Effective trading hours extension

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Abstract

To uncover the complex feature of the effect of extending trading hours, we analyze what kind of the extension is effective on price efficiency and price stability, by utilizing an agent-based market model. Specifically, we examine whether the partial extension of trading hours—namely, implementing the pre-market session and the after-hours session—and what duration of the session is effective. The simulation result reveals that the implementation of both sessions could have a negative impact on price efficiency and stability if investors' participation during the session is limited; it could result in more concentrated trading in the opening session, wider divergence between market prices and the fundamental value, and lower price stability. In addition, longer sessions are less beneficial (or more harmful). However, we find that the implementation of the pre-market session is far more beneficial than that of the after-hours session; specifically, the implementation of the short-term pre-market session could induce higher price efficiency and higher price stability regardless of the number of market participants during the session.

Keywords; Extended-hours trading, Agent-based market model, Price efficiency, Price stability
JEL code; D40, D53, G17, G19

1. Introduction

Periodical market closures are said to have a negative impact on trading activity and price efficiency. First, considerable studies, for example, Kyle (1985), Glosten and Milgrom (1985), Foster and Viswanathan (1990), and Easlay and O'Hara (1992), show that public and private information accumulates overnight while information asymmetry declines over the course of trading periods. These studies suggest market closures may induce a delay in the incorporation of information into stock prices, which can widen divergence between stock prices and their fundamental values. Second, periodical market closure may reduce price stability, especially at the beginning and end of the trading session on an intraday basis. Wood et al. (1985) and Harris (1986) find that a standard

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deviation of returns is especially high at the open and close of trading. Finally, the studies reveal that periodical market closures can cause skewed trading activity, that is, trading is concentrated at the open and close of sessions. Jain and Joh (1988) document a U-shaped intraday pattern of trading volume; trading volume is especially high at the open and close of the regular-hours session.

It seems that this problem can easily be solved by extending trading hours. In fact, the extension of trading hours for stocks to improve trading opportunities and price efficiency has increasingly been discussed (Osaki, 2014). In several markets (e.g., NYSE and NASDAQ), both pre-market and after-hours sessions have already been introduced, and the Tokyo Stock Exchange is considering extending trading hours by implementing these extended-hours sessions and/or shortening the midday recess. However, the problem is not so simple. Miwa and Ueda (2016) show that the extension of trading hours could disturb price formation and trading activity if investors' participation during the extended-hours session is limited. Actually, there are a limited number of market participants during the extended-hours session: there is less than 5% as much trading per time unit in after-hours sessions versus regular-hours sessions in the U.S. stock market (Barclay and Hendershott, 2004). Thus, it is possible that the extension of trading hours has a negative impact on trading activity and price formation; the effect of extending trading hours is not straightforward.

To uncover the complex feature of the effect of extending trading hours, it is necessary to analyze what kind of the extension is effective on price efficiency and price stability. In particular, while Miwa and Ueda (2016) consider a quite simplified case where an extended-hours session ranges from the close of the regular-hours session to the opening of the next day's regular-hours session, many stock exchanges are considering or have implemented a more complicated extension of trading hours; in the U.S. stock markets, extended-hours trading occurs in two sessions: the pre-market session (from 8:00 AM to 9:30 AM) and the after-hours session (from 4:00 PM to 6:30 PM). Thus, we should address the effect of implementing the pre-market session and the after-hours session, and the most beneficial session length. The analysis can definitely give an important indication of the design of trading hours for exchange markets.

In this study, we expand the model of Brock and Hommes (1998) and Miwa and Ueda (2016) by incorporating the pre-market session and the after-hours session. We analyze whether implementing the after-hours session and pre-market session can improve price efficiency and trading activity, and investigate the duration of the most beneficial (or the least harmful) session length.

We evaluate the effect of the implementation of those sessions on price efficiency and trading activity by the following three factors: the deviation between stock prices and fundamental values (as a proxy for price efficiency), volatility of stock returns especially at the open and close of regular hours session (as a proxy for price instability), and trading volume especially at the open and close (as a proxy for trade concentration). As above-mentioned, if the extension mitigates the negative

effects induced by periodical market closure, the deviation from fundamental value, return volatility, and trade concentration at the open and close should be smaller than when there is no extended-hours session.

The paper is constructed as follows. Section 2 derives the market model with limited pre-market and after-hours sessions trading; section 3 presents the simulation settings; section 4 presents simulation results for whether and when the extension of trading hours is beneficial; section 5 discusses the results; and section 6 concludes.

2. Market Model

In this section, we explain the agent-based market model used. The model is based on that of Brock and Hommes (1998) and Miwa and Ueda (2016). However, we modify the model to incorporate the pre-market and after-hours session trading.

2.1 Price Determination Process

Investors (Traders) can invest in both a risk-free asset and a risky asset. The risk free asset has perfect supply elasticity. Since the interest rate and dividend yield are irrelevant in the short-term investment horizon, we assume that the interest rate is zero and that the risky asset (e.g., a stock) pays no dividend. Let P_t be the price per share of the risky asset at time *t*.

Agents are myopic mean-variance maximizers; therefore, when an investor *i* participates in the market (investor *i* trades the asset), the demand $z_{i,t}$ per investor *i* for the risky asset is calculated:

$$z_{i,t} = \frac{E_{i,t}[\frac{\mathbf{p}_{t+1}}{p_t} - 1]}{a_i V_{i,t}[\frac{\mathbf{p}_{t+1}}{p_t} - 1]}$$

$E_{i,t}$ and $V_{i,t}$ denote the "beliefs" (forecasts) of trader i about conditional expectation and conditional variance of return $\boldsymbol{p}_{t+1}/p_t$-1, and a_i is the risk aversion parameter. Bold face variables denote random variables at date $t + 1$. The conditional variance is assumed to be equal and constant for all investors ($V_{i,t} = \sigma^2$).

$$z_{i,t} = \frac{E_{i,t}\left[\frac{\mathbf{p}_{t+1}}{p_t} - 1\right]}{a_i \sigma^2} \quad (1)$$

On the other hand, when an investor *i* does not participate in the market (investor *i* does not trade the asset), the demand $z_{i,t}$ is set to be unchanged ($z_{i,t} = z_{i,t-1}$), so that the investor does not place an order.

We suppose the case of zero supply of outside shares. Equilibrium of demand and supply yields:

$$\sum_{i \in X_{trade}} \frac{E_{i,t}[\mathrm{p}_{t+1} / p_t] - 1}{a_i \sigma^2} + \sum_{i \notin X_{trade}} z_{i,t-1} = 0$$

where X_{trade} denotes investors who trade the asset.

Thus, the Walrasian market clearing price satisfies:

$$p_t = \frac{\sum_{i \in X_{trade}} \frac{E_{i,t}[\mathrm{p}_{t+1}]}{a_i}}{\sum_{i \in X_{trade}} \frac{1}{a_i} - \sigma^2 \sum_{i \notin X_{trade}} z_{i,t-1}} \quad (2)$$

Note that the market clearing price p_t is always well defined (strictly positive).*

The volume of trade *TV* at time *t* is given by:

$$TV_t \equiv \frac{1}{2} \sum_i \left| z_{i,t} - z_{i,t-1} \right|$$

2.2 Overnight Risk

As discussed in the arguments of Brock and Kleidon (1992) and Hong and Wang (2000), investors assume overnight risk just before they periodically stop trading. Specifically, investors who trade only during the regular hours assume overnight risk at the end of the regular-hours session; further, investors who also trade during the after-hours session assume overnight risk at the end of the after-hours session. The risk aversion parameter for these investors could be higher than usual. To incorporate this possibility, we define the risk aversion parameter a_i ($a_i >= b$) at the close by:

$$a_i = \begin{cases} b \cdot B_i & i \in X_{\text{ovemight}} \\ b & i \notin X_{\text{ovemight}} \end{cases} \quad (3)$$

where $X_{overnight}$ denotes investors who assume overnight risk. We assume that the overnight risk aversion parameter varies across investors†. B_i-1 is assumed to follow an exponential distribution:

$$(B_i - 1) \sim Exp(1/\lambda) * \frac{NT_{overnight_i}}{(1 - NT_{regular})}$$

$NT_{\text{Overnight}_i}$ denotes the length of the periods during which investor *i* stops trading; NT_{regular} denotes the length of the regular-hours session. Investors' overnight risk could be higher as the length of market closure for them is longer. Thus, the risk aversion parameter for investors who

* The proof is shown in Appendix of Miwa & Ueda (2016).

† The implication of our simulation result is irrelevant to the distribution type of the overnight risk parameters.

assume overnight risk is associated with $NT_{\text{overnight}_i}$.

2.3 Heterogeneous Beliefs and Evolutionary Selection of Strategies

All traders are assumed to be able to derive the fundamental price p_t* that would prevail in a perfectly rational world. The fundamental price continuously reflects the upcoming fundamental news; the value is assumed to be varied over time as:

$$p_{t+1}^{*} = p_t^{*} e^{W_t} \tag{4}$$

W_t is a sequence of $N(0, \sigma_f)$ i.i.d. random variables.[‡] Traders believe that, in a heterogeneous world, prices may deviate from their fundamental value p_t*. Following the "four investor type" model of Brock and Hommes (1998), we assume that the model has four investor types: fundamentalists (denoted as X_1), trend followers (denoted as X_2) who allow price deviation from fundamental value, and two investor types with purely biased beliefs: optimists (denoted as X_3) and pessimists (denoted as X_4), who expect a constant price above (optimists) or below (pessimists) the fundamental price.

$$E_i[p_{t+1}] = p_{t+1}^{*} \qquad i \in X_1 \text{ Fundamentalists} \tag{5}$$

$$E[p_{t+1}] = p_{t+1}^{*} \cdot (p_t / p_t^{*})^{g} \quad i \in X_2 \text{ Trend followers} \tag{6}$$

$$E[p_{t+1}] = p_{t+1}^{*} \cdot (1+d) \qquad i \in X_3 \text{ Positively biased (optimists)} \tag{7}$$

$$E[p_{t+1}] = p_{t+1}^{*} \cdot (1-d) \qquad i \in X_4 \text{ Negatively biased (pessimists)} \tag{8}$$

where $1 > d \geq 0$, $g \geq 0$.[§]

The evolutionary part of the model, describing how beliefs are updated over time, follows the endogenous selection of forecasting rules introduced by Brock and Hommes (1997); each investor chooses one of the four investor types (strategies) at every time period.[**] The choice probability of investor i for strategy s $\Pr_t\{i \in X_s\}$ (s=1,2,3,4) is given by:

[‡] The dynamics of fundamental value in equation (4) is not a part of Brock and Hommes (1997, 1998). This exogenous process of the fundamental value is introduced because the assets pay no dividend and the interest rate is zero in our model.

[§] The manner in which each type of investor forecasts the future price is slightly different from that of Brock and Hommes (1997), in order to keep $E[P_{t+1}]$ (and asset price) positive.

[**] While Brock and Hommes (1997) only address forecasts (beliefs) of different investor types, we tackle the formulation of each individual investor's belief, because we need to analyze the condition that some individual investors do not trade during the extended-hours session.

$$\Pr_t\{i \in X_s\} = \begin{cases} P_{upd} \cdot \dfrac{\exp(\beta U_{s,t-1})}{\sum_s \exp(\beta U_{s,t-1})} + (1 - P_{upd}) & \text{if } i \in \mathrm{X_s} \text{ at time } t-1 \\ P_{upd} \cdot \dfrac{\exp(\beta U_{s,t-1})}{\sum_s \exp(\beta U_{s,t-1})} & \text{if } i \notin \mathrm{X_s} \text{ at time } t-1 \end{cases} \quad (9)$$

where Pr_{upd} ($0 \leq Pr_{upd} \leq 1$) represents the update speed of investors' strategy[††]; $U_{s,t}$ is the fitness measure of strategy s evaluated at time t; and $\beta \in (0, \infty)$ is the intensity of choice in the discrete choice model: a higher β means that investors are more inclined towards the strategy with a high fitness measure. A natural candidate for evolutionary fitness is realized profits, given by:

$$U_{s,t} \equiv (p_t / p_{t-1} - 1)\left(\frac{E[p_t]/ p_{t-1} - 1}{b\sigma^2}\right) \quad (10)$$

2.4 The Extended-Hours Session

Now, we consider the case when there is a periodic market closure or the extended-hours session might has lower market liquidity. We suppose the case that transactions continuously take place during the extended-hours session, as during the regular-hour session[‡‡]. A daily trading session consists of 4 sessions: the pre-market session (during which there are NT_{pre} successive time steps); the regular-hours session (during which there are $NT_{regular}$ successive time steps); the after-hours session (during which there are NT_{after} successive time steps); and the period between the after-hours session and next day's pre-market session, in which the market is closed. We call the first and the last time steps of the regular-hours session the "opening session" and the "closing session," respectively. In addition, we call a step subsequent to the opening session the "subsequent session." All the investors trade the asset during the regular-hours session; on the other hand, only a limited number of investors trade during two extended-hours sessions (i.e., the pre-market session and the after-hours session). R_e denotes the ratio of investors who trade the asset during the extended-hours session.

During the regular-hours session, the demand $z_{i,t}$. can be given by (1), except for the last time step of the regular-hours session. Thus, the formula of an equilibrium price expressed by (2) can be simplified as:

$$p_t = \frac{1}{N}\sum_i E_{i,t}[\mathbf{p}_{t+1}] \quad (11)$$

[††] The implementation of the update speed is not a part of Brock and Hommes (1997, 1998). The update speed parameter is introduced because we formulate a strategy update for each individual.

[‡‡] Some markets have implemented a so-called pre-opening or a pre-closing period, during which orders accumulate without trading then execute transaction at one price. These pre-opening or a pre-closing period can be regarded as short-term (one time step) pre-market session and after-hours session, whatever the length of the period is.

where N is the number of investors in the market (N>0). At the last time step of the regular-hours session, the formula of an equilibrium price expressed by (2) can be simplified as:

$$p_t = \sum_i \frac{E_{i,t}[\mathrm{p}_{t+1}]}{a_i} \Big/ \sum_i \frac{1}{a_i} \tag{12}$$

During the two extended-hours sessions, except for the last time step of the after-hours session, some investors' demand $z_{i,t}$ for the risky asset is constant. The formula of an equilibrium price expressed by (2) can be simplified as:

$$p_t = \frac{\sum_{i \in X_{trade}} E_{i,t}[\mathrm{p}_{t+1}]}{N_{trade_t} - b\sigma^2 \sum_{i \notin X_{trade}} z_{i,t-1}} \tag{13}$$

where N_{trade_t} denotes the number of investors who trade the asset at time t.

Finally, at the last time step of the after-hours session, the equilibrium price can be given by (2).

3. Simulation Settings

3.1 Parameter Setting

We define 24 time steps as comprising one trading day (1 time step per hour). We denote each time step as T_1, $T_2 \ldots T_{24}$. The first six successive time steps of each day (T_1, $T_2 \ldots T_6$) comprise the regular-hours session; the successive NT_{after} time steps ($NT_{after} \leq 9$) after the regular-hours session ($T_7 \ldots T_{6+NT_{after}}$) comprise the after-hours session; the successive NT_{pre} time steps ($NT_{pre} \leq 9$) before the next day's regular-hours session ($T_{24-NT_{pre}+1} \ldots \mathrm{T}_{24}$) comprise the pre-market session (for the next day's regular-hours session); the rest of the time steps ($T_{7+NT_{after}} \ldots T_{24-NT_{pre}}$) are included in the non-trading session. We call the first time step of the regular-hours session (T_1) and the last time step of the regular-hours session (T_6) the "opening session" and the "closing session," respectively.

The number of trading day is set at 1000, the number of investors is set at 100, the risk aversion parameter in equation (3) (b) at 1, the volatility parameter for the fundamental value (σ_f) at 0.01, and the constant estimated return volatility in equation (1) (σ^2) at 1. Let R_e (the ratio of investors who trade during the extended-hours sessions) ={0.1, 0.2, 0.3, 0.5, 0.7}. In terms of the initial conditions of the simulation, p_0=1 and each investor's types are randomly assigned to one of the types.

Other parameters, that is, d (the parameter of biased estimation in equation (7) and (8)), g (the parameter of trend chasing in equation (6)), β (the parameter of evolutionary updating of strategies in equation (9)), and P_{upd} (the parameter of strategy update speed in equation (9)), λ (the risk aversion parameter for overnight risk), are set so that the model replicates the existing stylized facts

regarding the intraday price and volume, namely, the U-shaped intraday pattern of trading volume and return volatility[§§]. In addition, in accordance with Chen et al. (2012), the parameters are set so that the model replicates the two famous stylized facts, namely fat-tailed returns and clustered volatility. We evaluate the fat-tailed return distribution by calculating the kurtosis of single period returns (returns for 1 hour), and examine whether the kurtosis of the stock returns is higher than 3. To evaluate the volatility clustering, we examine whether square returns (and the absolute return) have a significantly positive autocorrelation.

The result of simulation test is as follows[***]. In terms of the trading concentration, the concentration is observed under the condition that Pr_{upd} is not large ($P_{upd} < 0.5$), there is biased investor ($d > 0$), and the investors do concern overnight risk. In terms of U-shaped intraday volatility pattern, the pattern is observed when $Pr_{upd} > 0$, and the investors concern overnight risk. The result reveals that the volatility clustering is observed when there are strong trend followers ($g > 0.5$). In terms of the fat-tailed return distribution, all the runs replicate the fat tails.

In sum, the model can replicate these patterns under the following conditions:

(1) There are strong trend followers (at least, $g > 0.5$)

(2) There are biased traders ($d>0$)

(3) Investors update their strategy, but not excessively (at least $0< P_{upd} < 0.5$)

(4) Investors assume overnight risk at the end of trading sessions.

We regard parameters that satisfy the above conditions as adequate model parameters verified by the stylized facts, intraday volume, and volatility patterns, and the gradual incorporation of fundamental information. Specifically, following the study of Miwa and Ueda (2016), we show the result when we set d, g, λ, β, and P_{upd} at 0.2, 1.25, 5, 1, and 0.2, respectively. However, the implication of the simulation result is invariant regardless of the parameter settings, as long as the settings satisfy the above-mentioned conditions.

We analyze whether the effect of implementing the after-hours and pre-market sessions can improve price efficiency and trading activity, and determine the most beneficial (or the least harmful) length of the session. All the statistics and the figures use an average of 100 simulation runs, except the extreme (highest and lowest) results.[†††] We summarize the simulation settings in Table 1.

[§§] The evaluation method of the U-shaped intraday pattern of trading volume and return volatility is explained in Session 3.2.

[***] The detail of the result is available upon request. The simulation result regarding the parameter setting is also shown in the study of Miwa and Ueda (2016).

[†††] We also test the case when we set an initial learning phase (e.g., 1000 and 10000 times periods). We find that the implication of our simulation result is irrelevant to whether the initial phase is implemented or not.

3.2 Evaluation of the Improvement

In this section, we explain the criteria to evaluate the effect of the partial extension.

3.2.1 Price Efficiency

Since considerable studies (e.g., Kyle, 1985; Glosten and Milgrom, 1985; Foster and Viswanathan, 1990; and Easlay and O'Hara, 1992) suggest that market closures may lead to a delay in the incorporation of information into stock prices, the effect of the extension on price efficiency should be examined. In this study, price efficiency is evaluated by how much a price is closer to the fundamental value. Thus, to evaluate the effect of the extension of trading hours on price efficiency, we examine whether the deviation from fundamental values is lowered by extending the trading hours. We calculate a difference between market price and fundamental value, which is defined by $|\log(P_t/P_t^*)|$, during the regular-hours sessions, the pre-market session, the after-hours session, and the non-trading session, respectively. Specifically, we examine whether the deviation during the regular-hours sessions is lowered by the implementation of the pre-market session and the after-hours session.

3.2.2. Price Stability

Since periodical market closure may cause excessive price fluctuations, especially at the beginning and end of the trading session on an intraday basis (Wood et al., 1985; and Harris,1986), the effect on price stability during the regular session and relative price stability at the open and close of the session should be examined. Thus, we examine whether return volatility, defined by a standard deviation of asset returns, is decreased by the partial extension during the regular-hours session, even if investors rarely trade during the extended-hours sessions. In addition, we calculate return volatility in the opening session and in the closing session, respectively. To test the price instability at the open (close) of the regular-hours session, we calculate the ratio of return volatility in the opening (closing) session to that in other regular-hours sessions (defined by the time steps during regular-hours sessions except in the opening, subsequent, and closing sessions), and examine whether the ratio is higher than one.

3.2.3 Trade Concentration

Since periodic market closure results in trading concentration at the open and close of the regular-hours session (Jain and Joh, 1988), we should address the effect on the trading concentration. To evaluate trade concentration at the open and close of the regular-hours session, we calculate a ratio of average volume at the opening (closing) to that during the other regular-hours session. Then, we examine whether those ratios are higher than one.

4. Simulation Result

4.1. Implementation of the Pre-market Sessions

We examine whether implementing the pre-market session increases price efficiency and price stability, and reduces (eases) the trade concentration. We change the ratio of investors that trade during a pre-market session (R_e ={0.1, 0.2, 0.3, 0.5, 0.7}). Then, we change the length of the pre-market session from zero to nine (hours), and examine the effect of implementing the pre-market session on price efficiency, price stability and trading concentration. We examine the most beneficial duration of the session for each setting regarding extended-hours session participants.

Figure 1 shows the time-series average of the difference between market price and fundamental value. Implementation of the pre-market session reduces the difference between market prices and the fundamental values when there are sufficient market participants during the pre-market session (R_e>=30%); on the other hand, the implementation could increase the difference when there are a limited number of market participants (R_e<=20%). In addition, a longer pre-market session induces higher price divergence during the regular-hours session, especially when investors' participation during the pre-market session is limited. On the other hand, the price divergence is smaller when the length of the pre-market session is set to be one (hour) than when there is no periodic market closure, regardless of the ratio of market participants during the session. In other words, as long as the duration of the pre-market session is short, the implementation could improve price efficiency. As argued in previous sections, a pre-opening period, during which orders accumulate without trading then execute transaction at one time, can be regarded as brief pre-market session. Thus, the result also supports effectiveness of implementing such a pre-opening period.

Figure 2 (a) and (b) show the return volatility during the regular-hours session and price instability in the opening session. The figures show that the implementation of the pre-market session increases return volatility and increases price instability at the opening when there are a limited number of market participants during the session. On the other hand, it reduces these factors when there are sufficient market participants. In addition, a longer session induces higher return volatility during the regular-hours session and lower price stability in the opening session. However, the result also reveals implementation of the pre-market session lowers return volatility during the regular-hours session and increases the price stability at the opening when the duration of the pre-market session is short; the short-term pre-market session can improve price stability during the regular-hours session and ease the price instability at the opening of the session.

Figure 3 shows trade concentration during the opening session. The figures show that the trade concentration at the opening could be eased by implementing the pre-market session when there are sufficient market participants. On the other hand, the longer session results in higher trade concentration; specifically, the implementation of the long-term pre-market session could worsen trade concentration at the opening when there are a limited number of market participants.

Finally, the analysis regarding the trade concentration and price instability at the close of the regular-hours session reveals that the implementation of the pre-market session does not affect the trade concentration and price instability at the close[‡‡‡].

In sum, our result indicates that the implementation of the pre-market session could reduce price efficiency and price stability (especially at the opening), if there are not sufficient participants during the pre-market session. However, as long as the duration of the pre-market session is short, it could improve price efficiency and price stability without worsening the trade concentration, regardless of the number of market participants during the session.

4.2. Implementation of the After-hours Sessions

We examine whether implementing an after-hours session mitigates the price inefficiency, the price instability, and the trade concentration, and analyze the most beneficial session duration. We change the length from zero to nine hours, under the condition that there is no pre-market session.

Figure 4 shows the time-series average of the difference between market prices and the fundamental values. The implementation of the after-hours session could increase the difference, especially when there are a limited number of market participants during the after-hours session. Furthermore, the result reveals that the longer duration of session results in the wider divergence during the regular-hours session. Overall, the implementation of the after-hours session has a more negative impact on price efficiency than the pre-market one.

Figure 5 (c), which shows relative price instability in the closing session, reveals that price instability at the close tends to decrease when there are sufficient market participants during the session. However, the implementation of the after-hours session tends to have a negative impact on price stability during the regular-hours session (especially, at the opening). Figure 5 (a) and (b) show the time-series average of return volatility during the regular-hours session and price instability in the opening session, respectively. The figures show that the implementation increases price instability especially at the opening when there are a limited number of market participants during the session. In addition, the longer session results in higher return volatility during the regular-hours session and lower price stability in the opening session.

Figure 6 shows trade concentration in the opening and closing sessions. Figure 6 (b) reveals that the trade concentration at the close can be eased by implementing the after-hours session. On the other hand, Figure 6 (a) shows that the trade concentration is worsened by implementing the after-hours session when there are a limited number of market participants.

Our analysis reveals that the implementation of the after-hours session could reduce trade concentration and increase price stability at the close, as long as there are sufficient market

[‡‡‡] The detail of the result is not shown in Figures 2 and 3, but it is available upon request.

participants during the session. However, on the other hand, it induces lower price efficiency, lower price stability, and greater trade concentration at the opening, if investors' participation during the session is not sufficiently significant. In addition, it does not improve price efficiency, price stability, and trade concentration at the opening, even if there are sufficient market participants. Compared with the pre-market session, the implementation of the after-hours session has little benefit.

4.3. Simultaneous Implementation of Pre-market and After-hours Sessions

We examine price efficiency and trading activity when the pre-market and after-hours sessions are implemented simultaneously. We change the length of the extended-hours session from zero to nine[§§§]. Figure 7 (a) shows the time-series average of the difference between market prices and the fundamental values. The divergence is narrowed if there are sufficient market participants during the extended-hours session. On the other hand, the divergence is widened if investors' participation during the session is limited. Unlike the result of the implementation of the pre-market session, the short-term extended hours also widen the price divergence during the regular-hours session.

Figure 8 shows that return volatility during the regular-hours session and price instability at the opening are increased by implementing the sessions, if investors' participation during the session is limited. On the other hand, those are decreased when there are sufficient market participants during the session.

Figure 9 (a) shows that the trade concentration at the opening could be worsened by the simultaneous implementation if investors' participation during the extended-hours session is limited. On the other hand, the concentration at the opening could be eased if investors' participation is enough. In addition, the trade concentration at the close is eased by the simultaneous implementation, especially when there are sufficient market participants during the extended-hours session.

The simultaneous implementation of both sessions could improve relative price stability and trade concentration at the close. However, compared with the implementation of the pre-market session, there are many drawbacks in the simultaneous implementation. Specifically, while the implementation of a short-term pre-market session increases price efficiency and price stability, the simultaneous implementation of both of these brief sessions could worsen them. As discussed in the previous session, there are several drawbacks in the implementation of the after-hours session. The negative impact from implementing the after-hours session exceeds the positive impact from implementing the pre-market session.

5. Discussion

5.1. Effectiveness of the Short-term Pre-market Session

[§§§] The length of the after-hours session is equal to that of the pre-market session.

In this section, we discuss the reason why the implementation of a short-term pre-market session is effective for increasing price efficiency and stability.

Table 2 (a) and (b) show the influence of the implementation of the pre-market session on price efficiency when R_e=0.1 and 0.7, respectively. At the beginning of the pre-market session, a divergence of market prices from the fundamental values is significantly reduced, regardless of the number of market participants during the pre-market session. When the pre-market session begins, investors (which include fundamental investors) start to trade. Since they place the buy (sell) orders when the market price is lower (higher) than the fundamental value, their transactions at the beginning of the pre-market session drive market prices to the fundamental values. However, illiquidity during the pre-market session gradually lowers price efficiency. Therefore, the divergence between market prices and the fundamental values has been gradually widened during the pre-market session, especially when investors' participation during the session is limited. Thus, a longer pre-market session results in a greater price divergence at the opening of the regular-hours session. In other words, the implementation of the brief pre-market session is the most effective way to improve price efficiency during the regular-hours session (especially at the opening of the regular-hours session).

Furthermore, lower divergence of market prices from the fundamental values at the opening results in smaller heterogeneousness in investors' price expectations (especially that between fundamentalists and trend followers). Since smaller heterogeneousness in price expectations induces higher price stability, the implementation of the brief pre-market session increases price stability during the regular-hours session (especially at the opening).

On the other hand, the implementation of the pre-market session does not affect price stability and trading activity at the end of the regular-hours session. Since the pre-market session takes place just before regular-hours session, the implementation of the pre-market session affects price and trading behavior at the beginning (opening) of the regular-hours session rather than at the end (close) of the session.

5.2. Effectiveness of the After-hours Session

Our analysis shows that the implementation of the after-hours session eases trade concentration and increases price stability at the close of the regular-hours session. Since the implementation reduces the ratio of investors who assume overnight risk at the close of the regular-hours session, heterogeneousness in investors' expected price at the close is reduced by the implementation. Since the decrease in the heterogeneousness results in lower trading activity and higher price stability, the implementation of the after-hours session has a positive effect on price stability and trading activity at the close.

On the other hand, our analysis reveals that the implementation could reduce price efficiency and

price stability during the regular-hours session, and could increase trade concentration at the opening of the regular-hours session. Table 3 shows the influence of the implementation of the after-hours session on price efficiency and return volatility, when there are a limited number of market participants during the session (R_e=0.1). As shown in the table, prices are extremely unstable and asset prices diverge significantly from fundamental values at the end of the after-hours session, when investors' participation during the session is limited. At the end of the session, both overnight risk (specifically, difference in tolerance to overnight risk between investors) and an insufficiency in market participants increase heterogeneousness in investors' expected prices; and this increased heterogeneousness induces extreme price instability and great divergence from the fundamental value. Since the increased divergence at the end of the after-hours session induces greater divergence from fundamental values (especially at the opening of the regular-hours session), the implementation also results in greater trade concentration and price instability at the opening, due to greater divergence from the fundamental value at that time. Thus, the implementation of the after-hours session has a negative impact on price efficiency and stability during the regular-hours session and trade concentration at the opening.

5.3 Distribution of Investor Types

We assume that the investors select one of the investor types, namely: fundamentalist, trend follower, optimist, and pessimist. According to the simulation result, the distribution of the investor types is frequently changed, especially at the opening and closing of the regular-hours session. However, in the long run, the fractions of each investor type are approximately equal (fluctuate around 25%), regardless of the initial distribution of investor types.[****]

The fitness of each strategy is determined by a recent price change of the risky asset. This price change is attributed to a change in the fundamental value, illiquidity during extended-hours, overnight risk, and a distribution change in the investor types. These factors might cause temporal distribution bias in the investor types, since they cause a temporal difference in the fitness between the strategies. However, in the long run, neither factor is favorable to a specific strategy. Moreover, since a change in fundamental value (which is assumed to be a normal random variable) causes noise in the fitness measures for each strategy, it reduces the persistence of a distribution bias in the investor types. Therefore, the fraction of each investor type eventually fluctuates around 25%, regardless of the initial distribution of investor types.

5.4 Wealth of Investors

In this session, we analyze whether transactions during the extended-hours session affect

[****] Thus, the implication of the simulation result is unchanged whatever initial condition is applied

investors' wealth. For each simulation run, we calculate each investor's wealth growth wg_i by multiplying investors' position and subsequent asset return as

$$wg_{i,t} = \frac{1}{ND}\sum_{t}\left(\frac{p_{t+1}}{p_t} - 1\right) z_{i,t}$$

where *ND* is the number of trading session for one simulation run. Then, for each simulation run, we examine whether wealth growth for the extended-hours session participants is statistically significantly higher than that for non-extended-hours session participants.

The simulation result[††††] reveals that there is no significant difference in wealth growth between the extended-hours session participants and the non-participants, no matter what parameters are. In addition, there is no significant difference in trading volume. Considering that the extended-hours session participants should pay trading costs for the extended-hours trading which is usually higher than those for regular-hour session, wealth could be lower for the extended-hours participants. Thus, there is little incentive to trade during the extended-hours session. The lack of incentive might be one of reasons for low liquidity during the extended-hours session in actual markets.

6. Conclusion

Periodical market closures are said to have a negative impact on price efficiency (e.g., Kyle, 1985; Glosten and Milgrom, 1985; Foster and Viswanathan, 1990; Easlay and O'Hara, 1992), price instability (Wood et al., 1985; Harris, 1986), and the skewed trading activity (Jain and Joh, 1988). It has been suggested that this problem could be solved easily by extending the trading hours. However, Miwa and Ueda (2016) show that the effect of the extension of trading hours is not straightforward.

To uncover the complex feature of the effect of extending trading hours, it is necessary to analyze what kind of extension is effective. Specifically, while Miwa and Ueda (2016) consider a simplified case where the extended-hours trading is defined as trading outside of the regular-hours session, extended-hours trading in many stock exchanges are available during a limited period of time, namely, during the pre-market session and the after-hours session. Therefore, we analyze the effect of implementing the pre-market session and the after-hours session, and the most beneficial session length. To this end, we built an extended agent-based market model, which includes the pre-market and after-hours sessions.

Our simulation analysis reveals that the implementation of the short-term pre-market session is effective for improving price efficiency and price stability, regardless of whether sufficient market participants trade during the session. On the other hand, the implementation of the after-hours session could reduce trade concentration and price instability at the close of the regular-hours session; however, it could reduce price efficiency and price stability during the regular-hours session

[††††] The detail of the result is available upon request.

(especially at the opening), and could worsen trade concentration at the opening, when investors' participation during the after-hours session is limited. Since investors' participation during the after-hours session is actually quite limited (Barclay and Hendershott, 2004), our analysis indicates that the implementation of the after-hours session is likely to have such a negative impact.

Undoubtedly, our findings present an important indication regarding the extension of trading hours. My analysis further uncovers a complex feature of the effect of the extension; our results show significant drawbacks of a drastic extension (implementing long-term extended-hours session) and superiority of the pre-market session to the after-hours session.

Although the implementation of a brief pre-market session might be beneficial without drawbacks, in most case, the extension carries the risk of lowering price efficiency and price stability, especially when there is not enough liquidity during the session. In addition, our result also shows a lack of benefit of extended-hours trading, which results in illiquidity during the session. Thus, our findings suggest that the extended-hours trading has a structural weakness which causes illiquidity during the session and lowers price inefficiency and price instability during the regular-hour session.

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Table 1 Parameter Settings

Table 1 shows the parameter settings of our simulation.

# of trading days	250
# of simulation runs	100
# of investors	100
# of time steps during one regular-hours session	6
# of time steps during one pre-market session	{0, 1, 2…8, 9}
# of time steps during one after-hours session	{0, 1, 2…8, 9}
b (the risk aversion parameter)	1
σ_f (the volatility parameter for the fundamental value)	0.01
σ^2 (the constant estimated return volatility)	1
R_e (the ratio of investors who trade during the extended-hours sessions)	{0, 0.1, 0.2, 0.3, 0.5, 0.7}
d (the parameter of biased estimation)	0.2
g (the parameter of trend chasing)	1.25
β (the parameter of evolutionary updating of strategies)	1
Pr_{upd} (the parameter of strategy update speed)	0.2
λ (the parameter of overnight risk)	5

Table 2 The Effect of the Implementation of the Pre-market Session

Table 2 shows the difference between market prices and the fundamental values when we change the length of the pre-market session from 0 to 9. The rows "Regular-hours" and "Extended-hours" represent the time-series average of the deviation during the regular-hours and extended-hours sessions, respectively. Rows "T1," "T2"…"T24" represent the average value of the deviation in periods T1, T2…T24, respectively.

(a)R_e=0.1

		Length of the pre-market session									
		0	1	2	3	4	5	6	7	8	9
	Regular-hours	1.63%	1.62%	1.64%	1.66%	1.66%	1.68%	1.68%	1.69%	1.68%	1.69%
	Extended-hours	2.96%	2.95%	2.96%	2.99%	3.06%	3.12%	3.21%	3.31%	3.44%	3.55%
Regular-hours Session	T1(Opening)	1.88%	1.81%	1.92%	2.00%	2.05%	2.10%	2.13%	2.16%	2.16%	2.18%
	T2	1.57%	1.56%	1.57%	1.59%	1.58%	1.61%	1.62%	1.60%	1.60%	1.61%
	T3	1.54%	1.53%	1.53%	1.54%	1.53%	1.54%	1.55%	1.54%	1.53%	1.54%
	T4	1.54%	1.52%	1.53%	1.53%	1.53%	1.53%	1.54%	1.54%	1.53%	1.53%
	T5	1.53%	1.53%	1.52%	1.53%	1.53%	1.53%	1.53%	1.54%	1.52%	1.54%
	T6 (Closing)	1.75%	1.76%	1.75%	1.74%	1.75%	1.74%	1.74%	1.75%	1.75%	1.75%
Extended-hours Session	T7	1.92%	1.93%	1.92%	1.90%	1.93%	1.91%	1.92%	1.92%	1.93%	1.92%
	T8	2.08%	2.08%	2.09%	2.07%	2.10%	2.07%	2.08%	2.07%	2.09%	2.09%
	T9	2.22%	2.23%	2.23%	2.22%	2.24%	2.23%	2.22%	2.22%	2.24%	2.24%
	T10	2.36%	2.37%	2.37%	2.37%	2.37%	2.36%	2.35%	2.35%	2.37%	2.37%
	T11	2.49%	2.50%	2.50%	2.49%	2.49%	2.49%	2.48%	2.48%	2.50%	2.50%
	T12	2.61%	2.62%	2.62%	2.62%	2.63%	2.62%	2.62%	2.60%	2.63%	2.63%
	T13	2.73%	2.74%	2.75%	2.74%	2.74%	2.72%	2.73%	2.71%	2.75%	2.74%
	T14	2.85%	2.86%	2.86%	2.85%	2.84%	2.84%	2.84%	2.82%	2.86%	2.86%
	T15	2.96%	2.97%	2.97%	2.96%	2.94%	2.95%	2.96%	2.93%	2.97%	2.97%
	T16	3.06%	3.08%	3.08%	3.06%	3.05%	3.05%	3.06%	3.03%	3.09%	3.33%
	T17	3.15%	3.18%	3.18%	3.16%	3.15%	3.15%	3.16%	3.13%	3.36%	4.02%
	T18	3.27%	3.28%	3.27%	3.25%	3.24%	3.25%	3.26%	3.37%	4.04%	4.41%
	T19	3.36%	3.38%	3.37%	3.35%	3.34%	3.35%	3.37%	4.02%	4.40%	4.66%
	T20	3.46%	3.47%	3.46%	3.43%	3.44%	3.38%	4.01%	4.40%	4.65%	4.82%
	T21	3.56%	3.56%	3.55%	3.53%	3.42%	4.01%	4.38%	4.62%	4.81%	4.97%
	T22	3.65%	3.64%	3.65%	3.42%	4.05%	4.39%	4.64%	4.83%	4.94%	5.06%
	T23	3.75%	3.73%	3.42%	4.05%	4.40%	4.64%	4.82%	4.98%	5.05%	5.13%
	T24	3.85%	3.46%	4.03%	4.41%	4.64%	4.82%	4.96%	5.12%	5.14%	5.22%

(b)R_e=0.7

		Length of the pre-market session									
		0	1	2	3	4	5	6	7	8	9
	Regular-hours	1.63%	1.58%	1.58%	1.58%	1.58%	1.58%	1.57%	1.57%	1.57%	1.57%
	Extended-hours	2.96%	2.86%	2.77%	2.66%	2.58%	2.51%	2.43%	2.36%	2.30%	2.26%
Regular-hours Session	T1(Opening)	1.87%	1.60%	1.57%	1.58%	1.60%	1.61%	1.61%	1.60%	1.60%	1.61%
	T2	1.58%	1.54%	1.53%	1.54%	1.55%	1.55%	1.54%	1.54%	1.54%	1.55%
	T3	1.54%	1.54%	1.54%	1.53%	1.53%	1.54%	1.53%	1.53%	1.53%	1.53%
	T4	1.53%	1.53%	1.53%	1.53%	1.53%	1.53%	1.53%	1.53%	1.53%	1.52%
	T5	1.53%	1.53%	1.53%	1.54%	1.53%	1.53%	1.52%	1.53%	1.53%	1.53%
	T6 (Closing)	1.75%	1.75%	1.75%	1.73%	1.73%	1.71%	1.72%	1.71%	1.68%	1.68%
Extended-hours Session	T7	1.92%	1.92%	1.93%	1.90%	1.91%	1.90%	1.89%	1.88%	1.87%	1.86%
	T8	2.08%	2.08%	2.10%	2.06%	2.06%	2.07%	2.04%	2.04%	2.03%	2.03%
	T9	2.23%	2.23%	2.24%	2.21%	2.22%	2.22%	2.19%	2.20%	2.18%	2.19%
	T10	2.37%	2.35%	2.38%	2.35%	2.35%	2.36%	2.34%	2.33%	2.32%	2.33%
	T11	2.50%	2.49%	2.50%	2.48%	2.48%	2.49%	2.47%	2.47%	2.45%	2.46%
	T12	2.62%	2.61%	2.62%	2.61%	2.61%	2.62%	2.59%	2.59%	2.58%	2.58%
	T13	2.73%	2.72%	2.74%	2.73%	2.72%	2.73%	2.72%	2.70%	2.69%	2.71%
	T14	2.85%	2.84%	2.86%	2.84%	2.84%	2.84%	2.83%	2.82%	2.81%	2.81%
	T15	2.95%	2.95%	2.98%	2.95%	2.95%	2.93%	2.94%	2.92%	2.93%	2.92%
	T16	3.06%	3.06%	3.08%	3.05%	3.06%	3.04%	3.05%	3.02%	3.02%	1.91%
	T17	3.16%	3.16%	3.17%	3.15%	3.16%	3.14%	3.15%	3.12%	1.94%	1.93%
	T18	3.26%	3.26%	3.27%	3.25%	3.26%	3.24%	3.25%	1.96%	1.93%	2.00%
	T19	3.36%	3.36%	3.37%	3.36%	3.34%	3.33%	1.99%	1.95%	2.01%	2.06%
	T20	3.45%	3.45%	3.46%	3.45%	3.43%	2.01%	1.97%	2.01%	2.08%	2.10%
	T21	3.54%	3.53%	3.55%	3.54%	2.03%	1.95%	2.03%	2.08%	2.10%	2.14%
	T22	3.62%	3.62%	3.64%	2.03%	1.96%	2.02%	2.08%	2.12%	2.13%	2.17%
	T23	3.70%	3.70%	2.06%	1.95%	2.02%	2.08%	2.13%	2.15%	2.16%	2.20%
	T24	3.79%	2.07%	1.97%	2.02%	2.08%	2.13%	2.16%	2.17%	2.19%	2.21%

Table 3 The Effect of the Implementation of the After-hours Session

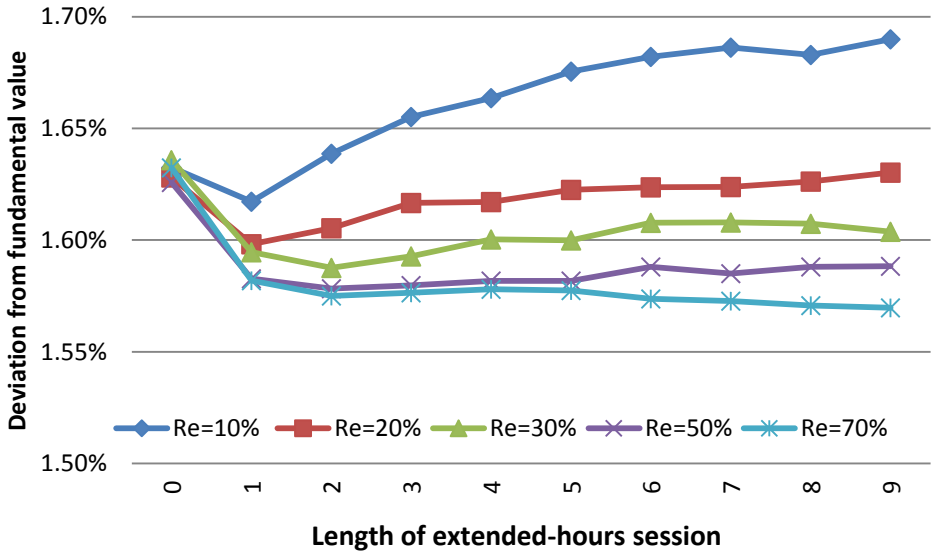
Table 3 (a) and (b) show the difference between the market prices and fundamental values and return volatility when we change the length of the pre-market session from 0 to 9, under the condition that R_e is set at 0.1. The rows "Regular-hours" and "Extended-hours" represent the time-series average during the regular-hours and extended-hours sessions, respectively. Rows "T1," "T2"…"T24" represent the average value in periods T1, T2…T24, respectively.

(a) The difference between market prices and fundamental values

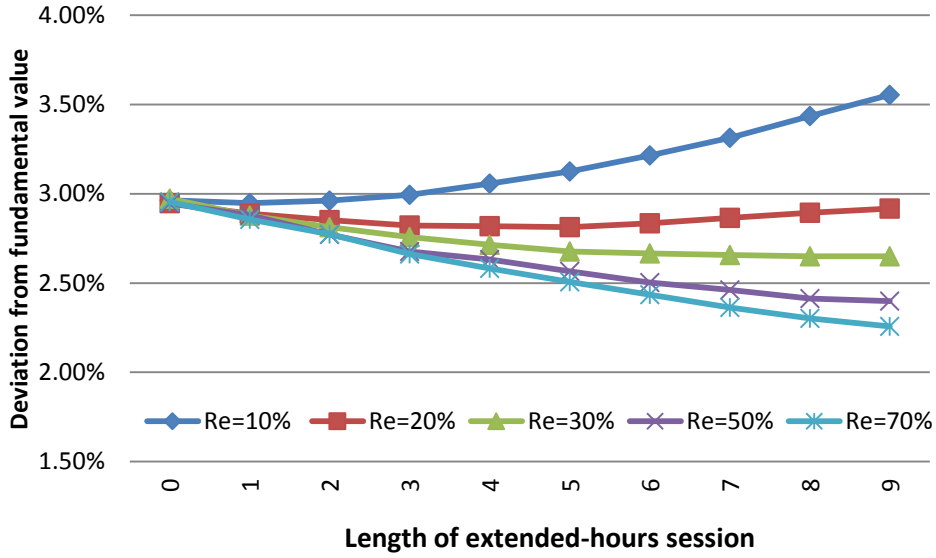
		Length of the after-hours session									
		0	1	2	3	4	5	6	7	8	9
	Regular-hours	1.65%	1.89%	1.92%	1.98%	2.01%	2.02%	2.06%	2.07%	2.08%	2.09%
	Extended-hours	2.97%	8.65%	8.88%	9.36%	9.53%	9.33%	9.45%	9.21%	9.02%	8.74%
Regular-hours Session	T1(Opening)	1.88%	3.18%	3.32%	3.58%	3.75%	3.79%	3.99%	4.01%	4.08%	4.09%
	T2	1.59%	1.77%	1.80%	1.85%	1.90%	1.90%	1.95%	1.96%	1.97%	1.98%
	T3	1.55%	1.55%	1.56%	1.56%	1.58%	1.57%	1.57%	1.58%	1.59%	1.59%
	T4	1.54%	1.53%	1.54%	1.54%	1.53%	1.53%	1.53%	1.54%	1.54%	1.55%
	T5	1.54%	1.54%	1.54%	1.53%	1.54%	1.52%	1.53%	1.54%	1.53%	1.54%
	T6 (Closing)	1.76%	1.78%	1.78%	1.80%	1.79%	1.78%	1.78%	1.77%	1.77%	1.78%
Extended-hours Session	T7	1.93%	8.28%	2.67%	2.70%	2.68%	2.68%	2.70%	2.68%	2.70%	2.71%
	T8	2.09%	8.33%	8.97%	3.57%	3.58%	3.55%	3.57%	3.56%	3.59%	3.57%
	T9	2.24%	8.38%	9.00%	9.89%	4.25%	4.23%	4.28%	4.26%	4.29%	4.25%
	T10	2.38%	8.42%	9.04%	9.92%	10.53%	4.75%	4.83%	4.81%	4.80%	4.78%
	T11	2.51%	8.47%	9.08%	9.95%	10.57%	10.72%	5.23%	5.20%	5.21%	5.19%
	T12	2.62%	8.52%	9.10%	9.98%	10.60%	10.74%	11.32%	5.52%	5.54%	5.50%
	T13	2.74%	8.56%	9.14%	10.01%	10.62%	10.78%	11.35%	11.48%	5.78%	5.74%
	T14	2.87%	8.59%	9.16%	10.05%	10.65%	10.80%	11.37%	11.52%	11.74%	5.89%
	T15	2.97%	8.63%	9.21%	10.09%	10.68%	10.83%	11.40%	11.55%	11.76%	11.84%
	T16	3.07%	8.67%	9.25%	10.12%	10.70%	10.87%	11.43%	11.58%	11.78%	11.87%
	T17	3.17%	8.71%	9.28%	10.15%	10.73%	10.90%	11.46%	11.62%	11.81%	11.90%
	T18	3.27%	8.75%	9.32%	10.19%	10.77%	10.93%	11.50%	11.64%	11.84%	11.92%
	T19	3.36%	8.79%	9.35%	10.22%	10.79%	10.97%	11.52%	11.66%	11.86%	11.95%
	T20	3.46%	8.84%	9.38%	10.26%	10.83%	10.99%	11.55%	11.70%	11.88%	11.97%
	T21	3.55%	8.88%	9.40%	10.30%	10.86%	11.01%	11.59%	11.72%	11.91%	12.00%
	T22	3.64%	8.91%	9.43%	10.33%	10.88%	11.04%	11.63%	11.74%	11.92%	12.03%
	T23	3.73%	8.95%	9.47%	10.35%	10.90%	11.07%	11.65%	11.78%	11.94%	12.07%
	T24	3.81%	9.00%	9.51%	10.37%	10.93%	11.11%	11.70%	11.81%	11.97%	12.10%

(b)Return volatility

		Length of the after-hours session									
		0	1	2	3	4	5	6	7	8	9
	Regular-hours	1.93%	3.68%	3.87%	4.15%	4.36%	4.41%	4.62%	4.66%	4.73%	4.80%
Regular-hours Session	T1(Opening)	3.32%	7.96%	8.42%	9.10%	9.61%	9.75%	10.24%	10.35%	10.51%	10.69%
	T2	1.64%	2.83%	2.99%	3.18%	3.33%	3.35%	3.54%	3.53%	3.61%	3.63%
	T3	1.38%	1.58%	1.61%	1.65%	1.69%	1.69%	1.76%	1.74%	1.76%	1.77%
	T4	1.36%	1.37%	1.38%	1.37%	1.39%	1.40%	1.40%	1.40%	1.39%	1.41%
	T5	1.35%	1.36%	1.34%	1.36%	1.35%	1.36%	1.35%	1.35%	1.36%	1.36%
	T6 (Closing)	1.73%	1.78%	1.76%	1.76%	1.76%	1.74%	1.77%	1.74%	1.76%	1.76%
Extended-hours Session	T7	0.00%	11.89%	3.00%	2.98%	2.98%	2.99%	3.00%	2.99%	3.01%	3.00%
	T8	0.00%	0.00%	11.75%	3.09%	3.11%	3.06%	3.07%	3.07%	3.07%	3.08%
	T9	0.00%	0.00%	0.00%	11.76%	3.08%	3.08%	3.08%	3.10%	3.10%	3.10%
	T10	0.00%	0.00%	0.00%	0.00%	11.78%	3.10%	3.09%	3.10%	3.11%	3.10%
	T11	0.00%	0.00%	0.00%	0.00%	0.00%	11.20%	3.10%	3.11%	3.10%	3.14%
	T12	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	11.40%	3.12%	3.11%	3.13%
	T13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.97%	3.13%	3.17%
	T14	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.99%	3.16%
	T15	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.89%
	T16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T17	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T18	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T21	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T22	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T23	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	T24	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%



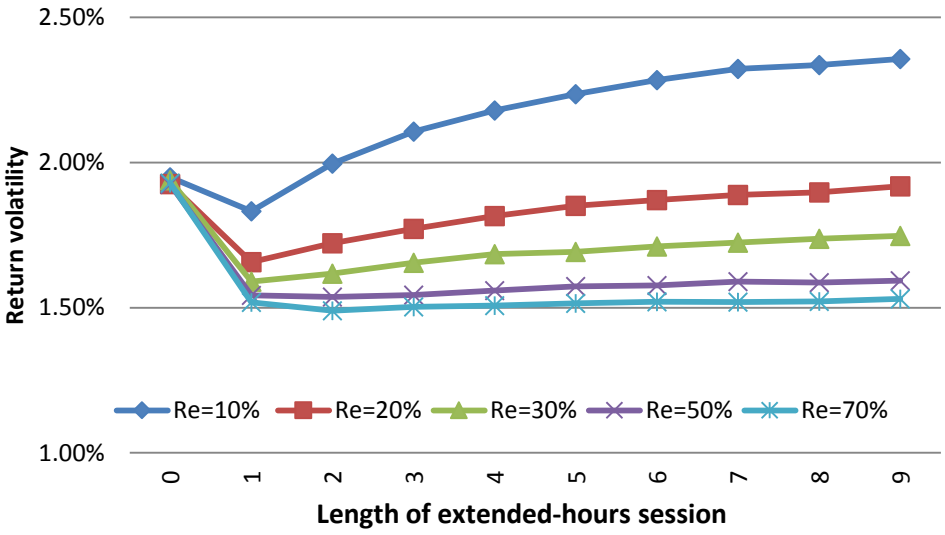
(a) Regular-hours session



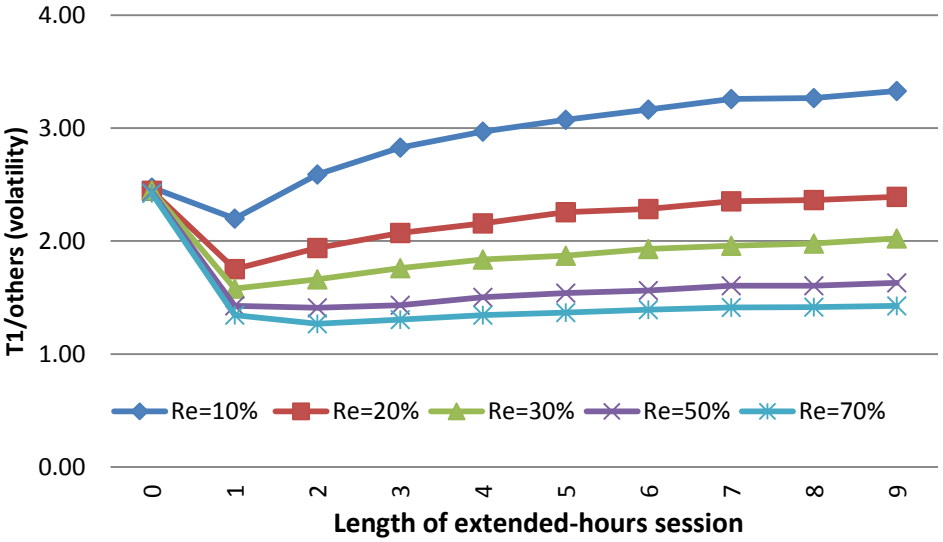
(b) Pre-market session

Figure 1 Deviation from Fundamental Value: Pre-market Session

Figure 1 shows the time-series average of the difference between market prices and fundamental values when we change the length of the pre-market session from 0 to 9, under the condition that an after-hours session does not take place. Figure 1 (a) and (b) show the deviation during the regular-hours session and the pre-market session, respectively. "R_e=10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e (the ratio of investors who trade during the extended-hours sessions)=10%, 20%, 30%, 50%, and 70%, respectively.



(a) Regular-hours session



(b) Price instability at the opening

Figure 2 Return Volatility: Pre-market Session

Figure 2 shows the average volatility when we change the length of the pre-market session from 0 to 9, under the condition that an after-hours session does not take place. Figure 2 (a) shows the volatility during the regular-hours session. Figure 2 (b) shows the ratios of average volatility in the opening session (T1) to that during the other regular-hours sessions. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.

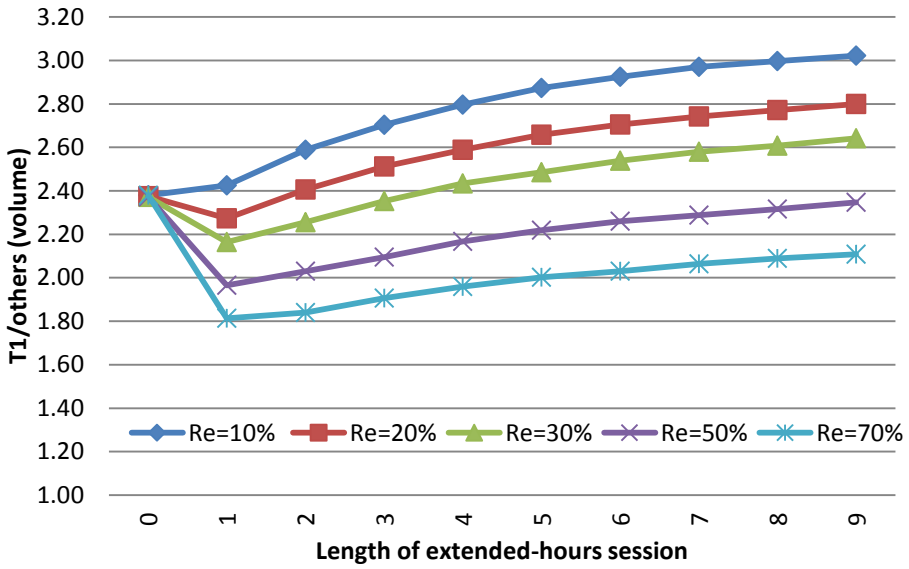
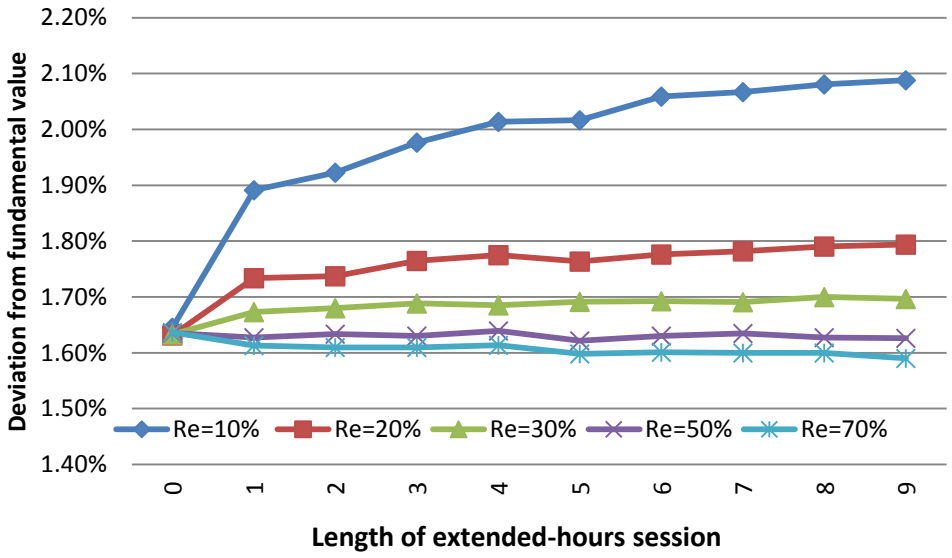
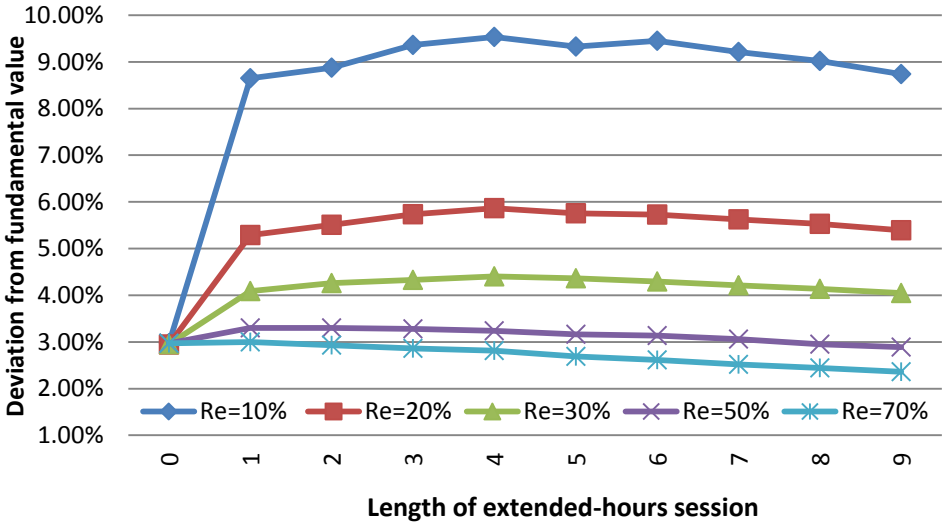


Figure 3 Trade Concentration: Pre-market Session

Figure 3 shows the ratios of average volume in the opening session (T1) to that during the other regular-hours sessions when we change the length of the pre-market session from 0 to 9, under the condition that an after-hours session does not take place. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.



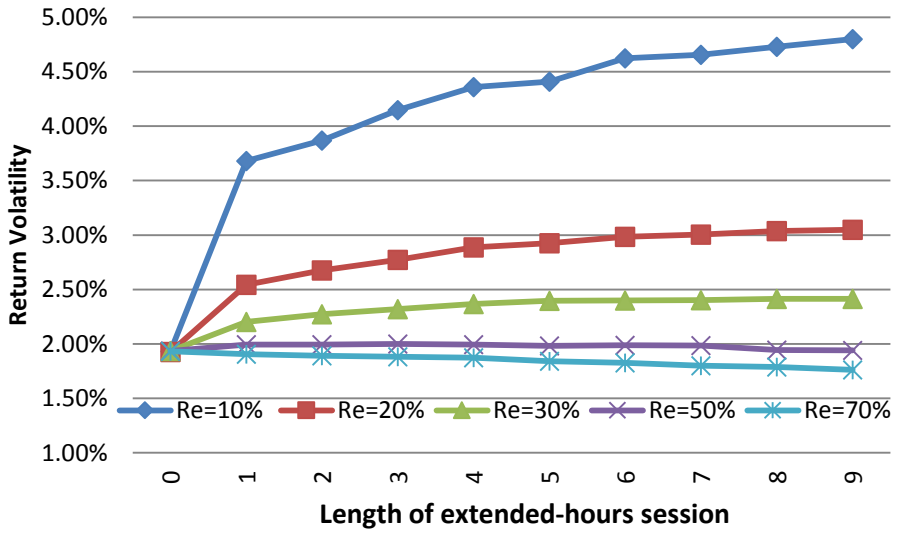
(a) Regular-hours session



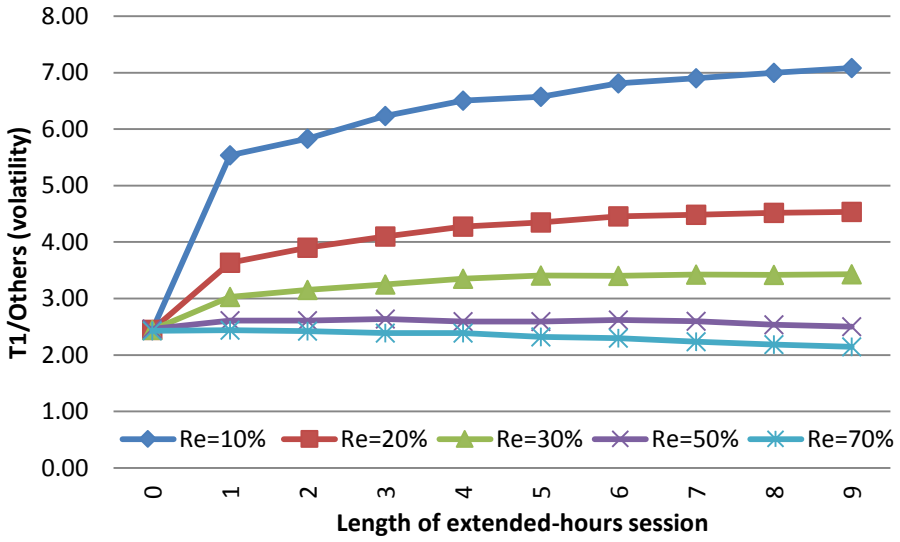
(b) After-hours session

Figure 4 Deviation from Fundamental Value: After-hours Session

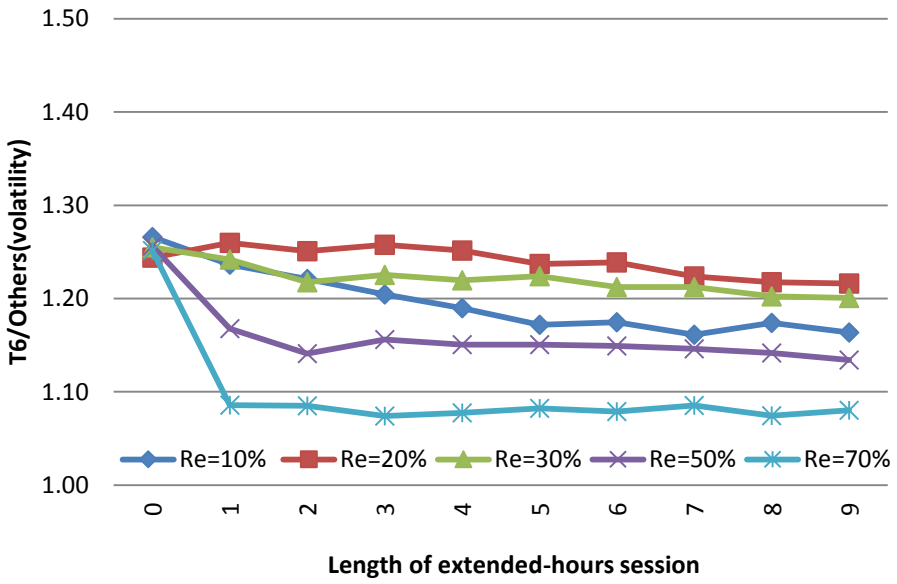
Figure 4 shows the time-series average of the difference between market prices and the fundamental value when we change the length of the after-hours session from 0 to 9, under the condition that a pre-market session does not take place. Figure 4 (a) and (b) show the deviation during the regular-hours session and the after-hours session, respectively. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e (the ratio of investors who trade during the extended-hours sessions)=10%, 20%, 30%, 50%, and 70%, respectively.



(a)Regular-hours session



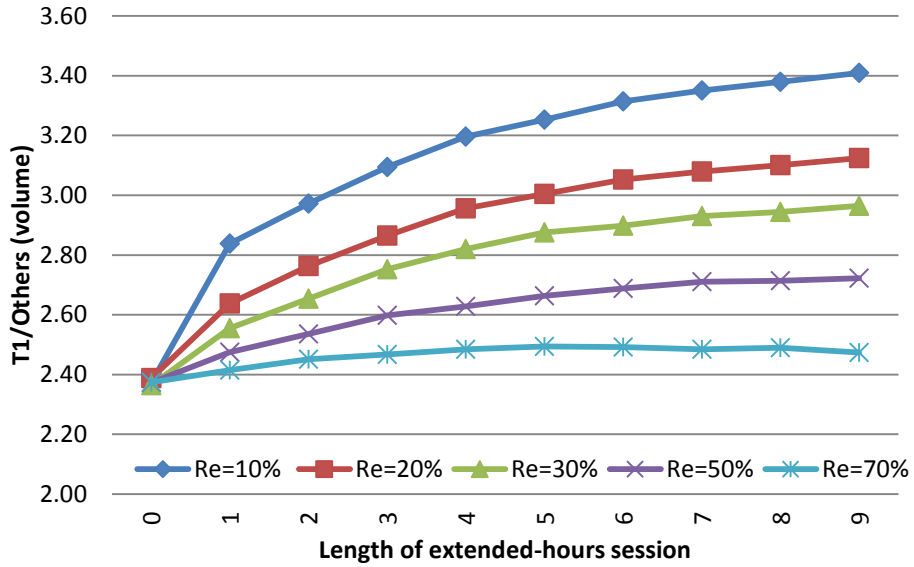
(b) Price instability at the opening



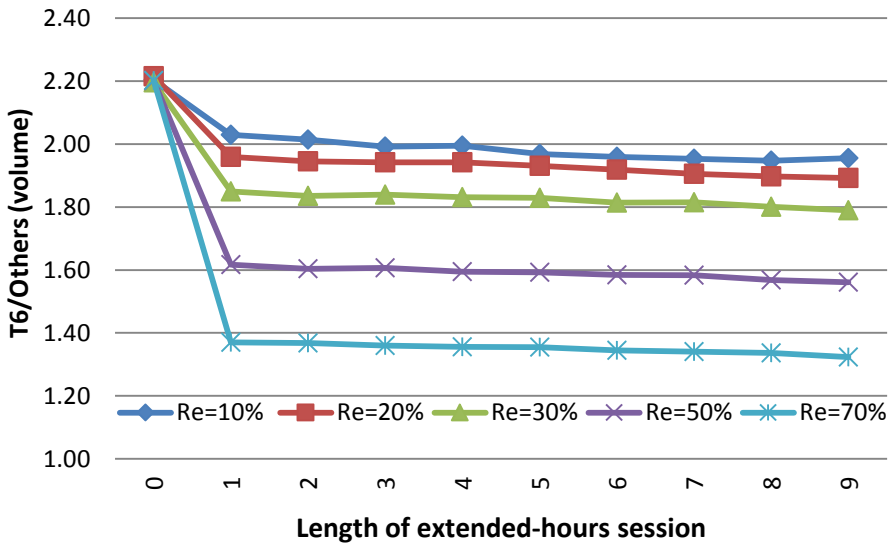
(c) Price instability at the close

Figure 5 Return Volatility: After-hours session

Figure 5 shows the average volatility when we change the length of the after-hours session from 0 to 9, under the condition that a pre-market session does not take place. Figure 5 (a) shows the volatility during the regular-hours session; Figure 5 (b) shows the ratios of average volatility in the opening session (T1) to that during the other regular-hours sessions; Figure 5 (c) shows the ratios of average volatility in the closing session (T6) to that during the other regular-hours sessions. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.



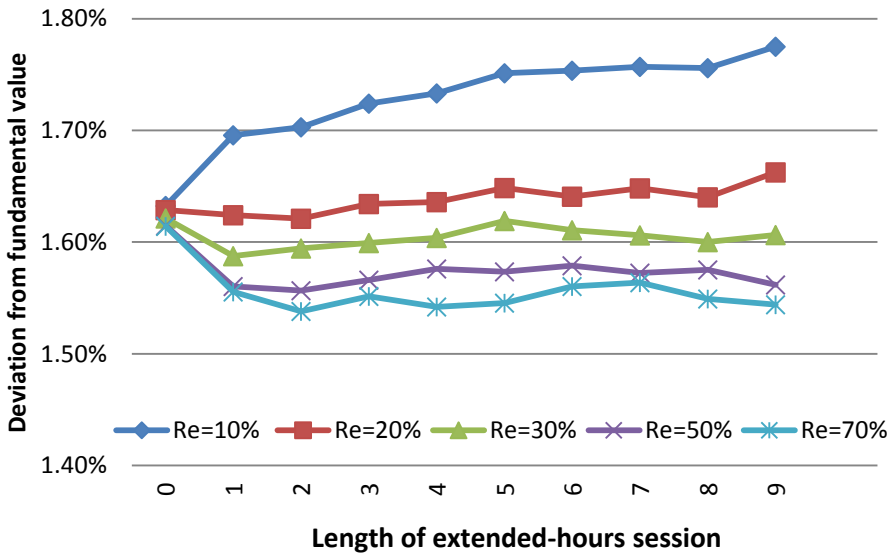
(a) Trade concentration at the open



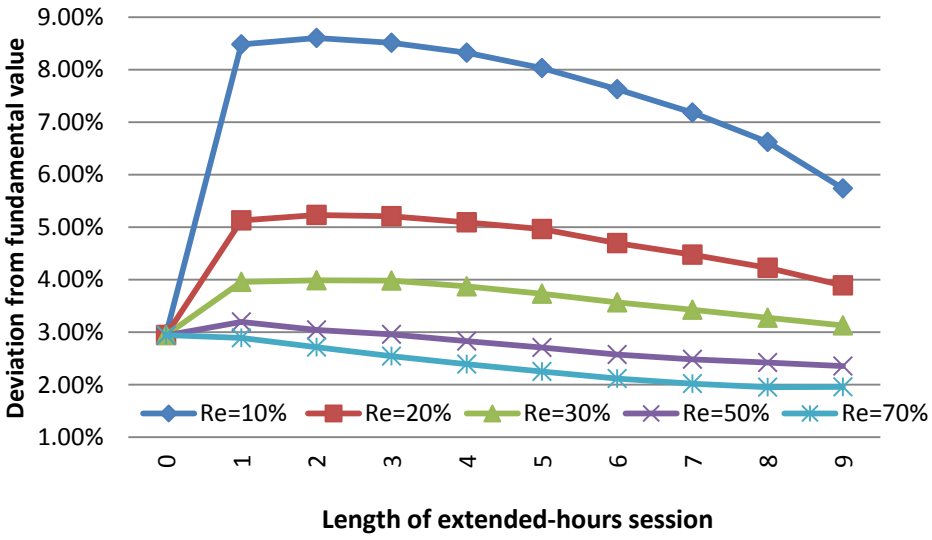
(b) Trade concentration at the close

Figure 6 Trade concentration: After-hours session

Figure 6 shows the average volume when we change the length of the after-hours session from 0 to 9, under the condition that a pre-market session does not take place. Figure 6 (a) shows the ratios of average volume in the opening session (T1) to that during the other regular-hours sessions. Figure 6 (b) shows the ratios of average volume in the closing session (T6) to that during the other regular-hours sessions. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.



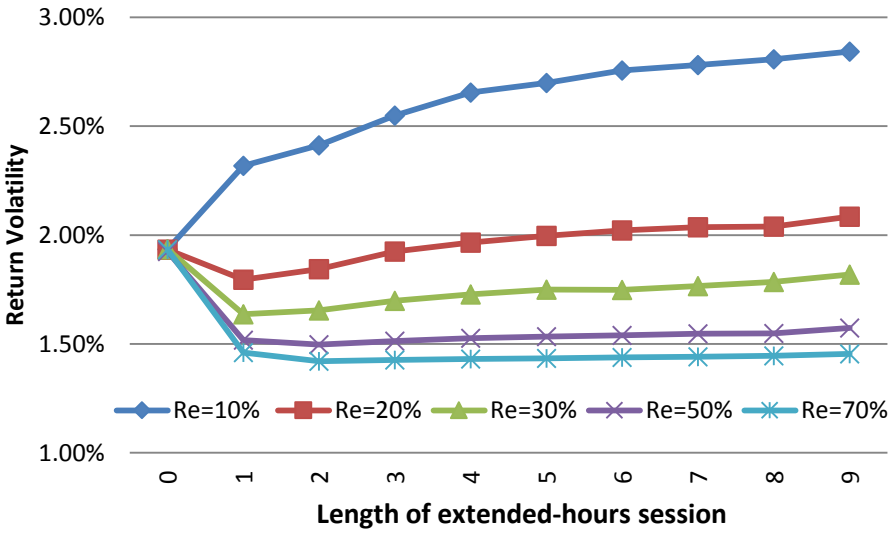
(a) Regular-hours session



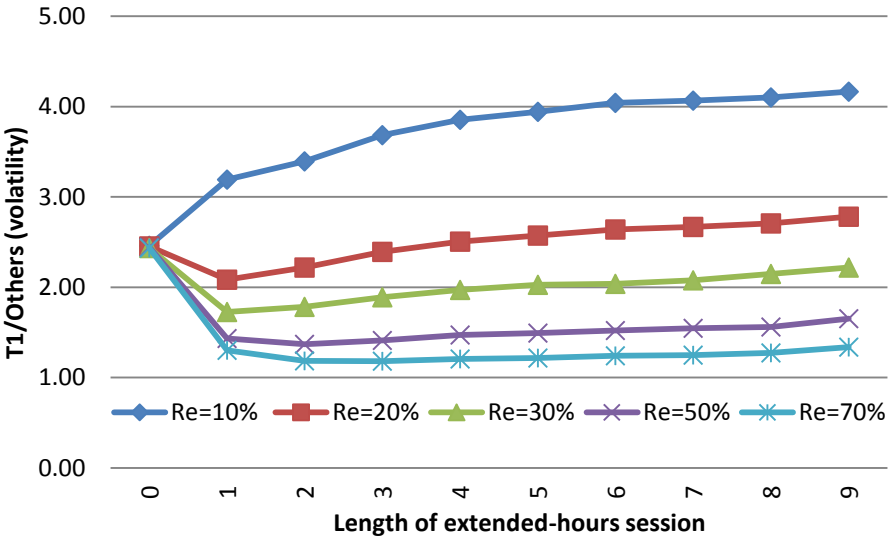
(b) Extended-hours session

Figure 7 Deviation from Fundamental Value: The Simultaneous Implementation

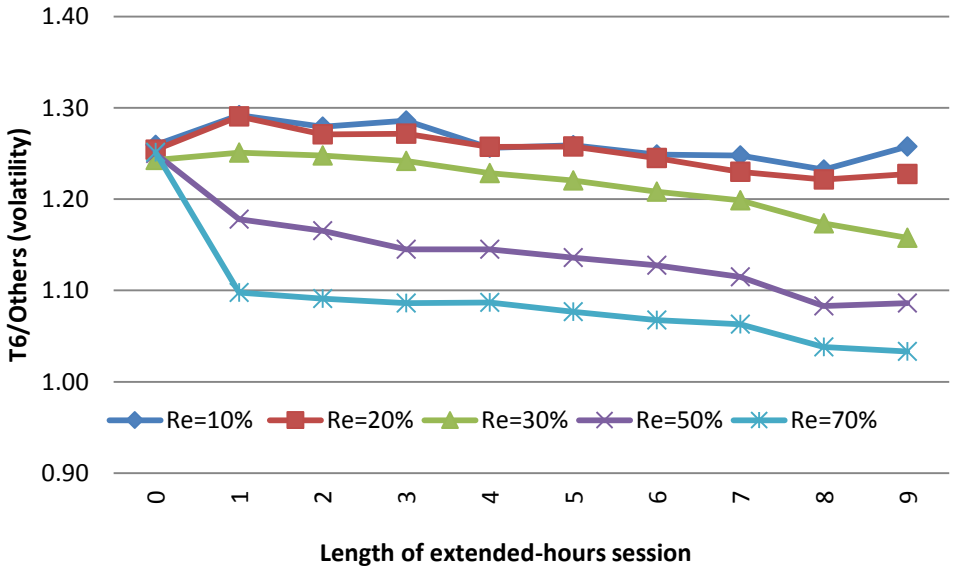
Figure 7 shows the time-series average of the difference between market price and fundamental value when we change the length of both the session from 0 to 9. Figure 7 (a) and (b) show the deviation during the regular-hours session and the extended-hours session, respectively. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e (the ratio of investors who trade during the extended-hours sessions)=10%, 20%, 30%, 50%, and 70%, respectively.



(a) Regular-hours session



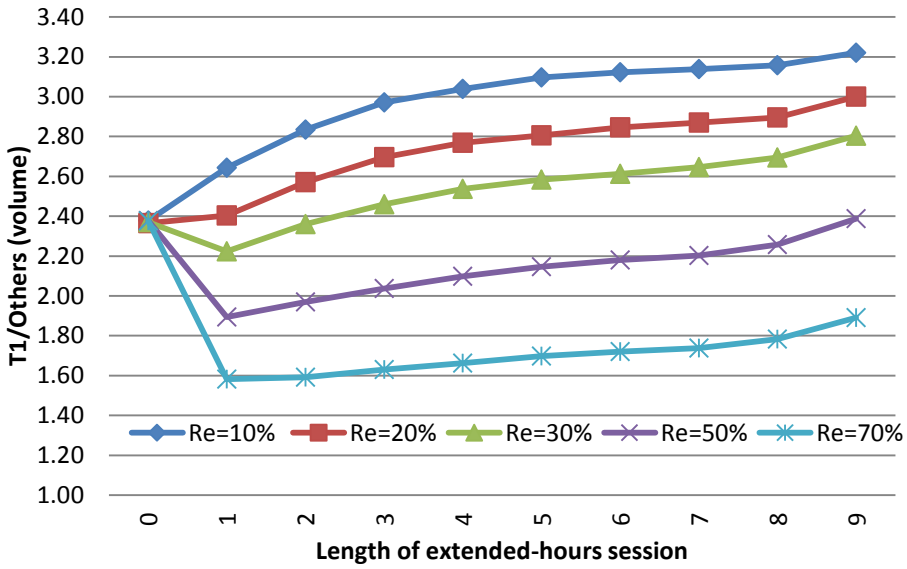
(b) Price instability at the opening



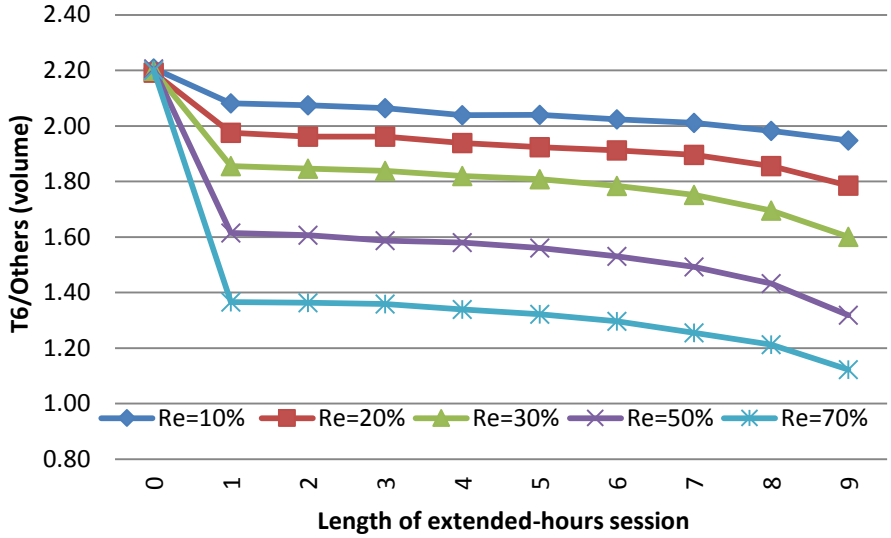
(c) Price instability at the close

Figure 8 Return Volatility: The Simultaneous Implementation

Figure 8 shows the average volatility when we change the length of both the session from 0 to 9. Figure 8 (a) shows the volatility during the regular-hours session; Figure 8 (b) shows the ratios of average volatility in the opening session (T1) to that during the other regular-hours sessions; Figure 8 (c) shows the ratios of average volatility in the closing session (T6) to that during the other regular-hours sessions. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.



(a) Trade concentration at the opening



(b) Trade concentration at the close

Figure 9 Trade Concentration: The Simultaneous Implementation

Figure 9 shows the average volume when we change the length of both the session from 0 to 9. Figure 9 (a) shows the ratios of average volume in the opening session (T1) to that during the other regular-hours sessions. Figure 9 (b) shows the ratios of average volume in the closing session (T6) to that during the other regular-hours sessions. "R_e =10%," "R_e =20%," "R_e =30%," "R_e =50%," and "R_e =70%" represent the time-series average when R_e =10%, 20%, 30%, 50%, and 70%, respectively.