Firing Costs and Labor Market Fluctuations: A Cross-Country Analysis*

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Abstract

Using a novel dataset, we document large differences across OECD countries in business cycle fluctuations of labor supply margins. Countries with larger fluctuations in employment relative to hours per worker tend to display larger fluctuations in total hours worked. We then present a quantitative framework that features both margins of labor supply as well as costs to the adjustment of employment. Cross-country differences in dismissal costs can account for a large fraction of the patterns observed in the data. Moreover, the range of dismissal costs supported by our experiments is almost one order of magnitude smaller than estimates found in the literature, where the intensive margin of labor supply is typically ignored.

JEL classification: E32, F44, J22

Keywords: Firing Costs, Labor Market Fluctuations, Hours Worked

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1 Introduction

Motivated by the observation that aggregate labor market outcomes differ significantly across OECD countries, a large literature has emerged that seeks to assess the effects of policies on such outcomes. Much of this literature, however, has focused on either steady-state differences or divergences in secular trends.[1] This paper continues the effort to understand the connection between labor market policies and labor market outcomes, but focuses on cross-country differences in fluctuations of labor supply margins over the business cycle.

We first document three novel facts exploiting a recently-constructed quarterly dataset on employment, hours per worker, and total hours for several OECD countries (see Ohanian and Raffo (2011) for details). First, there are substantial differences in the relative volatility of total hours worked over the business cycle across countries. Second, countries that have lower fluctuations in employment relative to output tend to have higher fluctuations in hours per worker relative to output. Third, countries with relatively lower fluctuations in employment relative to hours per worker also exhibit lower fluctuations in total hours worked.

One plausible candidate to account for these patterns is policies that affect the size of worker's dismissal costs. Indeed, a large literature has documented significant differences in these costs across countries. Bentolila and Bertola (1990) estimate that firing costs in France, Germany, and Italy amounted, on average, to almost a full year of wages but only one quarter of wages in the United Kingdom. In their view, these cross-country differences in firing costs explain much of the differrent behaviour of unemployment after the oil shocks of the 1970s. Garibaldi and Violante (2005) estimate that firing policies in Italy impose a dismissal cost of about 18 months of wages. Similarly, a 2012 report by Deloitte finds that firing costs in Europe typically amount to about 6-9 months of wages. Intuitively, holding

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[1] See Prescott (2004), Rogerson (2006), Ohanian *et al.* (2008) among others.

all else constant, higher firing costs would be expected to lead to relatively more fluctuations along the intensive margin and less along the extensive margin. In this paper we assess the extent to which differences in firing costs can quantitatively account for the nature of differences in labor market fluctuations observed across countries.

We carry out a quantitative assessment of the effects of firing costs on labor market outcomes using a standard real business cycle model extended to allow for labor supply fluctuations along both the intensive and extensive margin. Previous quantitative analyses of firing costs have typically assumed that labor is indivisible, implicitly specifying that all adjustment takes place only on the employment margin. We calibrate the model so that it is consistent with empirical regularities for the U.S. economy as well as the nature of fluctuations along the extensive and intensive margins. We then vary the magnitude of dismissal costs and examine how they affect labor market fluctuations. Specifically, we ask whether differences in these costs can quantitatively generate the differences in labor market fluctuations that we document in the cross-country data. While it is intuitive that dismissal costs could account for some of the differences, there is no presumption that dismissal costs are the sole factor responsible for these differences, or even that it is the most important.

Our first quantitative result is that abstracting from the presence of an intensive margin of adjustment, as typically assumed in the literature, severely underestimates the effects of firing policies on employment fluctuations. In this experiment, where we assume that there are no costs associated with employment adjustment in the United States, we show that firing costs as small as one month of wages virtually shut down fluctuations in employment, as firms adjust largely along the intensive margin.

Our second quantitative result is that differences in dismissal costs can account for most of the empirical differences in labor input fluctuations across countries. Following the literature that has estimated small (hiring) costs in the U. S. labor market, we incorporate a modest friction in the U.S. technology to adjust employment. Using this benchmark we then revisit the issue of how cross-country differences in firing costs affect business cycle fluctuations. We show that, after including such a small employment adjustment cost, our calibrated model quantitatively captures the salient features of the cross-country data. In addition, we find that the range of firing costs required to reproduce the empirical cross-country variation in labor supply fluctuations is almost one order of magnitude smaller than estimates discussed in the literature. This result highlights the importance of explicitly considering the intensive margin of adjustment in the analysis of the effects of firing costs, a dimension typically ignored in the literature. In the presence of higher firing costs, firms opt to adjust labor input largely along the intensive margin.

2 Cross-Country Patterns in Labor Market Fluctuations

In this section we document three novel facts about labor market fluctuations across countries. In a recent contribution, Ohanian and Raffo (2011) construct a dataset for total hours worked at the quarterly frequency that covers 14 OECD countries and spans, in most cases, the period 1960 to 2013.[2] This dataset is particularly useful because, to date, analysis of fluctuations across countries omitted the intensive margin and focused on employment as a summary statistics of labor market outcomes over the business cycle. This limitation has affected not only the conventional view about cyclical changes in labor, but also the study of the implications associated with different policies. For instance, the seminal work of Hopenhayn and Rogerson (1993) about the effects of hiring and firing costs suggested that European labor markets represent a laboratory for evaluating and quantifying the impact of these policies,

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[2] This paper makes use of the updated version of the dataset, which is available on the authors' website.

but lack of data on the intensive margins severely limited the potential for these analysis.

We briefly summarize here the main elements of the Ohanian and Raffo (2011) dataset and invite the reader to consult the original article for a more detailed description of their methodology. Data on employment, working-age population, and output are obtained from the OECD. These authors then construct series on quarterly hours worked per worker implementing the following three steps. First, they collect *annual* data on hours worked per worker that are comparable across countries and are consistent with national accounts from the Conference Board Total Economy Database (TED).[3]. This dataset has been the benchmark in the literature for analysis on trends in total hours worked (see Rogerson (2006), Ohanian *et al.* (2008), Rogerson and Shimer (2010)). Ohanian and Raffo then build a dataset of indicators for hours per worker for all the 14 countries considered drawing on a variety of international sources, including data from national statistical offices, establishment surveys, and household surveys. Finally, they adjust these quarterly indicators of hours worked per worker so that they conform with the annual TED series following the methodology proposed in Denton (1971).

Table 1 presents summary statistics for the standard deviations of total hours, employment, and hours per worker relative to the standard deviation of output, as well as the correlation for each of the series with output. As is standard in the business cycle literature, statistics refer to the cyclical components of each series obtained applying a Hodrick-Prescott filter with smoothing parameter equal to 1600 to the (log of the) series.

[Insert Table 1 Here]

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[3] Data contained in the TED are adjusted to reflect most sources of cross-country variation in hours worked, such as statuatory holidays and sick days.

We start by noting three basic facts that are apparent from this table. First, there is significant variation in the business cycle volatility of all margins of labor supply across countries. Total hours are about as volatile as output in Canada, Australia, and the United States, but much less volatile in Germany, Japan, and Austria, with the median value of this ratio just below 0.90. Second, there are substantial fluctuations along the intensive margin in all countries. In Australia, Canada, and the United States, for example, the intensive margin is almost half as volatile as the extensive margin. In France, Korea, and Japan, the intensive margin is even more volatile than the extensive margin. These patterns suggest that abstracting from fluctuations along the intensive margin, as typically assumed in the literature, necessarily eliminates a salient feature of labor market adjustment across countries. For completeness, we also report cyclical correlations for these variables and note that total hours and its margins are highly procyclical in all countries, consistent with the typical business cycle facts reported in the literature.

We next turn to the patterns in the data that serve to motivate our analysis. Using the values in Table 1, we first construct the ratio of the volatility of employment to the volatility of hours per worker. As mentioned before, this ratio turns out to vary quite dramatically across countries, from a low of 0.37 in the United States to as high as 1.63 in Japan. Figure 1 plots the ratio of the standard deviations of total hours and output against the ratio of the standard deviations of hours per worker and total employment. The variation in the relative volatility along the two labor supply margins is negatively correlated to the magnitude of total hours fluctuations. This suggests that countries in which there is more adjustment along the extensive margin relative to the intensive margin tend to also display a higher volatility of total hours relative to output.

[Insert Figure 1 Here]

In a standard real business cycle model with technology shocks as the sole driving force of business cycles, the volatility of total hours relative to output is largely determined by the elasticity of labor supply. Similarly, in a model that features variation along both the intensive and extensive margins, the volatility of employment and hours per worker reflects labor supply elasticities along the two margins. In view of this, one interpretation of the evidence just presented is that labor supply elasticities along the two margins differ across countries. To the extent that one is reluctant to ascribe differences in business cycle fluctuations across countries to differences in preference parameters, however, alternative explanations ought to be considered.

Labor market policies that influence how different economies react to the same shock represent a natural candidate to account for the observed patterns in labor market fluctuations. A long literature has pursued this idea to understand the very different secular evolutions of labor market outcomes across OECD economies (See, for example, Bruno and Sachs (1985), Krugman (1994), Blanchard and Wolfers (2000), Ljungqvist and Sargent (1998) and Hornstein et al. (2007)). In this paper, we pursue this approach in the context of business cycle fluctuations and consider employment protection policies that increase the costs associated with worker dismissals. As shown in Figure 2 the volatility of labor supply along the extensive margin relative to the volatility of labor supply along the intensive margin appears positively correlated with an index of employment protection legislation (EPL) obtained from the OECD. That is, economies with a higher EPL relay more heavily on the intensive margin rather than the extensive margin for labor input adjustment over the business cycle. This relationship appears quite intuitive: As the cost of changing labor input along the extensive margin increases countries tend to adjust more along the intensive margin relative to the extensive margin. The remainder of this paper presents a quantitative assessment of this idea in the context of a relatively standard real business cycle model.

[Insert Figure 2 Here]

3 Model

We consider a standard neoclassical growth model extended to allow for labor supply decisions along both the intensive and the extensive margin. There is a representative household that consists of a large number of members. The household has preferences defined by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, e_t, h_t)$$

where c_t is average consumption per household member at time t, e_t is the fraction of individuals that are employed in period t, and h_t is the amount of work per employed member in period t. Following Cho and Cooley (1994), we assume that the period utility function $U(c_t, e_t, h_t)$ takes the form:

$$U(c_t, e_t, h_t) = \log c_t - \frac{A}{1+\gamma} h_t^{\gamma+1} e_t - \frac{B}{1+\eta} e_t^{\eta+1}$$

where $\gamma > 0$, $\eta > 0$ and A and B are positive constants. In can be easily shown that this utility function can generate interior solutions for labor supply along both the intensive and extensive margins in steady state, implying that labor supply adjusts along both margins in response to shocks. Cho and Cooley (1994) motivated this form of household utility function by positing a distribution of fixed costs across household members associated with employment along the extensive margin.[4]

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[4] Other papers in the literature (see, e.g., Kydland and Prescott (1991), French (2005), Rogerson and Wallenius (2009) and Erosa *et al* (2012)) start with preferences in which individuals care only about total hours worked, and generate allocations in which labor supply adjusts along the intensive and extensive margins by positing non-convexities in the market technology and heterogeneity among household members. We view these two approaches as complementary, and have chosen the Cho and Cooley (1994) specification because it offers a more tractable and transparent model for our purposes.

As is standard in the real business cycle literature, there is an aggregate production function that uses capital and labor to produce output and is subject to persistent productivity shocks. Although we distinguish between employment and hours in the household's utility function, we assume that employment and hours per worker are perfect substitutes in terms of producing output. However, motivated by the desire to consider the effects of employment protection policies, we allow for adjustment costs associated with changes in employment between consecutive periods.(HERE WE SHOULD ADDRESS THE ISSUE THAT WE TAX GROSS FLOWS AND PROVIDE A DEFENSE FOR THIS APPROACH). Specifically, we assume that:

$$y_t = z_t k_{t-1}^{\theta} N_t^{(1-\theta)} - \digamma \max\{e_{t-1} - e_t, 0\}$$

where z_t is the technology shock in period t, k_t is input of capital in period t, and $N_t = e_t h_t$ is total hours in period t, and $\digamma \geq 0$ is the per-worker cost associated with reductions in employment, measured in units of output. The techonology shock follows an AR(1) process in logs:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}$$

where ε_t is an iid normally distributed innovation with mean μ_ε and standard deviation σ_ε. In our sensitivity analysis, we also consider other disturbances, including government spending shocks and investment-specific technology shocks.

Finally, the resource constraint posits that output can be used for either consumption or investment, with capital depreciating at rate δ

$$c_t + k_t - (1 - \delta) k_{t-1} = y_t$$

Note that the costs associated with employment reductions result in output losses. It follows that, in our specification, the adjustment cost function captures both technological costs associated with workforce management as well as legislated employment protection costs. This representation is consistent with the idea that employment protection legislation imposes administrative burdens on employers. Much of the literature, however, has studied firing taxes as a way to focus on the distortionary effects of legislated severance payments. Our formulation is also consistent with this alternative interpretation if one assumes that the government expenditures financed by these revenues enter the representative agent's utility function in a separable way. Since the main goal of the analysis is to focus on the distortion between intensive and extensive margin of labor supply over the business cycle, this issue is likely to be less relevant in the current context.

4 Calibration

Our general strategy is to calibrate the model to match some key features of the U.S. economy, and then study how business cycle fluctuations are affected when we vary the level of costs associated with employment reductions, as captured by the parameter F. As shown in Table 2, we consider two alternative calibrations for the United States. In the first calibration, we set the value of F in the United States equal to zero. This specification is consistent with the view that F is purely associated with employment protection legislation and that these provisions essentially do not exist in the United States. This view figures prominently in the literature that uses explicit models to assess the effects of employment protection on aggregate labor market outcomes (see, for example, Hopenhayn and Rogerson (1993), Alvarez and Veracierto (1999), and Veracierto (2001, 2008)). In the second calibration, we assume that the value of F is also positive, albeit small, in the United States. This calibration is consistent with the view that that even if the U.S. economy does not have legislation that imposes adjustment costs, there may still be costs associated with employment changes that reflect technological constraints. A large literature documents the presence of hiring and firing costs for individual establishments (ADD REFERENCE). In a business cycle context with a representative firm, hiring and firing costs are broadly similar in their effect, and so rather than separately modeling the two components we bundle these costs into the firing cost parameter F. In this second specification, we set this parameter to 4.5% of quarterly wages, consistent with the estimates of hiring costs found in Hagedorn and Manovskii (2006).

We follow a similar appraoch to calibrate the preference parameters associated with the labor choice. The curvature parameters γ and η, in particular, critically determine both the overall magnitude of fluctuations in total hours of work as well as how these fluctuations are divided along the intensive and extensive margin. Understanding the underlying foundations for these labor supply parameters is an important issue, but is not the focus of this paper. Rather, our goal is to analyze the extent to which a specific labor market policy can account for cross-country differences in the nature of labor market fluctuations. For this purpose, we first start with a model that captures the nature of fluctuations in the U.S. economy and then examine how the nature of fluctuations is affected by changes along employment protection policies. We therefore follow Cho and Cooley (1994) and calibrate the two labor supply elasticity parameters by setting η equal to 0.60 and choosing γ to match the volatility of employment relative to hours per worker. One of the advantages of our specification of the utility function is that it is sufficiently flexible to allow us to match properties of labor market fluctuations.

The resulting parameter values for our two different calibrations are summarized in top section of Table 2. When the U.S. labor market is assumed to be free of any labor market friction, our calibration strategy yields $\gamma = 0.71$, that is, a labor supply elasticity along the intensive margin that is lower than the labor supply elasticity along the intensive margin, consistent with the empirical evidence. In our second experiment - where the U.S. labor market faces a small technological cost of adjusting employment - the curvature on hours per worker is even higher ($\gamma = 1.71$), thus further reducing the implied elasticity along the intensive margins. Finally, once γ and η are calibrated, U.S. data on long-run averages for the levels of employment and hours per worker pin down the utility parameters A and B.

In each of the two different calibrations, the remaining parameters are chosen according to the following procedure. As is standard, we calibrate the values of β, θ, and δ so that the model's steady state matches some first moments from the U.S. data. Specifically, data on the capital's share of income and the investment to output ratio imply a value of 0.36 for θ and 0.025 for the depreciation rate δ. The discount factor β is set equal to 0.99, consistent with a steady state annual interest rate of about 4 percent.

Our quantitative analysis investigates how variation in the level of employment reduction costs influence the optimal response of labor supply along the intensive and extensive margins in reaction to aggregate shocks. As noted in the previous section, we begin by restricting our attention to aggregate technology shock. This choice is motivated by the fact that it is well known that technology shocks are able to generate business cycle fluctuations that capture many of the properties of business cycle fluctuations in the data. Consistent with most of the literature, we consider a process for technology shocks that is quite persistent, with the value of ρ set to 0.98. One of the key issues in the business cycle literature is the magnitude of fluctuations, which in the context of technology shock driven business cycles is intimately related to the standard deviation of the innovations (σ_{ε}). As the business cycle statistics of interest in our analysis of cross-country patterns are all measured relative to output fluctuations, the actual size of fluctuations in output turns out not to be of key importance in our study. Consequently, we assume that both the persistence and magnitude of shocks are the same across economies.

[Insert Table 2 here]

We conclude this section with a brief discussion about our solution algorithm. In our benchmark model, firing costs introduce an occasionally-binding costraint which prevents us from using standard log-linearization techniques. We find the solution for our model using Galerkin method, as discussed in Judd (1998).[5] As other projection methods, Galerkin uses polynominals to approximate the decision rules of the original problem. Here we apply Chebyshev polynomials basis, with their parameters identified by minimizing the approximation errors. Appendix A presents more details about our procedure as well as accuracy test statistics.

5 Results

5.1 First Experiment: $F^{US} = 0$

We begin by presenting results for the calibration in which the U.S. economy is assumed to have no adjustment costs. Table 3 presents the properties of business cycle fluctuations for both the $F = 0$ economy as well as several other values of F. We measure F in units of (quarterly) wages, so that $F = 1.00$ *represents a firing tax equal to one percent of quarterly steady-state wages.*

[Insert Table 3]

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[5] See also Fabrice Collard's notes for further information: http://fabcol.free.fr/index.php?page=notes

Table 3 confirms the intuition anticipated earlier: Higher dismissal costs lead to less adjustment along the employment margin and more adjustment along the intensive margin. However, two additional findings are worth noting. First, most of the effect of dismissal costs is realized for relatively small values of F. In other words, almost all of the variation in Table 3 occurs as we move from the frictionless economy to firing costs of only 10 percent of quarterly wages, with the volatility of employment falling by almost three quarters (from 0.75 to 0.18) and the volatility of hours per worker doubling (from 0.26 to 0.52). Although these patterns continue as dismissal costs are increased further, the changes to higher dismissal costs are small. Second, as we move from $F = 0$ to $F = 0.10$, changes in the intensive margin do not offset the decline in volatility of employment, resulting in lower overall total hours fluctuations. Intuitively, since the policy only affects adjustment along the extensive margin, as the adjustment on the extensive margin becomes sufficiently small, further increases in the dismissal cost do not have significant effects on the economy. While not our focus in this paper, we also note that dismissal costs have relatively small impact on the volatility of consumption and investment.

[Insert Figure 3 here]

Next we ask whether the magnitudes of the effects documented in Table 3 regarding volatility of total hours and its composition in terms of intensive and extensive margins are in line with the magnitudes of differences found in the cross-country data presented earlier. We focus on the relationship between the volatility of total hours relative to output and the volatility of total hours relative to the volatility of employment. Figure 3 plots the relationship between these values in the data as well as the relationship implied by the model as we increase dismissal costs starting from $F = 0$. In the figure, we have normalized both ratios to one for the U.S. economy, so that we are focusing on the values of these ratios relative to the U.S. values. In addition, we have added the fitted curve based on a logarithmic regression using the actual values (black dotted line).

Under our first calibration, the model cannot fully account for the variation observed in the data. The regression line obtained from the data is much more steeply sloped than the line generated by the model where the only variation across countries is the size of the dismissal cost. That said, both lines are negatively sloped. We also note that the range of dismissal costs that are represented in the figure is very small, with a maximum firing cost of 4 percent of quarterly wages.

Nonetheless, we interpret these findings as a key contribution and, at the same time, a fundamental critique of the previous literature on evaluating the consequences of dismissal costs. The current norm in the literature is to assume that the U.S. economy has zero adjustment costs on employment and that the extensive margin is the only margin through which labor input can be adjusted.[6] In these studies, increasing dismissal costs has a significant but much more modest effect on employment fluctuations. In these models, the extensive margin is the only margin of adjustment and it is used even when it becomes costlier to do so. However, if firms can respond to shocks along a second margin of labor adjustment that is not subject to the dismissal cost, the response in employment fluctuations is dramatically increased. A key message from the above analysis is that abstracting from the intensive margin when assessing the impact of dismissal costs on labor market fluctuations has very significant implications for the quantitative analysis.

[Insert Table 4]

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[6] Both of these features are present, for example, in the recent analysis of Veracierto (2008).

Table 4 provides additional quantitative evidence on this intuition. This table compares the moments of the main variables considered in Table 3 with those generated by a model that abstracts from the intensive margin of labor supply.[7] As shown by the statistics on employment, in the model without intensive margin the increase in dismissal costs reduces the volatility of employment remarkably less than in our benchmark model with the intensive margin. Not including the intensive margin also affects the assessment on the impact of dismissal costs on other properties of the business cycle. For instance, the fall in output volatility induced by an increase of dismissal costs from zero to 10 percent without the intensive margin is XXX times the fall with the intensive margin.

5.2 Second Experiment: $F^{US} = 0.045$

We now turn to the second calibration, in which we assume that even in the U.S. labor market there is a cost associated with employment adjustment along the extensive margin ($F^{US} = 0.045$). We interpret this friction to be indicative of technological costs associated with adjustment, rather than costs that are induced by policy.[8] We now repeat the earlier analysis and report results in Table 5.

[Insert Table 5]

As in Table 3, dismissal costs lead to substitution towards the intensive margin of labor supply. In this experiment, however, the elasticity of the response in employment volatility to increases in dismissal costs is much smaller than in our first calibration. The intuition for

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[7] To ease the comparison, the model without the intensive margin has the same steady state as the benchmark model. The only difference is that in the former, $h_t = \bar{h} \; \forall \; t$.

[8] Recall that we calibrate preference parameters in this case so that the model still generates an empirically reasonable amount of fluctuations along the two labor supply margins. Given the previous results, it is clear that if we kept the previous parameterization and increased F to the value that we use in this specification for the United States, employment fluctuations would fall dramatically.

this feature of model is that when the level of adjustment costs is high then a small increase due to policy represents a much smaller percent increase in costs and is likely to have smaller consequences.

We again ask whether variation in dismissal costs can capture the key variation in labor market fluctuations found across countries. Figure 4 repeats the same exercise that we previously presented in Figure 3.

[Insert Figure 4 here]

To ease comparison with the earlier results, Figure 4 also includes the curve implied by the model under the first calibration ($F^{US} = 0$). Interestingly, we see that the line generated by the second calibration has a steeper slope than the one in the first calibration and gets much closer to the empirical regression line. We conclude that if we assume a small but positive cost of employment adjustment in the U.S. labor market, then the model can capture the patterns in the data, both qualitatively and quantitatively.

It is also of interest to examine how this alternative calibration affects the responsiveness of hours to changes in the dismissal cost. In this regard, we emphasize that the new line in Figure 4 is generated by varying dismissal costs between 0.045 to XX of quarterly wages. These estimates are almost one order of magnitude lower than typically reported in the literature.[9]

5.3 Sensitivity Analysis

5.3.1 Sources of Business Cycle Fluctuations

In our analysis, we have emphasized that studies of labor market policies that abstract from the intensive margin may severely underestimate the quantitative response of employment to

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[9] See, for example, Bentolila and Bertola (1990) and Garibaldi and Violante (2005).

these policies. So far, however, we have focused on the adjustment of labor supply margins in a standard model where productivity shocks generate business cycle fluctuations. In this section, we investigate the extent to which this substitution between intensive and extensive margin is also operative in response to other aggregate shocks. Specifically, we focus on government expenditure shocks and investment-specific technology (IST) shocks as they have received large attention in the literature.

We assume that the government faces a simple budget constraint stating that lump-sum taxes are collected to finance exogenous spending shocks

$$\tau_t = g_t$$

where g_t follows an AR(1) process (in logs) of the form:

$$\log g_{t+1} = \rho^g \log g_t + \varepsilon^g_{t+1}$$

In this expression, ε^g_t is an iid normally distributed innovation with zero mean and standard deviation σ^g_ε.

Under this assumption, the resource constraint becomes:

$$c_t + k_t - (1-\delta)k_{t-1} + g_t = y_t$$

We calibrate the government expenditure shock as in Backus, Kehoe and Kydland (1993). We set the value of steady-state government spending to 20 percent of output, the autoregressive parameter ρ^g to 0.95, and σ^g_ε to 2 percent of steady-state government expenditure.

In the second case, we model IST shocks as in Greenwood, Hercowitz and Huffman (1988). We then assume that the stock of capital evolves according to the law of motion:

$$k_t = (1-\delta)\, k_{t-1} + \mu_t i_t$$

where μ_t is the IST shock and follows an AR(1) process (in logs):

$$\log \mu_t = \rho^{\mu} \log \mu_{t-1} + \varepsilon_t^{\mu}$$

In this expression, ε_t^{μ} is an iid normally distributed innovation with zero mean and standard deviation $\sigma_{\varepsilon}^{\mu}$. We set the value of ρ^{μ} equal to 0.7 and σ_{ε}^{g} equal to 0.02, following Justiano, Primiceri, and Tambalotti (2010).

[Insert Table 6 here]

Table 6 presents our main findings. For all shocks, we report the volatility of output as well as the volatility of total hours, employment, and hours per worker for different values of the firing cost. Our quantitative results indicate that higher firing costs induce more substitution between intensive and extensive margins of labor supply *independent* of the sources of business cycle fluctuations. When firing costs increase from zero to 10 percent of steady-state wages, the volatility of hours per worker almost doubles in all three specifications. If anything, we note that the volatility of employment decreases much faster when we consider government spending and IST shocks than under productivity shocks. This observation suggests that neglecting the intensive margin of adjustment from the policy analysis has even larger quantitative implications when shocks other than productivity are considered.

5.3.2 Labor Supply Elasticity

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In our benchmark calibration, we follow Cho and Cooley (1994) and calibrate the two labor supply elasticity parameters by setting η equal to 0.60 while choosing γ to match the volatility of employment relative to hours per worker. However, there is a great deal of uncertainty surrounding estimates for the labor supply elasticity, with important implications for the analysis of alternative policies. To guard against this concern, we assess the ability of our model in reproducing the cross-country evidence under different values for η, while maintaning the same calibration strategy for γ.

[Insert Figure 5 here]

Figure 5 compares the simulation results for a calibration of $\eta = 0.43$ (blue lines, labelled "Low η") with the baseline model (red line). Here we calibrate η following estimates reported in Chetty et al. (2011), which present the average of values typically reported in macroeconomic studies. As shown in the picture, the fit of the model line is not greatly affected and, if anything, it gets closer to the data regression line.

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6 Conclusion

Using a novel dataset, we document that labor market fluctuations display very different patterns across OECD countries. Specifically, countries differ greatly in terms of both the relative volatility of hours compared to output as well as the relative volatility of labor supply along the intensive and extensive margins.

Given the large differences in labor market policies and institutions across countries, it is natural to ask to what extent these differences in fluctuations are due to specific policies and/or institutions. This paper represents a first step in this broader research agenda. We introduce dismissal costs into an otherwise standard real business cycle model that also features intensive and extensive margins of labor supply and study quantitatively how fluctuations are affected by changes in such costs.

Our main finding is that differences in dismissal costs can quantitatively account for the patterns found in the cross-country data. As we increase firing costs, economies tend to adjust more along the intensive margin and the overall volatility of total hours declines, as in the data. This quantitative finding relies on several key contributions developed in our analysis. First, it is very important to allow for an empirically reasonable response along the intensive margin when studying the effects of policies such as dismissal costs. Almost all studies of the effects of firing costs assume that the only margin of adjustment is the extensive margin. We show that this assumption turns out to severely underestimate the quantitative response of employment to shocks. In our model, firms substitute away from the taxed margin and adjust largely along the intensive margin. Second, the range of firing costs required by our benchmark calibration to reproduce the cross-country differences observed in the data is almost one order of magnitude smaller than estimates typically found in the literature.

7 References

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8 Appendix A.

9 Model

Consider the problem:

$$\max_{\{c_t,h_t,e_t,k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{\sigma}}{\sigma} - \frac{A}{1+\gamma} h_t^{1+\gamma} e_t - \frac{B}{1+\eta} e_t^{1+\eta} \right]$$

with $0 < \sigma < 1$, $\gamma > 0$, $\eta > 0$, $A > 0$, $B > 0$ and subject to a resource constraint,

$$z_t k_{t-1}^{\theta} \left(e_t h_t \right)^{1-\theta} + (1-\delta) k_{t-1} - c_t - k_t - \mathcal{F} \max\left(e_{t-1} - e_t, 0 \right) \geq 0 \tag{1}$$

where z_t (in logs) follows an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma_{\varepsilon}^2\right),\ 0 < \rho < 1$$

and k_{-1}, e_{-1} and z_{-1} are given.

The first order conditions for c_t, e_t, h_t, and k_t are:

$$c_t^{\sigma-1} - \lambda_t = 0 \tag{2}$$

$$-A h_t^{\gamma} e_t + \lambda_t \left[(1-\theta) \frac{z_t k_{t-1}^{\theta} \left(e_t h_t \right)^{1-\theta}}{h_t} \right] = 0 \tag{3}$$

$$-\frac{A}{1+\gamma} h_t^{1+\gamma} - B e_t^{\eta} + \lambda_t \left[(1-\theta) \frac{z_t k_{t-1}^{\theta} \left(e_t h_t \right)^{1-\theta}}{e_t} \right] + \mathcal{F} \lambda_t \mathbb{I}_t^{(e_t < e_{t-1})} - \beta \mathcal{F} \mathbb{E}_t \left[\lambda_{t+1} \mathbb{I}_{t+1}^{(e_{t+1} < e_t)} \right] = 0 \tag{4}$$

$$-\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \left(\theta \frac{z_{t+1} k_t^{\theta} \left(e_{t+1} h_{t+1} \right)^{1-\theta}}{k_t} + (1-\delta) \right) \right] = 0 \tag{5}$$

where λ_t is the Lagrange multiplier on the resource constraint and function $\mathbb{I}_t$ is an indicator value function that takes the value of 1 when current employment is below its previous level, e.g. $e_t < e_{t-1}$, and 0 otherwise.

Note from equation (4) that the firing cost $\mathcal{F}$ is weighted by the household's marginal utility of consumption, i.e. higher consumption makes it easier to fire workers in that period.

In general for $\mathcal{F} > 0$, $\lambda_t \geq 0$, $\mathbb{I}_t \geq 0 \; \forall t$ and $\mathbb{I}_t > 0$ and $\lambda_t > 0$ for some states of nature, then $\mathbb{E}_t \left[\mathcal{F} \lambda_{t+1} \mathbb{I}_{t+1}^{(e_{t+1} < e_t)} \right] > 0$. This implies that expected firing costs are always present in the first order conditions. Note that e_{t-1} appears in the system only when workers are fired, i.e. $e_t < e_{t-1}$ (see equations (1) and (4)). This implies nonlinearities in the employment policy rule that are not present in the standard RBC model, i.e. $\mathcal{F} = 0$.

We apply two alternative methods to approximate the solution. The first method replaces the firing cost function $F \max[0, e_{t-1} - e_t]$ by a differentiable penalty function and the the dynamic solution is approximated via standard log-linearization around the steady state. The second method uses the model's first order conditions and then solves these equations via Galerkin's non-linear approximation method. We use the log-linearized model only to calibrate some parameter values. The results presented in the main text are obtained from the non-linear approximation method.

9.1 Log-linearization

9.1.1 Penalty function

The function $F \max[0, e_{t-1} - e_t]$ presents a kink at the point $e_t = e_{t-1}$. To circumvent this problem, we replace the firing cost function with a differentiable penalty function $\chi_t(\cdot)$.

$$F \max[0, e_{t-1} - e_t] \approx \chi_t(e_t, e_{t-1}) = \mathcal{F} \left(\frac{1}{2} (e_{t-1} - e_t) + \frac{1}{2} \sqrt{(e_{t-1} - e_t)^2 + \kappa} \right)$$

where κ is a positive real number determining the accuracy of the approximation: the lower κ, the better the approximation. If $\kappa \rightarrow 0$ and $e_{t-1} > e_t$, then $\chi_t (\cdot)$ is equal to $\mathcal{F} (e_{t-1} - e_t)$. For $e_{t-1} \leq e_t$, the sumands cancel each other out and $\chi_t (\cdot)$ equals zero.

The penalty function also implies:

$$\frac{\partial \chi_t(e_t, e_{t-1})}{\partial e_t} = -\mathcal{F} \left(\frac{1}{2} + \frac{1}{2} \frac{(e_{t-1} - e_t)}{\sqrt{(e_{t-1} - e_t)^2 + \kappa}} \right)$$

Setting $\kappa \to 0$ provides an accurate approximation to the "marginal" firing cost. If $\kappa \to 0$ and $e_{t-1} > e_t$, then $\partial \chi_t / \partial e_t$ equals $-\digamma$. For $e_{t-1} < e_t$, then $\partial \chi_t / \partial e_t$ equals zero. At the kink, i.e. $e_t = e_{t-1}$, $\partial \chi_t / \partial e_t$ equals $-\digamma/2$. The latter is the only point where the penalty function provides an innacurate approximation as $\kappa \to 0$.

9.1.2 Alternative problem

Given the penalty function, we replace the original problem by the following:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{\sigma}}{\sigma} - \frac{A}{1+\gamma} h_t^{1+\gamma} e_t - \frac{B}{1+\eta} e_t^{1+\eta} \right]$$

s.t.

$$c_t + k_t - (1-\delta) k_{t-1} = z_t k_{t-1}^{\theta} (e_t h_t)^{1-\theta} - (\chi_t - \chi(\bar{e}, \bar{e}))$$

where $\bar{e}$ is the steady state level of employment. We include $\chi(\bar{e}, \bar{e})$ to eliminate the penalty function in steady state. The focs remain the same except for the foc (4). Under the penalty function, the equivalent foc can be written as:

$$-\frac{A}{1+\gamma} h_t^{1+\gamma} - B e_t^{\eta} + \lambda_t \left[(1-\theta) \frac{z_t k_{t-1}^{\theta} (e_t h_t)^{1-\theta}}{e_t} \right] + \lambda_t \frac{\partial \chi_t}{\partial e_t} - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{\partial \chi_{t+1}}{\partial e_t} \right] = 0 \qquad (6)$$

The model is log-linearized around its deterministic steady state, i.e., $z_t = 1 \ \forall t$. The unique and stable log-linearized rational expectation equilibrium is computed numerically via standard methods. This approximation preserves one important property of the original model, i.e. as $\digamma \to 0$, the model collapses to the standard frictionless model. However, one particular issue with this method is that the log-linearized approximation of $\partial \chi_t / \partial e_t$ and $\partial \chi_{t+1} / \partial e_t$ approach to infinity as $\kappa \to 0$. Given this we set $\kappa = 10^{-3}$.

9.2 Galerkin

We solve the problem using Galerkin method, see Judd (1998).[10] As other projection methods, Galerkin uses polynominals to approximate the decision rules of the original problem. Here we apply Chebyshev polynomials basis. The parameters of these polynomials are identified by minimizing the approximation errors. The conditional expectations are computed via Gauss-Hermite quadrature. Next we describe the solution method in more detail.

We approximate the policy functions for λ_t and e_t via Chebyshev polynomials. Specifically, we assume:

$$\lambda_t \cong \Psi\left(x_t, \alpha^{\lambda}\right) \qquad e_t \cong \Theta\left(x_t, \alpha^{e}\right)$$

where x_t a Chebyshev basis of the state vector $y_t \equiv (k_{t-1}, e_{t-1}, z_t)$, and α^c and α^e are vectors of parameters. The basic idea of the method is that $\Psi(\cdot)$ and $\Theta(\cdot)$ are functions of Chebyshev polynomials of x_t. We restrict the state vector y_t to be in the space $\left[\underline{y}, \overline{y}\right]$. This implies subspaces for each state variable: $k_{t-1} \in \left[\underline{k}, \overline{k}\right]$, $e_{t-1} \in \left[\underline{e}, \overline{e}\right]$, $z_t \in \left[\underline{z}, \overline{z}\right]$. Thus, $\left[\underline{y}, \overline{y}\right] = \left[\underline{k}, \overline{k}\right] \times \left[\underline{e}, \overline{e}\right] \times \left[\underline{z}, \overline{z}\right] \subset \mathbb{R}^3_+$.

Each state variable has its Chebyshev polynomial with a specific degree. That is, $T_{n_k}\left(\varphi\left(\log k_{t-1}\right)\right)$ is the n_k-degree Chebyshev polynomial of k_{t-1} defined over $\left[\underline{k}, \overline{k}\right]$. In the same way, we have $T_{n_e}\left(\varphi\left(\log e_{t-1}\right)\right)$ and $T_{n_z}\left(\varphi\left(\log z_t\right)\right)$ defined over $\left[\underline{e}, \overline{e}\right]$ and $\left[\underline{z}, \overline{z}\right]$, respectively. For k_{t-1} we set $n_k = 3$; for e_{t-1} we set $n_e = 3$; and for z_t we set $n_z = 2$.

Given the above, we can form a multi-dimensional Chebyshev polynomial of x_t. We use a complete polynomial basis which limits the degree of the Chebyshev polynomial of x_t to n_x.[11] This means that the maximum sum of powers in the polynomial is n_x, e.g. $k^i_{t-1} e^j_{t-1} z^{\ell}_t$

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[10] See also Fabrice Collard's notes for further information: http://fabcol.free.fr/index.php?page=notes

[11] Alternatively, we could form a multi-dimensional Chebyshev polynomial using a tensor product basis. For example,
$\Psi\left(x_t, \alpha^{\lambda}\right) = \exp\left(\sum_i \sum_j \sum_{\ell} \alpha^{\lambda}_{ij\ell} T_i\left(\varphi\left(\log k_{t-1}\right)\right) T_j\left(\varphi\left(\log k_{t-1}\right)\right) T_{\ell}\left(\varphi\left(\log z_t\right)\right)\right)$

where $i+j+\ell \leq n_x$. We set $n_x = 3$. Therefore, the approximating functions $\Psi\left(x_t, \alpha^\lambda\right)$ and $\Theta\left(x_t, \alpha^e\right)$ take the form:

$$\begin{aligned}
\Psi\left(x_t, \alpha^\lambda\right) &= \exp\left(\sum_{\substack{i=0 \\ i+j+\ell\leq n_x}}^{n_k}\sum_{j=0}^{n_e}\sum_{\ell=0}^{n_z}\alpha^\lambda_{ij\ell}T_i\left(\varphi\left(\log k_{t-1}\right)\right)T_j\left(\varphi\left(\log e_{t-1}\right)\right)T_\ell\left(\varphi\left(\log z_t\right)\right)\right) \\
\Theta\left(x_t, \alpha^e\right) &= \exp\left(\sum_{\substack{i=0 \\ i+j+\ell\leq n_x}}^{n_k}\sum_{j=0}^{n_e}\sum_{\ell=0}^{n_z}\alpha^e_{ij\ell}T_i\left(\varphi\left(\log k_{t-1}\right)\right)T_j\left(\varphi\left(\log e_{t-1}\right)\right)T_\ell\left(\varphi\left(\log z_t\right)\right)\right)
\end{aligned}$$

We collect the Chebyshev polynomials included in the above expressions in $T\left(x_t\right) = T\left(\varphi\left(y_t\right)\right)$.

Galerkin method requires us to evaluate the approximating functions around different points (or nodes) within the state space. Given the degree of the Chebyshev polynomials, we compute $m_k\left(> n_k\right)$, $m_e\left(> n_e\right)$ and $m_z\left(> n_z\right)$ nodes from $T_{n_k}\left(\varphi\left(\log k_{t-1}\right)\right)$, $T_{n_e}\left(\varphi\left(\log e_{t-1}\right)\right)$ and $T_{n_z}\left(\varphi\left(\log z_t\right)\right)$ respectively. These nodes form a triplet $\widetilde{y} \equiv \left(k_a, e_b, z_c\right)$, $a = 1, ...m_k$, $b = 1, ...m_e$ and $c = 1, ..., m_z$. Each element of $\widetilde{y}$ has its corresponding Chebyshev nodes, i.e. $\widetilde{x} = \varphi\left(\widetilde{y}\right)$. The fact that the number of nodes is greater than the degree of approximation guarantees that the number of coefficients $\left(\alpha^\lambda, \alpha^e\right)$ to be solved are overindentified in the Galerkin method.

Given the above preliminaries, we proceed sequentially as follows:

1. Guess α^e and α^λ.

2. Set the state variables $y_t = \left(k_{t-1}, e_{t-1}, z_t\right)$ at any arbitrary node from $\widetilde{y}$.

3. Compute the value of λ_t and e_t from $\Psi\left(x_t, \alpha^\lambda\right)$ and $\Theta\left(x_t, \alpha^e\right)$, respectively, where $x_t = \varphi\left(y_t\right)$.

Under this approach the number of coefficients increases exponentially.

4. Compute c_t, h_t, k_t from equations (2), (3) and (5), respectively.

5. Draw next period productivity from:

$$z_{t+1} = \rho z_c + \sqrt{2\sigma_\varepsilon}\xi_q$$

   where ξ_q is the q^{th} node of a Hermite polynomial $q = 1, ..., m_\xi$. Each node has a weight $\omega(\xi)$ associated with it. We set $m_\xi = 20$. This implies that we are computing m_ξ next period productivity shocks z_{t+1}.

6. Given the possible values for z_{t+1} and the unique values for k_t and e_t, form $x_{t+1} \equiv \varphi\left((k_t, e_t, z_{t+1})\right)$.

7. Compute the possible values for next period multiplier and employment, i.e. λ_{t+1} and e_{t+1}, from $\Psi\left(\varphi(x_{t+1}), \alpha^\lambda\right)$ and $\Theta\left(\varphi(x_{t+1}), \alpha^e\right)$.

8. Compute the possible values for next period hours from (3) one period forward.

9. Compute the residual :

$$\begin{aligned} R_t\left(\alpha^\lambda, \alpha^e\right) &\equiv \begin{bmatrix} R_t^1\left(\alpha^\lambda, \alpha^e\right) \\ R_t^2\left(\alpha^\lambda, \alpha^e\right) \end{bmatrix} \\ &= \begin{bmatrix} \lambda_t - \left(\beta \mathbb{E}_t\left[\lambda_{t+1}\left(\theta \frac{z_{t+1}k_t^\theta (e_{t+1}h_{t+1})^{1-\theta}}{k_t} + (1-\delta)\right)\right]\right) \\ Be_t^\eta + \frac{A}{1+\gamma}h_t^{1+\gamma} - \left(\lambda_t\left[(1-\theta)\frac{z_t k_{t-1}^\theta (e_t h_t)^{1-\theta}}{e_t}\right] + \digamma \lambda_t \mathbb{I}_t^{(e_t < e_{t-1})} - \beta \digamma \mathbb{E}_t\left[\lambda_{t+1}\mathbb{I}_{t+1}^{(e_{t+1} < e_t)}\right]\right) \end{bmatrix} \end{aligned}$$

   The conditional expectations are computed using Gauss-Hermite quadrature over the nodes ξ_q, $q = 1, ..., m_\xi$.

10. Repeat steps 2 through 9 for each node in $\widetilde{y} \equiv (k_a, e_b, z_c)$, $a = 1, ...m_k$, $b = 1, ...m_e$ and $c = 1, ..., m_z$.

11. Finally, solve Galerkin's loss criterium:

$$\min_{\alpha^\lambda, \alpha^e} \sum_{x \in \widetilde{x}} R\left(\alpha^\lambda, \alpha^e\right) T\left(\varphi(x)\right)$$

The final step of the solution algorithm is equivalent to the method of moments where the expectation of residuals coonditioned by specific variables is minimized. The numerical solution is obtained via standard optimization algorithms.

In the paper we are interested in studying the sensitivity of the business cycle to different degrees of firing cost. We define a grid of values for the firing cost parameter: $\mathcal{F} = [F_1, F_2, ...F_J]$ where $F_j < F_{j+1}$ and $F_1 = 0$. The outlined algorithm is applied sequentially for each element of $\mathcal{F}$ starting from F_1. The initial guess for F_{j+1} is the solution under F_j. For F_1, we use α^{λ} and α^{e} from the frictionless model. In the case of the frictionless model, the initial guesses are obtained from the log-linearized version of the model without a firing cost and improved sequentially through Galerkin method by increasing the degree of approximation n_x.

9.3 Accuracy tests

The use of Galerkin method is allegedly justified on the grounds of accuracy. To confirm this, we compare the accuracy of the Galerkin method and log-linearization using Judd's (1992) accuracy criteria and Den Haan and Marcet (1994) accuracy test. We analyze the accuracy tests of the frictionless model. Table X reports the results. Overall, these results confirm that Galerkin method provides a better approximation than log-linearization.

Table X. Accuracy Statistics, Frictionless Model

	Log-linearization	Galerkin
Judd (1992) accuracy measures		
$\mathcal{E}_1 \equiv \log_{10}\left(\frac{1}{T}\sum_{t=0}^{T}\left\|\frac{R_t^1}{c_t}\right\|\right)$	-4.10	-7.23
$\mathcal{E}_2 \equiv \log_{10}\left(\frac{1}{T}\sum_{t=0}^{T}\left\|\left\|\frac{R_t^1}{c_t}\right\|\right\|^2\right)$	-7.95	-14.28
$\mathcal{E}_3 \equiv \log_{10}\left(\max\left\|\frac{R_t^1}{c_t}\right\|\right)$	-3.31	-6.30
Den Haan and Marcet (1994) test*		
below 5% critical value	4.00	4.50
above 95% critical value	6.20	4.10

* The model is simulated 1000 times for 5000 periods. For each simulation, Den Haan and Marcet's statistic is computed by conditioning down the expectation errors of the consumption Euler equation to vector $h(y_t) \equiv [1, k_{t-1}, e_{t-1}, z_t]'$. The reported numbers refer to the percentage of times the computed statistics is lower or greater than the 5 and 95 percentile of $\chi_{(4)}$ distribution.

Judd's (1992) accuracy criteria are applied over the residual of the consumption Euler equation, i.e. R_t^1 in the Appendix. For each criterium, the accuracy is expressed in units of consumption.[12]. Judd's criteria measure the approximation error from different angles.[13] From the table we confirm that Galerkin method is better than log-linearization, with an improvement of accuracy of the order of magnitude of 75 to 90 percent.

Table X also reports Den Haan and Marcet (1994) accuracy test. According to this test, the approximation is accurate if it satisfies the rational expectation hypothesis (null hypothesis), i.e. the expectation errors are uncorrelated with variables in the information set that the individuals use to formulate expectations. Den Haan and Marcet (1994) showed that under the null hypothesis, the test-statistic is distributed chi-squared.[14] To assess the

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[12] In others words, we can interpret the measures as the loss of consumption agents would suffer from by using an approximation rather than the true. For example, if $\mathcal{E}_1 = -6$, then the agent would make a mistake of 1$ for every 1,000 000 $ she consumes.

[13] The first indicator $\mathcal{E}_1$ measures the average error an agent would make using the approximate solution. The second indicator $\mathcal{E}_2$ measures a similar information, although slightly different as it can be interpreted in terms of volatility. The last indicator $\mathcal{E}_3$ measures the maximal error the agent would make.

[14] In other words, the expectation errors satisfy:

$$E\left(u_{t+1} \otimes h\left(y_t\right)\right) = 0$$

accuracy of the solution, we compute Den Haan and Marcet's test-statistic by simulating the model and computing the percentages of times in which the simulated statistic belongs to the upper and lower critical 5 percent of the theoretical distribution under null hypothesis. Table X shows that both methods generate probabilities that are close to 5 percent. However, Galerkin method underestimates the probabilities by less than one percent while the log-linearized method under or overestimates the probabilities by one percent or more. Thus, Galerkin method is more accurate than log-linearization from the point of view of rationality.

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where u_{t+1} is m-dimensional expectation error vector, y_t is a n-dimensional vector that belongs to the information set and $h(\cdot)$ is any q-dimensional real-valued function, i.e. $h(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^q$. Den Haan and Marcet (1994) showed that under the null hypothesis, the test-statistic satisfies,

$$TB_T' A_T^{-1} B_T \rightarrow \chi^2_{qm} \text{ as } T \rightarrow \infty$$

where

$$B_T \equiv \frac{\sum^T \hat{u}_{t+1} \otimes h(\hat{x}_t)}{T} \qquad A_T \equiv \frac{\sum^T \hat{u}^2_{t+1} h(\hat{x}_t) h(\hat{x}_t)'}{T}$$

In this application, we apply the test to the consumption Euler equation:

$$u_{t+1} \equiv \lambda_t - \beta\lambda_{t+1} \left(\theta \frac{z_{t+1} k_t^\theta (e_{t+1} h_{t+1})^{1-\theta}}{k_t} + (1-\delta) \right)$$

Table 1. Business Cycle Statistics

	Standard Deviation relative to Output			*Correlation with Output*		
	Total Hours	Employment	Hours per Worker	Total Hours	Employment	Hours per Worker
Australia	1.02	0.79	0.45	0.62	0.50	0.40
Austria	0.64	0.58	0.48	0.53	0.59	0.26
Canada	1.03	0.78	0.34	0.77	0.74	0.55
Finland	0.91	0.70	0.57	0.61	0.67	0.12
France	0.81	0.48	0.64	0.47	0.73	0.08
Germany	0.67	0.57	0.40	0.75	0.67	0.40
Ireland	0.89	0.78	0.35	0.65	0.62	0.22
Italy	0.77	0.51	0.44	0.60	0.58	0.35
Japan	0.70	0.31	0.51	0.62	0.48	0.54
Korea	0.88	0.50	0.58	0.55	0.64	0.28
Sweden	0.94	0.75	0.51	0.66	0.59	0.20
U.K.	0.80	0.57	0.33	0.68	0.54	0.65
U.S.	0.98	0.80	0.30	0.84	0.80	0.67
Median	0.88	0.62	0.45	0.62	0.62	0.35

Note. Statistics are based on Hodrick-Prescott filtered data.

Table 2. Calibration

Alternative calibrations	First	Second
Firing cost (F) in units of quarterly wages	0.00	0.045
Curvature in utility function (γ)	0.71	1.71
Utility scale parameter (A)	5.17	30.06
Utility scale parameter (B)	0.27	0.84

Fixed parameters	
Discount factor (β)	0.99
Capital share (θ)	0.36
Depreciation rate (δ)	0.025
Curvature in utility function (η)	0.60
Persistence of technology shock (ρ)	0.98
Standard deviation of technology shock (σ_ε)	0.0092

Table 3. Results, first calibration

Standard deviation (percent), $\digamma = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Output	1.86	1.82	1.76	1.70	1.66	1.66
Consumption	0.73	0.71	0.70	0.67	0.66	0.66
Investment	5.43	5.29	5.07	4.87	4.76	4.75
Capital	0.46	0.45	0.43	0.41	0.40	0.40
Total hours	1.01	0.94	0.84	0.74	0.69	0.68
Employment	0.75	0.62	0.45	0.29	0.19	0.18
Hours per worker	0.26	0.32	0.39	0.46	0.51	0.52
Productivity	1.22	1.22	1.22	1.22	1.22	1.22

Correlation with output, $\digamma = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Consumption	0.95	0.95	0.95	0.95	0.95	0.95
Investment	0.99	0.99	0.99	0.99	0.99	0.99
Capital	0.34	0.34	0.34	0.34	0.34	0.34
Total hours	0.98	0.98	0.98	0.98	0.98	0.98
Employment	0.98	0.98	0.97	0.94	0.87	0.86
Hours per worker	0.98	0.98	0.98	0.98	0.99	0.99
Productivity	1.00	1.00	1.00	1.00	1.00	1.00

Note. Firing costs ($\digamma$) are reported in percent of the steady-state real wage (w).

Table 4. The Role of the Intensive Margin of Labor Supply

Standard deviation (percent),	$\mathcal{F} = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Output	w/ intensive	1.86	1.82	1.76	1.70	1.66	1.66
	wo/ Intensive	1.81	1.75	1.66	1.56	1.51	1.45
Consumption	w/ intensive	0.73	0.71	0.70	0.67	0.66	0.66
	wo/ Intensive	0.71	0.69	0.66	0.63	0.61	0.59
Investment	w/ intensive	5.43	5.29	5.07	4.87	4.76	4.75
	wo/ Intensive	5.24	5.03	4.76	4.43	4.26	4.09
Capital	w/ intensive	0.46	0.45	0.43	0.41	0.40	0.40
	wo/ Intensive	0.44	0.43	0.41	0.38	0.36	0.35
Total hours	w/ intensive	1.01	0.94	0.84	0.74	0.69	0.68
	wo/ Intensive	0.92	0.82	0.68	0.52	0.44	0.35
Employment	w/ intensive	0.75	0.62	0.45	0.29	0.19	0.18
	wo/ Intensive	0.92	0.82	0.68	0.52	0.44	0.35

Note. Firing costs ($\mathcal{F}$) are reported in percent of the steady-state real wage (w).

Table 5. Results, second calibration

Standard deviation (percent), $\digamma = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Output	1.74	1.70	1.64	1.59	1.52	1.50
Consumption	0.70	0.68	0.66	0.65	0.62	0.62
Investment	5.00	4.88	4.66	4.49	4.27	4.23
Capital	0.42	0.42	0.40	0.38	0.36	0.36
Total hours	0.81	0.75	0.64	0.56	0.46	0.44
Employment	0.69	0.61	0.48	0.38	0.26	0.23
Hours per worker	0.13	0.14	0.17	0.18	0.21	0.22
Productivity	1.22	1.22	1.22	1.22	1.22	1.22

Correlation with output, $\digamma = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Consumption	0.95	0.95	0.95	0.96	0.96	0.96
Investment	0.99	0.99	0.99	0.99	0.99	0.99
Capital	0.34	0.34	0.34	0.34	0.34	0.34
Total hours	0.98	0.98	0.98	0.98	0.97	0.97
Employment	0.98	0.98	0.98	0.98	0.94	0.93
Hours per worker	0.98	0.98	0.98	0.98	0.98	0.98
Productivity	1.00	1.00	1.00	1.00	1.00	1.00

Table 6. Firing Costs and Aggregate Shocks

Standard deviation (percent),	$F = (x/100)w$ with $x =$	0.00	1.00	2.50	5.00	7.50	10.00
Productivity	Output	1.86	1.82	1.76	1.70	1.66	1.66
	Total hours	1.01	0.94	0.84	0.74	0.69	0.68
	Employment	0.75	0.62	0.45	0.29	0.19	0.18
	Hours per worker	0.26	0.32	0.39	0.46	0.51	0.52
Government Spending	Output	0.05	0.04	0.04	0.03	0.04	0.03
	Total hours	0.09	0.07	0.06	0.05	0.05	0.05
	Employment	0.06	0.03	0.02	0.01	0.01	0.01
	Hours per worker	0.02	0.04	0.04	0.05	0.05	0.05
Investment-Specific Technology	Output	2.20	1.93	1.60	1.41	1.34	1.30
	Total hours	3.49	3.05	2.52	2.23	2.11	2.06
	Employment	2.58	1.89	1.06	0.60	0.43	0.35
	Hours per worker	0.90	1.16	1.47	1.64	1.71	1.74

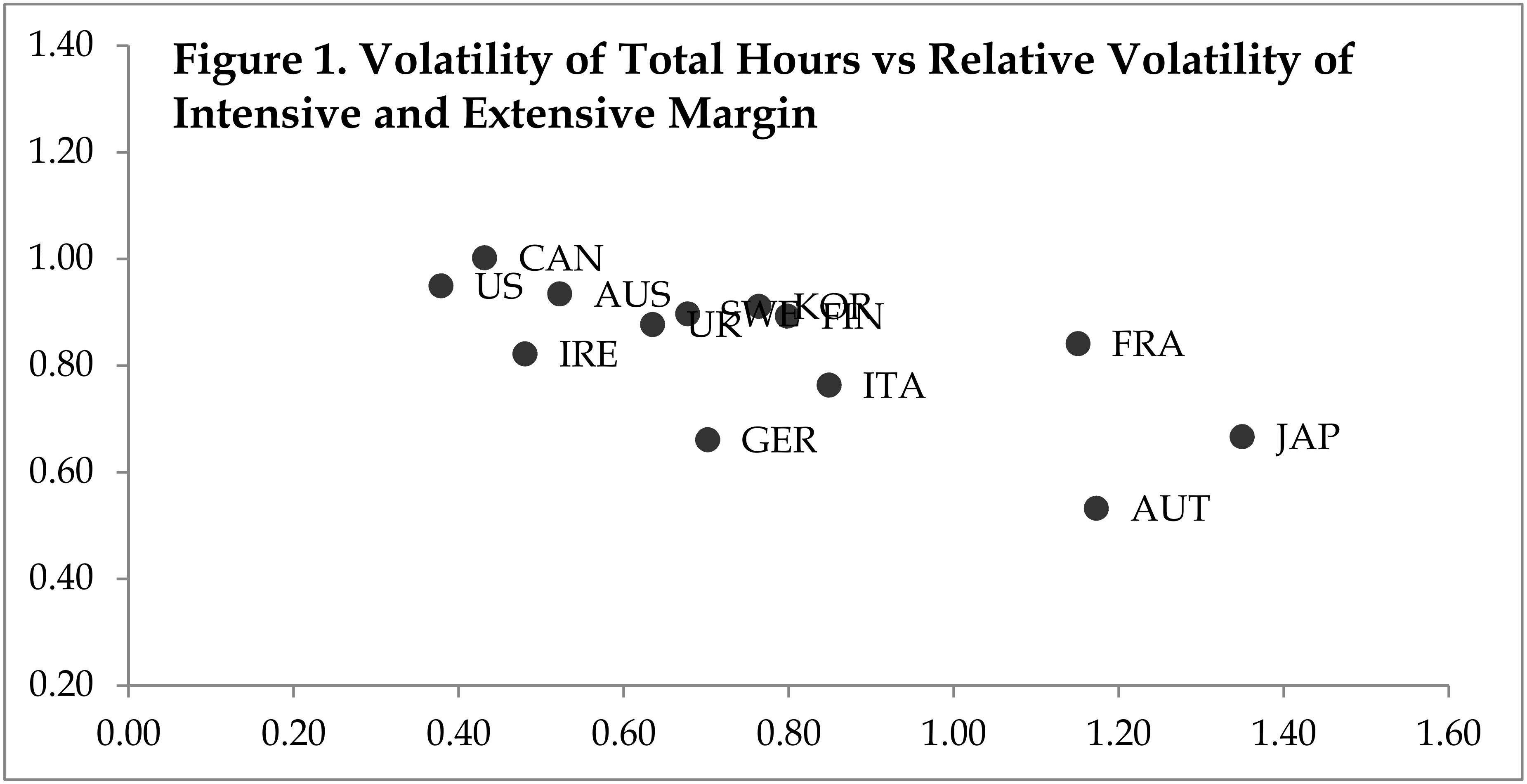

Figure 1. Volatility of Total Hours vs Relative Volatility of Intensive and Extensive Margin

Figure 2. EPL and Relative Volatility of Intensive and Extensive Margin

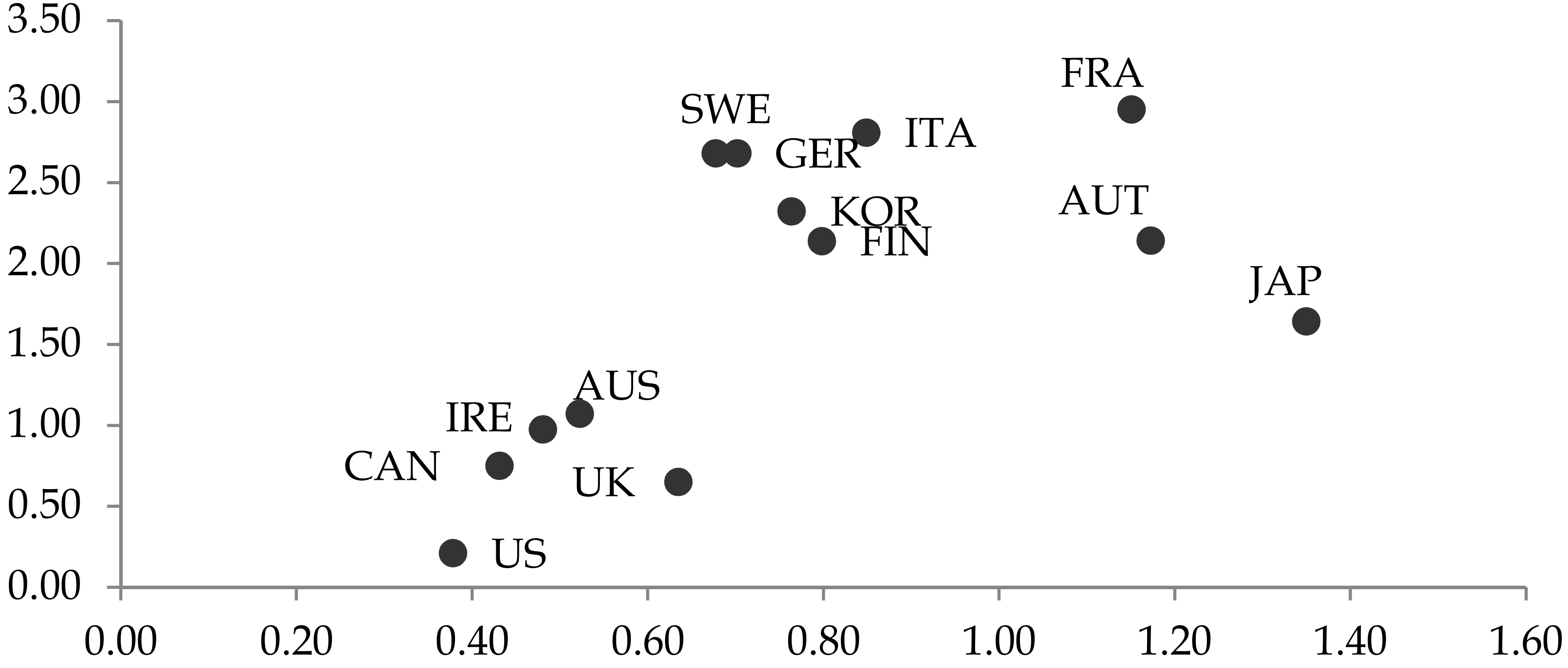

Figure 3. Results, first calibration

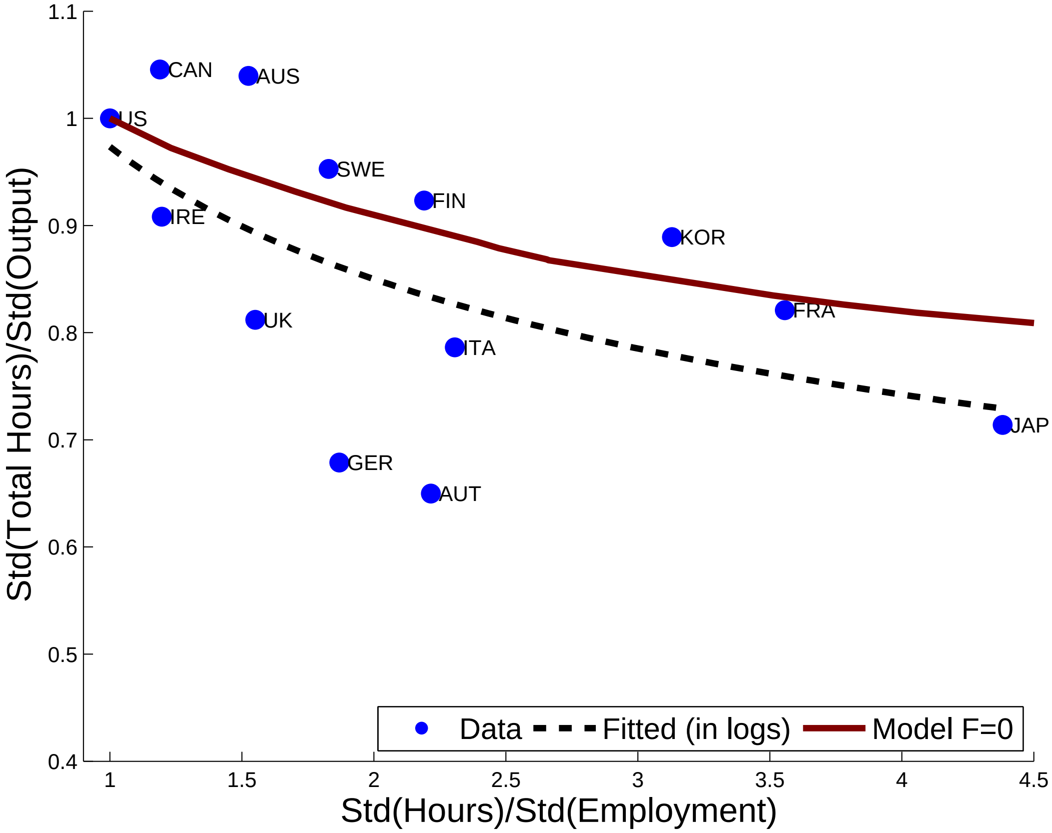

Figure 4. Results, second calibration

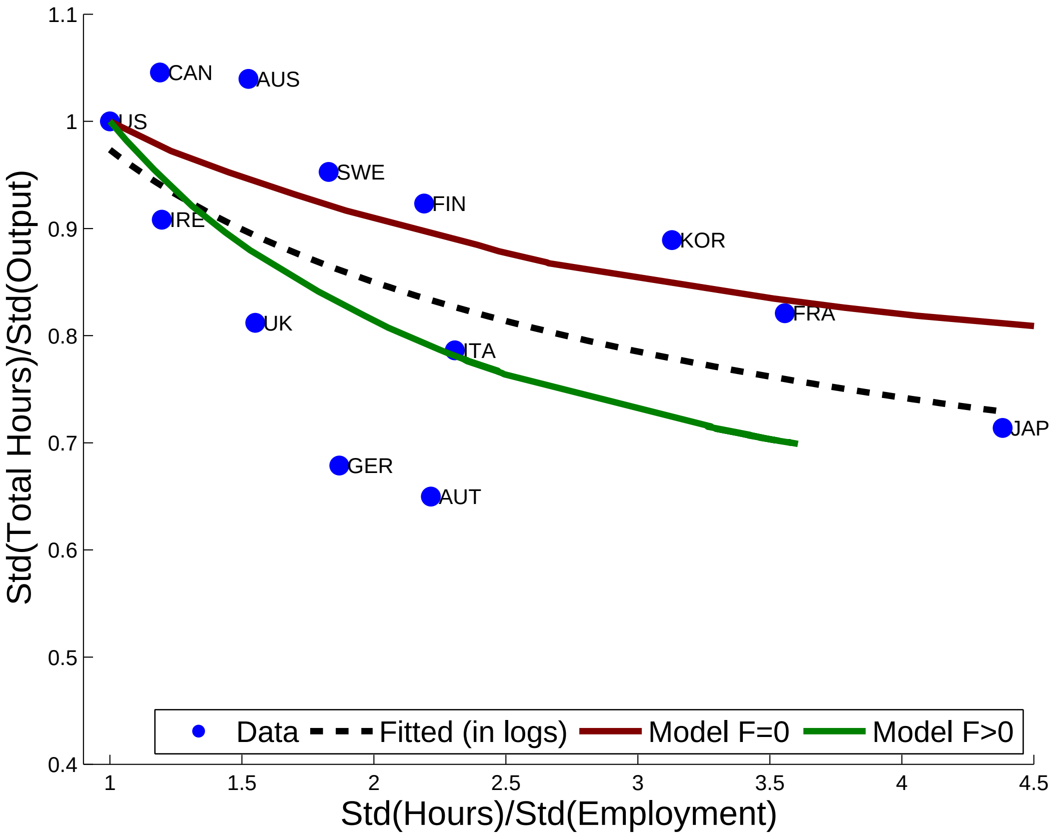

Figure 5. Sensitivity Analysis, second calibration (UPDATE)