All Shook Up: International Shocks and Firm-level Volatility*

Christopher Kurz
Federal Reserve Board

Mine Senses
Johns Hopkins University

Andrei Zlate
Federal Reserve Bank of Boston

February 2016

Abstract

A large body of theoretical and empirical literature provides insight into the implications of international trade for macroeconomic dynamics. In contrast, there is little work and a lack of consensus regarding the impact of international shocks on firm-level characteristics through international trade. More recently, Kurz and Senses (2016) empirically investigate the link between a firm's international status and its volatility. However, despite the vast theoretical literature on the macroeconomic dynamics that result from international trade, there is little theoretical research that rationalizes the relationship between a firm's trading patterns and its volatility. Our paper attempts to fill this gap by exploring the relationship between firms' exporting and importing status and firm-level characteristics—namely the volatility of output and employment—in a stochastic, dynamic, general equilibrium model of international macroeconomics and trade. We augment the framework with heterogeneous firms and endogenous exporting from Ghironi and Melitz (2005) to allow for international input sourcing. In this framework, we examine the firm-level volatility generated by the model for a cross-section of firm types, which are defined to reflect the rich heterogeneity in firms' international activities highlighted in Kurz and Senses (2016). In line with the empirical evidence, the model predictions are: (1) Exporters display lower volatility than non-exporters, whereas importers display higher volatility that non-importers. (2) Firms that trade for longer durations display

*Contact: christopher.j.kurz@frb.gov, msenses1@jhu.edu, andrei.zlate@bos.frb.org. The views in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Federal Reserve Bank of Boston, the Board of Governors of the Federal Reserve System, or of any other person associated with the Federal Reserve System.

lower volatility than firms switching in and out of international trade. (3) Firms that export to a larger number of uncorrelated foreign markets are less volatile, whereas firms importing from a more diversified range of uncorrelated foreign suppliers are more volatile. The model rationalizes these findings by highlighting the asymmetry in the way that diversification across uncorrelated trading partners affects exporters and importers: While diversification reduces the volatility of exporters, as positive and negative shocks in destination markets offset each other, it enhances the volatility of importers, since disruptions from one single supplier affects the entire production process reliant on complementary inputs.

JEL classification: F16, F23, F41, L25

Keywords: firm-level volatility; exporting; international sourcing; heterogeneous firms; trade intensity; trade duration; diversification.

1 Introduction

A large body of theoretical and empirical literature provides insight into the implications of international trade for macroeconomic dynamics. In contrast, there is little work and a lack of consensus regarding the impact of international shocks on firm-level characteristics through international trade. More recently, Kurz and Senses (2016) empirically investigate the link between a firm's international status and its volatility. However, despite the vast theoretical literature on the macroeconomic dynamics that result from international trade, there is little theoretical research that rationalizes the relationship between a firm's trading patterns and its volatility.

Our paper attempts to fill this gap by exploring the relationship between firms' exporting and importing status and firm-level characteristics—namely the volatility of output and employment—in a stochastic, dynamic, general equilibrium model of international macroeconomics and trade. We augment the framework with heterogeneous firms and endogenous exporting from Ghironi and Melitz (2005, henceforth, GM05) to allow for international input sourcing, like in Zlate (2016). Thus, our model consists of two economies, the North and the South. Each economy includes one representative household and a continuum of firms—with each firm producing a different variety—that are monopolistically competitive and heterogeneous in labor productivity. In both the North and the South, each firm produces for its domestic market. In addition, some firms also produce domestically and export like in GM05. Nonetheless, some of the Northern firms can choose to produce either domestically or offshore and import to the North.

In this framework, we examine the firm-level volatility generated by the model for a cross-section of firm types, which are defined to reflect the rich heterogeneity in firms' international activities highlighted in Kurz and Senses (2016). For this purpose, we define three types of aggregate firms according to their exporting and importing status. The resulting firms produce multiple varieties, and their aggregate output is obtained by integrating over the varieties produced and endogenously exported or imported. Based on export status, the three firms are: (1) the firm that never exports, (2) the firm that sometimes exports, and (3) the firm that always exports. Based on import status, the three firms are: (1) the firm that never imports, (2) the firm that sometimes imports, and (3) the firm that always imports. As such, the model with heterogeneous firms becomes a model with heterogeneous varieties, where the idiosyncratic productivity factor applies to the variety rather than to the firm itself. To compare the model implications for firm-level volatility to those from the data, we simulate the model under various assumptions for the two-country bivariate process of aggregate productivity, and compute model-generated measures of volatility for the three types of exporting/importing firms.

In line with the empirical evidence, the model implies that, first, exporters display lower volatility than non-exporters, whereas importers display higher volatility that non-importers. Second, firms that trade for longer durations display lower volatility than firms switching in and out of international trade. Third, firms that export to uncorrelated foreign markets are less volatile, whereas firms importing from uncorrelated foreign suppliers are more volatile. The model rationalizes these findings by highlighting the asymmetry in the way that diversification across uncorrelated trading partners affects exporters and importers: While diversification reduces the volatility of exporters, as positive and negative shocks in destination markets offset each other, it enhances the volatility of importers, since disruptions from one single supplier affects the entire production process reliant on complementary inputs.

This paper provides a theoretical rationale for the empirical evidence on the link between exporting, importing, and firm-level volatility documented in Kurz and Senses (2016). In addition, our paper adds to a recent stream of the theoretical literature that rationalizes the impact of firms'decisions to export and/or import on either firm-level or macro-level characteristics. However, most of this theoretical literature is silent on the impact of firms' exporting and importing decisions on firm-level characteristics, although it devotes ample attention to the impact on macro-level variables (see GM05, Alessandria and Choi, 2007; Contessi, 2015; Fattal Jaef and Lopez, 2014; Liao and Santacreu, 2015; Mandelman, 2016; Mandelman and Zlate, 2015; Zlate, 2016). Among the former, Fillat and Garetto (2015) model the impact of firms' international status on firms' financial characteristics, with firms engaging in exports and horizontal FDI displaying higher stock returns and earning yields. Fillat, Garetto, and Oldenski (2015) also model the impact of host economy characteristics on the risk premia of multinational firms.

The rest of the paper is organized as follows: Section 2 introduces the baseline model with heterogeneous firms, describes the firms' endogenous decisions to export and import, and translates the model into an equivalent framework with three aggregate firms defined according to their trade status. Section 3 presents the calibration. Section 4 discusses the results, including impulse responses and moments describing the link between the firms' trade status and volatility of output growth. Section 5 concludes.

2 Model with Exporting and Importing Firms

The model builds on the setup with heterogeneous firms, endogenous firm entry, and endogenous exporting in Ghironi and Melitz (2005, henceforth GM05). This setup is augmented with endogenous imports as in Zlate (2016). Thus, the model consists of two economies, the North and the South. Each economy includes one representative household and a continuum of firms that are monopolistically competitive and heterogeneous in labor productivity. In both the North and the South, each firm produces a different variety of goods for its domestic market, and some firms also produce domestically for the export market like in GM05. In addition, some of the Northern firms can choose to produce either domestically or offshore for their home market, like in Zlate (2016). Offshore production results in the firm producing in the South and importing back to the North. All Southern firms produce domestically for their home market due to the steady-state asymmetry in the cost of effective labor across countries, which is higher in the North.

This section describes the problem of the representative household and firms from the North, for the baseline model with financial integration.[1] The case for the South looks similar, except

[1] "Baseline" refers to the model with fixed labor supply and no capital. Also, "financial integration" refers

for the fact that Southern firms do not produce offshore in the North and thus do not import.

2.1 Household's Problem

The representative household maximizes expected lifetime utility: $\max_{\{B_{t+1},\, x_{t+1}\}} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \right]$, where $\beta \in (0,1)$ is the subjective discount factor, C_t is aggregate consumption, and $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution. The budget constraint is:

$$
\begin{aligned}
&(\widetilde{v}_t + \widetilde{d}_t) N_t x_t + w_t L + (1 + r_t) B_{N,t} + (1 + r_t^*) Q_t B_{S,t} + T_t \\
&\geqslant \widetilde{v}_t (N_t + N_{E,t}) x_{t+1} + C_t + B_{N,t+1} + Q_t B_{S,t+1} + \frac{\pi}{2} (B_{N,t+1})^2 + \frac{\pi}{2} Q_t (B_{S,t+1})^2 ,
\end{aligned}
\tag{1}
$$

The household starts every period with share holdings x_t in a mutual fund of N_t firms whose average market value is $\widetilde{v}_t$.[2] It also holds risk-free, country-specific real bonds from the North and the South, $B_{N,t}$ and $B_{S,t}$, denominated in units of the issuing country's consumption basket. The holdings of Southern bonds are converted into units of the Northern basket through the real exchange rate Q_t.[3] The household receives dividends equal to the average firm profit $\widetilde{d}_t$ in proportion with the stock of firms N_t, the real wage w_t for $L \equiv 1$ supplied inelastically, and real rates of return r_t and r_t^* from the North and South-specific bonds.

Every period, the household purchases two types of assets. First, it purchases x_{t+1} shares in a mutual fund of Northern firms, which includes N_t incumbent firms producing either domestically or offshore at time t, and also $N_{E,t}$ new firms that enter the market in period t. (Firm entry is discussed in Section 2.2.) On average, each of these firms is worth its market value $\widetilde{v}_t$, equal to the net present value of the expected stream of future profits. The household also purchases the risk-free bonds $B_{N,t+1}$ and $B_{S,t+1}$. The budget allows for quadratic costs of adjustment for bond holdings $\frac{\pi}{2} (B_{N,t+1})^2$ and $\frac{\pi}{2} Q_t (B_{S,t+1})^2$, which are rebated to the household as T_t. Parameter π is set at a small value to ensure stationarity for net foreign assets in the presence of shocks.

The consumption basket includes varieties produced by the Northern firms domestically ($\omega \in \Omega_t^{NN}$), varieties produced by the Northern firms offshore and imported ($\omega \in \Omega_t^{NS}$), as well as varieties produced by the Southern exporters ($\omega \in \Omega_t^{SS}$), with the symmetric elasticity of

to the presence of risk-free, country-specific bonds traded internationally.

[2] Since stocks are not traded across countries, the equilibrium condition is $x_t = x_{t+1} = 1$.

[3] The real exchange rate $Q_t = P_t^* \varepsilon_t / P_t$ is the ratio between the price indexes in the South and the North expressed in the same currency, where ε_t is the nominal exchange rate.

substitution $\theta > 1$:

$$C_t = [\underbrace{\int_{z_{\min}}^{z_{V,t}} y_{D,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{NN}} + \underbrace{\int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{NS}} + \underbrace{\int_{z_{X,t}^*}^{\infty} y_{X,t}^*(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{SS}}]^{\frac{\theta}{\theta-1}}, \tag{2}$$

As explained in Section 2.2 below, $[z_{\min}, \infty)$ is the support interval for the idiosyncratic productivity of Northern firms, and only the more productive firms (with productivity above the endogenous cutoff $z_{V,t}$) choose to produce offshore and import back to the North.[4] Since the number of firms is time-variant and firms re-optimize their offshoring and exporting strategies every period, the composition of the consumption basket changes over time. With the consumption basket C_t set as numeraire, the price index for North is $1 = \left[\int \rho_t(\omega)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}$, in which $\rho_t(\omega)$ is the real price of each variety and $\omega \in \Omega_t^{NN} \cup \Omega_t^{NS} \cup \Omega_t^{SS}$.

The Euler equations for bonds are:

$$1 + \pi B_{N,t+1} = \beta(1 + r_{t+1}) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right], \quad 1 + \pi B_{S,t+1} = \beta(1 + r_{t+1}^*) E_t \left[\frac{Q_{t+1}}{Q_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right], \tag{3}$$

with the market-clearing conditions $B_{N,t+1} + B_{N,t+1}^* = 0$ and $B_{S,t+1} + B_{S,t+1}^* = 0$, in which the asterisk denotes holdings by the Southern household of each type of bond. The Euler equation for stocks is below, with the rate of firm exit δ described in Section 2.2:

$$\widetilde{v}_t = \beta(1 - \delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (\widetilde{d}_{t+1} + \widetilde{v}_{t+1}) \right] \tag{4}$$

2.2 Firm Entry

Firm entry takes place every period in the North and the South, following the mechanism in GM05. In the North, firm entry requires a sunk entry cost equal to f_E units of Northern effective labor, which reflects headquarter activities in the country of origin.[5] After paying the sunk entry cost, each firm is randomly assigned an idiosyncratic labor productivity factor z, which is drawn independently from a common distribution $G(z)$ with support over the interval

[4] In the South, $[z_{\min}^*, \infty)$ is the support interval for the idiosyncratic productivity of Southern firms, and $z_{X,t}^*$ is the endogenous productivity cutoff for Southern exporters.

[5] The sunk entry cost is equivalent to $f_E w_t / Z_t$ units of the Northern consumption basket.

$[z_{min}, \infty)$, and which the firm keeps for the entire duration of its life. Thus, $N_{E,t}$ new firms are created every period t and start producing at $t+1$. However, all existing firms, including the new entrants, are subject to a random exit shock that occurs with probability δ at the end of every period, irrespective of their idiosyncratic productivity. The law of motion for the number of active firms is: $N_{t+1} = (1-\delta)(N_t + N_{E,t})$.

The potential entrants anticipate their expected post-entry value $\widetilde{v}_t$, which depends on the expected stream of future profits $\widetilde{d}_t$, the stochastic discount factor, and the exogenous probability δ of exit every period. The forward iteration of the Euler equation for stocks from (4) generates the following expression for the expected post-entry value of the average firm:

$$\widetilde{v}_t = E_t \left\{ \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t} \right)^{-\gamma} \widetilde{d}_s \right\}. \tag{5}$$

Thus, every period, the unbounded pool of potential entrants face a trade-off between the sunk entry cost and the expected stream of monopolistic profits. In equilibrium, firm entry takes place until the expected value of the average firm is equal to the sunk entry cost: $\widetilde{v}_t = f_E \frac{w_t}{Z_t}$.

2.3 Firms' Choice of Markets and Production Strategies

Every period, the active firms N_t choose endogenously the destination market(s) that they serve and the location of production, as follows: (1) All firms serve their home market. For this purpose, the Northern firms can produce either at home or offshore. Offshoring offers the advantage of a lower cost of production but is subject to fixed and trade costs every period. Importantly, the firms' choice between producing at home or offshore concerns output intended for the home market only, and is not guided by access to the foreign market. (2) A subset of firms from each economy also serve the foreign market. For this purpose, they produce domestically and export subject to a fixed cost as in GM05. Each of these two problems (the offshoring decision of firms serving their home market, and the exporting decision of firms serving the foreign market) are described next.

2.3.1 Domestic vs. Offshore Production for the Northern Market

Every period, the Northern firm with idiosyncratic productivity z chooses between the two possible production strategies to serve its home market: (a) Produce domestically, with output

$y_{D,t}(z) = Z_t z l_t$ as a function of aggregate productivity Z_t, the firm-specific labor productivity z, and domestic labor l_t. (b) Alternatively, produce offshore to obtain $y_{V,t}(z) = Z_t^* z l_t^*$. Thus, the firm producing offshore uses Southern labor l_t^* and becomes subject to the aggregate Southern productivity Z^*, but carries its idiosyncratic labor productivity z abroad.

Under monopolistic competition, the firm with idiosyncratic productivity z solves the profit-maximization problem for the alternative scenarios of domestic and offshore production:

$$\max_{\{\rho_{D,t}(z)\}} d_{D,t}(z) = \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z), \tag{6}$$

$$\max_{\{\rho_{V,t}(z)\}} d_{V,t}(z) = \rho_{V,t}(z) y_{V,t}(z) - \tau \frac{w_t^* Q_t}{Z_t^* z} y_{V,t}(z) - f_V \frac{w_t^* Q_t}{Z_t^*}, \tag{7}$$

where $\rho_{D,t}(z)$ and $\rho_{V,t}(z)$ are the prices associated with each of the two production strategies, w_t and w_t^* are the real wages in the North and the South, and Q_t is the real exchange rate. Thus, the cost of producing one unit of output either domestically or offshore varies not only with the cost of effective labor w_t/Z_t and $w_t^* Q_t/Z_t^*$ across countries, but also with the idiosyncratic labor productivity z across firms. In addition, the Northern firms producing offshore incur a fixed cost equal to f_V units of Southern effective labor, which reflects the building and maintenance of the production facility offshore, and also an iceberg trade cost $\tau > 1$ associated with the shipping of goods produced offshore back to the country of origin.[6]

The demand for the variety of firm z produced either domestically or offshore is $y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t$ or $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t$ respectively, where C_t is the aggregate consumption in the North. Profit maximization implies the equilibrium prices $\rho_{D,t}(z) = \frac{\theta}{\theta-1} \frac{w_t}{Z_t z}$ and $\rho_{V,t}(z) = \frac{\theta}{\theta-1} \tau \frac{w_t^* Q_t}{Z_t^* z}$ for the alternative scenarios of domestic and offshore production. The corresponding profits are $d_{D,t}(z) = \frac{1}{\theta} \rho_{D,t}(z)^{1-\theta} C_t$ and $d_{V,t}(z) = \frac{1}{\theta} \rho_{V,t}(z)^{1-\theta} C_t - f_V \frac{w_t^* Q_t}{Z_t^*}$.

When deciding upon the location of production every period, the firm with productivity z compares the profit $d_{D,t}(z)$ that it would obtain from domestic production with the profit $d_{V,t}(z)$ that it would obtain from producing the same variety offshore. As a particular case, we define the productivity cutoff level $z_{V,t}$ on the support interval $[z_{\min}, \infty)$ such that the firm at

[6] The fixed offshoring cost is equivalent to $f_V w_t^*/Z_t^*$ units of the Southern consumption basket.

the cutoff obtains equal profits from producing domestically or offshore:

$$z_{V,t} = \{ z \mid d_{D,t}(z) = d_{V,t}(z) \} . \tag{8}$$

The model implies that only the relatively more productive Northern firms find it profitable to produce their varieties offshore. Despite the lower cost of effective labor in the South, only firms with idiosyncratic productivity above the cutoff level ($z > z_{V,t}$) obtain benefits from offshoring that are large enough to cover the fixed and iceberg trade costs. This implication is consistent with the empirical evidence in Kurz (2006), who shows that the U.S. plants and firms using imported components in production are larger and more productive than their domestically-oriented counterparts, as the larger idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.[7]

In addition, the productivity cutoff $z_{V,t}$ responds to fluctuations in the relative cost of effective labor across countries, and thus affects the extensive margin of offshoring over the business cycle. For any given level of firm-specific productivity, a relatively lower cost of effective labor abroad implies lower prices, higher revenues, and higher profits from offshoring, and therefore leads to a larger fraction of offshoring firms in equilibrium. This implication is consistent with the empirical evidence on the determinants of offshoring in Hanson, Mataloni, and Slaughter (2005), who show that U.S. multinationals attract larger shares of their foreign affiliates's sales when the latter benefit from lower trade costs and lower wages abroad.

In equilibrium, the existence of productivity cutoff $z_{V,t}$ requires a cross-country asymmetry in the cost of effective labor, which ensures that some of the Northern firms have an incentive to produce offshore. To illustrate this point, Figure 1 shows the per-period profits from domestic and offshore production as functions of the idiosyncratic productivity $z^{\theta-1}$ over the support interval $[z_{min}, \infty)$, expressed as $d_{D,t}(z) = M_t \left(\frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1}$ and $d_{V,t}(z) = M_t \left(\tau \frac{w_t^* Q_t}{Z_t^*}\right)^{1-\theta} z^{\theta-1} - f_V \frac{w_t^* Q_t}{Z_t^*}$, where $M_t \equiv \frac{1}{\theta}\left(\frac{\theta}{1-\theta}\right)^{1-\theta} C_t$ measures demand in the North. The vertical intercept is zero for domestic production; it is equal to the negative of

[7] A useful implication of the Melitz (2003) model is that more productive firms have larger output and revenue. Given two firms with idiodsyncratic productivity $z_2 > z_1$, their output and profit ratios are $\frac{y(z_2)}{y(z_1)} = \left(\frac{z_2}{z_1}\right)^{\theta} > 1$ and $\frac{d(z_2)}{d(z_1)} = \left(\frac{z_2}{z_1}\right)^{\theta-1} > 1$. This is consistent with the evidence that firms using imported inputs in production are more productive, obtain larger revenues, and employ more workers (Kurz, 2006).

the fixed cost $(-f_V \frac{w_t^* Q_t}{Z_t^*})$ for offshoring. In this framework, the productivity cutoff $z_{V,t}$ exists in equilibrium if the profit function from offshoring is steeper than the profit function from domestic production, $slope\{d_{V,t}(z)\} > slope\{d_{D,t}(z)\}$. When this condition is met, offshoring generates larger profits than domestic production for the subset of firms with idiosyncratic productivity z along the upper range of the support interval $(z > z_{V,t})$. The inequality of profit slopes is equivalent to $\tau TOL_t < 1$, with the "terms of labor" $TOL_t = \frac{Q_t w_t^*/Z_t^*}{w_t/Z_t}$ defined as the ratio between the cost of effective labor in the South and the North expressed in the same currency. The condition implies that the effective wage in the South must be sufficiently lower than in the North, so that the difference covers the fixed and iceberg trade cost $(\tau > 1)$, and thus provides an incentive for some of the Northern firms to produce offshore. (Note that an appreciation of the terms of labor for the North is equivalent to a decline in TOL_t.) The model calibration and the magnitude of macroeconomic shocks ensure that this condition is satisfied every period.[8]

2.3.2 Exporting

In addition to serving their domestic market, firms from each economy can choose to serve the foreign market through exports, as in GM05. In the North, the firm with idiosyncratic productivity z would use an amount of domestic labor $l_{X,t}(z)$ to produce for the Southern market, $y_{X,t}(z) = Z_t z l_{X,t}(z)$. The Southern firms that choose to export to the North face a similar problem. Profit maximization implies the following equilibrium price: $\rho_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* \frac{w_t Q_t^{-1}}{Z_t z}$ and profit function: $d_{X,t}(z) = \frac{1}{\theta}\rho_{X,t}(z)^{1-\theta} C_t^* Q_t - f_X \frac{w_t}{Z_t}$ for the Northern exporter with productivity factor z, where C_t^* is aggregate consumption in South. Producing for the foreign market generates additional profits, but involves a fixed exporting cost equal to f_X units of Northern effective labor, and also an iceberg trade cost τ^*. The model implies that only the subset of Northern firms with idiosyncratic labor productivity above the productivity cutoff $z_{X,t}$ find it profitable to produce in the North and export to the Southern market, as they can afford the fixed and iceberg trade costs of exporting. Thus, the time-varying productivity cutoff for exporters is:

$$z_{X,t} = \inf \{z \mid d_{X,t}(z) > 0\}. \tag{9}$$

[8] A second condition necessary to avoid the corner solution when all firms would produce offshore is $d_{D,t}(z_{\min}) > d_{V,t}(z_{\min})$. It ensures that $z_{V,t} > z_{\min}$ in all periods.

To illustrate the exporting productivity cutoff, Figure 2 plots the profit function for exports $d_{X,t}(z) = M_t^* \left(\tau^* \frac{w_t Q_t^{-1}}{Z_t}\right)^{1-\theta} z^{\theta-1} - f_V \frac{w_t^* Q_t}{Z_t^*}$, where $M_t^* \equiv \frac{1}{\theta}\left(\frac{\theta}{1-\theta}\right)^{1-\theta} C_t^* Q_t$ is a measure of demand in the South. The exporting productivity cutoff $z_{X,t}$ is such that only firms above it obtain positive profits from exporting to the South.

2.4 Aggregation over Heterogeneous Firms

Like in Zlate (2016), the model is solved as an equivalent framework with *three representative Northern firms*: one produces domestically, another produces offshore and imports back to the North, while a third firm produces domestically and exports to the Southern market. There are only *two representative Southern firms*: one produces for the local market, and the other exports to the North. Assuming that the firm-specific labor productivity draws z are Pareto-distributed, with p.d.f. $g(z) = k z_{min}^k / z^{k+1}$ and c.d.f. $G(z) = 1 - (z_{min}/z)^k$ over the support interval $[z_{min}, \infty)$, the average productivity levels of the two representative Northern firms that produce domestically and offshore for the Northern market are:

$$\widetilde{z}_{D,t} = \nu z_{\min} z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right]^{\frac{1}{\theta-1}} \quad \text{and} \quad \widetilde{z}_{V,t} = \nu z_{V,t}, \tag{10}$$

where the cutoff is $z_{V,t} = z_{min}(N_t/N_{V,t})^{(1/k)}$, with parameters $\nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $k > \theta - 1$. Like in GM05, the average productivity of exporting firms from the North is:

$$\widetilde{z}_{X,t} = \nu z_{\min} \left(\frac{N_t}{N_{X,t}}\right)^{1/k}. \tag{11}$$

Using the average productivity levels for the domestic, offshoring, and exporting firms, the average prices for each representative firm from the North are $\widetilde{\rho}_{D,t}$, $\widetilde{\rho}_{V,t}$, and $\widetilde{\rho}_{X,t}$; the profits are $\widetilde{d}_{D,t}$, $\widetilde{d}_{V,t}$, and $\widetilde{d}_{X,t}$. There are similar expressions for the Southern firms.

2.5 Aggregate Accounting

Under financial integration, aggregate accounting implies that households spend their income from labor, stock, and bond holdings on consumption and investment in new firms:

$$C_t + N_{E,t}\widetilde{v}_t + B_{N,t+1} + Q_t B_{S,t+1} = w_t L + N_t \widetilde{d}_t + (1 + r_t) B_{N,t} + (1 + r_t^*) Q_t B_{S,t}, \tag{12}$$

$$C_t^* + N_{E,t}^* \widetilde{v}_t^* + Q_t^{-1} B_{N,t+1}^* + B_{S,t+1}^* = w_t^* L^* + N_{D,t}^* \widetilde{d}_t^* + (1 + r_t) Q_t^{-1} B_{N,t}^* + (1 + r_t^*) B_{S,t}^*. \tag{13}$$

The balance of international payments requires that the current account balance (i.e., the trade balance, repatriated profits of offshore affiliates, and income from investments) equals the change in bond holdings:

$$TB_t + \underbrace{N_{V,t}\widetilde{d}_{V,t}}_{\text{Repatriated profits}} + \underbrace{r_t B_{N,t} + r_t^* Q_t B_{S,t}}_{\text{Income from bonds}} = \underbrace{(B_{N,t+1} - B_{N,t}) + Q_t (B_{S,t+1} - B_{S,t})}_{\text{Change in bond holdings}}, \tag{14}$$

where the trade balance is given by:

$$TB_t = \underbrace{N_{X,t} (\widetilde{\rho}_{X,t})^{1-\theta} C_t^* Q_t}_{\text{Exports}} - \underbrace{N_{V,t} (\widetilde{\rho}_{V,t})^{1-\theta} C_t}_{\text{Offshoring imports}} - \underbrace{N_{X,t}^* (\widetilde{\rho}_{X,t}^*)^{1-\theta} C_t}_{\text{Regular imports}} \tag{15}$$

Thus, the baseline model with financial integration for the Northern economy is characterized by 18 equations in 18 endogenous variables: N_t, $N_{D,t}$, $N_{V,t}$, $N_{X,t}$, $N_{E,t}$, $\widetilde{d}_t$, $\widetilde{d}_{D,t}$, $\widetilde{d}_{V,t}$, $\widetilde{d}_{X,t}$, $\widetilde{z}_{D,t}$, $\widetilde{z}_{V,t}$, $\widetilde{z}_{X,t}$, $\widetilde{v}_t$, r_t, w_t, C_t, $B_{N,t+1}$, and $B_{S,t+1}$. Since the Southern firms do not produce in the high-cost North, the Southern economy is described by only 13 equations in 13 endogenous variables; there are no Southern counterparts for N_t, $N_{V,t}$, $\widetilde{d}_{V,t}$, $\widetilde{z}_{D,t}$ and $\widetilde{z}_{V,t}$. Finally, the real exchange rate Q_t and the balance of international payments close the model. In this framework, aggregate output in the North and the South are: $Y_t = C_t + N_{E,t}\widetilde{v}_t + TB_t$ and $Y_t^* = C_t^* + N_{E,t}^* \widetilde{v}_t^* + TB_t^*$, respectively.

2.6 Firms that Never, Sometimes, Always Export or Import

The model with heterogeneous firms allows us to define three types of aggregate firms according to their exporting and importing status. The aggregate firms produce multiple varieties, and their output is defined by integrating over the support interval for idiosyncratic productivity

z. As such, the model with heterogeneous firms becomes a model with heterogeneous varieties, where the idiosyncratic productivity z applies to the variety rather than to the firm itself. As shown in Figure 1, for importers, these firms are: (1) the firm that never imports, (2) the firm that sometimes imports, and (3) the firm that always imports. In Figure 2, the three types of exporters are: (1) the firm that never exports, (2) the firm that sometimes exports, and (3) the firm that always exports. The boundaries of these firms are set according to two intermediary cutoffs, which are chosen at 1 percent below and 1 percent above the steady-state value of the importing and exporting cutoffs $z_{V,t}$ and $z_{X,t}$. These intermediary cutoffs are shown by the dashed red lines below and above the steady-state importing and exporting cutoffs in Figures 1 and 2. For both importers and exporters, the two intermediary cutoffs are chosen in such a way that, in the presence of stochastic shocks to aggregate productivity Z, the importing and exporting cutoffs move without breaching the intermediary cutoffs.

The output of each firm is obtained by integrating over the support interval for idiosyncratic productivity z in between the intermediary cutoffs. For the firms that never export, sometimes export, or always export, output is:

$$Y_{EX\_NEV,t} = \underbrace{N_{EX\_NEV,t}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\widetilde{z}_{EX\_NEV,t}}\right]^{-\theta}C_t}_{\text{Varieties sold at home}},$$

$$Y_{EX\_SOM,t} = \underbrace{N_{EX\_SOM,t}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\widetilde{z}_{EX\_SOM,t}}\right]^{-\theta}C_t+}_{\text{Varieties sold at home}}$$

$$+\underbrace{(N_{X,t}-N_{EX\_ALW,t})\left[\frac{\theta}{\theta-1}\frac{\tau^* w_t Q_t^{-1}}{Z_t\widetilde{z}_{EX\_SOM(z>z_{X,t}),t}}\right]^{-\theta}C_t^* Q_t}_{\text{Varieties exported when } z>z_{X,t}},$$

$$Y_{EX\_ALW,t} = \underbrace{N_{EX\_ALW,t}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\widetilde{z}_{EX\_ALW,t}}\right]^{-\theta}C_t}_{\text{Varieties sold at home}} + \underbrace{N_{EX\_ALW,t}\left[\frac{\theta}{\theta-1}\frac{\tau^* w_t Q_t^{-1}}{Z_t\widetilde{z}_{EX\_ALW,t}}\right]^{-\theta}C_t^* Q_t}_{\text{Varieties always exported}},$$

where $N_{EX\_NEV,t}$, $N_{EX\_SOM,t}$, and $N_{EX\_ALW,t}$ are the number of varieties between the intermediary cutoffs represented by the red dashed lines in Figure 2; $\widetilde{z}_{EX\_NEV,t}$, $\widetilde{z}_{EX\_SOM,t}$, and $\widetilde{z}_{EX\_ALW,t}$ are the corresponding average productivities; and $\widetilde{z}_{EX\_SOM(z>z_{X,t}),t}$ is the average productivity for varieties between the variable exporting cutoff $z_{X,t}$ and the fixed intermediary cutoff just above it, as the cutoff is affected by shocks, only the varieties with productivity

$z > z_{X,t}$ but z below the intermediary cutoff are exported.

For the firms that never import, sometimes import, or always import, output is defined as:

$$Y_{IM\_NEV,t} = \underbrace{N_{IM\_NEV,t} \left[\frac{\theta}{\theta - 1} \frac{w_t}{Z_t \widetilde{z}_{IM\_NEV,t}} \right]^{-\theta} C_t,}_{\text{Varieties produced at home}}$$

$$Y_{IM\_SOM,t} = \underbrace{(N_t - N_{IM\_NEV,t} - N_{V,t}) \left[\frac{\theta}{\theta - 1} \frac{w_t}{Z_t \widetilde{z}_{IM\_SOM(z<z_{V,t}),t}} \right]^{-\theta} C_t}_{\text{Varieties produced at home when } z<z_{V,t}} +$$

$$+ \underbrace{(N_{V,t} - N_{IM\_ALW,t}) \left[\frac{\theta}{\theta - 1} \frac{\tau w_t^* Q_t}{Z_t \widetilde{z}_{IM\_SOM(z>z_{V,t}),t}} \right]^{-\theta} C_t,}_{\text{Varieties imported when } z>z_{V,t}}$$

$$Y_{IM\_ALW,t} = \underbrace{N_{IM\_ALW,t} \left[\frac{\theta}{\theta - 1} \frac{\tau w_t^* Q_t}{Z_t \widetilde{z}_{IM\_ALW,t}} \right]^{-\theta} C_t,}_{\text{Varieties always imported}}$$

where $N_{IM\_NEV,t}$, $N_{IM\_SOM,t}$, and $N_{IM\_ALW,t}$ are the number of varieties between the intermediary cutoffs represented by the red dashed lines in Figure 1; $\widetilde{z}_{IM\_NEV,t}$, $\widetilde{z}_{IM\_SOM,t}$, and $\widetilde{z}_{IM\_ALW,t}$ are the corresponding average productivities; $\widetilde{z}_{IM\_SOM(z<z_{X,t}),t}$ is the average productivity between the importing cutoff $z_{V,t}$ and the intermediary cutoff below it; and $\widetilde{z}_{IM\_SOM(z>z_{X,t}),t}$ is the average productivity between the importing cutoff $z_{V,t}$ and the intermediary cutoff above it.

Finally, we compute the growth of each type of output in the presence of shocks as $grY_t = \ln Y_t - \ln Y_{t-1}$.

3 Calibration

We use a standard quarterly calibration by setting the subjective rate of time discount $\beta = 0.99$ to match an average annualized interest rate of 4 percent. The coefficient of relative risk aversion is $\gamma = 2$. Following GM05, the intra-temporal elasticity of substitution is $\theta = 3.8$ and the probability of firm exit is $\delta = 0.025$. The quadratic adjustment cost parameter for bond holdings is $\pi = 0.0025$. The Pareto distribution parameter k, the iceberg trade cost τ, and the fixed costs of offshoring (f_V) and exporting (f_X and f_X^*) are calibrated so that the model in steady state matches the importance of offshoring for the Mexican economy, as illustrated by three empirical moments: (1) The maquiladora value added represents about 20 percent of Mexico's manufacturing GDP (Bergin, Feenstra, and Hanson, 2009) compared to 15 percent in the model in steady state. (2) The maquiladora sector provided about 55 percent of Mexico's manufacturing exports on average from 2000 to 2006 (INEGI, 2008) compared to about 61 percent in the model. (3) The maquiladora sector accounts for about 25 percent of Mexico's manufacturing employment (Bergin, Feenstra, and Hanson, 2009) and 20 percent in the model. To this end, I set $k = 4.2$, $\tau = 1.2$, $f_V = 0.095$, $f_X = 0.040$, and $f_X^* = 0.025$.[9] Without loss of generality, the lower bound of the support interval for firm-specific productivity in the North and the South is $z_{min} = z_{min}^* = 1$.

To obtain an asymmetric cost of effective labor across countries in steady state, the sunk entry cost, which reflects headquarter costs sensitive to the regulation of starting a business in the firms' country of origin, is set to be larger in the South than in the North ($f_E^* = 4f_E$ and $f_E = 1$). As a result, the steady-state output, the number of firms, the labor demand, and the effective wage are relatively lower in the South. The calibration reflects the considerable variation in the monetary cost of starting a business across economies, which was 2.8 times higher in Mexico than in the United States in purchasing power parity terms in 2010 (World Bank, 2011). The asymmetric sunk entry costs, along with the values for k, τ, f_V, f_X, and f_X^* discussed above, generate a steady-state value for the terms of labor that is less than unit ($TOL = \frac{Qw^*/Z^*}{w/Z} = 0.75$). In other words, the steady-state cost of effective labor in the South is 75 percent of the cost of effective labor in the North. Thus, the calibration provides an incentive for some of the Northern firms to produce offshore in steady state.

4 Results

4.1 Impulse Responses

To illustrate the mechanism of offshoring and its aggregate implications, we log-linearize the model around the steady state and compute impulse responses for key variables to a transitory one-percent increase in aggregate productivity in the North. Aggregate productivity follows the autoregressive process $\log Z_{t+1} = \rho \log Z_t + \xi_t$, with persistence $\rho = 0.9$.

[9] The resulting exports-to-GDP ratios in steady state are 27 perecent for the North and 41 percent for the South. In the South, offshoring exports represent 61 percent of total exports and 25 percent of GDP.

As shown in Figure 3, the positive shock to aggregate productivity in the North leads to an increase in the entry of new varieties and the number of incumbent varieties. The entry of new varieties puts upward pressure on the wage in the North, which causes the terms of labor to appreciate (i.e., the cost of effective labor in the North to increase relative to the South), as shown by the terms of labor falling below the steady state. As a result, the exporting cutoff $z_{X,t}$ increases, which implies that the North exports fewer varieties. At the same time, the importing cutoff $z_{V,t}$ falls, which implies that the North relocates the production of more varieties to the low-cost South and import the resulting output. The number of varieties exported increases by less than the number of total varieties, while the number of varieties increases by more. Since the exporting and importing of varieties around the cutoffs are most affected by the shocks, output fluctuates by more for the firms that "sometimes export" and "sometimes import" than for the firms that never or always export/import, respectively.

4.2 Moments

Table 1 presents the moments from model simulations based on the assumption that aggregate productivities Z_t and Z_t^* follow the bivariate autoregressive process:

$$\begin{bmatrix} \log Z_t \\ \log Z_t^* \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{ZZ^*} \\ \rho_{Z^*Z} & \rho_{Z^*} \end{bmatrix} \begin{bmatrix} \log Z_{t-1} \\ \log Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix}, \tag{16}$$

with the persistence parameters ρ_Z and $\rho_{Z^*} < 1$, spillovers ρ_{ZZ^*} and $\rho_{Z^*Z} \geqslant 0$, and normally-distributed, zero-mean technology shocks ξ_t and ξ_t^*.

4.2.1 TFP Calibration

The bivariate productivity process is calibrated like in Backus, Kehoe, and Kydland (1992, henceforth BKK92), with the symmetric persistence and spillovers are $\rho_Z = \rho_{Z^*} = 0.906$ and $\rho_{ZZ^*} = \rho_{Z^*Z} = 0.088$, the variance of innovations is $\text{var}(\xi_t) = var(\xi_t^*) = 0.00852^2$, and the correlation of innovations is $\text{corr}(\xi_t, \xi_t^*) = 0.258$. In an alternative case, the correlation of innovations is set at negative the value from BKK92, namely $\text{corr}(\xi_t, \xi_t^*) = -0.258$.

4.2.2 Firm-Level Volatility

In Table 1, we report the standard deviations of output growth for the three types of firms from the North defined according to their trade status. First, the firm that always exports is less volatile than the firm that never exports, while the firm that always imports is more volatile than the non-importer. Like in the data, diversification reduces the volatility of exporters but enhances the volatility of importers.

Second, the firm that sometimes exports (or imports) is more volatile than the firm that always exports (or imports). Thus, the adjustment along the extensive margin of varieties exported or imported enhances the volatility of less persistent exporters and importers (which are closer to the cutoffs) than that of firms that consistently export and import.

Third, the firm that always exports is more volatile when it exports to a foreign destination that is positively correlated with the home economy (column 1) than when it exports to a foreign destination that is negatively correlated (column 2). As such, diversification across uncorrelated export destinations reduces the exporters' volatility, such as when positive shocks abroad may offset negative shocks at home.

Fourth, the firm that always imports is less volatile when it imports from foreign economies that are positively correlated with the home economy (column 1) than when it imports from foreign economies that are negatively correlated (column 2). The result suggests that diversification enhances importers' volatility. In our model, the result hinges on the CES structure of consumption in which varieties produced at home and abroad are substitutes. In next steps, the implications of importing for firm-level volatility in a setup that allows for complementarity between domestic and imported inputs should be a helpful extension.

5 Conclusion

This paper is predicated upon a set of stylized facts presented in Kurz and Senses (2016) which point to considerable heterogeneity in the volatility firms that differ in terms of the level of engagement in international trade, the type of products they trade, and the characteristics of their trading partners. In particular, Kurz and Senses (2016) find that importers experience higher levels of volatility compared to non-trading firms. This relationship is mainly driven by firms that switch in and out of importing. Firms that only export experience lower levels of volatility. These results are complemented with findings that indicate firm-level volatility increases with a larger share of exports and imports and is depended on the number of products traded and the number of countries traded with.

In contrast to the large body of theoretical and empirical literature on international trade's implications for macroeconomic dynamics, there is little theoretical work regarding the transmission of international shocks to firm-level volatility. Our paper attempts to fill this gap by exploring the relationship between firms' exporting and importing status and firm-level characteristics—namely the volatility of output and employment—in a stochastic, dynamic, general equilibrium model of international macroeconomics and trade. We augment the heterogeneous firms and endogenous exporting framework of Ghironi and Melitz (2005) to allow for international input sourcing and multiple trading partners. More specifically, we examine the firm-level volatility generated by the model for a cross-section of firm types, which are defined to reflect the rich heterogeneity in firms' international activities.

In line with the empirical evidence, the model predictions are: (1) Exporters display lower volatility than non-exporters, whereas importers display higher volatility that non-importers. (2) Firms that trade for longer durations display lower volatility than firms switching in and out of international trade. (3) Firms that export to a larger number of uncorrelated foreign markets are less volatile, whereas firms importing from a more diversified range of uncorrelated foreign suppliers are more volatile. The model rationalizes these findings by highlighting the asymmetry in the way that diversification across uncorrelated trading partners affects exporters and importers: While diversification reduces the volatility of exporters, as positive and negative shocks in destination markets offset each other, it enhances the volatility of importers, since disruptions from one single supplier affects the entire production process reliant on complementary inputs.

6 List of References

References

[1] Alessandria, George and Horag Choi. 2007. "Do sunk costs of exporting matter for net export dynamics?" *Quarterly Journal of Economics*, 122(1): 289-336.

[2] Backus, David K.; Patrick J. Kehoe; and Finn E. Kydland, 1992. "International real business cycles." *Journal of Political Economy*, 100(4): 745-775.

[3] Contessi, Silvio. 2015. "Multinational firms' entry and productivity: Some aggregate implications of firm-level heterogeneity." *Journal of Economic Dynamics and Control*, 61(2015): 61-80.

[4] Fattal Jaef, Roberto N. and Jose Ignacio Lopez. 2014. "Entry, trade costs and international business cycles." *Journal of International Economics*, 94: 224-238.

[5] Fillat, Jose L. and Stefania Garetto. 2015. "Risk, returns, and multinational production." *Quarterly Journal of Economics*, 130(4): 2027-2073.

[6] Fillat, Jose L.; Stefania Garetto; and Lindsay Oldenski. 2015. "Diversification, cost structure, and the risk premium of multinational corporations." *Journal of International Economics*, 96: 37-54.

[7] Ghironi, Fabio and Marc J. Melitz. 2005. "International trade and macroeconomic dynamics with heterogeneous firms." *Quarterly Journal of Economics*, 120(3): 865-915.

[8] Hanson, Gordon H.; Raymond J. Mataloni; and Matthew J. Slaughter. 2005. "Vertical production networks in multinational firms." *Review of Economics and Statistics*, 87(4): 664-678.

[9] Kurz, Christopher. 2006. "Outstanding outsourcers: A firm- and plant-level analysis of production sharing." FEDs Working Paper No. 2006-04.

[10] Kurz, Christopher and Mine Z. Senses. 2016. "Importing, exporting, and firm-level employment volatility." *Journal of International Economics*, 98: 160-175.

[11] Liao, Wi and Ana Maria Santacreu. 2015. "The trade comovement puzzle and the margins of international trade." *Journal of International Economics*, 96: 266-288.

[12] Mandelman, Federico. 2016. "Labor market polarization and international macroeconomic dynamics." *Journal of Monetary Economics*, forthcoming.

[13] Mandelman, Federico and Andrei Zlate. 2016. "Offshoring, low-skilled immigration, and labor market polarization." Federal Reserve Banks of Atlanta and Boston, mimeo.

[14] Melitz, Marc. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica*, 71(6): 1695-1725.

[15] Zlate, Andrei. 2016. "Offshore production and business cycle dynamics with heterogeneous firms." *Journal of International Economics*, forthcoming 2016.

Table 1: Moments

	TFP calibration as in BKK92	
	$corr(\varepsilon, \varepsilon^*) > 0$	$corr(\varepsilon, \varepsilon^*) < 0$
St. dev. (%), aggregate variables, levels	**(1)**	**(2)**
Y	0.97	0.92
C (relative to Y)	0.67	0.55
N_E (relative to Y)	3.52	4.37
St. dev. (%), exporting firms' output, growth		
grY_{NEV} (never export)	0.44	0.34
grY_{SOM} (sometimes export)	5.30	4.17
grY_{ALW} (always export)	0.42	0.33
St. dev. (%), importing firms' output, growth		
grY_{NEV} (never import)	0.44	0.34
grY_{SOM} (sometimes import)	11.29	9.00
grY_{ALW} (always import)	4.18	5.42
Correlations		
Y, Y^*	0.42	-0.08
Z, Z^*	0.31	-0.20
$Y, TB/Y$	-0.18	-0.14

Figure 1: Importing firms.

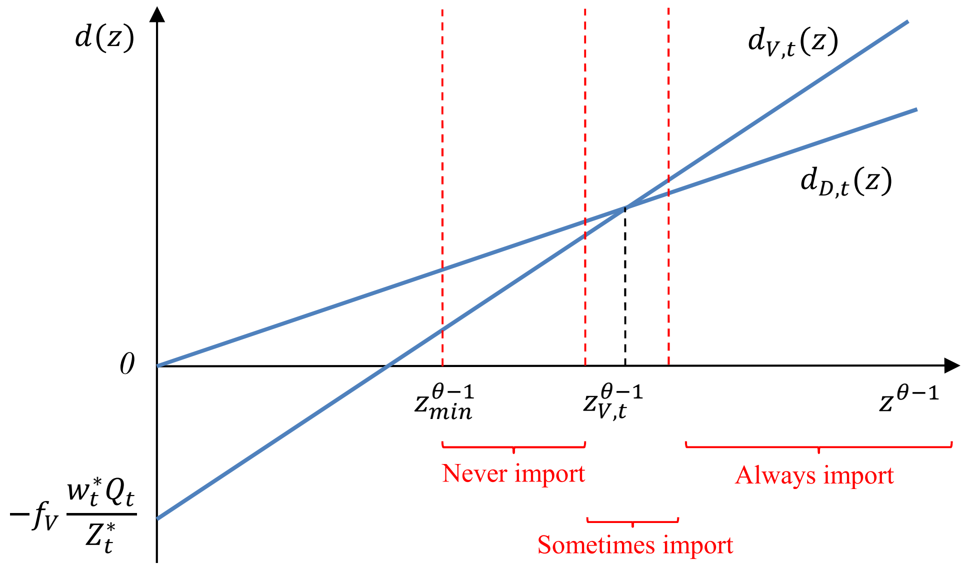

Figure 2: Exporting firms.

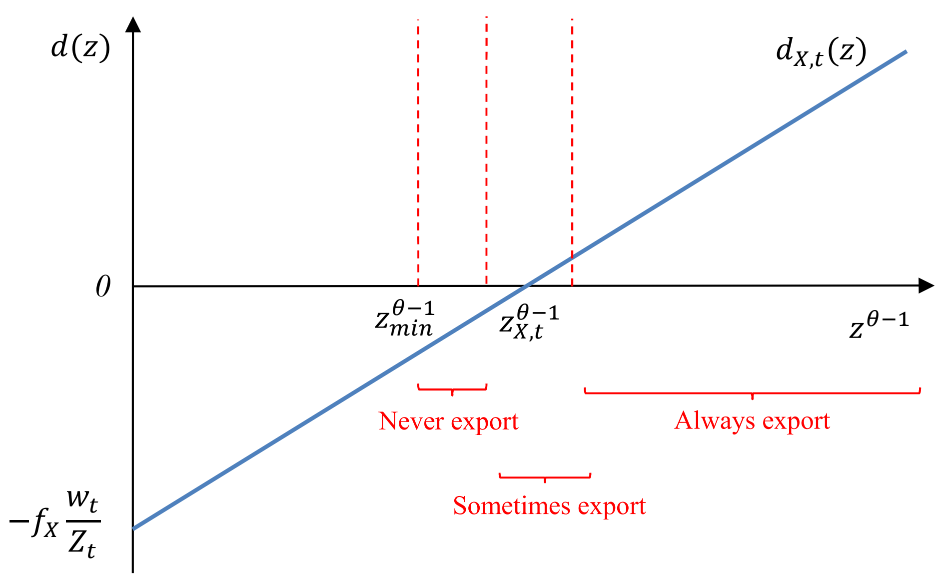

Figure 3: Impulse responses to a temporary shock to aggregate productivity.