Growing through the Merger and Acquisition*

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Abstract

The paper studies with an endogenous growth model how the merger and acquisition (M&A) affects the aggregate growth rate. We model the M&A as a capital reallocation process, which can increase both productivity and growth rates of firms. The model is tractable and greatly consistent with patterns observed in the M&A at the micro level. Matching our model to the data, we find that prohibiting the M&A would lead to the reduction of the aggregate growth rate of US economy by 0.1% and the reduction of the aggregate TFP by 5%.

Keywords: Merger and Acquisition, Two-sided Matching, Complementarity, Growth, Capital Reallocation

JEL Codes: O49, G34, E10

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1 Introduction

There are two capital reallocation processes on the market. The first is that firms can buy or sell individual machines, in other words, internal investment; the other is that firms can buy or sell individual firms through the merger and acquisition (M&A). Many papers focus on the first capital reallocation process and its aggregate effect, such as Hsieh and Klenow (2009). By contrast, this paper studies the second process, M&A, and its aggregate effect.

Macroeconomists typically do not distinguish the two capital allocation processes and neglect the M&A. M&A is considered as a process, in which talented managers acquire assets or employees, like buying machines. Macroeconomists assume that new acquired firms directly get acquiring firms' productivity and then they run together, but do not specify how the mechanism works (Manne (1965), Lucas (1978)).[1] However, acquirers need to absorb the organization capital of acquirees in M&A, such as management systems and selling channels. As a report from Toyota says, "(the acquired firm) is an integrated system and difficult to digest". Therefore, it is necessary to distinguish the M&A from the other capital reallocation process.

Furthermore, it is important to understand the M&A from an aggregate economy perspective. Macroeconomists investigate how firms grow because firm growth is a key determinant of the macroeconomic growth (Luttmer (2007)). Firms can either grow "in house" through the internal investment (getting more machines) or grow "externally" through the M&A.[2] The latter, M&A, has become a very common firm growth strategy. In US, approximately 30% of firms are involved in the M&A in the last a few decades.[3] Totally, the M&A expenditures have averaged around 5% of annual GDP.[4] Thus the M&A is not only critical at the firm level, but also significant at the aggregate level.

The goal of this paper is to quantify the effect of M&A to the aggregate growth rate. We build an endogenous growth model, in which firms are allowed to invest through the M&A or internal investment. The technology of internal investment is conventional: firms get new machines by paying convex costs. However, the M&A technology is different: the M&A

[1] The assumption is reasonable if buying firms is considered the same as buying machines. Recent researches often use this assumption, such as to understand how financial friction (Eisfeldt and Rampini (2006), Midrigan and Xu (2014)) and how asymmetric information (Eisfeldt and Rampini (2008)) affects capital reallocation.

[2] In this paper, "internal investment" means creating new capital, while M&A is a process of ownership change of existing capital.

[3] Source: Compustat dataset from 1978-2012.

[4] Source: SDC M&A database from 1978-2012.

costs depend on what kind of firms to buy. In other words, it is easier for acquiring firms to digest targets with similar productivity. We make this complementary M&A technology assumption based on M&A patterns observed in the data, that (un)productive firms are more likely to buy (un)productive targets (a positive assortative matching pattern).

As costs of internal investment are increasing and convex, firms can enjoy lower investment costs by smoothing the total investment on M&A and internal investment. Therefore, the existence of M&A offers firms another way to expand with lower costs. M&A leads to a higher firm growth rate, and further improves the aggregate growth rate. Our model predicts that the aggregate growth rate would decrease by 0.1% if firms can only grow through internal capital accumulation.

The paper contributes to the existing literature in two aspects. First, we contribute to the growth literature by answering a fundamental question: Should firms expand through internal investments or M&A? Most existing growth models neglect the latter, M&A. In our model, we fill this gap: the model distinguishes M&A and internal investments by introducing the M&A technology, which is consistent with existing discussion. Quoting from Prescott and Visscher (1980), "Organization capital is not costlessly moved, however, and this makes the capital organization specific."[5] Moreover, Rob and Zemsky (2002) show that the cost of transferring organization capital is low when two firms are similar. The model, taking these theories as the microfoundations, discusses the growth effect of M&A.

Second, we contribute to the corporate finance literature by extending M&A research from firm level to aggregate level. Corporate finance researchers are extremely interested in whether M&A can increase firms' efficiency. Most research concludes that M&A can increase firms' efficiency, but some research finds that after the M&A, firms' efficiency may be lowered. In section 6, we extend our model to include this consideration that M&A may hurt firm's efficiency. We find that the M&A is still beneficial to the aggregate economy even if some M&A transactions hurt the firms' efficiency.

The paper is structured as follows. Section 2 discusses the related literature. Further in section 3, we develop the model, which is analyzed in section 4 and whose quantitative results are explored in section 5. After section 6 discusses some robustness, we conclude the paper in section 7.

[5] Atkeson and Kehoe (2005) claim the accumulation of organization capital within the firm can account 8% of US output. Our paper suggests that transferring organization capital across firms may be also important.

2 Related Literature

We are going to mention several other related papers in the literature. First, the paper relates to the theoretical research of assignment model, developed by Roy (1951). Eeckhout and Kircher (2012) and Geerolf (2013) extend the research into "one to many" assignment model. They study the matching of one firm with multiple workers in a static environment. We use this framwork to study the matchng between the acquirer and target firms. Even though the model of Eeckhout and Kircher (2012) is close to ours, our model differs from theirs in two aspects: (1) our model is dynamic; (2) our model endogenizes the status choice of acquiring firms and target firms.

Second, the paper relates to certain theoretical papers modeling M&A and studying its benefits and costs.[6] Jovanovic and Rousseau (2002) explain M&A as a simple capital reallocation process. Rhodes-Kropf and Robinson (2005) build a theory of M&A based on an asset's complementarity assumption. Perhaps, the most related paper to ours is David (2013), which develops a structural model that M&A gains come from (1) the complementarity between acquiring and target firms' assets and (2) capital reallocation. We also use the complementarity and capital reallocation assumptions, but go beyond the David (2013) by exploring how M&A gains and costs vary with firms' productivity and size. Another difference is that David (2013) studies the M&A market with search frictions, through which prices are determined by bargaining. By contrast, we model the M&A market as a competitive market in which prices are determined by market clearing conditions. In the real world, acquiring firms often buy targets from the stock market, consistent with our assumption.

Third, the paper relates to a series of empirical papers studying the productivity change after M&A. Schoar (2002) and Braguinsky et.al (2013) document that the productivity of acquiring firms drops temporarily during the M&A and then recovers, while the productivity of target firms increases. The M&A technology assumptions in our model fit these empirical findings.

Fourth, if we consider the M&A as a process to increase targets' productivity, the paper relates to recent literature on the technology spread and economic growth. When studying

[6] Some empirical papers in the finance literature report that stock prices of acquirers fall on the M&A annoucement day and take this as evidence that M&A reduces efficiency. However, Jovanovic and Braguinsky (2004) show that even M&A increases efficiency, the acquirer's stock price may still fall. Furthermore, Masulis et al. (2007) show that stock prices increase if the M&A is a cash transaction or the target is a private firm.

how technology is spread, Perla and Tonetti (2014) and Lucas and Moll (2014) assume that unproductive firms can raise productivity by imitating productive firms. We explore another channel of technology spread: M&A.

Lastly, starting from the seminal paper by Hsieh and Klenow (2009), many papers argue that resource reallocation can explain aggregate TFP differences across countries. This paper, by modeling a particular way of capital reallocation, points out that capital reallocation can not only result in huge TFP differences but also generate large differences in growth rates.

3 Model

3.1 Household Problem

A representative consumer, who consumes aggregate consumption C_t each period, maximizes the lifetime utility

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t), \quad \beta \in (0,1)$$

The optimal intertemporal optimization condition yields

$$\frac{1}{1+r_t} = \beta \frac{U'(C_{t+1})}{U'(C_t)} \tag{1}$$

where r_t is the equilibrium interest rate at time t. We assume that there is no aggregate uncertainty and hence the consumer has a deterministic consumption path.

3.2 Firm Problem

There is a continuum of risk neutral firms which produce one homogeneous good. Each firm is endowed with a firm specific productivity z and some capital when it is born. Productivity z is fixed over time. At time t if the firm has capital k on hand, the firm's output is $y = zk$.[7]

Firms have two technologies to expand k. First, they can expand through internal investment i. This "organic" growing technology is conventional in a classical growth model. The cost of investing i is $\Phi^I(i,k)$. We assume that $\Phi^I(i,k)$ is the same across all

[7] We would like the readers to think k as physical capital, while the difference of z is due to intangible assets.

Table 1: Output Change before and after M&A

	t	t+1	t+2
Target	$z_T k_T$		
Acquirer	$(1-s)\,zk$	$zk+\hat{z}_T k_T$	$zk+\hat{z}_T k_T$

firms, increasing and convex on i. The second technology for firms to expand is to acquire other firms.

In this paper, we build an endogenous acquisition cost function. We will continue to show you how we construct it. Basically, we construct this cost function through two steps. First, we introduce the M&A technology, which defines the output of the new firm after acquisition. Second, we construct an M&A market, which determines the price of target firms.

3.2.1 M&A Technology

A Simple Example Let us start from a simple example. Consider two firms (z, k) and (z_T, k_T). We suppose $z > z_T$ and there is no depreciation or further investment. In period t, z starts to acquire z_T. To do so, the manager of the acquiring firm needs to spend time $s_t \in [0, 1]$ to digest the target firm. There is a forgone cost $s_t z$ for the acquiring firm in period t, and the output of the acquirer is $(1-s_t)\,zk$. At the end of period t, the acquirer owns the target.

Then in $t+1$, the productivity of the acquirer jumps back to its original level z, while the productivity of the target changes from z_T to $\hat{z}_T$. If the M&A process can create value, $\hat{z}_T$ should be greater than z_T. The target belongs to the acquirer, and the output of the acquirer after M&A is $zk+\hat{z}_T k_T$. From period $t+2$, we assume that the output is the same as that in period $t+1$ and does not change in the future. The output of the target and acquiring firm is presented in table 1. In period t, the output of target firm is $z_T k_T$ and the output of the acquiring firm is $(1-s)\,zk$. In period t+1, the target firm disappears and the output of the acquiring firm has two components. The first part is the capital controlled by the acquirer before $t+1$, zk. The second component is the output from the target's capital $\hat{z}_T k_T$. In period $t+2$, the output of the acquiring firm is the same as that in the period $t+1$.

Table 2: Output Dynamics before and after M&A

	t	t+1	t+2
Target	$z_T k_T$	$z'_T k'_T$	
Acquirer	$(1-s)\, zk$	$(1-s')\,(zk+\hat{z}_T k_T)$	$zk+\hat{z}_T k_T+\hat{z}'_T k'_T$
Acquirer (the Hayashi Insight)	$(1-s)zk$	$(1-s')\, z\,(k+k_M)$	$z\,(k+k_M+k'_M)$

The General Case Table 2 shows a more complicated example: in period $t+1$, the acquiring firm gets another target firm (z'_T, k'_T) and spends s' time to absorb the new target firm. In $t+2$, the output of the acquirer has three components: the capital before period t, the capital from the first target firm and the capital from the second target firm.

At first glance, the problem seems complicated that we need to track the productivity distribution within the acquiring firm. To avoid it, we use Hayashi insight (1982, 1991): we transform the contribution of target output into efficiency units of capital, k_M. k_M is defined as the capital level which gives the same output level as target firm if we impose the productivity level as z.

$$k_M = \frac{\hat{z}_T}{z} k_T$$

In the third row of table 2, we show another way of writing the output of the acquirer. The output of the acquirer after M&A can be rewritten as $zk+\hat{z}_T k_T = z\,(k+k_M)$. Hence through M&A, the acquirer expands its capital from k units to $k+k_M$ units. This is what we call "growing through the M&A". [8]

Functional Form of M&A Technology In this paper, we assume the functional form of $\hat{z}_T$ as

$$\hat{z}_T = b\,(s)\,\chi\left(\frac{k_T}{k}\right) f\,(z, z_T) \in (z_T, z) \tag{2}$$

where $b' \geq 0, \chi' \leq 0$; f is increasing on both z and z_T. We also assume $\chi\left(\frac{k_T}{k}\right) k_T$ is increasing and concave on k_T.

Armed with the above functional forms, we have the M&A technology that transforms

[8] It is worthwhile to point out the measurement problem here. In the M&A transaction, the capital acquired is adjusted by the capital value of the replacement. We believe this adjustment can capture the process that k_T changes to k_M in the acquisition.

target's capital into the acquirer's capital as

$$k_M = \frac{\hat{z}_T}{z} k_T \leq k_T \tag{3}$$

Micro Evidence from Related Literature The functional form of the M&A technology is disciplined by the micro evidence. The key assumption in our M&A technology is that the acquirer can improve the productivity of target firm. This is consistent with the empirical evidence from Schoar (2002) and Braguinsky et al. (2013) that study the productivity change after M&A.[9] Their findings are summarized in the left graph of figure 1: (1) During the M&A process, the productivity of acquiring firms drops and then recovers in a few years;[10] (2) Targets' productivity $\hat{z}_T$ increases but can not catch up with acquirers' productivity. Both are consistent with our M&A technology. The right graph of figure 1 shows the prediction of our M&A technology. During the M&A period, the productivity of acquiring firm drops temporarily due to the forgone cost sz and then recovers. The productivity of target firms increases but does not exceed z since f is a CES function and s is smaller than 1.[11]

In terms of the functional form of $\hat{z}_T$, as we assume in the equation (2), $b' \geq 0$ implies that the acquiring firm can spend more time s and increase z_T more. $\chi' \leq 0$ implies that if a large acquiring firm buys a small target firm (small $\frac{k_T}{k}$), it is easier for the acquiring firm to absorb the target productivity.[12] Several special cases help to understand the M&A technology.

Case 1 $\hat{z}_T = z$: In this case, the acquirer uses its productivity to replace the targets' productivity, which represents the M&A technology in many capital reallocation literature.[13]

Case 2 $\hat{z}_T = hs^\theta z_T$:[14] This function says that the acquiring firm can spend time s

[9] In Schoar (2002), she actually focuses on diversified M&A, i.e. M&A in different industries. In our paper, we use the M&A within the same industry. However, the target productivity increase in a diversified M&A is usually smaller than the productivity increase in a horizontal M&A. Hence you can take Schoar's result as a lower bound of the change of productivity. It makes our result even stronger.

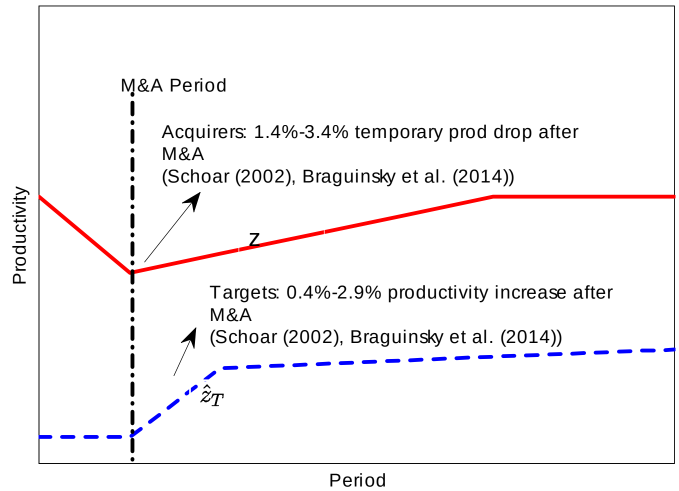
[10] Some finance literature suggests that the ROA of the acquiring firm drops after the acquisition. From Schoar's finding, it is a temporary phenomenon. In our exercise, we use the forgone cost to capture it.

[11] The recovery of z and the increase of z_T in the model are in one period. It is not consistent with the data. We do a robusness check by assuming the productivity changes take several periods, same as the data. The new assumption does not change our results too much.

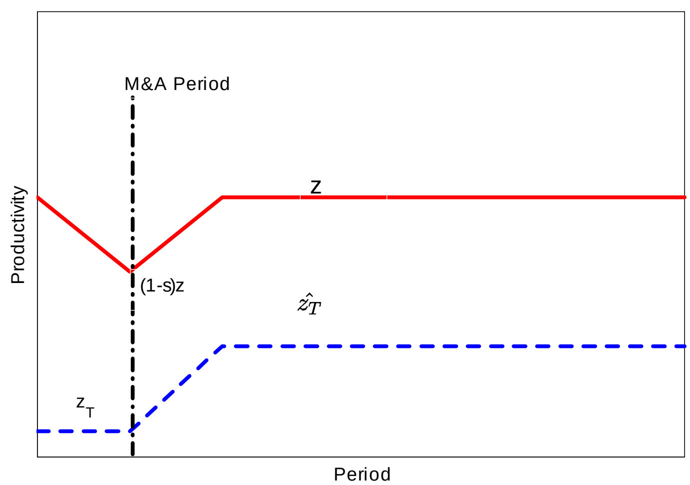
[12] This assumption is consistent with Carlin, et al. (2012) which finds that M&A is most valuable if one large firm acquires a similar but small target firm.

[13] Look at footnote 4.

[14] In this case, we need to assume $h > 1$.



Data



Model

Figure 1: Productivity before and after M&A

Notes: This figure compares productivity of acquiring and acquired firms before and after M&A in the data and the model. Productivity change in the data comes from Schoar (2002) and Braguinsky et al. (2013). They can distinguish the targets' and the acquirers' output after M&A because both of them use plant level data. Their main findings are: (1) The productivity of acquiring firms temporarily drops by 1.4%-3.4%; (2) The productivity of target firms increases 0.4%-2.9% but can not catch up with the productivity of acquiring firms.

to increase the targets' productivity. This assumption is used broadly in human capital literature, such as Ben-Porath (1967).

Case 3 $\hat{z}_T = f(z, z_T)$ and f is a CES function: This assumption is consistent with papers by Rhodes-Kropf and Robinson (2005) and David (2013). They explore the complementarity property between acquiring firm and target firm. Rob and Zemsky (2002) study the optimal design of firms' organization and conclude that the cost of two firms merging depends on the productivity distance between acquiring and target firms (equation (7)).[15]

Thus the functional form of assumption in (2) is general. Many existing models are nested as special cases of our model.

In this paper, we assume the following functional forms $b(s) = hs^{\theta}$ while $h \in (0,1)$ and $\theta \in (0,1)$, $\chi\left(\frac{k_T}{k}\right) = \left(\frac{k_T}{k}\right)^{-(1-\alpha)}$ with $\alpha \in (0,1)$ and $f(z, z_T) = \left[(1-\varepsilon) z^{\psi} + \varepsilon z_T^{\psi}\right]^{\frac{1}{\psi}}$ with $\varepsilon \in (0,1)$ and $\psi < 1$.

3.2.2 M&A Market Structure

Given the M&A technology, we then construct an M&A market to endogenize the price of the target firms. There is a continuum of competitive and frictionless M&A markets. Acquirers (targets) optimally choose the market to participate in and the amount of capital

to buy (sell). Technically, each M&A market is indexed by the target firm's productivity

in this market, z_T. At time t, under the market clearing condition, target firm can get a price $P_t(z_T)$ per each unit of capital. Notice that we do not assume that capital markets are indexed by both target productivity and amount of capital. Hence targets with the same productivity pool their capital in one market and acquirers can choose the desirable amount of capital.

The endogenous acquisition cost is defined as

$$\Phi_t^M(s, z, z_T, k, k_M) = szk + P_t(z_T) k_T \tag{4}$$

where k_M follows equations 2 and 3. A nice property of the endogenous acquisition cost is that it is homogeneous of degree 1 on k and k_M and it is increasing and convex on k_M.

[15] More generally, this function is also used in human capital literature, such as Cunha et al. (2010, equation (2.3) and (2.4)). They study the complementarity between parents' and children's abilities.

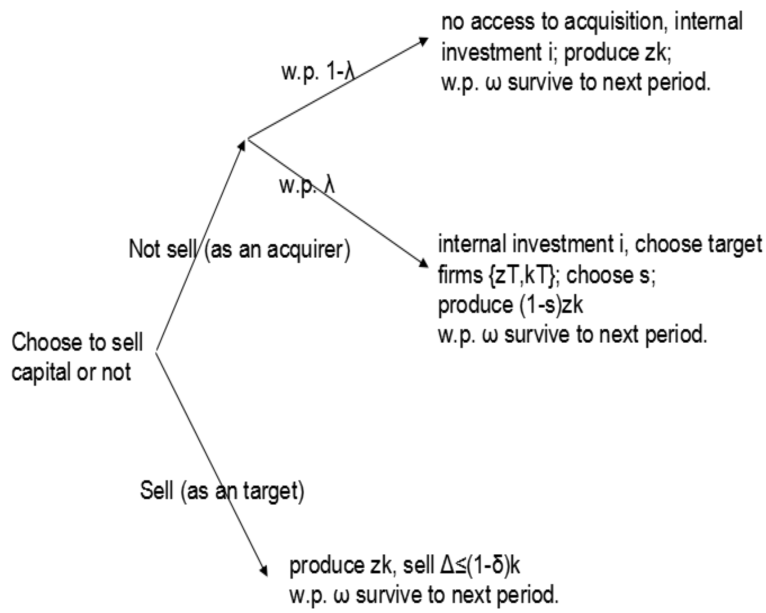


Figure 2: Timing

Hence we can rewrite Φ^M as

$$\Phi_t^M (s, z, z_T, k, k_M) = \phi_t^M \left(s, z, z_T, \frac{k_M}{k} \right) k \tag{5}$$

3.2.3 Timing

In figure 2, we summarize the timing of the firm problem. At the beginning of each period, the firm needs to choose whether to become a target firm (sell its capital) or an acquiring firm (get new capital). If the firm chooses to sell its capital, it produces first and then optimally choose the amount of capital Δ to sell. At the end of the period, there is a death shock: with probability $1 - \omega$, it dies and all its capital is burnt. If the firm chooses to become an acquirer, it receives an iid random shock: with probability λ the firm has a chance to acquire target firms. If the firm has access to M&A, it can choose the target firm's productivity level, z_T, the amount of capital it wants to buy from the target, k_T, and the time s_t. If the acquiring firm does not have the opportunity to engage in M&A, it can only accumulate capital internally.

3.2.4 Firm Value Functions

Define V_t^A as the acquiring firm's value, V_t^I as the value of a firm investing internally only and V_t^T as the value of a target firm at time t. If the acquiring firm has a chance to acquire targets, we have

$$V_t^A(z,k) = \max_{s,z_T(j),k_T(j),i} \left\{ \begin{array}{c} zk - \Phi_t^M(s,z,z_T,k,k_M) - \Phi^I(i,k) \\ +\frac{\omega}{1+r_t} \max\left[\lambda V_{t+1}^A(z,k') + (1-\lambda) V_{t+1}^I(z,k'), V_{t+1}^T(z,k')\right] \end{array} \right\} \tag{6}$$

$$\text{s.t. } k' = (1-\delta)k + i + k_M, \text{ and (5)}, i \geq 0, k_T \geq 0, s \in [0,1]$$

Equation (6) says that the acquiring firm optimally chooses (1) the productivity of its target, z_T, (2) the capital it buys from the target firm, k_T, (3) the time it would like to spend on M&A, s, and (4) internal investment i. The current output is zk and the cost of investment is $\Phi_t^M(s,z,z_T,k,k_M) + \Phi^I(i,k)$.[16] Hence the first row in equation (6) is the current profit. The firm discounts future by $\frac{\omega}{1+r_t}$. In the next period, the firm needs to choose whether to become an acquirer or a target. If it becomes an acquirer, the firm has a chance to acquire target firms with probability λ. With probability $1-\lambda$, the firm can expand only through internal capital accumulation. Hence the expected value of the acquirer is $\lambda V_{t+1}^A + (1-\lambda) V_{t+1}^I$. The firm optimally chooses between the maximum of $\lambda V_{t+1}^A + (1-\lambda) V_{t+1}^I$ and V_{t+1}^T.

If the acquiring firm does not have a chance to acquire targets, it optimally chooses internal investment and receives value:

$$V_t^I(z,k) = \max_i \left\{ \begin{array}{c} zk - \Phi(i,k) \\ +\frac{\omega}{1+r_t} \max\left[\lambda V_{t+1}^A(z,k') + (1-\lambda) V_{t+1}^I(z,k'), V_{t+1}^T(z,k')\right] \end{array} \right\} \tag{7}$$

$$\text{s.t. } k' = (1-\delta)k + i, \; i \geq 0$$

Equation (7) is very similar to equation (6) except $k_T = 0$. It says that the acquiring firm can only invest through internal capital accumulation i.

If a firm chooses to become a target, we have

[16] We can allow firms choose different types of targets. However, as we show later, in the equilibrium, one acquiring firm only buys one type of target firm.

$$V_t^T(z,k) = \max_{k' \geq 0} \left\{ \begin{array}{c} zk + P_t(z)\Delta \\ +\frac{\omega}{1+r_t} \max\left[\lambda V_{t+1}^A(z,k') + (1-\lambda) V_{t+1}^I(z,k'), V_{t+1}^T(z,k')\right] \end{array} \right\}) \tag{8}$$

$$\text{s.t } k' = (1-\delta)k - \Delta$$

Equation (8) defines the value of the target firm at time t. The firm's current profit at time t includes output zk and income from selling capital $P_t(z)(k' - (1-\delta)k)$. Capital in the next period becomes k'.

To close the model, we define the entry problem as follows. In period t, there is a mass of entrants e_{t+1} that pay the entry cost and draw productivity from a distribution with PDF $m(z)$ whose support is $[z_{\min}, z_{\max}]$. There is one period of time-to-build: new entrants start to produce in the next period. Each new entrant is endowed with initial capital $\tilde{k}_{t+1}$, which is a fixed fraction μ of average firm capital $\bar{K}_t$ in the economy. That is $\tilde{k}_{t+1} = \mu \bar{K}_t$. The cost of entry *per unit* of capital is q and the entry process satisfies the free entry condition

$$q\tilde{k}_{t+1} = \frac{1}{1+r_t} \int V_{t+1}\left(z, \tilde{k}_{t+1}\right) m(z)\, dz \tag{9}$$

We simplify the model by making the following assumption.

Assumption 1: $\Phi^I(i,k) = \phi\left(\frac{i}{k}\right) k$

Proposition 1 *Given assumption 1, firm value functions are constant returns to scale on capital k:* $J_t^A(z) = \frac{V_t^A}{k}, J_t^T(z) = \frac{V_t^T}{k}, J_t^I(z) = \frac{V_t^I}{k}$

Proof. From equation (6) to equation (8), we guess all value functions are linear on k. Then we define $J_t^A(z) = \frac{V_t^A}{k}, J_t^T(z) = \frac{V_t^T}{k}, J_t^I(z) = \frac{V_t^I}{k}$. Substituting them into equation (6) to equation (8), we can verify this guess. ■

Define $\hat{x} = \frac{x}{k}$. Then the investment rate of the firm is $\hat{k} = \frac{k_M + i}{k}$. Equations (6) to (8) can be rewritten as

$$J_t^A(z) = \max_{\hat{k} \geq 0} \left\{ z - c_t^A\left(z, \hat{k}\right) + \frac{\omega}{1+r_t}\left(1 - \delta + \hat{k}\right) J_{t+1}(z) \right\} \tag{10}$$

$$s.t.\ \ c_t^A\left(z,\hat{k}\right) = \min_{z_T(j),\hat{k}_T(j),s\in[0,1]} \left\{\phi_t^M\left(s,z,z_T,\hat{k}_M\right)+\phi\left(\hat{\imath}\right)\right\} \tag{11}$$
$$\hat{k}_M \geq 0, \hat{\imath}\geq 0 \text{ and } \hat{k}=\hat{\imath}+\hat{k}_M$$

$$J_t^I\left(z\right)=\max_{\hat{k}\geq 0}\left\{z-\phi\left(\hat{k}\right)+\frac{\omega}{1+r_t}\left(1-\delta+\hat{k}\right)J_{t+1}\left(z\right)\right\} \tag{12}$$

$$J_t^T\left(z\right)=z+\left(1-\delta\right)P_t\left(z\right) \tag{13}$$

$$J_{t+1}=\max\left(\lambda J_{t+1}^A+\left(1-\lambda\right)J_{t+1}^I, J_{t+1}^T\right) \tag{14}$$

Equation (10) defines J_t^A. We decompose the firm problem into two steps. First, we solve the investment cost of firm z, $c_t\left(z,\hat{k}\right)$. It is defined in (11). The first term in (11) sz is the forgone cost of M&A. The second term $\int P_t\left(z_T\left(j\right)\right)\hat{k}_T\left(z_T\left(j\right)\right)dj$ is the price paid to target firms and the third term $\phi\left(\hat{\imath}\right)$ is the cost of internal investment. In (11), we optimally choose target z_T, $\hat{k}_T$ and $\hat{\imath}$ to minimize the cost of investment. Second, we solve the optimal investment rate of firm z in equation (10). $z-c_t^A\left(z,\hat{k}\right)$ is the profit in t. In next period, the firm expands by $1-\delta+\hat{k}$. It survives with probability ω and the firm value is $\left(1-\delta+\hat{k}\right)J_{t+1}$, otherwise the firm dies and gets 0. As we show later, there is only one z_T acquired by the acquiring firm z in the equilibrium.

From (11), we can see how M&A can improve the firm growth rate. The M&A technology in section 2 gives us an endogenous M&A cost $\phi_t^M\left(s,z,z_T,\hat{k}_M\right)$. It is increasing and convex in $\hat{k}_M$. In other words, firms have two technologies to expand: through M&A or through internal investment. Both of them have convex cost functions. The existence of M&A helps firms to smooth the cost of growth, hence reducing the cost of growth, as shown in equation (11).

Equation (12) is similar to (10), except that the firm can not acquire capital from the target hence $\hat{k}_M=0$. $\phi\left(\hat{\imath}\right)$ is the cost of internal capital investment.

Equation (13) describes the value of a target firm. Notice that when the firm chooses to become a target, it sells all its capital since the firm's value function is linear in k.[17]

[17] In this paper, the sales of individual machines is included in the internal investment process. However, M&A always dominates the sales of individual machines in the model since if firms only sell individual machines, the value of intangible asset is lost. Mathematically, it means that the price of any target firm $P\left(z_T\right)\geq 1$. 1 is the value of physical capital. In principle, we can assume the price of capital greater than

The free entry condition can be simplified as

$$q = \frac{1}{1+r_t}\int J_{t+1}(z)\,m(z)\,dz \tag{15}$$

The economic mechanism of the model can be seen from equation (11) and (15). Because the existence of M&A reduces the cost of firm growth, the expected firm value $\int J_{t+1}(z)\,m(z)\,dz$ increases. From household's Euler equation, we see that interest rate is positively correlated with aggregate growth, thus the M&A increases the aggregate growth rate.

4 Equilibrium

A competitive equilibrium is defined as follows.

Definition 2 *A competitive equilibrium includes: (i) two occupation sets A_t, T_t. If $z \in A_t$ (or T_t), the firm chooses to be acquirer (target); (ii) a matching function $z_{T,t}(z)$; (iii) prices $P_t(z)$ and r_t; (iv) number of entrants e_t; (v) distribution of firm size and productivity $\Gamma_t(k,z)$; (vi) aggregate consumption C_t, such that (a) firm and household problems are solved given prices; (b) distributions are consistent with firm decisions; (c) capital markets are cleared: $\forall$ measurable subset $A' \subseteq A_t$,its image set defined by the matching function $z_{T,t}$ is $z_{T,t}(A') \subseteq T_t$, then*

$$\lambda \int_{z\in A',k} \hat{k}_{T,t}(z)\,kd\Gamma_t(k,z) = \int_{z\in z_{T,t}(A'),k} (1-\delta)\,kd\Gamma_t(k,z) \quad \forall A' \subseteq A \tag{16}$$

(d) goods market clears

$$Y_t = C_t + \int \Phi_i d_i + qe_{t+1}\tilde{k}_{t+1} \tag{17}$$

In equation (16), the left hand side is the total demand of capital from acquiring firms $z \in A'$ at time t. $\hat{k}_{T,t}(z)$ is the demand of acquiring firms z *per unit* of capital. Among z, there is only a share λ that can acquire firms. Hence after multiplying $\hat{k}_{T,t}(z_{T,t}(z))$ by firm size k and λ, we have the demand of targets' capital from acquiring firms (z,k). Then we sum across all possible k and get the demand of targets' capital from acquiring firms, conditional on productivity z. Integrating across all firms in set A', we get total demand of

1, and some low z firms prefer to selling individual machines.

capital for acquiring firms, whose productivity is in set A'. The right hand side of equation (16) is the total supply of the capital from target firms. The set of targets' productivity is given by the image set $z_{T,t}(A')$, and the total capital of those firms is given by the right hand side.[18]

4.1 Equilibrium at the Micro Level

From the definition of equilibrium, we can see that the capital market clearing condition in our model is much more complicated than that in standard models: we have infinite capital markets and all of them should satisfy condition (16). The following two propositions show that we can simplify the capital market clearing condition under some assumptions.

Given the functional form we assume in section 3.2.2, the next proposition shows a sorting pattern in M&A in the equilibrium.

Proposition 3 *There exists a $\hat{\psi} < 0$ such that if $\hat{\psi} < \psi \leq 0$, a cutoff value z_t^* exists such that $\lambda J_t^A(z_t^*) + (1-\lambda) J_t^I(z_t^*) = J_t^T(z_t^*)$. If $z > z_t^*$ then firm chooses to be acquirer; if $z < z_t^*$, it chooses to become target. There is a positive assortative matching between acquiring firms' productivity and target firms' productivity: z_T increases on z.*

Proof. See appendix ■

The above proposition says that acquiring firms' productivity is higher than target firms' productivity. Intuitively, in our M&A technology, there are two parts: $f(z, z_T)$ measures the productivity change after M&A while v is the efficiency of absorbing target firms. If an unproductive firm acquires a productive firm, then potential output of M&A, $f(z, z_T)k_T$, is smaller than the target's initial output $z_T k_T$. Given the efficiency of absorbing v smaller than 1, there is no gain when an unproductive firm acquires a productive target.

Figure 3 shows the equilibrium matching pattern. When $\psi \leq 0$, our model equilibrium is summarized as: in each period when new entrants enter, fewer productive firms

[18] To complete the definition of the equilibrium, we also need to define the off-equilibrium price. If the firm $z \notin T$ chooses to become a target, the deviation price is defined as

$$P_t(z) = \sup \left\{ \begin{array}{l} p : \text{there exists an acquirer } (z_A, k_A) \text{ if matched} \\ \text{with } z \text{ at price } p\text{, payoff is same as } V_t^A(z_A, k_A) \end{array} \right\}$$

In other words, the deviation price is defined as the best price that firm z can get to make some acquiring firms indifferent.

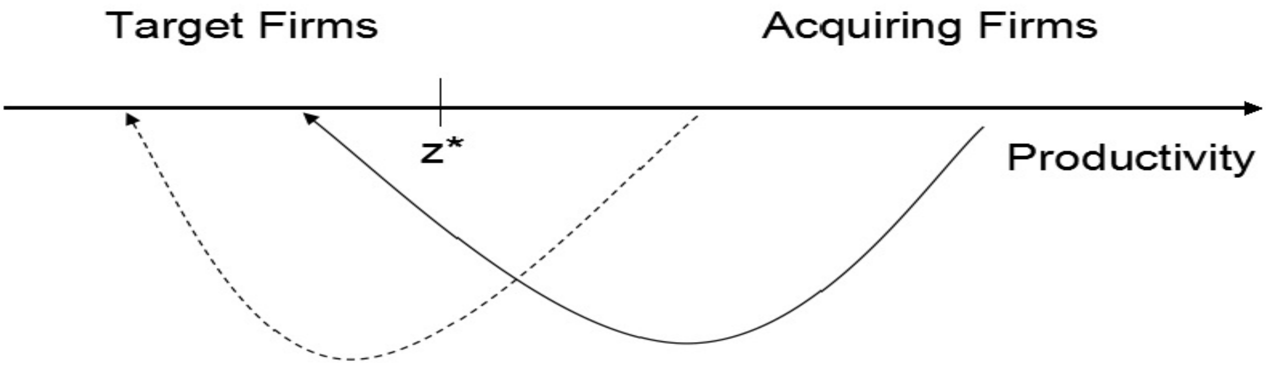


Figure 3: Matching Pattern of the Model

are acquired while productive firms survive. More productive acquiring firms buy more productive target firms.

In the following parts, we assume $\hat{\psi} < \psi \leq 0$. From the market clearing condition (16) and the positive sorting condition, we have

$$\lambda \int_{z}^{z_{\max}} \hat{k}_{T,t}\left(z\right) k d\Gamma_t(k,z) = \int_{z_{T,t}(z)}^{z_t^*} \left(1-\delta\right) k d\Gamma_t(k,z) \quad \forall z \geq z_t^* \tag{18}$$

Comparing the above equation to condition (16), we find that (18) is much simpler: first, z only chooses a unique target firm z_T; second, we do not need to solve market clearing conditions for any possible set A' but only need to check the subsets above z.

Equation (18) defines the matching function. We also need two boundary conditions

$$z_{T,t}\left(z_t^*\right) = z_{\min}, z_{T,t}\left(z_{\max}\right) = z_t^* \tag{19}$$

The above two equations say that acquiring firm z_t^* matches with $z_{\min}$, and $z_{\max}$ matches with firm z_t^*.

In proposition 3, there are two conditions. First, ψ should be smaller than 0. In a unidimensional sorting model (as Becker, 1973), positive sorting arises if in the M&A technology function f has positive cross partial derivative, $f''_{z_T z} > 0$. Given f is a CES function, f satisfies this condition for any $\psi \leq 1$. In our model, acquiring firms trade off between buying a small amount of capital from productive targets and buying a large amount of capital from unproductive targets.[19] Proposition 5 says that to obtain the positive sorting on acquiring firms' productivity and target firms' productivity, we need stronger complementarity than that in Becker's model.

Second, ψ can not be too small. Consider the extreme case that $\psi = -\infty$.[20] Acquiring firms never buy firms that have different productivity. The equilibrium pattern in figure 3 will collapse.

4.2 Balanced Growth Path

The aggregate capital in this economy is defined as

$$K_t = \int k d\Gamma_t(k, z) \tag{20}$$

and we can define the total output of the economy as

$$Y_t = \int_{z \geq z^*} [1 - \lambda s(z)] zkd\Gamma_t(k, z) + \int_{z < z^*} zkd\Gamma_t(k, z) \tag{21}$$

where $\lambda s(z)$ is the expected productivity loss of acquiring firm.

In the following parts, we focus on the balanced growth path (BGP) equilibrium, which is defined as:

Definition 4 *A Balanced growth path (BGP) equilibrium is a competitive equilibrium with a constant $g_K > 1$ such that (i) all value functions $J^A(z), J^T(z), J(z), P(z)$ and policy functions do not depend on time t; (ii) Y_t, K_t and C_t grow with same speed g_K.*

Let us define the firm growth rate as $g^A(z) = 1 - \delta + \hat{k}(z)$ if the firm has access to the acquisition market, and $g^I(z) = 1 - \delta + \hat{\imath}(z)$ if the firm does not have access to the acquisition market. The following proposition shows that a BGP exists in the model.

Proposition 5 *The model has a BGP with constant growth rate g_K such that g_K is implicitly defined by*

$$\int_{z \geq z^*} \frac{m(z)}{1 - \frac{\omega}{g_K}\left(\lambda g^A(z) + (1 - \lambda) g^I(z)\right)} dz + M(z^*) = \frac{g_K}{e\mu} \tag{22}$$

[19] Eeckout and Kircher (2012) studies this "quality vs quantity" tradeoff in a static environment.
[20] Then function f collapses into Leontief function.

Aggregate output is determined by

$$Y_t = ZK_t \tag{23}$$

Z is the aggregate TFP

$$Z = \int_{z \geq z^*} \frac{(1-\lambda s(z))\, z}{1 - \frac{\omega}{g_K}\left(\lambda g^A(z) + (1-\lambda)\, g^I(z)\right)} m(z)\, dz + \int_{z^*}^{z_{\max}} z m(z)\, dz \tag{24}$$

Proof. See appendix. ■

We can interpret the BGP in this way: the cutoff z^* is a constant on the BGP. Firms with productivity above z^* always choose to invest. New entrants, if their productivity is below z^*, always produce only one period and then sell all their capital. Therefore, acquiring firms are more productive, larger and older than target firms in a BGP equilibrium. Firms above z^* gradually become larger with growth rates $g^A(z)$ if they have access to acquisitions and $g^I(z)$ if they do not have access to acquisitions.

On the BGP, the productivity distribution is fixed and only the firm size grows. The shape of the size distribution is unchanged, but the distribution shifts to the right with a constant rate. The next proposition shows that the firm size distribution has a Pareto tail.

Proposition 6 *Define the average firm size as $\bar{K}_t$ and the relative size of firm j as $\frac{k_t(j)}{\bar{K}_t}$, and then the distribution of the relative size conditional on productivity has a Pareto tail*

$$\lim_{x \to \infty} \frac{\Pr\left(\frac{k_t(j)}{\bar{K}_t} \geq x | z\right)}{x^{-\Theta(z)}} = constant \tag{25}$$

and $\Theta(z)$ satisfies

$$\omega\left[(1-\lambda)\, g^I(z)^{\Theta(z)} + \lambda g^A(z)^{\Theta(z)}\right] = g_K^{\Theta(z)} \tag{26}$$

and the unconditional distribution of relative firm has a Pareto tail with tail index $\Theta(z_{\max})$

$$\lim_{x \to \infty} \frac{\Pr\left(\frac{k_t(j)}{\bar{K}_t} \geq x\right)}{x^{-\Theta(z_{\max})}} = constant \tag{27}$$

Proof. See appendix. ■

The intuition of the proposition 6 is as follows: conditional on the productivity, the firm growth rate does not depend on the size. Hence our model follows the Gibrat's law conditional on the productivity. It is well known that Gibrat's law generates a size

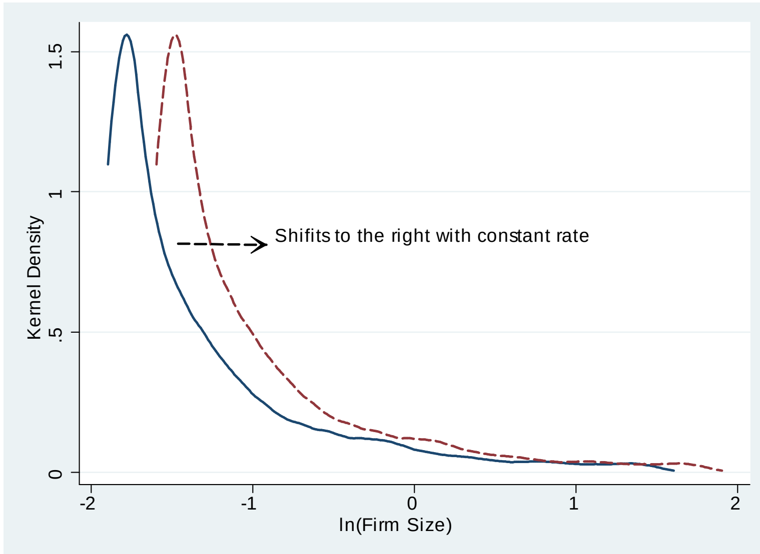


Figure 4: Distribution of Firm Size

Notes: The parameters are shown in table 3.

distribution with Pareto tail (Garbaix (2009)). Thus conditional on productivity, the firm size distribution has a Pareto tail. If all firms are pooled together, the most productive firm determines the tail of the size distribution.

Tonetti and Perla (2014) studies a growth model in which unproductive firms can imitate productive firms. They start with a Pareto productivity distribution and get an equilibrium Pareto size distribution. However, in our model, productive firms try to raise the productivity of unproductive firms and the price is determined endogenously. In addition, starting from any productivity distribution, our model can generate a Pareto size distribution. Figure 4 draws the shift of distribution of firm size.[21] The distribution has a right tail and shifts to the right with a constant rate, which is the aggregate growth rate of the economy.

[21]The parameters are used as the benchmark case in section 5.2.

5 Quantitative Analysis

In this section, we provide some empirical evidence of our model's implications. This section is organized as follows. We first calibrate the parameters of the model from the M&A data at the micro level and compare our model with M&A pattern. Then we get more evidence from information of new-startups. Finally, we provide some evidence from aggregate data.

5.1 Data

We use two datasets. The first one is the Compustat dataset. The second one is M&A transaction data from the Thomson Reuters SDC Platinum database (SDC). SDC collects all M&A transactions in US that involve at least 5% of the ownership change of a company where the transaction is valued at $1 million or more (after 1992, all deals are covered) or where the value of the transaction is undisclosed. We download all US M&A transactions from 1978 to 2012. In this paper, we only focus on M&A within the same industry. For most transactions, SDC contains a limited number of pre-transaction statistics on the merging parties, such as sales, employee counts and property, plant and equipment. In order to get more statistics, we merge the SDC dataset with the Compustat dataset. However, direct merging these two datasets is not possible since Compustat data only records most recent CUSIP codes while SDC data uses CUSIP codes at the time of M&A. Hence we first use historical CUSIP information in the CRSP dataset and merge SDC data with CRSP data. Then we use CRSP identifier to link with Compustat data. 77901 transactions are directly downloaded from the SDC dataset. After matching CRSP translator, we get 6608 transactions, for which we can find CRSP identifier (permno) of both acquirers and targets. After merging with Compustat data, we have 3255 transactions remaining without any missing information on sales, employee counts or total assets.

5.2 Calibration

To calibrate the model, we assume that consumer's utility is $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$ with $\gamma = 3$. The model period is 1 year. And we choose the depreciation rate $\delta = 0.1$, the probability of survival rate $\omega = 0.85$, the size of new entrants $\mu = 0.15$ (Thorburn (2000)) and the discount factor $\beta = 0.95$.

We assume that the internal investment has a cost function as $\phi(\hat{\imath}) = \frac{v_i}{2}\hat{\imath}^2$. We choose v_i

to match the M&A intensive margin: the share of M&A in total investment $\left(\frac{P(z_T)k_T}{P(z_T)k_T+\phi(i)}\right)$.

The productivity distribution of entrants $m(z)$ is a truncated log-normal distribution. We normalize the the mean of log productivity to be 1 and the standard deviation to match the firm growth rate dispersion. The log $z_{\max}$ and log $z_{\min}$ are two standard deviations away from the mean. q is calibrated to match the firm growth rate.

The rest six parameters are related to the M&A technology: h, ψ, θ, α, ε and the probability of accessing M&A market λ. The idea of our analysis is to use the micro pattern in the M&A data to calibrate those parameters in the M&A technology. We calibrate them to jointly match the M&A share in total output, sales difference between acquiring and target firms, the productivity difference between target and acquiring firms $\frac{z_T}{z}$, the productivity matching function slope, extensive margin of the M&A and the slope of intensive margin. Extensive margin is the percentage of firms with acquisitions>0 in the Compustat database. The slope of intensive margin is the slope of regressing log M&A intensive margin on $\log(z)$. The parameters are shown in table 3.

Intuitively, M&A/output tells us the level of M&A cost. It helps us to calibrate h. The relative sales between targets and acquirers shed light on the forgone cost sz. We use this moment to calibrate θ. Next, the slope of intensive margin implies the slope of price $P(z_T)$. It is helpful to calibrate ε.[22] Finally, $\frac{z_T}{z}$ and the slope of $\frac{z_T}{z}$ tell us how to transform k_T into k_M. We calibrate ψ and α to match these two moments.

$\varepsilon = 0.35$ indicates that in M&A transactions, only 65% of the acquirers' productivity would be passed to newly merged firms. $\alpha = 0.55$ means that there is a strong decreasing returns to scale on absorbing large target firms: when the relative size of the target increases by 1%, the absorbing efficiency decreases by 45%.

Table 4 reports the target moments of the data and the model. The model replicates the data moments reasonably well. We can see that target firms are smaller and less productive than acquiring firms,[23] consistent with the model prediction.

It is useful to compare the parameters in our model with those in the human capital literature. We take acquiring firms as parents and the target firms as children. There is a large amount literature studying how the parents' investment change the human capital of children. Ben-Porath (1967, equation (2)) assumes that children can spend time to increase their human capital. He uses a functional form s^{θ}, while θ ranges from 0.5 to 0.8. Our θ

[22] In equation (11), taking the first order condition with respect to z_T and assuming $\psi = 0$, we can get that $P_t(z_T) = X_t z_T^{\frac{\varepsilon}{\alpha}}$, where X_t is a constant that is determined in the equilibrium.

[23] David (2013) also documents this fact.

Table 3: Parameters

Parameters	Value	Moments
M&A Tech		
h	0.81	M&A Intensive margin
θ	0.05	Sales dif
ϵ	0.35	z_T/z
$\frac{1}{1-\psi}$	0.67	Slope of M&A intensive margin
$1-\alpha$	0.45	Slope of z_T/z
Other Params		
λ	0.35	M&A extensive margin
v_i	54.3	M&A/Output
q	4.80	Firm growth rate
σ_z	0.47	Firm growth rate std.
ω	0.85	Thorburn (2000)
μ	0.15	Dunne et al. (1988)

Notes: This table reports the parameters used. M&A extensive margin = percentage of firms whose acquisitions>0; M&A intensive margin $=\frac{P(z_T)k_T}{P(z_T)k_T+\phi(i)}$.

Table 4: Moments of the Data and Model

	Data	Model
Target sales/Acquirer sales	0.20	0.18
$\frac{z_T}{z}$	0.65	0.59
Slope of $\frac{z_T}{z}$	0.85	0.93
Extensive margin	0.30	0.29
Intensive margin	0.62	0.63
Slope of Intensive margin	0.14	0.21
M&A/Output	0.05	0.06
Firm growth rate	0.065	0.070
Firm growth rate std.	0.12	0.10

is much smaller. It is because the temporary productivity drop of acquiring firms is not large. In Cunha et al. (2010, equation (2.3) and (2.4)), they study the complementarity between parents' and children's abilities using a CES functional form, similar to what we use. Their elasticity of substitution $\frac{1}{1-\psi}$ ranges from 0 to 5 (Cunha et al. 2010). Our choice of parameter is within this range.

5.3 Positive Sorting Pattern in M&A

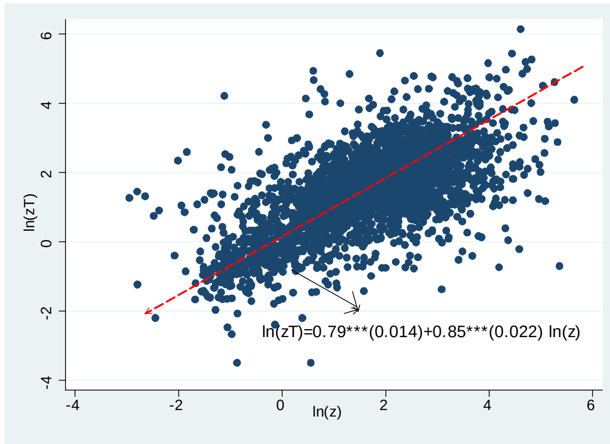
Our model predicts the positive sorting pattern between productivity of acquirers and targets. Figure 5 plots the sorting matching pattern of acquiring and target firms. The top graph plots the sorting pattern of productivity, which is measured by log sales minus log assets. The horizontal line is the productivity of the acquiring firm and the vertical line, the productivity of the target firm. We can see a strong positive assortative matching pattern on productivity: more productive acquirers tend to buy more productive targets. The linear fit function has a significant slope coefficient of 0.85 while the intercept is 0.79. The bottom graph plots the matching pattern of log productivity in the model. We plot log z on the x-axis and log z_T on the y-axis. There are two lines in the graph. The solid blue line is the matching function implied by the model. We can see that when log z is approximately 0.9, the firm is indifferent between target and acquirer choice (the x-axis starts at 0.9 while y-axis ends at 0.9). The dashed red line is the linear fit function, with a slope of 0.93 and an intercept of -1.05.

5.4 Growth Decomposition of US Economy

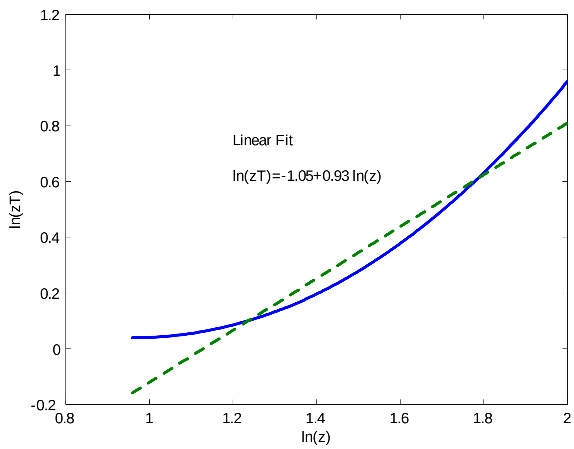
In this section, we explore a counterfactual experiment to understand how M&A can affect the growth rate. We shut down internal investment channel and M&A channel one by one. The results are shown in table 5. The first column is an economy in which firms can grow only through M&A. The second column is an economy where firms can grow only through internal capital accumulation. The third column is the benchmark model: firms can grow through both channels. We can see that when only M&A exists, the growth rate is about 2.08%, while when only internal capital accumulation exists, the growth rate is about 2.90%. Combining them together, we get the growth rate about 3.01%.

We interpret the model with only internal investment (second column) as an exercise to evaluate the contribution of internal capital accumulation to growth. We find that 95% of the aggregate growth rate in our model can be explained by internal investment. Greenwood et al. (1997) has emphasized the importance of internal investment. They find that internal investment can account for about 60% of US GDP growth rate. Our model, without the innovation of productivity z, predicts a larger effect of internal investment. By comparing the second and third columns, we find that when shutting down M&A the change of growth rate is as high as 0.1%.

We interpret the model with only M&A (first column) as an exercise to evaluate the importance of technology spillover. It is interesting to compare our paper with Perla and Tonetti (2014) and Lucas and Moll (2014). In their models, productivity is imitated on costly contact. The growth in their models is solely driven by the improvement in the



Data



Model

Figure 5: Productivity Sorting Pattern in M&A

Notes: This figure presents the log productivity matching patterns in the data and the model. Productivity in data is defined as ln(z)=ln(sales)-ln(assets). The dashed lines are the linear fits of the matching functions. *** denotes statistically significant at 1% level and standard errors are reported in brackets. Data source: SDC M&A database.

Table 5: Growth Contribution of M&A and Internal Capital Accumulation

	Only M&A	Only Internal Investment	Both
Growth Rate	1.81%	2.90%	3.01%
Firm growth rate	4.81%	5.61%	7.05%
TFP	5.85	4.89	5.21

Notes: This table shows the aggregate gains in three cases: firms can grow only through M&A, firms can grow only through internal investments and firms can growth through both channels.

productivity distribution: unproductive firms can increase their productivity by paying a contact cost. However, it is difficult to quantify the effect of this channel on aggregate growth. In our model, we consider the M&A as a means of improving productivity. The technology spillover is not driven by immitation, but caused by improving unproductive firms' productivity in the M&A. Under an appropriate M&A cost function, our model should be isomorphic with their models. Our results suggest that the technology spillover is a significant contributor to the aggregate growth. It can explain about 60% (1.8%/3%) of the GDP growth rate.

Besides the growth rate, the third row compares aggregate TFP. Literature on capital reallocation has discussed how misallocation of resources can decrease the aggregate TFP, such as Klenow and Hsieh (2009), Midrigan and Xu (2014), David (2013). Our paper confirms this perspective. We can see that when shutting down the whole M&A process, TFP decreases by about 5%.

To understand the magnitudes of these effects, we can think from equations (1) and (15). Combining them together, we can get

$$(1+r) \propto \ g_K^{\gamma} \propto \ \int J(z)\, m(z)\, dz$$

The formular shows that the change of growth rate is determined by the change of firm expected value. Because the M&A can smooth the investment cost, compared to solely internal investment, M&A can increase the firm expected value. The magnitude of the M&A effect depends on the curvature of the internal investment cost and the M&A cost. Table 4 shows that conditional on doing M&A, the intensive margin of M&A is over 60%, which implies that the curvature of the M&A cost is small. As such, the M&A has a significant effect on aggregate growth rate.

6 Robustness

Our model implies a significant impact of the M&A on the aggregate growth rate. In this section, we would like to find some more evidence.

6.1 Productivity Drop after M&A

Some corporate finance literature finds that the productivity drops after the M&A and argues that the M&A may hurt the efficiency.[24] To address this concern, we extend our model: we assume that after the M&A, the productivity of the new firm may drop. With probability η, the productivity of the new firm changes to $(1-x)z$, while with probability $1-\eta$, the productivity of the new firm is z, the same as what we assume in the benchmark case. We use the parameters in table 3 again.

Figure 6 plots the aggregate growth rate against η and x. There are three lines on figure 6. Each denotes a choice of x. For example, the solid line denotes that the firm productivity may drop by 1% after acquisition. As we can see, when η is close to 0, the model is similar as the benchmark case: the aggregate growth rate would drop by 0.1% if we shut down M&A. When η and x become larger, the contribution of the M&A to the aggregate growth rate becomes smaller. If η approaches to 1, the M&A gradually disappears in the economy. Thus the elasticity of aggregate growth rate to the M&A approaches to 0. Interestingly, we can see that the contribution of the M&A to the aggregate growth rate can be as large as 0.02%, even if the productivity after M&A declines by 5% with 50% chance.

6.2 Evidence across Sectors

We continue to look at the sector growth within US. Our model predicts that if the dispersion of firm productivity is larger, the M&A has a greater effect on the growth. So we use the following regression to test this prediction.

$$\begin{aligned}\text{sector growth} &= a_0 \times \text{M\&A/Sales} \times \text{Dispersion of firm sales} + \\ &\quad a_1 \times \text{Initial sector sales} + \text{other controls+error}\end{aligned}$$

The first two columns of table 6 report the result. Each observation is a 4 digit SIC industry. The dependent variable is the sector's average sales growth rate from 1990-2005.

[24] In our model, we have already included this concern: the productivity of the acquiring firms temporarily drops after the acquisition because the choice of s.

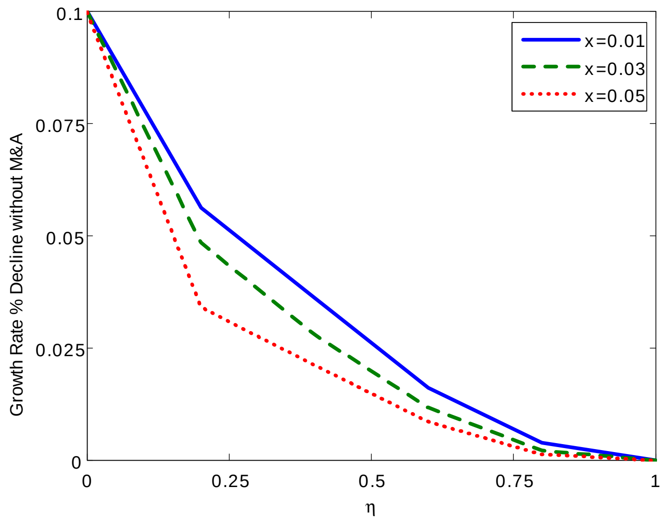


Figure 6: Aggregate Growth Rate Decline without M&A

There are two measures of dispersion of firm sales: the first column uses the standard deviation of firm sales growth rate and the second column uses the interquartile of firm sales growth rate. Other controls include M&A/Sales and dispersion of firm sales. We can see that in both two setups, the interaction term is positively correlated with the growth rate.

6.3 Evidence across Countries

We explore some suggestive evidence in the aggregate data to support our model. Following Barro (1991), we do the following regression:

$$g_i = \beta_0 + \beta_1 \frac{M\&A}{GDP}_{1995} + \beta_2 GDP_{1995} + \beta_3 School_{1995} + \text{other controls} + error \tag{28}$$

g_i = average real GDP per capita growth rate from 1995 to 2005 of country i.

$\frac{\text{M\&A}}{\text{GDP}}_{1995}$ = initial M&A value in GDP in 1995.

GDP_{1995} = initial GDP per capita in 1995.

School_{1995} = initial human capital, measured by percentage of population who have primary (PRIM) or secondary degrees (SEC). This information is obtained from Barro-Lee

Table 6: Sector Growth Rate and M&A

	(1)	(2)
Interaction ($\frac{M\&A}{Sales} \times \sigma$)	0.039**	
	(0.022)	
Interaction ($\frac{M\&A}{Sales} \times iqr$)		0.012***
		(0.005)
$\frac{Sales}{GDP}$ in 1990	-0.899***	-0.683***
	(0.253)	(0.201)
N	448	448
Adj R square	0.29	0.28

Notes: Dependent variable is the sector's sales growth rate from 1990-2005. σ=std. of firm sales growth rate; In the first two columns, we also control M&A/Sales and dispersion of sales. Standard errors are reported in brackets. Data Source: Compustat.

database.[25]

Other controls include life expectancy, fertility rate and government consumption ratio. Table 7 shows the results. In the first column, we can see if the initial M&A value increases by 1%, the growth rate increases by 0.6%. The effect is significant at 5% level. The second and third columns add new controls: stock market value in GDP and the total bank loan value in GDP.[26] Both of them control for the development of capital market in a country. We can see that after when these two variables are controlled, M&A is still positively correlated with growth.[27]

7 Conclusion

In this paper, we study how M&A can affect the aggregate economy. In particular, we highlight the positive effects of M&A process on aggregate growth rate. Applying the model to the data, we argue that M&A is a quantitatively important driving force of aggregate growth, which has been neglected in previous academic research. Moreover, we assume that the cost of M&A depends on the relative distance between acquiring and acquired firms. This assumption can help us to understand the relation between M&A

[25] The website of the database is http://www.barrolee.com/data/dataexp.htm

[26] The data is obtained from World Bank financial sector database. See http://data.worldbank.org/indicator/FS.AST.DOMS.GD.ZS/countries

[27] We do not want to say that our regression can clearly identify the causality between M&A and cross country growth rates. You can take our regression result as a suggestive evidence that is consistent with the model.

Table 7: M&A and Growth Rates across Countries

	(1)		(2)		(3)	
$\frac{M\&A}{GDP}_{1995}$	0.592**	(0.244)	0.600**	(0.250)	0.589**	(0.232)
GDP_{1995}	-0.003*	(0.002)	-0.002	(0.002)	-0.001	(0.002)
$PRIM_{1995}$	-0.011	(0.018)	-0.016	(0.017)	-0.009	(0.017)
SEC_{1995}	0.030**	(0.012)	0.034**	(0.015)	0.026*	(0.014)
Life expectancy	-0.001***	(0.000)	-0.001***	(0.000)	-0.001***	(0.000)
Fertility rate	-0.007***	(0.002)	-0.007***	(0.002)	-0.007***	(0.002)
Gov/GDP	-0.001**	(0.000)	-0.001***	(0.000)	-0.001***	(0.000)
$\frac{\text{Stock Mkt Value}}{\text{GDP}}$			-0.002	(0.003)		
$\frac{\text{Bank Loan}}{\text{GDP}}$					-0.001**	(0.000)
Constant	0.125***	(0.031)	0.130***	(0.034)	0.132***	(0.031)
N	75		63		74	
Adj R square	0.29		0.31		0.36	

Notes: This table reports the results of analyzing the M&A share and real GDP per capita growth rate across countries. The dependent variable is real GDP per capita growth rate. $\frac{\text{M\&A}}{\text{GDP}}_{1995}$ = initial M&A value in GDP in 1995. GDP_{1995} = initial GDP per capita in 1995. $PRIM_{1995}$ = percentage of population who have primary degrees. SEC_{1995} = percentage of population who have secondary degrees. Fertility rate = births per woman. Standard errors are reported in brackets. ***, ** and * denote statistically significant at the 1%, 5% and 10% levels, respectively.

pattern and growth across countries, as well as some industry dynamics.

In our model, the M&A process is solely driven by the consideration of efficiency, while in reality M&A can increase the market power thereby harming some aspects of the market efficiency, which we do not explicitly model in the paper. Although our model may exaggerate the efficiency gain of M&A, it is useful to take our paper as a benchmark. To fully understand how M&A affects the aggregate economy, future research may introduce market power and strategic concern into the model.

In this paper, we focus solely on US M&A. As cross-border M&A is increasingly popular, it may also be interesting to study how M&A affect the cross-country differences in an open economy.

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8 Appendix

The appendix has two parts. The first part shows proofs of propositions; the second part explains the numerical method to solve the model.

8.1 Proof of Propositions

8.1.1 Proof of Propositions 3

Proof: First, we show that in the acquisition process, the productivity of the acquirer is higher than the productivity of the target. This comes from the assumption of the M&A technology. The productivity of the target after the acquisition is $\hat{z}_T$, which is smaller than acquirer's productivity z and larger than target's original productivity z_T. Hence it must be the case that a more productive firm acquires less productive firms otherwise there is no gain in the acquisition.

Second, we argue that in the acquisition process, the matching pattern is positive sorting if $\psi \leq 0$. The proof is to verify whether in a positive sorting equilibrium, the second order condition holds. Define $\hat{f}\left(\frac{z_T}{z}\right) = \left[1-\varepsilon+\varepsilon\left(\frac{z_T}{z}\right)^{\psi}\right]^{\frac{1}{\psi}}$. In equation (11), the first order condition of s yields

$$s = \left[\frac{\theta}{\alpha}\frac{P_t\left(z_T\right)}{z}\left(\frac{\hat{k}_M}{h\hat{f}}\right)^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{\theta+\alpha}} \tag{29}$$

Then we have the cost of acquisition is

$$\begin{aligned}\phi_t^M\left(s,z,z_T,\hat{k}_M\right) &= sz+P_t\left(z_T\right)\left(\frac{\hat{k}_M}{hs^{\theta}\hat{f}}\right)^{\frac{1}{\alpha}} \\ &= Hz^{\frac{\theta}{\theta+\alpha}}P\left(z_T\right)^{\frac{\alpha}{\theta+\alpha}}\left(\frac{\hat{k}_M}{\hat{f}}\right)^{\frac{1}{\theta+\alpha}} \end{aligned} \tag{30}$$

where H is a constant $H = \frac{\left(1+\frac{\theta}{\alpha}\right)\left(\frac{\alpha}{\theta}\right)^{\frac{\theta}{(\theta+\alpha)}}}{h^{\frac{1}{\theta+\alpha}}}$.

Given $J(z)$, the choice of investments can be written as two separate problems

$$\max_{z_T,\hat{k}_M}\left[\frac{\omega}{1+r}J\left(z\right)\hat{k}_M - Hz^{\frac{\theta}{\theta+\alpha}}P\left(z_T\right)^{\frac{\alpha}{\theta+\alpha}}\left(\frac{\hat{k}_M}{\hat{f}}\right)^{\frac{1}{\theta+\alpha}}\right] \tag{31}$$

and

$$\max_{\hat{\imath}} \left[\frac{\omega}{1+r} J(z)\,\hat{\imath} - \phi(\hat{\imath}) \right] \tag{32}$$

The equation (32) is the optimal decision of internal investment and the equation (31) is the optimal decision problem of M&A. To discuss M&A pattern, we only need to focus on (31). We redefine some new variables to make equation (31) cleaner: $Hz^{\frac{\theta}{\theta+\alpha}}\left(\frac{\hat{k}_M}{\hat{f}}\right)^{\frac{1}{\theta+\alpha}} = x$, $F(z, z_T, x) = \frac{\omega}{1+r} J(z)\,\hat{k}_M$ and $w(z_T) = P(z_T)^{\frac{\alpha}{\theta+\alpha}}$. The problem can be written in a short way such that

$$\max_{z_T, x} F(z, z_T, x) - w(z_T)\,x$$

Lemma 7 *The equilibrium has a positive sorting pattern iff*

$$F_{xx}F_{zz_T} - F_{xz}F_{xz_T} + F_{xz}\frac{F_{Z_T}}{x} \geq 0 \tag{33}$$

Proof. See Eeckhout and Kircher (2012). ■

Our next job is to verify that the above condition (33) is right iff $\psi \leq 0$. After substituting all equations into condition (33), we can show that

$$F_{xx}F_{zz_T} - F_{xz}F_{xz_T} + F_{xz}\frac{F_{Z_T}}{x} \propto \hat{f}\frac{d^2\hat{f}}{dzdz_T} - \frac{d\hat{f}}{dz}\frac{d\hat{f}}{dz_T}$$

Hence $F_{xx}F_{zz_T} - F_{xz}F_{xz_T} + F_{xz}\frac{F_{Z_T}}{x} \geq 0 \Leftrightarrow \hat{f}\frac{d^2\hat{f}}{dzdz_T} \geq \frac{d\hat{f}}{dz}\frac{d\hat{f}}{dz_T}$. This condition is true if $\psi \leq 0$.

As we argue in the paper, as long as ψ is large enough and smaller than 0, a positive sorting equilibrium exists.

8.1.2 Proof of Proposition 5

Proof: From proposition 3, we can see that if firms are in the acquirer set A then they quit the market only via exogenous death shocks. The mass of new entrants with productivity z is $em(z)$. Then after $t-\tau$ periods, only $\omega^{t-\tau}$ fraction survives. Hence at time t, the mass of firms that enters at period τ with productivity z is

$$n_{t,\tau}(z) = e\omega^{t-\tau}m(z) \quad \text{when } z \geq z^* \tag{34}$$

$$n_{t,\tau}(z) = \begin{cases} e \text{ if } \tau = t \\ 0 \text{ if } \tau < t \end{cases} \quad \text{when } z < z^* \tag{35}$$

Firm's growth rate is $g^A(z)$ when the firm can acquire target firms and $g^I(z)$ if it can not. If $z \geq z^*$, the aggregate capital of firms that enters at period τ with productivity z is

$$\sum_{j \in z} S_{t,\tau}(j) = \tilde{k}_\tau n_{t,\tau}(z) \sum_{n=0}^{t-\tau} \binom{t-\tau}{n} \lambda^n (1-\lambda)^{t-\tau-n} g^A(z)^n g^I(z)^{t-\tau-n} \tag{36}$$

$$= \tilde{k}_\tau n_{t,\tau}(z) \left[\lambda g^A(z) + (1-\lambda) g^I(z)\right]^{t-\tau} \tag{37}$$

The above equation says that in period t the aggregate capital of firms, whose productivity is z and age is $t-\tau$, is equal to the initial capital of entrants $\bar{k}_\tau$ multiplied by the expected growth rate and the number of firms. Then we can simplify the aggregate capital in equation (18) as

$$K_t = e \int_{z \geq z^*} \sum_{\tau=0}^{t} \tilde{k}_\tau \omega^{t-\tau} \bar{g}(z)^{t-\tau} m(z)\, dz + eM(z^*)\, \tilde{k}_t \tag{38}$$

where $\bar{g}(z) = \lambda g^A(z) + (1-\lambda) g^I(z)$. Aggregate capital has two parts in (38). The first part is the capital of the acquiring firms. $S_{t,\tau}(z)$ is the total capital of the acquiring firms z at time t. The second part is the capital of target firms that only live one period. Their size is $S_{t,t}(z) = \tilde{k}_t n_{t,t}(z)$ and they have a mass $n_{t,t}(z) = em(z)$. Guess that K_t grows with a constant rate g_K. Then

$$K_t = e \int_{z \geq z^*} \sum_{\tau=0}^{t} \mu K_t g_K^{\tau-t} \omega^{t-\tau} \bar{g}(z)^{t-\tau} m(z)\, dz + eM(z^*)\, \mu K_t \tag{39}$$

From consumer problem, we can see if $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$, then

$$\frac{1}{1+r_t} = \beta \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} = \frac{\beta}{g_K^\gamma}$$

When γ increases, we can see that $\frac{1}{1+r_t}$ decreases. The growth rate of the firm decreases as well. Given parameters, we numerically verify

$$\frac{\omega}{g_K}\bar{g}(z) < 1,\ \forall\ z$$

Then (39) can be simplified as equation (22).

8.1.3 Proof of Proposition 6

Proof: Let us denote firm as j and its size as $k_t(j)$. We then have

$$\frac{k_t(j)}{\bar{K}_t} = g(j)\frac{k_{t-1}(j)}{\bar{K}_{t-1}} + \varepsilon \tag{40}$$

In equation (40),

$$g(j) = \begin{cases} \frac{g^A(j)}{g_K} & \text{with prob } \omega\lambda \\ \frac{g^I(j)}{g_K} & \text{with prob } \omega(1-\lambda) \\ 0 & \text{with prob } 1-\omega \end{cases}$$

ε denotes the capital of new entrant $\varepsilon = \mu$ if $g(j) = 0$. Otherwise $\varepsilon = 0$. Notice that $E(g(j)) = \omega\left(\lambda g^A(z) + (1-\lambda)g^I(z)\right) < 1$ from proposition 8. Then we have the following lemma.

Lemma 8 *If $g^A(z) > 1$, then there exists $\Theta(z) > 0$ such that*

$$\omega\left(\lambda g^A(z)^{\Theta(z)} + (1-\lambda)g^I(z)^{\Theta(z)}\right) = g_K^{\Theta(z)} \tag{41}$$

and the conditional distribution of firm size satisfies

$$\lim_{x\to\infty}\frac{\Pr\left(k_t(z)/\bar{K}_t > x|z\right)}{x^{-\Theta(z)}} = c(z) \text{ for } z \text{ such that } g^A(z) > 1 \tag{42}$$

where $c(z)$ is a constant.

Proof. See Kesten (1973). ■

The above lemma says that conditional on firm productivity z, the firm's size distribution has a Pareto tail. Hence the distribution $\Pr\left(\frac{k_t(j)}{\bar{K}_t} > x\right)$ is a mixture of different Pareto distributions.

Denoting $\Theta_{\min} = \min\{\Theta(z)\}$, we have

$$\begin{aligned}\frac{\Pr\left(k_t(j)/\bar{K}_t > x\right)}{x^{-\Theta_{\min}}} &= \int \frac{\Pr\left(k_t(j)/\bar{K}_t > x|z\right)}{x^{-\Theta_{\min}}} f(z)\,dz \\ &= \int_{g^A(z)\le 1} \frac{\Pr\left(k_t(j)/\bar{K}_t > x|z\right)}{x^{-\Theta_{\min}}} m(z)\,dz + \int_{g^A(z)>1} \frac{\Pr\left(k_t(j)/\bar{K}_t > x|z\right)}{x^{-\Theta_{\min}}} m(z)\,dz\end{aligned} \tag{43}$$

In the first part, when $x \to \infty$, $\lim_{x\to\infty} \frac{\Pr(S_t(z)>x|z)}{x^{-\Theta_{\min}}} = 0$ since firms enter with size ε with a boundary support, while growth rate is less than 1 for these firms. Their size will shrink. Hence when x is larger than the upper bound of ε support, $\Pr\left(\frac{k_t(j)}{\bar{K}_t} > x|z\right) = 0$. In the second part, if $z \in \arg\min\{\Theta(z)\}$, we have $\lim_{x\to\infty} \frac{\Pr\left(k_t(j)/\bar{K}_t>x|z\right)}{x^{-\Theta_{\min}}} = c(z)$ otherwise $\lim_{x\to\infty} \frac{\Pr(S_t(z)>x|z)}{x^{-\Theta_{\min}}} = 0$. Then we have

$$\lim_{x\to\infty} \frac{\Pr\left(k_t(j)/\bar{K}_t > x\right)}{x^{-\Theta_{\min}}} = \int_{z\in\arg\min\{\Theta(z)\},g(z)>1} c(z)\,m(z)\,dz \tag{44}$$

Lemma 9 *$\Theta(z)$ is decreasing on z. Hence $z_{\max} = \arg\min\{\Theta(z)\}$ and $\Theta_{\min} = \Theta(z_{\max})$*

Proof. Taking derivative in equation (41), we have

$$\frac{d\Theta}{dz} = -\frac{\lambda\Theta g^{A\Theta-1}\frac{dg^A}{dz} + (1-\lambda)\Theta g^{I\Theta-1}\frac{dg^I}{dz}}{\lambda g^A(z)^{\Theta(z)}\ln g^A + (1-\lambda)g^I(z)^{\Theta(z)}\ln g^I}$$

The numerator is greater than 0 since g^A and g^I are strictly increasing in z. Denote $F(\Theta) = \omega\lambda g^{A\Theta} + \omega(1-\lambda)g^{I\Theta} = 1$. The denominator is $\frac{dF}{d\Theta}$. Considering a small $\Delta > 0$, we can see $F(\Theta+\Delta) = \omega\lambda\left(g^{A\Theta}\right)^{\frac{\Theta+\Delta}{\Theta}} + \omega(1-\lambda)\left(g^{I\Theta}\right)^{\frac{\Theta+\Delta}{\Theta}}$. $\frac{\Theta+\Delta}{\Theta} > 1$, and hence from Jensen inequality, we have

$$1 = F(\Theta)^{\frac{\Theta+\Delta}{\Theta}} < F(\Theta+\Delta)$$

We have $\frac{dF}{d\Theta} > 0$. Thus $\frac{d\Theta}{dz} < 0$. ■

Then we can simplify equation (44) as $\lim_{x\to\infty} \frac{\Pr\left(k_t(j)/\bar{K}_t>x\right)}{x^{-\Theta_{\min}}} = c(z_{\max})\,m(z_{\max})$

8.2 Numerical Algorithm to Solve BGP

In equation (11), the first order condition of z_T yields

$$\alpha \frac{P_t'(z_T)}{P_t(z_T)} = \frac{\left(\frac{z_T}{z}\right)^{\psi}}{1-\varepsilon+\varepsilon\left(\frac{z_T}{z}\right)^{\psi}} \frac{1}{z_T} \tag{45}$$

From equation (18), taking the derivative with respect to z on both sides, we get

$$\begin{aligned} z_{T,t}'(z) &= \frac{\lambda \int_k \hat{k}_{T,t}(z)\, k d\Gamma_t(k,z)}{(1-\delta)\int_k k d\Gamma_t(k, z_T(z))} \\ &= \frac{\lambda \frac{m(z)}{1-\frac{\omega}{g_K}\left(\lambda g^A(z)+(1-\lambda) g^I(z)\right)}}{(1-\delta)\,\mu m(z_T(z))} \end{aligned} \tag{46}$$

The second equality uses the property that the growth rate of acquiring firms is time invariant on the BGP. The above two ODEs, (45) and (46), and the two boundary conditions determine the equilibrium. To solve the equilibrium, we follow the steps:

(1) Guess the interest rate r and $P(z_{\min})$

(2) Guess the firm growth rates $g^A(z)$ and $g^I(z)$, as well as the cutoff productivity z^*

(3) Solve the price function $P(z_T)$ and matching function $z_T(z)$ from two ODEs (45) and (46), with boundary conditions $P(z_{\min})$ and $z_T(z^*) = z_{\min}$

(4) Solve the firm problem (10) to problem (14). Update the firm growth rates, $g^A(z)$ and $g^I(z)$, as well as z^*. Go back to step 2 until convergence.

(5) From the free entry condition (15) and the boundary condition $z_T(z_{\max}) = z^*$, we can update the guess of r and $P(z_{\min})$.

(6) The measure of new entrant e, aggregate output Y and aggregate consumption C are determined by equations (17), (22) and (23).