The calm policymaker: an information-based theory of the price level*

John Barrdear†
Bank of England and the Centre for Macroeconomics

15th February 2016

Abstract

I present a theory of the price level that assumes passive fiscal policy and permits 'violations' of the Taylor principle, with determinacy instead achieved through a combination of dispersed information and complementarity in firms' price setting, in the style of Morris and Shin (2001). The model nests the New Keynesian framework, but extends beyond it to admit a unique, stable solution for the price level, and not just the rate of inflation, when the monetary authority responds by less than one-for-one to inflation, including when the interest rate remains pegged at its steady-state value. The model circumvents Cochrane's (2011) critique of the New Keynesian framework and, by permitting a weak response of monetary policy to inflation, captures the well-established fact of policy gradualism. The persistence of the price level following a shock is increasing in the central bank's marginal response to inflation. When the Taylor principle is satisfied, the price level exhibits a unit root, but when it is violated, the price level is stationary. Consequently, the systematic response of policy to the state of the economy is shown to lessen the on-impact effect of monetary shocks, but at a cost of increasing the duration of the subsequent slump.

JEL classification: D82, D84, E31, E52
Key words: Dispersed information, Imperfect Common Knowledge, New Keynesian, Indeterminacy, FTPL, Blanchard-Kahn, Taylor rules, Taylor principle, Global Games

1 Introduction

The activist policymaker lies at the heart of modern central banking practice, resting on the contention that nominal stability – typically, the achievement of an inflation target – requires not only that policymakers respond to the state of the economy, but that they do so with sufficient strength. When central banks use interest rates as their policy instrument, the idea of an interventionist policymaker finds expression as the Taylor principle: that when (expected) inflation increases, the nominal interest rate should be raised sufficiently to ensure that the real interest rate will rise, thus damping demand and lowering inflation.

This paper challenges this narrative, arguing that beyond the successful establishment of an appropriate steady state, which I interpret as the successful anchoring of long run inflation expectations,

*An early version of this paper, circulated under the title "Towards a New Keynesian Theory of the Price Level", benefited from the comments, gratefully received, of Wouter den Haan, Paul Levine, Michael McMahon, James Cloyne and Roland Meeks, together with various seminar participants. The views expressed in this paper are those of the author and not necessarily those of the Bank of England or its committees.

†Email: john.barrdear@bankofengland.co.uk

it is not strictly necessary for a central bank to respond to temporary deviations from that long run trend. Welfare may be improved by the application of a systematic response of monetary policy to the state of the economy, but it is not formally required: nominal stability can still be achieved even when the nominal interest rate remains pegged at its steady-state value.

The model presented below achieves determinacy of the price level, and not just the rate of inflation, without restrictions on the interest rate rule when price-setting firms face *Imperfect Common Knowledge* (ICK) in the style of Woodford (2003a) – that is, with firms observing idiosyncratically noisy signals of the underlying state of the economy while facing strategic complementarity in their price-setting – so long as firms never discover past values of the price level with certainty. The model starts from (and therefore nests) the standard New Keynesian framework, but also extends beyond it to accommodate flexible prices, non-neutrality of monetary policy from information frictions and the systematic 'violation' of the Taylor principle. With the policymaker's coefficient against inflation able to fall below unity, the model can also deliver a unique steady-state equilibrium, avoiding the possibility of a deflationary trap as an absorbing state, as explored by Benhabib, Schmitt-Grohe, and Uribe (2001).

The imperfect common knowledge literature joins those of *sticky information* and *rational inattention* in seeking to reintroduce the ideas of Lucas (1972) and Phelps (1984): that information frictions are crucial to explaining the dynamics of aggregate variables following a shock.[1] Woodford (2003a) invoked the central insight of Townsend (1983) – that with heterogenous information and strategic interaction, rational agents become interested in an infinite regress of higher-order beliefs – in order to demonstrate sluggish aggregate behaviour following a shock to nominal spending, despite price flexibility. Significant work has followed Woodford (2003a),[2] but the work most similar to the current paper is that of Nimark (2008, 2015). Nimark extends Woodford's approach to incorporate a standard demand side to the economy (an Euler equation and a Taylor rule), but grants firms perfect knowledge of the previous period's price level. This maintains the possibility of indeterminacy and so requires approaches like the Taylor principle to address it. The link between the dispersed information and global games literatures should also be emphasised. The uniqueness result obtained here, in particular, amounts to an application of the the results of Morris and Shin (1998, 2001).

The possibility of a unique and stable path for the price level in the absence of interventionist monetary policy has potential implications for ongoing debates in variety of strands of literature. I therefore briefly[3] describe those of perhaps most direct relevance before moving on to the bulk of the paper.

Inertial policymaking

A simple Taylor rule that satisfies the Taylor principle implies substantial volatility in interest rates, in striking contrast to the gradualism observed in the data. Indeed, as noted by Coibion and Gorodnichenko (2012), if the econometrician uses Federal Reserve Green Book forecasts to capture central bank expectations, the Taylor principle was not satisfied during the Greenspan era under 'naïve' specifications of the Taylor rule. Given the commonly assumed requirement for the Taylor principle, a debate has ensued over the origins of the observed inertial behaviour of monetary policy.

[1] The field of *sticky information* began with Mankiw and Reis (2002, 2006), while that of *rational inattention* dates to Sims (2003).

[2] In addition to Nimark's papers, see, for example, Lorenzoni (2009), Angeletos and La'O (2009, 2010), Graham and Wright (2010), Graham (2011a,b), Melosi (2014) and Kohlhas (2014).

[3] Too briefly. Each of these literatures is vast and I only touch lightly on their most significant points of contention.

One empirical option, and the one advocated by Coibion and Gorodnichenko (2012), is to suppose the presence of interest rate smoothing (i.e. lags of the interest rate) in the Taylor rule. But many policymakers do not recognise 'blind' interest rate smoothing as a fair description of their decision making process. Perhaps most famously, Rudebusch (2002, 2006) argues instead that the objects to which policymakers respond, such as their beliefs about the economy, or monetary shocks themselves, are generally slow moving. Woodford (1999) offers a model in which a concern for the private sector's expected path of future interest rates serves to stay the central banker's hand.

The model presented here naturally contributes to this debate by accommodating the possibility of far smaller coefficients in an interest rate rule than have previous been considered admissible. Relatedly, I also identify a new trade-off faced by policymakers in the presence of standard demand shocks. The systematic response of policy to the economy lessens the depth of recessions on impact, but also serves to lengthen the duration of any ensuing slump. With both effects strengthening as the magnitude of the systematic response increases, this may justify the choice of smaller responses.

The Great Inflation and the Great Moderation

The adoption of a muscular approach to monetary policy, independent of government interference, is commonly granted the lion's share of credit for the taming of inflation in the 1980s and the ensuing Great Moderation, against the alternative explanations of good luck and structural change in the economy (Bernanke, 2004). In their seminal work, for example, Clarida, Galí, and Gertler (2000) estimate a Taylor rule over multiple time periods and find that the coefficient against inflation was less than one prior to, and greater than one after, the 1979 appointment of Paul Volcker as chair of the Board of Governors of the Federal Reserve System. Against this, however, Orphanides (2003) argues that when estimated with data available in real time, Taylor rules for the Federal Reserve do not show significant variation over time and that, instead, the high inflation of the 1970s is better attributed to policymakers' poor estimates of the output gap. He argues that monetary policy in this period was not passive, but excessively active, being based on unwarranted confidence in estimates of productivity growth. Cochrane (2011) makes a more fundamental criticism, arguing that if the New Keynesian model is the true data generating process, then monetary shocks will be correlated with both right-hand variables in the Taylor rule and any potential instrument for the same. He further asks what is being estimated when the economy was, reputedly, indeterminate.

More recent work has estimated structural models that allow for these criticisms from Cochrane. Bianchi (2012, 2013) estimates structural Markov switching models that incorporate regime change between 'active' and 'passive' mixes of monetary and fiscal policy, in the style of Leeper (1991). He finds evidence that fiscal policy was active in the United States throughout the 1970s and '80s, and that monetary policy became active in 1979.[4] Bianchi's work limits attention to only three of the four possible regimes, however, expressly ruling out the possibility of both monetary and fiscal policy being passive as in his model this would imply an indeterminacy.

Since the framework outlined here describes a unique solution when both monetary and fiscal policy are 'passive', it permits two alternative approaches to examining the end of the Great Inflation. First, it allows for a maintained assumption of passive fiscal policy throughout, as implied in standard New Keynesian models, without having to be concerned about what is being measured when the coefficient against inflation falls below one. Second, it allows for a more complete evaluation of the

[4] Bianchi also finds that US fiscal policy switched to passive somewhere around 1990.

possible interaction between monetary and fiscal policy by letting the econometrician grant her model the freedom to explore greater regions of the parameter space.

Determinacy and equilibrium selection

It is well known that interest rate rules can produce indeterminacy in forward-looking models with rational expectations and full information (Sargent and Wallace, 1975). In models that retain Ricardian equivalence, the standard response has been to adopt a mathematical variant of the Taylor principle by imposing a lower bound on the policymaker's marginal response to deviations of inflation from target.[5] This ensures that the eigenvalue restriction of Blanchard and Kahn (1980) is satisfied and, together with a transversality condition, serves to select the equilibrium in which, absent further fundamental shocks to the economy, agents exhibit perfect foresight.

Cochrane (2011) questions the use of this solution method for New Keynesian models, noting that (i) while a transversality condition is reasonable for real variables, imposing one for nominal variables is much less defensible, as hyperinflations do happen (and, in any event, a model that axiomatically imposes a no-bubble condition then becomes unsuitable for determining the conditions under which dynamic instability emerges); and (ii) by being set in order to deliver an eigenvalue greater than unity, the Taylor principle amounts to an *ex ante* commitment to "blow up the economy" (a commitment to deliver explosive inflation, or deflation, if agents ever make a forecast error without a new structural shock), which cannot be credible if policymakers retain *ex post* options to arrest high inflation without ruling out the deviation in the first place. He concludes that New Keynesian models, as typically formulated, do not have a well-determined equilibrium.

A variety of alternative equilibrium selection criteria have been proposed that avoid some of these criticisms,[6] but to date none have been established that address all of them. In a pair of recent papers, Cochrane (2015a,b) has emphasised that one admissible equilibrium is that which is "backward stable" (i.e. non-explosive as t goes backward in time). He notes that such an equilibrium selection would support the 'neo-Fischerian' viewpoint, in which an interest rate pegged below its (original) steady state value forever will not cause explosive inflation but that, instead, inflation will fall to accommodate the change. García-Schmidt and Woodford (2015) describe this as a paradox of perfect foresight and propose a deviation from rational expectations which avoids it. Kocherlakota (2016) argues that it is the infinite horizon in such models that drives such counter-intuitive results.

Under imperfect common knowledge, by contrast, price-setting firms may remain rational and consider the infinite horizon, while still producing a unique solution without restrictions on the central bank's interest rate rule, so long as they retain uncertainty about current and past values of their decision problem.[7] In the limit, as the idiosyncratic noise in firms' signals approaches zero, the solution converges to the Minimum State Variable solution of McCallum (1999).[8]

Remainder of the paper

[5]See Woodford (2003b). It bears noting that the mathematical and English language versions of the Taylor principle do not always perfectly coincide. In the event of a demand shock, moving both output and inflation together, a Taylor rule with a coefficient less than one against inflation can still raise the real rate on impact if the coefficient on output is sufficiently high.

[6]Perhaps most significantly, the E-stability criterion of Evans and Honkapohja (2001) does not require the imposition of a transversality condition or that agents in the model have knowledge of the policymaker's structural parameters, although issues with their identifiability may remain for the econometrician.

[7]This is, in effect, a restatement of the uniqueness result of Morris and Shin (1998, 2001).

[8]This is conditional on firms' signal vectors being instantly invertible under full information.

To illustrate the basic points of the paper, section 2 begins by presenting a highly simplified variant of the main model. Section 3 then formalises the problem of solving forward-looking models with rational expectations before presenting generalised model of an economy under incomplete common knowledge and characterising its solution, together with conditions for uniqueness and stability. Section 4 then presents the main model of the paper and provides sample simulations. Section 5 concludes.

2 A simple model

Before presenting a full version of the model, I here present a simplified version that illustrates its main points. It is a variation of the canonical New Keynesian model, log-linearised around a common-knowledge steady state, where firms' full information and price stickiness has been replaced with incomplete information and price flexibility. The representative household and central bank both retain full information.

The household Euler equation is standard:

$$y_t = E_t^{\Omega}\left[y_{t+1}\right] - \sigma\left(i_t - E_t^{\Omega}\left[p_{t+1}\right] + p_t\right) \tag{1}$$

where σ is the elasticity of intertemporal substitution and $E_t^{\Omega}[\cdot] = E[\cdot|\Omega_t]$ is the mathematical expectation conditioned on all information in existence in period t.

The central bank decision rule responds not at all to inflation and with a coefficient on expected future output that is contrived in order to remove its effect in the HH Euler equation. There is also an exogenous monetary shock that follows an AR(1) process:

$$i_t = \frac{1}{\sigma}E_t^{\Omega}\left[y_{t+1}\right] + x_t \tag{2a}$$

$$x_t = \rho x_{t-1} + u_t \quad \text{where } \rho \in (0,1) \text{ and } u_t \sim N\left(0, \sigma_u^2\right) \tag{2b}$$

Equation (2a) is clearly not a realistic depiction of actual central bank behaviour. It is imposed here only to ease exposition; more typical Taylor-type rules are included in the full model below.

Individual firms have complete flexibility in the setting of their prices, but limited information on which to base any such decision. In log-linear form, firm i's optimal price is given by

$$p_t(i) = E_t(i)\left[p_t\right] + \kappa E_t(i)\left[y_t\right] \tag{3a}$$

where κ is a function of the various elasticities of intertemporal substitution, demand, labour supply and marginal cost; and $E_t(i)[\cdot] \equiv E[\cdot|\mathcal{I}_t(i)]$ is firm i's expectation based on an incomplete information set: $\mathcal{I}_t(i) \subset \Omega_t$. The aggregate price level is then just the average of individual prices:

$$p_t = \overline{E}_t\left[p_t\right] + \kappa\overline{E}_t\left[y_t\right] \tag{3b}$$

where $\overline{E}_t[\cdot] = \int E_t(i)[\cdot]\,di$ is the average firm's expectation.

Substituting the Euler equation and the CB decision rule into the average pricing rule, and making use of the law of iterated expectations (in particular, that $\overline{E}_t\left[E_t^{\Omega}[\cdot]\right] = \overline{E}_t[\cdot]$) then produces the following competitive equilibrium condition:

$$p_t = (1-\sigma\kappa)\,\overline{E}_t\left[p_t\right] + \sigma\kappa\,\overline{E}_t\left[p_{t+1}\right] - \sigma\kappa\,\overline{E}_t\left[x_t\right] \tag{4}$$

where I assume that $\sigma\kappa \in (0,1)$. Price-setting firms therefore face a co-ordination problem (a global game), with the equilibrium degree of strategic complementarity given by $(1-\sigma\kappa)$. Note that with flexible prices and the central bank not responding to current inflation, there is no mention of p_{t-1} in the equilibrium condition. The solution is found in terms of p_t rather than in terms of $\pi_t \equiv p_t - p_{t-1}$.

To solve the model, we must consider how firms' expectations are formed. I describe solutions under (i) *full information*, when all firms observe x_t; (ii) *incomplete and common information*, when all firms observe the same noisy signal about x_t; and (iii) *dispersed information*, when each firm observes its own conditionally independent signal. I suppose that the signal noise shocks — v_t and $v_t(i)$ — are all transitory and independent, and distributed $N(0, \sigma_v^2)$. Note that models (ii) and (iii) each nest the full information case as $\sigma_v^2 \to 0$ and that as information becomes common or complete, the equilibrium condition correspondingly simplifies.

Model	Information is ...	Firm i's signal	Competitive equilibrium condition
(i)	Full	$s_t(i) = x_t$	$p_t = \overline{E}_t[p_{t+1}] - x_t$
(ii)	Incomplete and common	$s_t(i) = x_t + v_t$	$p_t = \overline{E}_t[p_{t+1}] - \overline{E}_t[x_t]$
(iii)	Dispersed	$s_t(i) = x_t + v_t(i)$	$p_t = (1-\sigma\kappa)\overline{E}_t[p_t] + \sigma\kappa\overline{E}_t[p_{t+1}] - \sigma\kappa\overline{E}_t[x_t]$

Table 1: The basic model under different information regimes.

2.1 Solving the model under full information

The Minimum State Variable (MSV) solution under full information is of the form $p_t = ax_t$, so that $E_t[p_{t+1}] = a\rho x_t$. Substituting this into the equilibrium condition, it is readily apparent that this is indeed a solution, with $a = -1/(1-\rho)$.

It is not the only solution, however. For example, a solution of the form $p_t = bx_t + cp_{t-1}$ is also valid. In this case, it follows that $E_t[p_{t+1}] = b\rho x_t + c(bx_t + cp_{t-1})$ and, hence, that $b = 1/\rho$ and $c = 1$. The problem is therefore *indeterminate* under full information, with little (or, arguably, nothing) to choose between the admissible solutions.[9]

2.2 Solving the model under incomplete and common information

Under incomplete and common information[10] the model has a solution of the form

$$p_t = -\frac{1}{1-\rho}\overline{E}_t[x_t] \tag{5}$$

which corresponds to the full-information MSV solution. With incomplete information, it is also necessary to derive firms' estimator for x_t. Since the model is linear and all shocks are Gaussian, the optimal estimator is a Kalman filter, in which expectations update recursively:

$$E_t(i)[x_t] = E_{t-1}(i)[x_t] + k_t(i)\{s_t(i) - E_{t-1}(i)[s_t(i)]\} \tag{6}$$

where $k_t(i)$ is a projection matrix for the orthogonalised new information in the period-t signal. Since x_t is stationary and all agents' problems are symmetric, this is common and converges to a constant

[9] There are an infinite number of valid solutions to this model, featuring various lags of x_t and p_t.

[10] A setting explored, for example, by Currie, Levine, and Pearlman (1986).

value $k_t(i) = k_t \to k = q/(q+\sigma_v^2)$ where q is the time-invariant variance of firms' prior expectation errors. Substituting in the law of motion for the shock and the signal then gives

$$\overline{E}_t[x_t] = (1-k)\rho\overline{E}_{t-1}[x_{t-1}] + k\{x_t + v_t\} \tag{7}$$

Substituting (7) backwards and combining the result with (5) arrives at

$$p_t = -\frac{1}{1-\rho}\left(k\sum_{s=0}^{\infty}((1-k)\rho)^s\{x_{t-s}+v_{t-s}\}\right) \tag{8}$$

which simplifies to the full-information MSV solution as $\sigma_v^2 \to 0$ (so that $k \to 1$).

Regardless of the Kalman filter, however, the model with incomplete and common information remains indeterminate, as (5) is not the only reduced-form solution to the equilibrium condition. Equivalently to the full-information case, a solution also exists with $p_t = p_{t-1} + \frac{1}{\rho}\overline{E}_t[x_t]$, so that

$$p_t = p_{t-1} + \frac{1}{\rho}\left(k\sum_{s=0}^{\infty}((1-k)\rho)^s\{x_{t-s}+v_{t-s}\}\right) \tag{9}$$

is equally valid. Adding incomplete and common information certainly introduces richer dynamics to the model (since past signals affect current behaviour), but does not address the question of determinacy. The same equilibrium-selection criteria that are used in the full-information case are therefore still necessary and any criticisms that may be made of them under full information apply equally well when information is incomplete and common.

2.3 Solving the model under dispersed information

Under dispersed (that is, incomplete and heterogeneous) information, it becomes necessary to consider the hierarchy of firms' (average) expectations. Let the 0-th order expectation of a variable be the variable itself; the 1-st order expectation be firms' average expectation about the variable; the 2-nd order expectation be firms' average expectation about the 1st-order expectation, and so on:

$$x_t^{(0)} \equiv x_t \tag{10a}$$

$$x_t^{(k)} \equiv \overline{E}_t\left[x_t^{(k-1)}\right] \quad \forall k \geq 1 \tag{10b}$$

Further define the *full state* (X_t) to be the complete, infinite-dimension hierarchy of expectations regarding x_t, including the variable itself:

$$X_t \equiv \begin{bmatrix} x_t \\ \overline{E}_t[X_t] \end{bmatrix} = \begin{bmatrix} x_t^{(0)} & x_t^{(1)} & x_t^{(2)} & \cdots \end{bmatrix}' \tag{11}$$

A solution is then a reduced-form expression for p_t as a function of X_t (potentially with lags) and a law of motion for X_t that describes firms' average belief formation.

Proposition 1. *For the simple model under dispersed information (that is, with $s_t(i) = x_t + v_t(i)$),*

a) the solution is of the form:

$$p_t = \gamma' X_t \tag{12a}$$

$$X_t = FX_{t-1} + Gu_t \tag{12b}$$

b) the solution (12) is unique; and

c) the solution (12) converges to the MSV solution as firms obtain full information ($\sigma_v^2 \to 0$).

Proposition 1 is a special case of proposition 2 below. I nevertheless provide a full derivation of the solution in the appendix and sketch the most important points here. For part (c) of the proposition, note that it will be the case that $\boldsymbol{\gamma}$ is of the form $\boldsymbol{\gamma} = -\frac{1}{1-\rho}(1-\phi)\begin{bmatrix}0 & 1 & \phi & \phi^2 & \cdots\end{bmatrix}'$ with $\phi \in (0,1)$. Since $X_t \to x_t \begin{bmatrix}1 & 1 & \cdots\end{bmatrix}'$ as $\sigma_v^2 \to 0$, it trivially follows that the dispersed information solution converges to the MSV solution under full information.

The reduced-form expression for p_t

To solve for $\boldsymbol{\gamma}$, substitute (12) into (4) to obtain

$$\boldsymbol{\gamma}' X_t = (1-\sigma\kappa)\,\overline{E}_t\left[\boldsymbol{\gamma}' X_t\right] + \sigma\kappa\,\overline{E}_t\left[\boldsymbol{\gamma}'\left(F X_t + G u_{t+1}\right)\right] - \sigma\kappa\,\overline{E}_t\left[x_t\right] \tag{13}$$

Define S and T as selection matrices such that $SX_t = x_t$ and $TX_t = \overline{E}_t\left[X_t\right]$. Then after some straightforward manipulation this becomes

$$\boldsymbol{\gamma}' = -\sigma\kappa S T\left(I - HT\right)^{-1} \qquad \text{where } H = (1-\sigma\kappa)\,I + \sigma\kappa F \tag{14}$$

Since T is just a shift operator, then the inverse will exist so long as the spectral radius of H is less than one. As I note below, the largest eigenvalue of F is ρ. An upper bound on the spectral radius of H is therefore $1-\sigma\kappa(1-\rho)$, which is indeed less than one.

The law of motion for X_t

With a state-space representation and Gaussian shocks, the optimal estimator is a Kalman filter. In the appendix I show that the hierarchy of firms' expectations will obey the following law of motion, with F lower-triangular (the logic of this identical to that of Woodford, 2003a):

$$X_t = F X_{t-1} + G u_t \tag{15a}$$

$$F = \rho\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ k_1 & (1-k_1) & 0 & 0 & \cdots \\ k_2 & (k_1-k_2) & (1-k_1) & 0 & \cdots \\ k_3 & (k_2-k_3) & (k_1-k_2) & (1-k_1) & \\ \vdots & \vdots & \vdots & & \ddots \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ k_1 \\ k_2 \\ k_3 \\ \vdots \end{bmatrix} \tag{15b}$$

where k_q is the kalman gain applied in firms' q-th order expectation. The eigenvalues of F are then just ρ and $\rho(1-k_1)$, with the latter repeated *ad infinitum*. Since $k_1 \in (0,1)$, the hierarchy of beliefs will therefore be stable. Note that the evolution and stability of agents' expectations are independent of the degree of strategic complementarity $(1-\sigma\kappa)$. The derivation of the kalman gains is provided in the appendix, although it bears noting that:

- They are functions of only ρ and σ_v^2/σ_u^2
- $1 > k_1 > k_2 > \cdots > 0$
- $\lim_{\sigma_v^2 \to 0} k_q = 1\ \forall q$

Uniqueness

While a full analysis is presented in the appendix, I here consider two important cases: the inclusion of a lagged price level and the addition of a pure sunspot shock.

Ruling out a solution with a lagged price level

Consider a candidate reduced-form solution of the form

$$p_t = \boldsymbol{\delta}' X_t + \phi p_{t-1} \tag{16}$$

that includes a term in the lagged price (it should be clear that without a change in firms' signals, there can be no change in the law of motion for the hierarchy of expectations). Substituting (16) into the CE condition (4) and rearranging gives

$$p_t = \left\{ (1 - \sigma\kappa)\, \boldsymbol{\delta}' + \sigma\kappa \boldsymbol{\delta}' F - \sigma\kappa S \right\} T X_t + \phi\, (1 - \sigma\kappa + \sigma\kappa\phi)\, \overline{E}_t \left[p_{t-1} \right] \tag{17}$$

This will only be a solution if $\overline{E}_t \left[p_{t-1} \right] = p_{t-1}$, which cannot be true given the recursive nature of firms' belief formation: firms can never be perfectly certain about anything that is a function of the underlying state.

Ruling out a solution with a pure sunspot shock

Let z_t be a sunspot shock. That is, z_t is an unforecastable, mean-zero stochastic process, orthogonal to x_t and $v_t\,(i)$, that is added to all firms' signal vectors:

$$\boldsymbol{s}_t\,(i) = \begin{bmatrix} x_t + v_t\,(i) \\ z_t \end{bmatrix} \tag{18}$$

The addition of z_t cannot change firms' beliefs about x_t, as the two are orthogonal (by construction) and so the optimal weight to place on the new signal in the Kalman filter will be zero. Since firms are rational, the law of motion for X_t will therefore also be independent of z_t.

But common knowledge of z_t may still provide a co-ordination point for a different reduced-form solution for p_t. That is, there may potentially exist a reduced-form solution of the form

$$p_t = \boldsymbol{\delta}' X_t + a z_t \tag{19}$$

Substituting this into the equilibrium condition (4) and rearranging gives

$$p_t = \left\{ (1 - \sigma\kappa)\, \boldsymbol{\delta}' + \sigma\kappa \boldsymbol{\delta}' F - \kappa S \right\} T X_t + (1 - \sigma\kappa)\, a z_t + \sigma\kappa a \overline{E}_t \left[z_{t+1} \right] \tag{20}$$

The candidate (19) can therefore only be a solution if

$$\boldsymbol{\delta}' = \left\{ (1 - \sigma\kappa)\, \boldsymbol{\delta}' + \sigma\kappa \boldsymbol{\delta}' F - \kappa S \right\} T \tag{21a}$$

$$\text{and} \quad a z_t = (1 - \sigma\kappa)\, a z_t + \sigma\kappa a \overline{E}_t \left[z_{t+1} \right] \tag{21b}$$

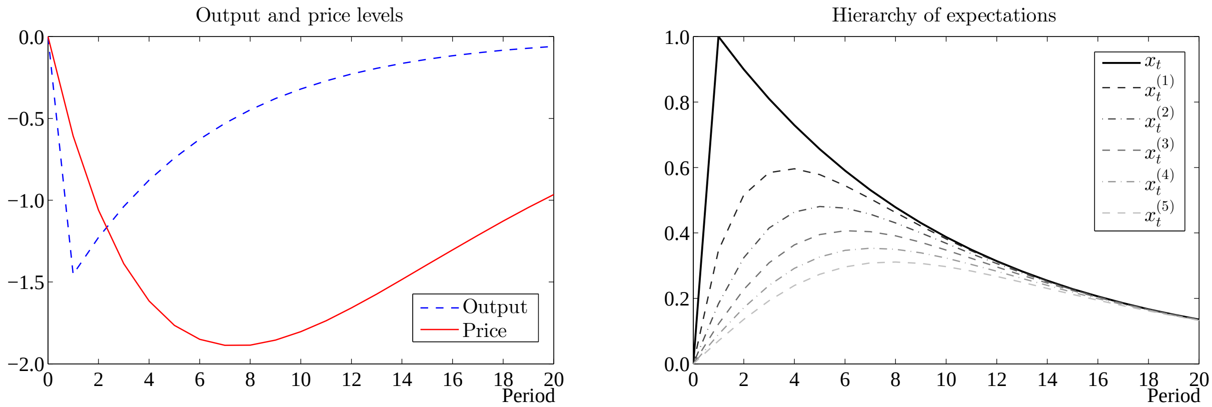
Of these, condition (21a) is the same as the MSV-equivalent reduced-form solution for $\boldsymbol{\gamma}$ shown above in equation (14). For condition (21b), subtracting $a z_t$ from both sides and rearranging gives

$$\overline{E}_t \left[z_{t+1} \right] = z_t \tag{22}$$

which says that (19) can only be a solution if z_t is a random walk. If z_t is transitory, or merely persistent, then (19) cannot be a solution. But if z_t is a random walk, it can only *remain* mean zero in the trivial case that $z_t = 0\ \forall t$, which is to say that there is no sunspot shock.

2.4 Simulations

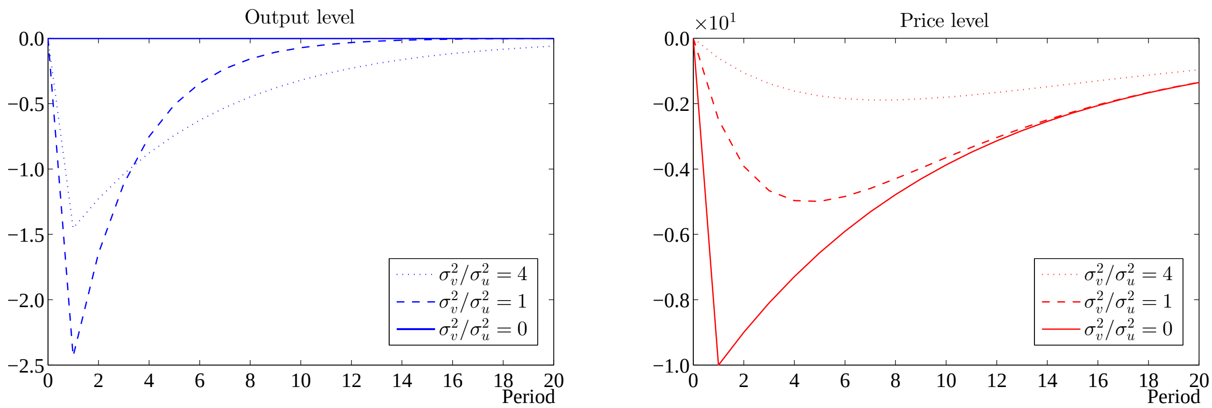
Figure 1 plots impulse responses of the simple model following a 1 s.d. monetary shock for a baseline calibration of $\rho = 0.9$, $\sigma = 1$ (log utility), $\kappa = 0.4$ and $\sigma_v^2/\sigma_u^2 = 4$. Firms' incomplete information causes their expectations of the shock to adjust only slowly. With strategic complementarity, firms must consider the average expectation of other firms, and with heterogeneous information this is not common knowledge. A hierarchy of expectations ensues, with each more sluggish than the last. Given their sluggish beliefs, firms are slow to adjust their prices and output therefore persistently deviates from trend, despite the complete flexibility of prices: money is non-neutral.



Note: $x_t^{(k)} \equiv \overline{E}_t\left[x_t^{(k-1)}\right]$ $\forall k \geq 1$. Parameters are: $\left\{\rho\,,\,\sigma\,,\,\kappa\,,\,\sigma_v^2/\sigma_u^2\right\} = \{0.9\,,\,1\,,\,0.4\,,\,4\}$.

Figure 1: Impulse responses following a monetary shock in the simple model

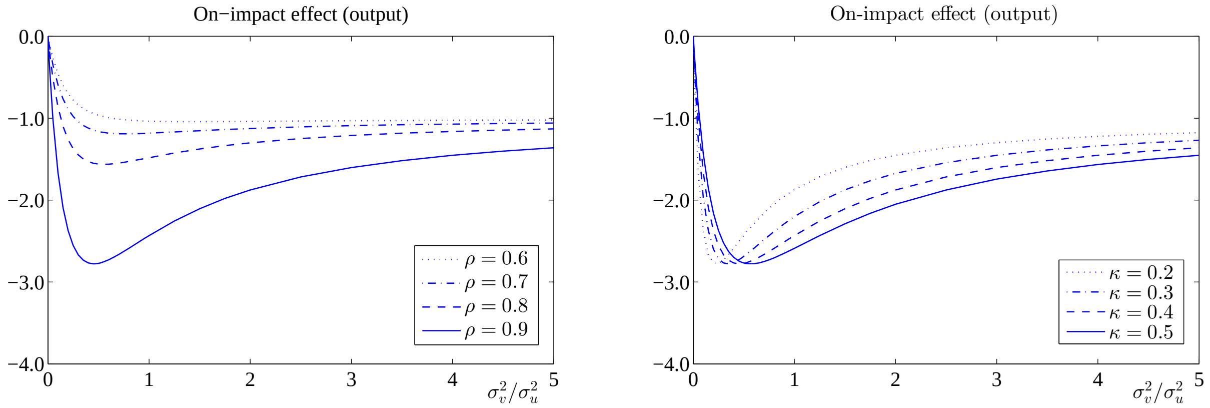
Figure 2 next plots various impulse responses for output and prices under different degrees of firm uncertainty. When $\sigma_v^2 = 0$, firms observe the shock perfectly. Since prices are flexible, output therefore doesn't change and everything happens through the price level (the MSV solution). As the noise in firms' signals increases, output moves and the persistence of both price and output deviations correspondingly rises (since the kalman gains fall). Notice, however, that as the relative variance increases, the on-impact effect on output goes from 0 to -2.4 and back to -1.5.



Note: When $\sigma_v^2/\sigma_u^2 = 0$, firms have full information, so that output does not change, but the price level is given by $p_t = -\frac{1}{1-\rho}x_t$ (the MSV solution). Other parameters are: $\{\rho\,,\,\sigma\,,\,\kappa\} = \{0.9\,,\,1\,,\,0.4\}$.

Figure 2: Varying relative variance in the simple model

Figure 3 explores this point further, plotting the *on-impact* effect on output for various levels of relative variance. Regardless of other parameter choices, when firms have access to full information ($\sigma_v^2/\sigma_u^2 = 0$) there is zero effect on output from a monetary shock. For very large values of σ_v^2/σ_u^2, the on-impact effect is close to -1. For intermediate amounts of noise, however, there is a noticeable hump in the response, with the size of the hump increasing in the persistence of the monetary shock.



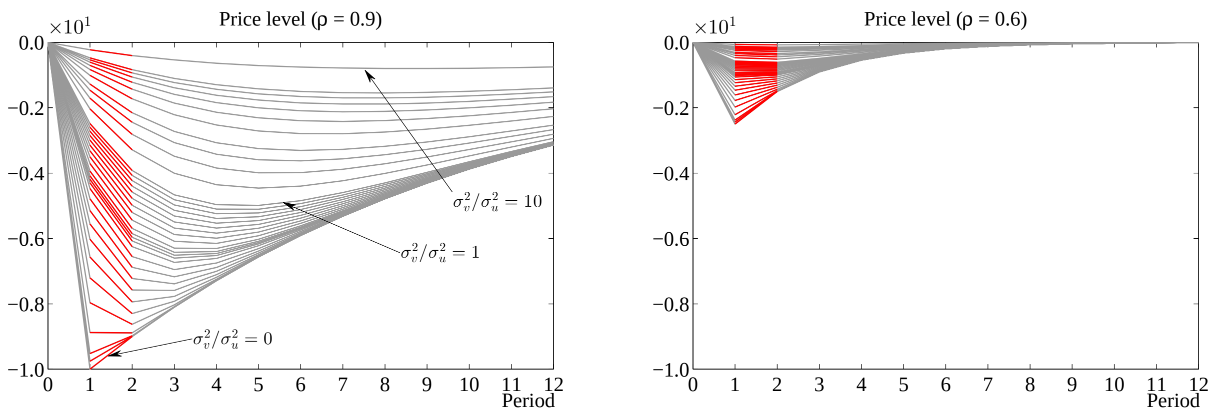
Note: Unless otherwise specified, other parameters are: $\{\rho\,,\,\sigma\,,\,\kappa\} = \{0.9\,,\,1\,,\,0.4\}$.

Figure 3: On-impact effect of monetary shocks on output

This hump-shaped variation follows directly from the increasingly sluggish price responses that occur as the amount of noise increases. To understand this, note that for a 1 s.d. shock in period 1, the value of output on impact (in period 1) is here simply given by

$$y_1 = -1 + p_2 - p_1 \tag{23}$$

The on-impact effect on output is just a function of the period 2 rate of inflation. Figure 4 plots IRFs for the price level following a monetary shock for a wide range of relative variances, emphasising the change from period 1 to period 2. For very low levels of noise, firms' information is sufficient to almost completely adjust their prices on impact. Then, on average, prices *rise* from period 1 to period 2 as the shock starts to dissipate. As the noise increases, however, prices initially fall for several periods so the trough in the price level occurs later. For moderate amounts of noise, the second-period fall is large as firms' beliefs are able to "catch up" with the true shock. But for large amounts of noise, firms' learning is so slow that prices adjust very little at all. The same process works for less persistent shocks, but the magnitudes are smaller as everything is multiplied by $\frac{1}{1-\rho}$.



Note: The price change from period 1 to period 2 determines the level of output in period 1.

Figure 4: Price level IRFs following a monetary shock for various values of σ_v^2/σ_u^2

3 Solving models with Imperfect Common Knowledge

3.1 Solving rational, forward-looking models under full information

Let $\boldsymbol{z}_t$ be the aggregate endogenous state of the economy in period t and $\boldsymbol{x}_t$ be a vector of mean zero disturbances. Most macroeconomic models, once linearised, are presented in the following form:

$$\boldsymbol{z}_t = \Phi E_t \left[\boldsymbol{z}_{t+1}\right] + \Theta \boldsymbol{x}_t \tag{24}$$

where Φ and Θ are matricies of parameters and $E_t\left[\cdot\right] \equiv E\left[\cdot|\Omega_t\right]$ is the mathematical expectation conditional on all information available in period t. Such models are indeterminate, however: we cannot use equation (24) to determine the exact values $\boldsymbol{z}_t$ will take for a given sequence of exogenous shocks because the process by which expectations are formed has not been specified. Imposing rational expectations in the sense of Muth (1961) – that is, supposing that agents (i) know the functional form and parameters of (24); and (ii) do not make systematic errors – then gives

$$\boldsymbol{z}_t = \Phi \boldsymbol{z}_{t+1} + \Theta \boldsymbol{x}_t - \Phi \boldsymbol{\delta}_{z,t+1} \text{ with } E\left[\boldsymbol{\delta}_{z,t+1}\right] = 0 \tag{25}$$

where $\boldsymbol{\delta}_{z,t+1} \equiv \boldsymbol{z}_{t+1} - E_t\left[\boldsymbol{z}_{t+1}\right]$ is the one-step-ahead forecast error. Although the set of admissible solutions is greatly reduced, the model remains indeterminate: *any* mean-zero process for $\boldsymbol{\delta}_{z,t+1}$ will represent a rational expectations equilibrium and it remains necessary to select between them. The most common approach to closing such models is to impose *saddle-path stability*, a process – most famously associated with Blanchard and Kahn (1980)[11] – that imposes two further assumptions:

Assumption 1. *Those eigenvalues of Φ^{-1} that relate to forward-looking variables lie outside the unit circle, making the system explosive along those dimensions.*

Assumption 2. *A no-bubble constraint in expectation:* $\lim_{s\to\infty} E_t\left[\Phi^s \boldsymbol{z}_{t+s}\right] = 0 \; \forall t$.

Assumption 1 supposes that the system is dynamically explosive, while assumption 2 supposes that agents know that it cannot be explosive in the ultimate long-run. Consequently, there remains only one solution: $\boldsymbol{\delta}_{z,t+1} = 0 \; \forall t$, conditional on $\boldsymbol{x}_t$. Agents are thus granted (conditionally) perfect foresight and, following a shock, forward-looking variables jump immediately to the equilibrium saddle path. Backward-looking variables adjust slowly and, each period, the forward-looking variables jump to remain on the saddle path until the steady state is achieved. Subsequent work has developed or improved various methodologies for finding the implied solution, but all of them retain these two defining assumptions.

When all eigenvalues of Φ^{-1} lie within the unit circle (contra assumption 1), the system is dynamically stable, but there remain a continuum of valid, rational expectation solutions to choose between. Some branches of literature have sought alternative methods of pruning the set of admissible solutions (for example, McCallum (2012) argues that plausible solutions must be continuous as coefficients against forward-looking variables approach zero), while others have embraced the multiplicity of valid solutions and sought to describe the process of selecting between them. In this latter strand, work on sunspot shocks, dating to Cass and Shell (1983), has supposed that the selection is made by an extrinsic process (i.e. one external to the system), while other work, such as the belief function formulation of Farmer (2013a,b), has sought to describe an endogenous process for selection.

[11] Subsequent work in developing algorithms for finding this solution has included Uhlig (1997), King and Watson (1998), Klein (2000) and Sims (2002).

3.2 Solving rational, forward-looking models under imperfect common knowledge

As I demonstrate below, these matters need not arise in models with imperfect competition, as a simple assumption that agents do not discover the past with certainty is sufficient to ensure a unique solution. To begin, note that a generalisation of (24) to a setting of incomplete common knowledge is the following, where I explicitly separate out the possibility of backward-looking variables for clarity:

Definition 1. *Let $\boldsymbol{z}_t$ be the $(m \times 1)$ vector of endogenous aggregate variables necessary to describe an economy's equilibrium conditions in a given period; $\boldsymbol{x}_t$ be the $(n \times 1)$ vector of aggregate exogenous variables; $\boldsymbol{s}_t(i)$ be the $(q \times 1)$ vector of signals observed in period t by agent $i \in [0,1]$; and $\boldsymbol{u}_t$ and $\boldsymbol{v}_t(i)$ be aggregate and idiosyncratic vectors of transitory, mean-zero and jointly-orthogonal innovations.*[12]

A linear economy with ***imperfect common knowledge*** *is one that evolves as*

$$\boldsymbol{z}_t = A_1 \overline{E}_t[\boldsymbol{z}_{t-1}] + A_2 \overline{E}_t[\boldsymbol{z}_t] + A_3 \overline{E}_t[\boldsymbol{z}_{t+1}] + B\overline{E}_t[\boldsymbol{x}_t] + C\boldsymbol{z}_{t-1} + D\boldsymbol{x}_t \tag{26a}$$

$$\boldsymbol{x}_t = P\boldsymbol{x}_{t-1} + \boldsymbol{u}_t \tag{26b}$$

$$\overline{E}_t[\cdot] = \int_0^1 E[\cdot|\mathcal{I}_t(i)]\, di \tag{26c}$$

$$\mathcal{I}_t(i) = \{\mathcal{I}_{t-1}(i), \boldsymbol{s}_t(i)\} \tag{26d}$$

$$\boldsymbol{s}_t(i) = M\boldsymbol{z}_{t-1} + N\boldsymbol{x}_t + O\boldsymbol{v}_t(i) \tag{26e}$$

where equation (26a) embodies all competitive equilibrium conditions, combining agents' individually optimised decision rules, market-clearing conditions and economy-wide resource constraints.

Note that the model nests the basic scenario of (24) by assuming that the signal equation (26e) is fully invertible (e.g. $M = 0$ and $N = I$) and taking the variance of $\boldsymbol{v}_t(i)$ to zero.[13] Note, too, that either (or both) of $\boldsymbol{x}_t$ and $\boldsymbol{v}_t(i)$ might include both 'fundamental' shocks (such as productivity shocks) and 'noise' shocks (such as measurement error).

Next, given the inclusion of heterogeneous information, so that the average expectation is distinct from agents' individual expectations, I also make use of the consequent hierarchy of higher-order expectations. The internally recursive definition listed here is used to ease with the solution below.

Definition 2. *In a linear economy with imperfect common knowledge, the* ***hierarchy of higher-order expectations*** *regarding $\boldsymbol{x}_t$ and $\boldsymbol{z}_{t-1}$ is defined as*

$$X_t \equiv \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{z}_{t-1} \\ \overline{E}_t[X_t] \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_{t|t}^{(0)\prime} & \boldsymbol{z}_{t-1|t}^{(0)\prime} & \boldsymbol{x}_{t|t}^{(1)\prime} & \boldsymbol{z}_{t-1|t}^{(1)\prime} & \boldsymbol{x}_{t|t}^{(2)\prime} & \boldsymbol{z}_{t-1|t}^{(2)\prime} & \cdots \end{bmatrix}' \tag{27a}$$

where

$$\boldsymbol{x}_{t|t}^{(0)} \equiv \boldsymbol{x}_t \qquad \boldsymbol{z}_{t-1|t}^{(0)} \equiv \boldsymbol{z}_{t-1} \tag{27b}$$

$$\boldsymbol{x}_{t|t}^{(k)} \equiv \overline{E}_t\left[\boldsymbol{x}_{t|t}^{(k-1)}\right] \ \forall k \geq 1 \qquad \boldsymbol{z}_{t-1|t}^{(k)} \equiv \overline{E}_t\left[\boldsymbol{z}_{t-1|t}^{(k-1)}\right] \ \forall k \geq 1 \tag{27c}$$

Further, the matrices S_x, S_z and T are defined to select $\boldsymbol{x}_t$, $\boldsymbol{z}_{t-1}$ and $\overline{E}_t[X_t]$ from X_t respectively (that is, such that $S_x X_t = \boldsymbol{x}_t$, $S_z X_t = \boldsymbol{z}_{t-1}$ and $TX_t = \overline{E}_t[X_t]$).

[12] So that $VCov(\boldsymbol{u}_t) = \Sigma_{\boldsymbol{u}}$, $VCov(\boldsymbol{v}_t(i)) = \Sigma_{\boldsymbol{v}}\ \forall i$, $Cov(\boldsymbol{u}_t, \boldsymbol{v}_t(i)) = \boldsymbol{0}\ \forall i$ and $Cov(\boldsymbol{v}_t(i), \boldsymbol{v}_t(j)) = \boldsymbol{0}\ \forall i \neq j$.

[13] Define $\widetilde{\boldsymbol{z}}_t \equiv \begin{bmatrix} \boldsymbol{z}_t' & \boldsymbol{z}_{t-1}' \end{bmatrix}'$. Then under full information, (26a) reduces to $\widetilde{\boldsymbol{z}}_t = \Phi E_t[\widetilde{\boldsymbol{z}}_{t+1}] + \Theta \boldsymbol{x}_t$ where $\Phi = \begin{bmatrix} A_3^{-1}(I - A_2) & -A_3^{-1}(A_1 + B) \\ I & 0 \end{bmatrix}^{-1}$ and $\Theta = \Phi \begin{bmatrix} A_3^{-1}(C + D) \\ 0 \end{bmatrix}$, subject to the respective inverses existing.

Note that the matrix T amounts to a *shift* operator. Post-multiplying a matrix with T shifts its elements to the right by $m+n$ places, while pre-multiplying a matrix by T' will shift the elements down by $m+n$ places. Finally, it remains only to define rationality in this context.

Definition 3. *A **rational** economy with heterogeneous information is one in which*

(i) all agents know the structure and parameters of the economy and this is common knowledge;

(ii) agents' expectations are unbiased; and

(iii) agents' expectations are optimal in the sense that they minimise their mean square errors.

This is a more restrictive definition of rationality than that of Muth (1961), which is normally applied in macroeconomic models and only formally imposes (i) and (ii).[14] In a linearised setting, the optimal mechanism for signal extraction is a Kalman filter,[15] in which agent i's period-t expectation about an unknown vector (X_t) is given by:

$$E_t(i)[X_t] = E_{t-1}(i)[X_t] + K_t(i)\{\boldsymbol{s}_t(i) - E_{t-1}(i)[\boldsymbol{s}_t(i)]\} \tag{28}$$

where $K_t(i)$ is a projection matrix (the Kalman gain). When agents' signals are symmetric, as in (26e), the Kalman gain will be common to all agents. Provided that X_t is stationary, a time-invariant Kalman gain will emerge so that $K_t(i) = K \; \forall i, t$.

With these definitions in place, I am in a position to state the following result.

Proposition 2. *For a rational, linear economy with imperfect common knowledge,*

a) the solution is given by

$$\boldsymbol{z}_t = \Gamma X_t \tag{29a}$$

$$X_t = F X_{t-1} + G\boldsymbol{u}_t \tag{29b}$$

where the matrices Γ, F and G are implicitly given by

$$\Gamma = \begin{bmatrix} D & C & B & A_1 & \mathbf{0}_{m\times\infty} \end{bmatrix} + A_2\Gamma T + A_3 \Gamma F T \tag{29c}$$

$$F = \sum_{k=0}^{\infty} (T')^k F^* (T)^k \tag{29d}$$

$$G = \begin{bmatrix} I_m \\ \mathbf{0}_{n\times m} \\ K\Lambda G \end{bmatrix} \tag{29e}$$

with

$$\Lambda = \begin{bmatrix} N & M & \mathbf{0}_{q\times\infty} \end{bmatrix} \tag{29f}$$

$$F^* = \begin{bmatrix} \begin{bmatrix} P & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times\infty} \end{bmatrix} \\ \Gamma \\ K\Lambda(I-T) \end{bmatrix} \tag{29g}$$

$$K = (FV\Lambda' + G\Sigma_{\boldsymbol{u}}\Theta')\left(\Lambda V \Lambda' + \Theta\Sigma_{\boldsymbol{u}}\Theta' + \Sigma_{\boldsymbol{v}}\right)^{-1} \tag{29h}$$

$$V = FVF' + G\Sigma_{\boldsymbol{u}}G' - K(\Lambda V F' + \Theta\Sigma_{\boldsymbol{u}}G') \tag{29i}$$

[14] Although, of course, (iii) is commonly imagined to hold implicitly.

[15] Optimal in the sense of minimising mean square error. In a linear model with Gaussian shocks, the Kalman filter represents the full Bayesian estimator.

b) the solution (29) is unique unless agents share common knowledge of lagged values of X_t or $\boldsymbol{z}_t$.

c) the solution (29) converges to the minimum state variable (MSV) solution as $Var(\boldsymbol{v}_t(i)) \to 0$, provided that agents' information sets would at that point be instantly invertible.

The intuition for uniqueness is straightforward: co-ordination around another solution requires common knowledge. No lagged variable, $\boldsymbol{y}_{t-s}$, where $\boldsymbol{y}_{t-s}$ might be any endogenous variable, exogenous state or period $t-s$ belief, can feature in a reduced-form solution unless agents can agree on what value it takes, and know that they agree, so that $\overline{E}_t[\boldsymbol{y}_{t-s}] = \boldsymbol{y}_{t-s}$.

Although proposition 2 establishes a unique equilibrium, it does not provide for the stability of that equilibrium. Instead, stability requires the following condition, the proof of which follows immediately from the vectorisation of (29c).

Proposition 3. *The equilibrium identified in proposition 2 will be stable if, and only if, $(I - T' \otimes A_2 - (T'F') \otimes A_3)$ is invertible.*

4 An Information-Based Theory of the Price Level

I now move on to describe a variation of the New Keynesian model for which: *in steady-state* the rate of inflation may be uniquely determined as a choice of the monetary authority, while *out of steady-state* not just the rate of inflation but also the deviation of the *price level* is determinate, despite the monetary authority only responding to deviations of inflation.

The model features Ricardian equivalence and lump sum taxes to eliminate any influence of fiscal policy. There is a continuum of firms, indexed $i \in [0,1]$, that supply differentiated goods to a representative household, who values them via a Dixit-Stiglitz aggregator. The household provides labour to the firms, with decreasing marginal productivity, in a competitive labour market. There is no capital. Firms are subject to Calvo (1983) pricing and information frictions. The household and the central bank each possess full information. Since the only material difference between the model presented here and the textbook three equation model of Woodford (2003b) or Galí (2008) is in the information structure of the firms, I only present here the log-linearised version of the model, with an emphasis on the information structure, and leave a full derivation for the appendix.

The representative household's Euler equation, combined with a market clearing assumption, is:

$$y_t = E_t^{\Omega}[y_{t+1}] - \sigma\left(i_t - E_t^{\Omega}[p_{t+1} - p_t]\right) + \sigma\left(x_t^c - E_t^{\Omega}[x_{t+1}^c]\right) \tag{30}$$

where y_t is output, p_t is the aggregate price level, i_t is the nominal interest rate, σ is the elasticity of intertemporal substitution, x_t^c is a persistent shock to the marginal utility of consumption and $E_t^{\Omega}[\cdot] = E_t[\cdot|\Omega_t]$ is the mathematical expectation conditional on all period-t information.

The central bank also has full information and makes use of a contemporaneous Taylor rule:

$$i_t = \phi_y y_t + \phi_\pi (p_t - p_{t-1}) + x_t^m \tag{31}$$

where x_t^m is a persistent monetary shock. I also consider forward-looking Taylor rules below.

Individual firms have an independent probability, θ, of not being able to update their price in each period, so that the aggregate price level evolves as:

$$p_t = \theta p_{t-1} + (1-\theta) g_t \tag{32}$$

where $g_t \equiv \int_0^1 g_t(j)\, di$ is the average reset price chosen in period t. Firms' individual reset prices are given by their expectations of their optimal reset prices:

$$g_t(j) = (1-\beta\theta) E_t(j) [p_t + \kappa y_t] + (\beta\theta) E_t(j) [g_{t+1}] \tag{33}$$

where β is the household discount factor, κ is a function of the various elasticities of intertemporal substitution, demand, labour supply and marginal cost; and $E_t(j)[\cdot] \equiv E[\cdot|\mathcal{I}_t(j)]$ is firm j's expectation based on an incomplete information set: $\mathcal{I}_t(j) \subset \Omega_t$. Taking an average of (33) and combining it with (32) then gives the following expression for the price level:

$$p_t = \theta p_{t-1} + (1-\theta(1+\beta)) \overline{E}_t [p_t] + (\beta\theta) \overline{E}_t [p_{t+1}] + (1-\theta)(1-\beta\theta) \kappa \overline{E}_t [y_t] \tag{34}$$

where $\overline{E}_t[\cdot] \equiv \int_0^1 E_t(j)[\cdot]\, dj$ is the average firm expectation.

For reference, note that this may be readily rearranged (using $\pi_t \equiv p_t - p_{t-1}$) to give

$$\begin{aligned} \pi_t = (1-\theta) \overline{E}_t [\pi_t] + (1-\theta) \left\{ \overline{E}_t [p_{t-1}] - p_{t-1} \right\} \\ + (1-\theta)(1-\beta\theta) \kappa \overline{E}_t [y_t] \\ + (\beta\theta) \overline{E}_t [\pi_{t+1}] \end{aligned} \tag{35}$$

This is the *Incomplete Information New Keynesian Phillips Curve*, first presented by Nimark (2008) although expanded slightly here to allow for (i) uncertainty about the previous period's price-level; and (ii) curvature in firms' production function. With full information, the terms in p_{t-1} drop out and expectations around period-t variables become accurate, leading to the canonical NKPC.

4.1 Timing and firms' information

Unlike in models of full information, where all variables are jointly determined by a Walrasian auctioneer, I instead suppose that each period proceeds in two stages:

1. In stage one ("overnight"), firms observe their signals and, when able, adjust their prices accordingly, thereby determining inflation.
2. In stage two ("the working day"), the household and monetary authority jointly determine the market-clearing nominal interest rate and average nominal wage. The household reveals the quantity demanded from each firm at the given prices, firms discover their current-period marginal costs and produce the goods. The household consumes the goods entirely.

Firms have only incomplete and heterogeneous access to information about the state of the economy. They each observe a set of signals about the aggregate economy and use these to update their beliefs. Note that equation (33) implies that there is strategic complementarity in firms' decision-making, so that each of them will care about not only the real marginal cost they will face but also the decisions (and beliefs) of all other firms.

One distinction to be made is between what information a firm is exposed to in principle, and what information is actually used to inform their decision making. The exogenous imposition of an information processing constraint in the style of Sims (2003), or the imposition of a finite flow cost to be paid for each signal incorporated (analogous to an informational menu cost), for example, might explain why firms with potential exposure to truly enormous torrents of information might instead form their beliefs – and so base their decision making – on the conditionally rational combination of only a small number of signals. As a baseline, I assume that each firm observes:

- A noisy signal regarding the previous period aggregate level of output.
- A noisy signal regarding the previous period aggregate price level.

That is,

$$\boldsymbol{s}_t(i) = \begin{bmatrix} y_{t-1} + x_t^{n_y} + v_t^{n_y}(i) \\ p_{t-1} + x_t^{n_p} + v_t^{n_p}(i) \end{bmatrix} \tag{36}$$

where $x_t^{n_y}$ and $x_t^{n_p}$ are public noise shocks to firms' signals regarding the level of output and the price level respectively, in order to capture the effect of imperfect measurement by national statistical agencies. The idiosyncratic noise may be interpreted as firms' failure to directly observe the public signal (perhaps instead getting an impression from newspaper coverage), an error of judgement, or as the imperfect applicability of national public signals to the aggregation level most relevant to each firm (e.g. at an industry or sector level).

Note that if the variance of idiosyncratic shocks in (36) were to be zero, agents' information sets would be common but still incomplete, as (i) signals would be received with a lag and (ii) they would still include public noise. To replicate full information, firms' signal vectors can be expanded to

$$\boldsymbol{s}_t(i) = \begin{bmatrix} y_{t-1} + x_t^{n_y} + v_t^{n_y}(i) \\ p_{t-1} + x_t^{n_p} + v_t^{n_p}(i) \\ x_t^c + v_t^c(i) \\ x_t^m + v_t^m(i) \end{bmatrix} \tag{37}$$

and the variance of both idiosyncratic and public noise shocks then taken to zero.

4.2 Stochastic processes

I gather the economy's shocks together, including the two "noise" shocks, and refer to them as the *underlying state* of the economy. I suppose that they follow an AR(1) process:

$$\boldsymbol{x}_t \equiv \begin{bmatrix} x_t^c & x_t^m & x_t^{n_y} & x_t^{n_p} \end{bmatrix}' \tag{38a}$$

$$= P\boldsymbol{x}_{t-1} + \boldsymbol{u}_t \tag{38b}$$

where $\boldsymbol{u}_t$ is a vector of period-t innovations identically and independently distributed as $N(\mathbf{0}, 1)$, while P is a diagonal matrix of fixed and commonly known parameters. The set of idiosyncratic shocks for each firm ($\boldsymbol{v}_t(i)$) is assumed to be entirely transitory, fully independent and jointly distributed as $N(\mathbf{0}, \sigma_v^2 I)$, with $Cov(\boldsymbol{x}_t, \boldsymbol{v}_t(i)) = \mathbf{0}\ \forall i, t$.

4.3 Solving the model

Equations (30), (31) and (34) may be combined to produce a single equilibrium condition:

$$\begin{aligned} p_t = \boldsymbol{b}_p' \overline{E}_t[\boldsymbol{x}_t] + \theta p_{t-1} + \zeta_{-1}\overline{E}_t[p_{t-1}] & \\ + \zeta_0\ \overline{E}_t[p_t] & \\ + \zeta_1\ \overline{E}_t[p_{t+1}] & \\ + \zeta_{2+}\overline{E}_t\left[\sum_{s=0}^{\infty}\lambda^{s+1}p_{t+s+2}\right] & \end{aligned} \tag{39}$$

where, for a contemporaneous Taylor rule, $\lambda \equiv 1/(1+\sigma\phi_y)$ (details of the other coefficients, which are functions of underlying parameters are provided in the appendix). The expressions for λ and the ζ_* coefficients differ with a forward-looking Taylor rule, but this functional form remains.

Together with the signal structure above, this satisfies the requirements of proposition 2. Defining the full state as $X_t \equiv \begin{bmatrix} \boldsymbol{x}_t' & \boldsymbol{p}_{t-1} & \boldsymbol{y}_{t-1} & \overline{E}_t[X_t]' \end{bmatrix}'$, the unique solution is therefore of the form:

$$X_t = FX_{t-1} + G\boldsymbol{u}_t \tag{40a}$$

$$p_t = \boldsymbol{\gamma}_p' X_t \tag{40b}$$

$$y_t = \boldsymbol{\gamma}_y' X_t \tag{40c}$$

where F embodies the average Kalman filter of firms' signal extraction problem.

4.4 Price stability

Recall that T is the selection matrix such that $TX_t = \overline{E}_t[X_t]$. I show in the appendix that

$$\boldsymbol{\gamma}_p' = \Phi \sum_{k=0}^{\infty} (HT)^k \tag{41a}$$

where

$$\Phi = \begin{bmatrix} \mathbf{0} & \theta & 0 & \boldsymbol{b}_p' & \zeta_{-1} & 0 & \mathbf{0} & \cdots \end{bmatrix} \tag{41b}$$

$$H = \zeta_0 I + \zeta_1 F + \zeta_{2+} F \sum_{s=1}^{\infty} (\lambda F)^s \tag{41c}$$

Since T is just a shift operator, then this sum will converge – that is, the model will exhibit stationarity in prices – if, and only if, all eigenvalues of H lie within the unit circle. I denote by $\rho\{H\}$ the spectral radius of H (that is, the largest absolute eigenvalue) and refer to it as firms' *equilibrium strategic complementarity*, since higher values of $\rho\{H\}$ represent higher weight placed by individual firms on the their beliefs about other firms' prices (and, hence, other firms' beliefs).

For values of $\rho\{H\}$ less than one, firms place decreasing weight on higher-order expectations and the stable solution is expressible as

$$\boldsymbol{\gamma}_p' = \Phi (I - HT)^{-1} \tag{42}$$

while, for values greater than or equal to one, firms place increasing weight on higher-order beliefs so the solution is explosive and the inverse will not exist.

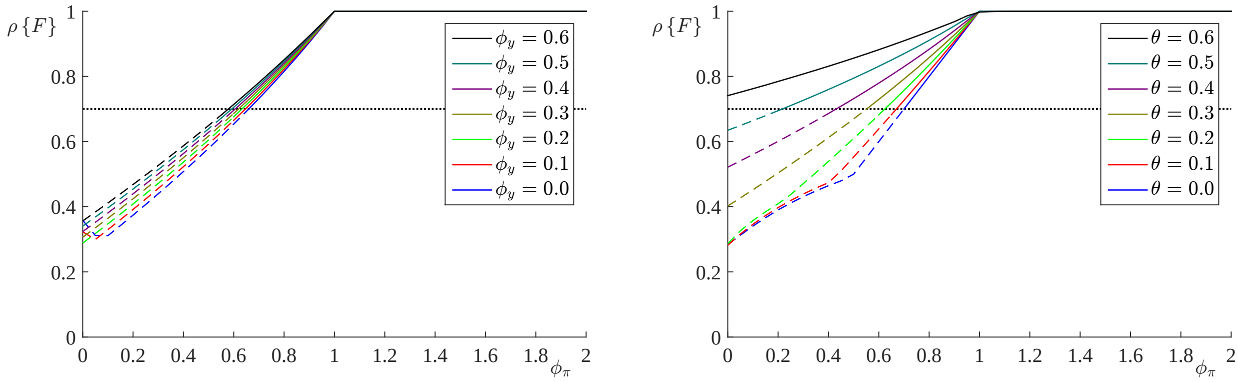
An inspection of (41c) makes it clear that having $\rho\{H\} < 1$ relies, itself, on a transversality condition: the sum $\sum_{s=1}^{\infty} (\lambda F)^s$ must converge. Since X_t includes the lagged price level, if prices exhibit a unit root then this will manifest as F having an eigenvalue on the unit circle ($\rho\{F\} = 1$). This leads to the following result:

Proposition 4. *When prices exhibit a unit root, a necessary (but not sufficient) condition of price stability is that the central bank respond to output.*

A value of $\phi_y > 0$ ensures that $\lambda \in (0,1)$ and, hence, that $\rho\{\lambda F\} < 1$. However, if prices are stationary and do not exhibit a unit root, then a value of $\phi_y = 0$ (so that $\lambda = 1$) would be admissible. Indeed, with unique solutions able to be found when $\phi_\pi < 1$, the following result emerges.

Proposition 5. *The persistence of the aggregate price level following a shock is increasing in ϕ_π and achieves a unit root when $\phi_\pi \geq 1$.*

Figure plots the two largest eigenvalues of F – one corresponding to the underlying shock and one corresponding to the price level and higher-order beliefs about the price level – as ϕ_π increases for different values of ϕ_y and θ. Three regions emerge: (i) when ϕ_π is very low, the persistence of the price level is lower than the persistence of the underlying shock, so the largest eigenvalue is simply that of the underlying shock; (ii) when $\phi_\pi > 1$, the price level exhibits a unit root and this is the largest eigenvalue; and (iii) an intermediate case, where the price level is stationary, but nevertheless more persistent than the underlying shock. Firms' equilibrium strategic complementarity remains less than one in all of these cases (except, as mentioned above, when $\phi_\pi > 1$ and $\phi_y = 0$).



Note: The charts plot the largest eigenvalue of F for different values of ϕ_π. Parameter choices are: $\beta = 0.994$ (HH quarterly discount factor), $\rho\{P\} = 0.7$ (AR(1) coefficient of the underlying shock process), $\sigma = 1$ (elasticity of intertemporal substitution); $\kappa = 1$ (slope of firms' pricing equation with respect to output); and $\sigma_v^2/\sigma_u^2 = 1$ (amount of idiosyncratic noise in firms' signals). For the left-hand chart, $\theta = 0.25$ (average duration = 4 months) while, for the right-hand chart, $\phi_y = 0.5$. The horizontal dotted line is the eigenvalue associated with the underlying shock (here 0.7).

Figure 5: Largest eigenvalue of F

The monetary authority does not need to intervene

Strikingly, the following result also emerges, the proof of which is given in the appendix:

Proposition 6. *When price-setting firms have incomplete common knowledge about current and past deviations from steady state, a sufficient condition for price stability when the interest rate remains pegged to its steady-state value, or deviates only due mean-zero, orthogonal shocks, is*

$$(1-\theta)(1-\kappa\sigma(1-\beta\theta)) < 1 + \beta\theta(1-\rho) \tag{43a}$$

or, when prices are flexible ($\theta = 0$),

$$\kappa\sigma < 1 \tag{43b}$$

4.5 Simulation results

Table 2 lists the parameters used for the simulations presented below. All models have been approximated with $k^* = 100$.[16] Given the benchmark parameter choices, the results presented here are not

[16] That is, the first 100 orders of higher-order expectations are included in the estimated solutions. With four variables in the underlying state ($\boldsymbol{x}_t$), this implies a total of 604 variables in the estimated full state (X_t).

appreciably sensitive to increasing the k^* threshold. The discussion below focusses on the dynamics of the economy following a monetary shock (that is, to x_t^m). Charts showing impulse responses following other shocks are provided in the appendix.

Parameter	Value	Description
β	0.994	Household discount factor (quarterly)
θ	0.5	Calvo paramettter (av. price duration = 6 months)
σ	1.0	Elasticity of intertemporal substitution (log utility)
κ	0.9	Slope of the Phillips curve
ρ	0.7	The AR(1) coefficient for each underlying shock
σ_v^2/σ_u^2	1.0	The relative variance of idiosyncratic shocks

Table 2: Baseline parameterisation

A monetary shock under the baseline parameterisation

Figure 6 plots impulse response functions (IRFs) of the shock, output level, interest rate and price level following a monetary shock under three different interest rate rules in the style of the Taylor rule. In each case, the left-hand panel plots IRFs under near-full information, with $\sigma_v^2 = 10^{-8}$, while the right-hand panels plot IRFs under dispersed information, with $\sigma_v^2 = 1$.

The top-left panel reproduces the IRFs of the textbook New Keynesian model (Galí, 2008; Woodford, 2003b). The shock seeks to raise the interest rate, which reduces demand and prices. The endogenous response from the central bank is sufficient to ensure that the interest rate falls on impact despite the shock being positive. As the shock dissipates, output, the interest rate and inflation (not shown) all return steadily to zero. The cumulative sum of deviations of inflation from target remain permanently in the price level, which exhibits a unit root. The top-right panel reexamines this setting under dispersed information. In addition to experiencing stickiness in their prices, firms are also subject to incomplete and heterogenous information. Their beliefs are slow to update and prices consequently deviate by less. The reduced price response induces a larger response in output.

The middle two panels depict the unique solution when the central bank's marginal response to inflation is more subdued, at only 0.5 instead of 1.5. With a marginal response less than one, the aggregate price level itself becomes stationary, requiring that inflation deviate above trend after several quarters. The above-target inflation induces an endogenous response from the central bank to raise the interest rate above the path it would follow with a standard Taylor rule and, after several periods, even above its steady-state value. This then causes output to fall by further on impact, as households retain full information and correctly anticipate this.

The bottom two panels show the unique solution when the central bank does not respond to the state of the economy at all. The interest rate is freely permitted to follow the path of the exogenous shock, jumping up on impact and then gradually falling over time. The price response is both smaller and less persistent, causing the response of output to be substantially larger than under the subdued interest rate rule.

As one might expect, a systematic response of policy to the state of the economy serves to lessen the depth of recessions following a monetary shock. However, the systematic response also serves to *increase the duration* of the ensuing slump. This fact is illustrated more clearly in figure 7, where the output IRFs from figure 6 are gathered together. In all cases, the magnitude of the output response is larger when firms have dispersed information, but the recovery back to trend is more rapid when the systematic response is weaker and, indeed, features a period of output above its trend.

This emerges because of the fact that the persistence of the aggregate price level is increasing in ϕ_π. The peak price response is asymptotic under a standard Taylor rule and occurs after four quarters under the subdued rule shown here, but takes only two quarters under a state-invariant rule. The subsequent above-trend inflation as prices rise back to their trend serves to stimulate demand. Since that price rise is more rapid the weaker the central bank response, the stronger the recovery in demand.

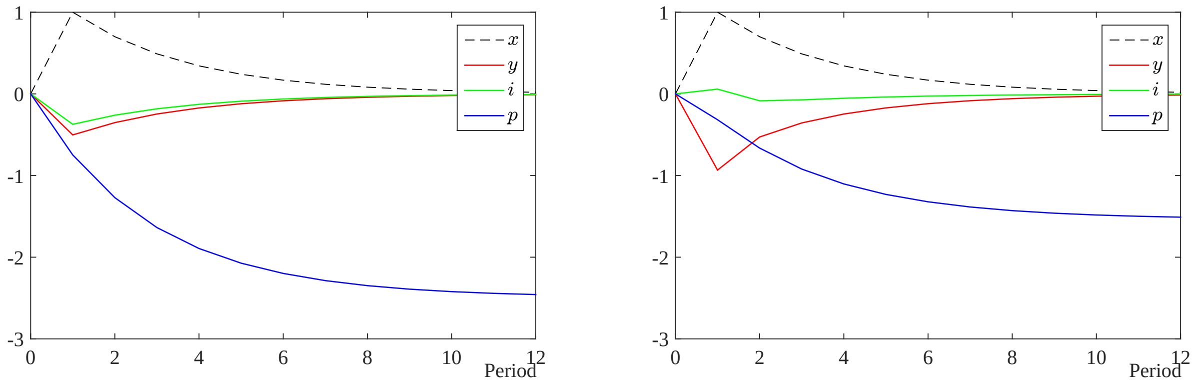
5 Discussion

This paper describes a model – based off, but extending beyond the New Keynesian model – with dispersed information among price-setting firms that features a unique and stable solution for the price level and not just the rate of inflation, despite also imposing a passive fiscal environment with Ricardian equivalence and lump-sum taxes. Determinacy is achieved by an assumption of imperfect common knowledge among price-setting firms. So long as firms' signals each contain *some* idiosyncratic noise, they will not be able to ever exactly agree on the current or past values of the aggregate price level. This eliminates all but the MSV solution, in what amounts to an application of the uniqueness result of Morris and Shin (2001).

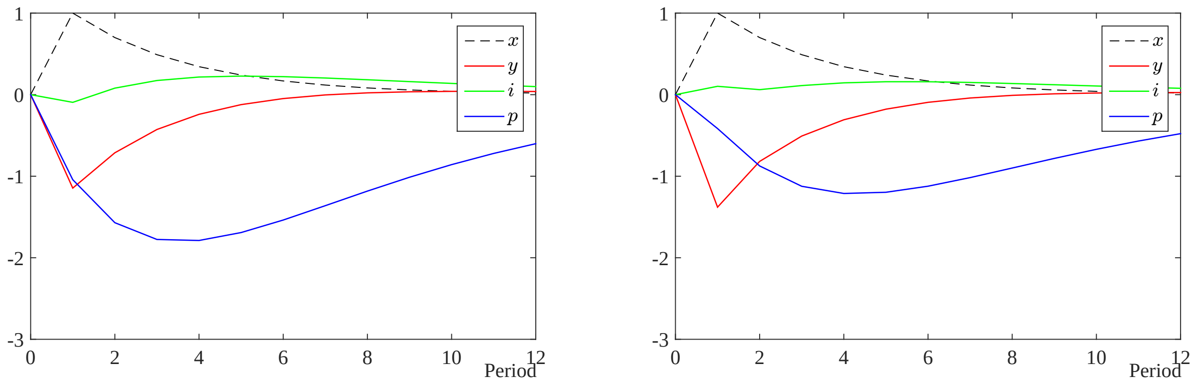
Restrictions on the coefficients of the central bank's interest rate rule are not required to deliver determinacy and unique solutions are able to be obtained under 'violations' of the Taylor principle. Indeed, the model admits the possibility of a state-invariant policy rule, with the interest rate either pegged to its steady-state value or varying only from shocks that are orthogonal to the rest of the economy. The persistence of deviations of the aggregate price level from its steady-state path is increasing in the central bank's marginal response to inflation (ϕ_π). When the Taylor principle is satisfied ($\phi_\pi > 1$), prices exhibit a unit root, but when $\phi_\pi < 1$ prices themselves become stationary. A monetary shock that raises the interest rate faced by households features both initial period of below-trend inflation and a later period of above-trend inflation.

A transversality condition emerges as a necessary (but not sufficient) condition of price stability, but it is satisfied when policymakers respond not at all to the state of the economy. When policymakers subscribe to the Taylor principle, so that prices exhibit a unit root, the central bank must also respond to output in order to achieve stability.

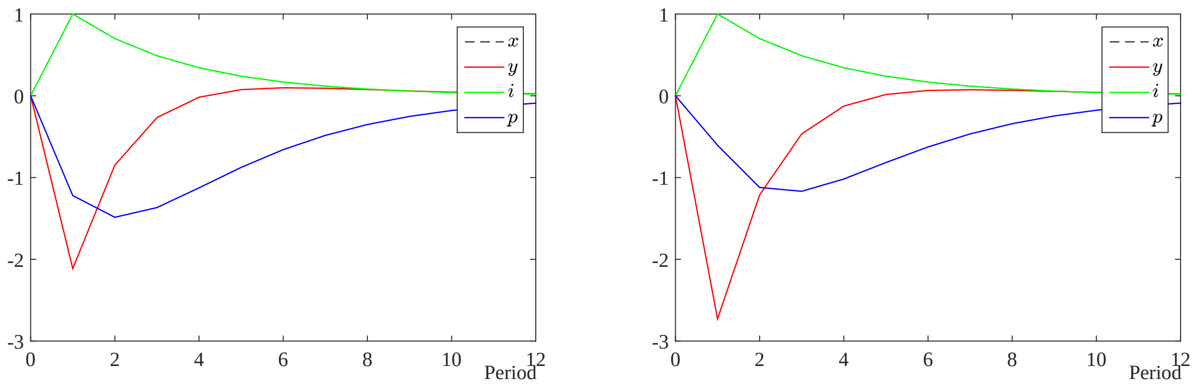
When the central bank systematically responds to the state of the economy at all, the on-impact effect of a monetary shock is lessened, but the duration of the ensuing slump is lengthened, thereby representing a previously unappreciated trade-off in the conduct of monetary policy. This emerges because of the increasing persistence of the price level. Without a systematic response, the peak price response occurs sooner and its return to trend is more rapid. Since this represents a period of above-trend inflation, it induces a "boom" during the recovery period. Stronger systematic responses to inflation serve to delay the peak response on prices (extending to an asymptote under the Taylor principle), meaning that the period of above-trend inflation is both smaller and more distant into the future.



(a) Standard Taylor rule: $\phi_\pi = 1.5$ and $\phi_y = 0.5$.



(b) Subdued rule: $\phi_\pi = 0.5$ and $\phi_y = 0.5$.



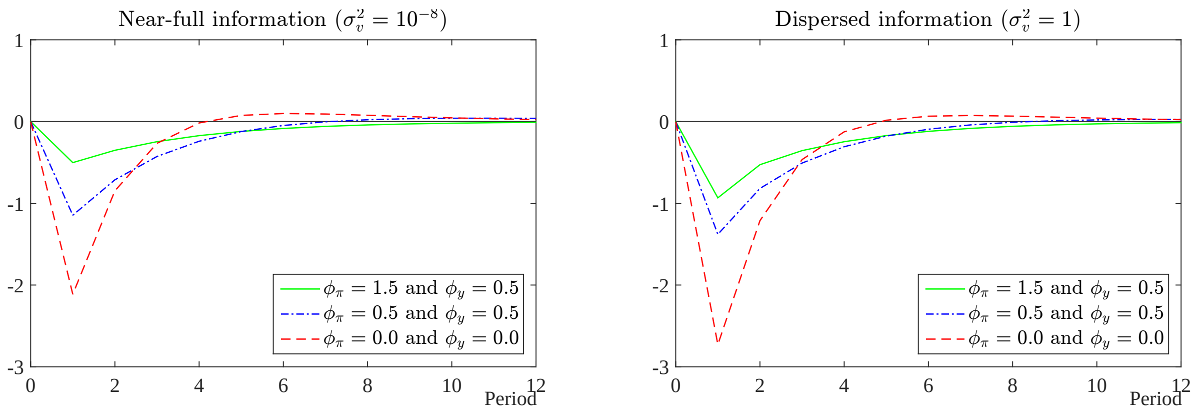
(c) State-invariant rule: $\phi_\pi = 0$ and $\phi_y = 0$.

Note: Each panel shows impulse response functions (IRFs) for the underlying shock, the output level, the interest rate and the price level following a monetary shock. The left-hand panels show IRFs under near-full information ($\sigma_v^2 = 10^{-8}$), while the right-hand panels show IRFs when firms are subject to idiosyncratic noise ($\sigma_v^2 = 1$). The top two panels adopt the standard Taylor rule, with $\phi_\pi = 1.5$ and $\phi_y = 0.5$ (so the top-left panel reproduces the basic New Keynesian model of Galí, 2008, while the top-right panel is quite similar to Nimark, 2008). The middle two panels show the setting under a subdued rule, when $\phi_\pi = \phi_y = 0.5$. The bottom two panels show the setting when policy is state invariant ($\phi_\pi = \phi_y = 0$).

Figure 6: Impulse responses following a monetary shock

5.1 Steady-state

The model, as implemented, is log-linearised around a deterministic steady state. This imposes an assumption that although firms do not possess common knowledge about deviations *from* the steady state, they do share common knowledge *of* the steady state. In effect, this amounts to an assumption that while firms' expectations about near-term inflation remain dispersed, their beliefs about long-run



Note: Each panel shows the impulse response functions (IRFs) for output following a monetary shock under three different monetary regimes: (i) a standard Taylor rule; (ii) a subdued interest rate rule; and (iii) an interest rate peg. The left-hand panel plots these under an assumption of near-full information, with $\sigma_v^2 = 10^{-8}$, while the right-hand panel does the same under dispersed information, with $\sigma_v^2 = 1$.

Figure 7: Impulse responses of output to a monetary shock under different regimes

inflation are perfectly anchored. Conditional on this assumption, nominal stability around that steady state need not require a systematic central bank response to the state of the economy.

This paper makes no comment on how agents might arrive at a consensus about the steady state of the economy. If, in practice, a systematic policy response is necessary to ensure that long-run inflation expectations remain well anchored then that would be in addition to the results discussed above. Nevertheless, it bears noting that when the central bank's marginal response to inflation is less than one, the full, non-linear model under full information features a unique, globally stable steady-state equilibrium even after allowing for the possibility of a lower bound on interest rates (albeit with indeterminacy following shocks from that steady state). This suggests that a learning model of the steady state, combined with the solution developed here for unique solutions around a given steady state, may prove fruitful in removing the deflationary trap explored by Benhabib, Schmitt-Grohe, and Uribe (2001).

References

Angeletos, G., and J. La'O (2009): "Incomplete Information, Higher-Order Beliefs and Price Inertia," *NBER Working Papers*, No. 15003.

——— (2010): "Noisy Business Cycles," in *NBER Macroeconomics Annual 2009*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, vol. 24, chap. 5, pp. 319–378. University of Chicago Press, Chicago.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96(1-2), 40–69.

Bernanke, B. (2004): "The Great Moderation," Speech, Board of Governors of the Federal Reserve System (U.S.).

Bianchi, F. (2012): "Evolving Monetary/Fiscal Policy Mix in the United States," *American Economic Review*, 102(3), 167–72.

——— (2013): “Regime Switches, Agents’ Beliefs, and Post-World War II U.S. Macroeconomic Dynamics,” *Review of Economic Studies*, 80(2), 463–490.

Blanchard, O., and C. Kahn (1980): “The Solution of Linear Difference Models under Rational Expectations,” *Econometrica*, 48(5), 1305–1311.

Calvo, G. (1983): “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12(3), 383–398.

Cass, D., and K. Shell (1983): “Do Sunspots Matter?,” *Journal of Political Economy*, 91(2), 193–227.

Clarida, R., J. Galí, and M. Gertler (2000): “Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory,” *The Quarterly Journal of Economics*, 115(1), 147–180.

Cochrane, J. (2011): “Determinacy and Identification with Taylor Rules,” *Journal of Political Economy*, 119(3), 565 – 615.

——— (2015a): “Do Higher Interest Rates Raise or Lower Inflation?,” Unpublished manuscript, University of Chicago.

——— (2015b): “The New-Keynesian Liquidity Trap,” Unpublished manuscript, University of Chicago.

Coibion, O., and Y. Gorodnichenko (2012): “Why Are Target Interest Rate Changes So Persistent?,” *American Economic Journal: Macroeconomics*, 4(4), 126–62.

Currie, D., P. Levine, and J. Pearlman (1986): “Rational expectations models with partial information,” *Economic Modelling*, 3(2), 90–105.

Evans, G., and S. Honkapohja (2001): *Learning and Expectations in Macroeconomics*. Princeton University Press, Princeton.

Farmer, R. (2013a): “Animal Spirits, Financial Crises and Persistent Unemployment,” *Economic Journal*, 0, 317–340.

——— (2013b): “Animal Spirits, Persistent Unemployment and the Belief Function,” in *Rethinking Expectations: The Way Forward for Macroeconomics*, ed. by R. Frydman, and E. Phelps, NBER Chapters, chap. 7, pp. 251–276. Princeton University Press.

Galí, J. (2008): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton.

García-Schmidt, M., and M. Woodford (2015): “Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis,” NBER Working Papers 21614, National Bureau of Economic Research, Inc.

Graham, L. (2011a): “Individual rationality, model-consistent expectations and learning,” CDMA Working Paper Series 201112, Centre for Dynamic Macroeconomic Analysis.

——— (2011b): “Learning, information and heterogeneity,” CDMA Working Paper Series 201113, Centre for Dynamic Macroeconomic Analysis.

Graham, L., and S. Wright (2010): "Information, hetereogeneity and market incompleteness," *Journal of Monetary Economics*, 57(2), 164–174.

King, R., and M. Watson (1998): "The Solution of Singular Linear Difference Systems under Rational Expectations," *International Economic Review*, 39(4), 1015–26.

Klein, P. (2000): "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control*, 24(10), 1405–1423.

Kocherlakota, N. (2016): "Fragility of Purely Real Macroeconomic Models," NBER Working Papers 21866, National Bureau of Economic Research, Inc.

Kohlhas, A. (2014): "Learning-by-Sharing: Monetary Policy and the Information Content of Public Signals," Job market paper, Stockholm University.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27(1), 129–147.

Lorenzoni, G. (2009): "A Theory of Demand Shocks," *American Economic Review*, 99(5), 2055–2084.

Lucas, R. (1972): "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4(2), 103–124.

Mankiw, G., and R. Reis (2002): "Sticky information versus sticky prices: A proposal to replace the New Keynsian Phillips curve," *The Quarterly Journal of Economics*, 117(4), 1295–1328.

——— (2006): "Pervasive Stickiness," *American Economic Review*, 96(2), 164–169.

McCallum, B. (1999): "Role of the Minimal State Variable Criterion in Rational Expectations Models," *International Tax and Public Finance*, 6(4), 621–639.

——— (2012): "A Continuity Refinement for Rational Expectations Solutions," NBER Working Papers 18323, National Bureau of Economic Research, Inc.

Melosi, L. (2014): "Estimating Models with Dispersed Information," *American Economic Journal: Macroeconomics*, 6, 1–31.

Morris, S., and H. S. Shin (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88(3), 587–97.

——— (2001): "Rethinking Multiple Equilibria in Macroeconomic Modeling," in *NBER Macroeconomics Annual 2000, Volume 15*, NBER Chapters, pp. 139–182. National Bureau of Economic Research, Inc.

Muth, J. (1961): "Rational Expectations and the Theory of Price Movements," *Econometrica*, 29(3), 315–335.

Nimark, K. (2008): "Dynamic pricing and imperfect common knowledge," *Journal of Monetary Economics*, 55(2), 365–382.

——— (2015): "A low dimensional Kalman filter for systems with lagged states in the measurement equation," *Economics Letters*, 127, 10–13.

Orphanides, A. (2003): “Historical monetary policy analysis and the Taylor rule,” *Journal of Monetary Economics*, 50(5), 983–1022.

Phelps, E. (1984): “The trouble with ‘rational expectations’ and the problem of inflation stabilization,” in *Individual Forecasting and Aggregate Outcomes: ‘Rational Expectations’ Examined*, ed. by R. Frydman, and E. Phelps, chap. 2, pp. 31–46. Cambridge University Press.

Rudebusch, G. (2002): “Term structure evidence on interest rate smoothing and monetary policy inertia,” *Journal of Monetary Economics*, 49(6), 1161–1187.

——— (2006): “Monetary Policy Inertia: Fact or Fiction?,” *International Journal of Central Banking*, 2(4).

Sargent, T., and N. Wallace (1975): ““Rational” Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy*, 83(2), 241–54.

Sims, C. (2002): “Solving Linear Rational Expectations Models,” *Computational Economics*, 20(1-2), 1–20.

——— (2003): “Implications of Rational Inattention,” *Journal of Monetary Economics*, 50(3), 665–690.

Townsend, R. M. (1983): “Forecasting the Forecasts of Others,” *Journal of Political Economy*, 91(4), 546–587.

Uhlig, H. (1997): “A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily,” in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott, Handbook of Macroeconomics, chap. 3, pp. 30–61. Oxford University Press.

Woodford, M. (1999): “Optimal Monetary Policy Inertia,” *The Manchester School Supplement*, 67(0), 1–35.

——— (2003a): “Imperfect Common Knowledge and the Effects of Monetary Policy,” in *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, ed. by P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford. Princeton University Press.

——— (2003b): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.