Macroeconomic Policy Games*

Martin Bodenstein
National University of Singapore and CAMA

Luca Guerrieri
Federal Reserve Board

Joe LaBriola**
UC Berkeley

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Abstract

Strategic interactions between policymakers arise whenever each policymaker has distinct objectives. Deviating from full cooperation can result in large welfare losses. To facilitate the study of strategic interactions, we develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games. Two examples for the use of our toolbox offer some novel results. The first example revisits the case of monetary policy coordination in a two-country model to confirm that our approach replicates well-known results in the literature, but extends these results by highlighting their sensitivity to the choice of policy instrument. In the second example, a central bank and a macroprudential regulator are assigned different objectives in a model with financial frictions. Lack of coordination leads to large welfare losses even if technology shocks are the only source of fluctuations.

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* The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. The toolbox and replication codes for the examples discussed in this paper are available from https://sites.google.com/site/martinbodenstein/ and from http://www.lguerrieri.com/games_code.zip.

** Contact information: Martin Bodenstein (corresponding author), Telephone +65 6516 6008, E-mail ecsbmr@nus.edu.sg; Luca Guerrieri, Telephone +1 202 452 2550, E-mail Luca.Guerrieri@frb.gov; Joe LaBriola: E-mail joelabriola@gmail.com.

1 Introduction

Both across countries and within countries regulators face the challenging task of finding the appropriate response to the actions of other regulators. This task has informed active research on the gains from monetary policy coordination across countries, as described in detail by Canzoneri and Henderson (1991). Strategic interactions also arise within a country when different regulators are assigned or pursue distinct objectives. For instance, the expansion and reorganization of regulatory responsibilities spurred by the Financial Crisis has been approached differently across countries. In the United States the Dodd-Frank Act substantially increased the macroprudential responsibilities of the Federal Reserve. In the United Kingdom, the Financial Services Act 2012 established an independent Financial Policy Committee as a subsidiary of the Bank of England, with some policymakers participating in both the Monetary and the Financial Policy Committee. By contrast, in the euro area monetary policy tasks are strictly separated from macro prudential and supervisory tasks, although both functions involve the European Central Bank. Other examples include the interaction between fiscal and monetary authorities or games between countries about improving global competitiveness by setting tariffs and taxes across countries.

To facilitate the study of strategic interactions between regulators, we develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games. The toolbox is designed to extend Dynare, a convenient and popular modeling environment.[1] Our work augments the single regulator framework of Lopez-Salido and Levin (2004).[2] The general framework for the policy games that we consider distinguishes between two groups of actors. The first group of private agents acts optimally given the (expected) path of the policy instruments. The second group consists of the policymakers who determine policies taking into account the private sector's response to the implemented

[1] See Adjemian, Bastani, Karam, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011).

[2] Given a characterization of the actions of private agents, the framework in Lopez-Salido and Levin (2004) facilitates the computation of the welfare-maximizing Ramsey policies for a single regulator that has one or several policy instruments.

policies. Taking as input a set of equilibrium conditions given arbitrary rules for the reactions of the policy instruments, our toolbox replaces those rules with either the welfare-maximizing Ramsey policies or with the policies for the open-loop Nash game.

To showcase the wide applicability of our toolbox, we consider two examples that provide some new results regarding the gains from cooperative policies. The first example is a two-country monetary model that closely follows Clarida, Gali, and Gertler (2002), Benigno and Benigno (2006), and Corsetti, Dedola, and Leduc (2010). These authors characterize the optimal monetary policies both with and without cooperation between two monetary policy authorities in a dynamic general equilibrium model with sticky prices. If we take a linear approximation to the policymakers' first-order conditions around the optimal deterministic steady state of the model, we confirm that our toolbox produces the same results as the linear-quadratic approach in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). A key advantage of our toolbox is the ability of characterizing the solutions numerically without additional analytical manipulations once the actions of the private agents are characterized. We replicate key analytical insights from Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010) with our numerical method and extend their results by considering alternative policy instruments. Beyond the replication of existing results, we show that the choice of inflation measure that is used as an instrument for monetary policy can imply quantitatively important differences for the gains from cooperation.

The second example considers the workhorse New Keynesian model with financial frictions of Gertler and Karadi (2011). An agency problem on financial intermediaries has two important effects. First, the problem inefficiently limits the provision of credit. Second, the agency problem also magnifies the reaction of the economy to shocks through familiar financial accelerator mechanisms. We extend the model of Gertler and Karadi (2011) to include a transfer tax between households and firms. Within that model, we consider a game between a financial regulator and a monetary policy authority. The policy instrument of the central bank is the inflation rate; the policy instrument of the financial regulator is the transfer tax. The objectives of the two regulators reflect the preferences of households, but in both cases include an extra term. The central bank has an objective biased towards stabilizing inflation. The financial regulator has an objective biased towards stabilizing the provision of credit. We characterize optimal cooperative Ramsey and open-loop Nash policies. We constrain the choice of biases so that the cooperative policies with the skewed objectives come close to replicating the allocations under policies that maximize the welfare of the representative household. Nonetheless, the strategic interaction between regulators lead to large and persistent deviations from cooperative outcomes and imply substantial welfare losses.

The usefulness of our toolbox is not limited to solving the particular examples above. Following the approach in Dixit and Lambertini (2003), differences in objectives are fertile ground to explore the strategic interactions between policymakers. For instance, the solution under coordinated optimal monetary and fiscal policies explored in Schmitt-Grohe and Uribe (2004) could be readily extended for strategic interactions after allowing for small differences in the objectives of the monetary and fiscal authorities. More recent examples of stylized models that set the stage for strategic interactions between policymakers include Costinot, Lorenzoni, and Werning (2014), who illustrate the use of capital controls to manipulate the terms of trade and Brunnermeier and Sannikov (2014), who show how capital controls may improve welfare in a model with financial frictions (but who do not consider a non-cooperative solution). Furthermore, our toolbox greatly facilitates the analysis of more fully articulated models. Examples include Bergin and Corsetti (2013), who introduce firm entry into a two-country model to study how the resulting production relocation externality influences monetary policy, and Fujiwara2013, who allow for nominal rigidities in loan contracts. Finally, the optimal policy implications for models with numerous empirically relevant features (such as consumption habits, capital accumulation, investment adjustment costs, incomplete financial markets, sticky wages) as in the two-country model of Coenen, Lombardo, Smets, and Straub (2007) can also be analyzed and extended with the help of our toolbox.

The rest of the paper is organized as follows. Section 2 outlines the algorithm for calculating cooperative optimal policy and extends the algorithm to the calculation of optimal policies in open-loop Nash games. Section 3 applies the algorithm to an open-economy model where each country wishes to maximize welfare through controlling inflation, and Section 4 considers the application of our algorithm to a model with a monetary authority and a macroprudential regulator. Section 5 concludes. An Appendix with details on the toolbox is provided.

2 Equilibrium Definitions and Solution Algorithms

This section covers three topics: 1) it defines an equilibrium under cooperative Ramsey policies; 2) it defines an equilibrium under an open-loop Nash game; and 3) it spells out the relationship between our solution approach and the linear-quadratic approach.

In maximizing the policy objectives subject to the structural equations of the private sector our toolbox employs a Lagrangian approach. The exact nonlinear first-order conditions that characterize the optimal policies under cooperation and the open-loop Nash game, respectively, are obtained by symbolic differentiation. Each system of equations is then approximated around its deterministic steady state using higher order perturbation methods. An alternative approach to characterizing optimal policies uses linear-quadratic (LQ) techniques. The LQ approach involves finding a purely quadratic approximation of each policymakers' objective function which is then optimized subject to a linear approximation of the structural equations of the model. Following Benigno and Woodford (2012), Levine, Pearlman, and Pierse (2008) and Debortoli and Nunes (2006) we show how the LQ approach relates to the approach underlying our numerical procedure and that the LQ approach delivers the same solution if the nonlinear output of our toolbox is approximated to the first order.

2.1 General Framework

Policy games distinguish between two groups of actors. We label the first group "private agents." Private agents act optimally given the (expected) path of the policy instruments. The second group consists of the policymakers who determine policies taking into account the private sector's response to the implemented policies. With more than one policymaker, strategic interaction between the policymakers can cause the outcomes of the dynamic game to deviate from the welfare-maximizing cooperative policy. For simplicity, we restrict the exposition to the case of two policymakers (or players). Furthermore, each policymaker is assumed to have only one instrument.

Let the $N \times 1$ vector of endogenous variables be denoted by x_t, which is partitioned as $x_t = (\tilde{x}_t', i_{1,t}, i_{2,t})'$. The variable $i_{j,t}$ is the policy instrument of player $j = [1, 2]$, respectively. The exogenous variables are captured by the vector ζ_t. For given sequences of the policy instruments $\{i_{1,t}, i_{2,t}\}_{t=0}^{\infty}$, the remaining $N-2$ endogenous variables need to satisfy the $N-2$ structural conditions that characterize an equilibrium

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0. \tag{1}$$

We assume that the system of equations in g is differentiable up to the desired order of approximation. Without loss of generality and to facilitate changes in the set of policy instruments for our toolbox, the block of structural equations (1) contains two definitions relating the generic instrument variables $i_{1,t}$ and $i_{2,t}$ to the desired instruments in the model. For example, if player 1 uses the (core) inflation rate $\pi_{1,t}$ as instrument as in Woodford (2003), then one of the equations in (1) simply reads $i_{1,t} - \pi_{1,t} = 0$.

To complete our framework, we need to describe how policies are determined. The intertemporal preferences of player j are given by $\mathcal{U}_j = E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ with the generic utility function $U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ required to be concave. Under cooperation, the two players maximise the joint welfare function $\omega_1 \mathcal{U}_1 + \omega_2 \mathcal{U}_2$ for given weights ω_1 and ω_2. We normalise the welfare weights to satisfy $\omega_1 + \omega_2 = 1$. Absent cooperation, each policymaker considers his own preferences only.

2.2 Definition of Equilibrium under Cooperation

The welfare-maximizing Ramsey policy with full commitment is derived from the maximization program

$$\begin{aligned} &\max_{\{\tilde{x}_t, i_{1,t}, i_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)\right] \\ &s.t. \\ &E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0. \end{aligned} \tag{2}$$

The first-order conditions for this problem can be obtained by differentiating the Lagrangian problem of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda_t' g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right]. \tag{3}$$

The $(N-2) \times 1$ Lagrange multipliers associated with the private sector equilibrium conditions in (1) are denoted by λ_t for any $t \geq 0$.

Taking derivatives of $\mathcal{L}_0$ with respect to the N endogenous variables in x_t delivers N first order conditions. Additionally, taking derivatives with respect to λ_t delivers again the $N-2$ private sector conditions. In total, there are $2N-2$ conditions and $2N-2$ variables. Since the generic instruments $i_{1,t}$ and $i_{2,t}$ are added to the model equations through definitions of the form $i_{j,t} = \tilde{x}_t^j$ where $\tilde{x}_t^j$ is player j's actual policy instrument, taking derivatives with respect of $i_{1,t}$ and $i_{2,t}$ returns the Lagrange multipliers associated with these definitions. Here, we assume that λ_t^j is the Lagrange multiplier attached to the definition of player j's instrument. In sum, the Ramsey equilibrium process $\{\tilde{x}_t, i_{1,t}, i_{2,t}, \lambda_t\}_{t=0}^{\infty}$ satisfies

$$\begin{aligned} &\sum_{j=1,2} \omega_j \{D_{\tilde{x}} U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_j(\tilde{x}_t, \tilde{x}_{t+1}, \zeta_{t+1})\} \\ &+\beta E_t \left\{\lambda_{t+1}' D_{\tilde{x}^-} g(x_t, x_{t+1}, x_{t+2}, \zeta_{t+1})\right\} + E_t \left\{\lambda_t' D_{\tilde{x}} g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right\} \\ &+\beta^{-1} \lambda_{t-1}' D_{\tilde{x}^+} g(x_{t-2}, x_{t-1}, x_t, \zeta_{t-1}) = 0 \end{aligned} \tag{4}$$

$$\lambda_t^1 = 0 \tag{5}$$

$$\lambda_t^2 = 0 \tag{6}$$

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \tag{7}$$

at each date $t > 0$. The notation $D_{\tilde{x}}$ denotes the vector of partial derivatives of any functions with respect to the elements of $\tilde{x}_t$; likewise do $D_{\tilde{x}^-}$ and $D_{\tilde{x}^+}$ for derivatives with respect to $\tilde{x}_{t-1}$ and $\tilde{x}_{t+1}$, respectively. Following equations (5) and (6), the multipliers λ_t^1 and λ_t^2 need to equal to zero for all $t \geq 0$. For $t = 0$, the set of equations in (4) is replaced by

$$\sum_{j=1,2} \omega_j \{ D_{\tilde{x}} U_j(\tilde{x}_{-1}, \tilde{x}_0, \zeta_t) + \beta E_0 D_{\tilde{x}^-} U_j(\tilde{x}_0, \tilde{x}_1, \zeta_1) \} + \beta E_0 \left\{ \lambda_1' D_{\tilde{x}^-} g(x_0, x_1, x_2, \zeta_1) \right\}$$
$$+ E_0 \left\{ \lambda_0' D_{\tilde{x}} g(x_{-1}, x_0, x_1, \zeta_t) \right\} = 0.$$

It is hard to argue, that the policymaker cannot commit to policies that would need to be implemented before the beginning of time. This problem creates a time-inconsistency problem at time $t = 0$.[3] Even without shocks, the endogenous variables are not constant (or grow at a constant rate). Although this system of equations can in general be solved, the equilibrium functions will not be time-invariant. The popular and computationally convenient approach of solving a system of locally approximated equations obtained by approximating the nonlinear equilibrium conditions around the model's deterministic steady state is not applicable. To obtain a recursive structure and to make the problem suitable for applying standard solution methods, we follow most of the literature in adopting the concept of optimality from a *timeless perspective*.[4] In short, this concept requires an initial pre-commitment to suitably chosen values λ_{-1} at time 0 so that the first-order conditions (4) to (7) apply to all $t \geq 0$. The timeless perspective implies that the optimal deterministic steady state $(\bar{x}, \bar{\lambda})$ needs to satisfy

$$\sum_{j=1,2} \omega_j \{ D_{\tilde{x}} U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0) + \beta D_{\tilde{x}^-} U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0) \}$$
$$+ \bar{\lambda}' \left(\beta D_{\tilde{x}^-} g(\bar{x}, \bar{x}, \bar{x}, 0) + D_{\tilde{x}} g(\bar{x}, \bar{x}, \bar{x}, 0) + \beta^{-1} D_{\tilde{x}^+} g(\bar{x}, \bar{x}, \bar{x}, 0) \right) = 0 \tag{8}$$

[3] See Benigno and Woodford (2012) for a discussion.

[4] In principle, the output of our toolbox can be used to compute a solution to the original problem. Yet, to make full use of the algorithms embedded in Dynare adopting the timeless perspective is key.

$$\bar{\lambda}^1 = 0 \tag{9}$$

$$\bar{\lambda}^2 = 0 \tag{10}$$

$$E_t g(\bar{x}, \bar{x}, \bar{x}, 0) = 0. \tag{11}$$

As the problem stated in equations (8) to (11) is linear in the Lagrange multipliers, the optimal steady state is easily computed. For arbitrary steady-state choices of the instruments i_1, i_2, we find the vector $\tilde{x}$ satisfying (11). To find the Lagrange multipliers, recognise that given a vector x, (8) can be written in the form $Y = X\beta + \varepsilon$ with $\beta = \lambda'$. We then compute the best linear fit by setting $\beta = (X'X)^{-1}X'Y$ and $\varepsilon = Y - X\beta$. Because there are N conditions and $N-2$ variables, ε does not necessarily equal 0 for arbitrary choices i_1, i_2. Hence, i_1, i_2 need to be varied until $Y = X\beta$, leading to the optimal steady state allocation under cooperation $\bar{x}$.

Equations (4) and (7) can now be replaced by a local approximation around the optimal steady state $\{\bar{x}, \bar{\lambda}\}$ of desired order. The resulting system of (higher-order) difference equations can easily be solved by standard algorithms.

2.3 Definition of Open-loop Nash Equilibrium

To define an open-loop Nash equilibrium, let $\{i_{j,t,-t^*}\}_{t=0}^{\infty}$ denote the sequence of policy choices by player j before and after, but not including period t^*. An open-loop Nash equilibrium is a sequence $\left\{i^*_{j,t}\right\}_{t=0}^{\infty}$ with the property that for all t^*, i^*_{j,t^*} maximises player $j's$ objective function subject to the structural equations of the economy for given sequences $\left\{i^*_{j,t,-t^*}\right\}_{t=0}^{\infty}$ and $\left\{i^*_{-j,t}\right\}_{t=0}^{\infty}$, where $\left\{i^*_{-j,t}\right\}_{t=0}^{\infty}$ denotes the sequence of policy moves by all players other than player j. Each player's action is the best response to the other players' best responses.

With policymakers needing to specify a complete contingent plan at time 0 for their respective instrument variable $\{i_{j,t}\}_{t=0}^{\infty}$ for $j = [1, 2]$, under the open-loop equilibrium concept, the problem can be reinterpreted as a static game allowing us to recast each player's optimization problem as an optimal control problem given the policies of the remaining players. As under the static Nash equilibrium concept, player j restricts attention to his own objective function and the maximisation program is given by

$$\begin{aligned}
&\max_{\{\tilde{x}_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \\
&\text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.
\end{aligned} \tag{12}$$

The first-order conditions for each player are obtained from differentiating the Lagrangian of the form

$$\mathcal{L}_{j,0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda'_{j,t} g(x_{t-1}, x_t, x_{t+1}, \zeta_t) \right] \tag{13}$$

for $j = [1, 2]$. Taking derivates of the $\mathcal{L}_{j,0}$ with respect to the $N-1$ choice variables $(\tilde{x}_t, i_{j,t})$, excluding the instrument of the other player, and the $N-2$ Lagrange multipliers $\lambda_{j,t}$ associated with the $N-2$ structural relationships $2N-3$ conditions for each player.

Notice that the full set of $4N-6$ equations includes the $N-2$ structural equations twice. Since in equilibrium all players face the same values of the non-policy variables $\tilde{x}_t$, an interior Nash equilibrium $\{\tilde{x}^*_t, i^*_{1,t}, i^*_{2,t}, \lambda^*_{1,t}, \lambda^*_{2,t}\}_{t=0}^{\infty}$ satisfies the following $3N-4$ conditions for $t > 0$

$$D_{\tilde{x}} U_1(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_1(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1}) + \beta E_t \left\{ \lambda^{*'}_{1,t+1} D_{\tilde{x}^-} g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1}) \right\}$$

$$+ E_t \left\{ \lambda^{*'}_{1,t} D_{\tilde{x}} g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) \right\} + \beta^{-1} \lambda^{*'}_{1,t-1} D_{\tilde{x}^+} g(x^*_{t-2}, x^*_{t-1}, x^*_t, \zeta_{t-1}) = 0 \tag{14}$$

$$\lambda^{1*'}_{1,t} = 0 \tag{15}$$

$$D_{\tilde{x}} U_2(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_2(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1}) + \beta E_t \left\{ \lambda^{*'}_{2,t+1} D_{\tilde{x}^-} E_t g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1}) \right\}$$

$$+ E_t \left\{ \lambda^{*'}_{2,t} D_{\tilde{x}} g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) \right\} + \beta^{-1} \lambda^{*'}_{2,t-1} D_{\tilde{x}^+} g(x^*_{t-2}, x^*_{t-1}, x^*_t, \zeta_{t-1}) = 0 \tag{16}$$

$$\lambda^{2*'}_{2,t} = 0 \tag{17}$$

$$E_t g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) = 0. \tag{18}$$

In a fashion similar to the case of cooperation, the first-order conditions with respect to $i_{1,t}$ and $i_{2,t}$ imply the restriction that the Lagrange multipliers associated with the definition of the policy instruments — here $\lambda_{1,t}^{1*'}$ and $\lambda_{2,t}^{2*'}$ for players 1 and 2, respectively — are zero.

Adopting the timeless perspective is again key to obtaining time-invariant decision rules. The optimal response of each player given the policies of the other player derived from the optimal control problem at time 0 is not necessarily time consistent. Last, the deterministic steady state is found as for the cooperative case by exploiting the linearity of the system (14)-(18) in the $2N-4$ Lagrange multipliers.

2.4 Relationship to Linear-Quadratic Approach

An alternative approach to solve optimal policy problems uses linear-quadratic (LQ) techniques. In the case of a single decision maker, the LQ approach involves finding a purely quadratic approximation of the policymaker's objective function which is then optimized subject to a linear approximation of the structural equations of the model.Benigno and Woodford (2012) and Levine, Pearlman, and Pierse (2008) and Debortoli and Nunes (2006) discuss necessary and sufficient conditions for a "correct LQ approximation" to the optimization problem stated in equation (2) to exist. In contrast to the early literature the approach followed here does not require the steady state of the model to be efficient.[5]

To see the connection between the LQ approach and the approach followed in our toolbox, assume we were interested in the solution to the problem stated in (2) obtained from the linear approximation of the first order conditions (4) to (7) around the optimal steady state. Under the timeless perspective, the first order conditions with respect to the endogenous variables can then be approximated by

$$\sum_{j=1,2} \omega_j \left\{ D^2_{xx^-}\bar{U}_j \hat{x}_{t-1} + \left[D^2_{xx}\bar{U}_j + \beta D^2_{x^-x^-}\bar{U}_j \right] \hat{x}_t + \beta D^2_{x^-x}\bar{U}_j E_t \hat{x}_{t+1} \right\}$$
$$+ \sum_{j=1,2} \omega_j \left\{ D^2_{x\zeta}\bar{U}_j \zeta_t + \beta D^2_{x^-\zeta}\bar{U}_j E_t \zeta_{t+1} \right\}$$
$$+ \beta\bar{\lambda} \left\{ D^2_{x^-x^-}\bar{g}\hat{x}_t + D^2_{x^-x}\bar{g} E_t \hat{x}_{t+1} + D^2_{x^-x^+}\bar{g} E_t \hat{x}_{t+2} + D^2_{x^-\zeta}\bar{g} E_t \zeta_{t+1} \right\}$$

[5] Rotemberg and Woodford (1998) popularized this approach in economics. To gain tractability they assumed the steady state to satisfy certain efficiency conditions.

$$+\bar{\lambda}\left\{D^2_{xx^-}\bar{g}\hat{x}_{t-1}+D^2_{xx}\bar{g}\hat{x}_t+D^2_{xx^+}\bar{g}E_t\hat{x}_{t+1}+D^2_{x\zeta}\bar{g}\zeta_t\right\}$$
$$+\beta^{-1}\bar{\lambda}\left\{D^2_{x^+x^-}\bar{g}\hat{x}_{t-2}+D^2_{x^+x}\bar{g}\hat{x}_{t-1}+D^2_{x^+x^+}\bar{g}\hat{x}_t\ +D^2_{x^+\zeta}\bar{g}\zeta_{t-1}\right\}$$
$$+\beta E_t D_{x^-}\bar{g}'\hat{\lambda}_{t+1}+D_x\bar{g}'\hat{\lambda}_t+\beta^{-1}D_{x^+}\bar{g}'\hat{\lambda}_{t-1}=0. \tag{19}$$

Note that we have augmented the partial derivatives of the utility functionals to include derivatives with respect to the instrument variables $i_{1,t}$ and $i_{2,t}$ — which are zero — to simplify notation. The notation $D^2_{xx^-}$ marks the matrix of second derivatives of a function with respect to x and x^-. $\bar{U}_j$ and $\bar{g}$ is used as short-hand to indicate that a function (or its partial derivatives) is evaluated at the steady state values $\{\bar{x},\bar{\lambda}\}$. 'Hatted' variables refer to the deviation of the original variable from its steady state value. Regrouping terms delivers

$$\bar{\lambda}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right]\hat{x}_{t-2}+\left\{\sum_{j=1,2}\omega_j D^2_{xx^-}\bar{U}_j+\bar{\lambda}\left[D^2_{xx^-}\bar{g}+\beta^{-1}D^2_{x^+x}\bar{g}\right]\right\}\hat{x}_{t-1}$$
$$+\left\{\sum_{j=1,2}\omega_j\left[D^2_{xx}\bar{U}_j+\beta D^2_{x^-x^-}\bar{U}_j\right]+\bar{\lambda}\left[D^2_{xx}\bar{g}+\beta D^2_{x^-x^-}\bar{g}+\beta^{-1}D^2_{x^+x^+}\bar{g}\right]\right\}\hat{x}_t$$
$$+\left\{\sum_{j=1,2}\omega_j\beta D^2_{xx^-}\bar{U}_j+\beta\bar{\lambda}\left[D^2_{xx^-}\bar{g}+\beta^{-1}D^2_{x^+x}\bar{g}\right]\right\}'E_t\hat{x}_{t+1}$$
$$+\beta^2\bar{\lambda}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right]'E_t\hat{x}_{t+2}+\left\{\sum_{j=1,2}\omega_j\beta D^2_{x^-\zeta}\bar{U}_j+\beta\bar{\lambda}D^2_{x^-\zeta}\bar{g}\right\}E_t\zeta_{t+1}$$
$$+\left\{\sum_{j=1,2}\omega_j D^2_{x\zeta}\bar{U}_j+\bar{\lambda}D^2_{x\zeta}\bar{g}\right\}\zeta_t+\beta^{-1}\bar{\lambda}D^2_{x^+\zeta}\bar{g}\zeta_{t-1}$$
$$+\beta E_t D_{x^-}\bar{g}'\hat{\lambda}_{t+1}+D_x\bar{g}'\hat{\lambda}_t+\beta^{-1}D_{x^+}\bar{g}'\hat{\lambda}_{t-1}=0 \tag{20}$$

which coincides with the first order conditions of the following LQ problem

$$\max_{\{\hat{x}_t\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}\hat{x}_t'A(L)\hat{x}_t+\hat{x}_t'B(L)\zeta_{t+1}\right]$$
$$s.t.$$
$$E_tC(L)\hat{x}_{t+1}+D(L)\zeta_t=0$$
$$C(L)\hat{x}_0=d_0 \tag{21}$$

where

$$
\begin{aligned}
A_2 &= \bar{\lambda}\left[\beta^{-1}D^2_{x^+x^-}\bar{g}\right] \\
A_1 &= \sum_{j=1,2}\omega_j D^2_{xx^-}\bar{U}_j + \bar{\lambda}\left[D^2_{xx^-}\bar{g} + \beta^{-1}D^2_{x^+x}\bar{g}\right] \\
A_0 &= \sum_{j=1,2}\omega_j\left[D^2_{xx}\bar{U}_j + \beta D^2_{x^-x^-}\bar{U}_j\right] + \bar{\lambda}\left[D^2_{xx}\bar{g} + \beta D^2_{x^-x^-}\bar{g} + \beta^{-1}D^2_{x^+x^+}\bar{g}\right] \\
A(L) &= A_0 + A_1 L + A_2 L^2 \\
B(L) &= \left\{\sum_{j=1,2}\omega_j \beta D^2_{x^-\zeta}\bar{U}_j + \beta\bar{\lambda}D^2_{x^-\zeta}\bar{g}\right\} + \left\{\sum_{j=1,2}\omega_j D^2_{x\zeta}\bar{U}_j + \bar{\lambda}D^2_{x\zeta}\bar{g}\right\} L \\
&\quad + \beta^{-1}\bar{\lambda}D^2_{x^+\zeta}L^2 \\
C(L) &= D_{x^-}\bar{g} + D_x\bar{g}L + D_{x^+}\bar{g}L^2 \\
D(L) &= D_\zeta \bar{g}.
\end{aligned}
$$

The constraint $C(L)\hat{x}_0 = d_0$ is added to implement the timeless perspective by an appropriate choice of d_0. Benigno and Woodford (2012) refer to the program in equation (21) as the "correct LQ approximation" and they show how to derive the correct LQ program directly from the original problem stated in (2) rather than going through the first order conditions associated with (2), which is the approach followed by Levine, Pearlman, and Pierse (2008). Using the above definitions, it is easy to compute the matrices for the LQ problem from our toolbox output numerically. Hence, to a first order approximation the output of our toolbox is equivalent to that of the LQ approach.

3 Monetary Policy in an Open-Economy Model

We first illustrate our toolbox for a two-country monetary model that closely follows Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). These authors characterize the optimal monetary policies both with and without cooperation between two central banks in dynamic general equilibrium models with sticky prices. To this end, they derive the true linear quadratic approximation of the model. As discussed in Section 2.4, for given choice of policy instruments and strategies of the players, the linear-quadratic approach delivers the same output as our toolbox if we take a linear approximation of the first-order conditions of the two central banks around the deterministic steady state.

3.1 Model Environment

The two countries are equal in size and symmetric in their economic structure. We only describe the economy of country 1 in detail.

3.1.1 Households

Following Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010) each country is populated by a continuum of households. Each of them engages in the production of a specific good for which the household uses its own labor as the sole input. The good produced by household h carries the index f. Before describing the production and pricing of goods in detail, we first set up the household's optimisation problem for given labor and production choices, $L_t(h)$ and $Y_t(f)$ with financial markets being complete at the domestic and the international level

$$\begin{aligned}
&\max_{\{C_t(h),B_{D,t+1}(h),B_{F,t+1}(h)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \chi_0 \frac{L_t(h)^{1+\chi}}{1+\chi} \right) \\
&s.t. \\
&P_{C,t}C_t(h) + \int_S Q_{D,t}B_{D,t+1}(h) + \int_S e_t Q_{F,t}B_{F,t+1}(h) + T_t(h) \\
&= P_t(f)Y_t(f) + B_{D,t}(h) + e_t B_{F,t}(h) \qquad (22)
\end{aligned}$$

Household f uses its income on consumption, $P_{C,t}C_t(h)$, on the acquisition of domestic bonds in domestic currency, $\int_S Q_{D,t}B_{D,t+1}(h)$, and foreign bonds priced in foreign currency, $\int_S e_t Q_{F,t}B_{F,t+1}(h)$, and on lump-sum taxes, $T_t(h)$. The nominal exchange rate is denoted by e_t. Income is derived from selling its product, $P_t(f)Y_t(f)$, as well as the payoffs from foreign and domestic bonds, $Q_{F,t}B_{F,t}(h) + Q_{D,t}B_{D,t}(h)$.

Consumption utility is derived from consuming a domestic good, $C_{D,t}(h)$, and a foreign good, $C_{M,t}(h)$, according to

$$C_t(h) = \left(\omega_c^{\frac{\rho_c}{1+\rho_c}} C_{D,t}(h)^{\frac{1}{1+\rho_c}} + (1-\omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}(h)^{\frac{1}{1+\rho_c}}\right)^{1+\rho_c} \tag{23}$$

with the goods price in domestic currency being denoted by P_t and $P_{M,t}$, respectively. Under the assumption of producer currency pricing, the law of one price holds absent transportation costs and the price of the imported foreign good equals the price of the foreign good in the foreign country adjusted by the nominal exchange rate, $P_{M,t} = e_t P_t^*$. The price of the final consumption good, $P_{C,t}$, is obtained from minimising the costs of obtaining final consumption, $C_t(h)$, subject to the constraint (23).

3.1.2 Production of Final Goods

Competitive producers of the domestic good, Y_t, aggregate a variety of intermediate goods, $Y_t(f)$, produced by the home country's households using the production technology

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\nu_p}} df\right]^{1+\nu_p}. \tag{24}$$

Profit maximisation delivers the well-known result for the price of the domestic good, P_t,

$$P_t = \left[\int_0^1 P_t(f)^{-\frac{1}{\nu_p}} df\right]^{-\nu_p} \tag{25}$$

and the demand function for each variety $Y_t(f)$

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\frac{1+\nu_p}{\nu_p}} Y_t. \tag{26}$$

3.1.3 Production by Households

Each household produces exactly one variety $Y_t(f)$ and engages in monopolistic competition with all other households. A household chooses its price so as to maximize its utility. Following Calvo (1983) the probability of adjusting prices in a given period is $1-\xi_p$. The variable $\tau_{p,t}$ captures a time-varying subsidy on sales to motivate the presence of mark-up shocks. It is not necessarily the case that the subsidy eliminates the price mark-up in the steady state. Assuming household h uses a linear technology to produce good f, it is

$$Y_t(f) = (e^{z_t})^{\frac{\chi}{1+\chi}} L_t(h), \tag{27}$$

where the country-wide technology shock, z_t, evolves according to $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$. The production and pricing problem of household h can be stated as

$$\max_{P_t(f),\{Y_{t+i}(f)\}_{t=0}^{\infty}} E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \left\{ (1+\tau_{p,t}) \frac{C_{t+i}(h)^{-\sigma}}{P_{C,t+i}} P_t(f) Y_{t+i}(f) - \chi_0 (e^{z_{t+i}})^{-\chi} \frac{Y_{t+i}(f)^{1+\chi}}{1+\chi} \right\}$$
$$s.t.$$
$$Y_{t+i}(f) = \left[\frac{P_{t+i}(f)}{P_{t+i}} \right]^{-\frac{1+\nu_p}{\nu_p}} Y_t. \tag{28}$$

3.1.4 Market Clearing

Aggregating over households, clearing the market for the domestic good requires

$$Y_t = C_{D,t} + C^*_{M,t} + G_t \tag{29}$$

where $C^*_{M,t}$ denotes the foreign country's demand for the domestic good and G_t is the demand for the domestic good due to government spending.

Bonds are in zero net-supply, requiring $B_{D,t+1} = 0$ and $B_{F,t+1} + B^*_{F,t+1} = 0$. Finally, the budget constraint of the government is balanced in every period by adjusting lump-sum taxes, T_t, to the stochastic government purchases, G_t. The share of government consumption in output, $\frac{G_t}{Y_t}$, evolves according to

$$\omega_{gy,t} = \rho_{gy} \omega_{gy,t-1} + \sigma_{gy} \varepsilon_{gy,t} \tag{30}$$

where $\omega_{gy,t}$ measures the deviation of $\frac{G_t}{Y_t}$ from its steady state value.

3.1.5 Equilibrium Conditions and Calibration

Appendix B displays the set of structural equations associated with the model in (22)-(30) that characterize the private sector equilibrium conditions. Using the notation introduced in Section 2.1, the endogenous variables are collected in the vector

$$\tilde{x}_t = \begin{pmatrix} C_t, C_{D,t}, C_{M,t}, Y_t, G_t, \frac{P_{C,t}}{P_t}, \pi_t, H_{p,t}, G_{p,t}, \frac{P_t^{opt}}{P_t}, \Delta_t, R_t^n, q_t, \\ C_t^*, C_{D,t}^*, C_{M,t}^*, Y_t^*, G_t^*, \frac{P_{C,t}^*}{P_t^*}, \pi_t^*, H_{p,t}^*, G_{p,t}^*, \frac{P_t^{opt*}}{P_t^*}, \Delta_t^*, R_t^{n*} \end{pmatrix}' \tag{31}$$

where the variables $Q_{D,t}, Q_{F,t}, B_{D,t+1}, B_{F,t+1}, T_t, \Pi_t, e_t$ and their foreign counterparts are omitted from $\tilde{x}_t$, since they assume the value of zero in equilibrium or are substituted out in Appendix B. The vector of endogenous variables includes producer price inflation, defined as $\pi_t = \frac{P_t}{P_{t-1}}$, and the nominal interest rate R_t^n. The exogenous variables are collected in vector

$$\zeta_t = (z_t, \tau_t, G_t, z_t^*, \tau_t^*, G_t^*)' . \tag{32}$$

For illustration, we assume as in Benigno and Benigno (2006) that the policymakers use producer price inflation rates π_t and π_t^* as instruments.[6] Augmenting the set of conditions (67)-(91) in Appendix B by the two definitions

$$i_t = \pi_t \tag{33}$$

$$i_t^* = \pi_t^* \tag{34}$$

we have cast the structural equations of the model into the form of (1)

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0.$$

The step of adding equations (33) and (34) is automated by our toolbox.

The parameterization of the model is provided in Table 1. The choices are comparable to those in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). Most notably, by setting the coefficient governing the intertemporal elasticity of substitution σ equal to 2 and fixing the elasticity of substitution between traded goods at 2, the home and foreign good are substitutes in the utility function

[6] For this class of models, the open-loop Nash equilibrium is not unique if policymakers opt for the nominal interest rate as instrument. See for example Coenen, Lombardo, Smets, and Straub (2007) for a discussion of this issue.

the household. Steady-state imports are about 15% of GDP, which reflects home-biased preferences, given that the two countries are equal in size and symmetric. Accordingly, the countries are equally weighted in the global welfare function.

3.2 Optimal Policy with and without Cooperation

The model results are well-known in the literature and provide a benchmark to assess the output of our toolbox. Below we review key insights from Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). All of these insights are matched by the output of our toolbox.

In the face of technology shocks the welfare-maximising policy under cooperation replicates the flexible price allocations for the two-country model laid out above. As in closed economy models, the "devine coincidence" applies for "efficient shocks" – see Blanchard and Gali (2007): technology shocks move quantities and prices in the same direction relative to the flexible price economy and the central bank does not face a trade-off between inflation and output gap stabilisation.

A different picture emerges when the economy experiences a markup or cost-push shock, i.e., an "inefficient disturbance." As is the case in a closed economy model, the cooperating policymakers cannot perfectly stabilise the economy. In response to a positive cost-push shock, the output gap turns negative, whereas inflation is positive.

If policymakers do not cooperate across borders, prices and quantities will in general differ from those under cooperation. Each country has the ability to influence the terms of trade through its monetary policy stance. Hence, except for specific parameter choices, the (open-loop) Nash equilibrium does not replicate the flexible-price allocations even for efficient shocks.

Figures 1 and 2 show the responses to a positive technology shock and a cost-push shock under the welfare-maximizing cooperative policy and under an open-loop Nash game. Figure 1 shows that the responses to a technology shock under the two policies are quite close. However, there are some notable differences. As in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010), output price inflation is perfectly stabilized under the cooperative policy and the output response coincides with its counterpart in a flexible price model (not shown) for both countries. In the open-loop Nash game, inflation and output gaps are not perfectly stabilized. Yet the differences are minor as commonly seen in the literature.

Under the cost-push shock, shown in Figure 2, the two policies differ both qualitatively and quantitatively. Neither policy completely stabilizes output price inflation and the output gaps.[7] As shown in Corsetti, Dedola, and Leduc (2010), for our parameterization the home country's real exchange rate appreciates and its terms of trade improve by more under the open-loop Nash policies than under the cooperative policy. Furthermore, the spillover effects are larger.

To assess the reliability of the results produced by our toolbox, we confirm that its output under a first-order approximation coincides with the results produced by the linear-quadratic approaches in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). See Appendix B.3 for a reconciliation of the notation in Corsetti, Dedola, and Leduc (2010) with ours.

3.3 Sensitivity to the Choice of Policy Instrument

Exploiting the flexibility of our toolbox, we can easily analyze how the choice of strategy space impacts the outcomes of the open-loop Nash game.[8] To this end we compare the baseline case, in which each country uses producer price inflation as its policy instrument, to a case in which both policymakers use consumer price inflation as the instrument.[9]

Figure 3 compares the impulse responses to a cost push shock under these possibilities. Strikingly, the differences in outcomes implied by the alternative choice of instruments in the two games can outsize the differences between the cooperative and open-loop Nash outcomes in Figure 2. Coordination on the choice of policy instrument may be worthwhile.

[7] The efficient output level does not move at all in response to a technology shock. Hence, any movements in actual output are equivalent with movements in the output gap.

[8] Compare Lombardo and Sutherland (2004) and Coenen, Lombardo, Smets and Straub (2007) for detailed comparisons.

[9] We focus on the open-loop Nash game, as the choice of instrument does not affect the outcomes under the cooperative policy in this model.

4 Macroprudential Regulation Model

Our toolbox can also be applied to policy games in a closed economy. We lay out a policy game between a central bank and a financial regulator in a model following Gertler and Karadi (2011). In addition to nominal rigidities, the economy features financial frictions. Non-financial firms are prevented from issuing equity to households directly, but have to go through financial intermediaries, referred to as "banks," in order to raise funds. Due to an agency problem, however, banks are limited in their ability to attract deposits and issue credit to non-financial firms. Accordingly, credit is under-supplied, and the reactions to shocks are amplified by the familiar financial-accelerator mechanism.

4.1 Model Environment

4.1.1 Households

The representative household consists of a continuum of members. A fraction $1-f$ of its members supplies labor to firms and returns the wage earned to the household. The remaining fraction f works as bankers. The household utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \right]. \tag{35}$$

The importance of internal habits in consumption is governed by the parameter γ. The budget constraint takes the form

$$P_t C_t = P_t W_t L_t + P_t \Pi_t - P_t T_t - P_t D_t + (1+R_t) P_t D_{t-1} \tag{36}$$

Households use their income to consume, C_t, make tax transfers to the government, T_t, and to save in terms of deposits with banks, D_t. Income is derived from returns on deposits, wages, and profits of banks, Π_t.

Financially constrained bankers have an incentive to retain earnings. To prevent the financial constraint from becoming irrelevant by the retention of bank earnings, a banker ceases operations next period with the i.i.d. probability $1-\theta$. Upon exiting, bankers transfer retained earnings to the households and become workers. Each period $(1-\theta)f$ workers are selected to become bankers. These new bankers receive a startup transfer from the family. By construction, the fraction of household members in each group is constant over time. Π_t is net funds transferred to the household from its banker members; that is, funds transferred from existing bankers minus the funds transferred to new bankers (measured by $\bar{\omega}$). See Appendix C for details.

4.1.2 Banks

Bank j takes in deposits, $D_t(j)$, from households and invests into non-financial firms through an equity contract. Continuing banks do not consume but accumulate all earnings. Due to taxes/subsidies on equity, the bank operates with the amount $(1-BT_t)N_t(j)$, where BT_t is the tax rate and $N_t(j)$ is the equity of bank j. Since assets equal liabilities on the bank balance sheet

$$Q_t S_t(j) = (1-BT_t)N_t(j) + D_t(j). \tag{37}$$

Let deposits $D_t(j)$ pay the non-state-contingent (real) return $(1+R_t)$ and let shares $S_t(j)$ pay the stochastic return $(1+R^s_{t+1})$ at time $t+1$. Net worth in $t+1$ is then determined as the difference between earnings on assets and interest payments on liabilities

$$N_{t+1}(j) = (1+R^s_{t+1})Q_t S_t(j) - (1+R_t)D_t(j) \tag{38}$$

or combing (37) and (38)

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1+R_t)(1-BT_t)N_t(j). \tag{39}$$

The expected terminal wealth of a bank is then given by

$$\max_{\{S_{t+i}(j)\}} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} N_{t+1+i}(j) \tag{40}$$

with the stochastic discount factor $\Lambda_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$.

Absent financial frictions, the bank expands its balance sheet when the expected discounted excess return on loans, $E_t \Lambda_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i} \right)$, is positive. To limit the ability of banks to attract deposits, Gertler and Karadi (2011) introduce the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets to his household. If the banker makes this transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households will deposit funds with bank j only if the expected terminal wealth, $V_t(j)$ exceeds the fraction of assets that can be diverted, $\lambda Q_t S_t(j)$, in period t

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{41}$$

If equation (41) binds a bank's ability to raise deposits is limited and expected positive excess returns can persist in equilibrium.

As shown in Appendix C a bank's ability to attract deposits is directly related to its net worth. At the aggregate level this relationship is shown to obey

$$Q_t S_t = \frac{\eta_t}{\lambda - \upsilon_t} (1 - BT_t) N_t. \tag{42}$$

The term $\frac{\eta_t}{\lambda - \upsilon_t}$ is the ratio of assets to equity. Condition (42) limits the aggregate leverage ratio to the point where the incentives to cheat are balanced by the costs for each bank. The marginal values of loans, υ_t, and of equity, η_t, are defined recursively as

$$\begin{aligned} \upsilon_t &= E_t (1 - \theta) \Lambda_{t,t+1} \left(R^s_{t+1} - R_t \right) \\ &\quad + \theta \Lambda_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - \upsilon_{t+1})}}{\frac{\eta_t}{(\lambda - \upsilon_t)}} \left[\left(R^s_{t+1} - R_t \right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t) \right] (1 - BT_{t+1}) \upsilon_{t+1} \quad (43) \\ \eta_t &= (1 - \theta) + \theta \Lambda_{t,t+1} \left[\left(R^s_{t+1} - R_t \right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t) \right] (1 - BT_{t+1}) \eta_{t+1}. \quad (44) \end{aligned}$$

Finally, aggregate net worth evolves according to

$$N_t = \theta \left[(R^s_t - R_{t-1}) \frac{\eta_{t-1}}{(\lambda - \upsilon_{t-1})} + (1 + R_{t-1}) \right] (1 - BT_{t-1}) N_{t-1} + \omega Q_t S_{t-1}. \tag{45}$$

4.1.3 Production of Goods

The representative firm uses capital and labor to produce its output

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}, \tag{46}$$

where technology evolves according to $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$. Each firm operates for only one period, but it must purchase the capital used in period $t+1$ one period in advance. To do so, the firm issues one share for each unit of capital purchased in period t to be used in period $t+1$. Absent arbitrage opportunities, the value of capital equals the value of shares

$$P_t Q_t K_{t+1} = P_t Q_t S_t. \tag{47}$$

The firm's revenues consist of output sales (priced at marginal costs) and the value of undepreciated capital. Payments for servicing the shares and for labor services enter the accounting as expenses. Hence, profits in period $t+1$ are given by

$$\Pi_{t+1}^{f} = MC_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} - (1 + r_{t+1}^{s}) P_t Q_t S_t. \tag{48}$$

With the decision on the capital stock made in period t and labor hired in the $t+1$ spot market, the firm's maximization problem taking prices as given satisfies

$$\begin{aligned} &\max_{S_t, K_{t+1}} E_t \left[\Lambda_{t,t+1} \max_{L_{t+1}} \Pi_{t+1}^{f} \right] \\ &s.t. \\ &Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \\ &Q_t P_t K_{t+1} = Q_t P_t S_t. \end{aligned} \tag{49}$$

The zero profit condition implies that the return on shares is given by

$$(1 + R_{t+1}^{s}) = \frac{1}{Q_t} \frac{\alpha MC_{t+1} Y_{t+1}}{P_{t+1} K_{t+1}} + \frac{(1-\delta)}{Q_t} Q_{t+1} \tag{50}$$

where

$$(1+R_t^s) = \frac{(1+r_t^s)}{\frac{P_t}{P_{t-1}}}. \tag{51}$$

The optimal choice of labor satisfies

$$L_t = (1-\alpha)\frac{Y_t}{W_t}\frac{MC_t}{P_t}. \tag{52}$$

To support an environment with nominal price rigidities, we introduce an intermediate layer of firms between producing-firms and firms that assemble the final goods. Each intermediate firm acquires the product of a producing firm and applies a stamp to it that differentiates it from those of others. In choosing the optimal resale price $P_t(f)$ an intermediate firm faces adjustment costs as in Rotemberg (1982)

$$\max_{P_{t+i(}(f)} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left\{(1+\tau_p) P_{t+i}(f) - MC_{t+i}\right\} \left(1-\phi_{P,t+i}(f)\right) Y_{t+i} \left(\frac{P_{t+i}(f)}{P_{t+i}}\right)^{-\frac{1+\nu_p}{\nu_p}}, \tag{53}$$

where $Y_{t+i}\left(\frac{P_{t+i}(f)}{P_{t+i}}\right)^{-\frac{1+\nu_p}{\nu_p}}$ is the demand schedule for good f. The adjustment cost for prices follows

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{P_t(f)}{\pi P_{t-1}(f)} - 1\right)^2. \tag{54}$$

4.1.4 Production of Capital

Physical capital accumulates according to

$$K_{t+1} = I_t^n + (1-\delta) K_t. \tag{55}$$

The capital stock is augmented by net investment, I_t^n, and requires gross investment in the amount, I_t^g

$$I_t^n = \left[1 - \frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_{t.}^g \tag{56}$$

Taking the price of capital, Q_t, as given, capital producing firms solve

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\psi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i}^g\right]. \tag{57}$$

4.1.5 Market Clearing

The aggregate resource constraint requires

$$Y_t = C_t + I_t^g + G_t \tag{58}$$

where government spending is set to be

$$G_t = \omega_{gy} Y_t. \tag{59}$$

4.1.6 Equilibrium Conditions and Calibration

Appendix C displays the set of structural equations associated with the model in (35)-(57) that characterize the private sector equilibrium conditions. Using the notation introduced in Section 2.1, the endogenous variables are collected in the vector

$$\tilde{x}_t = \left(\begin{array}{c} Y_t, L_t, K_{t-1}, W_t, R_t^s, \frac{MC_t}{P_t}, \lambda_t^c, C_t, R_t, S_t, N_t, v_t, \eta_t, \\ I_t^n, I_t^g, G_t, \pi_t, \phi_t, \frac{\partial \phi_t}{\partial P_t} P_t, \frac{\partial \phi_t}{\partial P_{t-1}} P_t, R_t^n, \Delta R_t^s, \left[\frac{QS}{N}\right]_t, \left[\frac{N}{Y}\right]_t \end{array} \right)' \tag{60}$$

where the nominal interest rate, R_t^n, the interest rate spread, ΔR_t^s, the loan to net worth ratio, $\left[\frac{QS}{N}\right]_t$, and the net worth to output ratio, $\left[\frac{N}{Y}\right]_t$, are defined in Appendix C. The exogenous vector ζ_t contains the technology shock

$$\zeta_t = z_t. \tag{61}$$

In the following, the central bank uses inflation, π_t, as instrument whereas the financial regulator uses the tax on bank capital, BT_t.[10] By augmenting the set of conditions (67)-(91) in Appendix C with the two definitions

$$i_t^{cb} = \pi_t \tag{62}$$

$$i_t^{mpr} = BT_t \tag{63}$$

[10] Similar to the case of the two-country model, the open-loop Nash equilibrium is indeterminate when the nominal interest is used as policy instrument.

we have cast the structural equations of the model into the form of (1)

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0.$$

Table 2 summarises the parameter choices for the subsequent experiments. Most parameters are set at values commonly found in the literature. The parameter ϕ_p in the adjustment cost function for prices is set at 1281. With this value in place the (linearized) Phillips curve features the same slope as that of a model with Calvo contracts and an expected contract duration of one year. Inflation is set to zero in the steady state and the subsidy to the intermediate goods producers is set to remove monopolistic distortions in the steady state. The parameters governing the banking sector mimic those in Gertler and Karadi (2011). The survival probability for banks is set at 0.95 implying an average horizon of bankers of ten years. The steady state ratio of loans to equity is set equal to 4. For ease of exposition, we abstract from steady state distortions by setting the interest rate spread between loans and deposits ($R^s - R$) equal to zero.[11] These choices imply that the resource transfer to new banks as a fraction of total loans, $\bar{\omega}$, is 0.0101 and the portion of net worth that the bank management can divert, λ, is 0.25.

When setting up the policy problem under cooperation, the objectives of the individual policymakers receive equal weight in the joint objective function, i.e., $\omega_{cb} = \omega_{mpr} = 0.5$. Positive values of the parameters μ_{cb} and μ_{mpr} introduce biases into the objective functions of the central bank and the macroprudential regulator as described below.

4.2 Analyzing the Gains from Cooperation

Figure 4 shows the responses to a contraction in technology under alternative policies. The shock considered brings down technology by 1 percent in the first quarter. Subsequently, technology follows its auto-regressive process.

[11] The financial frictions in the model will still imply inefficient allocations away from the state. At the expense of rendering the steady state inefficient, the steady state interest rate spread can of course be set at the value of one hundred basis points as in Gertler and Karadi (2011) (or any other value).

We first consider the cooperative policy between the two regulators that maximize the utility of the representative household defined in equation (35). The solid lines in Figure 4 denote the responses for this case. The instruments are so powerful that, for a technology shock, the policymakers replicate the allocations that obtain in the frictionless real business cycle model. Due to the financial friction, absent intervention from the financial regulator, banks are undercapitalized after the contractionary technology shock. An infusion of cash into the banks (i.e., a negative bank transfer BT_t) can prop up the equity position, N_t, and expand lending next period. At the same time, nominal rigidities call for a slight increase in the policy interest rate to prevent inflation from rising inefficiently. Notice that the welfare-maximizing cooperative policy completely stabilizes the expected spread between the bank return on investment and its cost of funding (the loan rate $E_t R^s_{t+1}$ minus the deposit R_t) in the next period and in all future periods. The same policy also achieves full inflation stabilization.

With identical objectives for the two regulators, the open-loop Nash and cooperative policies coincide. However, in practice, different regulators are assigned or pursue different objectives. We assume objectives for the two regulators that are biased versions of the preferences of the representative agent. Moreover, we restrict attention to a particular formulation of biased objectives that, under cooperative policies, yields minor differences relative to the welfare-maximizing policies (as quantified below). Accordingly, the objective of the monetary policy regulator is biased towards inflation stabilization

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{cb}(\pi_t - \bar{\pi})^2 \right], \tag{64}$$

where the parameter $\mu_{cb} = 1$ in our benchmark calibration governs the extent of the inflation bias, and where $\bar{\pi}$ is the steady-state level of inflation. Analogously, the objective of the macroprudential regulator is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{mpr} \left((R^s_t - \bar{R}^s) - (R_{t-1} - \bar{R}) \right)^2 \right], \tag{65}$$

where the parameter $\mu_{mpr} = 0.5$ in our benchmark calibration governs the extent of the bias towards stabilizing the interest rate spread for banks.[12][13]

As can be seen from Figure 4 the differences between the cooperative policies with biased and unbiased objectives are relatively minor. The bias implies that the macroprudential regulator is overzealous in stabilizing the interest rate spread for banks when the shock occurs. Conversely, the monetary policy regulator accepts small deviations from full stabilization of inflation. Similarly, all other allocations remain close to their counterparts under the welfare-maximizing cooperative policies with biased objectives.

By contrast, an open-loop Nash game with the same biased objectives yields outcomes that are drastically different. To understand the extent of these differences, consider the side effects of a policy that, in reaction to a decline in technology, pushes up the equity position of banks. Higher equity positions allow banks to expand credit and push up investment and aggregate demand. In the presence of nominal rigidities, this expansion in demand leads to higher resource utilization and higher marginal costs of production, which spill cause inflation to rise. In reaction to the same decline in technology, monetary policy will want to curb the inflationary effects of the shocks and increase policy rates. However, higher policy rates bring up the cost of funding for banks and by reducing profitability ultimately reduce the amount of funds available to support lending.

Accordingly, as the macroprudential regulator recognizes that the monetary policy regulator will move to push up rates, he counteracts that action by pushing up the transfer from households to banks (shown as a negative movement in Figure 4). In turn, the monetary policy regulator will have an incentive to increase policy interest rates by more, realizing that the macro prudential regulator will step up

[12] In analysing the strategic interaction between fiscal and monetary policy Dixit and Lambertini (2003) assume the central bank to be more aggressive about inflation stabilisation than the representative agent (and the fiscal authority) in order to obtain different objective functions for the fiscal and monetary authorities. Our formulation is more general, but reduces to the idea captured in Dixit and Lambertini (2003) for $\mu_{mpr} = 0$.

[13] As an alternative to the approach of biasing the objectives inspired by Dixit and Lambertini (2003), one could devise distinct objectives for the two policymakers based on a decomposition of the second order approximation to the utility function of the representative household in the spirit of the LQ approximation. For instance, competitive dynamics similar to the ones illustrated here would also arise by assigning the objective of inflation stabilization solely to the monetary authority and the remaining terms from the decomposition of the utility function to the financial regulator.

the recapitalization of banks. Effectively, the different biases in the objectives push each regulator to discount the reverberations of his own actions onto the objectives of the other regulator. Ultimately, as shown in Figure 4, the strategic interactions lead to an excessive recapitalization of banks, unnecessarily aggressive tightening in monetary policy, and stark deviations from the allocations under the welfare-maximizing cooperative policies and substantial welfare losses.

The top panel of Figure 5 confirms that the welfare losses from adopting biased objectives are small for cooperative policies for a broad range of the parameters that govern the biases. By contrast, the bottom panel of the figure shows that the welfare gains from cooperative policies increase substantially with the bias towards spread stabilization. With biased objectives, the welfare costs of open-loop Nash policies relative to the welfare maximizing policies can be orders of magnitude higher than the losses from allowing for biased objectives under cooperative policies relative to the case of unbiased objectives. Notice also that these welfare costs are orders of magnitudes larger than the welfare costs of business cycles reported in Lucas (2003).

Our results point to two implications for the design of institutional arrangements. Firstly, bringing different regulatory functions under the same institution fosters the recognition of alternative objectives and avoids potentially large welfare losses from strategic interaction. When this solution is politically not feasible, our results argue for devising broader objectives for each regulator as way to minimize the welfare-reducing impact of strategic behavior.

5 Conclusions

Studying strategic interaction between policymakers has a long tradition in macroeconomics. However, obtaining the relevant first order conditions that characterize the problem under consideration can be complicated. A popular approach is to solve the problem using linear-quadratic (LQ) techniques. Purely quadratic objective functions are derived for each policymaker; the first order conditions of the problem are then obtained by optimizing the quadratic objectives subject to linear approximations of the structural economic relationships. Unfortunately, this approach becomes laborious and potentially error-prone for larger models.

A more direct approach is to obtain the first order conditions by using the nonlinear structural equations of the model and the nonlinear objective functions assigned to the policymakers. Our toolbox fully automates this procedure using symbolic differentiation and its output is can be processed without additional changes by the Dynare modeling platform. The quadratic approximations to the policymakers' objective functions can in principle be retrieved from the output of our toolbox. However, there is no need to solve the analogous LQ problem any more as the resulting first order conditions of the LQ problem are identical with those obtained from the direct approach followed here to the first order of approximation. Any changes to an existing model such as allowing for cooperation between policymakers instead of playing out an open-loop Nash game or changing the policy instruments assigned to the policymakers imply a new set of first order conditions that is easily generated by our toolbox. Consequently, the implied (purely) quadratic loss function to be used in an LQ problem changes as well.

We apply the toolbox introduced in this paper to the well-known case of monetary policy coordination in a two-country model and replicate the features highlighted in the literature. Both the optimal monetary policies with and without coordination are characterized with the help of impulse response functions and we show how the choice of policy instruments influences the outcomes of an open-loop Nash game. We also apply the toolbox to address strategic interaction between a macroprudential regulator and a central bank in the a model with financial friction. The analysis points to potentially large welfare losses stemming from the lack of coordination between policymakers even if technology shocks are the only source of fluctuations.

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Table 1: Parameters for the Open Economy Model

Parameter	Used to Determine	Parameter	Used to Determine
$\beta = 1/1.01$	discount factor	$\sigma = 2$	intertemporal consumption elasticity
$\chi = 0.5$	labor supply elasticity	$\bar{L} = 1$	steady state labor supply to fix χ_0
$\frac{1+\rho^c}{\rho^c} = 2$	trade subst. elasticity	$\omega_c = 0.85$	home bias in consumption
$\xi_p = 0.75$	Calvo price parameter	$\frac{1+\nu_p}{\nu_p} = 10$	subst. elasticity of varieties
$\bar{\tau} = 1/9$	steady state subsidy to producers	$\bar{\pi} = 1$	steady state inflation
$\rho_z = 0.95$	persistence of tech. shock	$\sigma_z = 0.008$	std. of tech. shock
$\rho_\tau = 0$	persistence of cost push shock	$\sigma_\tau = 0.1$	std. of cost push shock
$\rho_{gy} = 0.99$	persistence of gov. spending shock	$\sigma_{gy} = 0.01$	std. of gov. spending shock
$\omega_{gy} = 0$	share of gov. spending	$\kappa_0 = 1$	
$\omega = 0.5$	weight on home country in Ramsey	$\omega^* = 0.5$	weight on foreign country in Ramsey

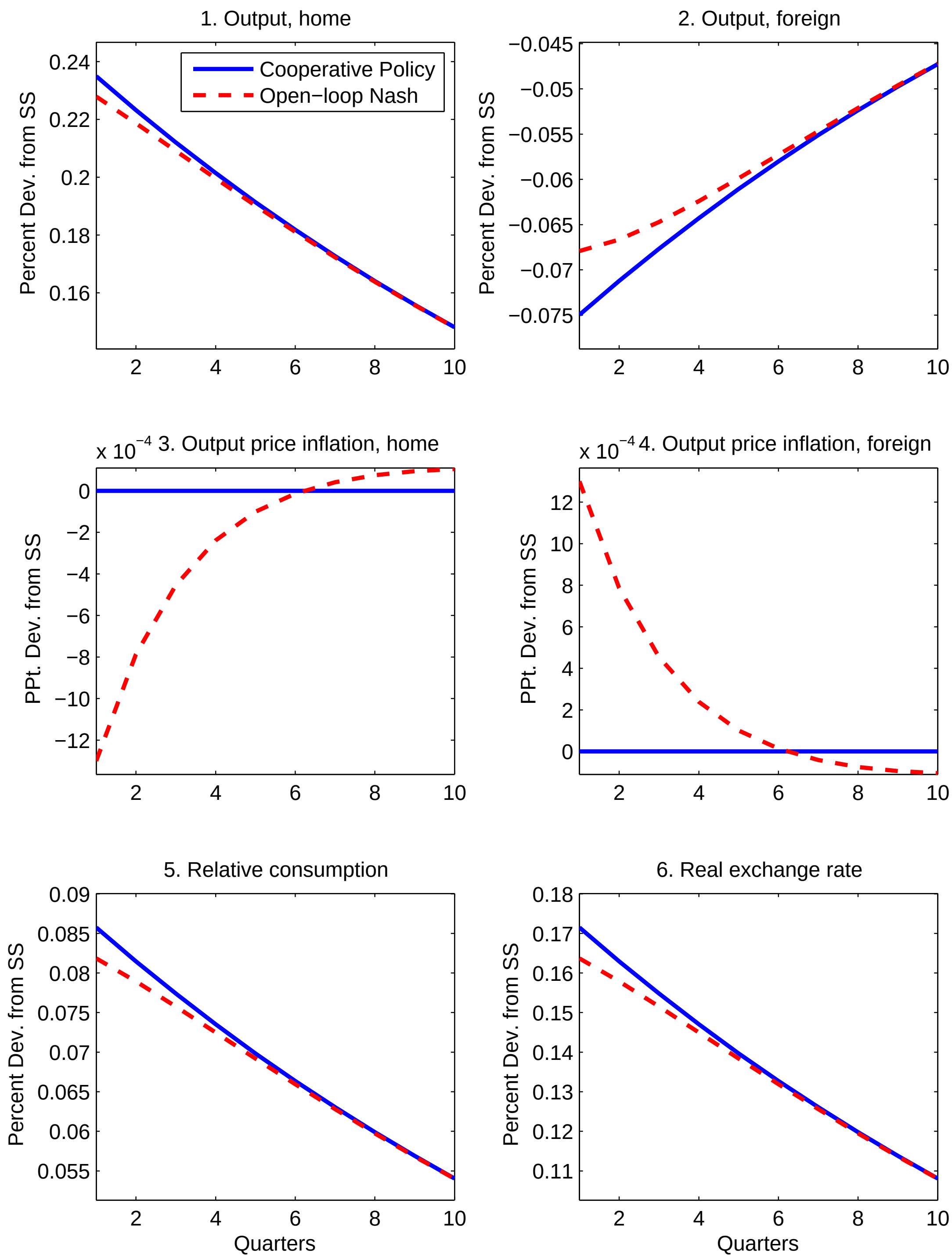
Note: This table summarizes the parameterization of the open economy model described in Section 3 at quarterly frequency.

Table 2: Parameters for the Macroprudential Regulation Model

Free Parameters			
Parameter	**Used to Determine**	**Parameter**	**Used to Determine**
$\beta = 0.99$	discount factor	$\gamma = 0.6$	consumption habits
$\chi = 1$	labor supply elasticity	$\bar{L} = 0.5$	steady state labor supply to fix χ_0
$\alpha = 0.3$	share of capital in production	$\delta = 0.025$	capital depreciation rate
$\frac{1+\nu_p}{\nu_p} = 11$	subst. elasticity of varieties	$\tau_p = 0.1$	subsidy to producers
$\phi_p = 1281$	price adjustment cost	$\bar{\pi} = 1$	steady state inflation
$\psi = 1$	investment adjustment cost	$\omega_{gy} = 0$	share of gov. spending
$\rho_a = 0.95$	persistence of tech. shock	$\sigma_a = 0.01$	std. of tech. shock
$\omega_{mpr} = 0.5$	weight of fin. reg. in Ramsey	$\omega_{cb} = 0.5$	weight of non. pol. in Ramsey
$\mu_{mpr} = 0.5$	add. term in fin. reg. utility	$\mu_{cb} = 1$	add. term in mon. pol. utility
$\left[\frac{QS}{N}\right] = 4$	steady state ratio loans to net worth	$\bar{R}^s - \bar{R} = 0$	steady state interest rate spread
$\theta = 0.95$	probability of bank survival		
Implied Parameters			
$\lambda = 0.25$	diversion parameter	$\bar{\omega} = 0.0101$	resource transfer to new banks
$\chi_0 = 3.6143$	shift parameter in utility function		

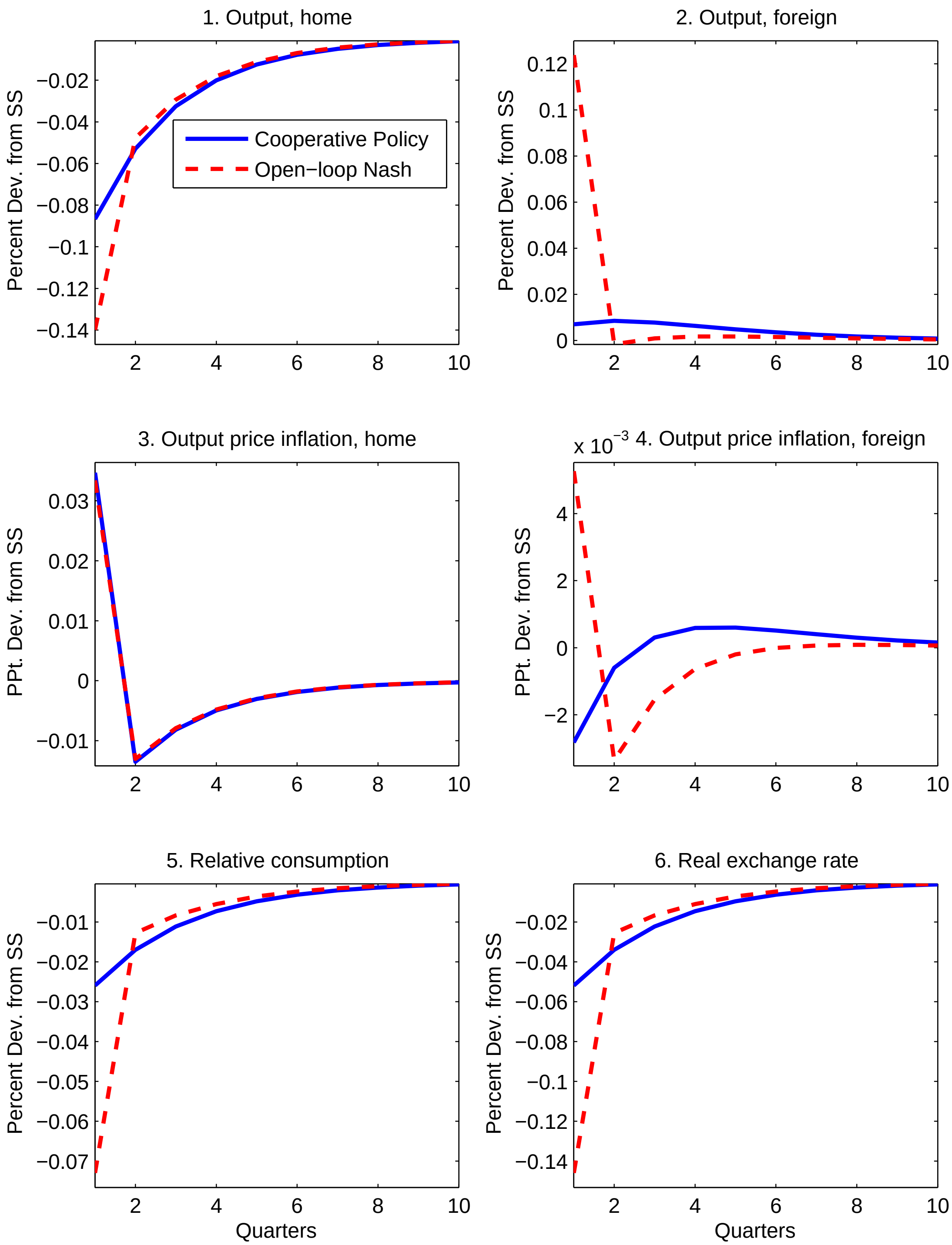
Note: This table summarizes the parameterization of the macroprudential regulation model described in Section 4 at quarterly frequency.

Figure 1: Cooperative and Open-loop Nash Policies in the Open Economy Model: Responses to a Technology Shock



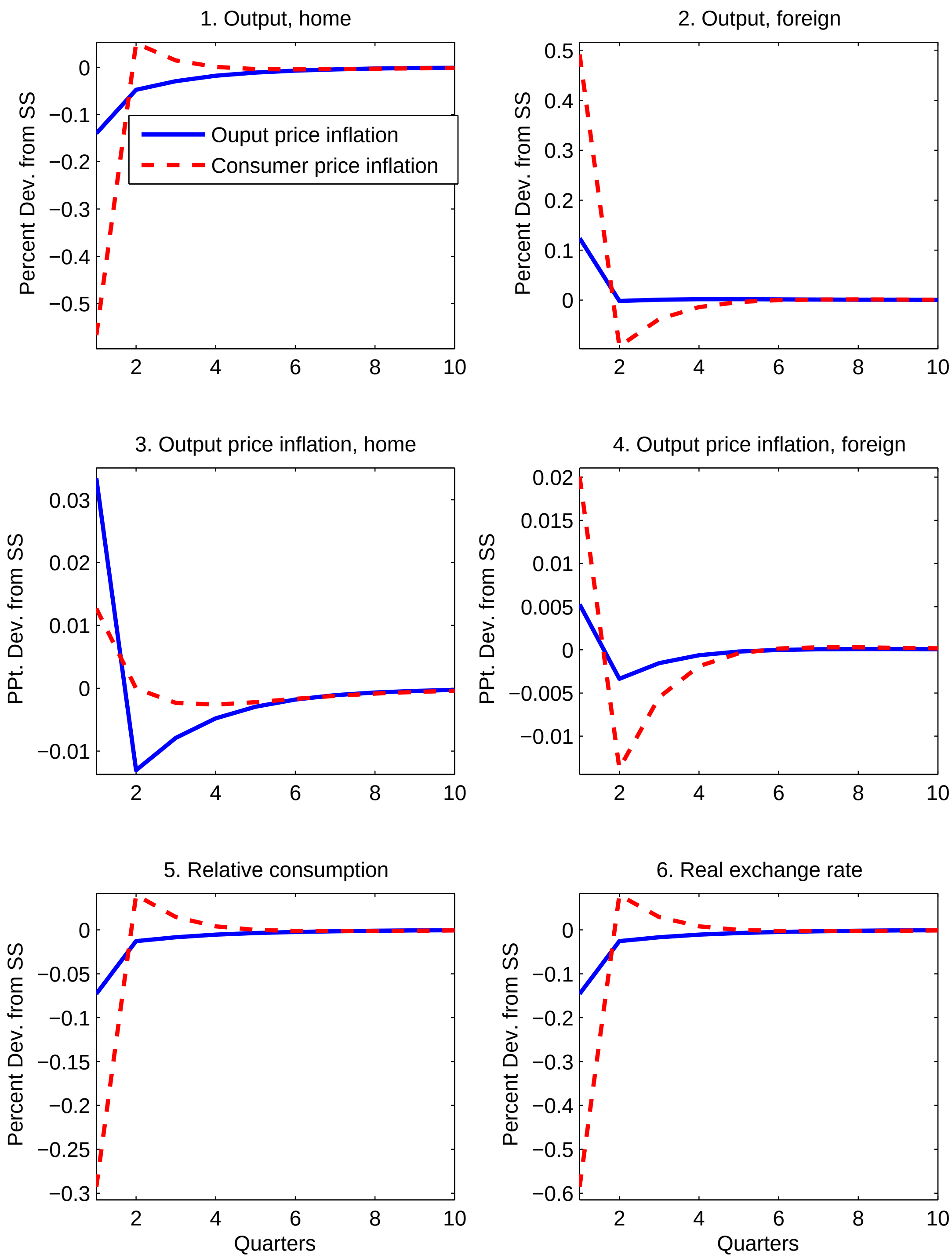
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation increase in technology in the home country. The two lines show the responses under cooperation with full commitment (Ramsey) and without cooperation (open-loop Nash) when policymakers use output price inflation in their respective country as the policy instrument.

Figure 2: Cooperative and Open-loop Nash Policies in the Open Economy Model: Responses to a Cost Push Shock



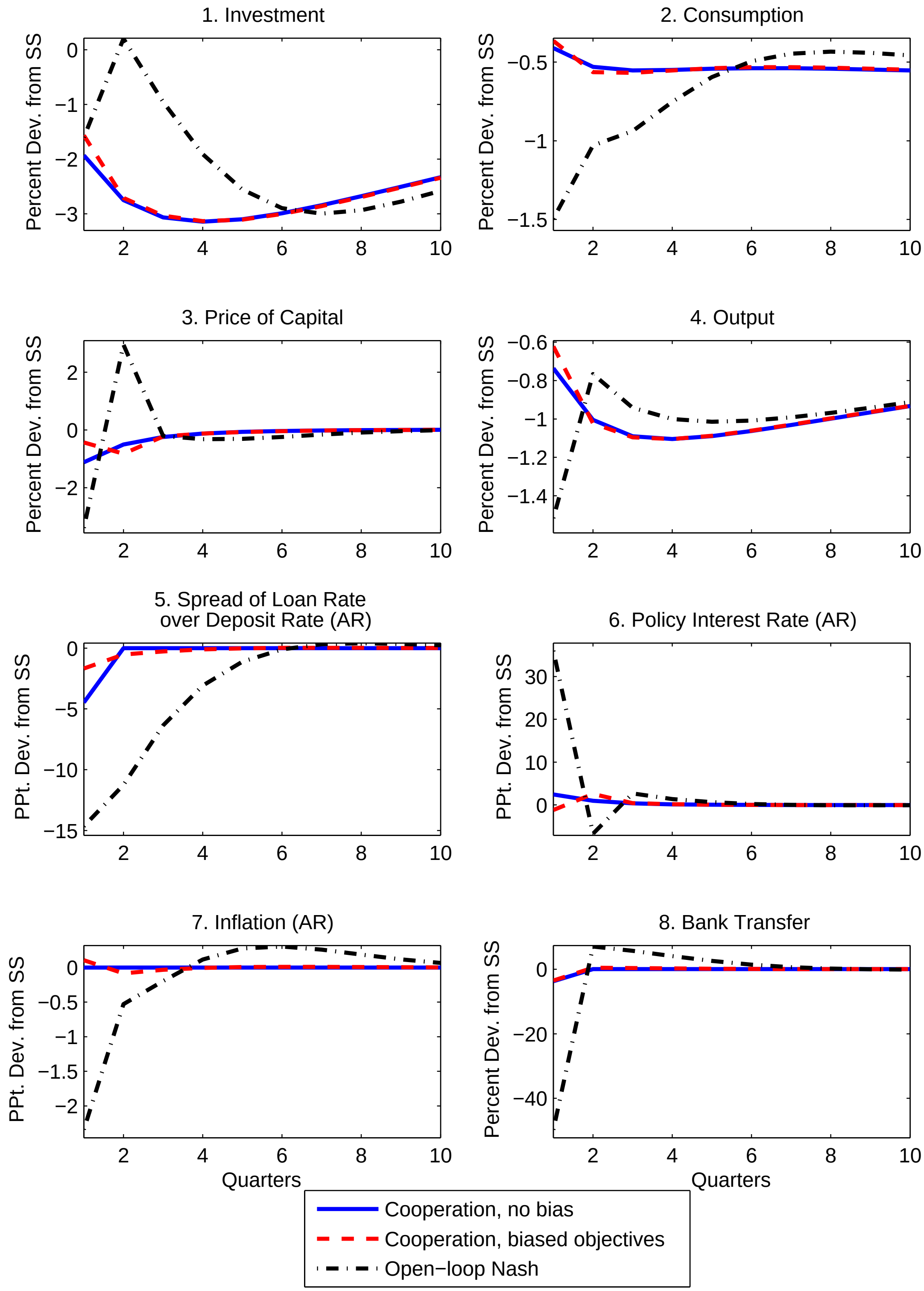
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation cost push shock that raises price markups in the home country. The two lines show the responses under cooperation with full commitment (Ramsey) and without cooperation (open-loop Nash) when policymakers use output price inflation in their respective country as the policy instrument.

Figure 3: Comparison of Instruments under Open-loop Nash policies in the Open Economy Model: Responses to a Cost Push Shock



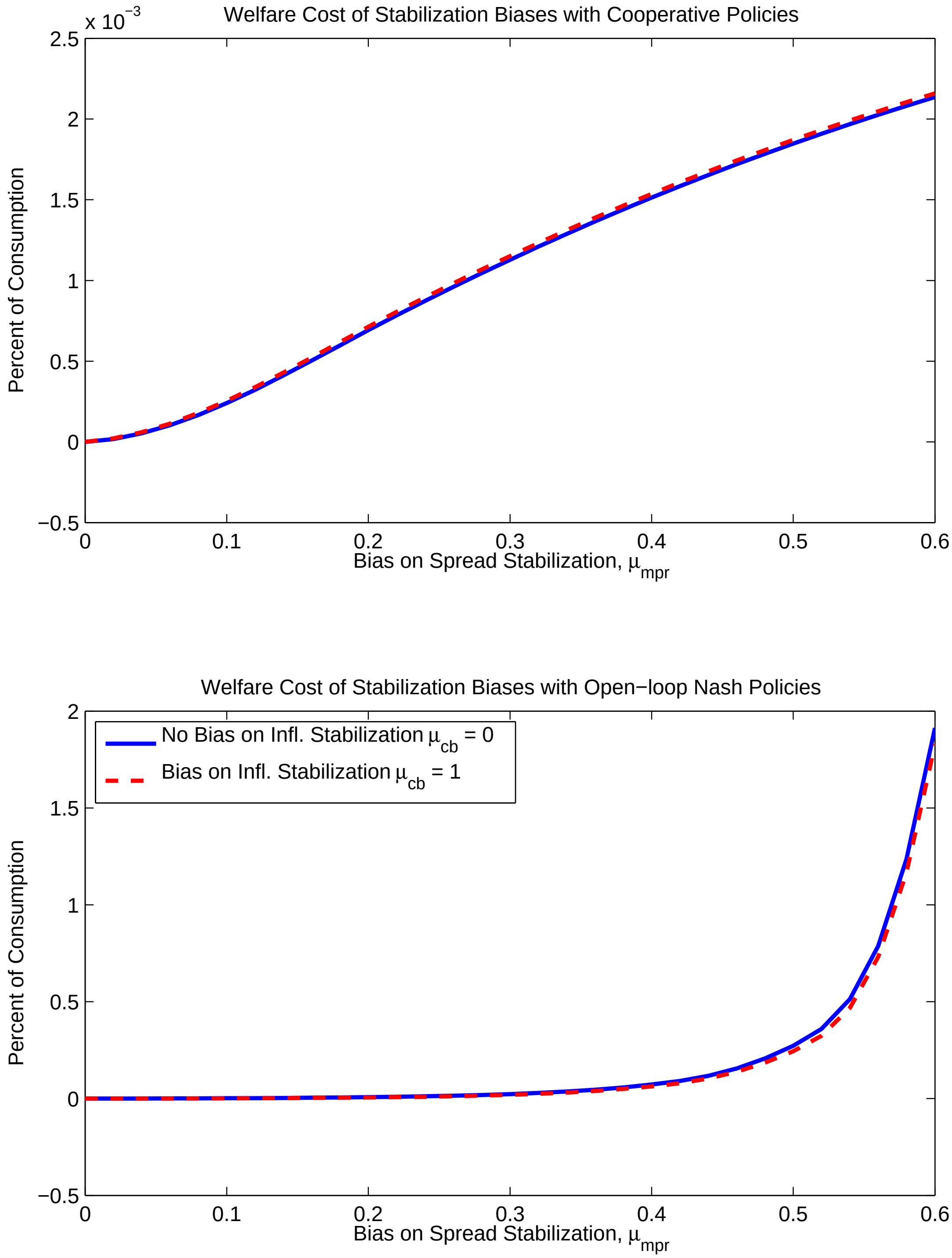
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation cost push shock that raises price markups in the home country. The two lines show the responses without cooperation (open-loop Nash) when policymakers use output price inflation and consumer price inflation as the policy instrument, respectively.

Figure 4: Cooperative and Open-loop Nash Policies in the Macroprudential Regulation Model: Responses to a Technology Shock



Notes: The figure plots the transition dynamics of the economy after a one-standard deviation decline in technology. The central bank uses inflation as instrument and the macroprudential regulator uses the tax on bank capital as instrument. The three lines show the responses for the cases of cooperation with unbiased policy preferences, cooperation with biased policy preferences, and without cooperation and biased policy preferences, respectively.

Figure 5: Cooperative and Open-loop Nash Policies in the Macroprudential Regulation Model: Responses to a Technology Shock



Notes: The figure plots the welfare costs as a function of the stabilization bias of the macroprudential regulator, μ_{mpr}. The model is simulated 10000 periods for each parameterization. The welfare gains of going from a given model to the model without stabilization bias and cooperation is expressed as a consumption equivalent variation. The top panel shows the welfare costs under cooperation but with stabilization biases for both regulators. The bottom panel plots the welfare costs, if policymakers have biased preferences and do not cooperate their activities.

A Description of Codes

The codes underlying this paper can be downloaded from
https://sites.google.com/site/martinbodenstein/ and from
http://www.lguerrieri.com/games_code.zip.
The zipped package includes five folders:

1. `nash_ramsey_toolbox` contains the codes for our toolbox,
2. `plot_support` contains plotting routines,
3. `BBCDL_model` contains the codes for the two-country model,
4. `GK_model` contains the codes for the macroprudential regulation model,
5. `LQ_BBCDL_model` contains the linear quadratic model by Corsetti, Dedola, and Leduc (2010) described in Appendix B.3.

A.1 Toolbox

The toolbox extends the functionality of Dynare which needs to be installed separately. We have verified that our toolbox is compatible with Dynare 4.4.2 and earlier versions on Mac, Windows, and Linux platforms. Before attempting to run the examples in `BBCDL_model`, `GK_model`, `LQ_BBCDL_model` the paths in `setpathdynare4.m` need reflect the local setup. The toolbox also requires access to the Matlab Symbolic Math Toolbox. The folder `nash_ramsey_toolbox` contains the codes of our toolbox. In order to generate the first-order conditions that characterize the optimal policies with and without cooperation using our toolbox, the user has to provide a Dynare-formatted model file. In addition to the structural equations derived from optimal behavior of households and firms, the file needs to specify the utility functions of the policymakers and an arbitrary description of the relevant policy rules (e.g., Taylor-style instrument rules in a two-country monetary model).[14] This input file is then used to generate an output file that contains the symbolic derivatives of the Lagrangian functions described in equation (3) for the Ramsey case and equation

[14] A primer on Dynare syntax can be found http://www.dynare.org/wp-repo/dynarewp001.pdf.

(13) for the open-loop Nash game. We first describe how to apply the toolbox; then we describe in more detail the key scripts of the toolbox.

A.1.1 Applying the Toolbox

Using our toolbox requires the user to follow a number of conventions. Through the rest of this section, we refer to the original Dynare-formatted model code as `example.mod`.

In `example.mod`:

1. Define the variables `Util1` and `Util2` in the `var` and ddd the objective functions of the policymakers in the `model` block. The equations defining `Util1` and `Util2` should be declared in the 'model' block as `Util1 = ...;` and `Util2 = ...;`
2. Break the `var` block into two `var` blocks so that the first block contains `Util1`, `Util2`, and all endogenous variables and the second block contains all exogenous variables (the shocks). Insert the line `// Endogenous variables` or `// Exogenous variables` before each block, as appropriate.
3. If parameter values are set directly in `example.mod`, remove them and save them as a separate script with the name `example_paramfile.m`.
4. In the `model` block, before the policy rule for each player, insert the line `// Policy Rule, agent 1` or `// Policy Rule, agent 2`, as appropriate.
5. If the steady-state values for the original N endogenous variables are set in the `initval` block delete the `initval` block and save the steady-state values for endogenous variables as a script in the same folder under the name `example_ss_defs.m`.
6. Collect the equations describing the paths of exogenous variables at the end of the `model` block, **after** all the structural equations.

Create a MATLAB function with the name `example_steadystate.m` in the same folder. Dynare will call this program to compute the steady-state of the model. The structure of `example_steadystate.m` should follow this template:

```
function [ys,check] = example_steadystate(junk,ys)
global M_
check = 0;

%% assign parameter values
example_paramfile

%% assign steady state values
example_ss_defs

%% send parameters and steady states to dynare
nparams = size(M_.param_names,1);
for icount = 1:nparams
eval(['M_.params(icount) = ',M_.param_names(icount,:),';'])
end

nvars = M_.endo_nbr;
ys = zeros(nvars,1);
for i_indx = 1:nvars
eval(['ys(i_indx)=',M_.endo_names(i_indx,:),';'])
end
```

The file `example_steadystate.m` first calls the scripts `example_paramfile.m` to set the parameter values; calling `example_ss_defs.m` assigns the steady-state values of the endogenous variables in the model. The values are saved in the vectors `M_.params` and `ys`, respectively, in order to be passed to Dynare.

Now the model can be processed to create the desired output files by calling the script `convertmodfiles` which is described in the next section.

A.1.2 Description of Toolbox Programs

The first order conditions to the various policy problems associated with the model file `example.mod` are created by executing the script `convertmodfiles.m`. For the open-loop Nash game, calling

```
convertmodfiles('example','nash','instrument1','instrument2')
```

generates the necessary output files `example_nash.mod`, `example_nash_steadystate.m`, `example_nash_ss_defs.m`, and `example_nash_paramfile.m`.[15]

The inputs into `convertmodfiles.m` are:

- `infilename`: a string containing the name of the Dynare file containing the model we want to analyze. Here, we set infilename = `example`, although `example.mod` also works.
- `policy_problem`: a string that must be `ramsey`, `nash`, or `one_agent_ramsey`
  - If `policy_problem = ramsey`, then `convertmodfiles.m` will output the model equations for the cooperative optimal policy (Ramsey).
  - If `policy_problem = nash`, then `convertmodfiles.m` will output the model equations for the open-loop Nash game.
  - If `policy_problem = one_agent_ramsey`, then one of the two players follows the optimal policy given that the other player will follow the arbitrary policy rule that was specified in the original file `example.mod`.
- `instrument1`: a string, giving the name of the instrument variable in the model for the first player. If `policy_problem = one_agent_ramsey`, this is the instrument used by the one player choosing the optimal policy for an arbitrary policy function of the other player.
- `instrument2`: a string, giving the name of the instrument for the second agent. If `policy_problem = one_agent_ramsey`, this should be `'1'` or `'2'`, representing the one player choosing the policy optimally.

Executing the file `convertmodfiles.m` calls the following sequence of scripts:

[15] The default names of the output files can be changed in to also reflect the names of the instruments.

1. `get_aux.m`

   - replaces lagged endogenous variables in the `model` block with auxiliary variables, which are also inserted under the `var` block as endogenous variables. Given endogenous variables `var_1,...,var_K` entering the structural equations or the utility functions with their lagged values, `get_aux.m` adds `var_1lag,...,var_Klag` to the end of the block of endogenous variables in the `var` block, and adds the equations
     `var_1lag = var_1(-1);`...`var_nlag = var_n(-1);`
     in the 'model' block.
   - given `policy_problem`, the script adds appropriate policy variables (`instr1` and `instr2`), parameters (`omega_welf1`, `omega_welf2`, `beta`), and welfare definitions to the Dynare model.The new temporary Dynare file is saved as `example_aux.mod`.
   - edits the existing files `example_paramfile.m`, `example_steadystate.m`, and `example_ss_defs.m` to account for the auxiliary and policy variables, parameters, and equations. The new files are `example_aux_paramfile.m`, `example_aux_steadystate.m`, and `example_aux_ss_defs.m`, respectively.

2. then, depending on the choice of `policy_problem.m`,

   - `get_nash.m` followed by `make_ss_nash` if `policy_problem = nash` to generate the first order conditions of the problem,
   - `get_ramsey.m` followed by `make_ss_ramsey` if `policy_problem = ramsey` to generate the first order conditions of the problem,
   - or, finally, `get_one_agent_ramsey.m` followed by `make_ss_one_agent_ramsey` if `policy_problem = ramsey` to generate the first order conditions of the problem.

We restrict the detailed description to the case of `policy_problem = nash`. The program `get_nash.m`, builds on the program `get_ramsey.m` originally provided by Lopez-Salido and Levin (2004) to find optimal Ramsey policies.[16] Taking the input

[16] Our version of `get_ramsey.m` extends the version distributed by Lopez-Salido and Levin (2004) by allowing lagged dependent variables in the objective functions.

`example_aux.mod`, `get_nash.m` outputs

1. `example_nash.mod` which contains the first order conditions of the players and removes the arbitrary policy rules from the model.
2. `example_nash_lmss.m` which contains the subset of first order conditions that is linear in the Lagrange multipliers evaluated in the steady state.

Next, the file `make_ss_nash.m` creates four auxiliary files

- `example_nash_steadystate.m`,
- `guess_example_nash_steadystate.m`,
- `example_nash_ss_defs.m`,
- `example_nash_paramfile.m`.

As we have introduced additional endogenous variables, the steady-state values of the existing endogenous variables may have changed and the steady-state values of the new endogenous variables are unspecified. `example_nash_steadystate.m` uses the values provided by `example_nash_ss_defs.m` and `example_nash_lmss.m` via `guess_example_nash_steadystate.m` to find the new steady-state values. To facilitate computation of the new steady state `example_nash_steadystate.m` allows for the choice of different algorithms. `example_nash_paramfile.m` sets the same parameter values as `example_paramfile.m`. In addition, the policy parameters are assigned the default values

$$\texttt{omega\_welf1} = 0.5$$
$$\texttt{omega\_welf2} = 0.5$$
$$\texttt{nbeta} = 0.99.$$

The toolbox includes additional programs that may be of use to researchers interested in comparing the effects of shocks across models:

- `add_welfare_vars.m` augments the Dynare model files that have been set up with period utility defined by `Util1` and `Util2` to define the variables `Welf1` and `Welf2` (cumulative welfare variables for each agent) along with `Util` and

Welf (joint utility and welfare variables using welfare weights omega_ welf1 and omega_welf2).

- edit_shocks.m takes in a character matrix of shocks (or the strings 'all' or 'none') and turns on those shocks in all Dynare model files in the current folder. This is helpful when running a program which compares the effects of different groups of shocks in a model.

A.2 Replication Codes

The replication codes for Figures 1 to 3 are stored in the folder BBCDL_model. The codes for Figures 4 and 5 are provided in the folder GK_model.

A.2.1 Open Economy Model

BBDCLmodelcomp.mod is the Dynare file containing the original model described in equations (67) to (91) with variables to be log-linearized where appropriate, i.e., the variables are surrounded by the expression exp(). This model file is ready for being processed by our toolbox. In particular, notice

- the separation of variables into the two blocks of // Endogenous variables and // Exogenous variables,
- the definition of the period-utility functions of the two policymakers as Util1 and Util2,
- the labelling of the policy rules by // Policy Rule,
- the ordering of putting the equations for the exogenous shock processes at the end of the model block.

Variables for the home country carry the prefix c1; variables for the foreign carry the prefix c2.

The model file is accompanied by three user-provided Matlab m-files

- BBCDLmodelcomp_paramfile sets the parameter values (via calling the parameter file stored in the folder parameterfiles labeled paramfile_BB which is common across all model files),

- `BBCDLmodelcomp_ss_defs` assigns the steady state values to all variables,
- `BBCDLmodelcomp_steadystate` which, after calling the previous two files, sends the parameter and steady state values to Dynare.

All relevant files for the Ramsey and the open-loop Nash problem are created by calling `convertmodfiles` via `CREATE_RAMSEY_AND_NASH` in the folder `BBCDL_model`. The first line in this script augments the Matlab path to include our toolbox. Output price inflation is denoted by `c1pid` and `c2pid` for countries 1 and 2, respectively. Consumer price inflation is labeled `c1dcore` and `c2dcore`. The files associated with any specific model carry the instrument labels in the file name.

For example, the files needed to compute the solution to the Nash problem using output price inflation as instruments are

- `BBCDLmodelcomp_nash_c1pid_c2pid.mod` containing the final model,
- `BBCDLmodelcomp_nash_c1pid_c2pid_paramfile` setting parameters by calling `paramfile_BB` and assigning values to `omega_welf1`, `omega_welf2`, `nbeta`,
- `BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` generating the new steady state,
- `guess_BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` computing the steady state using the steady state of `BBCDLmodelcomp.mod` as starting guess,
- `BBCDLmodelcomp_nash_c1pid_c2pid_ss_defs` initializing guess for steady state values of structural variables and via
- `BBCDLmodelcomp_nash_c1pid_c2pid_lmss` initialising the steady state guess for the Lagrange multipliers.

Notice, that our toolbox assigns the default values

$$\texttt{omega\_welf1} = 0.5$$
$$\texttt{omega\_welf2} = 0.5$$
$$\texttt{nbeta} = 0.99$$

to the policy parameters. The steady state of the new model may need to be computed numerically. `BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` allows for different algorithms to be employed by choosing the desired element of `algo` in the `options` variable.

The script `BBCDLfigure1` generates the impulse responses shown in Figures 1 and 2 and `BBCDLfigure2` generates Figure 3. The model names are set under the string variables `stem` and `modnam1` and `modnam2`. The variable `nperiods` fixes the number of periods for the impulse response functions. `titlelist` fixes the subplot titles, `ylabels` sets the labels for the y-axis. The desired shocks for computing impulse responses are set in `shocknamevector`. Finally, the variables to be plotted are picked in `line_ramsey` and `line_nash`, respectively.

The function `makeirfsecondorder` computes the impulse responses implementing pruning. The final argument in this function fixes the order of approximation (first (= 1) or second (= 2) order).

Finally, the folder `LQ_BBCDL_model` contains the model described in Appendix B.3. The file `call_LQBBCDL` computes the impulse responses to a cost push shock for the linear quadratic model stored in `LQBBCDL.mod` and compares them to those derived from the toolbox output `BBCDLmodelcomp_ramsey_c1pid_c2pid.mod`.

A.2.2 Macroprudential Regulation Model

`rbcb_monprud.mod` is the Dynare file containing the original model with biased objectives described in equations (107) to (132).[17] This model file is ready for being processed by our toolbox. In particular, notice

- the separation of variables into the two blocks of `// Endogenous variables` and `// Exogenous variables`,
- the definition of the period-utility functions of the two policymakers as `Util1` and `Util2`,
- the labelling of the policy rules by `// Policy Rule`,
- the ordering of putting the equations for the exogenous shock processes at the end of the model block.

[17] An additional model file with unbiased objectives is provided under the name `rbcb_monprud_nobias.mod`.

The model file is accompanied by three user-provided Matlab m-files

- `rbcb_monprud_paramfile` sets the parameter values (via calling the parameter files in the folder `parameterfiles`),
- `rbcb_monprud_ss_defs` assigns the steady state values to all variables,
- `rbcb_monprud_steadystate` which, after calling the previous two files, sends the parameter and steady state values to Dynare.

All relevant files for the Ramsey and the open-loop Nash problem are created by calling `convertmodfiles` via `CREATE_RAMSEY_AND_NASH` located in the folder `GK_model`. The first line in this script augments the Matlab path to include our toolbox. Inflation is denoted by `infl` and the bank transfer by `bt`. The files associated with any specific model carry the instrument labels in the file name.

For example, the files needed to compute the solution to the Nash problem using output price inflation as instruments are

- `rbcb_monprud_nash_infl_bt.mod` containing the final model,
- `rbcb_monprud_nash_infl_bt_paramfile` setting parameters by calling the parameter files located in the folder `parameterfiles` and assigning values to `omega_welf1`, `omega_welf2`, `nbeta`,
- `rbcb_monprud_nash_infl_bt_steadystate` generating the new steady state,
- `guess_rbcb_monprud_nash_infl_bt_steadystate` recomputing the steady state using the steady state of `rbcb_monprud.mod` as starting guess,
- `rbcb_monprud_nash_infl_bt_ss_defs` initializing guess for steady state values of structural variables and via
- `rbcb_monprud_nash_infl_bt_lmss` initialising the steady state guess for the Lagrange multipliers.

Notice, that our toolbox assigns the default values

$$\texttt{omega\_welf1} = 0.5$$

$$\texttt{omega\_welf2} = 0.5$$

$$\texttt{nbeta} = 0.99$$

to the policy parameters. Furthermore, the steady state of the new model may need to be computed numerically. `rbcb_monprud_nash_infl_bt_steadystate` allows for different algorithms to be employed by choosing the desired element of `algo` in the `options` variable.

The script `GKfigure1` generates Figure 4. The model names are set under the string variables `stem` and `modnam1` and `modnam2`. The variable `nperiods` fixes the number of periods for the impulse response functions. `titlelist` fixes the subplot titles, `ylabels` sets the labels for the y-axis. The desired shocks for computing impulse responses are set in `shocknamevector`. Finally, the variables to be plotted are picked in `line_ramsey` and `line_nash`, respectively.

The function `makeirfsecondorder` computes the impulse responses implementing pruning. The final argument in this function fixes the order of approximation (first (= 1) or second (= 2) order).

Figure 5 is generated by calling the script `GKfigure2`. The welfare gains from cooperation are expressed as the percent increase in consumption needed under the open-loop Nash game to make households equally well-off as they are under the Ramsey outcomes. The means of the welfare variables is computed by simulating each economy for a large number of periods using the Dynare command `stoch_simul` with `order=2`, and invoking `pruning`.

Changes in the value of the bias parameters μ_{cb} and μ_{mpr} are communicated through the global variables `overwrite_param_names` and `overwrite`. Overwriting the parameters set in the original parameter files occurs the respective steady state files.

Finally, some last words are in place when regenerating the model files by passing `rbcb_monprud.mod` through our toolbox. The default number of simulation periods in `stoch_simul` is set to zero. Furthermore, the block defining the variance of the innovations is commented out. To run stochastic simulations using `GKfigure2` these default feature need to be adjusted appropriately.

To preserve the option of passing parameter values through the global variables `overwrite_param_names` and `overwrite`, the steady state files created by the toolbox have to be edited manually following the template in rbcb_monprud_steadystate.[18]

[18] When creating the steady state files of the Ramsey and Nash model, our toolbox copies the content of `rbcb_monprud_steadystate` into `guess_rbcb_monprud_ramsey_infl_bt_steadystate` and `guess_rbcb_monprud_nash_infl_bt_steadystate`. The template for creating the steady state files `rbcb_monprud_ramsey_infl_bt_steadystate` and `rbcb_monprud_nash_infl_bt_steadystate` does not automatically create the ability to overwrite parameters.

B Equilibrium Conditions in the Open Economy Model

B.1 Baseline Model

Under complete financial markets, the endogenous variables are summarized in the vector

$$\tilde{x}_t = \begin{pmatrix} C_t, C_{D,t}, C_{M,t}, Y_t, G_t, \frac{P_{C,t}}{P_t}, \pi_t, H_{p,t}, G_{p,t}, \frac{P_t^{opt}}{P_t}, \Delta_t, R_t^n, q_t, \\ C_t^*, C_{D,t}^*, C_{M,t}^*, Y_t^*, G_t^*, \frac{P_{C,t}^*}{P_t^*}, \pi_t^*, H_{p,t}^*, G_{p,t}^*, \frac{P_t^{opt*}}{P_t^*}, \Delta_t^*, R_t^{n*} \end{pmatrix}'. \tag{66}$$

Without detailed derivations, we provide a complete list of the conditions characterising the private sector equilibrium for given policies in the model described in the main text.

The following equations result from the households' optimization problems:

1. derivatives with respect to C_t and C_t^* and $B_{D,t+1}$ and $B_{D,t+1}^*$ to define nominal interest rates

$$\beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_{C,t}}{P_t} \frac{P_{t+1}}{P_{C,t+1}} \frac{1}{\pi_{t+1}} \right) = \frac{1}{1+R_t^n} \tag{67}$$

$$\beta E_t \left(\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_{C,t}^*}{P_t^*} \frac{P_{t+1}^*}{P_{C,t+1}^*} \frac{1}{\pi_{t+1}^*} \right) = \frac{1}{1+R_t^{n*}} \tag{68}$$

2. derivatives with respect to B_{Ft}

$$\kappa_0 \left(\frac{C_t^*}{C_t} \right)^{-\sigma} = q_t \tag{69}$$

with q_t denoting the consumption based real exchange rate and $\kappa_0 = q_0 \left(\frac{C_0^*}{C_0} \right)^{\sigma}$

3. optimal choice of $C_{D,t}$, $C_{D,t}^*$ imply

$$C_{D,t} = \omega_c C_t \left(\frac{P_{C,t}}{P_t} \right)^{\frac{1+\rho_c}{\rho_c}} \tag{70}$$

$$C^*_{D,t} = \omega^*_c C^*_t \left(\frac{P^*_{C,t}}{P^*_t}\right)^{\frac{1+\rho_c}{\rho_c}} \tag{71}$$

4. optimal choice of $C_{M,t}$, $C^*_{M,t}$ imply

$$C_{M,t} = C_t(1-\omega_c)\left(\frac{P^*_{C,t}}{P^*_t}\frac{1}{q_t}\right)^{\frac{1+\rho_c}{\rho_c}} \tag{72}$$

$$C^*_{M,t} = C^*_t(1-\omega^*_c)\left(\frac{P_{C,t}}{P_t}q_t\right)^{\frac{1+\rho_c}{\rho_c}} \tag{73}$$

5. the definition of the consumption goods C_t, and C^*_t impose

$$C_t = \left(\omega_c^{\frac{\rho_c}{1+\rho_c}} C_{D,t}^{\frac{1}{1+\rho_c}} + (1-\omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}^{\frac{1}{1+\rho_c}}\right)^{1+\rho_c} \tag{74}$$

$$C^*_t = \left(\omega_c^{*\frac{\rho_c}{1+\rho_c}} C_{D,t}^{*\frac{1}{1+\rho_c}} + (1-\omega^*_c)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}^{*\frac{1}{1+\rho_c}}\right)^{1+\rho_c}. \tag{75}$$

Profit maximisation by the intermediaries implies the following set of conditions:

1. the optimal (relative) price set by adjusting firms $\frac{P_t^{opt}}{P_t}$ and $\frac{P_t^{opt*}}{P_t^*}$

$$\left(\frac{P_t^{opt}}{P_t}\right)^{1+\frac{1+\nu_p}{\nu_p}\chi} = \frac{H_{p,t}}{G_{p,t}} \tag{76}$$

$$\left(\frac{P_t^{opt*}}{P_t^*}\right)^{1+\frac{1+\nu_p^*}{\nu_p^*}\chi} = \frac{H^*_{p,t}}{G^*_{p,t}} \tag{77}$$

2. with $H_{p,t}$ and $H^*_{p,t}$ following

$$\begin{aligned} H_{p,t} &= \frac{1+\nu_p}{\nu_p}\chi_0\left(\frac{Y_t}{e^{z_t}}\right)^{\chi}\frac{P_{C,t}}{C_t^{-\sigma}P_t}Y_t \\ &+ \xi_p\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_{t+1}}{P_{C,t+1}}\frac{P_{C,t}}{P_t}\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)} H_{p,t+1}\right] \end{aligned} \tag{78}$$

$$H^*_{p,t} = \frac{1+\theta^*_p}{\theta^*_p}\chi^*_0\left(\frac{Y^*_t}{e^{z^*_t}}\right)^{\chi}\frac{P^*_{C,t}}{C_t^{*-\sigma}P^*_t}Y^*_t$$

$$+ \quad \xi_p^* \beta E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_{t+1}^*}{P_{C,t+1}^*} \frac{P_{C,t}^*}{P_t^*} \left(\frac{\bar{\pi}^*}{\pi_{t+1}^*} \right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)} H_{p,t+1}^* \right] \tag{79}$$

$\bar{\pi}$ is the steady state (gross) inflation rate

3. with $G_{p,t}$ and $G_{p,t}^*$ following

$$\begin{aligned} G_{p,t} &= \frac{1+\tau_{p,t}}{\nu_p} Y_t \\ &+ \xi_p \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_{t+1}}{P_{C,t+1}} \frac{P_{C,t}}{P_t} \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{1-\frac{1+\nu_p}{\nu_p}} G_{p,t+1} \right] \end{aligned} \tag{80}$$

$$\begin{aligned} G_{p,t}^* &= \frac{1+\tau_{p,t}^*}{\theta_p^*} Y_t^* \\ &+ \xi_p^* \beta E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_{t+1}^*}{P_{C,t+1}^*} \frac{P_{C,t}^*}{P_t^*} \left(\frac{\bar{\pi}^*}{\pi_{t+1}^*} \right)^{1-\frac{1+\nu_p^*}{\nu_p^*}} G_{p,t+1}^* \right] \end{aligned} \tag{81}$$

4. the evolution of prices

$$(1-\xi_p) \left(\frac{P_t^{opt}}{P_t} \right)^{-\frac{1}{\nu_p}} + \xi_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{-\frac{1}{\nu_p}} = 1 \tag{82}$$

$$(1-\xi_p^*) \left(\frac{P_t^{opt*}}{P_t^*} \right)^{-\frac{1}{\nu_p^*}} + \xi_p^* \left(\frac{\bar{\pi}^*}{\pi_t^*} \right)^{-\frac{1}{\nu_p^*}} = 1 \tag{83}$$

5. evolution of price dispersion

$$\Delta_t = (1-\xi_p) \left(\frac{P_t^{opt}}{P_t} \right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)} + \xi_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)} \Delta_{t-1} \tag{84}$$

$$\Delta_t^* = (1-\xi_p^*) \left(\frac{P_t^{opt*}}{P_t^*} \right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)} + \xi_p^* \left(\frac{\bar{\pi}^*}{\pi_t^*} \right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)} \Delta_{t-1}^* \tag{85}$$

The goods market clearing conditions are:

$$Y_t = C_{Dt} + C_{Mt}^* + G_t \tag{86}$$

$$Y_t^* = C_{Dt}^* + C_{Mt} + G_t^*. \tag{87}$$

Government spending is a fixed stochastic share of output:

$$G_t = \omega_{gy,t} Y_t \tag{88}$$

$$G_t^* = \omega_{gy,t}^* Y_t^*. \tag{89}$$

The period utility functions are:

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi_0 \left(e^{z_t}\right)^{-\chi} \frac{Y_t^{1+\chi}}{1+\chi} \Delta_t \tag{90}$$

$$U_t^* = \frac{C_t^{*1-\sigma}}{1-\sigma} - \chi_0^* \left(e^{z_t^*}\right)^{-\chi} \frac{Y_t^{*1+\chi}}{1+\chi} \Delta_t^*. \tag{91}$$

The policy rules, which will be replaced by the first order conditions of the policy-makers, are

$$R_t^n = (1+\bar{R}^n) \left(\frac{1+R_{t-1}^n}{1+\bar{R}^n}\right)^{\gamma_{R^n}} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\gamma_{R^n})\gamma_\pi} - 1 \tag{92}$$

$$R_t^{n*} = (1+\bar{R}^{n*}) \left(\frac{1+R_{t-1}^{n*}}{1+\bar{R}^{n*}}\right)^{\gamma_{R^n}^*} \left(\frac{\pi_t^*}{\bar{\pi}^*}\right)^{(1-\gamma_{R^n}^*)\gamma_\pi^*} - 1 \tag{93}$$

B.2 Extensions

We briefly describe the additional equations if consumer price inflation is used as instruments. Using consumer price inflation, $\pi_{C,t} = \frac{P_{C,t}}{P_{C,t-1}}$ as the policy instrument, we need to define consumer price inflation by relating the relative price of consumption $\frac{P_{C,t}}{P_t}$ to producer price inflation:

$$\pi_{C,t} = \left(\frac{P_{C,t}}{P_t}\right) \left(\frac{P_{t-1}}{P_{C,t-1}}\right) \pi_t \tag{94}$$

$$\pi^*_{C,t} = \left(\frac{P^*_{C,t}}{P^*_t}\right)\left(\frac{P^*_{t-1}}{P^*_{C,t-1}}\right)\pi^*_t. \tag{95}$$

Furthermore, the vector of endogenous variables is modified to include $\pi_{C,t}$ and $\pi^*_{C,t}$, i.e.,

$$\tilde{x}_t = \begin{pmatrix} C_t, C_{D,t}, C_{M,t}, Y_t, G_t, \frac{P_{C,t}}{P_t}, \pi_t, H_{p,t}, G_{p,t}, \frac{P^{opt}_t}{P_t}, \Delta_t, R^n_t, q_t, \pi_{C,t}, \\ C^*_t, C^*_{D,t}, C^*_{M,t}, Y^*_t, G^*_t, \frac{P^*_{C,t}}{P^*_t}, \pi^*_t, H^*_{p,t}, G^*_{p,t}, \frac{P^{opt*}_t}{P^*_t}, \Delta^*_t, R^{n*}_t, \pi^*_{C,t} \end{pmatrix}'. \tag{96}$$

B.3 Relationship with Linear-Quadratic Solution

Corsetti, Dedola, and Leduc (2010) deviate from the setup in Benigno and Benigno (2006) by allowing for home bias, but by eliminating government spending. In the following, we allow for home bias, abstract form government spending, and focus on the case of the efficient steady state in order to restate the model presented in Corsetti, Dedola, and Leduc (2010) using our notation. Absent home bias ($\omega_c = \omega^*_c = 0.5$), this model coincides with the one in Benigno and Benigno (2006) for equally-sized countries.

The set of relevant structural relationships of the economy can be reduced to the following set of equations if the model is (log-)linearised around its deterministic steady state

$$\pi_t = \kappa\left(\widetilde{y}_t + \frac{\tau}{\chi+\sigma}\widetilde{\delta}_t + u_t\right) + \beta E_t \pi_{t+1} \tag{97}$$

$$\pi^*_t = \kappa^*\left(\widetilde{y}^*_t - \frac{\tau}{\chi+\sigma}\widetilde{\delta}_t + u^*_t\right) + \beta E_t \pi^*_{t+1} \tag{98}$$

$$\widetilde{y}_t - \widetilde{y}^*_t = \frac{1-2\tau}{\sigma}\widetilde{\delta}_t \tag{99}$$

where

$$\lambda = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(1+\frac{1+\nu_p}{\nu_p}\chi\right)}$$

$$\lambda^* = \frac{\left(1-\beta\xi^*_p\right)\left(1-\xi^*_p\right)}{\xi^*_p\left(1+\frac{1+\theta^*_p}{\theta^*_p}\chi\right)}$$

$$\kappa = \lambda\left(\chi+\sigma\right)$$

$$\kappa^* = \lambda^* (\chi + \sigma)$$
$$\tau = -2\omega_c(1-\omega_c)\left(\sigma \frac{1+\rho_c}{\rho_c} - 1\right).$$

Following Corsetti, Dedola, and Leduc (2010) we assume symmetry, i.e., $\omega_c = \omega_c^*$. As before, the remaining parameters governing preferences over types and timing of consumption and leisure are identical across countries. For the home country π_t denotes the producer price inflation rate in deviation from its steady state, $\widetilde{y}_t$ is the output gap, and $\widetilde{\delta}_t$ stands for the terms of trade gap. The terms of trade are denoted as the price of imports divided by the price of exports. π_t^* and $\widetilde{y}_t^*$ are defined analogously.

Relative consumption and the real exchange rate gaps are determined as

$$\widetilde{q}_t = \sigma (\widetilde{c}_t - \widetilde{c}_t^*)$$
$$\widetilde{q}_t = (1-\omega_c-\omega_c^*)\widetilde{\delta}_t.$$

By taking the true linear-quadratic approximation to the utility function, Corsetti, Dedola, and Leduc (2010) show that the loss function under symmetry is given by

$$L_t = -\frac{1}{2}\left(\lambda_y (\widetilde{y}_t)^2 + \lambda_y^* (\widetilde{y}_t^*)^2 + \lambda_\pi (\pi_t)^2 + \lambda_\pi^* (\pi_t^*)^2 + \lambda_\delta \left(\widetilde{\delta}_t\right)^2\right) \tag{100}$$

where

$$\lambda_y = \chi + \sigma \tag{101}$$
$$\lambda_y^* = \chi + \sigma \tag{102}$$
$$\lambda_\pi = \frac{1}{\lambda}\frac{1+\nu_p}{\nu_p} \tag{103}$$
$$\lambda_\pi^* = \frac{1}{\lambda^*}\frac{1+\nu_p^*}{\nu_p^*} \tag{104}$$
$$\lambda_\delta = \frac{1-2\tau}{\sigma}\tau. \tag{105}$$

C Equilibrium Conditions in the Macroprudential Regulation Model

The endogenous variables are summarized in the vector

$$\tilde{x}_t = \begin{pmatrix} Y_t, L_t, K_{t-1}, W_t, R_t^s, \frac{MC_t}{P_t}, \lambda_t^c, C_t, R_t, S_t, N_t, v_t, \eta_t, \\ I_t^n, I_t^g, G_t, \pi_t, \phi_t, \frac{\partial \phi_t}{\partial P_t} P_t, \frac{\partial \phi_t}{\partial P_{t-1}} P_t, R_t^n, \Delta R_t^s, \left[\frac{QS}{N}\right]_t, \left[\frac{N}{Y}\right]_t \end{pmatrix}'. \tag{106}$$

We provide a complete list of the conditions characterising the private sector equilibrium for given policies for the model described in the main text. At the end of this appendix we will also provide the derivations for equations (42) to (45).

The following equations result from the households' optimization problem:

1. choice of optimal consumption

$$\lambda_t^c = \frac{1}{C_t - \gamma C_{t-1}} - E_t \beta \frac{\gamma}{C_{t+1} - \gamma C_t} \tag{107}$$

2. choice of optimal labor supply

$$\chi_0 L_t^{\chi} = \lambda_t^c W_t \tag{108}$$

3. choice of optimal deposit holdings

$$E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} = \frac{1}{\beta(1+R_t)}. \tag{109}$$

The following equations result from the banks:

1. leverage constraint

$$Q_t S_t = \frac{\eta_t}{(\lambda - v_t)}(1 - BT_t) N_t \tag{110}$$

2. bank capital evolves according to

$$N_t = \theta \left[(R_t^s - R_{t-1}) \frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1 + R_{t-1}) \right] (1 - BT_{t-1}) N_{t-1} + \bar{\omega} Q_t S_{t-1} \tag{111}$$

3. the marginal value of loans

$$v_t = E_t (1-\theta) \Lambda_{t,t+1} \left(R^s_{t+1} - R_t\right) + \theta \Lambda_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1+R_t) \right] (1 - BT_{t+1}) v_{t+1} \tag{112}$$

4. the marginal value of equity

$$\eta_t = E_t (1-\theta) + \theta \Lambda_{t,t+1} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1+R_t) \right] (1 - BT_{t+1}) \eta_{t+1}. \tag{113}$$

The following equations result from the basic producers:

1. equity financing for capital

$$K_{t+1} = S_{t.} \tag{114}$$

2. production function

$$Y_t = e^{z_t} {K_t}^{\alpha} L_t^{1-\alpha}. \tag{115}$$

3. choice of optimal labor input

$$L_t = (1-\alpha) \frac{Y_t}{W_t} \frac{MC_t}{P_t} \tag{116}$$

4. zero profit condition

$$(1 + R^s_t) = \frac{\alpha Y_t}{Q_{t-1} K_t} \frac{MC_t}{P_t} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{117}$$

The following equations result from the variety producers:

1. first order condition with respect to prices

$$E_t \begin{bmatrix} \left[-\frac{1}{\nu_p}\left(1+\tau_p\right)+\frac{1+\nu_p}{\nu_p}\frac{MC_t}{P_t}\right]\left(1-\phi_t\right)Y_t \\ -\left\{\left(1+\tau_p\right)-\frac{MC_t}{P_t}\right\}Y_t P_t \frac{\partial \phi_t}{\partial P_t} \\ -\Lambda_{t,t+1}\left\{\left(1+\tau_p\right)-\frac{MC_{t+1}}{P_{t+1}}\right\}Y_{t+1}P_{t+1}\frac{\partial \phi_{t+1}}{\partial P_t} \end{bmatrix} = 0 \tag{118}$$

2. with the price adjustment cost and its derivatives satisfying

$$\phi_t = \frac{\phi_p}{2}\left(\frac{\pi_t}{\bar{\pi}}-1\right)^2 \tag{119}$$

$$\frac{\partial \phi_t}{\partial P_t}P_t = \phi_p\left(\frac{\pi_t}{\bar{\pi}}-1\right)\frac{\pi_t}{\bar{\pi}} \tag{120}$$

$$\frac{\partial \phi_t}{\partial P_{t-1}}P_t = -\phi_p\left(\frac{\pi_t}{\bar{\pi}}-1\right)\frac{\pi_t}{\bar{\pi}}\pi_t. \tag{121}$$

The following equations result from the physical capital producers:

1. evolution of physical capital

$$K_{t+1} = I_t^n + \left(1-\delta\right)K_t \tag{122}$$

2. investment adjustment costs

$$I_t^n = \left[1-\frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right]I_t^g. \tag{123}$$

3. price of capital from optimal investment choice

$$Q_t\left[1-\frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2-\psi\left(\frac{I_t^g}{I_{t-1}^g}-1\right)\frac{I_t^g}{I_{t-1}^g}\right] + \Lambda_{t,t+1}Q_{t+1}\psi\left(\frac{I_{t+1}^g}{I_t^g}-1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 = 1 \tag{124}$$

The aggregate resource constraint requires

$$Y_t = C_t + I_t^g + G_t \tag{125}$$

where government spending is set to be

$$G_t = \omega_{gy} Y_t. \tag{126}$$

In addition, we define:

1. the loan rate spread

$$\Delta R_t^s = R_t^s - R_{t-1} \tag{127}$$

2. the ratio of loans to net worth

$$\left[\frac{QS}{N}\right]_t = \frac{\eta_t}{\lambda - \upsilon_t} \tag{128}$$

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3. the nominal interest rate

$$\frac{1}{(1+R_t^n)} = \beta \frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{1}{\pi_{t+1}} \tag{129}$$

4. the net worth to output ratio

$$\left[\frac{N}{Y}\right]_t = \frac{N_t}{Y_t} \tag{130}$$

The period utility functions are

$$U_t^{cb} = \log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{cb}(\pi_t - \bar{\pi})^2 \tag{131}$$

and

$$U_t^{mpr} = \log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{mpr}\left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\right)^2. \tag{132}$$

The policy rules followed by the central bank and the macroprudential regulator that will subsequently be replaced by the first order conditions of the policymakers are:

$$R_t^n = \bar{R}^n + \gamma_{R^n}\left(R_{t-1}^n - \left(\frac{\bar{\pi}}{\beta} - 1\right)\right) + (1-\gamma_{R^n})\gamma_\pi(\pi_t - \bar{\pi}) \tag{133}$$

and

$$BT_t = \gamma_{BT} BT_{t-1} + \gamma_S (S_t - S_{t-1}) \tag{134}$$

C.1 Details on Conditions (42) and (45)

We begin by restating the expected terminal wealth of a bank as

$$\max_{\{S_{t+i}(j)\}} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} N_{t+1+i}(j) \tag{135}$$

where

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1+R_t)(1-BT_t)N_t(j). \tag{136}$$

$V_t(j)$ can be split into two parts

$$\begin{aligned} V_t(j) &= E_t \left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) \right) \\ &\quad + E_t \left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} (1+R_{t+i}) N_{t+i}(j) \right). \end{aligned} \tag{137}$$

Defining $v_t(j)$ and $\eta_t(j)$

$$v_t(j) = E_t \left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} \right) \tag{138}$$

$$= E_t \left((1-\theta)\,\Lambda_{t,t+1} \left(R^s_{t+1} - R_t\right) + \Lambda_{t,t+1}\theta \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} v_{t+1}(j) \right) \tag{139}$$

$$\eta_t(j) = E_t \left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} (1+R_{t+i}) \frac{N_{t+i}(j)}{N_t(j)} \right)$$

$$= E_t \left((1-\theta) + \Lambda_{t,t+1}\theta \frac{N_{t+1}(j)}{N_t(j)} \eta_{t+1}(j) \right). \tag{140}$$

we arrive at

$$V_t(j) = v_t(j) Q_t S_t(j) + \eta_t(j) N_t(j). \tag{141}$$

In oder to aggregate over banks, we make use of the fact that all banks have access to the same investment opportunities as we will show now. $\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)}$ will be equalized across surviving firms, and similarly for $\frac{N_{t+1}(j)}{N_t(j)}$. Substitute

$$V_t(j) = v_t Q_t S_t(j) + \eta_t N_t(j) \tag{142}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_t S_t(j) \tag{143}$$

to obtain

$$v_t(j) Q_t S_t(j) + \eta_t(j) N_t(j) \geq \lambda Q_t S_t(j). \tag{144}$$

Assuming this constraint binds with equality and substituting $Q_t S_t(j) = \frac{\eta_t}{(\lambda - v_t)} N_t(j)$ into the evolution of net worth $N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1 + R_t) N_t(j)$ we arrive at

$$\frac{N_{t+1}(j)}{N_t(j)} = \left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t). \tag{145}$$

In turn, $\frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)}$ is given by

$$\begin{aligned} \frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)} &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \frac{N_{t+1}(j)}{N_t(j)} \\ &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right]. \end{aligned} \tag{146}$$

Consequently, v_t and η_t are identical for each bank and evolve according to

$$\begin{aligned} v_t &= E_t (1 - \theta) \Lambda_{t,t+1} \left(R^s_{t+1} - R_t\right) \\ &\quad + \theta \Lambda_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] v_{t+1} \end{aligned} \tag{147}$$

$$\eta_t = E_t (1 - \theta) + \theta \Lambda_{t,t+1} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \eta_{t+1}. \tag{148}$$

Finally, aggregate net worth is the sum of the net worth of two groups: old and new bankers. Bankers that survive from period $t - 1$ to period t will have aggregate net worth equal to

$$\theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right]N_{t-1}. \tag{149}$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, amounting to $(1-\ \theta)\,Q_t S_{t-1}$. Furthermore, let households transfers the fraction $\frac{\bar{\omega}}{(1-\ \theta)}$ of that amount to new bankers. Thus,

$$N_t^n = \frac{\bar{\omega}}{(1-\ \theta)}\,(1-\ \theta)\,Q_t S_{t-1} = \bar{\omega} Q_t S_{t-1}. \tag{150}$$

Current aggregate net worth is then the sum of net worth carried from the previous period by surviving firms plus the net worth of new entrants, or

$$N_t = \theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right]N_{t-1} + \bar{\omega} Q_t S_{t-1} \tag{151}$$

with υ_t and η_t as defined in equations (147) and (148).