts of shocks from theoretical models as pointed out by Ramey (2016). Having a monetary policy following a Taylor-type rule in mind, we would like to distinguish changes in the systematic part of monetary policy, for example, due to changes of weights on inflation or output in the central bank’s loss function, from innovations to the Taylor rule. A structural model that captures macroeconomic as well as financial variables reasonably and produces sizable and time varying risk premia is needed to address this goal. However, the impact of monetary policy on interest rates beyond the expectation hypothesis is not captured by the linear New Keynesian models commonly used in policy analysis and modelling approaches that go beyond the expectation hypothesis face significant computational challenges.

To this end, we estimate a medium scale New Keynesian macro-finance model in the present paper that fits the main macro and financial in the present paper, using the risk adjusted approximation of Meyer-Gohde (2016) that captures the salient features of risk while being linear in states to enable the estimation and posterior analysis using standard macroconometric techniques. Our risk adjusted linear New Keynesian macro-finance model is able to produce sizable and time varying risk premia, comparable to historical estimates from affine terms structure models (e.g. Kim and Wright, 2005; Adrian, Crump, and Moench, 2013). Specifically, the model has both nominal and real frictions (see e.g. Smets and Wouters, 2003, 2007; Christiano, Eichenbaum, and Evans, 2005) and is estimated using U.S. data from 1983:Q1 to 2007:Q4. We follow Meyer-Gohde (2016) and adjust a linear approximation of the model for risk out to the second moments of the underlying stochastic driving forces to capture both constant and time varying risk premia as well as the effects of conditional heteroskedasticity. Unlike standard perturbations (e.g. Andreasen, Fernández-Villaverde, and Rubio-Ramírez, 2016), our approximation maintains linearity in states and shocks, giving our method significant computational advantages over higher order polynomial approximations in iterative calculations, such as the Markov chain Monte Carlo method used to simulate the posterior density of the deep parameters. Moreover, this approach allows us to use the standard set of tools for estimation and analysis of linear models. Especially, this approach put us in the position to investigate forward guidance on the term structure of interest rates within fully structural model.

The role of monetary policy in shaping the term structure has gained particular prominence against the recent backdrop of unconventional monetary policy, with, e.g., forward guidance having become an important tool around the world of central banks constrained by the zero lower bound on nominal interest rates. However, forward guidance has long been a component of central banks’ toolkits and as such these polices are likely to remain important after the recent liftoff from the zero lower bound in the U.S. (see Akkaya, Gürkaynak, Kisacikoğlu, and Wright, 2015). Whereas Eggertsson and Woodford (2003) highlight,
among other points, the impact of forward guidance on long rates due to the expectation hypothesis, *Hanson and Stein* (2015) point out that monetary policy does not operate via the expectation hypothesis alone, but also operates via the term premia. Hence, a growing body of empirical papers investigates the effects of conventional and, also more recently unconventional, monetary policy on the term structure of interest rates.\footnote{See for example the pioneering work by *Kuttner* (2001), *Cochrane and Piazzesi* (2002), and *Gürkaynak, Sack, and Swanson* (2005a,b). More recent papers which focus also on unconventional monetary policy are, for example, *Nakamura and Steinsson* (2013), *Gertler and Karadi* (2015), *Gilchrist, López-Salido, and Zakrajšek* (2015), *Abrahams, Adrian, Crump, and Moench* (2016), and *Crump, Eusepi, and Moench* (2016).} Some differences in the empirical literature’s approaches in isolating the effects of monetary policy are explicitly intended. For example, *Hanson and Stein* (2015) focus on changes of the monetary stance in absence of forward guidance while *Nakamura and Steinsson* (2013) are explicitly interested in the effects of forward guidance. A comprehensive analysis of conventional and unconventional monetary policy would need a structural model that captures the nonlinearity behind the risk underlying financial variables yet is suited to answer questions about, e.g., forward guidance (see, for example, *van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez*, 2012; *Andreasen*, 2011; *Rudebusch and Swanson*, 2012).

We start our analysis by investigating the effects of conventional monetary policy on the term structure of interest rates. In particular, we investigate the effects of a monetary policy shock that affects the monetary stance and a shock to the residual of the Taylor rule. We find that a shock to the monetary stance has strong effects on risk premia. In the long run, moreover, real interest rates are mainly driven by those risk premia. Both results confirm the findings of *Hanson and Stein* (2015). Additionally, we find that both monetary policy actions affect risk premia at different maturities differently. When agents do not expect that a lower policy rate will along with an change of the monetary policy stance, then a lower policy rate lowers, on impact, the nominal term premia for shorter maturities and increases nominal term premia for longer maturities. This result is qualitatively comparable with the findings of *Nakamura and Steinsson* (2013). Overall, the effect of a unexpected monetary policy shock – a simple innovation to the Taylor rule – has limited effects on the term premia at all maturities. This finding in line with those of other structural models (see, for example, *Rudebusch and Swanson*, 2012). Simply put, an uncorrelated innovation to the Taylor-rule dies out too quickly to have substantial effects at business cycle frequencies. Therefore, the effects on risk premia, which vary primarily at lower frequencies (see, for example, *Piazzesi and Swanson*, 2008), are limited. In contrast, a shock to the monetary policy stance has much stronger effects on the term structure of interest rates across all maturities. The reason behind the strong effect on the risk premia, as laid out by *Rudebusch and Swanson* (2012), is that long-run nominal risk has strong effects on the nominal term premium. Therefore, a shock to the monetary stance is much longer lasting and so has stronger effects on business cycle and lower frequencies.

We find for both monetary policy shocks that risk premia tend to move opposite the policy rate in the long run, i.e., a looser monetary policy increases risk premia. In particular in our model, such a looser monetary policy increases the precautionary savings motive of agents as they expect more volatile inflation and output and, therefore, demand a higher risk premia. This finding is comparable to the empirical results of *Crump et al.* (2016) who investigate the same sample period as we do. However, this finding is in contrast to many
other recent papers (see, for example, Hanson and Stein, 2015; Abrahams et al., 2016; Gertler and Karadi, 2015) who find that a looser monetary policy goes along with decreasing risk premia. It is important to note that all of these studies use a different sample period, starting in 1999 or later and including recent data after the financial crisis, which marks an episode of potentially different systematic monetary policy. Nevertheless, it is important to highlight that the finding of those studies would be in line with theoretical predictions such as a “search-for-yield” channel of institutional investors (Rajan, 2005). While our model features a frictionless asset trade, a model featuring the former channel would need some kind of market segmentation to change the policy conclusions of our paper (see, for example, Fuerst, 2015). This notwithstanding, the task of the present paper is not to investigate different channels, but rather to provide a macroeconomic framework which is consistent with a wide variety of asset pricing facts and is therefore well suited to investigate the impact of monetary policy on term structure of interest rate.

With this in mind, we turn to the analysis of the effects of forward guidance on the term structure of interest rates. In particular, we follow the approach by Campbell et al. (2016) and analyze the effect of Odyssean forward guidance, where the nomenclature originates in Campbell, Evans, Fisher, and Justiniano’s (2012) delineation of Delphic and Odyssean forward guidance. The former refers to announcements about future movements of real variables while the latter announces a path of short term interest rates independent of macroeconomic conditions. Thus when examining Odyssean forward guidance, the central bank’s hands are tied to the allegoric mast. Distinguishing between these different kinds of forward guidance is a significant challenge in many empirical approaches (see, for example, the discussion in Nakamura and Steinsson, 2013; Campbell et al., 2016), which makes a structural model that can naturally distinguish between the two all the more desirable. Unfortunately, the commonly used workhorse model, the New Keynesian model shows often unreliable outcomes from forward guidance. In particular, Del Negro, Giannoni, and Patterson (2015) deem the phenomenon of incredulously strong output and inflation responses the “forward guidance puzzle”. Graeve, Ilbas, and Wouters (2014) show that the behavior of long rates can differ substantially in otherwise similar New Keynesian models, at odds with the transmission mechanism originally described by Eggertsson and Woodford (2003), who argue long term bond yields must fall in accordance with the expectation hypothesis.

We find that forward guidance affects the risk premia substantially, prying bond yields away from the expectations hypothesis. In particular, we find that forward guidance policy causes real term premia and inflation risk premia to rise because agents expect a more volatile inflation and output in the future. This finding is in line with the empirical finding of Akkaya et al. (2015). Similarly to most studies, we find that forward guidance reduces macroeconomic activity and substantially reduces inflation. In comparison to many standard New Keynesian models, however, our model does not generate a “forward guidance puzzle” as the effects on output and inflation are rather modest. Although we cannot pretend that our setup provides a solution to the forward guidance puzzle as illustrated by Del Negro et al. (2015), we provide evidence that properly taking into account the term premium component of long yields induces an important dampening factor to anticipated monetary policy actions.

The reminder of the paper reads as follows: Section 2 presents the model. Following, section 3 describes the solution method, the data, and the Bayesian estimation approach in greater detail. Section 4 presents the estimation results and discusses the model fit. Section
5 presents the effects of unexpected and expected monetary policy on the term structure as well as the forward guidance experiment. Section 6 concludes the paper.

2 Model

In the following section, we present our dynamic stochastic general equilibrium (DSGE) model. We study a New Keynesian model, where households have recursive preferences following Epstein and Zin (1989, 1991) and Weil (1989) and maximize their wealth from consumption, labor input, and investment. The nominal yield curve is derived from the micro-founded stochastic discount factor and no-arbitrage restrictions. Firms are monopolistic competitors selling differentiated products at prices that are allowed to adjust in a stochastic fashion as in Calvo (1983). The central bank follows a Taylor rule which sets the short-term nominal interest rate as a function of the inflation rate and output. The model has a similar structure like Smets and Wouters (2003, 2007) or Christiano et al. (2005) including nominal and real rigidities which have demonstrated success in replicating stylized facts of the macroeconomy. Additionally, the model incorporates real and nominal long-run risk (Bansal and Yaron, 2004; Gürkaynak et al., 2005b) which, together with recursive preferences, have been highlighted in the literature as important in order to explain many financial moments in a consumption-based asset pricing model.

2.1 Firms

A perfect competitive representative firm produces the final good $y_t$. This final good is an aggregate of a continuum of intermediate goods $y_{j,t}$ and given by the function

$$y_t = \left( \int_0^1 \frac{y_{j,t}^{\theta_p-1}}{y_j^{\theta_p}} \, dj \right)^{\frac{\theta_p}{1-\theta_p}},$$

with $\theta_p > 1$ the intratemporal elasticity of substitution across the intermediate goods. The competitive, representative firm takes the price of output, $P_t$, and the price of inputs, $P_t(j)$ as given. The resulting demand function for the intermediate good is

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t,$$

and the aggregate price level is defined as

$$P_t = \left( \int_0^1 P_{j,t}^{-\theta_p} \, dj \right)^{\frac{1}{1-\theta_p}}$$

and gross inflation is $\pi_t = P_t/P_{t-1}$.

The intermediate good $j$ is produced by a monopolistic competitive firm with the following Cobb-Douglas production function:

$$y_{j,t} = \exp \{a_t \} k_{j,t-1}^\alpha (z_{l,j,t})^{1-\alpha} - z_{l}^+ \Omega_t,$$
where \( k_{j,t} \) and \( l_{j,t} \) denote capital services and the amount of labor used for production by the \( j \)th intermediate good producer, respectively. The parameter \( \alpha \) denotes the output elasticity with respect to capital and \( \Omega_t \) the fixed costs of production. The variable \( \exp\{a_t\} \) refers to a stationary technology shock, where \( a_t \) is described by the following AR(1) process:

\[
a_t = \rho a_{t-1} + \sigma \epsilon_{a,t}, \quad \text{with} \quad \epsilon_{a,t} \overset{iid}{\sim} N(0,1) \quad (5)
\]

The variable \( z_t \) depicts an aggregate productivity trend. We include this non-stationary productivity shock to allow for a source of real long-run risk. As put forward by Bansal and Yaron (2004), the presence of real long-run risk is important in order to explain many financial moments in a consumption-based asset pricing model. We assume that \( \exp\{\mu_{z,t}\} = z_t/z_{t-1} \) and let

\[
\mu_{z,t} = (1 - \rho_z) \bar{\mu}_z + \rho_z \mu_{z,t-1} + \sigma_z \epsilon_{z,t}, \quad \text{with} \quad \epsilon_{z,t} \overset{iid}{\sim} N(0,1) \quad (6)
\]

Overall the economy has two sources of growth. Next to the aforementioned productivity trend in \( z_t \) the economy faces also a deterministic trend in the relative price of investment \( \Upsilon_t \) with \( \exp\{\bar{\mu}_\Upsilon\} = \Upsilon_t/\Upsilon_{t-1} \). Therefore, we follow Altig, Christiano, Eichenbaum, and Linde (2011) and define \( z_t^+ = \Upsilon_t^{\alpha_\Upsilon} z_t \), which can be interpreted as an overall measure of technological progress in the economy. The overall trend in the economy is characterized by

\[
\mu_{z^+,t} = \frac{\alpha_1 - \alpha}{1 - \rho_\Upsilon} \bar{\mu}_\Upsilon + \mu_{z,t}. \quad (7)
\]

Finally, we scale \( \Omega_t \) by \( z_t^+ \) to ensure the existence of a balanced growth path and let production costs be time-varying as proposed by Andreasen (2011). In our model, such variations in firms fixed production costs represent real supply shocks by assuming that

\[
\log \left( \frac{\Omega_t}{\Omega} \right) = \rho_\Omega \log \left( \frac{\Omega_{t-1}}{\Omega} \right) + \sigma_\Omega \epsilon_{\Omega,t}, \quad \text{with} \quad \epsilon_{\Omega,t} \overset{iid}{\sim} N(0,1). \quad (8)
\]

Following Calvo (1983), intermediate good firms are subject to staggered price setting, i.e., they are allowed to adjust their prices only with probability \( (1 - \gamma_p) \) each period. If a firm cannot re-optimize its price, the nominal price evolves according to the indexation rule:

\[
P_{j,t} = P_{j,t-1} \pi_t^{\xi_p}. \quad \text{When the firm is able to optimally adjust its price, the firm sets the price} \quad \tilde{p}_t = P_{j,t} \quad \text{to maximize the value of its expected future dividend stream subject to the demand it faces and taking into account the indexation rule and the probability of not being able to readjust. The first order conditions of this maximization problem are}
\]

\[
\mathcal{K}_t = y_t \tilde{p}_t^{-\theta_p} + \gamma_p E_t \left[ M_{t+1}^{\xi_p} \left( \frac{\pi_t^{\xi_p}}{\pi_{t+1}} \right)^{1-\theta_p} \left( \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \right)^{-\theta_p} \mathcal{K}_{t+1} \right] \quad (9)
\]
and
\[
\frac{\theta_p - 1}{\theta_p} K_t = y_t m c_t \hat{p}_t^{-\theta_p - 1} + \gamma_p E_t \left[ M_{t+1}^{\theta_p} \left( \frac{\pi_{t+1}^{\xi_p}}{\pi_t} \right)^{-\theta_p} \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right)^{-\theta_p - 1} - \frac{1}{\theta_p} K_{t+1} \right],
\]
which is the same for all firms that can adjust their price in period \( t \). Moreover, the variable \( M_{t+1}^{\theta_p} \) represents the real stochastic discount factor from period \( t \) to \( t+1 \) and \( m c_t \) the real marginal costs of the intermediate good firm. Finally, the aggregate price index evolves according to:
\[
1 = \gamma_p \left( \frac{\pi_{t+1}}{\pi_t} \right)^{1-\theta_p} + (1 - \gamma_p) (\hat{p}_t)^{1-\theta_p}
\]

### 2.2 Household

We assume that the representative household has recursive preferences as postulated by Epstein and Zin (1989, 1991) and Weil (1989). Following Rudebusch and Swanson (2012), the value function of the household can be written as
\[
V_t = \begin{cases} 
 u_t + \beta \left( E_t \left[ V_{t+1}^{1-\sigma_{EZ}} \right] \right)^{1-\sigma_{EZ}} & \text{if } u_t > 0 \text{ for all } t \\
 u_t - \beta \left( E_t \left[ (-V_{t+1})^{1-\sigma_{EZ}} \right] \right)^{1-\sigma_{EZ}} & \text{if } u_t < 0 \text{ for all } t 
\end{cases}
\]
where \( u_t \) is the household’s period utility kernel and \( \beta \in (0, 1) \) the subjective discount factor. For \( \sigma_{EZ} > 0 \), these preferences allow us to disentangle the household’s risk aversion from its intertemporal elasticity of the substitution (IES), which is one of the main advantages of Epstein-Zin-Weil preferences. For \( \sigma_{EZ} = 0 \), equation (12) reduces to standard expected utility functions.

Similar to Andreasen et al. (2016), the utility kernel has the following functional form
\[
u_t = \exp\{\varepsilon_{b,t}\} \left[ \frac{1}{1-\gamma} \left( \frac{c_t - bh_t}{z_t^+} \right)^{1-\gamma} - 1 \right] + \frac{\psi_L}{1-\chi} (1-l_t)^{1-\chi},
\]
with consumption \( c_t \), the predetermined stock of consumption habits \( h_t \), hours worked \( l_t \), and preference parameter \( \gamma, \chi \), and \( \psi_L \). The habit stock is external to the household, thus we set \( h_t = C_{t-1} \), the level of aggregate consumption in the previous period. The parameter \( b \in (0, 1) \) controls the degree of external habit formation. The presence of habit formation enables the model to match macroeconomic as well as asset pricing moments jointly as discussed in the literature (see, for example, Hördahl, Tristani, and Vestin, 2008; van Binsbergen et al., 2012). The variable \( \exp\{\varepsilon_{b,t}\} \) represents a preference shock, where \( \varepsilon_{b,t} \) evolves according to the process:
\[
\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \sigma_b \epsilon_{b,t}, \text{ with } \epsilon_{b,t} \overset{iid}{\sim} N(0,1)
\]
the utility kernel ensures a balanced growth path (see, for example, An and Schorfheide, 2007).

The households real period-by-period budget constraint reads

\[ c_t + \frac{I_t}{Y_t} + b_t + T_t = w_t l_t + \frac{b_{t-1} \exp\{r^f_{t-1}\}}{\pi_t} + \int_0^1 \Pi_t(j) \, dj, \] (15)

where the left-hand side represents the household’s resources spent on consumption, investment \( I_t \), a lump-sum tax \( T_t \), and holding of a one-period bond \( b_t \) which accrues the risk-free nominal interest \( r^f_t \) in the following period. The right-hand side of equation (15) describes the income of the household in period \( t \). It consists of labor income \( w_t l_t \) with \( w_t \) the real wage, income from capital services sold to firms \( r^k_t \) last period, the pay-off from bond holdings issued one period before \( b_{t-1} \). Finally, the term \( \Pi(j) \) represents the income from dividends of monopolistically competitive intermediate firms – indexed \( j \) – owned by households.

The households own the economy wide physical capital stock which accumulates according to the following law of motion

\[ k_t = (1 - \delta) k_{t-1} + \exp\{\varepsilon_{i,t}\} \left(1 - \frac{\nu}{2} \left( \frac{I_t}{I_{t-1}} - \exp\{\bar{\mu}_t + \bar{\theta}_t\} \right)^2 \right) I_t, \] (16)

where \( \delta \) is the depreciation rate and \( \nu \geq 0 \) introduces investment adjustment costs as in Christiano et al. (2005). The term \( \exp\{\bar{\mu}_t + \bar{\theta}_t\} \) ensures that the investment adjustment costs are zero along the balanced growth path. Following Justiniano, Primiceri, and Tam-balotti (2010), the variable \( \exp\{\varepsilon_{i,t}\} \) represents an investment shock which measures the exogenous variation in the efficiency with which the final good can be transformed into physical capital and thus into tomorrow’s capital input, where \( \varepsilon_{i,t} \) evolves according to the process:

\[ \varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \sigma_i \varepsilon_{i,t}, \text{ with } \varepsilon_{i,t} \sim_{i.i.d} N(0,1) \] (17)

### 2.3 Monetary policy

We follow Rudebusch and Swanson (2008, 2012) and assume that monetary policy sets the one-period nominal interest rate \( r^f_t \) by following a Taylor-type policy rule expressed annually

\[ 4r^f_t = 4 \cdot \rho_R r^f_{t-1} + (1 - \rho_R) \left( 4 \bar{r}^s + 4 \log \pi_t + \eta_y \log \left( \frac{y_t}{z_t \bar{y}} \right) + \eta_\pi \log \left( \frac{\pi_t}{\pi_0^\pi} \right) \right) + \sigma_m \epsilon_{m,t}, \] (18)

where \( \bar{r}^s \) is the real interest rate at the deterministic steady state and \( \rho_R, \eta_y, \) and \( \eta_\pi \) are policy parameters that characterize the systematic response of the central bank. The term \( \epsilon_{m,t} \) represents a shock to the nominal interest rate which is assumed to be iid normally distributed with mean 0 and variance 1. In particular, monetary policy aims to stabilize the inflation gap \( \log (\pi_t/\pi_0^\pi) \) and the output gap \( \log \left( y_t/z_t^\pi \bar{y} \right) \). The output gap is characterized by the deviation of actual output from its balanced growth path. The inflation gap is characterized by the deviation of inflation from the central bank’s inflation target \( \pi_0^\pi \). Rudebusch and Swanson (2012) interpret changes in the inflation target as long-run nominal (inflation) risk
and show that the existence of such long-run risk is helpful in explaining the historical U.S. term premium. To this end, we follow Gürkaynak et al. (2005b) and Rudebusch and Swanson (2012) and assume that the inflation target is time-varying and can be described by the following law of motion

$$\log \pi_t^* - 4 \log \bar{\pi} = \rho_\pi (\log \pi_{t-1}^* - 4 \log \bar{\pi}) + 4 \zeta_\pi (\log \pi_{t-1} - \log \bar{\pi}) + \sigma_\pi \epsilon_{\pi,t},$$

(19)

with $\epsilon_{\pi,t}$ representing a shock to the inflation target, assumed iid normal with mean 0 and variance 1.

2.4 Aggregation and market clearing

The aggregate resource constraint in the goods market is given by

$$p_t^+ y_t = \exp\{a_t\} k_t^{\alpha} (z_t l_t)^{1-\alpha} - z_t^+ \Omega_t,$$

(20)

where $l_t = \int_0^1 l(j,t) \, dj$ and $k_t = \int_0^1 k(j,t) \, dj$ are the aggregate labor and capital inputs, respectively. The term $p_t^+ = \int_0^1 (P_j^{i,t})^{-\theta_p} \, dj$ measures the price dispersion arising from staggered price setting. Price distortion follows the law of motion

$$p_t^+ = (1 - \gamma_p) (\bar{p}_t)^{-\theta_p} + \gamma_p \left( \frac{\pi_t^\epsilon}{\pi_t} \right)^{-\theta_p} p_{t-1}^+.$$

(21)

Finally, the economy’s aggregate resource constraint implies that

$$y_t = c_t + \frac{I_t}{Y_t} + g_t,$$

(22)

where $g_t = g^z_t \exp(\varepsilon_{g,t})$ represents government consumption expenditures, which are growing with the economy and are financed by lump-sum taxes $g_t = T_t$. The variable $\exp(\varepsilon_{g,t})$ represents an exogenous shock to government consumption with $\varepsilon_{g,t}$ evolving according to the following AR(1) process

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \sigma_g \epsilon_{g,t}, \text{ with } \epsilon_{g,t} \overset{iid}{\sim} N(0, 1).$$

(23)

2.5 The nominal and real term structure

The derivation of the nominal and real term structure in our model is identical to the procedure described, for example, by Rudebusch and Swanson (2008, 2012) or Andreasen (2012a). In particular, the price of any financial asset equals the sum of the stochastically discounted state-contingent payoffs of the asset in period $t+1$ following standard no-arbitrage arguments. For example, the price of a default free $n$-period zero-coupon bond that pays
one unit of cash at maturity satisfies

\[ P_{n,t} = E_t [M_{t,t+n+1}] \]
\[ = E_t [M_{t,t+1} P_{n-1,t+1}] , \]

where \( M_{t,t+1} \) is the household’s nominal stochastic discount factor, which has the following functional form

\[ M_{t,t+1} = \beta \lambda_{t+1} \lambda_{t+1} (V_{t+1})^{-\sigma_{\text{E}} E} E_t \left[ \frac{\sigma_{\text{E}}}{\sigma_{\text{E}}} \right], \]

with \( \lambda_{t} \) the marginal utility of consumption. Additionally, the continuously compounded yield to maturity on the \( n \)-period zero-coupon bond is defined as

\[ \exp \{-nR_{n,t}\} = P_{n,t} \]

Following the literature (e.g. Rudebusch and Swanson, 2012), we define the term premium on a long-term bond as the difference between the yield on the bond and the unobserved risk-neutral yield for that same bond. Similarly to eq. (24), this risk-neutral bond price, \( \hat{P}_{n,t} \), which pays also one unit of cash at maturity is defined as

\[ \hat{P}_{n,t} = \exp \left\{-R_{1,t} \right\} E_t \left[ \hat{P}_{n-1,t+1} \right]. \]

In contrast to eq. (24), discounting is performed using the risk-free rate (with \( R_{1,t} \) equal to the expression \( R_{1,t} \)) rather than the stochastic discount factor. Accordingly, the nominal term premium on a bond with maturity \( n \) is given by

\[ TP_{n,t} = \frac{1}{n} \left( \log \hat{P}_{n,t} - \log P_{n,t} \right). \]

Similarly, we can derive the yield to maturity of a real bond \( P_{n,t}^{\$} \) as well a the price of risk-neutral real bond. Hence, it is straightforward to solve also for the real term premium \( TP_{n,t}^{\$} \) of a bond with maturity \( n \). Finally, we follow the literature and define that inflation risk premia \( TP_{n,t}^{\pi} \) in our model are given by

\[ TP_{n,t}^{\pi} = TP_{n,t} - TP_{n,t}^{\$}. \]

3 Model Solution and Estimation

3.1 Risk-Adjusted Linear Approximation

The macro-finance literature has broadly taken two different approaches to solving models with the term structure of interest rates. One uses a joint nonlinear approximation of the macro and financial variables, with perturbation approaches being the favored choice of nonlinear approximation, and the other separates the macro and financial variables, generally using a (log) linear approximation of the former and an affine model for the latter. We will adopt the method of Meyer-Gohde (2016) that adjusts a linear in states approximation for
risk and provide derivations for the approximation around the means of the endogenous variables approximated out to the second moments of the underlying stochastic driving forces.

Before we turn to our derivations, we will outline the methods used in the literature. Beginning with the first approach, the macro model and the financial variables are modeled jointly, with the stochastic discount factor consistent with agents’ optimizing behavior in the macro model used as the pricing kernel to price financial variables. Third order perturbation approximations have recently been the most used, Rudebusch and Swanson (2008), van Binsbergen et al. (2012), Rudebusch and Swanson (2012), Andreasen (2011), as only at third order to variables such as the term premium become time-varying in this approach (and only at second order do they become nonzero, Hördahl et al. (2008) is an example of such a second order method and De Graeve, Emiris, and Wouters (2009) uses a purely linear model and the expectations hypothesis neglecting endogenous premia). Additionally, many recent perturbation approaches, Andreasen and Zabczyk (2015); Andreasen (2012a), Andreasen et al. (2016), adopt pruning techniques to ensure the asymptotic stability of the nonlinear perturbation commensurate with the local stability properties of the model. While den Haan (1995) uses global methods, policy function iteration and parameterized expectations for macro variables and quadrature to solve the integral in bond pricing, this approach would be computationally prohibitive in the medium scale model we examine here.

The second approach separates the macro and financial variables, using an affine approximation for the yield curve following the empirical finance literature. These price bonds in an arbitrage free setup using either the endogenous pricing kernel implied by households’ stochastic discount factors, as Dew-Becker (2014), Bekaert, Cho, and Moreno (2010), and Palomino (2012), or an estimated exogenously specified kernel, as Hördahl, Tristani, and Vestin (2006), Hördahl and Tristani (2012), Ireland (2015), Rudebusch and Wu (2007), Rudebusch and Wu (2008). In contrast to the methods above, the macro variables and financial variables are approximated separately, with the macro side usually approximated to first order around the deterministic steady state and, taking this as given, applying a log-exponential transformation or assuming a log normal distribution to price bonds (Doh (2011) is an exception, using a second order perturbation for the macro side and applying exponential-quadratic forms of multivariate normal random variables to derive the affine yield curve).

Closest to Meyer-Gohde’s (2016) approach that we adopt here are Dew-Becker (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015), who both approximate the nonlinear macro side of the model to obtain a linear in states approximation with adjustments for risk and then derive affine approximation of the yield curve taking this macro approximation as given. Yet it is not entirely clear what these risk adjustments on the macro side are and what risk adjustments the bond prices in the end encompass. The method we apply approximates the macro and financial variables with the same linear in states method that adjusts the coefficients out to the second moments in shocks around the mean of the endogenous variables, itself approximated out to the second moments in shocks. Thus, our method provides linear in state approximations of macro and financial variables around their means, both adjusted for the second moments in shocks.

The tension between the nonlinearity need to capture the time varying effects of risk underlying asset prices on the one hand and the difficulties bringing nonlinear estimation
routines such as the particle filter to bear on such models on the other is highlighted by van Binsbergen et al. (2012), who model inflation as exogenous in a New Keynesian model to make their Bayesian likelihood estimation tractable. The advantage of a linear in state approximation for estimation has been noted by, e.g., Ang and Piazzesi (2003), Hamilton and Wu (2012), Dew-Becker (2014) and our approach compromises between the goals of non-linearity in risk to capture financial variables and the endogenous stochastic discount factor to price financial variables consistent with the macroeconomy and the need for linearity in states to make the estimation of medium scale policy relevant models feasible. To further reduce the computational burden that becomes prohibitive with solving the model a multitude of times in order to apply the monte carlo methods needed to simulate the posterior density of the model, we apply the PoP method of Andreasen and Zabczyk (2015), formalizing the approach adopted by Hördahl et al. (2008), Rudebusch and Swanson (2012), and others, that recognizes that the perturbation of the model can be solved for in a two-step fashion. First the policy rules for the macro side, including the pricing kernel and the nominal short rate, are approximated and then the financial variables are solved for using this policy function. It is important to note that this is not a further approximation, but rather the recognition that the equations that price different maturities such as eq. (24) are forward recursions that do not enlarge the state space.

Here we develop the method for approximating the solution of our dynamic model from above.\textsuperscript{2} We adjust the points and slopes of the decision rules for risk out to the second moments of the underlying stochastics to capture both constant and time-varying risk premium, as well as the effects of conditional heteroskedasticity.\textsuperscript{3} Unlike standard perturbations, we will construct a linear approximation, giving our method significant computational advantages over higher order polynomial approximations for iterative calculations. Our approach differs from other methods for constructing an approximation centered around a risk-adjusted critical point, such as Juillard (2010), Kliem and Uhlig (2016), and Coeurdacier, Rey, and Winant (2011). First, our method is direct and noniterative relying entirely on perturbation methods to construct the approximation. Second, our method construct the approximation around (an approximation of) the ergodic mean of the true policy function instead of its stochastic or “risky” steady state, placing the locality of our approximation in a region with a likely high (model-based) data density.

Stacking our \(n_y\) endogenous variables into the vector \(y_t\) and our \(n_\varepsilon\) normally distributed exogenous shocks into the vector \(\varepsilon_t\), we collect our equations into the following vector of nonlinear rational expectations difference equations

\[
0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t)] = \hat{F}(y_{t-1}, \varepsilon_t)
\]

(30)

where \(f\) is an \((n_{eq} \times 1)\) vector valued function, continuously \(M\)-times differentiable in all its arguments and with as many equations as endogenous variables \((n_{eq} = n_y)\).

\textsuperscript{2} Meyer-Gohde (2016) provides derivations for adjustments around the deterministic and stochastic steady states, along with those around the mean that we derive and apply here, accuracy checks and formal justifications for the method.

\textsuperscript{3} See, e.g., van Binsbergen et al. (2012) and Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012).
The solution to the functional problem in (30) is the policy function

\[ y_t = g^0(y_{t-1}, \varepsilon_t) \tag{31} \]

Generally, a closed form for (31) is not available, so recourse to numerical approximations is necessary. We will approximate the model with a risk-sensitive linear approximation developed around the ergodic mean of \( y_t \) that we will derive in the following. This approximation maintains linearity, enabling standard linear methods such as impulse responses and likelihood calculations using the Kalman filter, while ameliorating difficulties with standard linearizations around the deterministic steady state, such as certainty equivalence and the lack of precautionary behavior.

We assume that the related deterministic model

\[ 0 = f(y_{t+1}, y_t, y_{t-1}, 0) = F(y_{t-1}, 0) \tag{32} \]

admits the calculation of a fix point, the deterministic steady state, which we define as

**Definition 1 Deterministic Steady State**

Let \( \bar{y} \in \mathbb{R}^{n_y} \) be a vector such that

\[ 0 = F(\bar{y}, 0) \tag{33} \]

We are, however, interested in the stochastic version of the model and will now proceed to nest the deterministic model, for which we can recover a fix point, and the stochastic model, for which we cannot, within a larger continuum of models, following standard practice in the perturbation DSGE literature.

We introduce an auxiliary variable \( \sigma \in [0, 1] \) to scale the stochastic elements in the model. The value \( \sigma = 1 \) corresponds to the “true” stochastic model and \( \sigma = 0 \) returns the deterministic model in (32). Accordingly, the stochastic model, (30), and the deterministic model, (32), can be nested inside the following continuum of models

\[ 0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \tilde{\varepsilon}_t)] = F(\sigma, y_{t-1}, \tilde{\varepsilon}_t), \tilde{\varepsilon}_t \equiv \sigma \varepsilon_t \tag{34} \]

with the associated policy function

\[ y_t = g(y_{t-1}, \tilde{\varepsilon}_t, \sigma) \tag{35} \]

Notice that this reformulation allows us to express the deterministic steady state in definition 1 as the fix point of (34) for \( \sigma = 0 \)

**Definition 2 Deterministic Steady State, Perturbation Formulation**

Let \( \bar{y} \in \mathbb{R}^{n_y} \) be a vector such that

\[ 0 = F(0, \bar{y}, 0) = F(\bar{y}, 0) = g(\bar{y}, 0, 0) \tag{36} \]

We use this deterministic steady state and derivatives of the policy function in (35), recovered by the implicit function theorem,\(^4\) evaluated at at \( \bar{y} \) (both in the deterministic model, (32),

\(^4\)See Jin and Judd (2002).
and towards our stochastic model, (30), to construct our approximation of and around the ergodic mean.

Since \( y \) in the policy function (35) is a vector valued function, its derivatives form a hypercube.\(^5\) Adopting an abbreviated notation, we write \( g_{z^i\sigma^i} \in \mathbb{R}^{n_y \times n_z} \) as the partial derivative of the vector function \( g \) with respect to the state vector \( z_t \) \( j \) times and the perturbation parameter \( \sigma^i \) \( i \) times evaluated at the deterministic steady state.

Instead of using the partial derivatives to construct a Taylor series as is the standard procedure,\(^6\) we would like to construct a more accurate linear approximation of the true policy function (31), centered at the mean of \( y_t \). This will allow us to maintain the linear machinery for estimation and analysis of the model while accounting for the average non-linearities implied by the model. Accordingly, we will construct a linear approximation of (31) around the ergodic mean, which we formalize in the following.

**Proposition 3 Linear Approximation around the Ergodic Mean**

Nest the means of the stochastic model (\( \sigma = 1 \)) and of the deterministic model (\( \sigma = 0 \)) through

\[
\tilde{y}(\sigma) \equiv E[g(y_{t-1}, \sigma \varepsilon_t, \sigma)] = E[y_t]
\]

Then for any \( \sigma \in [0, 1] \), the linear approximation of the policy function, (31), around the mean of \( y_t \) defined in (38) and that of \( \varepsilon_t \) is

\[
y_t \simeq \tilde{y}(\sigma) + y_p(\tilde{y}(\sigma), 0, \sigma)(y_{t-1} - \tilde{y}(\sigma)) + \varepsilon_p(\tilde{y}(\sigma), 0, \sigma) \varepsilon_t
\]

Furthermore, the mean of \( y_t \) defined in (38) and the two additional unknown functions in this linear approximation

\[
\tilde{y}_p(\sigma) \equiv g_p(\tilde{y}(\sigma), 0, \sigma) \quad (40)
\]

\[
\tilde{\varepsilon}(\sigma) \equiv g_{\varepsilon}(\tilde{y}(\sigma), 0, \sigma) \quad (41)
\]

can be approximated, assuming that they are all analytic in a neighborhood around \( \sigma = 0 \) with a radius of at least one,\(^7\) using the partial derivatives of (35) from the standard nonlinear perturbation around the deterministic steady state in definition 2.

\(^5\) We use the method of Lan and Meyer-Gohde (2012) that differentiates conformably with the Kronecker product, allowing us to maintain standard linear algebraic structures to derive our results, see Appendix A for further details.

\(^6\) The Taylor series approximation is, assuming (35) is \( C^M \) with respect to all its arguments, we can write a Taylor series approximation of \( y_t = g(\sigma, z_t) \) at a deterministic steady state as

\[
y_t = \sum_{j=0}^{M} \frac{1}{j!} \left( \sum_{i=0}^{M-j} \frac{1}{i!} g_{z^i\sigma^i} \right) (z_t - \bar{z})^\otimes[j]
\]

Exceptions to this methodology include Judd and Gun (1997) and Judd (1998), who also explore Páde approximations, Evers (2012), who expands the equilibrium conditions (34) first in \( \sigma \) and then constructs a Taylor series approximation in only \( z_t \), and Lombardo’s (2010) matched perturbation that produces a recursively linear solution in progressive orders of approximation.

\(^7\) This ensures that the Taylor series in these functions converge to the true functions for values of \( \sigma \) including the value of one that transitions to the true stochastic problem.
Proof. See the Appendix A. ■

Thus instead of either a linear certainty-equivalent or nonlinear non-certainty-equivalent approximation, we construct a linear non-certainty-equivalent approximation. By using all the higher order derivative of the policy function at the deterministic steady state, we construct approximations of the ergodic mean of \( y_t \) as well as of the first derivatives of the policy function around this ergodic mean. This allows us to use the standard set of tools for estimation and analysis of linear models, without limiting the approximation to the certainty-equivalent approximation around the deterministic steady state.

3.2 Data

We estimate the model with quarterly US data between 1983:q1 and 2007:q4. As such, our sample covers the Great Moderation, stopping right before the onset of the Great Recession. This period is chosen specifically because for two reasons. First, it is widely accepted in the literature that the US faced a systematic change in monetary policy after Paul Volcker became chairman of the Federal Reserve (e.g. Clarida, Galí, and Gertler, 2000). Second, the start of the Great Recession, the financial crisis of 2008, along with the zero interest policy rates that prevailed from December 2008 onward marks an another structural change in US monetary policy. While the systematic behavior of monetary policy is an important driver of the yield curve, as pointed out, for example, by Rudebusch and Swanson (2012), we chose a time episode which is characterized by a relatively stable monetary policy regime.⁸

In particular, the estimation of the model is based on four macroeconomic time series complemented by six time series on the nominal yield curve and two time series of survey data on interest rate forecasts.⁹ The macroeconomic dynamics are characterized by real GDP growth, real private investment growth, real private consumption growth, and annualized GDP deflator inflation rates. While the last is measured in levels the remaining variables are expressed in per capita log-differences using the civilian noninstitutional population over 16 years (CNP16OV) series from the U.S. Department of Labor, Bureau of Labor Statistics.

The nominal yield curve is measured by the 1-quarter, 1-year, 3-year, 5-year, and 10-year annualized interest rates of US Treasury bonds. With the exception of the 1-quarter interest rate, the data are from Adrian et al. (2013) which are identical to the otherwise often used time series by Gürkaynak, Sack, and Wright (2007). For the 1-quarter maturity, we use the 3-month Treasury Bill rate from the Board of Governors of the Federal Reserve System. To have a consistent description of the yield curve, we use this interest rate as the policy rate in our model instead of the effective Fed funds rate.

Survey data on interest rate forecasts have shown to be helpful to improve the identification of term structure models (see, for example, Kim and Orphanides, 2012; Andreasen, 2011). For this reason, we use incorporate expectations 1 and 4-quarters ahead on the 3-month Treasury Bill into the estimation. The data are taken from the Survey of Professional Forecasters.

⁸See, for example, Bikbov and Chernov (2013) and Bianchi, Kung, and Morales (2016) for an investigation of policy regime changes and the term structure of interest rates.

⁹See Appendix B for details on the source and a description of any data used in this paper.
3.3 Bayesian estimation

As shown in section 3.1, our construction of linear non-certainty-equivalent approximation results in a policy function eq. 39 which is linear in states and shocks. This characteristic allows us to use standard Bayesian estimation techniques, including a linear Kalman filter, commonly applied in the literature to estimate linear DSGE model (An and Schorfheide, 2007). Subsequently, this subsection discusses the prior choice for the estimated parameters as well as the calibration of the remaining parameters.

Given the choice of our observable variables and the characteristics of our model, for example, the highly stylized labor market, some of the model parameters can hardly be expected to be identified. These parameters are calibrated either following the literature or related to our observables. In particular, we calibrate the steady state growth rates, $\bar{z}$ and $\bar{\Psi}$ to 0.54/100 and 0.08/100 which implies growth rates of 0.54 and 0.62 percent for GDP and investment as in our sample. Moreover, we calibrate the depreciation of capital, $\delta$, to 10% per year and the share of capital, $\alpha$, in the production function to 1/3. We also assume that in the deterministic steady state, the labor supply $\bar{l}$ and government consumption over GDP $\bar{g}/\bar{y}$ are 1/3 and 0.19, respectively. The discount rate $\beta$ is set equal to 0.99 and the steady state of the elasticity of substitution between the intermediate goods $\theta_p$ is equal to 6 which implies a markup of 20%. Following Andreasen et al. (2016), we set the price indexation $\xi_p = 0$ and calibrate the Frisch elasticity of labor supply $FE$ to 0.5. Hence, we can solve recursively for $\chi = 1/FE \cdot (1/\bar{l} - 1)$. Table 1 summarizes the parameter calibration.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology trend in percent</td>
<td>$\bar{z}$</td>
<td>0.54/100</td>
</tr>
<tr>
<td>Investment trend in percent</td>
<td>$\bar{\Psi}$</td>
<td>0.08/100</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>1/3</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Price markup</td>
<td>$\theta_p/(\theta_p - 1)$</td>
<td>1.2</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\xi_p$</td>
<td>0</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Frisch elasticity of labor supply</td>
<td>$FE$</td>
<td>0.5</td>
</tr>
<tr>
<td>Labor supply</td>
<td>$\bar{l}$</td>
<td>1/3</td>
</tr>
<tr>
<td>Ratio of government consumption to</td>
<td>$\bar{g}/\bar{y}$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 1: Parameter calibration.

The remaining parameter of the model are estimated. Since the focus of the paper is to jointly explain macroeconomic as well as asset pricing facts, we pay special attention to selected first and second moments when estimating the DSGE model. As described in Kliem and Uhlig (2016), the practical problem boils down to having just one observation on the means, e.g., of the slope, curvature, and level of the yield curve, while there are many observations helping to identify parameters crucial for the macroeconomic dynamics of the model. To this end, we apply an endogenous prior approach similar to Del Negro and Schorfheide (2008) and Christiano, Trabandt, and Walentin (2011). In particular, we use a
set of initial priors, \( p(\theta) \), where the priors are independent across parameters. Then, we use two sets of first and second moments from a pre-sample. We treat the first and second moments of interest separably in two blocks to capture potentially different precisions of beliefs regarding first and second moments. Finally, the product of the initial priors, the likelihood of selected first moments, the likelihood of selected second moment form the endogenous prior distribution which we use for the estimation of the model. In the next paragraphs, we describe the method of endogenously formed priors regarding first and second moments as well as its practical application in the paper.

Following Del Negro and Schorfheide (2008), we assume \( \hat{F} \) to be a vector that collects the first moments of interest from our pre-sample and \( F_M(\theta) \) be a vector-valued function which relates model parameters and ergodic means

\[
\hat{F} = F_M(\theta) + \eta,
\]

where \( \eta \) is a vector of measurement errors. In our application, we assume that the error terms \( \eta \) are independently and normally distributed. Hence, we express Eq. 42 as quasi-likelihood function which can be interpreted as the conditional density

\[
L \left( F_M(\theta) | \hat{F}, T^* \right) = \exp \left\{ -\frac{T^*}{2} \left( \hat{F} - F_M(\theta) \right)' \Sigma^{-1}_\eta \left( \hat{F} - F_M(\theta) \right) \right\} \]

\[
= p \left( \hat{F} \mid F_M(\theta), T^* \right)
\]

This quasi-likelihood is small for values of \( \theta \) for which the DSGE model predicts first moments that strongly differ from the measures of the pre-sample. The parameter \( T^* \) captures, along with the standard deviation of \( \eta \), the precision of our beliefs about the first moments. In practice we set \( T^* \) to the length of the pre-sample.

For the application in this paper, we assume that the vector \( \hat{F} \) contains the mean of inflation and the means of proxies for the level, slope, and curvature factors of the yield curve. We include the mean of inflation because due to the non-linearities in our model imposes a strong the precautionary motive and, therefore, the predicted ergodic mean of inflation differs from its deterministic steady state, \( \bar{\pi} \). This effect of the precautionary motive is also discussed by Tallarini (2000) and Andreasen (2011). Regarding \( L \left( F_M(\theta) | \hat{F} \right) \), we assume that \( E_t [400\pi|\theta] \) is normally distributed with mean 2.5 and variance 0.1.

Moreover, we follow e.g. Diebold, Rudebusch, and Aruoba (2006) and specify common proxies for the level, slope, and curvature factors of the yield curve. In particular, the proxy for the level factor is \( (R_{1,t} + R_{8,t} + R_{40,t}) / 3 \), with all yields expressed in annualized terms and the nominal yield of the 1-quarter Treasury Bond equal to the policy rate in the model. Moreover, the proxies for the slope and curvature factors are defined as \( R_{1,t} - R_{40,t} \) and \( 2R_{8,t} - R_{1,t} - R_{40,t} \), respectively. Regarding \( L \left( F_M(\theta) | \hat{F} \right) \), we assume that the ergodic mean of each factor is normally distributed, with the mean equal to its empirical counterpart of the pre-sample. Moreover, we assume that the means of level, slope, and curvature have a

\[\text{In practice, we follow Christiano et al. (2011) and use the actual sample as our pre-sample as no other suitable data is available because of the monetary regime changes immediately before and after our sample.}\]
variance of 22, 12, and 9 basis points respectively. Thus, the mean and variances can be interpreted as \( \hat{F} \) value and the variance of the measurement error \( \eta \) in Eq. (42).

Additionally, we use selected second moments of macroeconomic variables, those regarding which we have a priori knowledge, to inform our prior distribution and apply the approach of by Christiano et al. (2011). This approach uses classical large sample theory to form a large sample approximation to the likelihood of the pre-sample statistics. Moreover, the approach is conceptually similar to the one proposed by Del Negro and Schorfheide (2008) but differs in some important respects. Specifically, Del Negro and Schorfheide (2008) focus on the model-implied p-th order vector autoregression, which implies that the likelihood of the second moments is known exactly conditional on the DSGE model parameters, and requires no large-sample approximation in contrast to the approach by Christiano et al. (2011). Yet, the latter approach is more flexible insofar as the statistics to target are concerned. Accordingly, let \( S \) be a column vector containing the second moments of interest, then, as shown by Christiano et al. (2011) under the assumption of large sample, the estimator of \( S \) is

\[
\hat{S} \sim N \left( S^0, \frac{\hat{\Sigma}_S}{T} \right),
\]

with \( S^0 \) the true value of \( S \), \( T \) the sample length, and \( \hat{\Sigma}_S \) the estimate of the zero-frequency spectral density. Now, let \( S_M(\theta) \) be a function which maps our DSGE model parameters \( \theta \) into \( S \). Then, for \( n \) targeted second moments and sufficiently large \( T \), the density of \( \hat{S} \) is given by

\[
p \left( \hat{S} | \theta \right) = \left( \frac{T}{2\pi} \right)^{\frac{n}{2}} \left\| \hat{\Sigma}_S \right\|^{-\frac{1}{2}} \exp \left\{ -\frac{T}{2} \left( \hat{S} - S_M(\theta) \right)' \hat{\Sigma}_S^{-1} \left( \hat{S} - S_M(\theta) \right) \right\}
\]

In our application, \( S \) is a set of variances of macroeconomic variables (GDP growth, consumption growth, investment growth, inflation, and the policy rate). In conclusion, the overall endogenous prior distribution takes the following form

\[
p \left( \theta | \hat{F}, \hat{S}, T^* \right) = C^{-1} p \left( \theta \right) p \left( \hat{F} | F_M \left( \theta \right), T^* \right) p \left( \hat{S} | \theta \right)
\]

where \( p \left( \theta \right) \) is the initial prior distribution and \( C \) a normalization constant. Two thing are noteworthy. First, while the initial priors are independent across parameters, as is typical in Bayesian analysis, the endogenous prior is not independent across parameters. Second, the normalization constant \( C \) is necessary for, e.g., posterior odds calculation but not for estimating the model. To this end, we do not calculate this constant, which has otherwise to be approximated (see, for example, Del Negro and Schorfheide, 2008; Kliem and Uhlig, 2016). So, the posterior distribution is given by

\[
p \left( \theta | X, \hat{F}, \hat{S}, T^* \right) \propto p \left( \theta | \hat{F}, \hat{S}, T^* \right) p \left( X | \theta \right)
\]

with \( p \left( X | \theta \right) \) the likelihood of the data conditional on DSGE model parameters \( \theta \).

Table 2 summarizes the initial prior distributions of the remaining parameters. While
<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Domain</th>
<th>Density</th>
<th>Para(1)</th>
<th>Para(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative risk aversion</td>
<td>$RRA/100$</td>
<td>$\mathbb{R}^+$</td>
<td>Uniform</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Calvo parameter</td>
<td>$\gamma_p$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Investment adjustment</td>
<td>$\nu$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>4.0</td>
<td>0.75</td>
</tr>
<tr>
<td>Habit formation</td>
<td>$b$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Intertemporal elas. substitution</td>
<td>$IES$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td>Steady state inflation</td>
<td>$100(\bar{\pi} - 1)$</td>
<td>$\mathbb{R}^+$</td>
<td>Uniform</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Interest rate AR coefficient</td>
<td>$\rho_R$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Interest rate inflation coefficient</td>
<td>$\eta_\pi$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>1</td>
<td>0.15</td>
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<tr>
<td>Interest rate output coefficient</td>
<td>$\eta_y$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Inflation target coefficient</td>
<td>$100\zeta_\pi$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>AR coefficient technology</td>
<td>$\rho_a$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
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<td>AR coefficient preference</td>
<td>$\rho_b$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>AR coefficient investment</td>
<td>$\rho_i$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>AR coefficient gov. spending</td>
<td>$\rho_g$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>AR coefficient inflation target</td>
<td>$\rho_\pi$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.95</td>
<td>0.025</td>
</tr>
<tr>
<td>AR coefficient long-run growth</td>
<td>$\rho_z$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>AR coefficient fixed costs</td>
<td>$\rho_\Omega$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>S.d. technology</td>
<td>$100\sigma_a$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. preference</td>
<td>$100\sigma_b$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. investment</td>
<td>$100\sigma_i$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. monetary policy shock</td>
<td>$100\sigma_m$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. government spending</td>
<td>$100\sigma_g$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. inflation target</td>
<td>$100\sigma_\pi$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>S.d. long-run growth</td>
<td>$100\sigma_z$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>S.d. fixed costs</td>
<td>$100\sigma_\Omega$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>ME 1-year T-Bill</td>
<td>$R_{4,t}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 2-year T-Bill</td>
<td>$R_{8,t}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 3-year T-Bill</td>
<td>$R_{12,t}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 5-year T-Bill</td>
<td>$R_{20,t}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 10-year T-Bill</td>
<td>$R_{40,t}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 1Q-expected policy rate</td>
<td>$E_t[R_{t,t+1}]$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
<tr>
<td>ME 4Q-expected policy rate</td>
<td>$E_t[R_{t,t+4}]$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGam</td>
<td>0.005</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 2: Initial prior distribution. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distributions and to the lower and upper bounds for the Uniform distribution.
the prior distributions for most of the parameters are chosen following the literature, it is noteworthy to highlight some deviations. First, we do not use a prior for the preference parameters, $\gamma$ and $\alpha_{EZ}$, directly, but rather impose priors for the intertemporal elasticity of substitution, $IES$, and the coefficient relative risk aversion, $RRA$, and solve for the underlying parameters. The intertemporal elasticity of substitution, $IES$, in our model with external habit formation is

$$IES = \frac{1}{\gamma} \left( 1 - \frac{b}{\exp(\bar{z}^+)} \right).$$

Moreover, we follow Swanson (2012) by using his closed-form expressions for risk aversion, $RRA$, which takes into account that households can vary their labor supply. Hence, our model implies

$$RRA = \frac{\gamma}{1 - \frac{b}{\exp(\bar{z}^+)} + \frac{2}{\chi} (1 - \bar{l}) \frac{\bar{w}}{\bar{c}}} + \alpha_{EZ} \frac{1 - \gamma}{1 - \frac{b}{\exp(\bar{z}^+)} - \left( 1 - \frac{b}{\exp(\bar{z}^+)} \right)^{\gamma \bar{c}^{-1}} + \frac{\bar{w}(1 - \bar{l})}{\bar{c}} \frac{1 - \gamma}{1 - \chi}},$$

where $\bar{l}$ is the steady state labor supply, while $\bar{c}$ and $\bar{w}$ are consumption and the real wage in the deterministic steady state, respectively. Given the wide range of different estimates for relative risk aversion in the macro- and finance literatures, we assume initially an uniform prior with 0 and 2000 being the lower and upper bounds; our endogenous prior approach, however, does impose an informative prior. We proceed analogously for the deterministic steady state of inflation and choose an uninformative initial prior distribution. Finally, we add measurement errors to the 1-year, 2-year, 3-year, 5-year, and 10-year Treasury bond yields as well as to the expected policy rate expected 1 and 4-quarters ahead. By adding measurement errors along the yield curve, we are following the empirical term structure literature (see, for example, Diebold et al., 2006) and the measurement errors on the expectations of the short rate align the imperfect fit of the data with the model’s rational expectation assumption.

4 Estimation Results

In the following section, we present the estimated parameters and discuss the predicted first and second moments of endogenous variables. Additionally, we compare the historical term premium predicted by our model with various estimates from the literature. We estimate the posterior mode of the distribution and employ a random walk Metropolis-Hasting algorithm to simulate the posterior distribution of the parameters and to quantify the uncertainty of our estimates of the same. In particular, we run two chains, each with 60,000 parameter vector draws where the first 75% have been discarded.

4.1 Parameter estimates

Table 3 provides posterior statistics of the estimated parameters, e.g., the posterior mode, posterior mean and the 90% posterior credible set. Figures 12 and 13 illustrate the posterior
distribution of each parameter in comparison to its initial prior distribution. The results indicate that the posterior distributions of all structural parameters are well approximated and differ from the initial prior distribution. In the following, we discuss some key parameters in greater detail.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Mode</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative risk aversion</td>
<td>$RRA$</td>
<td>89.86</td>
<td>88.08</td>
<td>74.34</td>
<td>103.54</td>
</tr>
<tr>
<td>Calvo parameter</td>
<td>$\gamma_p$</td>
<td>0.853</td>
<td>0.854</td>
<td>0.843</td>
<td>0.866</td>
</tr>
<tr>
<td>Investment adjustment</td>
<td>$\nu$</td>
<td>1.417</td>
<td>1.437</td>
<td>1.179</td>
<td>1.697</td>
</tr>
<tr>
<td>Habit formation</td>
<td>$b$</td>
<td>0.685</td>
<td>0.673</td>
<td>0.616</td>
<td>0.744</td>
</tr>
<tr>
<td>Intertemporal elas. substitution</td>
<td>$IES$</td>
<td>0.089</td>
<td>0.088</td>
<td>0.076</td>
<td>0.096</td>
</tr>
<tr>
<td>Steady state inflation</td>
<td>$100(\pi - 1)$</td>
<td>1.038</td>
<td>1.039</td>
<td>0.988</td>
<td>1.087</td>
</tr>
</tbody>
</table>

| Interest rate AR coefficient              | $\rho_R$ | 0.754  | 0.752  | 0.714 | 0.783  |
| Interest rate inflation coefficient       | $\eta_\pi$ | 3.124  | 3.166  | 2.842 | 3.497  |
| Interest rate output coefficient          | $\eta_y$ | 0.156  | 0.161  | 0.123 | 0.195  |
| Inflation target coefficient              | $100\zeta_\pi$ | 0.210  | 0.257  | 0.163 | 0.346  |

| AR coefficient technology                 | $\rho_a$  | 0.366  | 0.345  | 0.293 | 0.400  |
| AR coefficient preference                 | $\rho_b$  | 0.820  | 0.815  | 0.786 | 0.847  |
| AR coefficient investment                 | $\rho_i$  | 0.956  | 0.955  | 0.950 | 0.960  |
| AR coefficient gov. spending              | $\rho_g$  | 0.910  | 0.911  | 0.886 | 0.944  |
| AR coefficient inflation target           | $\rho_\pi$ | 0.934  | 0.927  | 0.902 | 0.948  |
| AR coefficient long-run growth            | $\rho_z$  | 0.630  | 0.613  | 0.501 | 0.736  |
| AR coefficient fixed cost                 | $\rho_\Omega$ | 0.928  | 0.928  | 0.922 | 0.933  |

| S.d. technology                           | $100\sigma_a$ | 2.333  | 2.472  | 1.878 | 3.025  |
| S.d. preference                           | $100\sigma_b$ | 4.878  | 5.014  | 4.415 | 5.041  |
| S.d. investment                           | $100\sigma_i$ | 2.516  | 2.529  | 2.350 | 2.699  |
| S.d. monetary policy shock                | $100\sigma_m$ | 0.561  | 0.566  | 0.501 | 0.631  |
| S.d. government spending                  | $100\sigma_g$ | 2.010  | 2.047  | 1.868 | 2.233  |
| S.d. inflation target                     | $100\sigma_\pi$ | 0.167  | 0.172  | 0.131 | 0.211  |
| S.d. long-run growth                      | $100\sigma_z$ | 0.345  | 0.346  | 0.241 | 0.435  |
| S.d. fixed cost                           | $100\sigma_\Omega$ | 9.766  | 9.776  | 9.183 | 10.461 |

| ME 1-year T-Bill                          | $400R_{4,t}$ | 0.185  | 0.187  | 0.160 | 0.224  |
| ME 2-year T-Bill                          | $400R_{8,t}$ | 0.084  | 0.084  | 0.070 | 0.099  |
| ME 3-year T-Bill                          | $400R_{12,t}$ | 0.078  | 0.081  | 0.067 | 0.097  |
| ME 5-year T-Bill                          | $400R_{20,t}$ | 0.152  | 0.155  | 0.125 | 0.176  |
| ME 10-year T-Bill                         | $400R_{40,t}$ | 0.287  | 0.300  | 0.261 | 0.337  |
| ME 1Q-expected policy rate                | $400E_t[R_{t+1}]$ | 0.456  | 0.472  | 0.425 | 0.524  |
| ME 4Q-expected policy rate                | $400E_t[R_{t+4}]$ | 0.738  | 0.764  | 0.683 | 0.848  |

Table 3: Posterior statistics. Posterior means and parameter distributions are based on a standard MCMC algorithm with two chains of 60,000 parameter vector draws each, 75% of the draws used for burn-in, and a draw acceptance rates about 1/3.

We find a low steady-state intertemporal elasticity of substitution ($IES = 0.089$) and a high relative risk aversion ($RRA \approx 90$). Both estimates are common in much of the existing
macro-finance literature. For example, van Binsbergen et al. (2012) estimate a relative risk aversion of 65, Rudebusch and Swanson (2012) find a value of 110, and Andreasen (2011) estimate a value of around 28 for the U.S. when accounting for variable labor supply as in Swanson (2012). In their benchmark model, which is closely related to our model, Andreasen et al. (2016) find an even higher value of over 600. However, it is difficult to compare these number with each other. First, all these studies use longer samples for the estimation, whereas our study covers just the Great Moderation. Second, the models differ regarding the underlying structural shocks of the economy. As pointed out by van Binsbergen et al. (2012), models which feature a higher volatility of shocks (higher risk) that increase the volatility of the stochastic discount factor need a smaller amount of, e.g., relative risk aversion to match average bond yields. Nevertheless, our estimates are high in comparison with risk aversion used in endowment economies or in comparison with micro-studies (Barsky, Juster, Kimball, and Shapiro, 1997). However, Malloy, Moskowitz, and Vissing-Jørgensen (2009) show that risk aversion estimated for stockholders in the U.S. is substantially lower than a representative agent using aggregate consumption. Tho this end, the authors find that the estimated relative risk aversion increases to 81 when using aggregate consumption. Alternatively, Barillas, Hansen, and Sargent (2009) argue that a small amount of model uncertainty can substitute for the large degree relative risk aversion often found in the literature.

We estimate a quarterly steady state inflation of 1.04% which is substantially higher than the mean observed inflation rate (0.64%). As mentioned before, the difference is related to the household’s precautionary motive in our model. For the inflation target, we estimate $\rho_\pi = 0.93$ and $\zeta_\pi = 0.002$. The latter coefficient is similar to Rudebusch and Swanson (2012), while the former coefficient is slightly smaller, implying an less persistent effect of nominal risk in our model. Moreover, we estimate a moderate size of investment adjustment costs ($\nu = 1.4$) and comparable estimates to the literature for price stickiness ($\gamma_p = 0.85$) and external habit formation ($b = 0.67$). Finally, we find that monetary policy puts more weight

![Figure 1: Observed and model implied nominal returns of treasury bills and returns of expected short rates.](image-url)
on stabilizing the inflation gap ($\eta_\pi = 3.13$) then on the output gap ($\eta_y = 0.16$) and smooths changes in the policy rate ($\rho_R = 0.75$).

Figure 1 shows the the historical time series (dash-dotted line) and the model-implied smoothed time series (solid line) for the seven variables estimated with measurement error. Note that we estimate small measurement errors along the yield curve. In particular, the measurement errors range between 7 and 29 basis points, implying a correlation between the smoothed model implied yields and the data of 0.99 or higher. The measurement errors for the 1-quarter ahead and 1-year ahead rate expectations of the 3-month T-Bill are 45 and 74 basis points, respectively, delivering high correlations (0.94 and 0.98) of our model based expectations with the data from the Survey of Professional Forecasters.

4.2 Predicted Moments

In the following subsection, we start our posterior analysis with respect to the predicted first and second moments. Figure 2 shows the predicted ergodic means of the nominal yields in relation to the means of the corresponding data. The figure illustrates the success of our estimation approach, with the a priori information about the level, slope, and curvature, based on only 3-month, 2-year, and 10-year nominal yields, sufficient to estimate first moments for all maturities.

Backus, Gregory, and Zin (1989) and den Haan (1995) formalized the bond-pricing puzzle by asking the question: Why is the yield curve upward sloping? This question refers to the idea that long-term bond should carry an insurance-like negative risk premium, and therefore the yield curve should be downward sloping. However, the data for nominal yields as well as estimates for the nominal term premium suggest the opposite as does our model (see Figure 3(b)). The mechanism behind this has already described by, e.g., Rudebusch
and Swanson (2012): supply shocks move consumption and inflation in opposite directions, imposing a negative correlation between the two and, therefore, generating a positive term premium. To this end, Piazzesi and Schneider (2007) show that for the U.S., consumption and inflation were negatively correlated form 1952-2004, which suggests that supply shocks play a relatively important role in generating the upward sloping nominal term structure in the data anf in their New-Keynsian DSGE model (see, for example, Rudebusch and Swanson, 2012).

This line of argumentation holds for the nominal term structure, but not for the real term structure. Nevertheless, as illustrated by Figures 3(a) and 3(c), our model predicts also a upward-sloping real term structure which is in line with the data (see, for example, Abrahams et al., 2016). The mechanism in our model follows the description by Hördahl et al. (2008), habit formation introduces a hump-shaped response of consumption. This makes consumption growth positively autocorrelated but at the same time ensures that the agents’ precautionary saving motive plays a smaller role for longer maturities. This circumstance is also illustrated in 3(a), while the red line shows the yield curve in absence of risk; the difference between the ergodic mean and the deterministic steady state illustrate the decreasing precautionary motive along the maturities.
Figure 3(d) shows that our model predicts an upward sloping inflation risk premium consistent with the data (see, for example, Abrahams et al., 2016). The ergodic mean of inflation risk is approximately half as big as the real term premia for all maturities. Consequentially, our results suggest that most of the average slope of the nominal term structure is related to real rather than to inflation risk. Again, this finding is consistent with recent estimates for the U.S. and is also qualitatively comparable to the results by Hördahl and Tristani (2012) for the Euro area.

<table>
<thead>
<tr>
<th>Name</th>
<th>Data Mean</th>
<th>Data S.d.</th>
<th>Model Mean</th>
<th>Model S.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.540</td>
<td>0.803*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.515,0.764]</td>
<td>[0.771,0.836]</td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.540</td>
<td>0.593</td>
<td>0.540</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.383,0.515]</td>
<td>[0.528,0.584]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption growth</td>
<td>0.610</td>
<td>0.435</td>
<td>0.540</td>
<td>0.559*</td>
</tr>
<tr>
<td></td>
<td>[1.796,2.744]</td>
<td>[2.147,2.436]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment growth</td>
<td>0.620</td>
<td>2.096</td>
<td>0.620</td>
<td>2.292*</td>
</tr>
<tr>
<td>Annualized inflation</td>
<td>2.496</td>
<td>1.022</td>
<td>2.469*</td>
<td>1.198*</td>
</tr>
<tr>
<td></td>
<td>[0.840,1.493]</td>
<td>[2.426,2.527]</td>
<td>[1.134,1.255]</td>
<td></td>
</tr>
<tr>
<td>Annualized policy rate</td>
<td>5.034</td>
<td>2.069</td>
<td>5.144*</td>
<td>2.861*</td>
</tr>
<tr>
<td></td>
<td>[1.521,3.927]</td>
<td>[5.064,5.217]</td>
<td>[2.724,3.055]</td>
<td></td>
</tr>
<tr>
<td>1-year T-Bill</td>
<td>5.577</td>
<td>2.334</td>
<td>5.515</td>
<td>2.574</td>
</tr>
<tr>
<td></td>
<td>[1.724,4.417]</td>
<td>[5.436,5.579]</td>
<td>[2.450,2.756]</td>
<td></td>
</tr>
<tr>
<td>2-year T-Bill</td>
<td>5.896</td>
<td>2.373</td>
<td>5.900*</td>
<td>2.257</td>
</tr>
<tr>
<td></td>
<td>[1.699,4.435]</td>
<td>[5.829,5.952]</td>
<td>[2.141,2.407]</td>
<td></td>
</tr>
<tr>
<td>3-year T-Bill</td>
<td>6.124</td>
<td>2.384</td>
<td>6.106</td>
<td>2.019</td>
</tr>
<tr>
<td></td>
<td>[1.699,4.580]</td>
<td>[6.032,6.159]</td>
<td>[1.915,2.154]</td>
<td></td>
</tr>
<tr>
<td>5-year T-Bill</td>
<td>6.460</td>
<td>2.311</td>
<td>6.359</td>
<td>1.662</td>
</tr>
<tr>
<td></td>
<td>[1.611,4.643]</td>
<td>[6.280,6.414]</td>
<td>[1.583,1.777]</td>
<td></td>
</tr>
<tr>
<td>10-year T-Bill</td>
<td>6.974</td>
<td>2.101</td>
<td>7.013*</td>
<td>1.150</td>
</tr>
<tr>
<td></td>
<td>[1.480,4.634]</td>
<td>[6.922,7.069]</td>
<td>[1.125,1.263]</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Predicted first and second moments of selected macro and financial variables. Bold moments are calibrated and moments marked with * where used directly or indirectly to form the endogenous prior. All variables are in annualized percentage.

Table 4 presents the first and second moments of the observables predicted by the model as well as those contained in the data. As the predicted moments from the model are population moments, we have calculated the corresponding population moments of the data by using an Bayesian vector autoregression model with two-lags. The results illustrate that our estimation approach delivers an ergodic mean of inflation comparable to the mean of the data as intended and, as a result, captures the household’s precautionary saving motive appropriately. Moreover, the predicted moments regarding the macroeconomic variables are in line with the data, highlighting the ability of our New Keynesian DSGE model to match

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11 We fit a BVAR(2) to the observables by assuming a weak Normal-Whishart prior for the coefficients adn covariance of the BVAR. For the comparison, we draw 1200 parameter vector draws from the posterior of the BVAR as well as 1200 parameter vector draws from posterior distribution of the DSGE model.
financial and macroeconomic moments jointly (see also Andreasen et al., 2016). Regarding the treasury bonds, our model misses the high volatility for longer maturities. This issue of general equilibrium models has been described in den Haan (1995) and is related to some missing persistence in the model (see Hördahl et al., 2008). We do not see this, however, as a fatal shortcoming of our analysis but rather as illustration of the competing goals the model face: matching highly volatile treasury bonds while predicting a very smooth inflation rate.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Mean 50%</th>
<th>5%</th>
<th>95%</th>
<th>50%</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year real T-Bill</td>
<td>$R^8_{1,t}$</td>
<td>2.68</td>
<td>2.57</td>
<td>2.76</td>
<td>2.13</td>
<td>2.02</td>
<td>2.26</td>
</tr>
<tr>
<td>2-year real T-Bill</td>
<td>$R^8_{2,t}$</td>
<td>3.00</td>
<td>2.90</td>
<td>3.08</td>
<td>1.84</td>
<td>1.74</td>
<td>1.95</td>
</tr>
<tr>
<td>3-year real T-Bill</td>
<td>$R^8_{3,t}$</td>
<td>3.16</td>
<td>3.07</td>
<td>3.24</td>
<td>1.63</td>
<td>1.54</td>
<td>1.73</td>
</tr>
<tr>
<td>5-year real T-Bill</td>
<td>$R^8_{5,t}$</td>
<td>3.32</td>
<td>3.22</td>
<td>3.41</td>
<td>1.32</td>
<td>1.25</td>
<td>1.40</td>
</tr>
<tr>
<td>10-year real T-Bill</td>
<td>$R^8_{10,t}$</td>
<td>3.72</td>
<td>3.63</td>
<td>3.83</td>
<td>0.87</td>
<td>0.82</td>
<td>0.92</td>
</tr>
<tr>
<td>1-year nominal term premium</td>
<td>$TP^1_{1,t}$</td>
<td>37.18</td>
<td>34.93</td>
<td>39.26</td>
<td>10.95</td>
<td>9.42</td>
<td>12.48</td>
</tr>
<tr>
<td>2-year nominal term premium</td>
<td>$TP^1_{2,t}$</td>
<td>76.98</td>
<td>73.10</td>
<td>80.22</td>
<td>24.44</td>
<td>21.41</td>
<td>27.29</td>
</tr>
<tr>
<td>3-year nominal term premium</td>
<td>$TP^1_{3,t}$</td>
<td>99.31</td>
<td>94.83</td>
<td>103.72</td>
<td>32.32</td>
<td>28.52</td>
<td>36.03</td>
</tr>
<tr>
<td>5-year nominal term premium</td>
<td>$TP^1_{5,t}$</td>
<td>128.64</td>
<td>123.47</td>
<td>133.90</td>
<td>40.61</td>
<td>36.17</td>
<td>45.29</td>
</tr>
<tr>
<td>10-year nominal term premium</td>
<td>$TP^1_{10,t}$</td>
<td>201.86</td>
<td>195.82</td>
<td>208.68</td>
<td>52.23</td>
<td>47.33</td>
<td>59.18</td>
</tr>
<tr>
<td>1-year real term premium</td>
<td>$TP^r_{r,t}$</td>
<td>23.95</td>
<td>21.78</td>
<td>26.10</td>
<td>5.92</td>
<td>4.84</td>
<td>7.08</td>
</tr>
<tr>
<td>2-year real term premium</td>
<td>$TP^r_{r,t}$</td>
<td>56.99</td>
<td>52.96</td>
<td>60.35</td>
<td>16.37</td>
<td>14.01</td>
<td>18.57</td>
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<tr>
<td>3-year real term premium</td>
<td>$TP^r_{r,t}$</td>
<td>74.65</td>
<td>69.55</td>
<td>79.33</td>
<td>22.60</td>
<td>19.86</td>
<td>25.35</td>
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<tr>
<td>5-year real term premium</td>
<td>$TP^r_{r,t}$</td>
<td>92.95</td>
<td>87.46</td>
<td>99.49</td>
<td>28.27</td>
<td>25.24</td>
<td>31.23</td>
</tr>
<tr>
<td>10-year real term premium</td>
<td>$TP^r_{r,t}$</td>
<td>138.18</td>
<td>131.76</td>
<td>146.82</td>
<td>36.05</td>
<td>32.66</td>
<td>39.90</td>
</tr>
<tr>
<td>1-year inflation risk premium</td>
<td>$TP^r_{1,t}$</td>
<td>13.28</td>
<td>12.18</td>
<td>14.39</td>
<td>5.17</td>
<td>4.50</td>
<td>5.86</td>
</tr>
<tr>
<td>2-year inflation risk premium</td>
<td>$TP^r_{2,t}$</td>
<td>20.13</td>
<td>17.35</td>
<td>21.93</td>
<td>8.19</td>
<td>7.21</td>
<td>9.36</td>
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<tr>
<td>3-year inflation risk premium</td>
<td>$TP^r_{3,t}$</td>
<td>25.07</td>
<td>21.38</td>
<td>27.82</td>
<td>9.94</td>
<td>8.58</td>
<td>11.42</td>
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<tr>
<td>5-year inflation risk premium</td>
<td>$TP^r_{5,t}$</td>
<td>35.92</td>
<td>30.99</td>
<td>39.97</td>
<td>12.54</td>
<td>10.67</td>
<td>14.69</td>
</tr>
<tr>
<td>10-year inflation risk premium</td>
<td>$TP^r_{10,t}$</td>
<td>63.53</td>
<td>55.86</td>
<td>70.38</td>
<td>16.59</td>
<td>13.95</td>
<td>19.81</td>
</tr>
</tbody>
</table>

Table 5: Predicted first and second moments for the real treasury bonds as well as the nominal and real term premia and inflation risk premia. The real treasury bills are in annualized percentage while the nominal and real term premia and inflation risk premia are in annualized basis points.

Table 5 provide the model predicted first and second moments for the real treasury bonds as well as the nominal and real term premia and the inflation risk premium. Our model predicts sizable and volatile risk premia. The mechanism behind is similar to those postulated in the recent literature (see, for example, Andreasen et al., 2016). Beside the important role of supply shocks in our model which generate a sizable term premium, the presence of lon-run nominal risk is important to generate a volatile term premium (see Rudebusch and Swanson, 2012). Additionally, our model captures a channel which was recently postulated by Andreasen et al. (2016), namely the role of steady-state inflation for mean and volatility of risk premia. In particular, steady-state inflation generates more heteroscedastisity in the
stochastic discount factor which eventually produces more volatile risk premia. This channel is present beside the fact that the shocks in our model are all homoscedastic. More specifically, the endogenously generated heteroscedasticity in the pricing kernel is a byproduct of heteroscedasticity in the price dispersion due to positive steady-state inflation.

4.3 Historical Term Premium

In the following subsection, we discuss our model implied historical term premium and contrast our estimate with various estimates from the literature. Following the majority of the empirical literature, we limit our discussion to 10-year nominal term premium.

For comparison, we use several different prominent estimates of the 10-year nominal term premium from the literature. In particular, we use estimates based on the models developed by Kim and Wright (2005), Rudebusch and Wu (2007, 2008), Bernanke, Reinhart, and Sack (2004), and three additional models of Rudebusch et al. (2007). As Rudebusch et al. (2007) show, all of the estimated term premia follow a similar pattern and are highly correlated. We extend this analysis by adding two more recent estimates by Adrian et al. (2013) and Bauer (2016), but restrict the analysis to the 1984:q1 until 2005:q4 episode analyzed in Rudebusch et al. (2007).

![Figure 4: Model implied 10-year nominal term premium (black line) and range of corresponding estimates in the literature (gray area).](image)

The gray area in Figure 4 presents range (maximum and minimum) of the estimates for the 10-year term premium from the aforementioned studies. All estimates reproduce the

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12 We are very thankful to Eric Swanson and Michael Bauer for sharing their estimates on the 10-year nominal term premium with us.

13 The estimates of the term premium based on these models where calculated by Rudebusch, Sack, and Swanson (2007) and Rudebusch, Swanson, and Wu (2006). A description of the estimates can be found there.

14 The estimates by Bauer (2016) start in 1990, so all calculation using this estimate are restricted to a shorter sample.
well established pattern of a steadily declining 10-year nominal term premium from the early 1980’s until 2005 (and beyond). The black line in Figure 4 represents the smoothed 10-year nominal term premium predicted by our model. It follows the same pattern as described above is visually in line with the range of estimates from the literature outlined above.

Table 6 presents the correlations between the six measures of the term premium. As already discussed in Rudebusch et al. (2007), the measures by Kim and Wright (2005) and Bernanke et al. (2004) show the highest correlation over the sample. Over the full sample, our estimate shows also the highest correlation with those measures, 0.94 and 0.90 respectively. The Rudebusch and Wu (2007, 2008) measure shows the lowest correlation with the aforementioned models and is related to the low volatility of the estimated term premia presented in the last column in Table 6. Hence, the model attributes more of the movements in the long rates to short-rate expectations and therefore needs a less volatile term premium to explain bond yields. With this in mind, it is somewhat surprising that the Rudebusch and Wu (2007, 2008) measure shows the highest correlation among all estimates with the measure by Adrian et al. (2013). Given that our model is arguably closest in structure to the model used by Rudebusch and Wu (2007, 2008), we would have expected our model to display a much higher correlation with their measure than is actually does.

## 5 Monetary policy through the lens of our model

We begin the posterior analysis of the effects of monetary policy by examining the impulse responses to surprise shocks to the policy rate and the inflation target. While the former shock does not affect the stance of monetary policy, the latter shock does and therefore
affects the agents’ perception of monetary policy. Moreover, we will discuss the effects of expected monetary policy shocks and thus shed light on the effects of forward guidance.

5.1 Impulse Response Functions - Unexpected Monetary Policy Shock

Figures 5(a) through 7 contain the IRFs of macro and financial variables to a surprise shock to the policy rate, normalized to yield a median lowering of the policy rate by 50 basis points.

![Graphs showing impulse response functions](image)

Figure 5: Posterior Impulse Responses of Macro Variables to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.

The responses of the macroeconomy to the surprise policy rate shock are contained in Figure 5. The standard effects of the expansionary policy, the lowering of the policy rate can be seen in Figure 5(b), are depicted in Figure 5(a), with the easing of monetary policy on impact leading to an increase in aggregate demand and its components as well inflation. As the policy rate begins to return to its mean level with inflation still positive, the resulting increase in expected real rates reverses the expansion and depresses aggregate demand and its components, before the macroeconomy then settles back to its mean position.

The first row of Figure 6 contains the responses of the nominal curve to the surprise lowering of the policy rate. The response on impact of the term structure, in Figure 6(a), becomes more muted with the maturity, as would be expected in accordance with the expectations hypothesis and the path of the policy rate, assumed identical with the short rate. The impulse responses of the short and long ends of the nominal yield curve are contained in Figure 6(b) and, again consistent with the expectations hypothesis, the entire dynamic response of the long end is muted in response to the surprise change in the short rate. The responses of the real yield curve to the surprise policy rate shock are illustrated in the second row of Figure 6. The response on impact of the real yield curve, contained in Figure 6(c), is driven primarily by the expectations hypothesis, see the response of the nominal yield curve in Figure 6(a), and the Fisher equation, see the impulse response of inflation in Figure 5(a).

The full set of impulse responses to monetary factors can be found in Appendix C. We summarize to the impacts on term structures for the sake of brevity and clarity of the exposition.
with the response becoming more muted with the maturity. This is likewise reflected in the impulse responses of the short and long ends contained in Figure 6(d).

The first row of Figure 7 contains the responses of the nominal term premia to the surprise lowering of the policy rate. On impact, bond holders demand higher total premia for holding nominal bonds for longer maturities and lower total premia for shorter maturities, as can be seen in Figure 7(a), which is in line with the findings of Nakamura and Steinsson (2013). The stimulative effects in the short run, see Figure 5(a), generate increased confidence in the absence of downside risks to the economy, reflecting the fall in the short run premia. The delayed contractionary effects of the loosening of monetary policy are reflected in the higher medium to long run premia demanded on impact. After the contractionary effects are realized two quarters after the shock, the economy faces a recovery and the downside risks are decreasing at all horizons, which is reflected in a reduction in the premia demanded at both the short and long ends of the term structure of total risk premia in Figure 7(b). Our findings are qualitative similar to those by Crump et al. (2016), who find a decreasing nominal term premium for longer maturities, but whose results for shorter maturities are statistically insignificant.

The responses of the real term premia to the surprise policy rate shock are contained
Figure 7: Posterior Impulse Responses of Nominal and Real Term Premia and Inflation Risk Premia to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets in the second row of Figure 7. The effects on impact, in Figure 7(c), and the dynamic responses of the short and long ends, Figure 7(d), qualitatively mirror those of the total term premia in 7, confirming that the primary driver of the total premia is indeed the real economy. The timing of the ten year real premium turning negative coincides with the onset of the realization of the contraction in the real economy and the overall level of the effects
on impact are shifted downward across all maturities relative to the impact response of the
total premia, reflecting the elevation in the inflation risk premia demanded by investors in
response to the inflationary effects of the expansionary monetary policy to which we turn
next.

The last row of Figure 7 contains the responses of the inflation premia to the surprise
lowering of the policy rate. The impact effect of the expansionary monetary shock, Figure
7(e), is an increase across all maturities in the premia demanded by investors to compensate
them for the upside risks in inflation associated with the surprise change in monetary policy.
This upside risk is quickly reversed as the delayed contractionary effects of the monetary
policy shock are realized and the inflation premia demanded at both the short and long ends
of the term structure are reduced, as can be seen in Figure 7(f).

Overall the effect of an unexpected monetary policy shock – a simple innovation to the
Taylor rule – has limited effects on the term premia along all maturities. This finding in line
with those of other structural models (see, for example, Rudebusch and Swanson, 2012). A
simple innovation to the Taylor-rule lacks persistence and dies out quite quickly, limiting its
effect on business cycle frequencies. Consequentially, the effects on risk premia, which vary
primarily at lower frequencies (see, for example, Piazzesi and Swanson, 2008), are likewise
limited.

5.2 Impulse response functions - Unexpected Inflation Target
Shock

The impulse responses to a surprise inflation target shock, normalized to yield a median
lowering of the policy rate by 50 basis points, are contained in Figures 8 through 10. The
size of the reduction in the inflation target can be seen in Figure 8(b), which is a persistent,
nearly two annualized percentage point reduction in the target, reflecting a substantial change
in the stance of monetary policy.

Figure 8 contains the responses of the macroeconomy to the surprise inflation target
shock. The lowering of the policy rate, see Figure 8(d), is hump shape with the maximal
decrease of about 110 annualized basis points occurring about a year after the lowering
of the inflation target. This lowering of the policy rate is not sufficient to overcome the
contractionary effects of the tightened monetary stance as can be seen by the negative
responses on aggregate demand and investment in Figure 8. The sharp drop in inflation
likewise reflects the contractionary shift in the stance of monetary policy, but as observed
inflation begins to exceed its target, as can be seen in Figure 8(a), investment and output
are stimulated. With the slowdown in output less than that of investment, the resulting
surplus in production is captured by an increase in consumption.

The responses of the nominal yield curve to the surprise inflation target shock are con-
tained in the first row of Figure 9. Commensurate with the shock effecting a change in
the stance of monetary policy, the entire yield curve, both the short and long ends as can
be seen in Figure 9(b), shift downwards. The effect on impact, see Figure 9(a), demon-
strates a hump shaped dependency on maturity, reflecting the expectations hypothesis and
the dynamic response of the short rate in Figure 8(d).

The second row of Figure 9 contains the responses of the real yield curve to the surprise
inflation target shock. In contrast to the nominal yield curve, the response of the real yields is decreasing in their maturity. This is consistent with a Fisher equation perspective on the real rates, noting the delayed reduction in the nominal short rate in response to the decrease in inflation. The short end of the real yield curve, in Figure 9(d), falls below zero as the nominal short rate recovers from its trough one year after the impact of the shock and remains there as the policy rate converges more quickly to its mean value than inflation. In the long-run, the effect of monetary policy shock on real rates is zero as theory would predict.

The responses of the nominal term premia to the surprise inflation target shock are contained in the first row of Figure 10. On impact, shorter maturities are associated with an increased premium and longer maturities with a decreased premium, as can be seen in Figure 10(a), by and large coinciding with the initial expansion and delayed contraction in the aggregate real economy. In contrast to the responses to the shock in the policy rate in Figure 7(b), the premia at the short and long ends of the term structure remain diverged in their entire dynamic responses to the change in the inflation target, see Figure 10(b), consistent with the interpretation of the shock to the inflation target as being a shift in the stance of monetary policy: long run downside risks to the economy are reduced by the more
aggressive stance of monetary policy at the cost of heightened short run risks. Furthermore, these risks are perceived to be an order of magnitude larger than those associated with a surprise shift in the policy rate, normalized to result in an identical median reduction of the policy rate by 50 basis points.

The second row of Figure 10 contains the responses of the real term premia to the surprise lowering of the inflation target. As was the case with the policy rate shock, the effects on impact, in Figure 10(c), and the dynamic responses of the short and long ends, Figure 10(d), largely reflect those of the nominal term premia in 10(b), confirming that the primary driver of the total premia is indeed the real economy. The downside risks to the real economy are perceived on impact to be longer lived than the nominal risks, which can be seen in the larger and for longer maturities positive impact effect of the reduction in the inflation target on the premia demanded by investors. The divergence in the short and long ends of the real premium term structure follows that of the nominal premia, emphasizing that the divergence in down vs. upside risks are driven by the real economy. As was the case with nominal premia, the risks associated with the reduction in the inflation target, recall this reduction was normalized to lead to an identical median fall in the policy rate as the surprise cut in the same, are an order of magnitude higher.
Figure 10: Posterior Impulse Responses of Nominal and Real Term Premia and Inflation Risk Premia to a Surprise Cut in the Inflation Target leading to a 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
The responses of the inflation risk premia to the surprise inflation target shock are contained in the last row of Figure 10. With both the inflation target and realized inflation reduced by the more aggressive stance of monetary policy towards inflation, investors’ perception of upside risks to inflation are ameliorated, leading to a reduction in the premia that they demand at all horizons on impact, see 10(e), as well as dynamically at the short and long ends of the associated term structure. The increase in the magnitude of the effects of this change in monetary policy are roughly twentyfold, higher than the shifts in magnitude associated with the total and real term premia, consistent with the interpretation of this experiment being not only a change in the stance of monetary policy, but more specifically a more aggressive stance towards inflation.

In contrast to the monetary policy shock before, a shock to the monetary policy stance has a much stronger effect on the risk premia of interest rates across all maturities. Moreover, our results confirm the finding of Hanson and Stein (2015) that real interest rates in the long run are mainly driven by real term premia. The reason behind the strong effect on the risk premia is described by Rudebusch and Swanson (2012), who show that long run nominal risk has strong effects on the nominal term premium. In contrast to a simple innovation to the Taylor-rule which quickly dissipates, a shock to the monetary stance is much more long lasting and therefore has stronger effects on business cycle and low frequencies.

5.3 Forward Guidance

In this section, we analyze the dynamic responses of macroeconomic variables, yields and term premia to anticipated monetary policy shocks. Typically, forward guidance amounts to announcements of lower than necessary nominal interest rates after the economy has exited the zero-lower bound. The starting point of the experiments is thus a central bank’s announcement to decrease the policy rate in the future assuming the policy rate to be held constant at its mean value until this decrease is implemented. We analyze the effects of different announcement horizons. We implement these scenarios using sequences of anticipated shocks to the Taylor rule to keep the policy rate upon announcement constant until the announced interest rate cut is implemented.

Formally the structure of the residuals affecting the Taylor rule in Eq. (18) is modified such that

$$ r_t^f = R \left( r_{t-1}^f, \pi_t, y_t \right) + \sigma_m \left( \epsilon_{m,t} + \sum_{k=1}^{K} \epsilon_{m,t+k} \right), $$

where $R(\cdot)$ characterizes the systematic response of monetary policy, $\epsilon_{m,t}$ is the usual contemporaneous policy shock and $\sum_{k=1}^{K} \epsilon_{m,t+k}$ a sequence of policy shocks known to agents at time $t$ but that affect the policy rule $k$ periods later, i.e., at time $t + k$. We assume as is usual that $\epsilon_{m,t+k} \overset{iid}{\sim} N(0, 1)$.

The blue lines in the Figures 11(a) and 11(b) show the evolution of the economic variables of interest after an announcement that the policy rate will be lowered by 25 basis points 4 quarters in the future while being kept constant until then. The former focuses on macroeconomic variables and the latter figure shows the evolution of yields and term premiums.
Figure 11: Impulse Resonse Function to an Expected 25 Basis Point Policy Rate Cut in Four Quarters (blue line) and Eight Quarters (green line) into the Future.

All variables responses are reported as percentage deviations from the ergodic mean, except inflation, the interest rate, the yields at each maturities and the term premia, which are expressed in annualized absolute deviations.

Output and inflation both increase on impact with output reaching its peak after 5 quarters and falling slightly below its mean value after 17 quarters. The responses to the announcement are driven by expectations of lower nominal short term interest rates and of future inflation, a mechanism familiar from the discussions of central bank forward guidance. Expected higher inflation leads to a rise in current inflation through forward looking price
setting, with a consequential fall in current and expected real interest rates and associated increase in economic activity on impact.

Forward guidance propagates through an additional channel, the movements in the nominal long rates, which the recent literature has argued that this channel plays a nontrivial role. From both a theoretical and empirical perspective, it is a priori not obvious which maturities the nominal term structure should fall. From the perspective of our model the dynamic responses of interest rates are driven by two countervailing effects. As in standard models in accordance with the expectations hypothesis, the dynamics of interest rates with longer maturities reflect the adjustment of the risk free short rate, determined by the monetary authority’s Taylor rule. The large effects on inflation and output imply that the policy rate rises quickly above its ergodic mean only few quarters after its fall. This explains, at least in part, why we do not observe any significant fall in the 10 year rate but only in the 2 year rate on impact (see Figure 11(b)).

Our model also incorporates precautionary motives and we find that bondholders demand higher nominal premia for 2 and 10 year maturities at all horizons to compensate them for the downside risks they perceive to the nominal economy (see again Figure 11(b)). While there is some increased medium term confidence in the real economy, as can bee seen by the fall in the premium demanded for two year real bonds, this is outweighed by the larger increase in inflation risk perceived by the bondholders. This overall increase in nominal premia pushes nominal rates higher than the expectations hypothesis would predict, dampening the expansionary effects of the announced cut in the policy rate.

The effects of increasing the horizon can be seen in the figures, the green lines depict the effects of the central bank announcing the 25 basis points interest rate cut 8 quarters as opposed to 4 quarters (blue lines) ahead. As is to be expected, the macroeconomic outcomes are magnified. Consistently with the first mechanism described above, nominal yields are now increasing at all maturities but movements in term premia are now much larger, with the 4 and the 10 ten year term premia increasing roughly an order of magnitude more on impact as bondholders perceive inflation risk to substantially outweigh even the increased confidence in the real economy in the medium run.

While we do not presuppose that we provide a solution to the forward guidance puzzle as put forth by Del Negro et al. (2015), we provide evidence that taking the term premium component of long yields and the risks from agents’ perception than underlie this into account introduces an important dampening factor into the responses to anticipated monetary policy actions.

6 Conclusion

The role of monetary policy in shaping the term structure has gained particular prominence. Yet the empirical literature has yet to reach a definitive conclusion on the effects of monetary policy on the term structure, with respect to not only the quantitative but also the qualitative effects, and the standard structural alternative – the linear New Keynesian model – has been criticized for lacking effects on interest rates beyond the expectation hypothesis (Hanson and Stein, 2015). Moreover, newer structural modelling approaches that go beyond the expectation hypothesis face significant computational challenges (van Binsbergen et al.,
We ameliorate these challenges by using the risk adjusted approximation of Meyer-Gohde (2016), allowing our model to capture the salient features of risk while remaining linear in states such that Bayesian estimation and posterior analysis using standard macroeconometric techniques is tractable. Our resulting macroeconomic framework is consistent with a wide variety of asset pricing facts and therefore well suited to investigate the impact of monetary policy on term structure of interest rates. In particular, our medium scale New Keynesian macro-finance model captures produces sizable and time varying risk premia comparable to historical estimates from affine terms structure models (e.g. Kim and Wright, 2005; Adrian et al., 2013) without sacrificing the fit of macroeconomic or other financial variables.

We find that a shock to the monetary stance has strong effects on risk premia and in the long run, moreover, real interest rates are driven chiefly by these premia. In contrast, the effect of an unexpected monetary policy shock – a simple innovation to the Taylor rule – has limited effects on the term premia at all maturities as such an uncorrelated innovation to the Taylor-rule dissipates too quickly to have meaningful effects at business cycle frequencies. Consequently, the effects on risk premia, which vary primarily at lower frequencies (see, for example, Piazzesi and Swanson, 2008), are limited. In contrast, a shock to the monetary policy stance has much stronger effects on the term structure of interest rates across all maturities. As laid out by Rudebusch and Swanson (2012), long-run nominal risk has strong effects on the nominal term premium and, as a shock to the monetary stance is much longer lasting, it falls under the category of long-run nominal risk and has strong effects on business cycle and lower frequencies. We find that after monetary policy shocks, risk premia tend to move opposite the policy rate in the long run, i.e., a looser monetary policy increases risk premia. In particular in our model, such a looser monetary policy increases the precautionary savings motive of agents as they expect more volatile inflation and output and, therefore, demand a higher risk premia. This finding is comparable to the empirical results of Crump et al. (2016) who investigate the same sample period as we do. Yet, this finding is in contrast to many other recent papers (see, for example, Hanson and Stein, 2015; Abrahams et al., 2016; Gertler and Karadi, 2015) who find that a looser monetary policy goes along with decreasing risk premia.

When turning our analysis to unconventional monetary policy, we follow the approach by Campbell et al. (2016) and analyze the effect of Odyssean forward guidance. We find that forward guidance affects risk premia substantially, disjointing bond yields from the expectations hypothesis. Similarly to most studies, we find that expansionary forward guidance increases macroeconomic activity and substantially increases inflation. In comparison to many standard New Keynesian models, however, our model does not generate a “forward guidance puzzle” as the effects on output and inflation are modest relative to most existing studies. While our setup does not provide a solution to the forward guidance puzzle as illustrated by Del Negro et al. (2015), we provide evidence that properly taking the term premium component of long yields into account induces an important dampening factor to anticipated monetary policy actions. Importantly, our approach does not suffer from the various identification problems in empirical forward guidance studies. (see, for example, the discussion in Nakamura and Steinsson, 2013; Campbell et al., 2016) Therefore, our results in and of themselves deliver new insights into the response of risk premia to forward guidance and our results extend the literature that raises doubts about the power of forward guidance
(McKay, Nakamura, and Steinsson, 2015) to stimulate the economy as strongly as initially expected.

References


A Proof of Proposition 3

Since \( y \) in the policy function (35) is a vector valued function, its derivatives form a hypercube. We use the method of Lan and Meyer-Gohde (2012) that differentiates conformably with the Kronecker product, allowing us to maintain standard linear algebraic structures to derive our results.

Definition 4 Matrix Derivatives

Let \( A(B) : \mathbb{R}^{s \times 1} \rightarrow \mathbb{R}^{p \times q} \) be a matrix-valued function that maps an \( s \times 1 \) vector \( B \) into an \( p \times q \) matrix \( A(B) \), the derivative structure of \( A(B) \) with respect to \( B \) is defined as

\[
A_B \equiv \mathcal{D}_{B^T} \{ A \} \equiv \left[ \frac{\partial}{\partial b_1} \ldots \frac{\partial}{\partial b_s} \right] \otimes A
\]

where \( b_i \) denotes \( i \)'th row of vector \( B \), \( T \) indicates transposition; \( n \)'th derivatives are

\[
A_{B^n} \equiv \mathcal{D}_{(B^T)^n} \{ A \} \equiv \left( \left[ \frac{\partial}{\partial b_1} \ldots \frac{\partial}{\partial b_s} \right] \otimes [n] \right) \otimes A
\]

Adopting the abbreviated notation above, we write \( g_{z_j \sigma_i} \in \mathbb{R}^{n_y \times n_y} \) is the partial derivative of the vector function \( g \) with respect to the state vector \( z_t \) \( j \) times and the perturbation parameter \( \sigma_i \) \( i \) times evaluated at the deterministic steady state. We will recover the first order partial derivatives by applying the implicit function theorem on (34) and higher order partials through successive differentiation.\(^{16}\)

Beginning with the unknown point of approximation, the ergodic mean, construct a Taylor series around the deterministic steady state

\[
\bar{y}(\sigma) = \bar{y}(0) + \bar{y}'(0)\sigma + \frac{1}{2}\bar{y}''(0)\sigma^2 \ldots
\]

under the assumption of analyticity, the ergodic mean \( \bar{y}(1) \) can be approximated by

\[
\bar{y}(1) \approx \bar{y}(0) + \bar{y}'(0) + \frac{1}{2}\bar{y}''(0) + \cdots + \frac{1}{n!}\bar{y}^{(n)}(0)
\]

(C-3)

Analogously for the two first derivatives of the policy function (31)

\[
\bar{y}_y(1) \approx \bar{y}_y(0) + \bar{y}'_y(0) + \frac{1}{2}\bar{y}''_y(0) + \cdots + \frac{1}{(n-1)!}\bar{y}^{(n-1)}_y(0)
\]

\[
\bar{y}_\varepsilon(1) \approx \bar{y}_\varepsilon(0) + \bar{y}'_\varepsilon(0) + \frac{1}{2}\bar{y}''_\varepsilon(0) + \cdots + \frac{1}{(n-1)!}\bar{y}^{(n-1)}\varepsilon(0)
\]

Note that the approximations of \( \bar{y}_\varepsilon(1) \) and \( \bar{y}_y(1) \) are expressed up to order \( n - 1 \), whereas the approximation of \( \bar{y}(1) \) is expressed up to order \( n \). As the first two are derivatives of the the third, terms of the order in the first two \( n - 1 \) are of the order \( n \) with respect to derivatives of the underlying policy function (35), from which we will construct the approx-

imations. Additionally, the assumption of analyticity, here in a domain encompassing both the deterministic steady state and ergodic mean of (35), while hardly innocuous, underlies standard perturbations methods that approximate the stochastic model using derivatives of the meta policy policy function (35) evaluated at the deterministic steady state in definition 2.

Now we will show that the Taylor series representations of (38), (40), and (41) can be recovered from the derivatives of the policy function (35) evaluated at the deterministic steady state used in standard perturbations. We will derive the expressions out to \( n = 3 \) order, consistent with the goals laid out in the main text.

We will start with (38), the point of approximation,

\[
\tilde{y}(1) \approx \tilde{y}(0) + \tilde{y}'(0) + \frac{1}{2} \tilde{y}''(0) + \frac{1}{6} \tilde{y}^{(3)}(0)
\]

we need the four terms on the right hand side—\( \tilde{y}(0) \), \( \tilde{y}'(0) \), \( \tilde{y}''(0) \), and \( \tilde{y}^{(3)}(0) \)—to construct this approximation. Proceeding in increasing order of differentiation, we begin with \( \tilde{y}(0) \).

From (38),

\[
\tilde{y}(0) = E[g(y_{t-1}, 0, 0)] = g(\bar{y}, 0, 0) = \bar{y}
\]

the first derivative, \( \tilde{y}'(\sigma) \), is

\[
\tilde{y}'(0) = D_\sigma \left\{ E[y_t] \right\}_{\sigma=0} = D_\sigma \left\{ E[g(y_{t-1}, \sigma \varepsilon_t, \sigma)] \right\}_{\sigma=0} = E[D_\sigma \left\{ g(y_{t-1}, \sigma \varepsilon_t, \sigma) \right\} ]_{\sigma=0}
\]

where the expectation is with respect to the infinite sequence of \( \{\varepsilon_{t-j}\}_{j=0}^\infty \) with invariant i.i.d. distributions, thus and assuming stability of \( y_t \), gives the final equality. Taking derivatives and expectations and evaluating at the deterministic steady state

\[
D_\sigma \left\{ E[y_t] \right\}_{\sigma=0} = g_y D_\sigma \left\{ E[y_{t-1}] \right\}_{\sigma=0} + g_\varepsilon E[\varepsilon_t] + g_\sigma
\]

where the second line follows from the assumption of \( \varepsilon_t \) being mean zero.\(^{17}\) Thus,

\[
\tilde{y}'(0) = 0
\]

as \( g_y \) has all its eigenvalues inside the unit circle. The second derivative, \( \tilde{y}''(\sigma) \), is

\[
\tilde{y}''(0) = D_\sigma^2 \left\{ E[y_t] \right\}_{\sigma=0} = E[D_\sigma^2 \left\{ g(y_{t-1}, \sigma \varepsilon_t, \sigma) \right\} ]_{\sigma=0}
\]

Taking derivatives and expectations, evaluating at the deterministic steady state, and re-

\(^{17}\)Thus, \( E[\varepsilon_t] = 0 \) follows directly and \( g_\sigma \) consequentially, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2012).
calling results from the first derivative above\textsuperscript{18}

\[
\mathcal{D}_{\sigma^2}\{E\{y_t\}\} = \left. E \left[ g_y \mathcal{D}_{\sigma^2}\{y_{t-1}\} \right] + g_y^2 \mathcal{D}_\sigma \{y_{t-1}\} \otimes [2] + 2g_y \varepsilon_t \otimes \mathcal{D}_\sigma \{y_{t-1}\} \right|_{\sigma=0} \\
+ 2g_y \mathcal{D}_\sigma \{y_{t-1}\} + 2g_\varepsilon \varepsilon_t + g_\varepsilon^2 \varepsilon_t \right|_{\sigma=0}
\]

\[
= g_y \mathcal{D}_{\sigma^2}\{E\{y_{t-1}\}\} \bigg|_{\sigma=0} + g_y^2 E \left[ \mathcal{D}_\sigma \{y_{t-1}\} \otimes [2] \right] \bigg|_{\sigma=0}
\]

\[
+ g_\varepsilon^2 E \left[ \varepsilon_t^\otimes [2] \right] + g_\sigma^2
\]

\[
= g_y \mathcal{D}_{\sigma^2}\{E\{y_{t-1}\}\} \bigg|_{\sigma=0} + g_y^2 \left( I_{n_y} - g_y^{[2]} \right)^{-1} g_\varepsilon^{[2]} E \left[ \varepsilon_t^\otimes [2] \right]
\]

\[
+ g_\varepsilon^2 E \left[ \varepsilon_t^\otimes [2] \right] + g_\sigma^2
\]

\[
\bar{y}''(0) = \mathcal{D}_{\sigma^2}\{E\{y_t\}\} \bigg|_{\sigma=0} = \left( I_{n_y} - g_y \right)^{-1} \left( g_\varepsilon^2 + \left( I_{n_y} - g_y^{[2]} \right)^{-1} g_\varepsilon^{[2]} \right) E \left[ \varepsilon_t^\otimes [2] \right] + g_\sigma^2
\]

where the second to last equality follows\textsuperscript{19}—taking expectations, evaluating at the deterministic steady state, and recalling results from the first derivative above—as

\[
E \left[ \mathcal{D}_\sigma \{y_t\} \otimes [2] \right] \bigg|_{\sigma=0} = E \left[ \left( g_y \mathcal{D}_\sigma \{y_{t-1}\} + g_\varepsilon \varepsilon_t + g_\sigma \right) \otimes [2] \right] \bigg|_{\sigma=0}
\]

\[
= g_y^{[2]} E \left[ \mathcal{D}_\sigma \{y_{t-1}\} \otimes [2] \right] \bigg|_{\sigma=0} + g_\varepsilon^{[2]} E \left[ \varepsilon_t \right] + g_\sigma^2
\]

Thus, \( \bar{y}''(0) \) adjusts the zeroth order mean \( \bar{y}(0) \) or deterministic steady state for the cumulative—\( \left( I_{n_y} - g_y \right)^{-1} \)—influence of the variance of shocks, directly through \( E \left[ \varepsilon_t \right] \) and indirectly through the influence of risk on the policy function captured by \( g_\sigma^2 \). The third derivative, \( \bar{y}^{(3)}(0) \), is

\[
\bar{y}^{(3)}(0) = \mathcal{D}_{\sigma^3}\{E\{y_t\}\} \bigg|_{\sigma=0} = E \left[ \mathcal{D}_{\sigma^3}\{g(y_{t-1}, \sigma \varepsilon_t, \sigma)\} \right] \bigg|_{\sigma=0}
\]

Taking derivatives and expectations, evaluating at the deterministic steady state, and recalling results from the first two derivatives above

\[
\mathcal{D}_{\sigma^3}\{E\{y_t\}\} \bigg|_{\sigma=0} = E \left[ \left( g_y^3 \mathcal{D}_\sigma \{y_{t-1}\} \right) \otimes [3] + 3g_y^2 \varepsilon_t \mathcal{D}_\sigma \{y_{t-1}\} \otimes [2] \otimes \varepsilon_t + 3g_y \varepsilon_t^2 \mathcal{D}_\sigma \{y_{t-1}\} \otimes [2] \otimes \varepsilon_t \otimes [2] \right]
\]

\[
+ 3g_y \mathcal{D}_\sigma \{y_{t-1}\} \otimes \varepsilon_t^{[3]} + 6g_y \varepsilon_t \mathcal{D}_\sigma \{y_{t-1}\} \otimes \varepsilon_t + 3g_y^2 \mathcal{D}_\sigma \{y_{t-1}\} \right|_{\sigma=0}
\]

\[
+ y_\varepsilon \varepsilon_t \mathcal{D}_\sigma \{y_{t-1}\} \otimes \varepsilon_t + 3g_\varepsilon \varepsilon_t^2 \mathcal{D}_\sigma \{y_{t-1}\} \otimes \varepsilon_t + g_\sigma^3 + 3g_y^2 \mathcal{D}_\sigma \{E\{y_t\}\} \otimes \mathcal{D}_\sigma \{E\{y_t\}\}
\]

\[
+ 3g_y \mathcal{D}_\sigma \{E\{y_t\}\} \varepsilon_t + 3g_y \varepsilon_t \mathcal{D}_\sigma \{E\{y_t\}\} \varepsilon_t + g_y \mathcal{D}_\sigma \{y_{t-1}\} \right|_{\sigma=0}
\]

\textsuperscript{18}The notation \( x^{\otimes [n]} \) represents Kronecker powers, \( x^{\otimes [n]} \) is the \( n \)’th fold Kronecker product of \( x \) with itself: \( x \otimes x \cdots \otimes x \).

\textsuperscript{19}The second line follows as \( g_y \) and \( g_\varepsilon \) are zero, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2012).
From our assumption of mean-zero, normally distributed shocks, it follows that

\[ \tilde{y}(3)(0) = D_\sigma^3 \{ E \left[ y_t \right] \} \bigg|_{\sigma=0} = 0 \]  

(C-18)

as third derivatives of \( g \) involving derivatives with respect of \( \sigma \) only once are zero,\(^{20}\) terms cubic in \( \varepsilon_t \) (either directly or through products involving \( D_\sigma \{ y_{t-1} \} \), which is linear in \( \varepsilon_t \), or \( D_\sigma^2 \{ y_{t-1} \} \), which is quadratic in \( \varepsilon_t \), and \( g_3 \) are all zero in accordance with the symmetry of the normal distribution.\(^{21}\)

Moving on to the derivative of the policy with respect to \( y_{t-1} \), (40), for small deviations of \( y_{t-1} \) and \( \varepsilon_t \) from their respective means

\[ \tilde{y}_y(1) \approx \tilde{y}_y(0) + \tilde{y}_y'(0) + \frac{1}{2} \tilde{y}_y''(0) \]  

(C-19)

we need the three terms on the right hand side—\( \tilde{y}_y(0) \), \( \tilde{y}_y'(0) \), and \( \tilde{y}_y''(0) \). Starting with \( \tilde{y}_y(0) \),

\[ \tilde{y}_y(0) = D_{y_{t-1}} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = D_{y_{t-1}} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} = g_y \]  

(C-20)

Turning to \( \tilde{y}_y'(0) \)

\[ \tilde{y}_y'(0) = D_{\sigma y_{t-1}} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = D_{\sigma y_{t-1}} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} \]

\[ = D_\sigma \left[ g_y(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \right] \bigg|_{\sigma,\varepsilon_t=0} \]

\[ = g_y D_\sigma \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \otimes I_{ny} + g_{\sigma y} \]

\[ = 0 \]  

(C-21)

The first term is zero as \( D_\sigma \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \) was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, \( \tilde{y}_y''(0) \)

\[ \tilde{y}_y''(0) = D_{\sigma^2 y_{t-1}} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = D_{\sigma^2 y_{t-1}} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} \]

\[ = D_{\sigma^2} \left[ g_y(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \right] \bigg|_{\sigma=0} \]

\[ = g_y^3 D_\sigma \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \otimes I_{ny} + 2g_{\sigma y^2} D_\sigma \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \otimes I_{ny} \]

\[ + g_y^2 D_{\sigma^2} \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \otimes I_{ny} + g_{\sigma^2 y} \]

\[ = g_y^3 D_\sigma \left\{ \tilde{y}(\sigma) \right\} \bigg|_{\sigma=0} \otimes I_{ny} + g_{\sigma^2 y} \]  

(C-22)


\(^{21}\)See Andreasen (2012b) for perturbations with skewed distributions.
The final equality follows as \( \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \) and \( g_{\sigma y^2} \) are both zero following the results and discussions above.

Finally, the derivative of the policy with respect to \( \varepsilon_t \), (41), follows analogously to the derivative with respect to \( y_{t-1} \),

\[
\tilde{y}_\varepsilon(1) \approx \tilde{y}_\varepsilon(0) + \tilde{y}_\varepsilon'(0) + \frac{1}{2} \tilde{y}_\varepsilon''(0)
\]

Again, we need the three terms on the right hand side—\( \tilde{y}_\varepsilon(0) \), \( \tilde{y}_\varepsilon'(0) \), and \( \tilde{y}_\varepsilon''(0) \). Starting with \( \tilde{y}_\varepsilon(0) \),

\[
\tilde{y}_\varepsilon(0) = \mathcal{D}_{\varepsilon_t} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = \mathcal{D}_{\varepsilon_t} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} = g_\varepsilon
\]

then \( \tilde{y}_\varepsilon'(0) \)

\[
\tilde{y}_\varepsilon'(0) = \mathcal{D}_{\sigma\varepsilon_t} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = \mathcal{D}_{\sigma\varepsilon_t} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} = \mathcal{D}_\sigma \{ g_\varepsilon(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} = g_{y\varepsilon} \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{a\varepsilon} = 0
\]

The first term is zero as \( \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \) was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, \( \tilde{y}_{y''}(0) \)

\[
\tilde{y}_{y''}(0) = \mathcal{D}_{\sigma\sigma\varepsilon_t} \{ y_t \} \bigg|_{\sigma,\varepsilon_t=0} = \mathcal{D}_{\sigma\sigma\varepsilon_t} \{ g(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma,\varepsilon_t=0} = \mathcal{D}_\sigma \{ g_{y\varepsilon}(\tilde{y}(\sigma), \tilde{\varepsilon}_t, \sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + 2 g_{\sigma y\varepsilon} \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma y^2} \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon} = g_{y\varepsilon} \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma y^2} \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon}
\]

The final equality follows as \( \mathcal{D}_\sigma \{ \tilde{y}(\sigma) \} \bigg|_{\sigma=0} \) and \( g_{\sigma y^2} \) are both zero following the results and discussions above.
B Data

In this paper we use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This series is *BEA NIPA table 1.1.5 line 1 (A191RC1)*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of Nominal GDP to Real GDP.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of the respective nominal values of the *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* and finally deflated by the deflator mentioned above.

Private Investment: Total real private investment is the sum of the respective nominal values of the series Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* and finally deflated by the deflator mentioned above.

Civilian Population: This series is calculated from monthly data of *civilian noninstitutional population over 16 years (CNP16OV)* from the U.S. Department of Labor: Bureau of Labor Statistics.

Policy Rate: The quarterly policy rates is the 3-Month Treasury Bill: Secondary Market Rate *TB3MS* provided by Board of Governors of the Federal Reserve System. The quarterly aggregation is end of period.

Treasury Bond Yields: The quarterly series for 1-year, 2-year, 3-year, 5-year, and 10-year zero-coupon bond yields re measured end of quarter. The original series are daily figures based on the updated series by Adrian et al. (2013).

Source: https://www.newyorkfed.org/research/data_indicators/term_premia.html

Nominal Interest Rate Forecasts: The quarterly series for 1-quarter (TBILL3) and 4-quarter (TBILL6) ahead forecasts of the nominal 3month Treasury Bill. The time series are the median responses by the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia.

C Supplementary Results

C.1 Prior vs Posterior Plots

Figure 12: Prior (gray) and posterior (black) distribution of the model parameters, the green dashed line indicates the posterior mode.

Figure 13: Prior (gray) and posterior (black) distribution of measurement errors, the green dashed line indicates the posterior mode.
C.2 Impulse response functions - Unexpected Monetary Policy Shock

![Graphs showing impulse response functions for various macroeconomic variables after a 50 basis point policy rate cut.](image)

Figure 14: Posterior Impulse Responses of Macro Variables to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 15: Posterior Impulse Responses of the Nominal Term Structure to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 16: Posterior Impulse Responses of the Real Term Structure to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 17: Posterior Impulse Responses of the Term Structure of Risk Premia to a Surprise 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
C.3 Impulse response functions - Unexpected Inflation Target Shock

Figure 18: Posterior Impulse Responses of Macro Variables to a Surprise Cut in the Inflation Target leading to a 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 19: Posterior Impulse Responses of the Nominal Term Structure to a Surprise Cut in the Inflation Target leading to a 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 20: Posterior Impulse Responses of the Real Term Structure to a Surprise Cut in the Inflation Target leading to a 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.
Figure 21: Posterior Impulse Responses of the Term Structure of Risk Premia to a Surprise Cut in the Inflation Target leading to a 50 Basis Point Policy Rate Cut. Shaded areas represent the 90% and 68% posterior credible sets.