Search Frictions and the Labor Wedge*

Andrea Pescatori†
International Monetary Fund

Murat Tasci‡
Federal Reserve Bank of Cleveland

Prior Version: May 2013
This Version: January 2016

Abstract

We show that search frictions embedded in a RBC model primarily manifest themselves at the extensive margin. The ability to distinguish between intensive and extensive margin, however, affects the measurement of the marginal rate of substitution (MRS). In fact, the correct measurement of the MRS, in terms of hours per worker, implies a less variable and procyclical labor wedge than the one found in Chari et. al. (2007), especially at low Frisch elasticity. The main result is very robust to alternative wage determination mechanisms, even though implications for employment fluctuations may differ.

Key words: Labor Market Search; Business Cycle Accounting; Labor Wedge

JEL classification: E24; E32; J64

*The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management as well as the Federal Reserve Bank of Cleveland or the Federal Reserve System. We would like to thank David Arseneau, , Anton Cheremukhin, Sanjay Chugh, Charles Carlstrom, Tim Fuerst, Fatih Guvenen, Paulina Restrepo-Echavarria, Antonella Trigari, an anonymous referee and seminar participants at the Federal Reserve Bank of Cleveland, Federal Reserve Bank of Boston, Board of Governors of the Federal Reserve System, IMF, Koc University, Central Bank of the Republic of Turkey, Oberlin College, Ohio State University, as well as conference participants at the SED Meetings (2011), Midwest Macro Meetings (2011), Southwest Search and Matching Workshop (Spring, 2012), Midwest Economic Association Meetings (2012), and CEF (2012). Finally, we are grateful to Andrea Raffo for providing us with the relevant data for Germany.

†Andrea Pescatori: Research Department, International Monetary Fund, 700 19th Street, N.W., Washington, D.C. 20431. E-mail: apescatori@imf.org

‡Murat Tasci (corresponding author): Research Department, P.O. Box 6387, Federal Reserve Bank of Cleveland, Cleveland OH 44101-1387. E-mail: Murat.Tasci@clev.frb.org.

1 Introduction

Real business cycle (RBC) models (Kydland and Prescott, 1982) impose a strong discipline on the choice of consumption and hours, both intertemporally and intratemporally. In these models, the optimal choice of hours is determined in equilibrium such that the marginal rate of substitution between consumption and leisure (mrs) is equal to the marginal productivity of labor (mpl). The data show, however, that there is a substantial *wedge* between these two quantities that strongly co-varies with the economic cycle. In their seminal work, Chari Kehoe and McGrattan (2007) (henceforth CKM) conclude that, along with the efficiency wedge, the labor wedge accounts for most of the fluctuations in output, putting it at the center of their business cycle accounting research program.[1]

We interpret this finding as an indication of a significant misspecification of the prototype RBC model as it relates to the labor market. Search and matching frictions (Mortensen and Pissarides 1994, Pissarides 2000) introduce a wedge between the wage and both the mpl and the mrs, providing a natural framework to address misspecification related to labor market imperfections. It is indeed tempting to think that these type of frictions will induce endogenous movements in the optimal choice of hours worked that could manifest themselves as labor wedge. In this paper, we first present a model with labor market frictions—in the form of search and matching—that nests a prototype RBC model *a la* CKM; then, we ask whether the labor wedge, as usually measured, could be an artifact of these frictions and if so, to what extent.

We show that search frictions primarily manifest themselves at the extensive margin, in employment fluctuations, but not at the intensive margin—which is the focus of the business cycle accounting literature. In other words, the equation used to back out the labor wedge is unaffected by search frictions directly or explicitly over the cycle. The reason is that the business cycle effects of these frictions are absorbed in the wage bargaining quite efficiently, inducing a demand and supply of hours per worker that is close to the social planner solution. More specifically, even though the Nash bargaining over hours and wage actually distorts the

[1] The business cycle accounting research program has the goal of identifying promising modeling avenues for dynamic general equilibrium models by measuring the "discrepancy" between the data and a prototype real business cycle model. CKM identify four wedges: the efficiency, labor, investment and government consumption wedge. The labor and efficiency wedges are measured to be the most important, suggesting that macro models that would like to explain real macro fluctuations should pay more attention to understanding what type of frictions could manifest themselves as these wedges.

firm's perceived benefit of increasing hours per worker, this distortion is constant over time. There is, nonetheless, an important difference between our search model and the prototype RBC model: backing out the labor wedge in the RBC model requires to accept the full employment implication and replace hours per worker with total hours, distorting the measurement of the *mrs* and introducing a procyclical bias in the labor market equilibrium condition. This is not the case for the search model that naturally distinguishes between intensive and extensive margin.

We show the potential impact of ignoring the extensive margin in a simulation exercise where we back out the labor wedge in the prototype RBC model, as in CKM, using the search model as the data generating process. Results show that even though there is no labor wedge in the simulated data by construction, one can falsely measure a significantly procyclical and variable labor wedge if the distinction between the intensive and the extensive margin is ignored. About 15 percent of the relative variation in the labor wedge and all its comovement with output and total hours could be explained by this misspecification.

We complement this result with a direct comparison of the labor wedge measures derived from the prototype RBC model and our model with search frictions. Using U.S. data, we show that the labor wedge we obtain is less variable and procyclical than the prototype labor wedge. This result is sensitive to the exact parameterization of the labor supply elasticity. We find that, for instance, when the Frisch elasticity is relatively high, such as 2.8, as in most macro models, we can get up to 20 percent decline in the variability of the labor wedge. Similarly, we find a reduction in the procyclicality. This result is even stronger, a 40 percent reduction, for Frisch elasticities that are more consistent with the micro estimates. Moreover, we show that our results are not an artifact of the generalized Nash bargaining. Analyzing the implications of alternative wage determination mechanisms used in the literature, such as right-to-manage bargaining or rigid wages, we conclude that our results are very robust, even though these assumptions have different implications over other dimensions of the model such as the behavior of the extensive margin. In fact, search frictions do not matter for the intratemporal condition that relates *mrs* and *mpl* but they are potentially very relevant at the extensive margin in relation to employment dynamics. We leave an exhaustive analysis of the labor wedge at the extensive margin for future work, but present some examples to highlight its importance in section 8.

The next section discusses the related literature, especially that on the business cycle accounting and the labor wedge. Section 3 and 4 present an extension of the prototype RBC model with search frictions and how it nests the prototype model. Section 5 discusses how search frictions imply a different labor wedge. Section 6 shows, quantitatively, how search frictions alter the labor wedge, first by using the model generated data and analyzing the behavior of labor wedge in these simulations, then by using the U.S. data and focusing on only one equilibrium condition. We present a discussion of the alternative mechanisms for wage determination and the link between the extensive margin and search frictions in Section 7 and 8, respectively. The last section concludes.

2 Related Literature

This paper is part of the vast literature that studies labor market imperfections in connection with the business cycle, such as in Mortensen and Pissarides (1994), Cole and Rogerson (1999), and Shimer (2005), among others. The extension of the standard growth model we use is most closely related to Andolfatto (1996) and Merz (1995, 1999), which embed search frictions into an otherwise standard RBC model.

The focus on the labor wedge makes the paper naturally related to the recent literature on business cycle accounting, which, in different forms, dates back before CKM.[2] For example, Hall (1997) identifies variations in the marginal rate of substitution as an important element to explain aggregate fluctuations. Several other studies in the literature focus on the same equilibrium condition in the labor market and provide different interpretations of it, ranging from changes in labor market institutions, competitive structure of the economy, price-wage markup, changes in regulation, and tax policy (see in particular, Cole and Ohanian 2002, Gali, Gertler and Lopez-Salido 2007, Mulligan 2002, Rotemberg and Woodford 1991 and 1999). Chang and Kim (2007), instead, argues that the apparent distortion in the labor market clearing condition, as measured in the aggregate data, might be partly due to aggregation bias. They show that a heterogenous-agent economy with incomplete capital markets and indivisible labor can generate

[2] One can trace the basic idea behind this exercise to early work in the RBC literature as in Prescott (1986) or Ingram, Kocherlakota and Savin (1994).

this observed wedge. Arseneau and Chugh (2010) also analyze a general equilibrium matching model with distortions that map into a labor wedge. Even though the focus is on optimal tax policy, they show that, as in our paper, the labor wedge takes two different forms with matching frictions, one intratemporal and one intertemporal.[3] They do not, however, have an intensive margin, but a labor force participation margin instead. One example of an RBC model with both margins is Cho and Cooley (1994). Variation in the extensive margin is introduced via a fixed cost of supplying labor. It is easy to show that even in Cho and Cooley (1994), one can get two distinct labor wedges as ours, and the labor wedge for the intensive margin will take the exact same form. Instead of an ad-hoc adjustment cost, we prefer search frictions to introduce both margins in a more conventional modelling environment which easily nests the prototype RBC model.[4]

More recently, Blanchard and Gali (2010), Cheremukhin and Restrepo-Echavarria (2010) and Shimer (2009, 2010) focus on the variation in the labor wedge. Shimer (2009) reviews the literature and makes a case for focusing on the labor wedge, as it is relatively immune to how the model environment is specified and the expectations are formed. Cheremukhin and Restrepo-Echavarria (2010) lays out an RBC model with search frictions and argues that most of the variation in the labor wedge is attributable to the residual shock to matching efficiency, rather than variations in job destruction or impediments to the bargaining process. Their modelling choice is less conventional than ours, hence less comparable to prototype RBC models, and lacks an intensive margin. The focus of their paper is to identify what specific friction - among possible labor market frictions - is causing variation in the labor wedge. Both Blanchard and Gali (2010) and Shimer (2010), differ in their formulation of the search frictions from us, and derive a neutrality result: In a model without capital, fluctuations in the unemployment rate are independent of aggregate productivity. In Blanchard and Gali (2010), this is due to the fact that recruitment of firms is not affected by aggregate productivity. As Shimer (2010) argues, this follows when one assumes that recruitment is only labor-intensive, not good-intensive.[5]

[3] See sections 5 and 8 of the current paper for more on this distinction in our model.

[4] The distortion for the extensive margin in Cho and Cooley (1994) is different from the one we have, since employment choice is still not subject to any intertemporal distortion in their work.

[5] We favor a more traditional approach, as in Andolfatto (1996) and Merz (1995), and model recruitment as a good-intensive technology. Hence firms respond to productivity shocks by increasing recruitment during booms. We find it reasonable to assume that, at least partially, firms cannot just bear the cost of recruitment by costlessly switching workers from production to recruitment in response to productivity shocks.

Shimer (2010) also arrives at a conclusion similar to ours; namely search frictions per se will not help explaining the labor wedge. In section 8 we discuss in more detail the similarities between our analysis and arguments in Shimer (2010).

3 Search Frictions and Employment Fluctuations

The model is a decentralized complete-market version of Andolfatto (1996) and Merz (1995). The labor market is non-Walrasian: Search and matching frictions—summarized by a matching function at the aggregate level—may prevent full employment and allow for movements in both the intensive and the extensive labor margin. The labor contract is determined through bargaining between households and firms. Consumption and capital goods, instead, are exchanged in perfectly competitive markets. Finally, the model is laid out such that it nests the prototype real business cycle (RBC) model described in CKM.

3.1 Households

The economy is populated by a continuum of infinitely-lived worker-households distributed uniformly along the unit interval implying a constant labor force normalized to one. At any point in time, only a mass $n_t \leq 1$ of households is employed while the remaining $1-n_t$ households are unemployed and searching for a job.

Households earn a hourly wage, w_t, from working and receive rental income, r_t, from capital that they rent out to firms. They can invest x_t units each period which adds to new capital next period, k_{t+1}, net of depreciation, δ. Wedges operate as a tax, following CKM we introduce a tax on investment and labor earnings, $[1 + \tau_{x,t}]$ and $[1 - \tau_{l,t}]$, respectively. Finally, in each period, households receive a lump sum transfer, T_t, from the government. The household utility function is expressed in terms of consumption (c_t) and hours worked (h_t)

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \psi G(h_t)], \qquad 0 < \beta < 1, \tag{1}$$

where β is the subjective discount factor, E_0 is an expectation operator, while, without loss of generality, we normalize $G(0) = 0$ and assume $G(x) < 0$, $\forall x > 0$.[6] Finally, we define the

[6] Additionally, $U_c(\cdot) > 0$, $U_{cc}(\cdot) \leq 0$, $G_h(\cdot) < 0$, and $G_{hh}(\cdot) \leq 0$. We also assume that standard Inada

marginal rate of substitution between consumption and leisure as

$$mrs_t \equiv -\psi G_h(h_t)/U_c(c_t) \tag{2}$$

Since markets are complete, providing perfect insurance against unemployment risk, consumption is equalized across households. In this case, it is possible to prove that we can recast the household problem in terms of a large representative household, as in Andolfatto (1996) and Merz (1995), that maximizes the sum of its members' expected utility[7]

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \psi n_t G(h_t)], \qquad 0 < \beta < 1. \tag{3}$$

Each unemployed worker exerts some effort, e_t, to find a job, which costs $c(e_t)$ units of resources. The cost of search function, $c(e)$, is assumed to be strictly convex and increasing in the effort.[8] As a result, the budget constraint of the large household can be written as

$$c_t + x_t [1 + \tau_{x,t}] + c(e_t)(1 - n_t) = w_t [1 - \tau_{l,t}] n_t h_t + r_t k_t + T_t \tag{4}$$

When the large household maximizes (3) subject to (4), wages, w_t, and hours worked h_t are taken as given, since they will be determined through Nash Bargaining (see next section).

Given the complete markets assumption, it is easy to write the law of motion for aggregate capital

$$K_{t+1} = (1 - \delta)K_t + X_t \qquad 0 \leq \delta \leq 1 \tag{5}$$

where uppercase letters denote aggregate variables;[9] i.e., K is aggregate capital stock, X is aggregate investment etc.

We assume that trade in the labor market is mediated by an aggregate matching function that determines the number of jobs formed in each period as a function of the number of job

conditions hold for $U(\cdot)$.

[7] We have made use of the normalization $G(0) = 0$ and dropped the term $(1 - n_t)\psi G(0)$ from the objective function.

[8] Inclusion of search effort in the model is not necessary for the main message of the paper. However, for technical reasons and a comprehensive treatment of the model we need search effort. See the Appendix E for the description of the technical problem, and how search effort is related.

[9] When no confusion is supposed to arise, we will keep using lower case letters to denote both individual and aggregate variables.

vacancies, V_t, and the aggregate search effort of the household/workers, $(1-N_t)\,E_t$

$$M_t = \tau_{\chi,t} V_t^{\gamma} \left[(1-N_t)\,E_t\right]^{1-\gamma} \tag{6}$$

where $0 < \gamma < 1$, $\tau_{\chi,t}$ is the period-t realization of a process that governs the efficiency of matching, and the following intuitive restriction applies $M_t \leq \min\{V_t, 1-N_t\}$. Using the definition of the matching function, we can write the probability of finding a job for an unemployed worker as $p_t = \frac{Mt}{(1-N_t)E_t}$, which is taken as given at household level.

Job matches that are formed in a period are assumed to become productive in the following period. As jobs are created, they are also being destroyed. Letting σ denote the period-t fraction of existing jobs destroyed, the law of motion for employment is given by the expression

$$N_{t+1} = (1-\sigma)\,N_t + M_t, \tag{7}$$

where $\sigma \in [0,1]$. At the household level, employment n_t evolves endogenously according to the following, slightly modified equation of motion

$$n_{t+1} = (1-\sigma)\,n_t + p_t(1-n_t)e_t. \tag{8}$$

We can finally write down the representative household problem recursively

$$W^h(\omega_t^h) = \max_{c_t,x_t,e_t,k_{t+1},n_{t+1}} \{U(c_t) + \psi n_t G(h_t) + \beta \mathrm{E}_t W^h(\omega_{t+1}^h|\omega_t^h)\} \tag{9}$$

$$s.t.\; c_t + x_t\left[1+\tau_{x,t}\right] + c(e_t)(1-n_t) = w_t\left[1-\tau_{l,t}\right] n_t h_t + r_t k_t + T_t \tag{10}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{11}$$

$$n_{t+1} = (1-\sigma)\,n_t + p_t(1-n_t)e_t$$

taking $w_t = w(\Omega_t)$, $r_t = r(\Omega_t)$, $p_t = p(\Omega_t)$, and equations of motion for aggregate state variables, K_t and N_t as given. For notational simplicity, let $\omega_t^h = \{k_t, n_t, \Omega_t\}$ and $\Omega_t = \{\tau_t, K_t, N_t\}$ denote individual and aggregate state variables for the household, respectively, with $\tau_t = [\tau_{l,t}, \tau_{x,t}, \tau_{z,t}, \tau_{g,t}, \tau_{\varkappa,t}]$ being the vector of exogenous processes with $\tau_{z,t}$ and $\tau_{g,t}$ to be defined later. This optimization problem leads to two conditions that determine the optimal savings and search effort recursively for the large household (details are in the appendix):

$$U_c(c_t)\left[1+\tau_{x,t}\right] = \beta \mathrm{E}_t\left[U_c(c_{t+1})(r_{t+1}+(1-\delta)\left[1+\tau_{x,t+1}\right])|\omega_t^h\right]. \tag{12}$$

$$U_c(c_t)c_e(e_t)/p_t = \beta \mathrm{E}_t\left[\begin{array}{c} U_c(c_{t+1})[w_{t+1}\left[1-\tau_{l,t+1}\right]h_{t+1}+c(e_{t+1})]+ \\ \psi G(h_{t+1})+\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}\left(1-\sigma-p_{t+1}e_{t+1}\right)|\omega_t^h \end{array}\right] \tag{13}$$

The first Euler equation is the standard consumption Euler equation. The second Euler equation determines the households' optimal search behavior. The left-hand side in (13) is the expected marginal cost of search for the households in current consumption units, which is equal to the expected marginal gains from search on the right-hand side. If search is successful, the household expects to get utility from the net wage payments, $w_{t+1}\left[1-\tau_{l,t+1}\right]h_{t+1}$, and from economizing on future search costs, $c(e_{t+1})$, and to get disutility from working $G(h_{t+1})$. The final term in brackets represents the net future benefit arising from the expected persistence of a job match.[10]

3.2 Firms

Firms operate a production technology, $\tau_{z,t}f(k_t,l_t)$, that is constant returns to scale with respect to capital and labor, and is governed by an exogenous aggregate productivity process $\tau_{z,t}$. The production function satisfies standard restrictions; furthermore, we assume that its curvature is constant such as $f_{ll}(k_t,l_t)l_t/f_l(k_t,l_t)=-\alpha$. The labor input is total hours, l_t, which is given by the product of employed workers and hours per worker $l_t=n_th_t$. We can, thus, define the marginal productivity of labor as

$$mpl_t \equiv \tau_{z,t}f_l(k_t,l_t) \tag{14}$$

While capital is rented in a perfectly competitive market, firms must undergo a costly search process before jobs are created and output is produced. For each job vacancy created in period-t, firms pay κ units of output resulting in period-t "vacancy-posting" costs of κv_t. Jobs must be

[10] Given that any single current-period match survives with probability $1-\sigma$, households' expected utility will increase simply by reducing expected future recruiting costs by the quantity $\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}(1-\sigma)$. The second term in this sum, $-p_{t+1}e_{t+1}\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}$, represents the reduction in the future job-finding rate, due to the current depletion of the unemployment stock.

posted as vacancies before they can be filled. We assume that vacancies adjust in equilibrium such that the value of an additional vacancy is driven to zero.

We are going to approach the firms' problem in two steps. In the first step, firms decide how much capital to rent and how many jobs to create taking the rental rate r_t, aggregate state variables, the employment contract $\{w_t, h_t\}$, and the probability of filling a vacancy, q_t, as given[11]

$$W^f(\omega_t^f) = \max_{k_t, v_t, n_{t+1}} \{\tau_{z,t} f(k_t, n_t h_t) - w_t n_t h_t - r_t k_t - \kappa v_t + \tilde{\beta}_t \mathrm{E}_t W^f(\omega_{t+1}^f | \omega_t^f)\} \tag{15}$$

$$s.t.\ f(k_t, n_t h_t) = k_t^\alpha (n_t h_t)^{(1-\alpha)} \tag{16}$$

$$n_{t+1} = (1-\sigma)\, n_t + q_t v_t \tag{17}$$

where $\omega_t^f = \{k_t, n_t, \Omega_t\}$, $r_t = r(\Omega_t)$, $w_t = w(\Omega_t)$ with $\Omega_t = \{\tau_{t,} K_{t,} N_t\}$ and $\tilde{\beta}_t = \beta \left[\frac{U_c(c_{t+1})}{U_c(c_t)}\right]$. Firms' problem in (15) implies a set of first order conditions that determines the optimal level of capital stock rented and the number of vacancies posted by firms.

$$\tau_{z,t} f_{k_t}(k_t, n_t h_t) - r_t = 0 \tag{18}$$

$$\frac{\kappa}{q_t} = \tilde{\beta}_t \mathrm{E}_t \left[\tau_{z,t+1} f_{l_{t+1}}(k_{t+1}, n_{t+1} h_{t+1}) h_{t+1} - w_{t+1} h_{t+1} + (1-\sigma)\, \frac{\kappa}{q_{t+1}} | \omega_t^f\right] \tag{19}$$

The first condition is the familiar relation between the rental rate and the marginal product of capital. The second condition determines the optimal number of vacancies posted by firms, equating the expected marginal cost of filling a vacancy and its expected marginal benefit.[12]

3.3 Employment Contract and the Nash Bargaining

Since negotiating a wage has an implicit opportunity cost due to search frictions, we need a mechanism to determine the surplus for each contracting party. We assume that each worker's employment contract, $\{w_t, h_t\}$, is determined through Nash bargaining between the firm and

[11] From the aggregate matching function in (6), we know that the probability of filling a vacancy must be $q_t = M_t / V_t$.

[12] On the left hand side, $\frac{\kappa}{q_t}$ is the expected marginal cost of filling one vacancy since the expected duration of a vacancy is $1/q_t$. On the right hand side, we have, the expected difference between the marginal productivity of an extra worker and his wage bill, for given hours per worker, while the last term represents the expected saving in terms of future recruitment costs.

the household (see for example Pissarides 2000).[13] In a setting like this, where there are multiple workers within a firm, it is not entirely clear how to formulate the bargaining problem. Fortunately, Stole and Zwiebel (1996) show that bargaining should happen over the marginal surplus for both parties.[14] Assuming that households (firms) have a bargaining power of $1-\lambda$ (λ) the generalized Nash bargaining problem takes the following form[15]

$$\max_{w_t,h_t} W^h_{n_t}(\omega_t)^{1-\lambda} W^f_{n_t}(\omega_t)^{\lambda} \tag{20}$$

where $W^h_{n_t}(\omega_t)$ is the household's net marginal surplus from having one more worker employed given the household's optimal behavior, and $W^f_{n_t}(\omega_t)$ is the firm's net marginal surplus from having one more employee given firm's optimal behavior. As long as each party can extract some surplus, i.e. $W^h_{n_t}(\omega_t) > 0$ and $W^f_{n_t}(\omega_t) > 0$, the Nash bargaining problem yields two conditions that determine equilibrium level of hours, h_t, and wage per hour, w_t (details of the solution are presented in the Appendix E)

$$\lambda W^h_n(\omega_t) - (1-\lambda)(1-\tau_{l,t})U_c(c_t)W^f_n(\omega_t) = 0 \tag{21}$$

$$U_c(c_t)\left[1-\tau_{l,t}\right]\left[\tau_{z,t} f_l(k_t, n_t h_t) + \tau_{z,t} f_{ll}(k_t, n_t h_t) n_t h_t\right] + \psi G_h(h_t) = 0 \tag{22}$$

Equation (21) is the starting point to determine the hourly wage and basically divides the total surplus of the marginal match among the household and the firm. Notice that $\tau_{l,t}$ exogenously reduces the household's share of surplus and, in the limit $\tau_{l,t} = 1$, the household's surplus is zero $W^h_{n_t} = 0$. Household risk aversion adds a time-varying dimension to the sharing rule: in times of high marginal utility households claim a relatively higher surplus. Equation (22) is the first order condition with respect to hours. This equation relates the marginal rate of substitution to the marginal productivity of labor. As we show later, this equation plays a crucial role in the analysis of the labor wedge.

[13] In section 7 we will explore alternative mechanisms of wage determination.

[14] For examples of this formulation, see Merz (1995), Beauchemin and Tasci (2008), Fujita and Nakajima (2009), and Elsby and Micheals (2010), among others. Note that, we assume the contracted hours apply to every worker. However, given that $G(\cdot)$ is convex, it is easy to prove that homogenous hours minimize the overall worker disutility.

[15] We drop the distinction between the state variables for the households and the firms, since in equilibrium, $\omega^h_t = \omega^f_t = \omega_t$.

These two conditions complete the necessary set of equations that fully characterize the equilibrium.

3.4 Equilibrium

Since all households and firms are identical, in equilibrium, individually efficient allocations coincide with the aggregate, i.e. $k_t = K_t$, $c_t = C_t$, $l_t = L_t$, $n_t = N_t$, therefore the relevant state is Ω_t. Then the competitive equilibrium of this economy is characterized by a list, $k_t(\Omega_t)$, $l_t(\Omega_t)$, $c_t(\Omega_t)$, $n_t(\Omega_t)$, $w_t(\Omega_t)$, $r_t(\Omega_t)$ that satisfies the equilibrium conditions implied by household and firm optimization, (12-13) and (18-19), Nash bargaining, (21-22), as well as equations of motion for aggregate states, (5) and (7).[16] Finally, $\tau_{g,t}$ represents a dissipative wedge that can, alternatively, be interpreted as a government spending shock. It is also possible to pin down the aggregate resource constraint by exploiting the free entry condition for firms which imposes that all the flow profits net of recruitment expenses are exhausted in equilibrium

$$y_t = c_t + x_t + \tau_{g,t} + \kappa v_t + c(e_t)(1 - n_t) \tag{23}$$

Note that, search frictions do not affect the consumption Euler equation and the resource constraint in a significant way.[17] However, fluctuations in the extensive and intensive margin, along with the search decision, give rise to additional equilibrium conditions for employment, search effort, e_t, and job vacancies, v_t, that are absent in the prototype business cycle model. For the purpose of this paper, we can focus our analysis on the static equilibrium conditions that describe the labor wedge in (21) and (22). Next sections explore this in detail.

[16] There is a possibility of multiple steady state equilibria in this model due to the complementarities between firms' recruitment effort and workers' search effort. Intuitively, if firms expect that workers will not search hard, the returns to firms' recruitment will diminish, hence the number of vacancies posted. This will, in turn, provide workers with the incentive to search less, thereby fulfilling firms' expectations in the first place. However, one can show that assuming a constant returns to scale matching function and enough convexity in $c(e)$, guarantees a unique steady state equilibrium in this model. We calibrate the model such that $c(e)$ has enough convexity in the numerical exercises.

[17] At least quantitatively for the resource constraint, since most calibrations do not imply big search costs relative to output.

4 The Perfectly Competitive Labor Market

Once search and matching frictions are relaxed and the labor market is assumed to be Walrasian, the model presented above nests the prototype RBC model described in CKM.

Since there is no surplus to share, the Nash bargaining is replaced by having firms and households optimally choosing hours. In the case of firms, the absence of frictions is equivalent to setting $\kappa = 0$ and choosing hours worked to maximize (15) subject to equation (16). Optimality requires the wage to be equal to the marginal productivity of labor[18]

$$w_t = \tau_{z,t} f_l(k_t, n_t h_t) = mpl_t.$$

In the case of households, the absence of frictions is equivalent to setting $c(e) \equiv 0$ and choosing hours worked to maximize (9) subject to equations (10) and (11). Optimality requires the net wage to be equal to the marginal rate of substitution between consumption and leisure, which deliver the labor supply schedule

$$-\psi G_h(h_t)/U_c = (1 - \tau_{l,t})\, w_t\, (c_t)\,.$$

Given the absence of frictions and the presence of a Walrasian auctioneer, the labor market is in equilibrium at all times and full employment is guaranteed, i.e., $n_t = 1\ \forall t$. Equilibrium in the labor market can, thus, be represented by the following equation

$$-\psi G_l(h_t)/U_c\,(c_t) = \tau_{z,t} f_l(k_t, n_t h_t)\,[1 - \tau_{l,t}] \tag{24}$$

Finally, in addition to equation (24) and setting $n_t = 1$ at all times, equations (5), (12), (16), (18), and (23) summarize the equilibrium which is identical to the equilibrium of a standard real business cycle model.

Once functional forms and a parametrization are suitably chosen, it is possible to evaluate the RBC model along the labor market dimension by focusing only on equation (24), using data on *total hours* (l_t), output (y_t), and consumption (c_t). This widely used strategy, however,

[18] Since the firm is indifferent between changing production through the extensive or intensive margin, equation (19) is always satisfied.

requires to accept the counterfactual RBC implication that $n_t = 1$ and replace hours per worker with total hours, $l_t = h_t$, in the evaluation of the marginal rate of substitution—attributing all labor variations to the intensive margin (h_t). In other words, equation (24) is usually brought to the data in the following form

$$-\psi G_l(l_t)/U_c(c_t) = \tau_{z,t} f_l(k_t, l_t)\left[1 - \tau_{l,t}\right] \tag{25}$$

incorporating the full employment implication of the Walrasian assumption. In the next sections we analyze the importance of this step and its effects on the model's ability to match the data.

5 The Labor Wedge

The emergence of a wedge is a symptom of model misspecification especially if we cannot provide a clear interpretation of it. In a reverse engineering exercise, we can easily back out the wedge arising from equation (25) when it is brought to the data (we introduce $\tilde{\cdot}$ when we specifically refer to the prototype RBC model)

$$\left[1 - \tilde{\tau}_{l,t}\right] = -\frac{\psi G_l(l_t)/U_c(c_t)}{\tau_{z,t} f_l(k_t, l_t)} \tag{26}$$

Since the actual labor tax rate does not vary enough at business cycle frequency to explain fluctuations in $[1 - \tilde{\tau}_{l,t}]$, the wedge is to be interpreted as a discrepancy between the model and the data.[19] Hence, given that this is the main equation that describes the labor market equilibrium of the prototype RBC model, the literature has dubbed this wedge the *labor wedge* stressing the fact that it *probably* refers to a model's misspecification along its labor market dimensions (Shimer 2009).

In this section we will compare the labor wedge of the prototype RBC model with the one stemming from the search model.

[19] The labor wedge seems to have a low frequency movement which might be explained by changes in the taxes (see for instance, Ohanian, Rogerson and Raffo, 2008) or changes in the composition of the workforce and the resulting imperfect household aggregation over-time (Cociuba and Ueberfeldt, 2010). However, at the business cycle frequency, there is a lot of variation in the measured labor wedge that is highly unlikely to be explained by high-frequency changes in labor tax. Moreover, the labor wedge, $[1 - \tau_{l,t}]$, is pro-cyclical (falling during recessions) and is positively correlated with per capita hours worked (See the appendix for the time-series plot of the measured labor wedge over the period 1959:I to 20010:III.).

5.1 The Wage Equation in the Search Model

To facilitate the discussion of what is the analog, in the search model, of the prototype-RBC labor wedge (equation 26), it is instructive to derive an explicit wage equation. This is accomplished using the equilibrium conditions from household and firm optimization along with the Nash bargaining solution, equation 21.[20]

$$w_t h_t = (1-\lambda)\left[\tau_{z,t} f_n(k_t, l_t) + \kappa\frac{(1-\sigma)}{q_t}\right] + \frac{\lambda}{1-\tau_{l,t}}\left[-\frac{\psi G(h_t)}{U_c(c_t)} - c(e_t) - \frac{(1-\sigma-p_t e_t)}{p_t} c_{e_t}(e_t)\right] \tag{27}$$

The wage bill for an additional worker $w_t h_t$ is a function of the marginal productivity of employment, $\tau_{z,t} f_n(k_t, l_t)$, search frictions and the (extensive) marginal rate of substitution between the number of employed workers and consumption,$-\psi G(h_t)/U_c(c_t)$. It is useful to express the extensive marginal rate of substitution and the marginal productivity of employment in terms of *mrs* and *mpl*: $-\psi G(h_t)/U_c(c_t) = mrs_t h_t \eta(h_t)$, with $\eta_t = G(h_t)/[h_t G_h(h_t)] \in [0,1]$, and $\tau_{z,t} f_n(k_t, l_t) = mpl_t h_t$. We can thus rewrite the wage equation in a compact form

$$w_t = (1-\lambda) mpl_t + \lambda\frac{mrs_t}{1-\tau_{l,t}}\eta_t + \Phi_t/h_t. \tag{28}$$

where $\Phi_t \lesseqgtr 0$ is the only term that involves search frictions explicitly.[21] If we combine the wage equation (28) with the condition on hours, equation (22), we can relate both the mrs_t and the mpl_t to the wage

$$w_t = mpl_t[1-\lambda(1-\eta_t(1-\alpha))] + \Phi_t/h_t \tag{29}$$

$$w_t = \frac{mrs_t}{1-\tau_{l,t}}\frac{[1-\lambda(1-\eta_t(1-\alpha))]}{(1-\alpha)} + \Phi_t/h_t \tag{30}$$

where we have used the definition of the output elasticity to total hours $(1-\alpha)$, which is approximately the labor share.[22] Those two equations are very instructive and resemble the labor supply and labor demand equations of the perfectly competitive model. However, search

[20] Details are in the Appendix E.

[21] In the text we have implicitly defined $\Phi_t \equiv (1-\lambda)\kappa\frac{(1-\sigma)}{q_t} - \frac{\lambda}{1-\tau_{l_t}}[c(e_t) + \frac{(1-\sigma-p_t e_t)}{p_t} c_{e_t}(e_t)]$. Notice that its sign is ambiguous in principle, in fact, we have $\Phi > 0$ when $\lambda = 0$ and $\Phi < 0$ when $\lambda = 1$.

[22] In the search model, due to the search frictions, $(1-\alpha)$ is not exactly the labor share.

frictions coupled with Nash bargaining introduce two time varying wedges between the wage and both the marginal productivity of labor and the marginal rate of substitution. In fact, even if $\tau_{l,t}$ were constant, the wage would not perfectly co-vary with the mpl and the mrs over the cycle, being affected by movements in the search frictions Φ_t and in the way the match surplus is split. In general, the wage will be between the mrs and the mpl, however, when λ is small, it might be possible to observe $w_t > mpl_t$. In part this is due to the presence of α which reduces the firm's perceived benefit of an extra labor unit. Firms value the benefit of increasing hours worked h across workers in terms of its effect on the marginal productivity of an extra worker not in terms of the average worker productivity. This creates a distortion since the marginal productivity of employment increases with h less than one-to-one (as long as $\alpha > 0$).

Finally, it is also interesting to note that the firm bargaining power parameter, λ, is inversely related to the wage and enters symmetrically in both equations (29) and (30). As we will see, this implies that the bargaining power per se does not affect the labor wedge, as long as there is an interior solution of the Nash Bargaining problem, i.e., $\lambda \in (0, 1)$.

5.2 The Labor Wedge in the Search Model

In the model with search frictions, to derive an equation that is similar in spirit to the one arising in the perfectly competitive model, equation (26), it is best to exploit the solution of the equilibrium employment contract, equation (22):

$$-\psi G_h(h_t)/U_c(c_t) = [1 - \tau_{l,t}]\, \tau_{z,t} f_l(k_t, l_t)(1 - \alpha). \tag{31}$$

After substituting the definitions of mrs and mpl we have[23]

$$mrs_t = mpl_t\, [1 - \tau_{l,t}]\, (1 - \alpha) \tag{32}$$

It is striking that while the bargaining process has created a wedge between mrs and mpl, search frictions *per se* do not appear directly in this equation, which is surprisingly similar to the one of the perfectly competitive model.

[23] Alternatively, eliminating the wage from (29) and (30) we could have derived the same expression.

Mechanically, this is simply because distortions enter symmetrically and additively in the labor demand and supply equations, hence, when those are equated search frictions perfectly offset and cancel out. The same is true also for λ and η which affect mrs and mpl exactly in the same proportion. Intuitively, one reason for this result is that the bargaining process internalizes search frictions through the wage rather than hours. In part this may be due to the fact that search frictions are inherently intertemporal—i.e., it takes time and resources to match unemployed workers with vacant positions—while the measured labor wedge is inherently intratemporal. The effects of intertemporal frictions are absorbed by movements in the wage and the *extensive* margin, and not in the equation of the labor wedge, which focuses on the *intensive* margin. It would be premature, however, to conclude that search frictions do not affect the equilibrium allocation. In fact, search frictions impose additional cross-restrictions to variables, mainly intertemporal and in relation to employment, that will be as well challenged by the data—introducing at least a new wedge that might be aptly dubbed the "extensive labor wedge". We will investigate this point further in Section 8.

The only distortion present in (32) is induced by the bargaining problem and summarized by $(1-\alpha)$. Since the production function takes the Cobb-Douglas form, this term is constant—making it irrelevant at business cycle frequency. In any case, at low frequency, this distortion acts like a tax on employment, which is observationally equivalent to a labor income tax; in the baseline calibration α takes a value of about 0.35 reducing substantially the steady state value required for τ_l.[24]

We can finally compare the two fundamental equations that are used to back out the labor wedge from the data in the competitive and non-competitive model

$$[1-\tilde{\tau}_{l,t}] = \frac{\widetilde{mrs}_t}{mpl_t} = -\frac{\psi}{mpl_t U_c(c_t)} G_h(n_t h_t) \tag{33}$$

$$[1-\tau_{l,t}] = \frac{mrs_t}{mpl_t}\frac{1}{1-\alpha} = -\frac{\psi}{mpl_t U_c(c_t)}\frac{G_h(h_t)}{1-\alpha} \tag{34}$$

The major difference between $[1-\tilde{\tau}_{l,t}]$ and $[1-\tau_{l,t}]$ is that in the perfectly competitive model the lack of distinction between the extensive and intensive margin has favored the use

[24] While a different functional form for the production function would imply a time-varying α, we prefer to stay as close as possible to the prototype RBC model by adopting the restrictions imposed by a Cobb-Douglas production function.

of *total hours* worked in the evaluation of the *mrs* in place of *hours per worker*. The presence of n_t in the prototype labor wedge equation, however, may artificially introduce a strongly procyclical element—given that $G_h > 0$. Employment is instead not present in (34) when we back out $[1 - \tau_{l,t}]$. Since movements in both margins are not equally significant in driving the business cycle frequency fluctuations in total hours, this distinction about the intensive-extensive margin is clearly important. Figure (1) shows the decomposition of total hours into its components. Total hours in the U.S. is clearly procyclical. When total hours is assumed to follow a path where one of the margins is fixed at its historical average and the other margin is assumed to follow the actual path in the data, one recognizes the well-known fact that most of the fluctuations in total hours comes from the extensive margin, not the intensive margin (Shimer 2009). This stylized fact is particularly relevant when interpreting the results of our exercise.

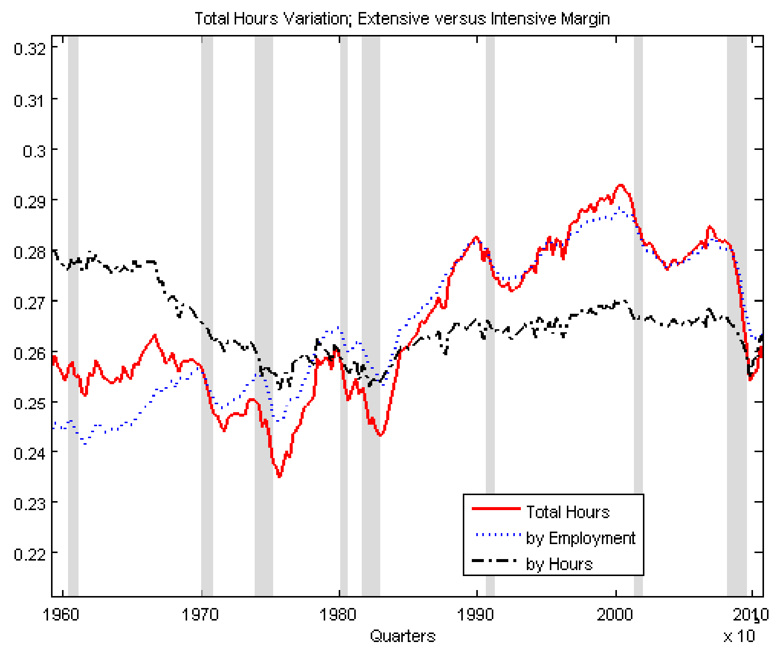

Figure 1: Total Hours decomposition into hours per worker versus employment. Plots for only employment and hours are generated, keeping the respective margin moving where the other margin is assumed to be fixed at the historical average. Shaded areas indicate NBER recession dates.

To see why this might affect the measured labor wedge, consider a simple constant Frisch elasticity utility function of the form $G(x) = -x^{(1+\phi)}/(1+\phi)$. Then we can write a simple relation describing the mapping between $[1 - \tilde{\tau}_{l,t}]$ and $[1 - \tau_{l,t}]$

$$[1 - \tilde{\tau}_{l,t}] = (1 - \tau_{l,t})(1 - \alpha)n_t^{\phi} \tag{35}$$

This shows how some of the observed procyclicality of $[1 - \tilde{\tau}_{l,t}]$ induced by employment does not have to be present in our labor wedge. In other words, even if we could interpret all the movements in τ_l as due to actual labor tax changes, equation (35) suggests that we would still find a strongly procyclical labor wedge $[1 - \widetilde{\tau}_{l,t}]$ in the prototype model merely because of the procyclicality of employment fluctuations in the data—the higher ϕ the stronger the procyclicality.

6 Numerical Results

In this section we present the results of two alternative experiments. In the first one, we generate artificial data from the search model, keeping $\tau_{l,t}$ constant, and compute the labor wedge as in CKM. Results show that, even though the data-generating process has no labor wedge, a prototype RBC model *a la* CKM would still measure a strongly procyclical labor wedge. This is very much consistent with the intuition given for the simple example described by equation (35). In the second experiment, we use U.S. data to draw a direct comparison between the behavior and statistical properties of the labor wedge in the prototype model and in the search model. We find that we can account for somewhere between 15 to more than 40 percent of the fluctuations in the prototype labor wedge, depending on the parameterization chosen.

6.1 Mapping with the Simulated Data

In this section, we use the model as the data generating process and analyze what it implies for the labor wedge of the prototype RBC model. More specifically, we calibrate our model with search frictions and simulate artificial data by shutting off all the exogenous shocks (i.e., wedges) except the productivity process, $\tau_{z,t}$. We set $[1 - \tau_{l,t}]$ to be constant at 0.6 over time. Then, using the simulated data on consumption, output and total hours, we measure the labor wedge as in CKM. The null hypothesis is no movements in the prototype labor wedge.

Calibration

We follow a standard approach for parameters that are not related to search frictions. More specifically, we set $\beta = 0.99$, $\delta = 0.025$, as standard in the literature, while we set $\alpha = 0.36$ to match the labor's share in the prototype model.[25]. We adopt a log-log-utility function to follow CKM. To calibrate parameters related to search frictions, we follow an approach that is similar to Merz (1995) and Andolfatto (1996), and target first moments of the labor market variables, n and h. The total recruiting cost share, i.e. $\kappa v/y$, is set to 1.5 percent. We follow Merz (1995) and assume a strictly convex search cost function $c(e) = c_0 e^{\mu}$, where c_0 is normalized to 1 and the output share of search costs born by workers, $c(e)(1-n)$, is targeted to be 0.5 percent. Unfortunately, there is no guidance in the literature over these parameters and we try to minimize the resource cost of search and recruitment by this calibration. It turns out this is not far from what has been done in the literature before and our results are not sensitive to the exact share of these costs (see Andolfatto 1996). What is more sensitive is the parameter μ, that determines the degree of convexity in cost of workers' search effort. As we have argued in section 4.4, in order to have a unique steady state equilibrium we need to have enough convexity in this function. In particular, as μ converges to 1 we will have multiplicity of equilibria. In our calibration μ is implied by the relative ratios of the search/recruitment costs in output, i.e. $\mu = 3$.[26] The elasticity of matching function and bargaining power of workers (firms) are calibrated such that the Hosios (1990) condition holds, which implies $\gamma = \lambda$. The elasticity of job matches with respect to vacancies, γ, is set to 0.5, which lies in the middle of the range of estimates reported by Petrongolo and Pissarides (2001). The quarterly rate of transition from employment to non-employment, σ, is set at 0.15, following calibration of Andolfatto (1996) and references therein. Five parameters, μ, ϕ, ψ, κ and χ are calibrated to match five moments, $\kappa v/y = 0.015$, $c(e)(1-n)/y = 0.005$, $n = 0.7074$, $h = 0.3752$, and $q = 0.9$.[27]

[25] Since total recruiting costs are a relatively small fraction of output, labor's share in the search model is very close to $1-\alpha$.

[26] Our results are essentially the same when $\mu = 1.5$.

[27] In this exercise, shocks to the matching efficiency are shut down, as well as for τ_l and $\tau_{g,}$. The latter two parameters are set to 2/5 and 0.16, respectively, following the averages in the data and the measured labor wedge from the previous section. Our targets for n and h are the sample averages for employment and hours per worker, respectively (the sample is described in Appendix A). The calibration target for q follows from the average duration of a posted vacancy based on van Ours and Ridder (1992).

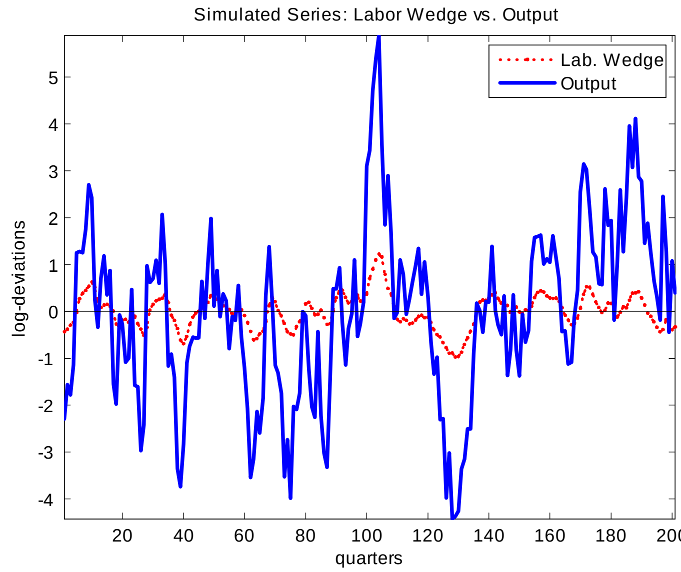

Figure 2: Labor wedge and output from the simulated data.

Results

Figure (2) shows simulated output series and the prototype labor wedge derived under the baseline calibration described above. The standard deviation of $\widetilde{\tau}_{l,t}$ relative to output is 0.15 which makes the null hypothesis of no fluctuations clearly rejected. In other words, this implies that doing the business cycle accounting as in CKM would falsely detect the presence of a labor wedge when reality is well described by the labor search model with no exogenous fluctuations in the labor wedge. Moreover, the correlation between the labor wedge is strongly procyclical: the correlation with output and total hours is about 0.85 and 0.96, respectively. Hence, a simple extension of the prototype RBC model with search frictions is consistent with a significantly procyclical labor wedge, which is entirely due to endogenous movements in both the extensive and the intensive margins. Those results support the view that search frictions may induce endogenous fluctuations in hours and employment that will manifest themselves as a procyclical and variable labor wedge as in CKM or Shimer (2009).

Table 1: Simulation Exercise		
	Simul. Data	Actual Data
$std([1-\widetilde{\tau}_{l,t}])/std(y_t)$	0.15	1.00
$corr([1-\widetilde{\tau}_{l,t}], y_t)$	0.85	0.51
$corr([1-\widetilde{\tau}_{l,t}], l_t)$	0.96	0.87

6.2 Mapping the Models to the U.S. Data

As we mentioned in previous sections, employment fluctuations make the measurement of $\widetilde{mrs}$ differ from the mrs of the search model. This difference is crucial for the labor wedge, because most of its variation is associated with $\widetilde{mrs}$ and not mpl fluctuations. Figure (3) plots the log-detrended $\widetilde{mrs}$ and mpl jointly with the log detrended prototype labor wedge. In line with the model, the log $\widetilde{mrs}$, mpl, and $[1-\widetilde{\tau}_{l,t}]$ has been detrended by subtracting the HP-trend of the log of consumption, output, and consumption-output ratio, respectively.[28] While some low-frequency movements in the labor wedge may be driven by the mpl, particularly between 1982-90, most of the business cycle movements are largely due to fluctuations in the $\widetilde{mrs}$. Hence, seen from the lens of the perfectly competitive model, labor supply increases less than implied by the model during expansions and it does not fall as much as required by the model during recessions. Given that the search labor wedge defined in (32) basically modifies the $\widetilde{mrs}$, reducing its procyclicality, the search model points in the right direction.

We recover the labor wedge in the prototype model $(1-\tilde{\tau}_{l,t})$ and in the model with labor market search frictions $(1-\tau_{l,t})$, from equations (33) and (34), respectively. In conducting this accounting exercise we partly rely on the calibration of the previous section; however, we use a general functional form for the disutility of labor: $G(x) = [(1-x)^{(1-\phi)} - 1]/(1-\phi)$.[29] Given the chosen form for $G(\cdot)$, we can write down two equations that implicitly define the prototype and search labor wedges[30]

$$\frac{\widetilde{\psi} c_t}{(1-l_t)^{\widetilde{\phi}}} = \frac{y_t}{l_t}(1-\alpha)\left[1-\widetilde{\tau}_{l,t}\right] \quad (36)$$

$$\frac{\psi c_t}{(1-h_t)^{\phi}} = \frac{y_t}{l_t}(1-\alpha)^2(1-\tau_{l,t}) \quad (37)$$

[28] The calibration strictly follows the one of CKM. Series are demeaned for ease of comparison.

[29] The commonly used CRRA form $G(x) = -x^{1+\phi}/(1+\phi)$ deliveres fairly similar results.

[30] The parameters such as ψ and ϕ, will take different values when calibrated in one of the two models. With a loose notation, we denote with a $\widetilde{\cdot}$ the parameter value calibrated for the perfectly competitive model.

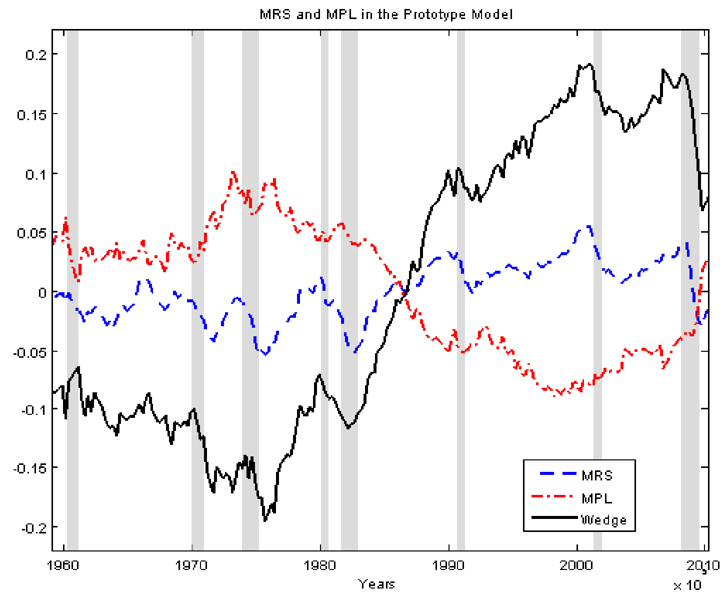

Figure 3: De-meaned log-detrended $\widetilde{mrs}$, mpl and the prototype labor wedge, $[1 - \widetilde{\tau}_{l,t}]$. They are detrended by subtracting the HP-filter trend of $\log(c_t)$, $\log(y_t)$ and $\log(c_t/y_t)$ respectively. Shaded areas indicate NBER recession dates.

Data on y_t, l_t, h_t and c_t pin down a unique labor wedge for each period in both equations once we calibrate $\widetilde{\phi}$ (ϕ), and $\widetilde{\psi}$ (ψ). We set the parameter $\widetilde{\psi}$ (ψ) such that in steady state $\widetilde{\tau}_l$ (τ_l) is 0.4, consistent with the tax wedge measured by Prescott (2004). Finally, we adjust $\widetilde{\phi}$, to get the same steady state labor elasticity in both models. In the baseline calibration, we choose the limiting case, $\widetilde{\phi} = 1$, which is also used by CKM and implies a steady state Frisch elasticity of about 2.8, then, in order to get the same elasticity in the search model, we set ϕ= 0.6.

Figure (4) shows the results for the 'low-frequency case' when variables are detrended using HP-trends in output and consumption—low-frequency movements are not necessarily filtered out. The upper panel presents the marginal rate of substitution for both models while the lower panel shows the related wedges. There is a clear low-frequency cycle in the wedges, mainly due to the mpl, that has not been captured by the trend in the consumption-labor ratio. The rise in the labor wedge in the second half of the sample is due to a declining mpl, whether we assume competitive labor markets or not. As suggested by Figure (4), the search labor wedge is less volatile than the prototype labor wedge, by about 23 percent (see Table 2). This is entirely due to our measurement of the marginal rate of substitution, which is about 40 percent

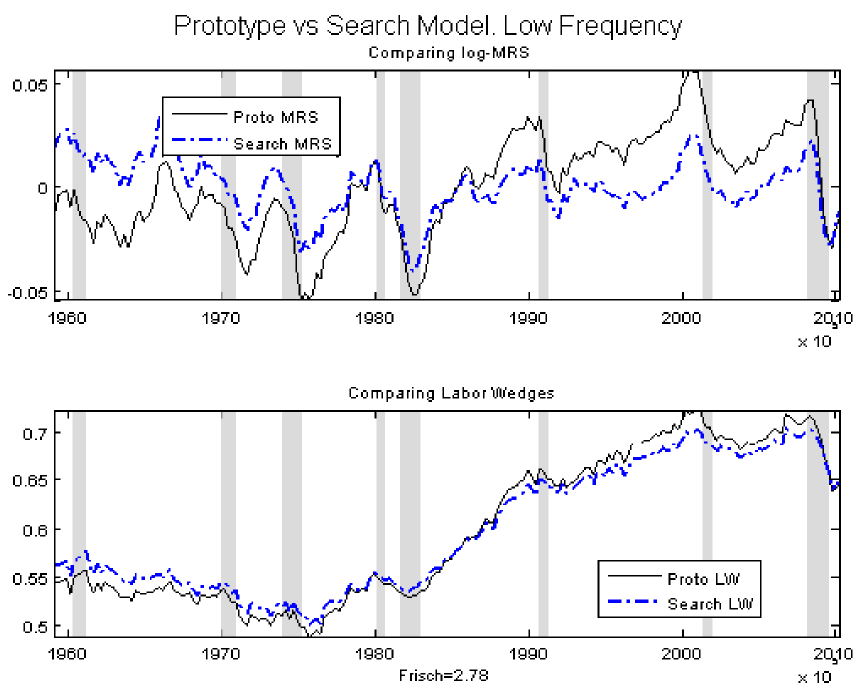

Figure 4: Prototype and search marginal rate of substitution and labor wedges, $[1-\widetilde{\tau}_{l,t}]$ and $[1-\tau_{l,t}]$, where Frisch elasticity is around 2.8. Detrending uses HP-filter trend of $\log(c_t)$ for mrs and $\log(c_t/y_t)$ for labor wedges. Shaded areas indicate NBER recession dates.

less volatile than the $\widetilde{mrs}$ measured through (36). This obviously follows from the fact that the intensive margin is the less variable margin in total hours, which makes the measurement of the marginal rate of substitution in the search model less variable than the one of the prototype model.

Table 2: Moments for High Frisch Elasticity (2.78)

	Y_t	$[1-\widetilde{\tau}_{l,t}]$	$[1-\tau_{l,t}]$	$\widetilde{mrs}_t$	mrs_t
Std. Dev.	0.015	0.015	0.012	0.013	0.010
	(0.015)	(0.074)	(0.057)	(0.024)	(0.014)
Corr(x_t, x_{t-1})	0.862	0.749	0.666	0.893	0.865
	(0.862)	(0.988)	(0.983)	(0.966)	(0.922)
Cross Correlations					
	Y_t	L_t	C/Y		
$[1-\widetilde{\tau}_{l,t}]$	0.514	0.866	-0.162		
	(0.183)	(0.992)	(-0.086)		
$[1-\tau_{l,t}]$	0.410	0.796	-0.043		
	(0.173)	(0.978)	(-0.066)		

All variables are HP-filtered, except L_t.
Moments in () are for the 'low-frequency' case.

As shown in Table 2 the correlations of the labor wedge with cyclical variables are substantially different from zero (excluding the consumption-output ratio). Results are particularly evident at business cycle frequency—in this case every variable in Table 2, but L_t, has been HP-filtered (shown in Figure 5). Given a relatively high Frisch elasticity (about 2.78), at business cycle frequency the search labor wedge is 20 percent less volatile than the prototype wedge (see Table 2). As in the low-frequency case, the nature of the decline stems from the measurement of the marginal rate of substitution. Moreover, the correlation with total hours is reduced by about 8 percent while the one with output by 20 percent.

While the calibration of α and ψ has no implication for our analysis, results are clearly affected by the choice of the parameter that governs the Frisch elasticity ϕ. So far, the Frisch elasticity we have used in the numerical exercises was 2.78, in line with most macro models and chosen to be comparable with CKM. However, this elasticity is at the high end of the estimates found in the micro literature (Blundell and MaCurdy 1999). In what follows we will show that our results are indeed amplified when the Frisch elasticity is chosen consistent with most of the micro estimates.

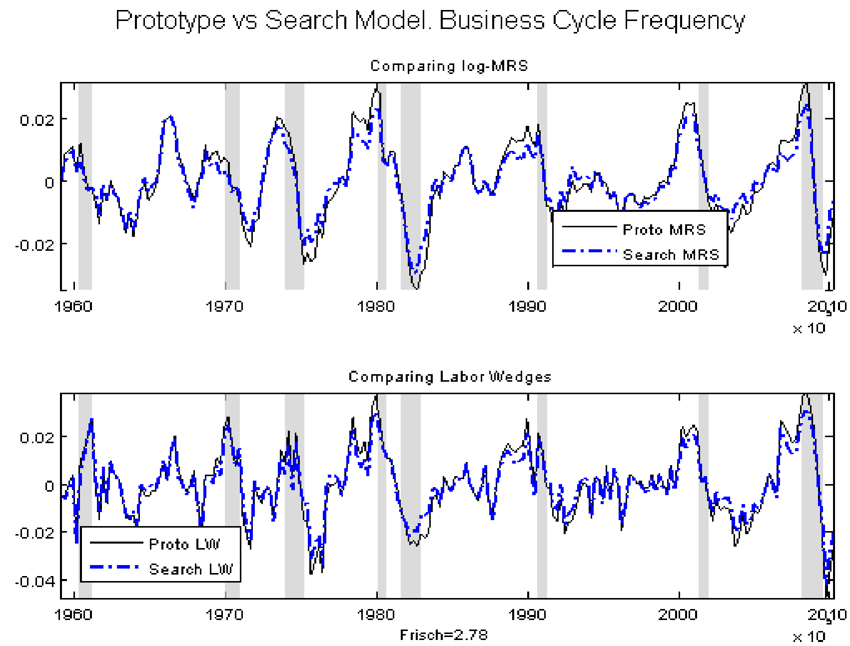

Figure 5: Prototype and search marginal rate of substitution and respective labor wedges, $[1-\widetilde{\tau}_{l,t}]$ and $[1-\tau_{l,t}]$ where Frisch elasticity is around 2.8. All expressed as deviations from their respective HP-filter trend. Shaded areas indicate NBER recession dates.

Low Frisch Elasticity The major problem behind the failure of the prototype model, as well as our extension of it, is that hours do not vary as much in the data as implied by our models. This has been well-recognized in the context of macro models. Higher aggregate micro elasticities tend to produce smaller wedges at the expense of a large micro literature that argues for lower individual labor elasticities. Here, we replicate our exercise by targeting a Frisch elasticity of 0.5 on average, which is more in line with these studies. Figures (6), (7) and Table 3 present our results.[31]

While the overall volatility is higher across variables, there is a dramatic reduction in the volatility of the labor wedge with search frictions of more than 40 percent, both at low frequency (Figure 6) and at business cycle frequency (Figure 7). Once again, this reduction is entirely due to the lower volatility of mrs relatively to $\widetilde{mrs}$. The reduction in correlation with output is about 22 percent at business cycle frequency. Note however, that a lower Frisch elasticity increases the overall standard deviation in both wedges relative to high elasticity case. Given that wages are not very volatile in the data, most macro models favor a high elasticity. However, while in

[31] This requires $\phi = 5.54$ in (36) and $\phi = 3.33$ in (37) for the given sample averages of L and h.

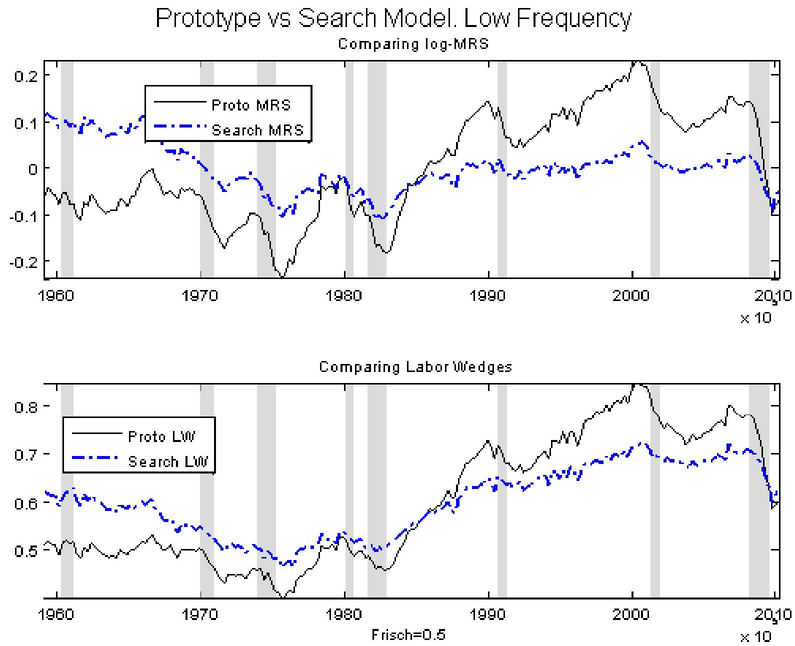

Figure 6: Prototype and search marginal rate of substitution and respective labor wedges, $[1-\widetilde{\tau}_{l,t}]$ and $[1-\tau_{l,t}]$ where Frisch elasticity is 0.5. Detrending uses HP-filter trend of $\log(c_t)$ for mrs and and $\log(c_t/y_t)$ for labor wedges. Shaded areas indicate NBER recession dates.

the prototype model the volatility of the $\widetilde{mrs}$ tripled, the low-elasticity mrs volatility is only 1.46 times the high elasticity $\widetilde{mrs}$, which makes the labor search model much more promising in reconciling micro and macro estimates.

Table 3: Moments for Low Frisch Elasticity (0.5)

	Y_t	$[1-\widetilde{\tau}_{l,t}]$	$[1-\tau_{l,t}]$	$\widetilde{mrs}_t$	mrs_t
Std. Dev.	0.015	0.037	0.021	0.035	0.019
	(0.015)	(0.165)	(0.082)	(0.114)	(0.052)
$\text{Corr}(x_t, x_{t-1})$	0.862	0.818	0.59	0.875	0.732
	(0.862)	(0.99)	(0.971)	(0.987)	(0.962)
Cross Correlations					
	Y_t	L_t	C/Y		
$[1-\widetilde{\tau}_{l,t}]$	0.721	0.975	-0.452		
	(0.238)	(0.998)	(-0.153)		
$(1-\tau_{l,t})$	0.56	0.869	-0.283		
	(0.229)	(0.813)	(-0.132)		

All variables are HP-filtered, except L_t.
Moments in () are for the 'low-frequency' case.

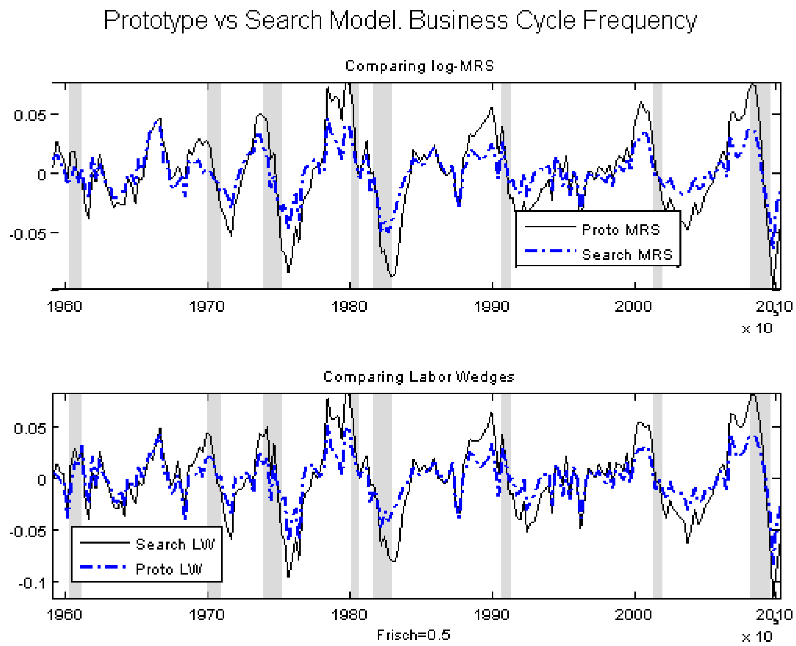

Figure 7: Prototype and search marginal rate of substitution and respective labor wedges, $[1 - \widetilde{\tau}_{l,t}]$ and $[1 - \tau_{l,t}]$ where Frisch elasticity is 0.5. All expressed as deviations from their respective HP-filter trend. Shaded areas indicate NBER recession dates.

7 Alternative Mechanisms for Wage Determination

In this section we show how the labor wedge implied by the search model is substantially robust to alternative bargaining protocols. We argued in the previous section that the labor wedge in the presence of search frictions is not directly or explicitly affected by search frictions. This was partly due to the form of the bargained wage. Hence, it is natural to ask whether this result is robust to alternative wage protocols. To give a comprehensive treatment of this issue, we analyze how equation (32) changes with right to manage bargaining, and rigid wages; two alternatives that are extensively used in the literature (i.e. Shimer 2010 and Trigari 2006). To provide a basic benchmark we start with the social planner's problem. This avoids the discussion of the wage bargaining altogether, focusing on the equilibrium optimal allocation of hours.

7.1 Social Planner's Problem

Wedges, by construction, operate as taxes, hence are distortionary. In order to formulate the decentralized problem in our model as a planner's problem, suppose we turn off all of the wedges. The social planner's problem is defined by maximizing the objective function in (3) subject to the aggregate resource constraint, (23), and the aggregate matching function, (6). Then, it is very easy to show that the optimal hours decision yields the following result

$$mrs_t = mpl_t. \tag{38}$$

This result highlights how bargaining over marginal surpluses generates a static wedge, $(1-\alpha)$, that a social planner would remove. In other words, socially optimal allocations cannot be decentralized under Nash bargaining, when we have both extensive and the intensive margin. We know that this optimality condition does not hold in the data, and one can back out the exact same labor wedge we recovered in our model with search frictions, with the exception of the static component $(1-\alpha)$.

7.2 Right-to-Manage Bargaining

An alternative to the Nash bargaining that has been used in the literature is the right-to-manage bargaining.[32] This form of bargaining is motivated by empirical observations that firms set hours unilaterally and potentially bargain over the wages.

Following Trigari (2006), we assume that under right-to-manage bargaining, firms are free to set hours per worker optimally and take the employment stock and wages as given. This means that $mpl_t = w_t$ at all times. Since hours are optimally chosen, firms are indifferent about hiring an additional worker.[33] Then, workers and firms bargain over the wage. This bargaining implies implicitly that $W^f_{n_t}(\omega_t)$ in (20) is zero for any wage, w_t. In other words, firms can always bring the marginal surplus to zero by adjusting the intensive margin. Hence, in setting the wage, workers will maximize their marginal surplus taking into account the optimal choice of hours $h_t = h(w_t)$. Since $h'(w_t) = -h_t/(\alpha w_t)$,[34] the wage has to satisfy the following equation

$$(1-\alpha)w_t = -\frac{\psi G_h(h_t)}{U_c(c_t)(1-\tau_{l,t})} \tag{39}$$

[32] Right-to-manage bargaining is used in the literature, mostly to get a lower elasticity of marginal costs with respect to output in the standard New Keynesian DSGE models (Trigari 2006, Sunakawa 2012).

[33] Notice that this results is related to assumption that the production function can be written solely in terms of capital and *total* hours worked—which is an assumption vastly common to the DSGE literature.

[34] In writing $h'(w)$ we have exploited the fact that $\alpha = -f_{ll}l/f_l$.

In other words, the wage is set as a constant markup, $1/(1-\alpha)$, over the marginal rate of substitution.[35] In fact, imposing firms' optimal hours choice in (39) implies that the equation that defines the labor wedge is identical to (32),

$$mrs_t = mpl_t(1-\tau_{l,t})(1-\alpha). \tag{40}$$

This result shows that a change in the bargaining protocol does not affect our main message that search frictions have minor, if any, cyclical implications for the labor wedge. The existence of both intensive and extensive margins still highlights that the correct marginal rate of substitution that appears in the labor wedge equation is mrs_t and not $\widetilde{mrs}_t$. The intuition behind this result is simple: Given a wage, hours are chosen optimally as in an RBC model. The wage setting, however, introduces only a static distortion, $(1-\alpha)$.[36]

7.3 Rigid Wages

The Nash bargaining outcome implied by equation (20) is only one of many wage offers that are potentially acceptable to both parties, as long as $W^f_{n_t}(\omega_t) \geq 0$, and $W^h_{n_t}(\omega_t) \geq 0$. This fact along with the inability of the benchmark labor market search model to generate the observed level of volatility in key labor market variables has led many researchers to propose wage rigidity as an alternative (Shimer (2005), Hall (2005), Hall and Milgrom (2008) and Gertler and Trigari (2009), among others). Since there is no consensus on the source of this rigidity, we do not want to take a stand on the 'true' macroeconomic model of wage rigidity. Instead, we follow Shimer (2010) and distinguish between a *target wage* $\dot{w}(\omega_t)$, which is determined by axiomatic Nash bargaining, which is similar to our formulation above, and the *actual wage* $w(\omega_t)$. The actual wage is a weighted average of the target wage and the actual wage observed in previous period.

$$w(\omega_t) = \Theta w(\omega_{t-1}) + (1-\Theta)\,\dot{w}(\omega_t) \tag{41}$$

[35] Notice that Trigari (2006) and Sunakawa (2012) have a time varying markup because the production function they use is linear in n. In our case, however, assuming a production function as $f(k,h)n$, would imply counterfactuallly high increasing returns to scale in capital and total hours at both firm and aggregate level.

[36] This is absent only if the elasticity of hours to wage is infinity. In other words, only if the firm is able to make a credible threat to the worker to set $h = 0$ if the workers asks for a markup above one.

where Θ determines the level of rigidity.

The target wage is given by the solution to an analog of the Nash bargaining problem we analyzed before,

$$\max_{w,h} \hat{W}^h_{n_t}(\omega_t, w)^{1-\lambda} \hat{W}^f_{n_t}(\omega_t, w)^{\lambda} \tag{42}$$

where $\hat{W}^h_{n_t}(\omega_t, w)$ and $\hat{W}^f_{n_t}(\omega_t, w)$ denote marginal surpluses at any wage, w, and are related to $W^h_{n_t}(\omega_t)$ and $W^f_{n_t}(\omega_t)$ in a simple way:

$$\begin{aligned}
\hat{W}^h_{n_t}(\omega_t, w) &= U_c(c_t)\left[1-\tau_{l,t}\right] h_t\left(w - w(\omega_t)\right) + W^h_{n_t}(\omega_t) \\
\hat{W}^f_{n_t}(\omega_t, w) &= h_t\left(w(\omega_t) - w\right) + W^f_{n_t}(\omega_t)
\end{aligned}$$

It turns out that the target wage and optimal hours decision leads to familiar conditions, even with the presence of rigid wages.

$$\lambda \hat{W}^h_{n_t}(\omega_t, w) - (1-\lambda)(1-\tau_{l,t}) U_c(c_t) \hat{W}^f_{n_t}(\omega_t, w) = 0 \tag{43}$$

$$U_c(c_t)\left[1-\tau_{l,t}\right]\left[\tau_{z,t} f_l(k_t, n_t h_t) + \tau_{z,t} f_{ll}(k_t, n_t h_t) n_t h_t\right] + \psi G_h(h_t) = 0 \tag{44}$$

The second equation, (44), gives precisely the same static condition for labor wedge measurement. What really changes, is the impact on the extensive margin, as (43) implies that the marginal surpluses at the observed wage will be influenced by the wage rigidity such that the following holds

$$\lambda W^h_{n_t}(\omega_t) = (1-\tau_{l,t}) U_c(c_t)\left[(1-\lambda) W^f_{n_t}(\omega_t) + \left(w(\omega_t) - \dot{w}(\omega_t)\right)\right] \tag{45}$$

and the wedge on the extensive margin is affected significantly, i.e. equation (48) changes to incorporate the wage rigidity. Finally, we can rewrite equation (45) by using the definition of the actual (41),

$$\lambda W^h_{n_t}(\omega_t) = (1-\tau_{l,t}) U_c(c_t)\left[(1-\lambda) W^f_{n_t}(\omega_t) + \Theta\left(w(\omega_{t-1}) - \dot{w}(\omega_t)\right)\right]. \tag{46}$$

As parties bargain over the wage, the equilibrium surplus sharing rule expressed in equation

(46) divides the total surplus of the marginal match among the household and the firm, as usual. The difference between the case with real wage rigidity and the baseline model with efficient bargaining is the last term, which changes the value of the marginal surplus each party extracts if the target wage is different from last period's wage. For instance, suppose the economy suffers an adverse shock to productivity, after a series of good realizations. Then the likelihood of current target wage being below the previous period's actual wage will be higher, transferring some extra surplus to households than they would otherwise get. This implies lower surplus to firms creating less incentives to post vacancies and further depressing job creation. The severity of this channel will be a function of the wage rigidity parameter, Θ. In the absence of wage rigidity , when $\Theta = 0$, this expression collapses to the outcome from efficient Nash bargaining we used in the baseline model, eq. (21).

The particular wage determination mechanism one uses affects the allocations. However, our discussion shows that the focal point on the labor wedge that we stressed, the optimal hours decision, is not affected. What alternative bargaining protocols actually do is to change the response of agents on the extensive margin, job creation decision, search effort choice and the resulting equilibrium unemployment. However, it does not, create a substantially different labor wedge from the one in equation (32).

8 Search Frictions and the Extensive Margin

Two main results of our paper seemingly contradict the recent work by Shimer (2010); the independence of the labor wedge from search frictions and the relative improvement in the volatility of the labor wedge due to the explicit distinction between the extensive and the intensive margin. Shimer (2010) points out that search frictions are essentially adjustment costs; hence, since standard calibrations of the search models suffer from an inability to generate enough volatility, he argues that adding adjustment costs this way into an RBC model cannot improve its performance. However, our results complement and reinforce the ones of Shimer (2010). To understand our reasoning, it is important to recall that our formulation of search frictions create two distinct labor wedges; one for the intensive margin, another one for the extensive margin. The former is the main focus of the business cycle accounting exercises.

Hence, most of the studies dealing with the labor wedge focus on this margin as a measure of the inability of different RBC models to match the data. However, we show that when we distinguish between these two margins, we can explain part of the cyclical behavior of the wedge measured at the intensive margin, at the cost, however, of introducing a second wedge at the extensive margin.

In order to understand the effects of search frictions on the extensive margin, it is crucial to observe that the variation in this margin is governed by mainly two decisions: workers' search effort and firms' vacancy posting decision. Optimality conditions for search effort and vacancies (see equations 13 and 19) along with the law of motion for employment will govern the movements at the extensive margin. Using equations (13) and (19), noting that the terms in expectations are each party's marginal surplus from a match, one can write down a simple equation that relates market tightness and search effort to movements in the value of expected surpluses:

$$\frac{\kappa\theta_t}{e_t c_e(e_t)} = \frac{\mathrm{E}_t\left[U_c(c_{t+1})W^f_{n_{t+1}}\right]}{\mathrm{E}_t\left[W^h_{n_{t+1}}\right]}. \tag{47}$$

Variations in the relative values of expected surpluses will determine fluctuations in market tightness, θ_t, and search effort, e_t, implying fluctuations in employment. Note also that the Nash bargaining outcome requires a specific sharing rule determining the relation between $W^f_{n_t}$ and $W^h_{n_t}$ for all t (see equation 21). Once this is taken into account, it is easy to see how fundamental parameters related to search frictions explicitly affect the movements in the extensive margin through search effort and market tightness. Up to a covariance term, equation (47) can be written as[37]

$$\frac{\kappa\theta_t}{e_t c_e(e_t)} \simeq \frac{\lambda}{1-\lambda} E_t \frac{1}{1-\tau_{l,t+1}} \tag{48}$$

Equation (48) shows how the labor wedge for the intensive margin, $1-\tau_l$, affects employment fluctuations. In principle, if this equation does not hold in the data, it will provide us with an estimate of an additional, and a distinctively different wedge - an extensive labor wedge - that primarily affects the intertemporal margin.

[37] This is similar to the 'intertemporal labor wedge' derived by Arseneau and Chugh (2010).

In other words, we conjecture that fluctuations in employment introduce an additional challenge for the model when it comes to matching the data. A well-defined and satisfactory test of this conjecture requires conducting a full blown business cycle accounting exercise that is beyond the scope of current paper and we leave it to future work.

We can provide, however, an illustrative simple accounting exercise using equation (48) under certain assumptions. To introduce enough degrees of freedom into the model for that purpose, assume that vacancy posting cost, κ, varies over time. More specifically, suppose that effective vacancy posting cost is $\kappa\,[1+\tau_{v,t}]$, where $\tau_{v,t}$ follows a stochastic process and operates like the investment wedge, $[1+\tau_{x,t}]$, but for vacancy creation. Assuming that agents correctly anticipate $1-\tau_{l,t+1}$ in equation (48), and search effort does not vary, we can back out a realization for $[1+\tau_{v,t}]$ that makes the unemployment data consistent with the evolution of the unemployment in the model. In principle, variations in the matching efficiency shock, $\tau_{\chi,t}$, that we introduced in our model in (6) or even changes in the bargaining power λ can map into an extensive labor wedge. Cheremukhin and Restrepo-Echavarria (2010) argue that the former is a promising avenue, whereas the latter does not help in generating fluctuations in the labor wedge. They are all observationally equivalent for us and we do not see our paper as a test on determining what specific friction maps into a labor wedge.

The resulting estimate of the extensive wedge, $[1+\tau_{v,t}]$, is presented in Figure (8) along with the labor wedge for the intensive margin $[1-\tau_{l,t}]$. Figure (8) shows a clearly countercyclical wedge for the vacancy creation that operates at the extensive margin. It implies that the cost of creating a vacancy increases quite sharply during recessions. This is consistent with the search models' inability to generate substantial unemployment fluctuations. In other words, the model requires significantly countercyclical and volatile shocks to vacancy creation cost to be able to replicate the US data. This wedge is a degree of magnitude more volatile (and cyclical) than $[1-\tau_{l,t}]$. This suggests that the argument in Shimer (2010) manifests itself at the extensive margin. Therefore, we have an interesting trade-off between the extent of the mismeasurement for the mrs_t at the intensive margin, and the contribution of employment to the total hours variation. As fluctuations in the extensive margin play more of a role over the business cycle, the mismeasurement problem in the prototype labor wedge becomes more severe. Hence, search frictions account for more of the variation in the labor wedge for the intensive margin. On the

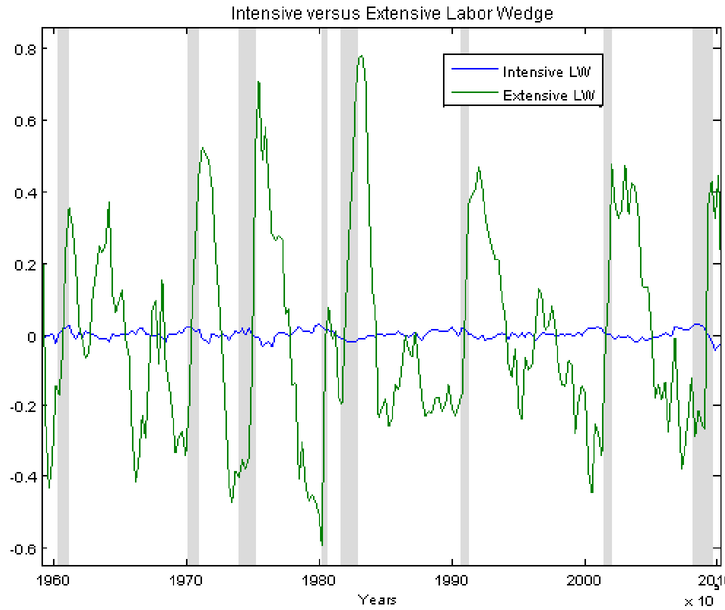

Figure 8: Intensive wedge refers to $1 - \tau_{l,t}$ whereas the extensive wedge refers to $[1 + \tau_{v,t}]$, described in the text. Both series are HP filtered.

other hand, since search frictions primarily manifest themselves at the extensive margin, as this margin becomes more important for the total labor input, the failure of the model as a whole becomes more severe.

To highlight the significance of the relative contribution of the extensive margin to total hours variation, consider an economy where the intensive margin plays a more significant role in accounting for variation in total hours. For instance, in Germany, intensive margin shows much more cyclicality than the extensive margin relative to the U.S. Data between 1960:Q1-2010:Q4 from Germany indicate that hours per worker is primarily driving the variations in total hours[38]. The standard deviations for total hours, l, hours per worker, h, and employment, n, are 0.0316, 0.0308, and 0.0076, respectively.[39] When total hours is assumed to follow a path where one of the margins is fixed at its historical average and the other margin is assumed to follow the actual path in the data, we see that the intensive margin accounts for almost all the low-frequency variation and a significant portion of the business cycle frequency variation. This is shown for Germany in figure (9), which is an analog of figure (1).

[38] Data we use come from Ohanian and Raffo (Forthcoming).

[39] Corresponding numbers for the U.S. are 0.0147, 0.0092, and 0.0380, respectively.

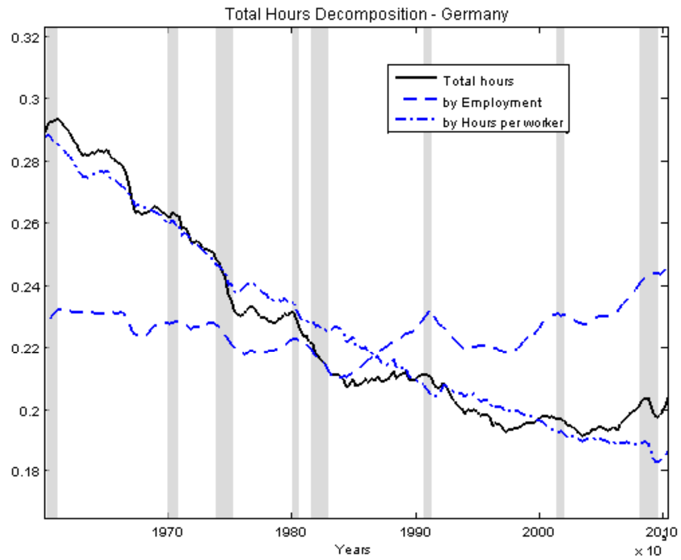

Figure 9: Total Hours decomposition into hours per worker versus employment. Plots for only employment and hours are generated, keeping the respective margin moving where the other margin is assumed to be fixed at the historical average. Shaded areas indicate U.S. recession dates, not those for Germany.

Consequently, when the labor wedge with search frictions, $[1 - \tau_{l,t}]$, is measured for Germany, relative to the U.S., the volatility at the business cycle frequency declines by about 10 percent as opposed to 33 percent. On the other hand, the relative improvement in terms of the decline in the measured wedge for Germany with lower Frisch elasticity case is 35 percent relative to 43 percent in the U.S.[40] In other words, as the extent of variation in the extensive margin increases, mismeasurement accounts for less of the measured labor wedge.

9 Conclusion

The business cycle accounting literature has identified the relation between the marginal productivity of labor (*mpl*) and the marginal rate of substitution (*mrs*) as a weakness common to many perfectly competitive RBC models. More precisely, the observed wedge between the *mrs* and *mpl* seems to conceal one of the keys that would allow us to improve our understanding of what drives economic cycles. This paper has explored the role of labor-market search models

[40] The detailed results of the this numerical exercise is reported in the appendix. We essentially conduct the same exercise in section 6.2 for Germany.

in addressing this weakness.

Our results indicate that labor-market search frictions *per se* do not provide a mechanism that could directly alter movements in the wedge between mpl and mrs, as observed in the data. The main reason is that the bargaining process between firms and workers internalize search frictions through the wage decision, rather than hours decision. However, search frictions impact the movements at the extensive margin more explicitly, which we also explore somewhat in the paper.

The advantage of the labor-search model is due to its natural ability to distinguish between fluctuations in total hours and hours per worker, leaving no doubt on the fact that the mrs has to be measured in terms of hours per worker. Since fluctuations in employment account for the majority of the movements in total hours, confounding hours per worker with total hours leads to a substantial mismeasurement of the mrs and, consequently, a serious misspecification of the model which appears as the labor wedge. Our findings show that, at business cycle frequency, about 20 percent of the observed volatility and most of the procyclicality of the labor wedge can be attributed to fluctuations in the extensive margin (employment) through their effect on the mrs.

In addition to that, we also cast additional light on how the divergent finding between macro and micro estimates of the labor supply elasticity is exacerbated by the use of total hours in the measurement of the mrs. The search model is able to take much lower values for the Frisch elasticity without dramatically increasing the volatility of the mrs—which is the main reason for why macro estimates give high values of the labor demand elasticity. We also show that these results are very robust to alternative assumptions about wage determination.

Finally we present some simple discussion and preliminary evidence on the importance of the apparent wedge for the extensive margin in the labor input. We show that search frictions primarily manifest themselves as a wedge on the extensive margin but the relative importance of it will impact the extent of mismeareument for the wedge at the intensive margin, the usual object of interest in the literature. We leave an exhaustive study of the role search frictions play on the extensive margin to future work.

References

[1] Andolfatto, David. 1996. "Business Cycles and Labor Market Search." *American Economic Review*, 86(1): 112–132.

[2] Arseneau, David M., and Sanjay K. Chugh. 2010. "Tax Smoothing in Frcitional Labor Markets." *mimeo.* Board of Governors of the Federal Reserve System.

[3] Beauchemin Kenneth, and Murat Tasci. 2008. "Diagnosing Labor Market Search Models: A Multiple-Shock Approach." *Macroeconomic Dynamics,* Forthcoming.

[4] Blanchard, Olivier Jean, and Jordi Gali. 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics*, 2: 1–30.

[5] Blundell, Richard and Thomas MaCurdy. 1999. "Labor Supply: A review of alternative approaches." in *Handbook of Labor Economics*, ed. Orley C. Ashenfelter and David Card, 1559–1695. Amsterdam: North-Holland.

[6] Chari, V.V., Patrick Kehoe, and Ellen R. McGratten. 2007. "Business Cycle Accounting." *Econometrica*, 75(3): 781-836.

[7] Chang, Yongsung, and Sun-Bin Kim. 2007. "Heterogeneity and Aggregation: Implications for Labor Market Fluctuations." *American Economic Review*, 97(5): 1939–1956.

[8] Cheremukhin, Anton A., and Paulina Restrepo-Echavarria. 2010. "The Labor Wedge as a Matching Friction." *Working Paper #1004.* Federal Reserve Bank of Dallas.

[9] Cho, Jang-Ok. and Thomas F. Cooley. 1994. "Employment and Hours over the Business Cycle." *Journal of Economic Dynamics and Control*, 18: 411–432.

[10] Cociuba, Simona., Alexander Ueberfeldt and Edward C. Prescott. 2009. "U.S. Hours and Productivity Behavior Using CPS Hours Worked Data: 1947-III to 2009-II." *Mimeo.* Federal Reserve Bank of Dallas.

[11] Cociuba, Simona, and Alexander Ueberfeldt. 2010. "Trends in U.S. Hours and the Labor Wedge." *Mimeo.* Federal Reserve Bank of Dallas.

[12] Cole, Harold L., and Richard Rogerson. 1999. "Can the Mortensen-Pissarides Matching model Match the Business Cycle Facts?" *International Economic Review*, 40(4): 933-959.

[13] Cole, Harold L., and Lee E. Ohanian. 2002. "The U.S. and U.K. Great Depressions through the Lens of Neoclassical Growth Theory" *American Economic Review: Papers and Proceedings*, 92 (2): 28–32.

[14] Elsby, Michael, Ryan Michaels. 2010. "Marginal Jobs, Heterogenous Firms, and Unemployment Flows." *Working Paper.* University of Michigan.

[15] Fujita, Shigeru and Makoto Nakajima. 2009. "Worker Flows and Job Flows: A Quantitative Investigation." *Working Paper #09-33.* Federal Reserve Bank of Philadelphia.

[16] Gali, Jordi, Mark Gertler, and J. David Lopez-Salido. 2007. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." *Review of Economics and Statistics*, 89(1): 44–59.

[17] Gertler, Mark, and Antonella Trigari. 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy.* 117(1): 38-86.

[18] Hall, Robert E. 1997. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics*, 15(1): 223–250.

[19] Hall, Robert E., and Paul R. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review*, 98(4): 1653-1674.

[20] Hosios, A. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*, 57(2): 279–298.

[21] Kydland Finn E., and Edward J. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50(6): 1345-1370.

[22] Ingram, B. F., Narayana Kocherlakota and N. Eugene Savin. 1994. "Explaining Business Cycles: A multiple-shock approach." *Journal of Monetary Economics*, 34(3): 415–428.

[23] Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*, 36(2): 269–300.

[24] Merz, Monika. 1999. "Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover." *Journal of Monetary Economics,* 43(1): 91-124.

[25] Mortensen, Dale and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 61(3): 397–415.

[26] Mulligan, Casey B. 2002. "A Century of Labor-Leisure Distortions." Working Paper 8774, National Bureau of Economic Research.

[27] Ohanian, Lee E., Richard Rogerson and Andrea Raffo. 2008. "Long-Term Changes in Labor Supply and Taxes: Evidence from OECD Countries, 1956-2004." *Journal of Monetary Economics*, 55(8): 1353–1362.

[28] Ohanian, Lee E., and Andrea Raffo. Forthcoming. "Aggregate Hours Worked in OECD Countries: New Measurement and Implications for Business Cycles." *Journal of Monetary Economics.*.

[29] Petrongolo, Barbara and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39(2): 390–431.

[30] Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* Cambridge: MIT Press.

[31] Prescott, Edward J. 1986. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review*, 10: 9–22.

[32] Prescott, Edward J. 2004. "Why Do Americans Work So Much More Than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review*, 28: 2–13.

[33] Rotemberg, Julio J. and Michael Woodford. 1991. "Markups and the Business Cycle." *NBER Macroeconomics Annual*, 6: 63–129.

[34] Rotemberg, Julio J. and Michael Woodford. 1999. "The Cyclical Behavior of Prices and Costs." in *Handbook of Macroeconomics*, ed. John Taylor and Michael Woodford, 1051–1135. Amsterdam: North-Holland.

[35] Shimer, Robert. 2005. "The Cyclical Behavior of Unemployment and Vacancies: Evidence and Theory." *American Economic Review.* 95(1): 25-49.

[36] Shimer, Robert. 2009. "Convergence in Macroeconomics: The Labor Wedge." *American Economic Journal: Macroeconomics*, 1(1): 280–297.

[37] Shimer, Robert. 2010. *Labor Markets and Business Cycles.* Princeton: Princeton University Press.

[38] Stole, Lars A. and Jeffrey Zwiebel. 1996. "Inrafirm Bargaining under Non-binding Contracts" *Review of Economic Studies*, 63(3): 375–410.

[39] Sunakawa, Takeki. "Optimal Monetary Policy with Labor Market Frictions and Real Wage Rigidity: The Role of the Wage Channel." *Mimeo.* Ohio State University.

[40] Trigari, Antonella. 2006. "The Role of Search FRictions and Bargaining for Inflation Dynamics." *Working Paper # 304.* IGIER.

[41] van Ours, Jan C., and Geert Ridder. 1992. "Vacancies and the Recruitment of New Employees." *Journal of Labor Economics*, 10(2): 138–155.

Appendix

A Data

We use data from NIPA tables to construct our measures for real output, y_t, consumption, c_t,, and government expenditures, $\tau_{g,t}$, which also includes net exports[41]. Labor market variables employment, n_t, and average hours per worker, h_t, are taken from Cociuba, Prescott and Ueberfeldt (2009). All the data are from 1959:Q2 through 2010:Q3 and seasonally adjusted at an annualized rate when relevant. Output y_t and some of its components c_t and $\tau_{g,t}$ are all deflated by the GDP deflator. Real output, y_t, is defined as the quarterly gross domestic product net of sales taxes. Consumption, c_t, is the sum of non-durable goods purchases and services. Finally, government expenditures, $\tau_{g,t}$, includes government consumption expenditures (including federal, state and local governments) and net exports of goods and services. In effect, we lump together government consumption and net exports. For the purpose of our exercises in this paper, this distinction is not important. For the labor market data, we follow Cociuba, Prescott and Ueberfeldt (2009) and use their data to incorporate military hours and employment into total hours and total employment figures as estimated by the Current Population Survey of the BLS.

B Prototype RBC Model

Equilibrium of the prototype RBC model described in Section 4 is determined by a set of first order conditions of the problem nested in our search model along with an exogenous process governing the set of exogenous shocks (wedges), $\tau_t = [\tilde{\tau}_{x,t}, \tilde{\tau}_{l,t}, \tilde{\tau}_{z,t}, \tilde{\tau}_{g,t}]$.

$$U_c(c_t)\left[1+\tilde{\tau}_{x,t}\right] = \beta \mathrm{E}_t[U_c(c_{t+1})[\tilde{\tau}_{z,t+1} f_k(k_{t+1}, l_{t+1}) + (1-\delta)\left(1+\tilde{\tau}_{x,t+1}\right)]|\omega_t] \tag{49}$$

$$\widetilde{mrs}_t(l_t) = -\psi G_h(l_t)/U_c\left(c_t\right) = mpl_t\left[1-\tau_{l,t}\right] \tag{50}$$

$$\tilde{\tau}_{z,t} f(k_t, l_t) = c_t + k_{t+1} - (1-\delta)k_t + \tilde{\tau}_{g,t} = y_t \tag{51}$$

$$\tilde{\tau}_{g,t} = T_t + x_t \tau_{x,t} + w_t l_t \tilde{\tau}_{l,t} \tag{52}$$

The final equation is implied by the assumption that the government runs a balanced budget in each period such that tax revenues are equal to government spending. Essentially, CKM uses these equilibrium conditions and data on output (y_t), consumption (c_t), total hours (l_t), and investment (x_t) to measure a unique realization of wedge for each period. This paper's focus is on the labor wedge which is given by the static equilibrium condition (50). Figure (10) plots $[1-\varepsilon_{l,t}]$ over the period 1959:I to 20010:III.

[41] We pulled this data from Haver Analytics database.

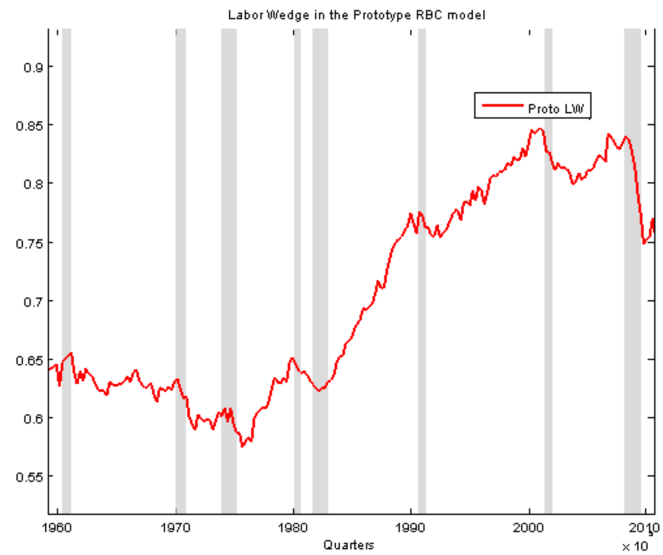

Figure 10: Historical realizations of $[1-\tilde{\tau}_{lt}]$ according to measurement equation (50) and data on y_t, c_t, and l_t over the period 1959:1 to 2010:III. Shaded areas indicate NBER recession dates.

C Households' Decision Problem

First order necessary conditions and costates for the household's problem (9)-(10) are,

$$x_t \quad : \quad -U_c(c_t)\left[1+\tau_{x,t}\right] + \beta \mathrm{E}_t W^h_{k_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial k_{t+1}}{\partial x_t} = 0 \tag{53}$$

$$e_t \quad : \quad -U_c(c_t)c_e(e_t)(1-n_t) + \beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial n_{t+1}}{\partial e_t} = 0 \tag{54}$$

$$k_t \quad : \quad W^h_{k_t}(\omega^h_t|\omega^h_{t-1}) = U_c(c_t)r_t + \beta \mathrm{E}_t W^h_{k_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial k_{t+1}}{\partial k_t} \tag{55}$$

$$n_t \quad : \quad W^h_{n_t}(\omega^h_t|\omega^h_{t-1}) = U_c(c_t)[w_t\left[1-\tau_{l,t}\right]h_t + c(e_t)] + \psi G(h_t) + \beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial n_{t+1}}{\partial n_t} \tag{56}$$

Combining (53) and (55) will give the consumption Euler equation in (12). Note that equation (54) implies that $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t) = \frac{U_{c_t}(c_t)c_{e_t}(e_t)}{p_t}$. Substituting this expression for $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)$ in (56) and taking expectations using (54) gives the households' choice of effort in (13).

D Firms' Decision Problem

First order necessary conditions and costates for the firms' problem (15)-(16) are,

$$k_t \quad : \quad \tau_{z,t} f_k(k_t, n_t h_t) - r_t = 0 \tag{57}$$

$$v_t \quad : \quad -\kappa + q_t \tilde{\beta} \mathrm{E}_t W^f(\omega^f_{t+1}|\omega^f_t) = 0 \tag{58}$$

$$k_{t+1} \quad : \quad W^f_{n_t}(\omega^f_t|\omega^f_{t-1}) = \tau_{z,t} f_k(k_t, n_t h_t)h_t - w_t h_t + (1-\sigma)\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_t|\omega^f_{t-1}) \tag{59}$$

Equation (57) gives us the equilibrium rental rate on capital in (18). Note that equation (58) implies that $\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t) = \frac{\kappa}{q_t}$. Substituting this expression for $\tilde{\beta}_t \mathrm{E}_t W^f(\omega^f_{t+1}|\omega^f_t)$ in (59) and taking expectations using (57) gives the firms' choice of vacancies in (19).

E Details of Employment Contract

Employment contract is given by the solution to the problem defined in (20). Note that we have the necessary expressions for each party's marginal surplus from respective decision problems. $W^h_{n_t}(\omega_t)$ is defined in (56) and $W^f_{n_t}(\omega_t)$ is defined in (59).

$$\begin{aligned} W^h_{n_t}(\omega^h_t) &= U_c(c_t)[w_t[1-\tau_{l,t}]h_t + c(e)] + \psi G(h_t) + (1-\sigma-p_t e)\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t) \quad (60) \\ W^f_{n_t}(\omega^f_t) &= \tau_{z,t} f_{l_t}(k_t, n_t h_t)h_t - w_t h_t + (1-\sigma)\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t) \quad (61) \end{aligned}$$

As long as there are gains from trade, first order conditions are given by two conditions that determine optimal hours and wage.

$$\begin{aligned} w_t &: \lambda \frac{W^f_{nw}(\omega^f_t)}{W^f_n(\omega^f_t)} + (1-\lambda)\frac{W^h_{nw}(\omega^h_t)}{W^h_n(\omega^h_t)} = 0 \\ h_t &: \lambda \frac{W^f_{nh}(\omega^f_t)}{W^f_n(\omega^f_t)} + (1-\lambda)\frac{W^h_{nh}(\omega^h_t)}{W^h_n(\omega^h_t)} = 0 \end{aligned}$$

where the cross-partial derivatives could be evaluated by using (60) and (61) to arrive at (21) and (22) in the text. Then one can use the firdt order condition for wage, (21), along with (60) and (61) to derive an explicit wage equation in (27). To do so, multiply both sides of (60) with λ and (61) with $(1-\lambda)U_c(c_t)[1-\tau_{l,t}]$.

$$\begin{aligned} \lambda W^h_n(\omega^h_t) &= \lambda U_c(c_t)[w_t[1-\tau_{lt}]h_t + c(e)] + \\ &\quad \lambda\psi G(h_t) + \lambda(1-\sigma-p_t e)\beta \mathrm{E}_t W^h_{n+1}(\omega^h_{t+1}|\omega^h_t) \\ (1-\lambda)U_c(c_t)[1-\tau_{l_t}]W^f_n(\omega^f_t) &= (1-\lambda)U_c(c_t)[1-\tau_{l,t}][\tau_{z,t} f_l(k_t, n_t h_t)h_t - w_t h_t] \\ &\quad +(1-\lambda)U_c(c_t)[1-\tau_{l,t}](1-\sigma)\tilde{\beta}_t \mathrm{E}_t W^f_{n+1}(\omega^f_{t+1}|\omega^f_t) \end{aligned}$$

Substituting for $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)$ and $\tilde{\beta}\mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t)$ and subtracting the second line from the first with some additional algebra gives the wage equation expressed in (27). Note that being able to substitute these expressions in the previous steps significantly simplifies the problem and provides us with an analytical expression for the wage that is tractable. This is the technical reason why we chose to have a search effort in the model, which enabled us to reduce the value of expected surplus from the marginal match to a simpler expression. This problem does not arise when $\tau_{l_t} = 0$, i.e. Andolfatto (1996). Finally, it is straightforward to see that the optimality in hours from Nash bargaining problem implies the labor wedge equation expressed in (32).

F Results for Germany

This section provides the results of the numerical exercise repeated for Germany. The data is from Ohanian and Raffo (forthcoming) and cover 1960:Q1-2010:Q4. Table A.1 and Table A.2. are analogous to Tables 2 and 3 in the text.

Table A.1: Moments for High Frisch Elasticity (3.51)

	Y_t	$(1-\tilde{\tau}_{l,t})$	$(1-\tau_{l,t})$	$\widetilde{mrs}_t$	mrs_t
Std. Dev.	0.015	0.011	0.01	0.014	0.013
	(0.015)	(0.175)	(0.174)	(0.044)	(0.043)
Corr(x_t, x_{t-1})	0.862	0.749	0.666	0.893	0.865
	(0.746)	(0.998)	(0.999)	(0.958)	(0.957)
Cross Correlations					
	Y_t	L_t	C/Y		
$(1-\tilde{\tau}_{l,t})$	0.396	0.656	0.233		
	(0.042)	(0.998)	(0.005)		
$(1-\tau_{l,t})$	0.273	0.531	0.372		
	(0.029)	(0.995)	(0.014)		

All variables are HP-filtered, except L_t.
Moments in () are for the 'low-frequency' case.

Table A.2: Moments for Low Frisch Elasticity (0.63)

	Y_t	$(1-\tilde{\tau}_{l,t})$	$(1-\tau_{l,t})$	$\widetilde{mrs}_t$	mrs_t
Std. Dev.	0.015	0.015	0.023	0.026	0.017
	(0.015)	(0.358)	(0.364)	(0.232)	(0.227)
Corr(x_t, x_{t-1})	0.746	0.637	0.800	0.795	0.624
	(0.746)	(0.999)	(0.999)	(0.997)	(0.998)
Cross Correlations					
	Y_t	L_t	C/Y		
$(1-\tilde{\tau}_{l,t})$	0.676	0.921	-0.234		
	(0.059)	(1.000)	(-0.024)		
$(1-\tau_{l,t})$	0.408	0.662	0.044		
	(0.023)	(0.989)	(-0.002)		

All variables are HP-filtered, except L_t.
Moments in () are for the 'low-frequency' case.