Optimal Public Debt with Life Cycle Motives *

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Abstract

In their seminal paper Aiyagari and McGrattan (1998) find that it is optimal for the government to hold debt equal to two thirds of output in an infinitely lived agent model. We revisit this result using a life cycle model and find that it is no longer optimal for the government to hold debt but instead to hold savings equal to 160% of output. In the infinitely lived agent model, government debt increases the interest rate, encourages agents to hold more savings and relaxes liquidity constraints. We find that this mechanism is quantitatively smaller in a life cycle model. In a life cycle model agents begin life without savings but quickly acquire assets not only to buffer against income shocks but also to finance post-retirement consumption. Therefore, changing the interest rate will only modestly affect how early in life young agents acquire enough savings to relax liquidity constraints. Through a series of counterfactual experiments we demonstrate that these differences in savings behavior explain the optimality of government debt versus savings across models. We also find that the optimal government policy is sensitive to the choice of social welfare function (SWF) in a life cycle model. We compare optimal policy across steady states of a SWF that maximizes the expected lifetime utility of a newborn agent (which is the consistent with the SWF in an infinitely lived agent model) to a SWF that maximizes the expected future lifetime utility of the entire population. We find that the optimal policy changes from saving 160% of output in the former to borrowing 90% of output in the latter, which is consistent with both Aiyagari and McGrattan's (1998) results and the current level of debt in the US economy.

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1 Introduction

How much public debt should a government hold? In their seminal paper, Aiyagari and McGrattan (1998) find that an optimal quantity of public debt must trade off enhanced liquidity provision with distorting the economy. In particular, the benefit to more public debt is that it increases the return to savings leading agents to hold more savings, thereby enhancing private liquidity. Enhanced liquidity means that these agents are less likely to be borrowing constrained after a series of adverse income shocks. However, an increase in government debt can have negative welfare impacts. A higher level of debt implies that the government must raise more tax revenue in order to service the debt. The distortions from these income taxes can have adverse welfare effects. Furthermore, government debt crowds out productive capital, which leads to lower per capita consumption. In an infinitely lived agent model, Aiyagari and McGrattan (1998) find the benefits of government debt dominate the welfare costs and that the optimal quantity of debt is equal to two-thirds of output.

In this paper, we revisit the question of optimal public debt in a life cycle setting and find that Aiyagari and McGrattan's (1998) mechanism is quantitatively mitigated. In particular, we find that in a life cycle model the benefits of government debt are weaker. In the steady state of an infinitely lived agent model, agents will hold savings in order to insure against idiosyncratic income shocks. In contrast, agents in a life cycle model are born with no savings but have strong incentives to quickly accumulate wealth. Their first incentive is to possess a buffer stock of savings to help partially insure against income risk. This mirrors the classic mechanism that boosts aggregate savings in the infinitely lived agent model. Their second incentive is to save in order to finance their consumption once they retire. Thus, agents have an additional motive to accumulate assets in a life cycle model. Furthermore, because agents start accumulating savings very early in their life, only the very youngest cohorts face tight liquidity constraints in the model. Although a higher interest rate may increase the amount of savings an agent accumulates by the middle of his life, incentives to save are already large and it will have less of an effect on how quickly young agents begin accumulating assets. Because liquidity constrained agents constitute a small fraction of the population in the life cycle model, the aggregate benefit from government debt can be small. In fact we find that the benefit is smaller than in an infinitely lived agent model.

In order to quantitatively study the question of optimal government debt, we develop a life cycle model that replicates the U.S. data on wealth, earnings, and government transfers. In particular, this baseline model includes mortality risk, idiosyncratic productivity shocks and unemployment risk, all of which can contribute to agents being liquidity constrained. Moreover, the model includes endogenous labor supply decisions, endogenous retirement decisions and a Social Security program, all of which can enhance the distortions arising from income taxes. Using a welfare criterion that maximizes the expected utility of an agent prior to being born in the model, our baseline life cycle model finds that it is optimal for the government to hold *savings* equal to 160% of output. In contrast, if we transform the model into an infinitely lived agent model by eliminating each life cycle element[1], we find that it is optimal for the government to hold debt in excess of 180% of output. We interpret this result as a argument for including life cycle features into models when determining the welfare consequences of the government debt policy.

Through a number of counterfactual experiments we demonstrate the motives for the government to save in the benchmark life cycle model. We examine the effects on optimal debt from particular life cycle elements, the effects of the enhanced distortions from government transfer programs, and the effects from the social welfare criteria. We find that the key elements leading to government savings being optimal in our benchmark model are life cycle features that limit the liquidity benefits from a higher interest rate. In particular, if the government holds more debt (or less savings) then there is a decrease in productive capital leading to an increase in the interest rate and a decrease in the wage rate. In an infinitely lived agent model this causes average savings to increase so agents are less liquidity constrained. In contrast, in the life cycle model the change in factor prices leads agents to want to accumulate assets more quickly. However, the lower wage generates a negative wealth effect that slows agents' asset accumulation and tightens liquidity constraints early in their lifetime.

To demonstrate that these channels are the key to the change in the optimal level of government debt we solve for the optimal level of debt in an infinitely lived agent model and solve for the optimal level of debt in counterfactual life cycle model where all the life cycle features are eliminated except for agents enter the economy with zero assets and live for 80 periods. In this counterfactual economy agents work throughout their whole lifetime so retirement savings is no longer a motive to accumulate assets. We find that as opposed to optimal debt being over 180 percent of output in our infinitely lived agent model, the government wants to hold savings equal to over 230 percent of output in this counterfactual life cycle model. These results demonstrate that the key feature leading to different optimal policies is agents entering the model with zero assets. Again, although less public savings increases the

[1] In particular, we remove mortality risk, retirement, Social Security, age-specific human capital, and the initial condition that agents enter the model with zero assets.

interest rate and leads agents to be more liquid once they reach the higher level of stationary precautionary savings midway through their life, it lowers wages making it more difficult for agents to accumulate these savings early on in their life.

We find that other potential reasons for the different optimal policy in the life cycle model do not have much effect on the optimal policy. However, interestingly we do find that the Social Security program produces a small motive for the government to hold more, not less, savings. In the benchmark life cycle model when the level of government savings or debt changes we assume that the payroll tax adjusts in order to ensure that the Social Security market clears. This leads to an interaction between the optimal level of public savings and Social Security. In particular, a small increase in the level of government savings leads to more productive capital in the economy and an increase in wages. Because the Social Security benefit formula is progressive, the average replacement rate from the Social Security program declines and the payroll tax rate necessary to clear the Social Security market also declines. The decline in the payroll tax rate helps agents when they are early in their life by easing liquidity constraints. Moreover, the larger Social Security benefits later in life also increases welfare for agents during these later years.[2] We quantify this interaction by solving for the optimal level of debt in a counterfactual life cycle model in which we relax the Social Security budget constraint and hold the payroll tax rates fixed regardless of the level of debt. We find that fixing the payroll tax rate causes the optimal level of government savings to fall from 210 percent of GDP to 185 percent of GDP. Thus, the interaction between Social Security and government debt or savings leads to small increase in the optimal level of government savings in a life cycle model.

Moreover we find that in the life cycle model, allowing agents to borrow has minimal effects on the optimal level of government savings or debt. In contrast, Winter and Roehrs (2015) find that when agents are allowed to borrow the government wants to increase its savings by twenty percent of GDP. Once again, the interaction of borrowing constraints and the optimal level of debt or savings is different in the life cycle model and the infinitely lived agent model. In a life cycle model allowing agents to borrow has much less of a general effect for two reasons. First, in a life cycle model agents' savings profile is humped shaped over their life in order to finance consumption once they retire. Therefore, allowing for borrowing will only affect agents who are early in their life before they have already accumulated savings. Second, in a life cycle model agents only work for approximately 50 years. Thus if they receive

[2] It might seem counterintuitive for this interaction with Social Security to help agents both early and later in their lifetime. However, by increasing government savings the size of the economy increases which leads to higher Social Security payments but a slight reduction in the relative size of the Social Security program. This leads to welfare improvements throughout an agent's lifetime.

an adverse income shock their expected total lifetime income falls and they react primarily by lowering their consumption. In contrast, in an infinitely lived agent model agents do not die. In this type of model if agents receive an adverse income shock it has a much smaller effect on their expected total lifetime income. Thus, instead of lower consumption, agents would prefer to use their savings (or borrow) in order to maintain their consumption. Overall, because allowing for borrowing has less of an effect on agents in a life cycle model it also has a smaller effect on the optimal level of government savings or debt in a life cycle model.

When considering the optimal policy in an infinitely lived agent model the social welfare function (SWF) is straightforward: maximize the ex-ante expected lifetime utility of agents. When comparing the optimal policy in a life cycle model we consider the analogue SWF in which the expected discounted lifetime utility of an agent prior to entering the economy is maximized. However, whether this SWF is the appropriate criteria in a life cycle model is less clear. Only valuing the expected lifetime utility of newborn agents places a high weight on the welfare agents receive when they are younger. This is because a newborn discounts highly the utility from later life, wherein the effective discount factor accounts for mortality risk in addition to time preference. If the government were elected by a current population that is heterogeneous with respect to age, one could imagine using a different SWF that maximizes the expected future utility for the whole population at a point in time (i.e. a specific steady state). This alternative social welfare function places higher weight on the older generations' utility. This is because the alternative SWF simultaneous values current older generations' utility *and* the utility that young generations expect once they grow old. Under the alternative social welfare function the government optimally holds debt equal to 90 percent of output. Therefore, this alternative social welfare function – which may very well be more consistent with a government beholden to a population of individuals at different stages in their life cycle – can reconcile the current level of US debt as well as Aiyagari and McGrattan's (1998) results.

Our paper is related to a well established literature that has examined the optimal level of government debt and savings in quantitative models. Following Aiyagari and McGrattan (1998), a number of studies examine the optimal level of debt in the steady state of an infinitely lived agent model. Floden (2001) finds that increasing government debt can provide welfare benefits if transfers are below optimal levels. Similarly, Dyrda and Pedroni (2015) find that it is optimal for the government to hold debt. However they find that when optimizing both taxes and debt at the same time leads to a smaller level of optimal debt than previous studies. In contrast, Winter and Roehrs (2015) find that when they include a skewed wealth distribution that more closely matches the upper tail of the U.S. wealth distribution, it is optimal for the government to save as opposed to holding debt. In contrast to these papers, we study optimal public debt and savings in a life cycle model in which individuals transition from liquidity constrained savers to retirees that run down their savings. We find that this transition can lead to different welfare effects from government savings and debt.

Our paper is also related to recent work by Dyrda and Pedroni (2015), Winter and Roehrs (2015), and Desbonnet and Weitzenblum (2012), that finds quantitatively large welfare costs of transitioning between steady states after a change in public debt. However, these studies focus on models inhabited by infinitely lived agent and do not incorporate the mechanisms prevalent in a life cycle setting. The present study does not consider steady state transitions, and instead focuses on steady state comparisons to more sharply highlight the contribution of the life cycle to the question of optimal debt and welfare.

The remainder of this paper is organized as follows. In section 2 we describe the the life cycle model environments and define equilibrium. Section 3 presents the calibration strategy, as well as functional forms and parameter values we use to quantify the model. In section 4 we present quantitative results from the calibrated model and counterfactual experiments. Section 5 concludes.

2 Life Cycle Model with Government Debt

In this section we extend Aiyagari and McGrattan's (1998) infinitely-lived heterogeneous agent model. We follow Peterman and Sommer (2014) in assuming overlapping generations of finitely-lived agents and incorporating government institutions such as social security and progressive income taxation. Relative to Peterman and Sommer (2014) we incorporate age-dependent unemployment shocks and a more detailed unemployment benefit formula to allow for earnings-dependent unemployment insurance. Lastly, the government's budget constraint follows Aiyagari and McGrattan (1998) in allowing (non-zero) government debt issuance.

2.1 Production

Assume there exist a large number of firms that sells goods in perfectly competitive product markets, rent inputs from perfectly competitive factor markets and each operate an identical constant returns to scale production technology, $Y = AF(K, L)$. These assumptions on primitives admit a representative firm fiction. The representative firm chooses capital and labor (K, L) to maximize profits given a capital rental rate r, a wage rate w and capital depreciation rate $\delta \in (0, 1)$.

2.2 Consumers

Demographics: Let time be discrete and let each model period represent a year. Each period, the economy is inhabited by J overlapping generations of individuals. Age J is each agent's exogenous terminal period of life. Before period J all living agents face mortality risk. Conditional on living, age-j agents have a probability s_j of living until age $j+1$ and face $s_J = 0$. Each period a new cohort is born and the size of each successive newly born cohort grows at a constant rate $g_n > 0$. The time invariant shares for cohorts are given by:

$$\mu_j = \left\{ \begin{array}{cc} s_{j-1}\dfrac{\mu_{j-1}}{1+g_n} & \text{for } j = 2, \ldots, J \\ 1 - \displaystyle\sum_{j=2}^{J} s_{j-1}\dfrac{\mu_{j-1}}{1+g_n} & \text{for } j = 1 \end{array} \right\}$$

Agents who die before age J may hold savings since mortality is uncertain. These savings are treated as accidental bequests and equally divided across each living agent in the form of a lump-sum transfer, denoted Tr.

Agents choose their retirement age, which is denoted by J_{ret}. Agents endogenously determine retirement age in the interval $j \in [\underline{J}_{ret}, \bar{J}_{ret}]$ and the decision is irreversible.

Preferences: Agents rank lifetime paths of consumption and labor, denoted $\{c_j, h_j\}_{j=1}^{J}$, according to the following preferences:

$$\mathbb{E} \sum_{j=1}^{J} \beta^j s_j \left[u(c_j) - \chi_1 \varphi(h_j) - \chi_2 \mathbb{1}(j < J_{ret}) \right]$$

where β is the time discount factor. Expectations are taken with respect to the stochastic processes governing labor productivity and unemployment shocks. Furthermore, $u(c)$ and $\varphi(h)$ are instantaneous utility functions over consumption and labor hours, respectively, satisfying standard conditions. Let $\mathbb{1}(j < J_{ret})$ denote an indicator variable that equals zero for each age that an agent is retired.

Labor Earnings: Agents are endowed with one unit of time per period, which they split between leisure and market labor. During each period of working life, an agent's labor earnings are given by:

$$y_j = we_j h_j (1 - \bar{h}_j)$$

where w represents a wage rate per efficiency unit of labor, e_j is the agent's idiosyncratic labor productivity drawn at age j, h is the time the agent chooses to work at age j, and $\bar{h}_j \in [0, 1]$ is an idiosyncratic shock to an agent's time endowment drawn at age j. We now elaborate on labor productivity shocks and time endowment shocks.

First consider shocks to agents' time endowments. In each period an agent may receive an exogenous unemployment shock which we denote with the idiosyncratic time endowment shocks $\bar{h} \in [0, 1]$ such that $1 - \bar{h}$ is the agent's available time allocation. As we will describe momentarily, let $\alpha \in \mathbb{R}$ be an agent's type. Assume that labor endowment shocks are binomially distributed with $\bar{h} \in \{0, d_j(\alpha)\}$ for age-dependent functions $d_j : \mathbb{R} \to (0, 1]$ with age-varying probability of unemployment $p_j(\alpha)$ for each j.

Next consider the stochastic process for idiosyncratic labor productivity shocks. Following Kaplan (2012), we decompose the stochastic process for e into four sources:

$$\log(e_j) = \alpha + \theta_j + \nu_j + \epsilon_j$$

where each component represents:

(i) $\alpha \overset{iid}{\sim} \mathcal{N}(0, \sigma_\alpha^2)$ is an individual-specific fixed effect that is drawn at birth,

(ii) $\{\theta_j\}_{j=1}^J$ is an age-specific fixed effect,

(iii) ν_j is a persistent shock that follows an autoregressive process

$$\nu_{j+1} = \rho \nu_j + \eta_{j+1}$$

with $\eta \overset{iid}{\sim} \mathcal{N}(0, \sigma_\nu^2)$ and $\eta_1 = 0$, and

(iv) $\epsilon_j \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$ is a per-period transitory shock.

For notational compactness, denote the stochastic state as a vector $\varepsilon = (\alpha, \nu, \epsilon, \bar{h})$, which contains each stochastic element necessary for computing contemporaneous labor earnings and forming expectations about future labor earnings. Denote the Markov process governing the process for ε by $\pi_j(\varepsilon'|\varepsilon)$ for each $j = 1, \ldots, J_{ret}$ and for all $J_{ret} \in [\underline{J}_{ret}, \bar{J}_{ret}]$.

2.3 Government Policy

Government: The government (i) consumes an exogenous amount G, (ii) collects linear Social Security taxes τ_{ss} on all pre-tax labor income below an amount $\bar{y}$, (iii) distributes lump-sum Social Security payments b_{ss} to retired agents, (iv) distributes unemployment benefits b_{ui} to agents who receive unemployment shocks, (v) distributes accidental bequests as lump-sum transfers Tr, and (vi) taxes each individual's taxable income according to an increasing and concave function $\Upsilon(\cdot)$.

Social Security: The Social Security system consists of the following objects,

$$\left(\{x_j\}_{j=1}^{J}, \{b_i^{ss}, \tau_{ri}\}_{i=1}^{3} \{\kappa\}_{i=1}^{2}, b_{base}^{ss}(x), b_{ss}\right)$$

such that

(i) an individual's average labor earnings, which are a state variable for future benefits:

$$x_{j+1} = \left\{ \begin{array}{ll} \frac{1}{j}\left(\min\{y_j, \bar{y}\} + (j-1)x_j\right) & \text{for } j \leq 35 \\ \max\left\{x_j, \frac{1}{j}\left(\min\{y_j, \bar{y}\} + (j-1)x_j\right)\right\} & \text{for } j \in (35, J_{ret}) \\ x_j & \text{for } j \geq J_{ret} \end{array} \right\}$$

where we recall that labor income is $y = weh(1 - \bar{h})$.

(ii) the pre-adjustment benefit for each retiree $b_{base}^{ss}(x_{J_{ret}})$ is computed as a piecewise-linear function over the individual's average labor earnings at retirement $x_{J_{ret}}$:

$$b_{base}^{ss}(x_{J_{ret}}) = \left\{ \begin{array}{lll} \tau_{r1} & \text{for} & x_{J_{ret}} \in [0, b_1^{ss}) \\ \tau_{r2} & \text{for} & x_{J_{ret}} \in [b_1^{ss}, b_2^{ss}) \\ \tau_{r3} & \text{for} & x_{J_{ret}} \in [b_2^{ss}, b_3^{ss}) \end{array} \right\}$$

and,

(iii) the age-dependent Social Security benefits are such that there is an early retirement penalty and delayed retirement credit, determined by functions $\{\kappa_i\}$

$$b_{ss}(x_{J_{ret}}) = \left\{ \begin{array}{lll} (1-\kappa_1(NRA - J_{ret}))b^{ss}_{base}(x_{J_{ret}}) & \text{for} & \underline{J}_{ret} \leq J_{ret} < NRA - 1 \\ (1+\kappa_2(NRA - J_{ret}))b^{ss}_{base}(x_{J_{ret}}) & \text{for} & NRA \leq J_{ret} < \bar{J}_{ret} - 1 \end{array} \right\}$$

where $k_i(\cdot)$ are functions governing the benefits penalty or credit, $\underline{J}_{ret}$ is the earliest age agents can retire, NRA is the "normal retirement age" and $\bar{J}_{ret}$ is the latest age an agent can retire retirement age.

Lastly, the social security payroll tax is given by τ_{ss} with cap $\bar{y}$. We assume that half of the social security contributions are paid by the employee and half by the employer. Therefore, the consumer pays a payroll tax given by: $(1/2)\ \tau_{ss} \min\{y, \bar{y}\}$.

Unemployment Insurance: If an agent receives an unemployment shock, their labor time allocation is reduced to $1 - \bar{h}$. The government distributes unemployment benefits with a earnings dependent replacement rate, $b_{ui}(\varepsilon)$. Total unemployment benefits for an agent with earnings ε and who spends a fraction $\bar{h}$ of the period unemployed are given by $b_{ui}(\varepsilon)we\bar{h}$.

Income Taxation: Taxable income is defined defined as labor income, capital income and unemployment benefits net of social security contributions from the employer. Denote taxable income by:

$$\tilde{y}(h, a, \varepsilon) \equiv weh(1-\bar{h}) + r(a + Tr) + b_{ui}(\varepsilon)we\bar{h} - \frac{\tau_{ss}}{2}\min\{weh(1-\bar{h}), \bar{y}\}$$

The government taxes each individual's taxable income according to the function $\Upsilon(\tilde{y}(h, a, \varepsilon))$.

Net Government Transfers: We will define the functions $T_j(\cdot)$ and $T^{ret}_j(\cdot)$ as the government's total transfers to agents in social security payments and unemployment benefits net of income taxation and social security payroll taxation. Define net transfers during an agent's working ages (if $j < J_{ret}$) as:

$$T_j(h, a, \varepsilon) = b_{ui}(\varepsilon)we\bar{h} - \Upsilon(\tilde{y}(h, a, \varepsilon)) - \frac{\tau_{ss}}{2}\min\{weh(1-\bar{h}), \bar{y}\}$$

and define net transfers during an agent's retirement (if $j \geq J_{ret}$) as:

$$T_j^{ret}(a,x) = b_{ss}(x) - \Upsilon(r(a+Tr))$$

Public Debt: The government issues debt, denoted B' and services the gross amount rB each period.

Budget Constraint: The model's Social Security system is self-financing and therefore do not appear in the governmental budget constraint. The resulting government budget constraint is:

$$G + UI + rB = B' - B + \Upsilon_y$$

where UI is the aggregate payout of unemployment benefits, and Υ_y is aggregate revenues from income taxation.

2.4 Consumer's Dynamic Program

Agents have access to a single asset, a non-contingent one-period bond denoted a_j with pre-tax return of r. Therefore, total pre-tax income is $w\varepsilon_j h_j(1-\bar{h}_j) + (1+r_j)a_j$. Agents are subject to a borrowing constraint in which savings are bounded below by $\underline{a} \in \mathbb{R}$. Agents are endowed with zero initial assets such that $a_0 = 0$.

The agent's state variables consist of asset holdings a, income shocks $\varepsilon \equiv (\alpha, \nu, \epsilon, \bar{h})$ and Social Security contribution status x. The agent's recursive problem is divided into three stages of life: working life, near-retirement and retirement. For age $j < \underline{J}_{ret}$ agents, the dynamic program during working life is:

$$v_j(a,\varepsilon,x) = \max_{c,a',h} \left[u(c) - \chi_1\varphi(h) - \chi_2\right] + \beta s_j \sum_{\varepsilon'} \pi_j(\varepsilon'|\varepsilon) v_{j+1}(a',\varepsilon',x')$$

$$\begin{aligned} \text{s.t.} \quad c + a' &\leq w\varepsilon h(1-\bar{h}) + (1+r)(a+Tr) + T(h,a,\varepsilon) \\ a' &\geq \underline{a} \end{aligned} \tag{1}$$

for age $j \in [\underline{J}_{ret}, \bar{J}_{ret}]$ agents who qualify for retirement but are not retired:

$$v_j(a,\varepsilon,x) = \max_{c,a',h} \left[u(c) - \chi_1\varphi(h) - \chi_2\right] + \beta s_j \sum_{\varepsilon'} \pi_j(\varepsilon'|\varepsilon) \max\left\{v_{j+1}(a',\varepsilon',x'), v_{j+1}^{ret}(a',x')\right\}$$

$$\text{s.t.} \quad c + a' \leq w\varepsilon h(1-\bar{h}) + (1+r)(a+Tr) + T(h,a,\varepsilon) \tag{2}$$
$$a' \geq \underline{a}$$

and for retired agents who either choose retirement during ages $j = J_{ret} \in [\underline{J}_{ret}, \bar{J}_{ret}]$ or who are forced to retire $(j > \bar{J}_{ret})$ the dynamic program is:

$$v_j^{ret}(a,x) = \max_{c,a'} \; u(c) + \beta s_j v_{j+1}^{ret}(a',x)$$

$$\text{s.t.} \quad c + a' \leq (1+r)(a+Tr) + T^{ret}(a,x) \tag{3}$$
$$a' \geq \underline{a}$$

2.5 Recursive Competitive Equilibrium

In this section we will define a stationary recursive competitive equilibrium. Let $\lambda_j(a, \varepsilon, x, \mathbb{1}_{ret})$ denote the joint distribution over wealth,income, average past earnings, retirement status and age, where $\mathbb{1}_{ret} = 1$ if the agent is retired and $\mathbb{1}_{ret} = 0$ otherwise. We will study stationary equilibria. A *stationary recursive competitive equilibrium* is a recursive competitive equilibrium in which aggregate quantities and prices are time invariant. We define equilibrium as follows.

Given a government policy $(G, B, b_{ui}, \Upsilon, \tau_{ss}, b_{ss})$, a *stationary recursive competitive equilibrium* is (i) an allocation for consumers described by policy functions $\{c_j, a'_j, h_j\}_{j=1}^{J}$, retirement decision J_{ret} and consumer value functions $\{v_j, v_j^{ret}\}_{j=1}^{J}$, (ii) an allocation for the representative firm (K, L), (iii) prices (w, r), (iv) accidental bequests Tr, and (v) the distribution over agents λ that satisfy:

(a) Given prices, policies and accidental bequests, $v_j(a, \varepsilon, x)$ and $v_j^{ret}(a, x)$ solve Bellman equations (1), (2) and (3) with associated policy functions $\{c_j(a, \varepsilon, x), a'_j(a, \varepsilon, x), h_j(a, \varepsilon, x)\}$ and J_{ret},

(b) Given prices (w, r), the representative firm's allocation minimizes cost:

$$r = \alpha A F_K(K, L) - \delta$$

$$w = (1-\alpha) A F_L(K, L)$$

(c) Accidental bequests satisfy:

$$Tr = \frac{\sum_{j=1}^{J}(1-s_j)\int a_j'(a,\varepsilon,x)d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret})}{\sum_{j=1}^{J}\int d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret})}$$

(d) Government policies satisfy budget balance:

$$G + UI + rB = B' - B + \Upsilon_y$$

aggregate income tax revenue is given by:

$$\Upsilon_y \equiv \sum_{j=1}^{J}\int \Upsilon\Big(\tilde{y}(h(a,\varepsilon,x),a,\varepsilon)\Big)\ d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret})$$

aggregate unemployment insurance income is given by:

$$UI \equiv \sum_{j=1}^{J}\int b_{ui}(\varepsilon)we\bar{h}\ d\lambda_j(a,\varepsilon,x,0)$$

(e) Social security is self-financing:

$$\sum_{j=1}^{J_{ret}-1}\int \tau_{ss}\min\{weh_j(a,\varepsilon,x)(1-\bar{h}),\bar{y}\}d\lambda_j(a,\varepsilon,x,0) = \int \sum_{j=J_{ret}}^{J} b_{ss}(x)d\lambda_j(a,\varepsilon,x,1)$$

(f) Given policies and allocations, prices clear asset and labor markets:

$$K + B = \sum_{j=1}^{J}\int a\ d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret})$$

$$L = \sum_{j=1}^{J}\int \varepsilon h_j(a,\varepsilon,x)\ d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret})$$

and the allocation satisfies the resource constraint:

$$\sum_{j=1}^{J}\int c_j(a,\varepsilon,x)d\lambda_j(a,\varepsilon,x,\mathbb{1}_{ret}) + G + K' = AF(K,L) + (1-\delta)K$$

(g) the distribution λ is a fixed point of the T^*-operator such that $\lambda_{j+1} = T^*(\lambda_j)$ or,

$$\lambda_{j+1}(\mathcal{A} \times \mathcal{E} \times \mathcal{X} \times \mathcal{R}) = \sum_{\mathbb{1}_{ret} \in \{0,1\}} \int_{A \times E \times X} Q_j\left((a, \varepsilon, x, \mathbb{1}_{ret}), \mathcal{A} \times \mathcal{E} \times \mathcal{X} \times \mathcal{R}\right) d\lambda_j$$

where the transition function Q_j gives the probability that an age-j agent with current state (a, ε, x) transits to the set $\mathcal{A} \times \mathcal{E} \times \mathcal{X} \times \mathcal{R}$ at age $j+1$. The transition function is given by:

$$Q_j\left((a, \varepsilon, x, \mathbb{1}_{ret}), \mathcal{A} \times \mathcal{E} \times \mathcal{X} \times \mathcal{R}\right) = \left\{ \begin{array}{ll} s_j \cdot \pi_j(\mathcal{E}|\varepsilon) & \text{if } a'_j(a, \varepsilon, x, \mathbb{1}_{ret}) \in \mathcal{A},\ x' \in \mathcal{X},\ J_{ret}+1 \notin \mathcal{R} \\ s_j & \text{if } a'_j(a, \varepsilon, x, \mathbb{1}_{ret}) \in \mathcal{A},\ x' \in \mathcal{X},\ J_{ret}+1 \in \mathcal{R} \\ 0 & \text{otherwise} \end{array} \right\}$$

(h) Aggregate capital, governmental debt, prices and the distribution over consumers are stationary, such that $K' = K$, $B' = B$, $w' = w$, $r' = r$, and $\lambda' = \lambda$.

3 Calibration

Demographics: We set the conditional survival probabilities $\{s_j\}_{j=1}^{J}$ according to Bell and Miller (2002) and impose $s_J = 0$. We set the populational growth rate to $g_n = 0.011$ to match annual population growth in the US.

Production: The production function is assumed to be Cobb-Douglas of the form $F(K, L) = K^{\zeta} L^{1-\zeta}$ where $\zeta = 0.36$. The depreciation rate is to $\delta = 0.26$ which allows the model to match the empirically observed investment-to-output ratio.

Preferences: The utility function is is separable in all three terms and features a constant Frisch elasticity:

$$U_j(c, h, J_{ret}) = \frac{c^{1-\sigma}}{1-\sigma} - \chi_1 \frac{h^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} - \chi_2 \mathbb{1}(j < J_{ret})$$

This functional form implies that the utility is constant relative risk aversion where σ controls the risk aversion. We set the coefficient of relative risk aversion $\sigma = 2$ consistent with Conesa et al. (2009) and Aiyagari and McGrattan (1998). We choose $\gamma = 0.5$ such that the Frisch elasticity is equal to 0.5 in line with the estimates in Kaplan (2012). We calibrate the labor disutility parameter χ_1 so that agents work one-third of their available time endowment prior to the normal retirement age. The calibrated value is $\chi_1 = 74.4$. Finally, χ_2 is a fixed utility cost of being eligible to earn labor income which is no longer paid once an agent retires. The fixed cost creates a non-convexity in the utility function, which is necessary to generate an extensive margin decision while respecting a realistic calibration for the Frisch elasticity. We set $\chi_2 = 1.095$ to match the empirically observation that seventy percent of the population has retired by the normal retirement age.

Labor Productivity Process: We take the labor productivity process from the estimates in Kaplan (2012) based on the estimates from the PSID data.[3] The deterministic labor productivity profile, $\{\theta_j\}$, is (i) smoothed by fitting a quadratic function in age, (ii) normalized such that the value equals unity when an agent enters the economy, and (iii) extended to cover ages 20 through 69 which we define as the last period in which agents are assumed to be able to participate in the labor activities ($\overline{J}_{ret}$).[4] The permanent, persistent, and transitory idiosyncratic shocks to individual's productivity are distributed normal with a mean of zero. The remaining parameters are also set in accordance with the estimates in Kaplan (2012): $\rho = 0.958$, $\sigma_\alpha^2 = 0.065$, $\sigma_\nu^2 = 0.017$ and $\sigma_\epsilon^2 = 0.081$. We discretize all three of the shocks in order to solve the model, representing the transitory shock with two states, the permanent shock with two states, and the persistent shock with five states. For expositional convenience, we refer to the two different states of the permanent shock as high and low ability types.

Unemployment Shocks and Insurance: Unemployment benefits qualifications and payouts are administered by individual states and, therefore, are not consistent throughout the US. Unemployment insurance programs vary by replacement rate of past income, maximum benefit caps and formulas that determine eligibility from income history. Because there is no single statutory replacement formula, we model a UI program that captures the average unemployment insurance program in the US.

Using CPS March Supplemental data from 1990 to 2005 we compute the age and ability type dependent probability of receiving an unemployment shock $p_j(\alpha)$, the age and type dependent unemployment duration $d_j(\alpha)$, and the earnings shock and type dependent unemployment insurance replacement rate $b_{ui}(\varepsilon)$. We compute the fraction of unemployed agents by age and educational attainment level (college or higher are assigned to high types) to obtain values for $p_j(\alpha)$. Then, conditional on being unemployed, we compute the average unemployment duration across individuals for each age and educational attainment level to obtain values for $d_j(\alpha)$. The upper panel of Figure 1 shows the aggregate unemploy-

[3]For details on estimation of this process, see Appendix E in Kaplan (2012).
[4]The estimates in Kaplan (2012) are available for ages 25-65.

ment probabilities and durations by age. We observe that while unemployment probabilities decline over the life cycle, unemployment duration increases.

We compute the unemployment insurance replacement rate as weekly unemployment benefit payments as a fraction of weekly wages while employed.[5] This provides us with values for the unemployment insurance replacement rate b_{ui}(past income). Note that actual unemployment insurance programs determine eligibility and benefits from past income, and the determination formulas are very heterogeneous across states in terms of length of income history used. Instead of increasing the size of the model's state space with income histories, we proxy income histories with the permanent and predetermined components of labor productivity that are highly serially correlated. Therefore, the weekly unemployment insurance replacement in the model are a function of agents' age-specific productivity $\{\theta_j\}_{j=1}^{J}$ and ability α. Accordingly, the model's unemployment benefits replacement rate is a function of $w \exp(\alpha + \theta_{j-1})$ and $b_{ui}(\varepsilon) we\bar{h}$ is the total benefit.[6] The lower panel of Figure 1 shows the aggregate weekly replacement rate by earnings, which we observe is a decreasing function. Average earnings imply a replacement rate of 35%.

Government: Consistent with Aiyagari and McGrattan (1998) we set government debt equal to two-thirds of output. We set government consumption equal to 15.5 percent of output consistent. This ratio corresponds to the average of government expenditures to GDP from 1998 through 2007.[7]

Social Security: We set the normal retirement age at 66. Consistent with the minimum and maximum retirement ages in the U.S. Social Security system, we set the interval in which agents can retire to between the ages of 62 and 70. The early retirement penalty and later retirement credits are set in accordance with the Social Security program. In particular, if agents retire up to three years before the normal retirement age agents benefits are reduced by 6.7 percent for each year they retire early. If they choose to retire four or five years before the normal retirement age benefits are reduced by an additional 5 percent for these years. If agents choose to delay retirement past normal retirement age then their benefits are increased by 8 percent for each year they delay. The marginal replacement rates in the progressive Social Security payment schedule $(\tau_{r1}, \tau_{r2}, \tau_{r3})$ are also set at their actual respective values of 0.9, 0.32 and 0.15. The bend points where the marginal replacement rates change (b_1^{ss}, b_2^{ss},

[5]The March CPS contains weekly wages. To find the weekly unemployment insurance benefit, we divide an individual's annual unemployment benefit income by their observed unemployment duration.

[6]Furthermore, state unemployment insurance programs define a maximum benefit. Because we construct the weekly replacement rate from observed payouts and because average unemployment durations are relatively short, our model does not allow benefits to exceed the average empirical maximum.

[7]We exclude government expenditures on Social Security since they are explicitly included in our model.

b_3^{ss}) and the maximum earnings ($\overline{y}$) are set equal to the actual multiples of mean earnings used in the U.S. Social Security system so that b_1^{ss}, b_2^{ss} and $b_3^{ss} = \overline{y}$ occur at 0.21, 1.29 and 2.42 times average earnings in the economy. We set the payroll tax rate, τ_{ss} such that the program's budget is balanced. In our benchmark model the payroll tax rate is 10.3 percent, roughly equivalent with the statutory rate.[8]

Income Taxation: The income tax function and parameter values are from Gouveia and Strauss (1994). The functional form is:

$$\Upsilon(y) = (1 - \tau_0)\left(y - \left(y^{-\tau_1} + \tau_2\right)^{-\frac{1}{\tau_1}}\right)$$

The authors find that $\tau_0 = 0.258$ and $\tau_1 = .768$ closely match the U.S. tax data. When calibrating the model we set τ_2 such that the government budget constraint is satisfied.

4 Quantitative Analysis

In this section, we demonstrate that governments in life cycle models have a strong incentive to accumulate capital instead of holding debt. We show this through a series of computational experiments. First we design an experiment to measure the pure life-cycle incentives for government debt issuance or savings. We show that the hump-shaped life cycle saving profile is crucial for explaining the planner's savings incentives. We then show how these incentives interact with changes in other fiscal policies. Moreover, we demonstrate that allowing agents, especially young liquidity constrained agents, to borrow does not alter our findings. However, we show that the optimal quantity of public debt or savings is particularly sensitive to the social welfare criterion in a life cycle model. We show that a simple voting-mechanism can be cast as a social welfare criterion with Pareto weights that place greater weight on older generations than does an ex ante Utilitarian criterion. The voting mechanism can rationalize the empirically observed level of US debt.

[8] Although the payroll tax rate in the U.S. economy is slightly higher than our calibrated value, the OASDI program includes additional features outside of the retirement benefits.

4.1 Optimal Public Debt

The government chooses debt B to maximize the expected lifetime utility of a newborn agent in the economy. The following defines the government's ex-ante Utilitarian social welfare function:

$$S(v, \lambda) \equiv \max_{B} \int v_0(a, \varepsilon, x; B)\ d\lambda_0(a, \varepsilon, x; B) \tag{4}$$

where the value function v_0 and distribution λ_0 are determined in competitive equilibrium and depend on the government's choice of debt.

When debt changes, the aggregate variables change along with total taxable income and the revenues from the tax policies. We adjust the payroll tax rate τ_{ss} to ensure Social Security is self-financing and adjust the income tax parameter τ_0 to ensure government budget is balanced. We choose to use τ_0 to balance the government budget instead of the other income taxation parameters (τ_1, τ_2) so that the average income tax rate is used to clear the budget, as opposed to changing in the progressivity of the income tax policy. Furthermore, the average tax rate is the closest analogue to the flat tax that Aiyagari and McGrattan (1998) use to balance the government's budget in their model.

In first panel of Figure 2, we see that while the model was calibrated to a level of debt corresponding to two-thirds of output, the optimal government policy is to accumulate savings. Optimal public savings is 1.6 times larger than output and the value of social welfare increases by 3.1%. In contrast, we solve for the optimal policy in a similar model that is only altered to extend agents' lifetime to the infinite horizon. Similarly to Aiyagari and McGrattan (1998), we find that the optimal policy in the infinitely lived agent model is to hold a considerable amount of debt (see section 4.2 below). Accordingly, the life cycle model leads to drastically different optimal government debt decisions than does an infinitely lived agent model.

In the bottom panels of Figure 2, we observe that relative to the calibrated level of debt, optimal public savings increases aggregate effective labor and productive capital. Because productive capital is an order of magnitude larger than aggregate effective labor, the interest rate decreases while the wage increases with public savings. As a result, total income rises and less income taxation is required to satisfy the government's budget constraint, which is consistent with the second panel in Figure 2. Notice that as public savings become larger than 0.9 times output, the budget balancing tax rate must increase. This is because public savings decreases the interest rate and eventually decreases total income. Once total income becomes sufficiently small, the tax base does not increase with wages and the government must tax income at a higher rate.

Figure 3 illustrates the average consumption, savings, hours worked and value function across each agent at each age. The figure shows substantial front-loading of consumption and leisure under the optimal policy relative to the baseline debt-to-output ratio of two-thirds. In fact, the optimal value function exceeds the baseline only during the first 10 periods of life. Lower interest rates largely account for the increased front-loading of consumption and leisure. Lastly, government savings crowds out private savings at each point during the life cycle.

4.2 Isolating Life Cycle Motives

Next we examine why optimal policy is different in a life cycle compared to an infinitely lived agent model. To what extent do life cycle motives encourage the government to accumulate capital instead of debt? In order to understand the government's optimal savings decision we find it instructive to compare the following two economies: (i) an economy in which agents live for 80 periods, with (ii) an economy in which agents live indefinitely. To make these economies comparable, we exclude social security, endogenous retirement and random death from the model. Instead, agents live for the maximum lifetime with certainty and may choose to work in each period. All other assumptions and model ingredients are unchanged.

In Figure 4 we immediately and clearly observe that while the government optimally chooses to save in the 80-period lifetime model, the government chooses to accumulate a large quantity of *debt* in the infinite horizon model. In other words, the length of an agent's lifetime matters and can cause the government to switch from significant savings to significant debt.

In the finite horizon (80-period lifetime) model agents enter the model liquidity constrained with zero assets. Figure 5 plots the life cycle profile of assets compared to the infinitely lived model. Unlike the infinitely lived agent model where agents already hold a stock of savings that fluctuates around a target level, agents that inhabit the finite life model must accumulate assets over time. If the government holds more debt the resulting higher interest rate will encourage agents to hold a larger stock of savings. However, a larger interest rate does not provide young agents with much incentive to save more. Young agents begin life with no endowment of assets and would borrow if they were not constrained. As a result, the government can increase young agent's utility by accumulating public savings, increasing the economy's capital stock and propping up the wage. The higher wage due to governmental savings is a wealth effect that is equivalent to loosening early life borrowing constraints.

Under the ex ante Utilitarian welfare criterion, the government discounts utility flows derived from later-in-life consumption and therefore, in accordance with the last paragraph, wishes to relax early-in-life borrowing constraints. On average, consumers face a hump-shaped life cycle consumption profile that delays consumption to middle-to-late life. The government optimally chooses maximal public savings to tilt the timing of consumption toward early life.

4.3 Liquidity Constraints

In this subsection, we study the effect of relaxing borrowing constraints.

Aiyagari and McGrattan (1998) find that in their infinitely lived agent model in which agents are disallowed from borrowing, the main channel through which the government holding additional debt (less savings) increases welfare is that it encourages agents to hold more private saving. This loosens borrowing constraints and increases private buffer stocks against which agents insure against negative income shocks.

Winter and Roehrs (2015) examine an alternative infinitely lived agent economy and find that it is optimal for the government to save as opposed to hold debt. They also find that in this model they relax private borrowing constraints then the optimal level of government savings decreases by the equivalent of 20 percent of GDP. Thus private liquidity constraints can have a quantitatively important effect on the optimal level of debt or savings in an infinitely lived agent model.

In order to isolate the quantitative effect that liquidity constraints play in our life cycle model, we determine the optimal government savings or debt in a model where agents are allowed to borrow. We relax the no borrowing constraint and impose an age-dependent natural debt limit that ensures agents do not borrow more than they can repay over their expected lifetime. Because agents face age-dependent mortality risk, the interest rate on debt accounts for survival probabilities.

We find that when agents are allowed to borrow it is still optimal for the government to hold savings equal to 160 percent of output, the same optimal policy as in the benchmark model when private borrowing is dissallowed. Hence relaxing potential private liquidity constraints by allowing agents to borrow plays a negligible role in the optimal level of government savings or debt in a life cycle model. Figure 6 depicts this result in the upper left panel, as well as showing that aggregate outcomes are approximately unchanged by relaxing borrowing constraints.

Relaxing borrowing constraints has a much smaller quantitative effect on the optimal level of government savings or debt in a life cycle model for two reasons: agents save to insure against adverse income shocks and to finance post-retirement consumption. In either case, agents begin accumulating savings early in their lifetime. Even if agents face a string of negative income shocks early in life, savings motives are strong enough to discourage agents from borrowing. Later in the life cycle, agents have a large stock of savings so borrowing has no value. In fact, if an agent receives a series of negative income shocks, the agent can always draw down savings meant for retirement. Figure 7 displays the hump shaped savings profile over the life cycle.

In contrast, agents in an infinite-life model do not need to save for retirement and thus will only hold enough savings to insure against adverse shocks. If they receive a particularly bad spell of income shocks they are more likely to deplete their savings. Hence, relaxing borrowing constraints in an infinitely live agent model benefits agents relatively more than in a life cycle model. The upshot is that government debt is more valuable in an infinitely lived agent model since public debt pushes up interest rates, thereby enhancing agent's liquidity.

Furthermore, since income shocks are mean reverting in an infinitely lived agent model, income shocks exert a small effect on total lifetime income. If an agent inhabiting an infinitely lived model receives a series of adverse shocks that depletes his savings, the agent wants to borrow in order to smooth consumption. This agent can effectively intertemporally substitute consumption from the future. In contrast, an agent inhabiting a life cycle model only works for fifty periods. The finitely lived agent's expected lifetime income is impacted more from receiving a series of adverse shocks throughout his working life. The finitely lived agent faces a larger opportunity cost intertemporally substitute consumption from the future and therefore values borrowing less. Therefore, since agents do not substantially increase borrowing, the government does not have an incentive to change its optimal level of savings.

4.4 Interaction with Government Transfers

Since one of the two motives for private savings in the life cycle model is to fund post-retirement consumption, we examine the interaction between Social Security and the optimal government policy. In order to examine this interaction we examine two policy counterfactuals. First, Social Security is not required to have a balanced budget except at the baseline level of debt. This is achieved by fixing payroll taxes and benefit formulas at their baseline values, when debt is two-thirds of output. In the second counterfactual, Social Security insurance is completely stripped from the model by setting the payroll tax and the retirement benefit to zero at each age. Figures 8 and 9 depict the results from these two policy counterfactuals.

Examining the first counterfactual, we find that that optimal level of savings decreases when the Social Security tax and replacement formulas are held fixed at their baseline levels. Thus, the current Social Security program creates a small incentive for more government savings. This is because holding more public savings interacts with the Social Security system in a way that increases utility for agents both when they are young and old. Increasing public savings leads to higher productive capital and wages. Since the Social Security benefits formula is progressive, higher labor income leads Social Security benefits to replace a smaller percentage of lifetime income. The relatively smaller size of the Social Security program implies that the payroll tax rate needed to clear the Social Security budget is also smaller. Agents appreciate the lower payroll taxes when they are young because it leaves them more after-tax income and eases liquidity constraints. Moreover, the interaction between Social Security and increasing public savings also increases the amount of utility agents receive later in their lifetimes. In particular, more public savings leads to higher wages and lifetime income. Thus, the level of the Social Security benefits that agents receive is higher implying more consumption for agents once they retire. Thus, Social Security interacts with government savings in such a way that leads to higher level of optimal government savings.

Although removing Social Security eliminates the previously mentioned interaction that leads to more government savings being optimal, completely removing the program also makes private savings less responsive to changes in the interest rate since agents need to fully fund from savings all post-retirement consumption. Figure 8 shows that as a result prices are more sensitive to changes in public savings in the absence of a Social Security system since there is less private crowd out. Thus, the government does not want to hold as much savings to avoid driving the interest rate to zero. Overall, these results demonstrate that the positive welfare interaction between Social Security and small changes in public savings is offset by the general equilibrium effects when Social Security is removed.

4.5 Alternative Welfare Criteria

We have shown that the fully articulated quantitative model, which resembles US fiscal policy and the distribution of consumer behavior, generates optimal public savings. However, the optimal policy in a life cycle model may be sensitive to our choice of ex ante Utilitarian social welfare function (SWF). In order to check robustness, we examine the effect on optimal policy if the government discounts the future differently from private agents. We show that optimal policy differs if one considers the whole living population in the steady state as opposed to maximizing the expected utility of an agent prior to entering the steady state.

Probabilistic Voting Mechanism: In order to demonstrate the effect of the social discount rate on optimal policy assume that private individuals discount with factor β and have a stream of utility given by:

$$v_j(a_j, \varepsilon_j, x_j) = \sum_{\tau=j}^{J} \beta^{\tau-j} \mu_\tau \mathbb{E}\left[U_\tau(c_\tau, h_\tau, J_\tau) \mid \varepsilon_j\right]$$

where $\mu_\tau \equiv \left(\prod_{i=1}^{\tau} s_i\right)$ is the unconditional probability that an agent lives to age τ. By a law of large numbers, μ_j is also the population share of age-j agents.

Now assume that the government uses welfare weights on each generation, with weights denoted by $\{\alpha_j\}_{j=1}^{J}$ such that $\alpha_j > 0$ for all j. Then the government values agents' welfare according to the following social welfare function:

$$\sum_{j=1}^{J} \alpha_j \mathbb{E}[v_j(a_j, \varepsilon_j, x_j) \mid \varepsilon_1] = \sum_{j=1}^{J} \left(\sum_{t=1}^{j} \alpha_t \beta^{j-t} \mu_j \right) \mathbb{E}\left[U_j(c_j, h_j, J_j) \mid \varepsilon_1\right] \tag{5}$$

This alternative SWF evaluates the welfare of the current population of agents by each age, instead of only valuing the newborn generation (which corresponds to weights of $\alpha_1 = 1$ and $\alpha_j = 0$ for all $j > 1$).

An important special case coincides with $\alpha_j = 1$ for all j. Under this selection of welfare weights, the SWF is equivalent to a *probabilistic voting mechanism*[9] in which the government uses its policy instrument to maximize the votes across all agents. The equivalence arises because an agent votes for a particular policy if it maximizes his value $v_j(a, \varepsilon, x)$ and because the government attempts to maximize votes by aggregating using the equilibrium distribution

[9]Several nice papers in the New Dynamic Public Finance literature have developed this equivalence, such as Farhi, Sleet, Werning, and Yeltekin (2012) who model probabilistic voting as an explicit political game consistent with Lindbeck and Weibull (1987).

(here, represented by the $\mathbb{E}$ operator).

If we define $\delta_j \equiv \sum_{t=1}^{j} \alpha_t \beta^{j-t} \mu_j$ then the discount factor satisfies:

$$\frac{\delta_{j+1}}{\delta_j} = \beta \mu_{j+1} + \frac{\alpha_{j+1}\mu_{j+1}}{\delta_j} > \beta \mu_{j+1}$$

so social preferences place greater value on older generations than do individual preferences.

Illustrative Example: In order to understand how the government's generational preferences impact optimal policy, we will consider a stark example. Suppose that the welfare weights belong to the class of geometric functions of age, satisfying $\hat{\beta}^j \mu_j \propto \sum_{t=1}^{j} \alpha_t \beta^{j-t} \mu_j$ which we can show attains under correctly specified parameter assumptions. Then the welfare criterion is:

$$S_{\hat{\beta}}(v, \lambda) = \max_B \sum_{j=1}^{J} \hat{\beta}^j \mu_j \mathbb{E}\left[U_j\Big(c_j, h_j, J_j; v_j(\cdot\,; B)\Big) \mid \varepsilon_1, \lambda_j(\cdot\,; B)\right]$$

Notice that when $\hat{\beta} = \beta$ then we recover the ex-ante Utilitarian objective of maximizing the value of newborn generations:

$$\begin{aligned} S_{\beta}(v, \lambda) &= \max_B \sum_{j=1}^{J} \beta^j \mu_j \mathbb{E}\left[U_j\Big(c_j, h_j, J_j; v_j(\cdot\,; B)\Big) \mid \varepsilon_1, \lambda_j(\cdot\,; B)\right] \\ &= \max_B \int v_0(a, \varepsilon, x; B) d\lambda_0(a, \varepsilon, x; B) \\ &= S(v, \lambda) \end{aligned}$$

Figure 10 plots optimal debt-to-output ratios under the $S_{\hat{\beta}}$ social welfare function for values of $\hat{\beta}$ between 0.975 and 1.025. The figure clearly shows that debt is optimal when $\hat{\beta}$ sufficiently larger than β. The mechanism behind this result can be seen from Figure 2 and Figure 3. When older generations receive a larger α_j weight, the government now wants to back-load instead of front-load utility, which is accomplished by using debt to push up interest rates in order to tilt forward the timing of consumption.

Comparing Social Welfare Criteria: Now we compare optimal debt under the competing social welfare functions:

- SWF in equation (4): the ex ante Utilitarian criterion that maximizes the expected present value of newborn generations, versus

- SWF in equation (5): the probabilistic voting criterion that maximizes the population weighted expected present value of each living agent.

Figure 11 plots each social welfare function as a function of government savings. We find that not only is debt optimal under the probabilistic voting mechanism, but also optimal debt is equivalent to 90 percent of output. The mechanism underlying this result echoes the previous section: when the government values the utility received from agents when they are older more, the government increases the expected present value of older agents by using debt to crowd out productive capital and exert upward pressure on the interest rate. With a higher interest rate, agents have an incentive to delay consumption to later life which increases the expected present value of consumption for older generations.

Strikingly, the alternative social welfare function generates a debt-to-output ratio that is very similar to the empirically observed two-thirds of output – which is also the approximate optimal quantity of debt in Aiyagari and McGrattan (1998). We interpret the result along two dimensions. First, relative to the ex ante Utilitarian maximization of newborns' utility, the probabilistic voting mechanism is arguably more consistent with the real-world pressures of government officials to please its constituency. In this sense, time inconsistency or lack of commitment of governments creates deviations from ex ante maximization of newborn generations' utility. Second, the probabilistic voting mechanism is seemingly more consistent with welfare evaluation in infinite horizon models. The implication is that ex ante Utilitarianism in life cycle models only considers within cohort variation in utility of young generations while in infinitely lived agent models ex ante Utilitarianism only considers within cohort variation in utility of the analogue of old generations.

5 Conclusion

In this paper we examine whether the optimal level of government debt or savings changes when determined in a life cycle model as opposed to an infinitely lived agent model. In an infinitely lived agent model the government holds debt in order to crowd out productive capital, increase the interest rate, and lead agents to hold more savings. In a life cycle model we find that it is not longer optimal for the government to hold debt but instead to hold savings equivalent to 160 percent of output. We demonstrate that this change in optimal policy is due to reduced benefit to encouraging agents to be more liquidity. In particular, in a life cycle model young agents accumulate savings early in life to prepare for their eventually retirement and thus few agents face binding borrowing constraints. Thus, introducing life cycle elements can have large affects on optimal policy.

Overall, we find that the optimal government policy in a life cycle model is fairly insensitive to government transfer policies like Social Security or to whether agents are allowed to borrow. However, we do find that in a life cycle model optimal policy is sensitive to the social welfare function. Although we find that optimal policy is to hold savings in te life cycle model when we use a social welfare funciton that is consistent with the criteria used in the infinitely lived agent experiments, we find that optimal policy is quite different under other social welfare functions. In particular, if the criteria is to maximize the expected future lifetime utility of the whole population in the steady state as opposed to just maximizing the utility of a newborn, then the optimal policy changes from the government holding saivngs to holding debt. We find under this alternative social welfare function that the government wants to hold debt equivalent to 90 perecent of output. This level of debt is in line both with current empirical observations in the U.S. and also the results in Aiyagari and McGrattan (1998).

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6 Appendix

6.1 Tables and Figures

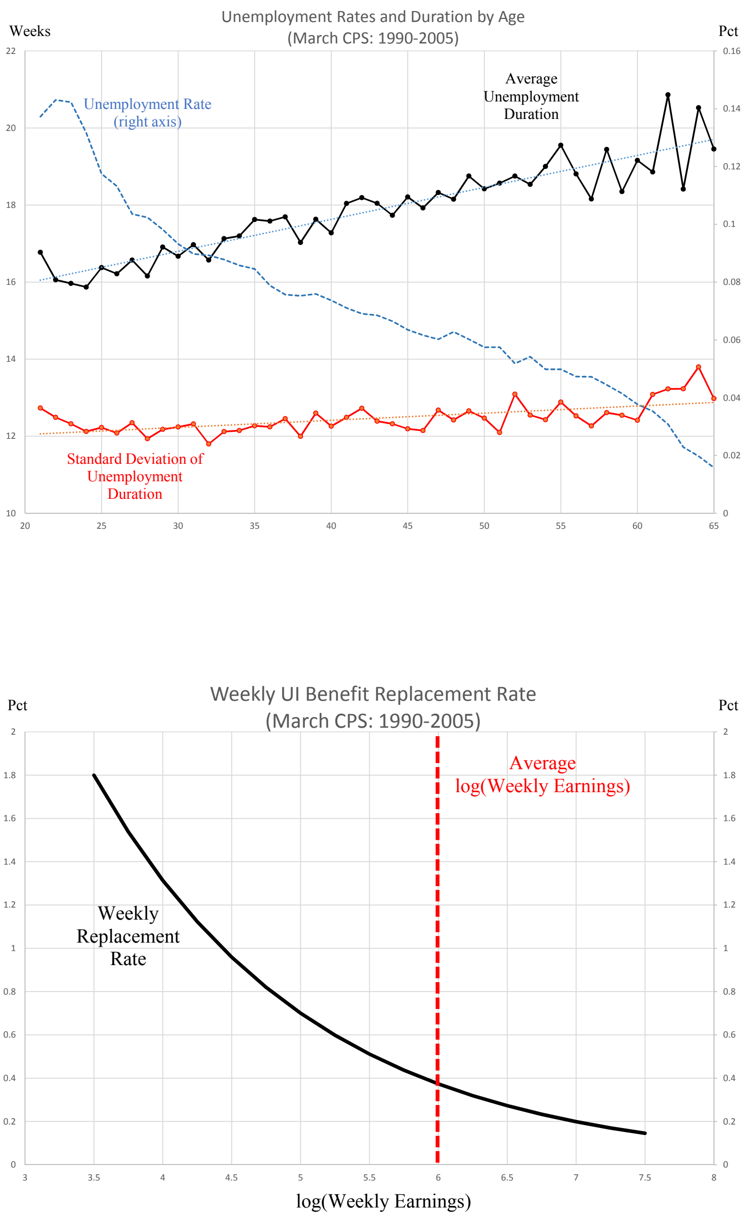

Figure 1: The top panel plots unemployment duration (average and standard deviation with fitted linear trend) and fraction of unemployed individuals by age. The bottom panel plots the computed weekly replacement rate against the logarithm of weekly earnings.

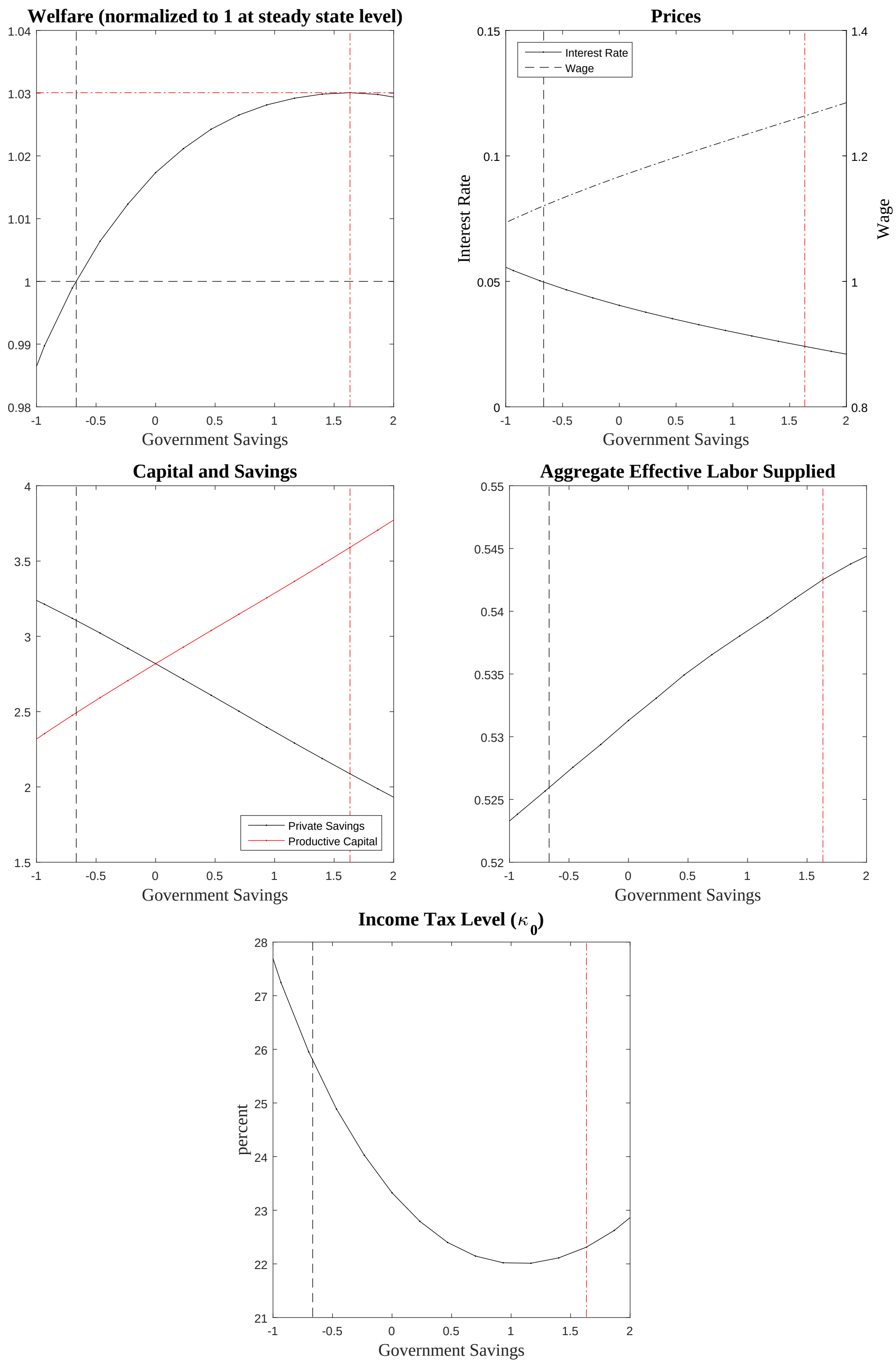

Figure 2: Baseline results compare aggregate outcomes (welfare, savings and taxation) across varying levels of government savings as a fraction of output. The vertical line at -0.67 marks the baseline level of government debt, while the vertical line at 1.4 marks the optimal level of government savings.

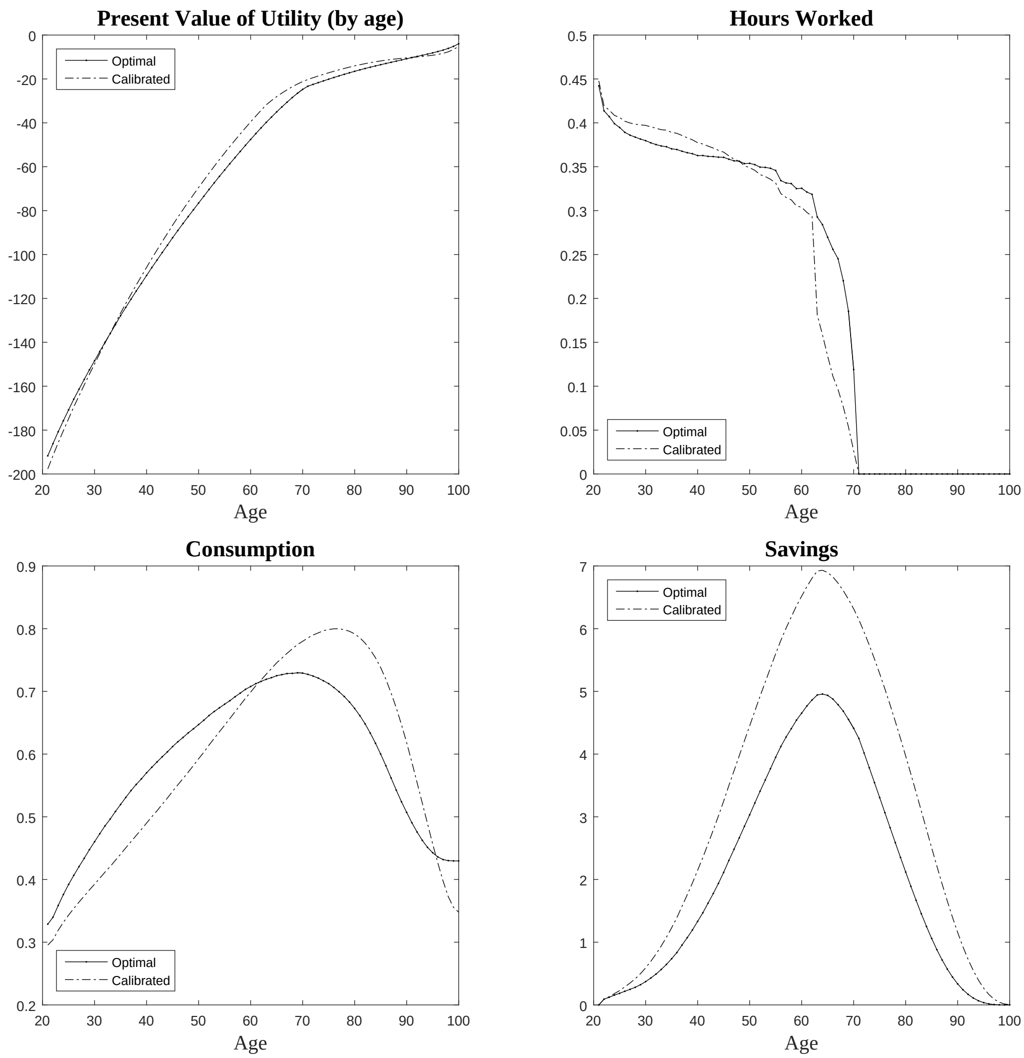

Figure 3: Baseline results compare cross-sectional averages for consumption, savings, hours and value functions. These averages are taken for two values of government debt: at the baseline and optimal debt to output ratios.

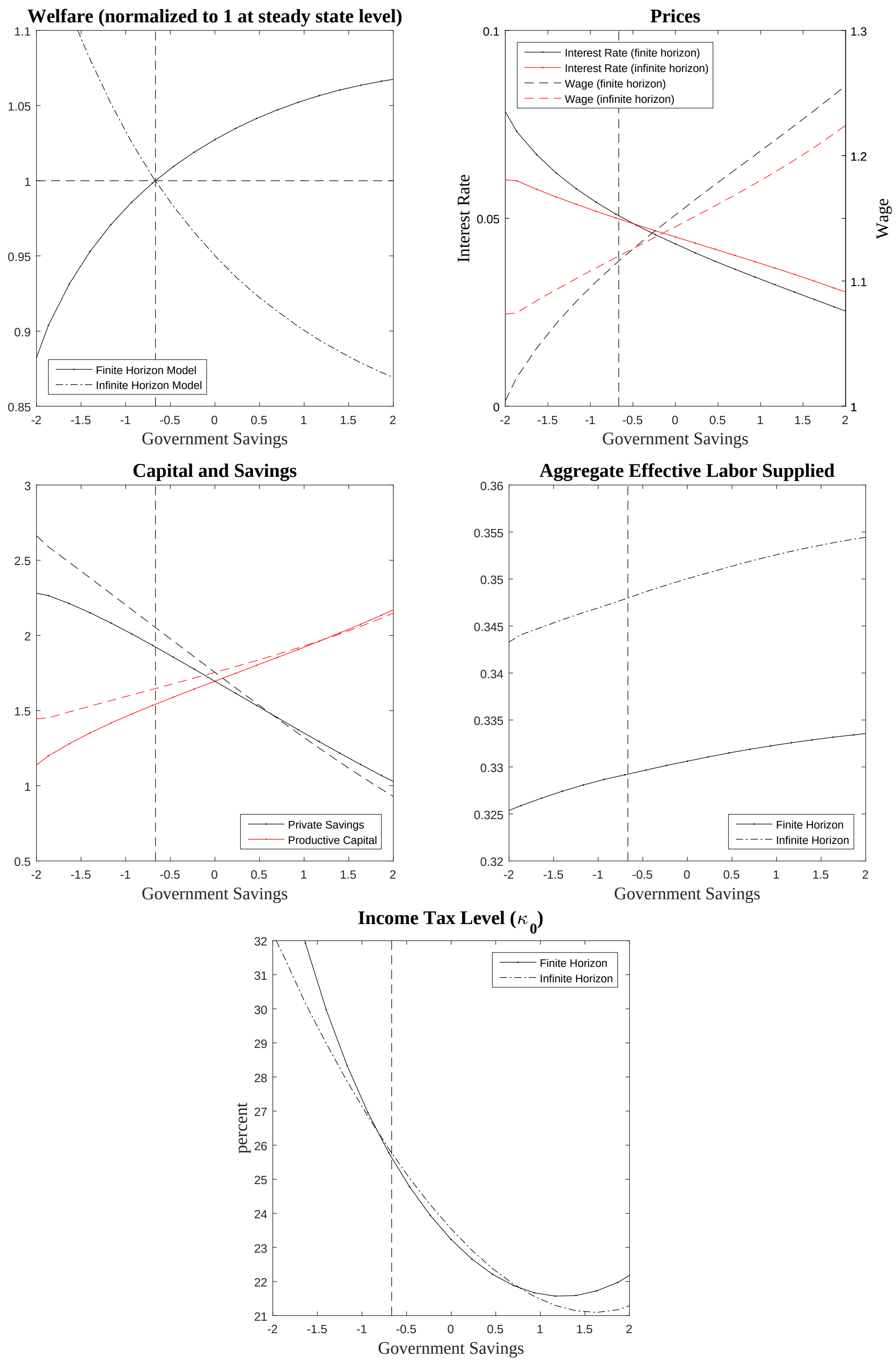

Figure 4: The above results compare aggregate outcomes (welfare, savings and taxation) across varying levels of government savings as a fraction of output. The vertical line at -0.67 marks the baseline level of government debt. When variables are plotted in solid lines, these variables are finite horizon economy outcomes; variables in dotted lines are infinite horizon economy outcomes.

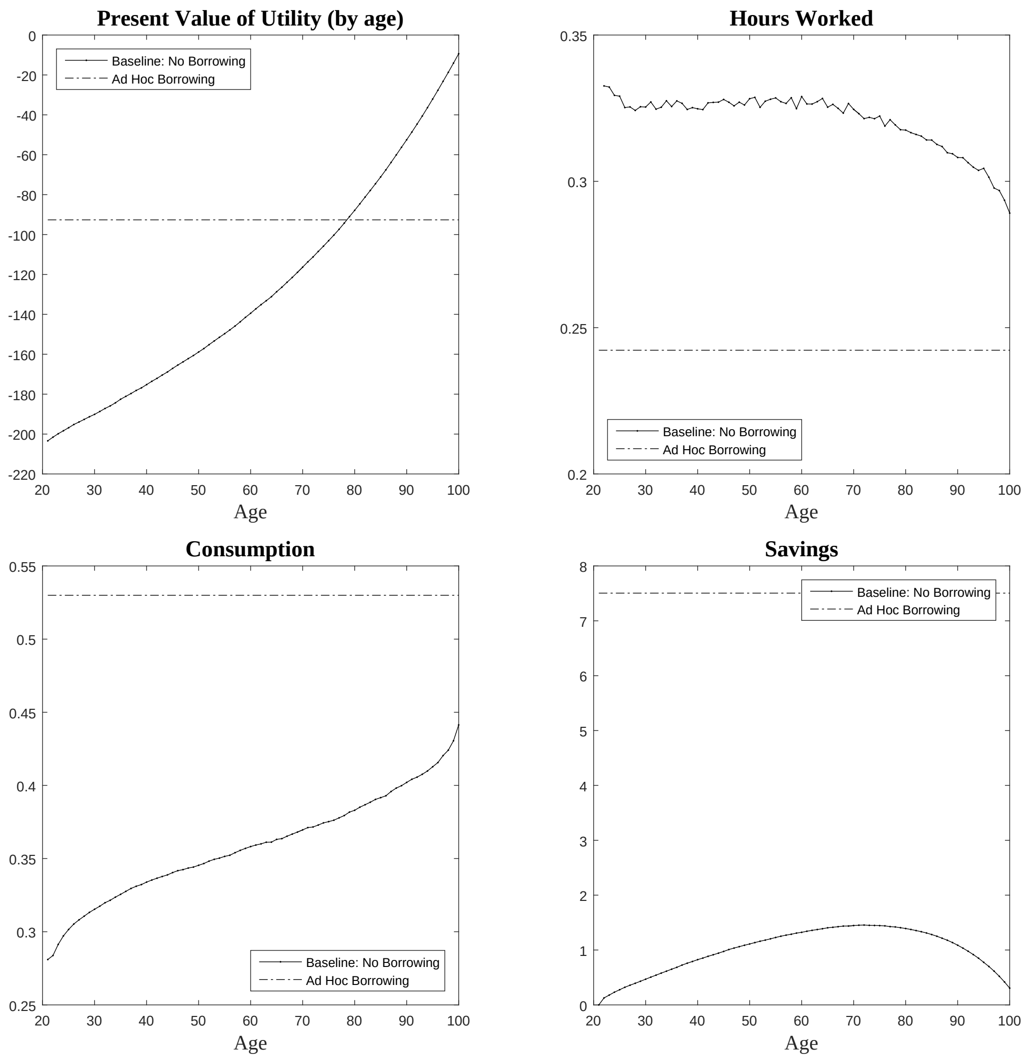

Figure 5: Baseline results compare cross-sectional averages for consumption, savings, hours and value functions. The finite horizon model (represented by solid lines) is evaluated at its optimal government savings-to-output ratio of 2, while the infinite horizon model (represented by dashed lines) is evaluated at its optimal government savings-to-output ratio of -2.

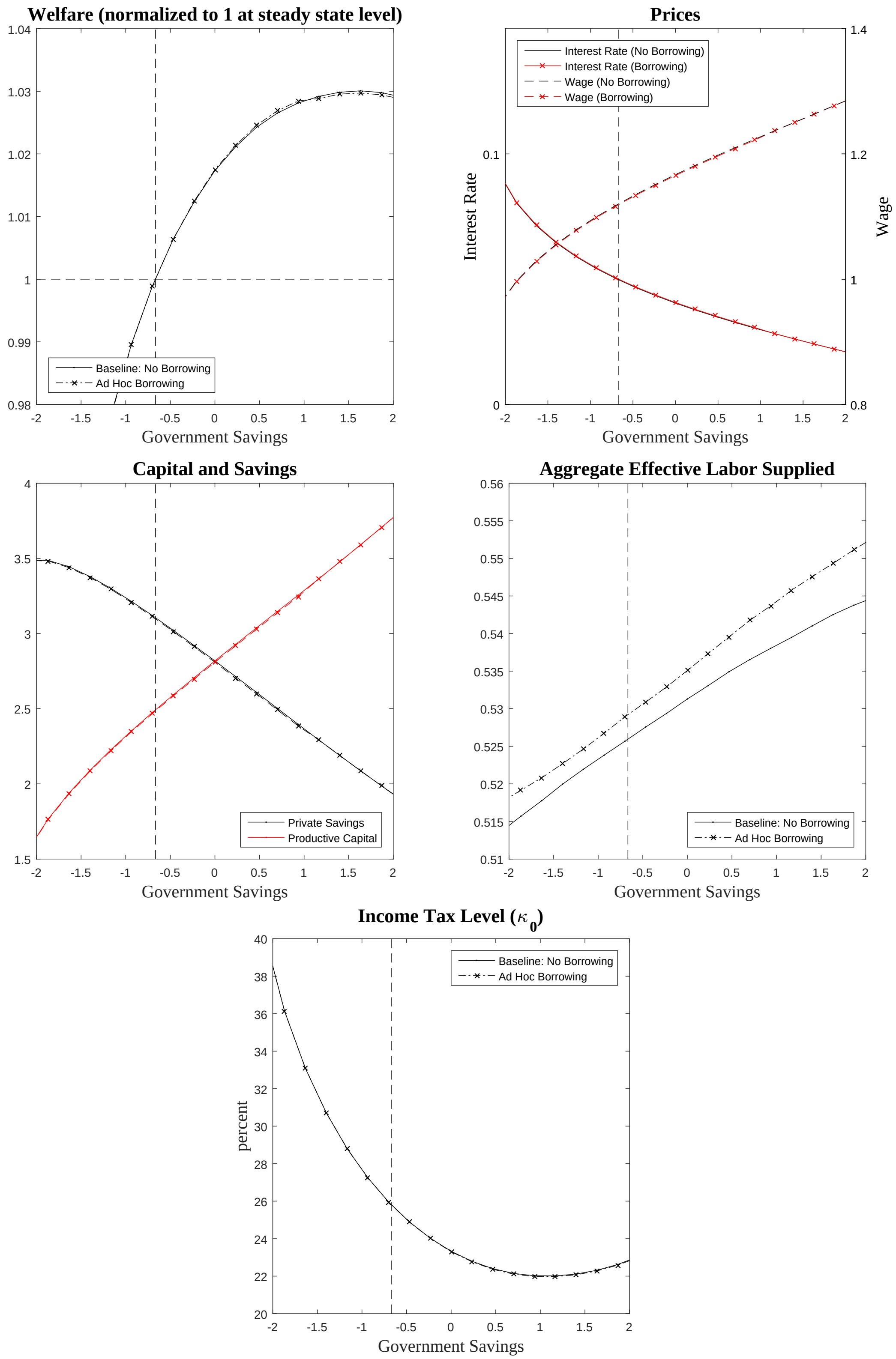

Figure 6: These graphs compare the baseline model aggregate outcomes (welfare, savings and taxation) with no borrowing to outcomes of a model that allows consumer borrowing. Outcomes are plotted for varying government savings-to-output ratios. The vertical line at -0.67 marks the calibrated level of government savings, $-2/3$.

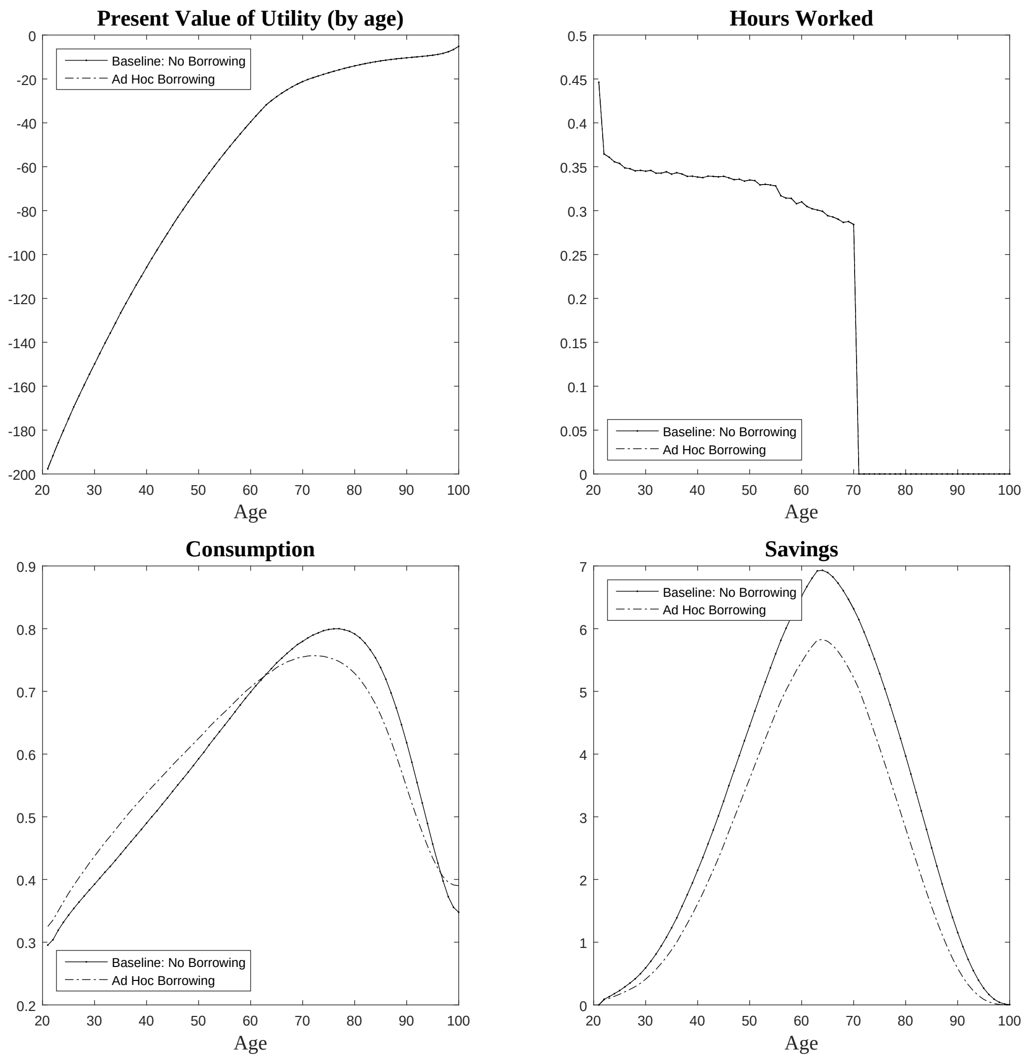

Figure 7: Baseline results compare cross-sectional averages for consumption, savings, hours and value functions. These averages are taken for two values of government savings: at the baseline and optimal government savings-to-output ratios.

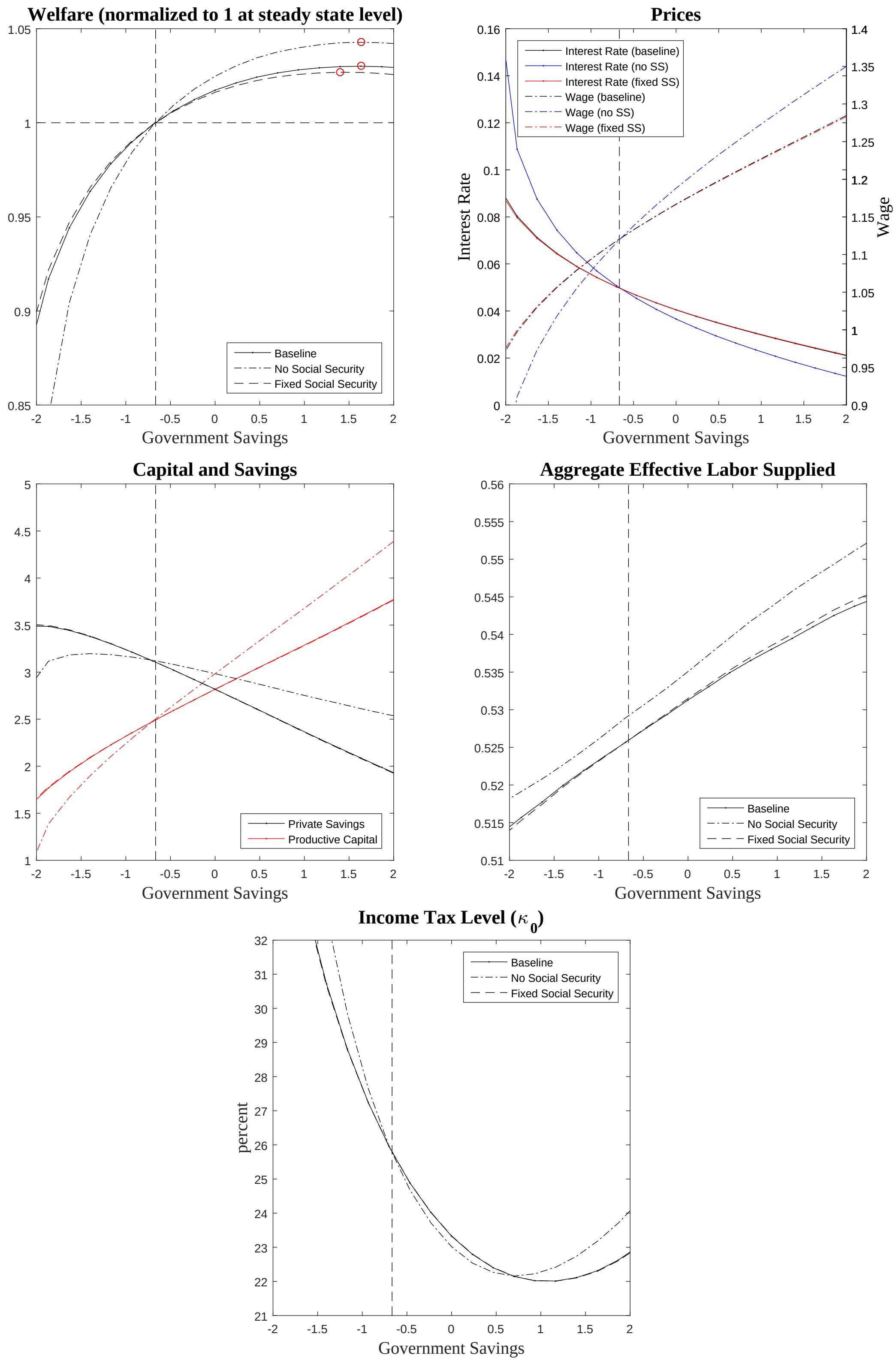

Figure 8: The above results compare aggregate outcomes (welfare, savings and taxation) across varying government savings-to-output ratios. The vertical line at -0.67 marks the baseline level of government debt.

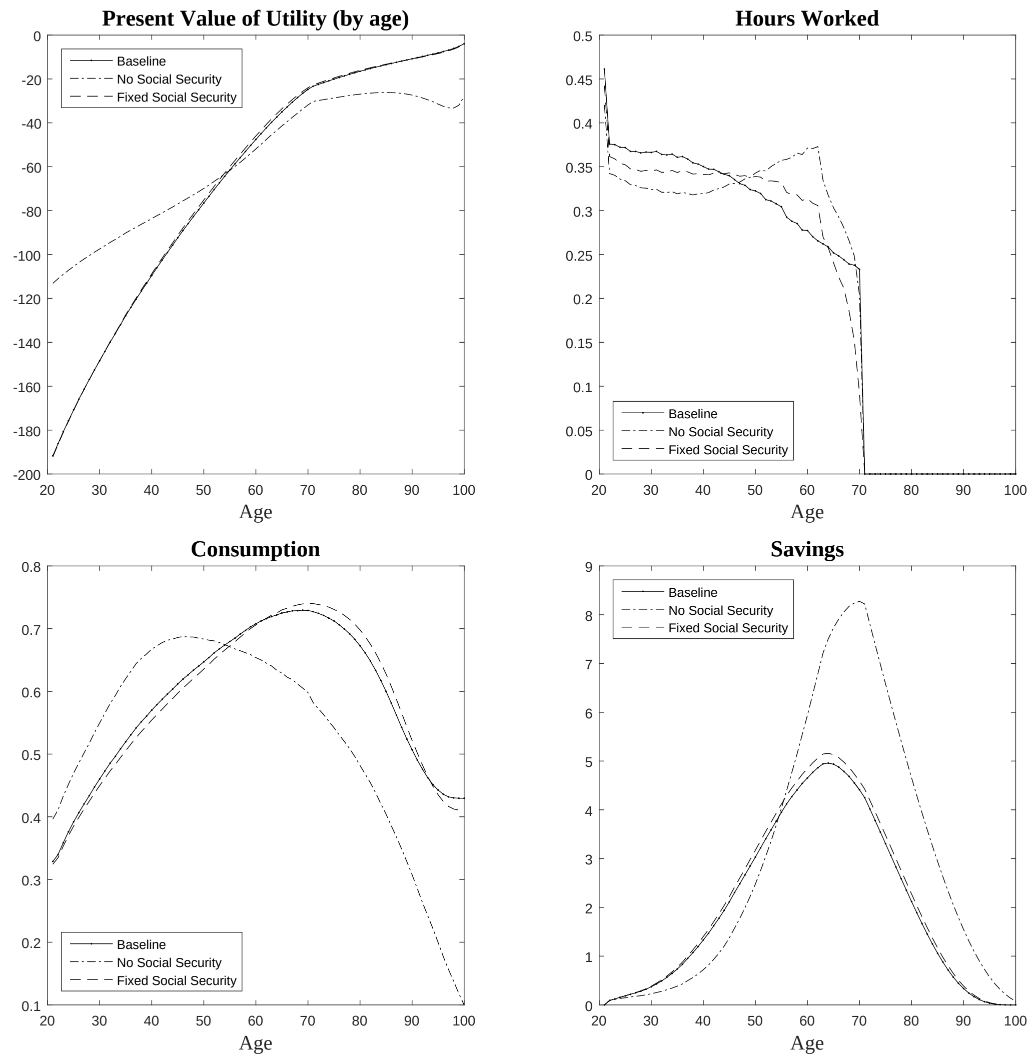

Figure 9: Baseline results compare cross-sectional averages for consumption, savings, hours and value functions when the government savings-to-output ratio is chosen to be optimal. Each line represents the outcome of a different policy counterfactual.

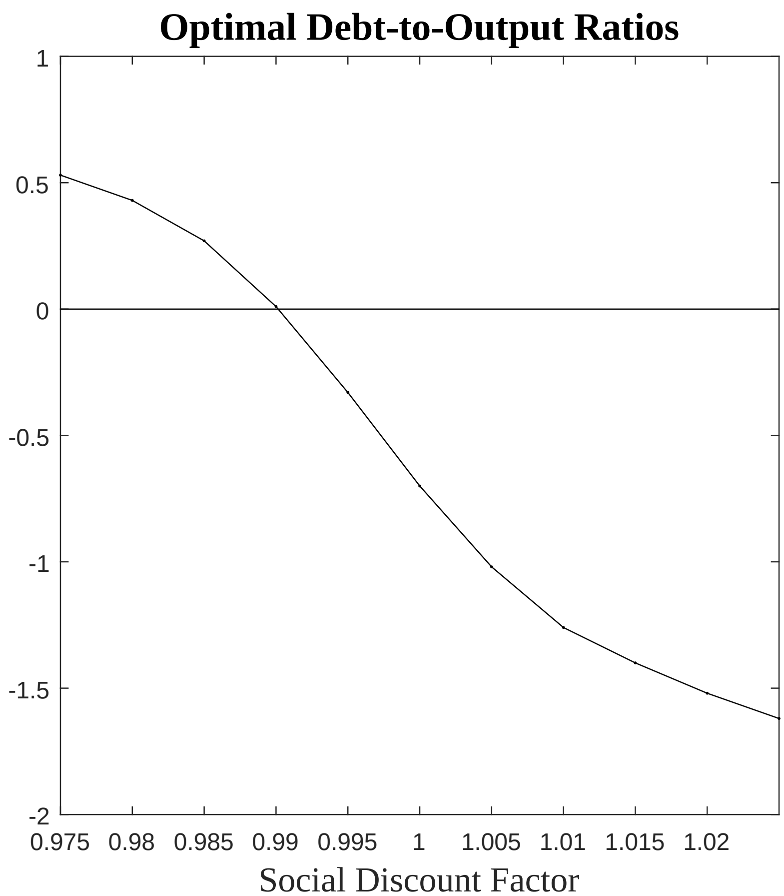

Figure 10: Optimal debt-to-output ratios plotted against values of $\hat{\beta}$.

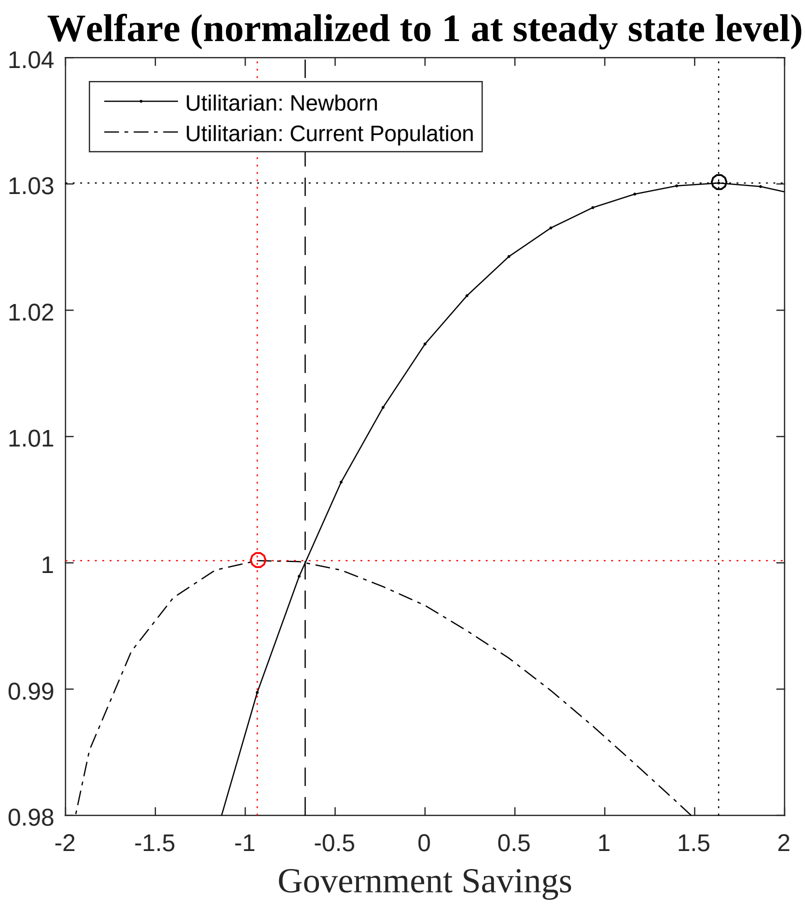

Figure 11: Utilitarian social welfare criteria: (i) for present value of newborns and (ii) for present value of each living agent in steady state. Welfare values are plotted for each level of government savings.