Commodity prices and labour market dynamics in small open economies *

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Abstract

We investigate the connection between commodity price shocks and unemployment in advanced resource-rich small open economies. First, we use a panel vector auto-regression to estimate the impact of commodity price shocks on macroeconomic aggregates, with a special focus on labour market variables. Second, we build a number of small open economy models enriched with search and matching frictions in the labour market that are capable of explaining the observed data. The results from the VAR serve as a yardstick against which to assess the performance of the model through an impulse response function matching exercise. From the panel VAR, we find that for advanced commodity exporters, an increase in the price of commodities raises GDP and its components. The real exchange rate appreciates. In the labour market, an 8 percent increase in commodity prices causes unemployment to fall by 8 basis points and leads to a 2 per cent increase in unfilled vacancies. The impulse response matching exercise sheds light on the search and matching models' ability to capture transmission mechanism of commodity price shocks. For commodity price shocks, there is very little difference between the standard Dimond, Mortensen and Pissarides model and the Alternating offer bargaining model suggest by Hall and Milgrom (2008). Both models are able to capture the implied dynamics of unemployment and vacancies.

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1 Introduction

How do commodity price shocks affect the labour market in resource rich economies? We attempt to answer this question in a two-pronged approach. First, we empirically document the effect of commodity price shocks on macroeconomic aggregates and labour market dynamics in advanced commodity producing countries. In so doing, we focus on an number of key issues. Are wealth effects associated with commodity price shocks large enough to affect real economic activity? Are aggregate labour market conditions improved following an increase in commodity prices? How are international relative prices and net trade affected by commodity price shocks? Second, we build a model capable of explaining the observed data. The model we propose is a simple, flexible price small open economy model enriched by search and matching frictions in the labour market as proposed by Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP).

Analysing the effects of commodity price shock in the DMP model restricts the available data to those countries that are net commodity exporters while having labour market institutions that can be usefully described by the search and matching framework. In practice this implies limiting our analysis to developed economies. Although most developed economies are net-importers of commodities, there is a set of developed countries with sizeable net-exports of commodities comprised of Australia, Canada, New Zealand and Norway. We leverage the information content of the data by combining the observations from the four economies into a panel.

Focussing on this particular set of countries in order to gain a better understanding of labour market dynamics bears several practical advantages. First, to identify commodity price shocks for small open economies it is uncontroversial to impose a recursive scheme or assuming full exogeneity of the commodity prices. Thus, structural vector auto-regressions (SVAR) deliver a clean yardstick against which to assess the performance of theoretical models through an impulse response function matching exercise. Second, high quality data in employment, unemployment, and vacancies are available. Finally, the labour markets in these countries are in liberalised and the DMP framework captures key features of the labour market institutions in place.

We estimate the impact of commodity price shocks on unemployment, vacancies, real wages, hours worked, output, consumption, investment, the trade balance, and the real exchange rate using a panel structural vector autoregressive model. Commodity price shocks are found to be expansionary for the non-commodity producing sectors of the economy. Following a positive shock to commodity prices, GDP and its components, including net trade, increase. The real exchange rate appreciates following a commodity price shock. In the labour market, unemployment falls and posted vacancies rise along with total hours worked.

We use the estimated impulse responses a yardstick against which to judge the performance of a number of small open economy models. By minimising the difference between data and model driven impulse responses, we estimate a number of deep model parameters and find empirical support for the mechanism in the search and matching model. A commodity price shock is akin to a wealth shock in our modelling framework, and thus our results enhance our understanding of the transmission mechanism of such shocks. The bond holding cost, a commonly used device used to close small open economy models, i.e. to make them stationary, plays a crucial role in the transmission of wealth shocks. In this respect, wealth shocks differ from supply or demand shocks. In response to commodity price shocks, a simple flexible price model can capture the the qualitative, but not quite the quantitative response of the real exchange rate.

2 Related literature

Our work makes contributions to two broad strands of the literature. The literature on the transmission of shocks in open economies and the literature on labour market dynamics in the presence of search and matching frictions. There are few studies analysing commodity price shocks from the point of view of advanced commodity exporters. One notable exception is Pieschacon (2012), who compares the effects of oil shocks in Norway and Mexico with emphasis on different taxation regimes. There is, however, a large literature on effects of oil price shocks from the oil importers' perspective, for example Leduc and Sill (2004) who analyse the monetary policy response to oil shocks.

In analysing the business cycle determinants of small open economies most contributors focus on movements in the terms of trade instead of the narrower concept of commodity prices. For *developing* economies, Mendoza (1995), Kose (2002), and Broda (2004) conclude that terms of trade shocks explain up to half the estimated volatility in aggregate output at business cycle frequency. Reexamining the conventional view, Schmitt-Grohe and Uribe (2015a) have uncovered substantial heterogeneity across countries with respect to the role of terms of trade shocks for the business cycle. In particular, economies that depend on commodity exports appear to be vulnerable as limited access to financial markets and an insufficient macroeconomic policy framework exasperate the impact of price movements. An important element of these studies is the assumption of exogenous terms of trade.

The majority of studies on small *developed* economies differ in this regard from those on developing economies. The terms of trade are considered to be an endogenous variable and, similar to the closed economy literature on the business cycle, fluctuations are viewed as the result of structural disturbances to technology and other sources.[1] Two exceptions in the literature, Correia et al. (1995) and Guajardo (2008) however, argue that shocks to the terms of trade can be helpful in accounting for aggregate fluctuations in developed small open economies just as they are in developing economies.

Our paper focusses on the labor market dynamics in commodity-exporting developed small open economies. More specifically, we investigate the empirical performance of the Diamond-Mortensen-Pissarides (DMP) search and matching framework conditional on commodity terms of trade shocks. Our selection of countries are a good laboratory for this purpose. First, the labor markets in these countries are in general liberalised and the DMP framework captures key features of the labor market institutions in place. Second, high quality data in employment, unemployment, and vacancies are available. Third, given the difficulties in identifying (domestic) structural shocks empirically, shifting focus to the open economy dimension may result in obtaining less controversial yardsticks against which to assess the performance of theoretical models.[2] If commodity prices are exogenous from the perspective of a country, no further econometric identification assumptions need to be made in order to gain insight on the labor dynamics conditional on commodity terms of trade shocks.

The search and matching framework has emerged as the leading approach for embedding labor markets into

[1] See, for example, Gali and Monacelli (2005), Justiniano and Preston (2010), Adolfson et al. (2007), and Aguiar and Gopinath (2007).

[2] Since Gali (1999) its has become standard to identify technology shocks in structural VARs by imposing the restriction that only technology shocks can impact labor productivity in the long run. However, this identification approach has not remained uncriticised. Faust and Leeper (1997) argue that structural VARs with long-run restrictions perform poorly in practice given sample size limitations. Furthermore, Lippi and Reichlin (1993) discuss how a short-ordered VAR may fail to provide a good approximation of the dynamics of the variables in the VAR if the true data-generating process has a VARMA representation. For a recent analysis of these issues and further details see Erceg et al. (2005).

macroeconomic equilibrium models. With few exceptions, most contributions building on the DMP framework assume the economy to be closed and to be driven by (labor) productivity shocks only. In this standard formulation, Shimer (2005) points to the difficulty of the DMP framework to generate unemployment and vacancy flows that are of comparable volatility as in the U.S. data. The strong response of the real wage to labor productivity shocks dampens the incentives of firms to post new vacancies. Shimer (2005) stimulated efforts to improve the propagation of technology shocks in the DMP framework.[3] The alternative of augmenting the standard model by additional sources of fluctuations has been pursued by Michelacci and Lopez-Salido (2007), Balleer (2012), and Christiano et al. (2013).[4]

Early work to embed the standard DMP framework into a model of the business cycle dates back to the mid-1990s, see Andolfatto (1996) and Merz (1995). However, probably due to the difference in focus, open economy models rarely feature search and matching frictions in the labor market. Hairault (2002) and Campolmi and Faia (2011) show how augmenting standard open economy model by the DMP framework impacts the transmission of shocks across countries. Christiano et al. (2011) develop a detailed small open economy DSGE model with search and matching frictions that can be employed for policy analysis. Finally, Boz et al. (2009) study search and matching frictions in a small open economy model calibrated to Mexican data. To our best knowledge, there are no published studies analysing the search and matching in the context of commodity price shocks.

3 Commodity price shocks in advanced small open economies

Among developed economies, net-exports of commodities are significant only for a small set of countries. According to the IMF (2012) net commodity exports account for more than 30% of total exports in Australia, Iceland, New Zealand and Norway, and around 20% in Canada. Furthermore, net commodity exports account for between 5% and 10% of GDP on average.[5] Because of data limitations Iceland is excluded from our analysis.

To quantify the impact of commodity price shocks on economic activity and labour markets we estimate structural vector autoregressive (SVAR) models. Most of the results reported below are derived from a panel SVAR to gain statistical efficiency. As the impulse response functions obtained in this section are ultimately used to estimate our small open economy model presented in Section 3 pooling the data delivers a single benchmark for the model estimation.

3.1 Data description

Our dataset consists of quarterly data for Australia, Canada, New Zealand and Norway spanning from 1994 Q3 to 2013 Q4.[6] For each country, we include a country-specific real commodity price index, expressed in US dollars and

[3] Candidate solutions to the DMP framework to overcome the shortcomings pointed out in Shimer (2005) are numerous: Shimer (2005) and Hall (2005) propose to real wage rigidities; Hagedorn and Manovskii (2008) argue that the opportunity cost of employment is too low in Shimer (2005); Christiano et al. (2013) suggest departures from Nash bargaining over wages. Yashiv (2007) provides a comprehensive summary of the debate and a broader assessment of the search and matching framework.

[4] The empirical analysis in these papers as well as in Canova et al. (2013) finds that neutral technology shocks with positive long run effects on labor productivity, raise unemployment in the short run. This finding is comparable to the hour-worked puzzle raised in Gali (1999).

[5] The same report shows that net commodity exports to total exports exceed 20% in South Korea, but net commodity exports account for less than 2.5% of GDP.

[6] The start of the data sample in our panel VAR is dictated by the availability of quarterly vacancy data for New Zealand. We also experimented with longer time series when estimating country-specific VARs depending on data availability. Details on the data used in our analysis and additional estimation results are presented in Appendix A.

nine country specific macroeconomic time series: GDP per capita, consumption per capita, investment investment per capita, the unemployment rate, the number of unfilled vacancies, net exports of goods and services, the real effective exchange rate, the real wage deflated by consumer prices and hours worked relative to population. With the exception of hours the unemployment rate and net trade, the data is transformed into logs. All data, logged or otherwise, are de-trended. For our baseline VAR, we subtract a quadratic trend from the data.

With the exception of Norway, we measure the commodity prices relevant to each country by its trade-weighted commodity price index. The commodity prices underlying each index are expressed in USD and deflated by the US CPI. As Norwegian commodity exports are dominated by oil, we use the price of Brent crude as the relevant commodity price. Figure 1 plots the quarterly percentage change in four commodity price series. For all countries, commodity prices have experienced high volatility over the sample. As documented in Table 2 commodity prices have been between 5 and 21 times more volatile than GDP on average.

Table 1 lists the three most important commodities for the four countries in the sample, based on net exports. Australia dominates the world trade in iron ores and concentrates, accounting for 50% of global net trade and 33% of Australia's net exports in 2014. Whereas 30% of Canada's net exports is accounted for by crude oil, it amounts to only 5% of global net trade. In New Zealand, the most important commodity is milk concentrate, accounting for 24% of net exports. NZ exports of milk powder account for 37% net trade in the commodity.

As seen in Table 1, some countries in our sample may have an outsized role in selected commodity markets. However, as we focus on a country's role across all commodities and use country-specific trade-weighted commodity price, each of the countries in our sample can be viewed as a price taker in the global market for commodities.

3.2 Estimation strategy

In our baseline setting, the effects of commodity price shocks on the labour market are estimated using a panel SVAR approach.[7] As our time series are relatively short, but the countries in our sample experience commonalities in their economic structure, combing the data across countries can improve the quality of our coefficient estimates. Furthermore, estimation of a panel provides us a single benchmark for our impulse response function matching approach.[8]

Our baseline specification assumes that heterogeneity across countries is constant, i.e., we conduct a pooled estimation with fixed effects of a reduced form VAR:

$$y_{i,t} = \tilde{A}_i + A(L)y_{i,t-1} + u_{i,t}. \tag{1}$$

The factor $A(L) \equiv A_0 + A_1L + A_2L^2 + ...$ denotes a lag polynomial where L is the lag operator. The vector $u_{i,t}$ summarizes the mean-zero, serially uncorrelated exogenous shocks with variance-covariance matrix Σ_u.

The prices of the commodities traded by the countries in our sample are determined in world markets. None of the four small countries has a dominating position in all of its relevant commodity markets; thus the impact of domestic developments in any of the countries in our sample on world commodity prices and thereby on a

[7] Canova et al. (2013) offers a comprehensive survey of panel VAR models used in macroeconomics.

[8] Our approach resembles Ravn et al. (2012) who estimate a structural panel VAR to obtain reference impulse response functions that are subsequently used in matching the impulse response functions of a stylised DSGE model with their empirical counterparts.

country's own relevant commodity price index is limited.

By imposing a recursive scheme to identify commodity price shocks, our identifying strategy does not rule out feedback from domestic shocks to commodity prices at all horizons, but only on impact. In other words, commodity prices are ordered first in the Cholesky decomposition and are thus not contemporaneously affected by country-specific shocks.[9] Finally, we allow for 2 lags in our VAR.

3.3 Estimation results

Figure 2 thus plots the median impulse responses to an eight per cent increase in commodity prices together with the 90% confidence intervals of the panel SVAR. The shock to commodity prices is both hump-shaped and persistent. Median commodity prices rise by about 8 per cent by the second quarter and have returned to steady state after 12 quarters. For our set of commodity producers, an increase in commodity prices induces a domestic boom.

Non-commodity output, consumption and investment rise on impact. Whereas output and investment increase gradually, eventually increasing by about 0.15% and 1%, respectively, the increase in consumption is front loaded. On impact, an 8% increase in commodity prices causes private sector consumption to increase by 0.17%. These results suggest that commodity price shocks have a significant, yet quantitatively modest effect on non-commodity output and the components of GDP.

An unexpected commodity price increase also leads to an improvement in the net trade position by as much as 0.6% of GDP. The dynamic patterns of the trade balance follow closely those of the commodity price index. Accounting for the share of commodity net-exports in GDP, we infer that the movements in the trade balance reflect primarily price rather than quantity changes suggesting a low (short-term) price elasticity of supply for commodities. Our measure of the real exchange rate appreciates following an increase in commodity prices, thus increasing the international purchasing power of domestic households and firms.[10]

Turning to variables reflecting labour market conditions, we note a sustained improvement as evidenced by the fall in unemployment and the rise in unfilled vacancies. Conditions improve on impact and continue to improve beyond the rise in commodity prices. At its peak, the median response of vacancies increases by just under 2% and the unemployment rate dropping by 8 basis points. CPI deflated real wages decline in response to a positive commodity price shock, whereas hours worked rises by about 0.2% on impact.

The effects of an increase in commodity prices in commodity-exporting countries mirror those conventionally found for commodity-importing countries. For example, Blanchard and Gali (2007) report that a shock that raises the price of oil unexpectedly leads to a contraction in economic activity with GDP falling and unemployment rising.

From a theoretical perspective, the empirical observations we derive for commodity exporting countries are not necessarily expected. An unexpected increase in commodity prices, raises the revenues from commodity exports. If the increase in revenues induces a strong (negative) wealth effect on the labour supply, employment, investment,

[9] This assumption is already weaker than what is assumed in many other studies. Authors studying the relationship between commodity prices (or the terms of trade) and domestic macroeconomic variables often assume commodity prices (or even the terms of trade) to be exogenous; see for example Pieschacon (2012) or Schmitt-Grohe and Uribe (2015b).

[10] The qualitative movements and overall magnitudes of these non-labour market variables are comparable to those shown in Schmitt-Grohe and Uribe (2015b) after a terms of trade shock in less developed economies.

and non-commodity output could contract depending on the role of capital and labour in producing commodities in the short-term. Our findings suggest a limited importance of such wealth effects on labour market variables. Furthermore, the fact that the trade balance inherits the shape of the commodity price movement suggests that the countries in our sample are limited in their capacity to share risk in international financial markets. Under a low supply elasticity, the commodity price increase (fall) constitutes a wealth transfer to (from) the commodity-producing country. If financial markets were complete these transfers and their effects on the macro economy should be much smaller in size.

In our baseline VAR, the data is transformed by subtracting a quadratic trend. As a robustness check, Figure 3 compares the baseline VAR to VARs estimated with data transformed by a linear as well as an Hedrick-Prescott filter. We find that our results are robust to different de-trending methods.

A complementary way of summarising the impact of commodity price shocks on macroeconomic variables in commodity-exporting countries is obtained from computing the conditional standard deviations of the variables of interest. Table 2 shows the unconditional volatilities, relative to the volatility of GDP, of our data; and in brackets, the volatilities conditional on commodity price shocks. We report data for individual country VARs. The conditional volatility of labour market variables is, in many cases, close to the data's unconditional moments. The same is true for consumption. The conditional volatilities of investment, the effective real exchange rate and net trade are somewhat larger in the VARs than in the data. For the price of commodities, the discrepancy between the conditional and unconditional volatility is even more pronounced. This reflects the relatively modest response of output to even a large change in price of commodities.

4 Baseline model

Our estimation results suggest that a commodity price boom is associated with a persistent fall in unemployment, a lasting increases in unfilled job vacancies and in consumption and investment. To gain a deeper theoretical understanding of the economic channels at work, we build a simple model of a small open economy that exports commodities. Given our focus on labour market dynamics, the model features search and matching frictions in the labour market to obtain satisfying concepts of unemployment and vacancies as introduced by the seminal contributions of Diamond (1982), Mortensen (1982), and Pissarides (1985).

To replicate the empirical findings in Section 3 our modelling strategy has to limit the wealth effect on the labour supply induced by the shock to commodity prices. At the expense of modelling the labour market variables of interest, we could specify preferences as in Greenwood et al. (1988). GHH preferences suppress the wealth effect on the labour supply and labour expands after a positive commodity price shock.

Although not essential for our results, our modelling of the labour market abstracts from the endogenous labour supply decision for an employed individual as in many papers that incorporate search and matching frictions into a full macroeconomic model. Aggregate labour supply and unemployment are thereby driven by the number of vacancies in the economy.

Apart from explicitly modelling the labour market, our model is standard. The small open economy is populated by a large number of household normalized to 1. Each households consists of a continuum of agents of measure one. Agents can be working (w) or unemployed (u). Employed agents supply labour inelastically and receive the real wage w_t. Unemployed households receive unemployment benefits in the amount of b^u. In order to be employed, an agents must first be matched to a specific job.

As employed and unemployed agents belong to the same household, they can share consumption risk by pooling their resources. This assumption has become the standard way of introducing labour market search into dynamic general equilibrium models and was first introduced by Andolfatto (1996) and Merz (1995).[11] A household consumes goods (a domestically produced traded good and an imported traded good) financed through wages, unemployment benefits and firm profits. The only asset that trades internationally is a foreign bond. Firms accumulate capital, produce goods and commodities. In Section 7 we extend the baseline model along several dimensions.

4.1 Labour flows

Firms post vacancies which are filled with workers looking for jobs. The number of new matches M_t resulting from this process is described by the constant returns to scale matching function:

$$m_t = \chi u_t^{\zeta} v_t^{1-\zeta}. \tag{2}$$

v_t is the number of vacancies and u_t is the number of unemployed workers searching for a job at the beginning of the period.

Newly formed matches increase the total number of employed workers immediately as in Christiano et al. (2013) and Gertler and Trigari (2009). Existing matches are destroyed at the exogenous rate ρ.[12] As a result, employment n_t evolves according to:

$$n_t = (1-\rho)n_{t-1} + m_t. \tag{3}$$

With the labor force being normalised to unity, u_t is given by:

$$u_t = 1 - (1-\rho)n_{t-1}. \tag{4}$$

The definition of u_t does not coincide with the concept used by statistical agencies. The conventional definition of unemployment in our framework is simply

$$\tilde{u}_t = 1 - n_t. \tag{5}$$

Finally, labour market tightness θ_t is defined as

$$\theta_t = \frac{v_t}{u_t}. \tag{6}$$

[11] Christiano et al. (2013), Gertler and Trigari (2009), Arseneau and Chugh (2012), or Ravenna and Walsh (2012) among many others follow this approach of risk pooling at the household level.

[12] Endogenous separation can be introduced by adapting the framework of firm-specific productivity shocks suggested by Ramey et al. (2000).

Using this definition, the matching function can also be related to the job finding probability, s_t:

$$s_t = \frac{m_t}{u_t} = \chi\theta_t^{1-\zeta} \tag{7}$$

or the job filling probability, q_t:

$$q_t = \frac{m_t}{v_t} = \chi\theta_t^{-\zeta}. \tag{8}$$

4.2 Households

Households are modelled following the early contributions of Andolfatto (1996) and Merz (1995). At any point in time n_t members of the household are employed and $1 - n_t$ members are unemployed. Each household maximises the weighted utility of the employed and unemployed households subject to a set of constraints.

The inter-temporal preferences of the household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[n_t U(c_t^w) + (1 - n_t) U(c_t^u) \right]. \tag{9}$$

The period utility functions for employed (w) and unemployed (u) household members are strictly concave and twice-continuously differentiable. At this stage, we refrain from the common assumption that the preferences of the individual household member over consumption feature constant relative risk aversion (CRRA) to obtain a general formulation of the value of employment.

Total consumption of the final consumption good by employed and unemployed household members is defined as:

$$c_t = n_t c_t^w + (1 - n_t) c_t^u. \tag{10}$$

The final consumption good c_t consists of a domestically produced good, c_t^h, and as imported good, c_t^f. More precisely, the final good is defined as a constant elasticity of substitution (CES) aggregate:

$$c_t = \left[v^{\frac{1}{\theta}} \left(c_t^h\right)^{\frac{\theta-1}{\theta}} + (1 - v)^{\frac{1}{\theta}} \left(c_t^f\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{11}$$

θ denotes the elasticity of substitution between these two types of goods and v is the share of the domestically produced good in final consumption. The price index of the final good, P_t, is chosen to be the numeraire. Consequently, all other prices are expressed relative to the home final good. For example, the relative price of domestically produced goods, p_t^h, denotes $\frac{P_t^h}{P_t}$.

The inter-temporal budget constraint of the household is defined as:

$$n_t c_t^w + (1 - n_t) c_t^u + p_t^f b_t = w_t n_t + (1 - n_t) b^u + (1 + r_{t-1}) p_t^f b_{t-1} + \pi_t + T_t. \tag{12}$$

Households smooth consumption by trading in one-period bonds, b_t, that pay out in units of the foreign intermediate good, $p_t^f b_t$. The interest rate payable on these bonds, r_t, is equal to the world interest rate adjusted for a debt elastic risk-premium. The spread (or discount) relative to the world interest rate r_t^* depends on the debt position of the economy:

$$1 + r_t = (1 + r_t^*)\, e^{-\phi^b \left(\frac{p_t^f b_t}{gdp_t} - \bar{b}\right)}. \tag{13}$$

With households owning all firms, profits from selling goods and commodities, π_t, are passed to the household. Additional income is derived from employment w_t and unemployment benefits b^u financed by lump-sum taxes T_t.

The household maximises (9) subject to the budget constraint (12), and equations (10) and (11) by choosing c^w, t, c^u, and b_t. The first order conditions associated with this problem can be written as:

$$\lambda_t = U_c(c_t^w) \tag{14}$$

$$\lambda_t = U_c(c_t^u) \tag{15}$$

$$\frac{1}{1 + r_t} = E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{p_{t+1}^f}{p_t^f}\right]. \tag{16}$$

λ_t denotes the Lagrange multiplier on the household budget constraint. Equations (14) and (15) reveal that due to efficient risk pooling marginal utility is equalised across household members irrespective of employment status. The optimal choices for c_t^h and c_t^f are derived from minimising the costs of obtaining one unit of the aggregate consumption good subject to condition (11):

$$c_t^h = v\left(p_t^h\right)^{-\theta} c_t \tag{17}$$

$$c_t^f = (1 - v)\left(p_t^f\right)^{-\theta} c_t. \tag{18}$$

The import price (expressed in terms of the final consumption good), p_t^f, and the price of domestically produced good, p_t^h, are related through:

$$1 = v\left(p_t^h\right)^{1-\theta} + (1 - v)\left(p_t^f\right)^{1-\theta}. \tag{19}$$

Note, that the household is not choosing the level of employment n_t or wages. Wages are set in a bargaining game between individual workers and firms after a match. However, the marginal value of employment to the household is a key component in determining the surplus of the match. Let $s_t = \frac{m_t}{u_t}$ denote the probability that an unemployed worker finds a new match. Applying this definition in equation (3)

$$n_t = (1 - \rho)n_{t-1} + s_t u_t = (1 - \rho)(1 - s_t)n_{t-1} + s_t \tag{20}$$

the marginal (monetary) value of employment to the household, H_t, is shown by applying the envelope theorem to evolve according to:

$$H_t = \frac{U(c_t^w) - U(c_t^u)}{\lambda_t} + w_t - b^u - (c_t^w - c_t^u) + (1 - \rho)E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} H_{t+1}(1 - s_{t+1})\right]. \tag{21}$$

Expression (21) is obtained from the value function of the household and the constraints (12) and (20) as shown in Cheron and Langot (2004) and Hall and Milgrom (2008) for arbitrary time-separable preferences. Moving one household member into employment affects utility of the overall household in three ways. First, the utility of the agent changing employment status rises by $U(c_t^w) - U(c_t^u)$. Second, total household resources rise by the difference between wages and replacement payment, $w_t - b^u$, and total expenditures change by the difference between consumption provided to working and unemployed household members. Household utility increase by the product of the net-increase in available resources, $w_t - b^u - (c_t^w - c_t^u)$, and the marginal utility of wealth to the household, λ_t. Finally, the gains from matching a household member with a firm also occur in future periods. To express the utility gain to the household in units of the final consumption good we divide by the marginal utility of wealth.

Most authors in the labour search DSGE literature assume a CRRA utility function:

$$U(c_t^i) = \frac{{c_t^i}^{1-\sigma}}{1-\sigma}. \tag{22}$$

Under CRRA-utility, it is not only true that all household members have the same marginal utility; in fact, each household member is assigned the same consumption level irrespective of employment status and the utility levels of employed and unemployed members are identical, compare equations (14) and (15). Thus, equation (21) reduces to the form commonly found in the literature:

$$H_t \quad = \quad w_t - b^u + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right]. \tag{23}$$

Assuming CRRA-utility in combination with efficient risk-pooling at the household level implies that the marginal value of employment to the household coincides with the value of employment in the standard DMP model with *risk-neutral* agents. The approach of Andolfatto (1996) and Merz (1995) has great appeal as it preserves the simplicity of the DMP model and allows embedding it into a standard business cycle framework with risk averse households.[13]

4.3 Firms

Domestic firms combine labour and capital to produce an intermediate good, y_t^h, with the relative price is p_t^h. The present discounted cash flow of these firms, π_t^h, is defined as:

$$E_0\sum_{t=1}^{\infty}\beta^t\lambda_t\pi_t^h = E_0\sum_{t=1}^{\infty}\beta^t\lambda_t\left(p_t^h y_t^h - w_t n_t - x_t - \kappa(v_t, v_{t-1})\right). \tag{24}$$

The real wage, w_t, is expressed in terms of the consumer price index. The firm's investment into its capital stock is captured by x_t. In order to hire new workers, the firm needs to first post vacancies. The cost function for posting a vacancy is denoted $\kappa(v_t, v_{t-1})$ with v_t measuring the number of vacancies. To improve the empirical fit of our model we allow the cost of posting vacancies to depend negatively on the rate at which vacancies are

[13]Without the construct of risk-sharing through the household, introducing risk averse agents into the DMP model complicates the analytics of the framework quickly; nonlinear numerical methods are required to obtain solutions to the model.

posted:

$$\kappa(v_t, v_{t-1}) = \kappa^v v_t \left(1 + \frac{\phi^v}{2}\left(\frac{v_t}{v_{t-1}} - 1\right)^2\right). \tag{25}$$

As with the aggregate consumption good, posting vacancies and investment into the capital stock requires the use of the domestically produced good and the imported good. We assume the functional forms for these aggregate goods to be identical to the one used to describe aggregate consumption, i.e.

$$x_t = \left[v^{\frac{1}{\theta}}\left(x_t^h\right)^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}}\left(x_t^f\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{26}$$

and similarly for vacancies.

Each firm maximises its present discounted cash flow (24) by subject to three constraints: its production function, the capital accumulation equation, and the evolution of employment. The Cobb-Douglas production function is defined over capital, total employment as well as total factor productivity:

$$y_t^h = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}. \tag{27}$$

The capital accumulation constraint is given by:

$$k_t = (1-\delta)k_{t-1} + \iota(x_t, x_t - 1) \tag{28}$$

with the conventional investment adjustment cost function

$$\iota(x_t, x_t - 1) = \kappa^x x_t \left(1 - \frac{\phi^x}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right). \tag{29}$$

Due to the presence of search and matching frictions in the labour market, firms are also constrained by the evolution of employment. Defining the probability of filling an open vacancy by $q_t = \frac{m_t}{v_t}$, equation (3) can be expressed as:

$$n_t = (1-\rho)n_{t-1} + q_t v_t. \tag{30}$$

The first order conditions with respect to capital and investment imply the usual restrictions:

$$tq_t = E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\alpha\frac{p_{t+1}^h y_{t+1}^h}{k_t} + (1-\delta)tq_{t+1}\right)\right] \tag{31}$$

$$1 = tq_t\frac{\partial\iota(x_t, x_{t-1})}{\partial x_t} + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}tq_{t+1}\frac{\partial\iota(x_{t+1}, x_t)}{\partial x_t}\right]. \tag{32}$$

Where tq_t denotes Tobin's q.

Pissarides (2009) and Christiano et al. (2011) assume that the firm has to pay a fixed cost, $\bar{\kappa}$, before the start of the bargaining process. Pissarides (2009) interprets these costs "as costs that are paid after the worker who is eventually hired arrives but before the wage bargain takes place; for example, they may be the costs of finding out about the qualities of the particular worker, of interviewing, and of negotiating with her. They are sunk before the wage bargain is concluded and the worker takes up the position, but this property is not important for volatility, because training costs that are not sunk play a similar role." At the aggregate level $\bar{\kappa}q_t$ units of the final good are used to pay for initialising the bargaining process. Introducing such a cost implies that the first order condition for vacancies can be written as:

$$q_t\left(J_t - \bar{\kappa}\right) = \frac{\partial\kappa(v_t, v_{t-1})}{\partial v_t} + E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\kappa(v_{t+1}, v_t)}{\partial v_t}\right]. \tag{33}$$

where J_t is the Lagrange multiplier on equation (30). The expected benefit from posting a vacancy, q_tJ_t, equals the marginal costs of posting a vacancy. Equation (33) is commonly referred to as the free entry into production condition. The optimal choices of producing the aggregate investment good and the payments for vacancy posting follow equations (17) to (19).

As shown next, J_t measures the value that the firm assigns to an additional unit of employment. Following similar steps as for households, J_t is obtained from the firm's value function associated with its optimisation problem. By the envelope theorem:

$$J_t = \left((1-\alpha)\frac{p_t^h y_t^h}{n_t} - w_t\right) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}J_{t+1}\right]. \tag{34}$$

By employing one additional worker, the firm raises profits in the current period when the marginal product of labour, $mpl_t = (1-\alpha)\frac{p_t^h y_t}{n_t}$, exceeds the wage payment, w_t. Furthermore, the firm receives a continuation value if the match survives.

Under standard assumption, the marginal costs of posting vacancies do not depend on past posting choices, i.e., $\phi^v = 0$, and there are no sunk costs of bargaining, i.e., $\bar{\kappa} = 0$. In this case, equations (33) and (34) reduce to the familiar system:

$$q_tJ_t = \kappa^v \tag{35}$$

$$J_t = mpl_t - w_t + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}J_{t+1}\right]. \tag{36}$$

4.4 Wage bargaining

When a match occurs between a worker and a firm, the two negotiate over the real wage w_t paid to the worker. The surplus of the match is given by:

$$H_t + J_t \tag{37}$$

where H_t and J_t measure the marginal values of employment to the household and the firm, respectively.

Assuming (efficient) Nash bargaining the solution of the bargaining game is derived from the optimisation program:

$$\max_{w_t} H_t^{\xi} J_t^{1-\xi} \tag{38}$$

subject to equations (21) and (34) which describe the evolution of the variables H_t and J_t over time. The term

$\xi \in (0,1)$ denotes the bargaining power of the household. The power of the firm is given by $1-\xi$. The surplus of the match is shared according to:

$$J_t = \frac{1-\xi}{\xi} H_t. \tag{39}$$

4.5 Real exchange rate

We define the real exchange rate, rer_t, in terms of consumer price indices. As standard in the small open economy literature, the negligible size of the domestic country relative to the rest of the world implies that the domestic import price roughly equals the consumption real exchange rate, $p_t^f \approx rer_t$. Equation (19) implies the following relationship between the price of the domestic good and the real exchange rate:

$$1 \quad = \quad v \left(p_t^h\right)^{1-\theta} + (1-v)\left(rer_t\right)^{1-\theta}. \tag{40}$$

4.6 Commodities

We model the production of commodities in the simplest possible way. The commodity supply of the small open economy in the world market is assumed to be price inelastic and fixed over time. In addition, we abstract from the use of commodities in domestic consumption or production as for the countries in our sample the share of domestic use is minuscule relative to commodity output.

Abstracting from endogenous movements in the supply of commodities focuses our work on the transmission of commodity prices changes through their impact on wealth. Changes in the supply of commodities are often slow to occur; unless the amount of excess capacity within the commodity-producing sector is sizeable in the case of a price increase or production turns unprofitable after a large price fall— maintaining production capacity often requires big expenses — the supply response is muted. However, to address the role of price-induced supply changes in the transmission of commodity price shocks, we consider different supply arrangements in Section 7.

We denote the price of the commodities by p_t^c and their supply by y_t^c. Profits from commodity extraction, $\pi_t^c = p_t^c y_t^c$ are distributed to the households. Commodity prices are determined in world markets and are set in foreign consumption units, p_t^{c*}. The domestic price of the commodity p_t^c is the related to its world price through the real exchange rate, rer_t:

$$p_t^c = rer_t p_t^{c*}. \tag{41}$$

4.7 Market clearing and net trade

Demand for the domestically produced good arises from consumption, investment, vacancy posting costs and from abroad. Given the relative price of the domestically produced good p_t^h and aggregate consumption demand c_t, the optimal consumption demand for the domestically produced good follows equation (17), i.e., $c_t^h = v\left(p_t^h\right)^{-\theta} c_t$. With similar relationships applying to the demand for the purpose of investment and covering vacancy posting costs, market clearing for the domestically produced good implies:

$$y_t^h = v\left(p_t^h\right)^{-\theta}\left(c_t + x_t + \bar{\kappa} q_t + \kappa\left(v_t, v_{t-1}\right)\right) + ex_t^h. \tag{42}$$

Export demand from abroad is assumed to be of the form:

$$ex_t^h = v^* \left(\frac{rer_t}{p_t^h}\right)^{\theta^*} y_t^* \tag{43}$$

with y_t^* denoting total foreign demand for the domestic good.

Finally, the evolution of the net foreign asset position of the domestic country is obtained from the budget constraint of the household (12) and the definition of profits by goods and commodity producers. Combining these equations yields:

$$p_t^f b_t = (1 + r_{t-1})\, p_t^f b_{t-1} + p_t^c y_t^c + p_t^h y_t^h - c_t - x_t - \bar{\kappa} q_t - \kappa(v_t, v_{t-1}). \tag{44}$$

5 Alternative models

To contrast the labour market dynamics after a commodity price shock in the DMP model with the dynamics derived under alternative approaches taken in the literature, we consider an equivalent model with a Walrasian labour market and the alternating offer bargaining model proposed in Hall and Milgrom (2008).

The search and matching framework is appealing not only because it is suitable for understanding the dynamics of unemployment and vacancies. In principle, the framework can also give rise to sticky real wages and volatile employment without requiring an unreasonably high labour supply elasticity. Under a Walrasian labour market, the flexible real wage adjusts instantly to induce market clearing. If the elasticity of the labour supply is set at the low values found in microeconometric studies, the real wage is too volatile a variable in comparison to the time series data. A high elasticity is necessary to match the high variability of hours worked, together with the low variability of the real wage.[14]

Shimer (2005) casts doubt whether the quantitative performance of a search and matching model is indeed superior to that of a model featuring a Walrasian labour market. At least simple versions of the search and matching approach have difficulty in accounting for the volatility of labour market variables under what Shimer (2005) considers a reasonable calibration of the model — a view challenged by Mortensen and Nagypal (2007) and Hagedorn and Manovskii (2008). To reduce the volatility in the real wage and thus to raise the volatility of unemployment and vacancies Hall and Milgrom (2008) propose replacing the idea of Nash bargaining between an employee-employer pair by an alternating offer bargaining game.

5.1 Walrasian labour market

Under a Walrasian labour market, the labour supply of each individual is taken to be elastic to account for the variation in the labour input in production at the aggregate level. Household preferences over consumption and leisure follow Greenwood et al. (1988). Using preferences without a wealth effect on the labour supply are key in replicating the expansion of employment after an increase in commodity prices.

[14]An alternative approach to overcome these tensions between theory and data and to preserve wage flexibility assumes that labour is indivisible as in Rogerson (1988).

All household members are employed and have preferences:

$$U(c_t, h_t) = \frac{\left(c_t - \frac{\phi_0}{1+\phi}\left(h_t\right)^{1+\phi}\right)^{1-\sigma}}{1-\sigma}. \tag{45}$$

Furthermore, we follow Blanchard and Gali (2007) in introducing real wage rigidities to improve the empirical performance of this model. The real wage evolves according to:

$$w_t = \eta w_{t-1} + (1-\eta)\, mrs_t \tag{46}$$

with the standard Walrasian model arising under the assumption of $\eta = 0$.

The optimality conditions pertaining to the labour market are:

$$\lambda_t = \left(c_t - \frac{\phi_0}{1+\phi} h_t^{1+\phi}\right)^{-\sigma} \tag{47}$$

$$mrs_t = \phi_0 h_t^{\phi} \tag{48}$$

$$w_t = (1-\alpha)\frac{p_t^h y_t^h}{n_t} \tag{49}$$

$$h_t = n_t \tag{50}$$

$$w_t = \eta w_{t-1} + (1-\eta)\, mrs_t. \tag{51}$$

5.2 Alternating offer bargaining model

In the work of Mortensen and Pissarides (1994) wages are determined through Nash bargaining. The threat points in the bargaining over wages are for the job-seeker to return to unemployment and for the firm to leave the vacancy unfilled.

Hall and Milgrom (2008) suggest replacing the assumption of Nash bargaining by a noncooperative alternating offer model which implies a change in the outside options of the negotiating parties.[15] While a breakdown in the negotiations still leads to unemployment for the worker and an unfilled vacancy for the firm, the main threat is to extend bargaining rather than to terminate it. Patience determines the threat points. By breaking the tight connection between between wages and outside conditions in Mortensen and Pissarides (1994), the alternating offer bargaining model implies higher volatility of unemployment than the standard DMP model for parameters that are deemed realistic by Hall and Milgrom (2008). Christiano et al. (2013) imbed the model by Hall and Milgrom (2008) into a standard monetary business cycle model and attest to it superior statistical performance based on a Bayesian procedure.

The main departure of the alternating offer bargaining model from Nash bargaining lies in the idea that a job-seeker and a firm negotiate over a finite time span with M^{aob} subperiods. The starting offer by the firm can be rejected by the job-seeker by formulating a counteroffer. γ^{aob} is the cost to the firm of making a counteroffer to the worker. This process continues until an agreement is reached, the time span for negotiation is over, or bargaining has broken down. The exogenous probability of a breakdown in bargaining is denoted by δ^{aob}. Christiano et al.

[15] The alternating offer bargaining model was introduced by Binmore et al. (1986).

(2013) show that the surplus sharing rule in the alternating offer bargaining model can be written as:

$$J_t = \beta_1 H_t - \beta_2 \gamma^{aob} + \beta_3 \left(mpl_t - b^u\right) \tag{52}$$

with $\beta_i = \alpha_{i+1}/\alpha_1$ for $i = 1, 2, 3$ and

$$\alpha_1 = 1 - \delta^{aob} + (1 - \delta^{aob})^{M^{aob}} \tag{53}$$

$$\alpha_2 = 1 - (1 - \delta^{aob})^{M^{aob}} \tag{54}$$

$$\alpha_3 = \alpha_2 \frac{1 - \delta^{aob}}{\delta^{aob}} - \alpha_1 \tag{55}$$

$$\alpha_4 = \frac{1 - \delta^{aob}}{2 - \delta^{aob}} \frac{\alpha_2}{M^{aob}} + 1 - \alpha_2 \tag{56}$$

where $mpl_t = (1-\alpha)\,\frac{p_t^h y_t}{n_t}$ denotes the marginal product of labour. Following Christiano et al. (2013), we assume in this version of the model that households have CRRA preferences which implies that consumption between employed and unemployed household members is equalized.[16] Following Pissarides (2009), Christiano et al. (2013) assume that the firm has to pay a fee $\bar{\kappa}$ to initialize the bargaining process just as we do — see equation (33).

In the limit, if the number of subperiods over which bargaining occurs, M^{aob}, is large and the cost for the firm to make a counter offer is low, γ^{aob}, the surplus sharing rule of the alternating offer bargaining model converges to the surplus sharing rule under Nash bargaining with $\xi = \frac{1-\delta^{aob}}{2-\delta^{aob}}$.

However, for a realistic value of M^{aob} — Christiano et al. (2013) suggest setting M^{aob} equal to 60 — the surplus sharing rules (52) can mimmic the surplus sharing rule (39) only if the bargaining power of the household under Nash bargaining, ξ, is low and the probability of bargaining breakdown under alternating offer bargaining, δ^{aob}, is high. To see this, choose δ^{aob} to satisfy the condition:

$$\beta_1 = \frac{1 - (1 - \delta^{aob})^{M^{aob}}}{1 - \delta^{aob} + (1 - \delta^{aob})^{M^{aob}}} = \frac{1 - \xi}{\xi}. \tag{57}$$

Notice that β_1 is increasing in δ^{aob}. Furthermore, the coefficient β_3 can be approximated as

$$\beta_3 \approx \frac{1}{2M^{aob}} + \frac{1 - \alpha_2}{\alpha_1}. \tag{58}$$

For a large value of δ^{aob} (say > 0.1) and $M^{aob} = 60$, β_3 will be close to zero. With the appropriate choices of γ^{aob} and δ^{aob}, the alternating offer bargaining model can nest the Nash bargaining model for arbitrary (even) values of M^{aob} provided the household's bargaining power under Nash bargaining, ξ, is sufficiently low.[17]

[16] As discussed below, modelling household preferences under the alternating offer bargaining model as in the model with Nash bargaining would lead to an identification problem.

[17] If preferences and the costs of initializing the bargaining process, $\bar{\kappa}$ are identical in the DMP and AOB model, the cost of making a counter offer, γ^{aob}, needs to be zero to achieve equivalence. If preferences differ, as assumed in our analysis, γ^{aob} will in general need to be a small positive number.

6 Reconciling model and VAR

Which of the labour market models is preferred by the empirical estimates provided in Section 3? To shed light on this question we estimate a number of model parameters for each model using a minimum distance estimation strategy.

The parameters are grouped into two groups: calibrated and estimated parameters. The parameters are grouped into two groups: calibrated and estimated parameters. The calibrated parameters are listed in Table 3. The first five calibrated parameters are common to all model in Section 3. The discount factor, β implies a real interest rate of 4% per annum. The share of capital in the production function, α is 0.33, the depreciation rate, δ is 2.5% per quarter. These are all standard values used in the literature. The elasticity of substitution between home and foreign-produced goods, θ is set a 0.8, which is within the relatively wide range of values commonly used in the literature. In Section , we allow this parameter to take on a higher value as robustness check. For the search and matching models, we set the probability that a match breaks up within a given quarter as 0.1, which is somewhat lower than the 0.15 suggested by Andolfatto (1996). We assume a replacement ratio of 40% of wages and a scale parameter in the matching function of 0.67. In the steady state, we employment to 0.95, which implied a steady state unemployment rate of 5%.

We estimate those parameters for which there is little or no empirical evidence. Given its importance in the transmission mechanism of wealth shocks, we estimate the bond holding cost parameter, ϕ^b. To allow the models to match the shape of vacancies, the vacancy adjustment cost parameter, ϕ^v is also estimated, as is the investment adjustment cost parameter. In the labour market, we estimate the share of unemployment in the matching function, ζ, the AOB parameter δ^{aob} and the unobservable consumption share of the unemployed. We calibrate the share of vacancy posting costs relative to output at 0.005, but estimate how these costs are allocated between fixed and variable costs. For a number of models, we also estimate the inverse of the Frisch elasticity as well as the parameter determining the degree of real wage rigidity.

Given the values of the calibrated parameters — stacked in the vector Θ^c — we estimate the remaining ones — stacked in the vector Θ^e — by minimising the weighted distance between the empirical impulse response functions from the VAR in Section 3, G and the impulse response function implied by one of our theoretical models, $G(\Theta^c, \Theta^e)$:

$$\hat{\Theta}^e = argmin_{\Theta^e}[G - G(\Theta^c, \Theta^e)]'\Omega^{-1}[G - G(\Theta^c, \Theta^e)]. \tag{59}$$

The diagonal weighting matrix Ω is obtained from the empirical variance-covariance matrix of the estimate impulse response functions Ψ by setting all off-diagonal elements in Ψ to zero.[18] Ω penalises those elements of the estimated impulse responses with wide error bands. We minimise the objective (59) over the first six periods of the VAR which allows us to match closely the initial response of the data, but leaves the subsequent dynamics of the model unrestricted. The values assigned to the calibrated parameters are provided in Table 3. The estimated values of the remaining parameters are collected in 4.[19]

[18] The estimate of the variance-covariance matrix Ψ is obtained by means of bootstrapping in Section 3.

[19] Standard errors are constructed from the the asymptotic covariance matrix of the estimator $\hat{\Theta}^e$ given by $\left[\Gamma(\Theta^e)'\Omega^{-1}\Gamma(\Theta^e)\right]^{-1}\Gamma(\Theta^e)'\left[\Omega^{-1}\Psi\Omega^{-1}\right]\Gamma(\Theta^e)\left[\Gamma(\Theta^e)'\Omega^{-1}\Gamma(\Theta^e)\right]^{-1}$ with $\Gamma(\Theta^e) = \frac{\partial G(\Theta^c, \Theta^e)}{\partial \Theta^e}$.

6.1 Performance of the DMP model

The red solid lines in Figure 4 denote the fitted impulse responses of the baseline model. By minimising the objective in (59), the DMP model is able to precisely replicate the behaviour of commodity prices and comes very close to matching the VAR's impulse response for net trade. Thus the implied transfer of resources to the domestic economy is of similar magnitude in both the model and the VAR.

Importantly, the DMP model matches the approximate paths of unemployment and vacancies. For vacancies, the estimation yields a value for the vacancy adjustment cost parameter of 1.38, which accounts for the slight "hump" shaped path of vacancies. the DMP model is able to reproduce the initial decline and the following gradual decrease in unemployment. Compared to the data, the model generates a path of unemployment that is somewhat more persistent. The DMP model cannot account for the initial decline or indeed the shape of the path of real wages and is silent (by construction) on the behaviour of individual hours.

The transmission mechanism of commodity price shocks is akin to a wealth effect as the shock increases the transfers to households. These transfers are used to finance additional consumption, to increase savings in the form of foreign bonds, and to raise domestic investment. Consumption rises by less that 0.1% and remains around this level for a long time. The DMP model captures the gradual increase in and initial magnitude of GDP, but under predicts the response of GDP in the first 5 quarters following a commodity price shock. We observe the same pattern for investment. The persistent rise in consumption pushes up demand for both the domestic and the foreign manufacturing good, although the appreciation in the real exchange rate holds back demand of the domestic good. In the short-run, the output expansion is driven by the increase in employment, while over time the gradual built-up of the capital stock also contributes to the modest rise in the domestic manufacturing good.

The shape and the magnitude of the impulse responses depends on the real interest rate movement induced by the shock and the household's decision how to allocate the additional transfers towards savings in foreign bonds, consumption, and investment. The interest rate faced by households and firms in our framework is equal to the world interest rate adjusted by a small risk premium that depends on the country's net foreign asset position. The elasticity of this risk premium to the net foreign asset position of the home country is estimated at 0.0078, which implies that the interest rate faced by agents in our model is largely exogenous. As a result, the model generates a virtually flat consumption profile following a commodity price shock. In the data as in the model, the rise in domestic consumption occurs along side a real exchange rate appreciation. The implied correlation between the real exchange rate and (relative) consumption suggests that country-specific consumption risk coming from wealth shocks cannot be effectively shared via relative price movements and trade in non state-contingent bonds. A more detailed discussion on nexus between interest rate movements, risk sharing, and the responses of macroeconomic aggregates is postponed until Section 7.

Table 4 reports the parameter estimates and standard errors that result from minimising the objective (59). For estimation purposes, we express the elasticity of the interest rate to deviations in the net foreign asset position from steady state as a percentage (multiplying by 100). The point estimate is 0.78, which, given the standard error is statistically different from zero and in line with commonly used values of this parameter in the open economy literature.

The adjustment cost function associated with firms' posting of vacancies has a positive point estimate, but the accompanying standard error is large. This feature of our framework accounts for the hump shaped response of vacancies in response to the shock. Initial attempts to estimate the investment adjustment cost parameter, ϕ^x, suggested that in order to minimise (59), a value of zero was required leading us to imposing $\phi^x = 0$ rather than including the parameter in the estimation. The share of unemployment in the matching function, ζ, is highly significant and comes out at around 0.7 across all specifications of the search and matching models we considered.[20] Implied by the estimated and calibrated parameters of the model is the bargaining power of households, ξ. It assumes the value of 0.4 which suggests that in the wage bargaining process, firms have a rather higher weight than workers. The Hosios condition, i.e., $\xi = \zeta$, does not apply in our setting.

Finally, we turn towards discussing those parameters that primarily govern the volatility of unemployment and vacancies: the share of consumption going to the unemployed, c^u/c^w, and the costs of getting the bargaining process started, $\bar{\kappa}$. The data is highly informative on the share of consumption going to the unemployed. Our estimates suggest that unemployed members of a household enjoy about 54% of the consumption that employed members of the household enjoy. With the replacement ratio set at 40% of wage income, there is a modest, but far from complete, requirement to reduce consumption inequality between household members.[21] By contrast, the estimate of the fixed cost that firms have to pay to start the bargaining process, $\bar{\kappa}$, is basically zero.

6.2 Critical assessment of the DMP model

Since Shimer (2005) argued that the textbook DMP model — in our context characterised by Nash bargaining and CRRA preferences — explained less than 10% of the volatility in U.S. unemployment and vacancies when fluctuations are driven by productivity shocks, lively discussion over the "correct value" of the replacement ratio b^u/w has erupted.

Although our focus is on commodity price shocks, the issues underlying the so-called Shimer-puzzle also arise in our context. To better understand our parameter estimates we consider a simplified version of the model, in which we abstract from several features:

- the vacancy adjustment costs are set to zero, i.e., $\phi^v = 0$ and thus $\kappa(v_t, v_{t-1}) = \kappa^v v_t$,
- the share of capital in production is zero, i.e., $\alpha = 0$.

In Appendix C, we show that the surplus sharing rule under Nash bargaining and the definitions of the marginal values of employment to the household and the firm yield the following log-linear approximation: :

$$
\begin{aligned}
& \zeta \frac{\kappa^v}{q_{ss}} \left(\hat{\theta}_t - E_t \hat{\theta}_{t+1} \right) + \left[\left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) [\zeta - (1-\rho)\beta (\zeta - q_{ss}\xi\theta_{ss})] - \zeta\bar{\kappa} [1 - (1-\rho)\beta (1 - q_{ss}\xi\theta_{ss})] \right] E_t \hat{\theta}_{t+1} \\
= \;& (1-\xi) mpl_{ss} \left(\hat{a}_t - \frac{1-\nu}{\nu} \widehat{rer}_t \right) + (1-\rho)\beta \left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) (1 - \xi q_{ss}\theta_{ss}) E_t \left(\hat{\lambda}_{t+1} - \hat{\lambda}_t \right). \qquad (60)
\end{aligned}
$$

In the steady state, equation (C.4) implies:

$$
\left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) [1 - (1-\rho)\beta (1 - q_{ss}\xi\theta_{ss})] = (1-\xi) [mpl_{ss} - (b^u + \Phi)]. \qquad (61)
$$

[20] Based on a variety of econometric studiesMortensen and Nagypal (2007) consider the range from 0.3 to 0.5 plausible for the elasticity of the matching function — in our notation $1 - \zeta$.

[21] In models that assume CRRA preferences, unemployed and employed agents enjoy the equal levels of consumption.

Our strategy of parameterizing the DMP model implies assigning a value to the replacement ratio, r^u, rather than unemployment benefits, b^u. The definition of the replacement ratio and the condition (34) yield the following unemployment benefits given the replacement ratio:

$$b^u = r^u w_{ss} = r^u \left(mpl_{ss} - (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \right). \tag{62}$$

Combing equations (62) and (61), the household bargaining weight, ξ,can be shown to satisfy::

$$\xi = \frac{1}{1 + \frac{(1-(1-\rho)\beta(1-q_{ss}\theta_{ss}))\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)}{(1-r^u)\left(mpl_{ss}-(1-(1-\rho)\beta)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\right)-\Phi}}. \tag{63}$$

ξ is decreasing in r^u and Φ. Finally, for convenience, we define the coefficient Υ as:

$$\Upsilon = \zeta + (1-\zeta) \frac{(1-\rho)\beta q_{ss}\xi\theta_{ss}}{1-(1-\rho)\beta\left(1-q_{ss}\xi\theta_{ss}\right)} - \zeta \frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}. \tag{64}$$

Only the second term in Υ depends on ξ; therefore Υ is bounded by $\zeta - \zeta\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ (for $\xi = 0$) and $1 - \zeta\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$.

With these expressions in hand, we can approximate the response of labour market tightness in response to a commodity price shock. To avoid complications from transition dynamics, we focus on the "medium run". The commodity price shock induces highly persistent movements in the endogenous variables, thus implying little change in the endogenous variables from one period to another the initial transition. Applying this reasoning to equation (C.7), the following relationship between labour market tightness and the real exchange rate emerges:

$$\hat{\theta} = \hat{v} - \hat{u} \approx -\frac{1}{\Upsilon} \frac{mpl_{ss}}{(1-r^u)mpl_{ss} + r^u\left(1-(1-\rho)\beta\right)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right) - \Phi} \frac{1-\nu}{\nu}\widehat{rer}. \tag{65}$$

The impact of a given change in the real exchange rate onto labour market tightness is determined by the interplay of the parameters r^u and Φ.[22] We abstract for now from the coefficients $\bar{\kappa}$ and κ^v. Although Υ is a function of ξ and therefore depends on the choices of r^u and Φ as is apparent from equation (C.12), the role of this coefficient in delivering big amplification of the exchange rate movement is limited since Υ is bounded. For $\bar{\kappa} = 0$, $\Upsilon \in [\zeta, 1]$. Much more powerful in delivering amplification are therefore the parameters r^u and Φ. An increase in the replacement ratio or in the consumption gap between employed and unemployed household members reduces the denominator of the second coefficient and results in bigger movements of labour market tightness for a given movement of the real exchange rate. It is for this interplay of the parameters r^u and Φ that our model can match the responses in unemployment and vacancies even for a moderate value of the replacement ration of $r^u = 0.4$.

The choice of the parameter $\bar{\kappa}$ also influences the transmission of the shock to labour market tightness. Our calibration strategy imposes a tight relationship with κ^v as we target a specific value of the total (expected) costs of vacancy posting, $\kappa^v v_{ss} + q_{ss}\bar{\kappa}$, relative to output in the steady state. Thus the term $\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ equals $\frac{s^{fixed}}{\frac{1-s^{fixed}}{v_{ss}}+s^{fixed}}$, where s^{fixed} is the share of vacancy posting costs that falls onto $q_{ss}\bar{\kappa}$. By increasing s^{fixed}, the term $\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ in

[22] Recall, that u_t measure the pool of job-seekers at the beginning of the period and not the unemployment rate which is measured by $\tilde{u}_t$. In the medium run, the two concepts are related via $\hat{u} = \frac{1-(1-\rho)n_{ss}-\rho}{1-(1-\rho)n_{ss}}\hat{\tilde{u}}$.

(C.13) increases and implies a *lower* value of Υ. However, it can also be shown that raising s^{fixed} implies a lower value of $\frac{\kappa^v}{q_{ss}} + \bar{\kappa}$ and a higher value of the Nash bargaining weight ξ. This effect in turn, raises the value of the second term in equation (C.13) and *raises* the value of Υ. Numerically, raising s^{fixed} fails to deliver sufficient amplification.

The most effective way of raising the impact of given change in the real exchange rate on labour market tightness is therefore to allow for preferences that are non-separable in consumption and leisure to justify a consumption gap between the employed and unemployed household members, i.e., $\Phi > 0$. This approach avoids having to set the replacement ratio to unrealistically high levels and is by far more effective than raising the share of the fixed costs of bargaining in the overall vacancy posting costs, s^{fixed}.

6.3 Walrasian labour markets

The green dashed lines show the fitted impulse responses for a standard model with Walrasian labour markets. Whereas the simple RBC is silent on key labour market variables such as unemployment and vacancies, it does assume an elastic individual labour supply that produces a data-consistent increase in hours worked in response to a positive commodity price shock. The key mechanism behind this result is the functional form assumed for household preferences. Because we have assumed GHH preferences, there is no wealth effect on labour supply which would lead to a reduction in hours worked and therefore output under CRRA preferences.

The fitted impulse responses for investment and the real exchange rate are almost identical to those of the DMP model. A key difference between the two models is found in the response of output and consumption. In both cases, the RBC model yields a larger initial response than the DMP model. Compared to the DMP mode, there is a larger increase in employment which allows both output and consumption to increase by more.

The dashed blue lines show the fitted impulse responses of the RBC model with a real wage rigidity. The estimated value of η is high at 0.80. Surprisingly, this does not appear to affect the model dynamics in significant way. On impact, the only difference between the RBC model with and without real wage rigidity appears to be the response of hours and consumption, which are slightly more volatile. The remaining variables, including the real wage, appear largely unchanged.

Table 4 reports the parameter estimates for the two RBC models. The bond holding cost parameter is slightly lower for the RBC than the DMP model with point estimates of 0.72 and 0.65 for the RBC model without and with real rigidities, respectively. The data suggests a modest amount of investment adjustment costs, but these are not statistically significant. The parameter determining the real wage rigidity, η has a point estimate of 0.8. Because of the large value of η, the estimated inverse of the labour supply elasticity, ϕ, is twice as large in the model with real rigidities than in the standard open economy RBC model.

6.4 Alternative search and matching models

Figure 5 reports fitted impulse responses for two alternative versions of the search and matching model. The blue dashed lines refer to a search and matching model with Hall and Milgrom (2008)'s bargaining structure. The green dashed lines correspond to our baseline model with an elastic labour supply. As in Christiano et al. (2013), there is virtually no difference between the estimated DMP model and the AOB version, apart from a modest increase in consumption and slight decrease in the volatility of investment. Consumption is somewhat more volatile because fitting the AOB model results is a slightly larger bond holding cost parameter, ϕ^b, such that the real interest and therefore consumption respond more to the wealth shock on impact. The lower volatility of investment on impact is explained by the larger estimate for the investment adjustment cost parameter. The data in not informative on the key parameter of the AOB model, δ^{aob}, which has a point estimate of 0.01, but is not statistically significant.

The estimated AOB and DMP models differ along two dimensions: the bargaining setup and the estimated parameters. To isolate the role of the differences in the bargaining process, we estimate two restricted versions of the AOB models. In the first version, all parameters that are common across models are set at their respective point estimates obtained from estimation of the DMP model. The remaining two free parameters in the AOB model are either calibrated, such as $M^{aob} = 60$, or estimated, such as δ^{aob}.[23] Given the values of M^{aob} and δ^{aob}, the steady state relationships pin down the value of γ^{aob}. The second version we consider is identical with the first one with the one exception of including the parameter $\bar{\kappa}$ into the estimation. Estimation of the DMP model assigned the value of zero to these fixed costs of starting bargaining.

Figure 6 plots the implied impulse responses of unemployment and vacancies of the DMP and AOB models and the two restricted versions of the AOB model. The first version of the model grossly understates the responses of the labour market variables. Changing the nature of the bargaining process is not sufficient to obtain empirically plausible responses in a model with search and matching frictions. In going from the DMP model to the first version of the restricted AOB model, we also abandoned the assumption that unemployed workers consume less than employed workers by reverting to the standard assumption of a CRRA utility function for the household members and simultaneously keeping the replacement ratio at 0.4.

Instead of estimating the replacement ratio, our second version takes the alternative approach of estimating the fixed cost of bargaining $\bar{\kappa}$. In terms of resolving the constraints imposed by the model's steady state relationships and the implied dynamics, $\bar{\kappa}$ plays a role similar to the one played by the replacement ratio.[24] Estimation of our second version of the restricted AOB model assign the value of 0.0131 to $\bar{\kappa}$ — close to its value of 0.0118 in the unrestricted AOB model. The dynamics of unemployment and vacancies of this second version of the AOB model are hard to distinguish from those obtained under Nash bargaining.

Allowing for an elastic labour supply does not alter the dynamics of the DMP model in significant way. Because of the endogenous response of hours; output, consumption and investment are somewhat more volatile on impact. The dynamics of hours worked are similar to those generated by the RBC models analysed. The remaining labour market variables are not affected by the inclusion of an endogenous labour supply. Indeed, amongst the estimated parameters, only the bond holding cost parameter ϕ^b differs from significantly from the baseline model. Implied by the full set of parameters of the estimated model is the bargaining share of households, ξ, which comes out at 0.3 and therefore somewhat lower than in the baseline model. The lower bargaining weight of the household accounts for the somewhat less volatile real wage in the DMP model with elastic labour supply, compared to the

[23] In moving from the DMP to the AOB model, the bargaining weight ξ and the consumption share of the unemployed become irrelevant. The latter one drops out under the assumption of CRRA preferences in the AOB model.

[24] When we investigated a third version of the AOB model by estimating δ^{aob} and the replacement ratio, the estimate of the latter was 0.856 and δ^{aob} was close to 1. In this case, the implied weight β_1 in the surplus sharing rule (52) approaches infinity and the AOB model resembles a version of the DMP model with very bargaining weight ξ for the household.

baseline model. Also implied by the parameters of the model is the inverse of the labour supply elasticity, which takes a value of 0.74. Compared to the RBC models, the DMP model with endogenous labour supply yields a significantly lower labour supply elasticity.

As for the DMP model, we show in Appendix C how to approximate the impact of a given change in the real exchange rate on labour market tightness over the medium term as:

$$\hat{\theta} = \frac{\Omega_2 - \frac{\beta_1}{1+\beta_1}\frac{\beta_2}{\beta_3}\gamma^{aob}}{\Omega_1 - (1-\rho)\beta q_{ss}\theta_{ss}(1-\zeta)\frac{\beta_3}{1+\beta_1}\left(mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}\right)} \frac{mpl_{ss}}{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}} \frac{1-\nu}{\nu}\widehat{rer} \tag{66}$$

where

$$\Omega_1 = \left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa}\right)\left[\zeta - (1-\rho)\beta\left(\zeta - \frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right] - \zeta\bar{\kappa}\left[1 - (1-\rho)\beta\left(1 - \frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right] \tag{67}$$

$$\Omega_2 = \left(\frac{\kappa^v}{q_s s} + \bar{\kappa}\right)\left[1 - (1-\rho)\beta\left(1 - \frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right]. \tag{68}$$

Given values of δ^{aob} and M^{aob}, the parameters β_1, β_2, β_3 are determined as demonstrated in Section 5.2. Furthermore, our calibration strategy of targeting q_{ss} and n_{ss} (and therefore θ_{ss}) implies that γ^{aob} is a function of κ^v and $\bar{\kappa}$. The parameter γ^{aob} is not a free parameter.

As under the DMP model, the impact of a given change in the real exchange rate on labour market tightness is bigger for higher values of the replacement ratio:

$$b^u = r^u w_{ss} = r^u\left(mpl_{ss} - (1-(1-\rho)\beta)\left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa}\right)\right). \tag{69}$$

The important question is, however, to what extent can the model amplify the response of labour market tightness for a moderate value of the replacement ratio? The free parameters in the alternating offer bargaining model are: δ^{aob}, M^{aob}, and s^{fixed}.[25]

To maximize the impact of the shock, it is advisable to:

- maximize the term $\frac{\beta_2}{\beta_3}\gamma^{aob}$ to raise $\frac{mpl_{ss}}{mpl_{ss}-b^u-\frac{\beta_2}{,}\beta_3\gamma^{aob}}$
- minimize β_1 to increase the term $\Omega_2 - \frac{\beta_1}{1+\beta_1}\frac{\beta_2}{\beta_3}\gamma^{aob}$,
- and maximize β_3 to reduce the term $\Omega_1 - (1-\rho)\beta q_{ss}\theta_{ss}(1-\zeta)\frac{\beta_3}{1+\beta_1}\left(mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}\right)$.

Setting a low probability of breakdown in bargaining, δ^{aob} achieves the second and third point. With regard to the first point, note from equation (C.16), the term $\frac{\beta_2}{\beta_3}\gamma^{aob}$ increases in the value of s^{fixed} for given M^{aob} and δ^{aob}.

Our estimation results confirm the conjecture that β_1 is ideally set close to zero given the calibration of the total cost of vacancy posting. Across different calibrations, our point estimate for the probability δ^{aob} is always close to zero.[26] If $\delta^{aob} \approx 0$, the relationship between the real exchange rate and labour market tightness reduces

[25] Recall that s^{fixed} measures the share of the overall vacancy posting costs apportioned to $q_s s\bar{\kappa}$ in our calibration.

[26] Using a different procedure, Christiano et al. (2013) also estimate the probability of breakdown in bargaining to be close to zero.

to:

$$J_t \approx -\beta_2 \gamma^{aob} + \beta_3 (mpl_t - b^u) \tag{70}$$

or log-linearized form:

$$\hat{\theta}_t = \frac{\beta_3 mpl_{ss}}{\zeta \frac{\kappa^v}{q_{ss}}} \frac{1-\nu}{\nu} \widehat{rer}_t. \tag{71}$$

In this case, the wage sharing rule is (close to) independent of the marginal value of employment to the household, H_t. Equation (C.22) reveals the importance of s^{fixed} in the alternating offer bargaining model. For a higher value of s^{fixed}, we obtain that the share of total vacancy posting costs accounted for by the variable component $\kappa^v v_s s$ is smaller. The resulting value of κ^v will also be smaller. For s^{fixed} sufficiently large, the model can deliver the desired amplification of the labour market tightness response of a given real exchange rate movement. Note, that restricting s^{fixed} to zero very much restricts the empirical performance of the model.

It is noteworthy, that had we assumed a much lower value of the total vacancy posting costs relative to output, $\frac{\kappa^v v_{ss} + q_{ss}\bar{\kappa}}{y^h_{ss}}$, the estimated value δ^{aob} would have been further away from zero, but s^{fixed} would have been close to zero. In the case of low vacancy posting costs, the implied value of κ^v might be so small — even for $s^{fixed} = 0$ — that the amplification of labour market tightness is too big relative to the data. In this case, the coefficient $\frac{\beta_3 mpl_{ss}}{\zeta \frac{\kappa^v}{q_{ss}}}$ needs to be lowered by reducing β_3, i.e., allowing for a larger value of δ^{aob}. Although in this case, H_t reenters the analysis and the simplified formula in equation (C.22) no longer applies directly, the intuition just laid out applies.

The sensitivity of the parameter estimates to the calibration of the total vacancy posting costs relative to output, $\frac{\kappa^v v_{ss} + q_{ss}\bar{\kappa}}{y^h_{ss}}$, is unique to the alternating offer bargaining model. In the DMP model a drastic reduction in these costs barely influences the parameter estimates; in particular the DMP model always favours s^{fixed} close to zero.

7 Sensitivity analysis

A key parameter in the transmission mechanism of commodity price shocks in our model is elasticity of the risk premium to the net foreign asset position, ϕ^b in equation (13). This parameter determines the degree to which changes in the net foreign asset position affect the real interest rate. An improvement in the trade balance, as occurs when there is an increase in commodity prices, will affect a reduction in the premium domestic agents face when holding foreign bonds and thus reduce the interest rate applicable to domestic households and firms.

Figure 7 compares our baseline estimated DMP model, where ϕ^b is estimated, to two alternative versions: one where $\phi^b \approx 0$ and one where ϕ^b is very large. The former equates to a model where the interest rate facing households and firms is exogenous and the latter one where there is financial autarky akin to Heathcote and Perri (2002).

With an exogenous, and in our case constant, interest rate a positive commodity price shock only affects household consumption. When ϕ^b is zero, the net foreign asset position does not revert to its pre-shock mean and the increase in commodity prices constitutes a rise in permanent income, leaving the real interest rate unchanged. With a constant real interest rate, there will be no effect on investment or vacancies.[27]

When bond holding costs are such that there is virtually no international trade in bonds, increases in net wealth arising from commodity price movements can not be smoothed across time and the trade remains balanced. In this case, the increase in household wealth is translated directly into an increase in consumption and an associated increase in real activity. Investment, unemployment and vacancies are significantly more volatile than the data, as is the real wage, which moves in the opposite direction to the VAR. For output, consumption and the real exchange rate, the fit is significantly improved relative to our baseline DMP model. This illustrates the role of ϕ^b in the transmission mechanism of wealth-type shocks. ϕ^b moves the economy along a spectrum of trade openness from a permanent income hypothesis model, where wealth shocks have a very limited impact, to a model of financial autarky, where wealth shocks have a more direct effect on the real economy. In contrast to Schmitt-Grohe and Uribe (2003), who only consider supply shocks, it does matter how the unit root in bond holdings is eliminated in small open economy models driven by wealth-type shocks.

Figure 8 examines the role of trade openness and the trade elasticity, θ in equation (40). The dashed blue lines denote the baseline model with a lower trade share than in the baseline model. Here, the share of exports in non-commodity production is 10%, while it is set to 20% in the baseline model. The dashed green lines denote a version of the baseline model where $\theta = 2$. Only the dynamics of the real exchange rate and to a lesser extent those of the real wage are affected in a meaningful way by these changes. A lower degree of trade openness adds volatility to the real exchange rate. The real exchange rate deviates form purchasing power parity in the model due to the presence of consumption home bias. For a small open economy, the less open an economy is to trade, the greater will be the degree of home bias and thus the more volatile will the real exchange rate.

The trade elasticity also affects the dynamics of the real exchange rate. At the limit, when the demand for home-produced goods is infinitely elastic, the relative price of home to foreign-produced goods, the real exchange rate in our case, is constant, because both goods are perfect substitutes for one another. At the other extreme, when goods are near compliments, the any change in relative demand requires a near infinite price response. For very low values of θ the real exchange rate becomes extremely volatile. Raising θ from 0.8 to 2, which implies that imported and home-produced goods are more substitutable, reduces the volatility of the real exchange rate in response to a commodity price induced increase in household wealth. The analysis of Figure 8 suggests that our baseline results are robust to different assumptions regarding the choice of trade openness and θ.

The supply of commodities in the model is assumed to be price inelastic and fixed over time. This assumption helps focus the attention on the transmission mechanism of wealth shocks, but comes at the cost of potentially under predicting the response GDP to commodity price shocks. If the supply of commodities reacts positively to an increase in price, we would expect the model to generate both a direct effect on GDP, through higher supply of commodities, as well as an indirect one working via the labour market that is observed in the model. If commodity production requires labour input, this too would amplify the effects on labour market variables such as unemployment and vacancies posted that the model already predicts for the non-commodity sector. Abstracting from the supply of commodities therefore makes fitting the model to the data somewhat more challenging, but

[27] When household preferences allow for wealth effects on the supply of labour, the increase in consumption is also associated with a decline in hours worked.

helps us to isolate the response of the non-commodity sector to commodity price shocks.

The effects of commodity price shocks are frequently analysed in a modelling framework that includes traded as well as non-traded goods; often in the context of the 'Dutch disease' literature. In these models, the real appreciation that results from a positive commodity price shock also leads to a resource shift away from the traded goods to the non-traded goods sector. Apart from being able to account for this internal resource reallocation effect, adding a non-traded goods sector to our model would not significantly affect our results. Indeed, the appreciation of the real exchange rate in our model already attenuates the effects of the commodity price boom on non-commodity output. In the presence of non-traded goods a positive commodity price shock appreciates the real exchange rate via both the home bias channel, present in our model, and also via the internal real exchange rate channel. Whereas the latter effect adds to the former, it is likely that the increase in real exchange rate volatility from adding a non-traded goods sector is modest and could even drive the model away from the data in terms of the response of GDP.

8 Conclusion

We analyse the effects of commodity price shocks on a set of advanced-economy commodity exporters. A particular focus of our analysis is the labour market. Using a panel VAR framework, we find that a positive commodity price shock leads to an 8 basis point decline in unemployment and a 2% increase in vacancies posted. The same commodity price shock is also associated with an expansion in the private sector components of GDP. Consumption, investment and net trade all increase. The non-commodity terms of trade, the relative price of imported to home-produced goods, appreciates. The impulse responses from the VAR are used as a yardstick against with to assess a number of small open economy models with search and matching frictions in the labour market.

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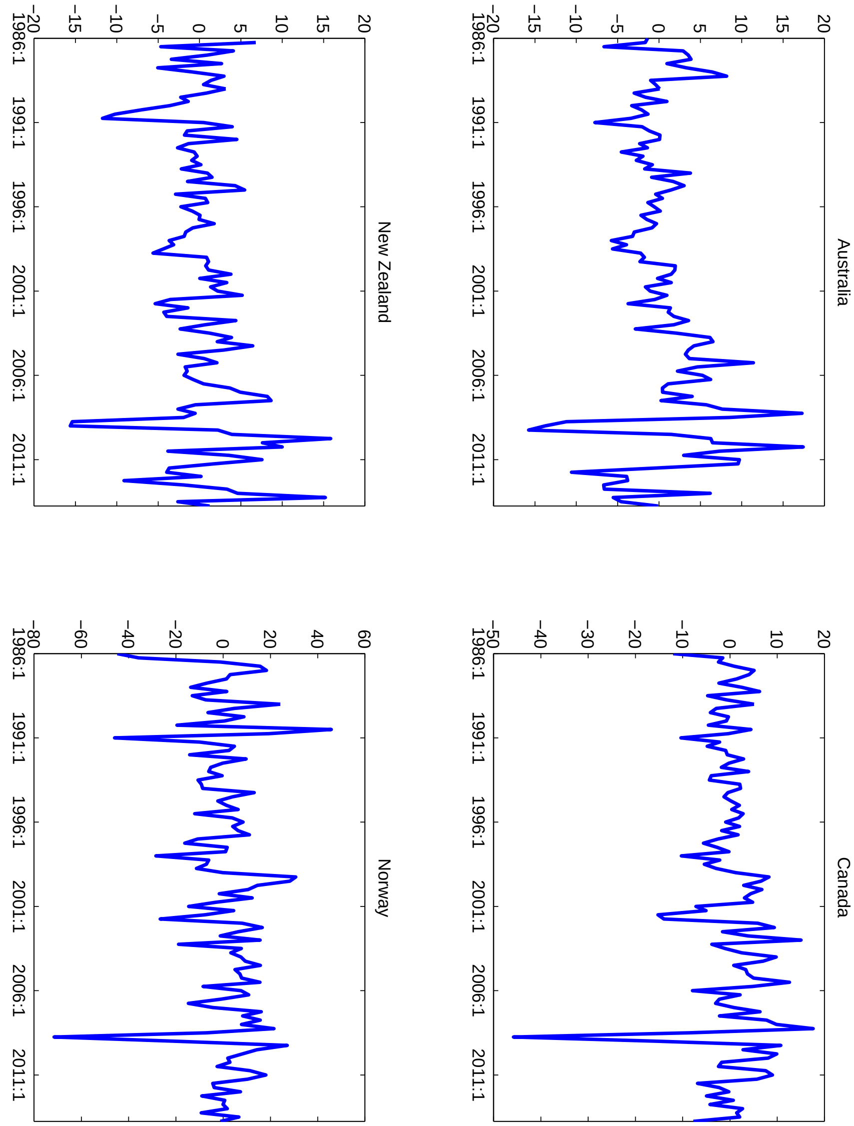
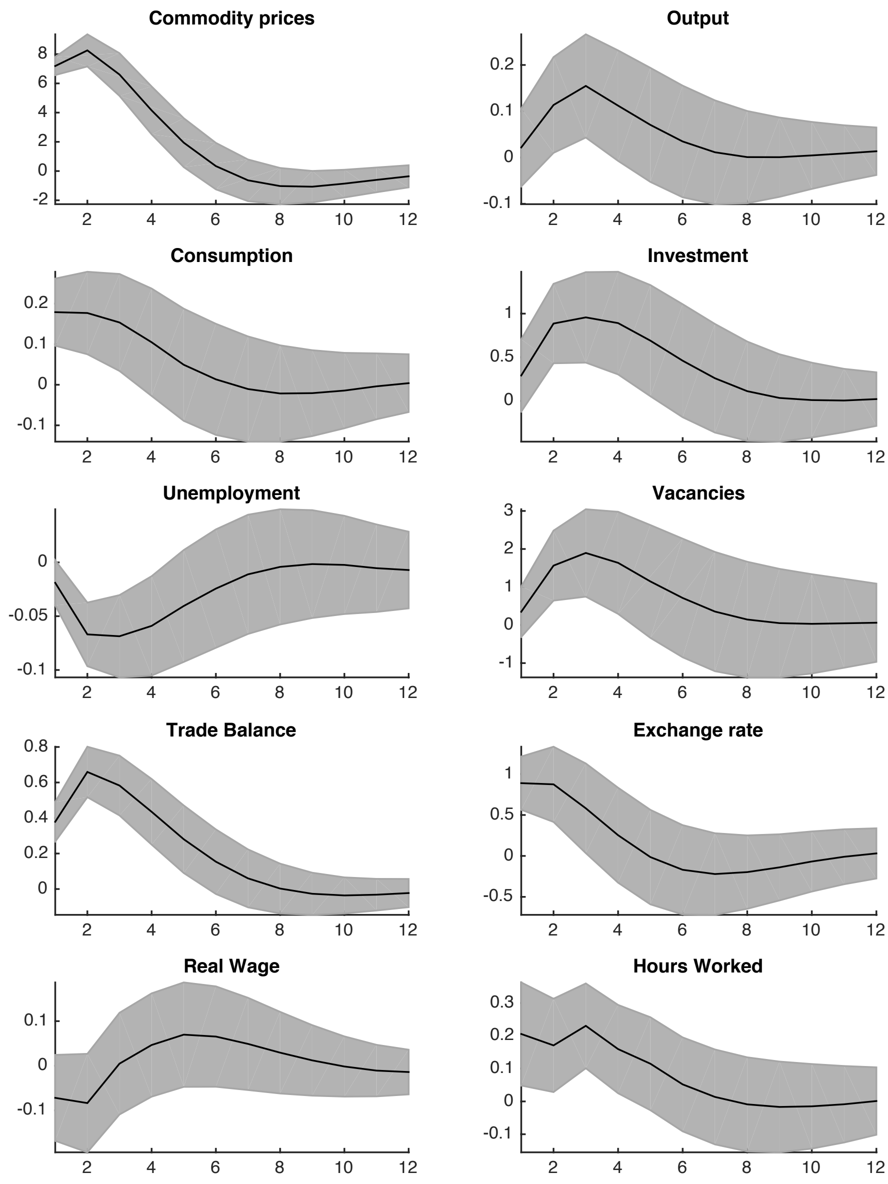
Figure 1: Commodity price indices



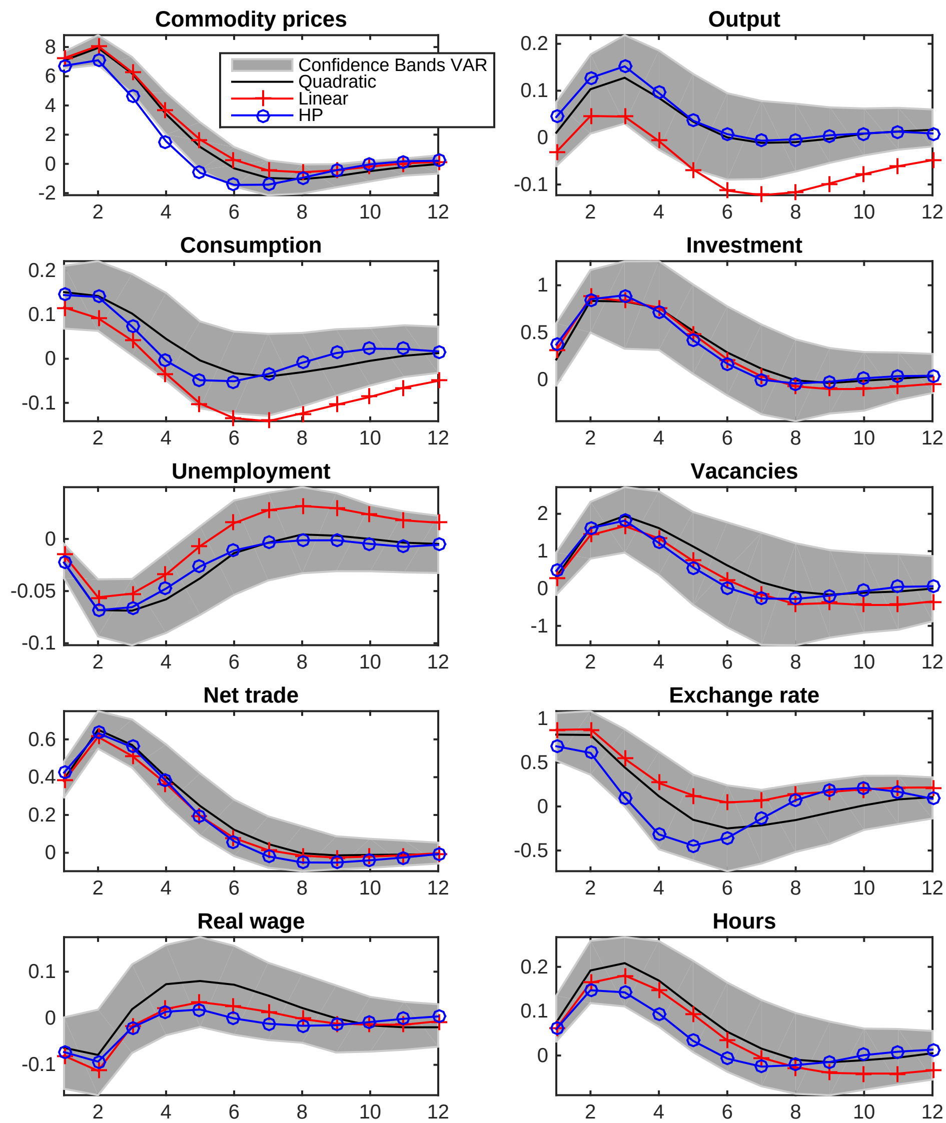
Note: Quarterly percentage change in the trade weighted commodity price index, except for Norway which uses the price of Brent crude.

Figure 2: Estimated impact of a commodity price shock



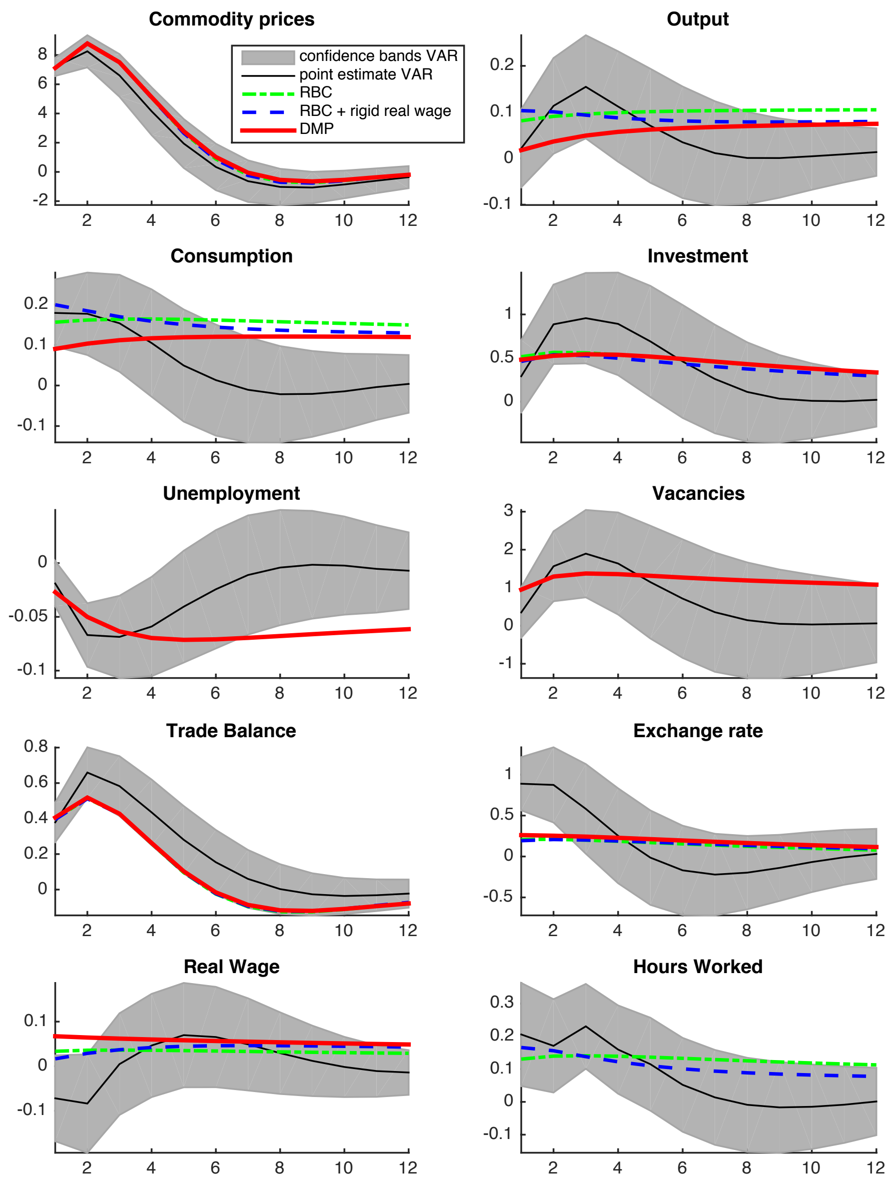
Note: The grey shaded area are the 90% confidence bands, the solid black line shows the median impulse response.

Figure 3: Estimated impact of a commodity price shock - alternative filters



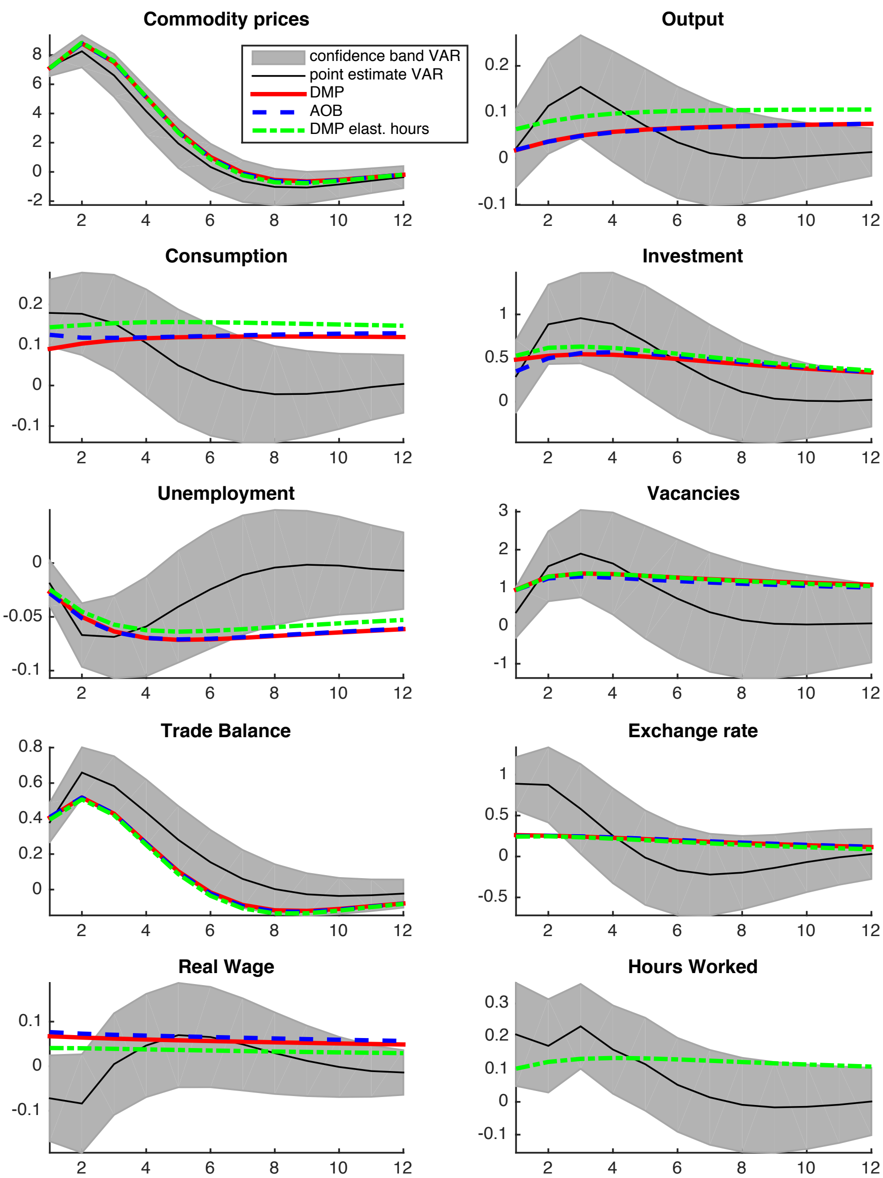
Note: The grey shaded area are the 90% confidence bands for the baseline quadratic de-trended VAR.

Figure 4: RBC and DMP model versus VAR



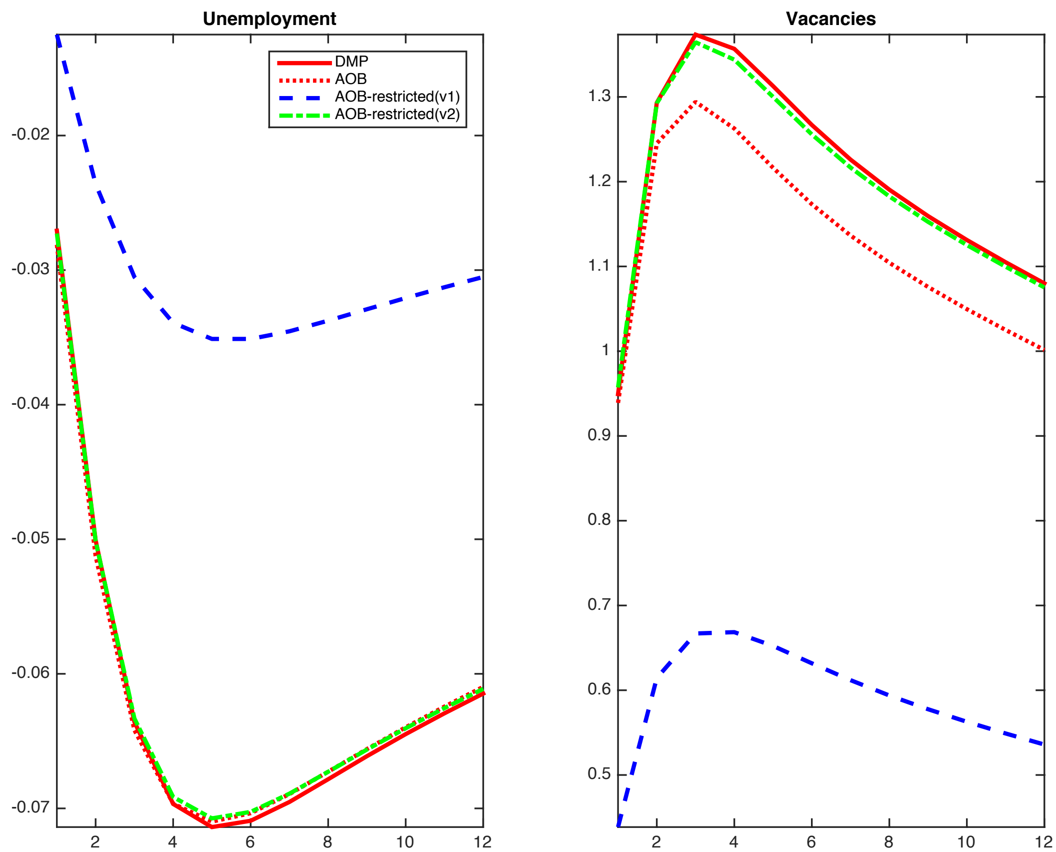
Note: Fitted impulse responses of the RBC model with GHH preferences and with and without rigid real wage, and the DMP model versus the baseline VAR estimates. All variables plotted are considered in minimising the objective (59). The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3. The grey shaded area and the black solid lines show the panel VAR median impulse response along with the 90% confidence bands. The red lines show the fitted model impulse responses.

Figure 5: DMP, AOB, and DMP model with elastic hours worked and versus VAR



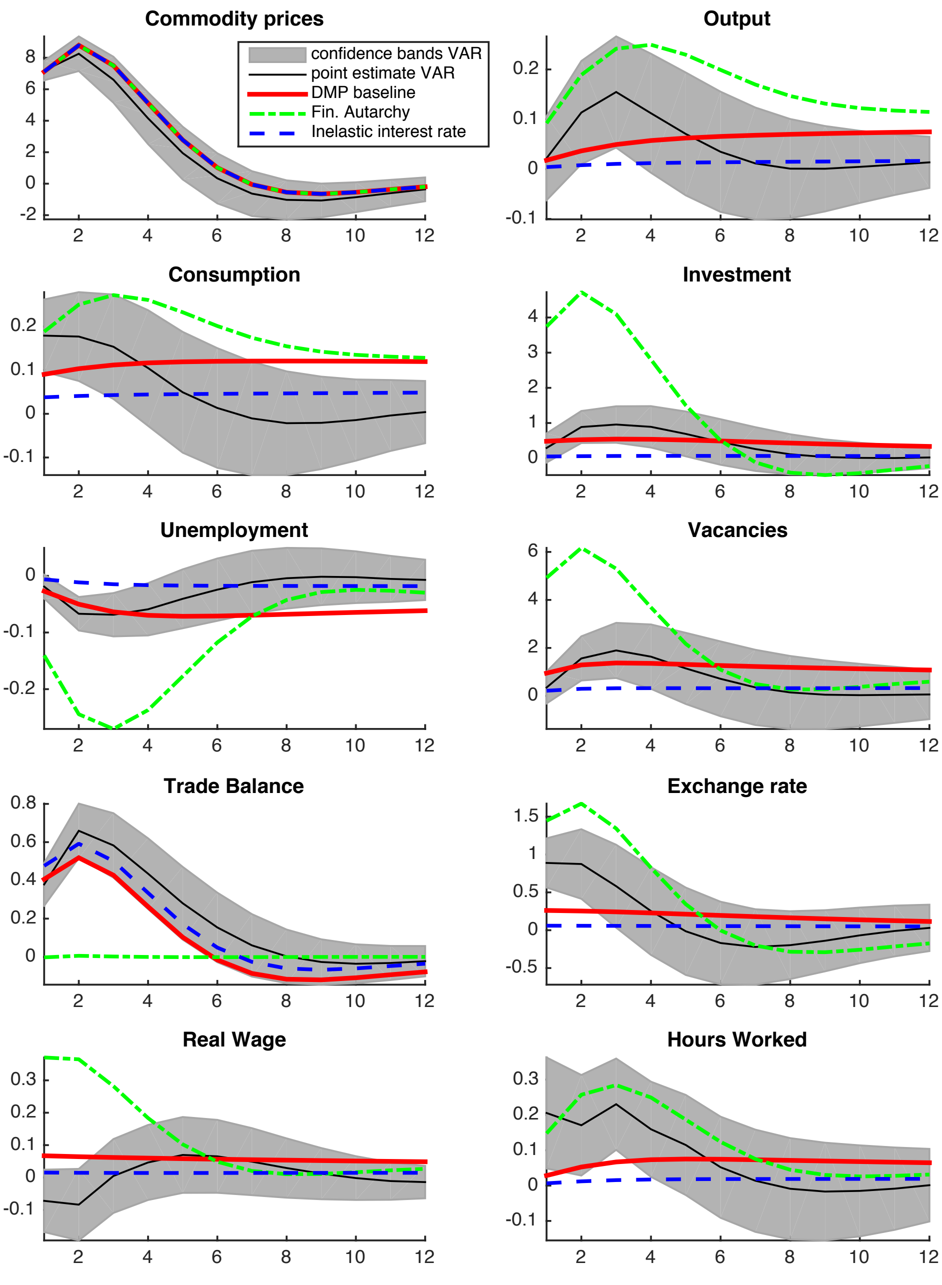
Note: Fitted impulse responses of the DMP model, the Alternating Offer Bargaining model, and the DMP model with elastic hours worked versus the baseline VAR estimates. All variables plotted are considered in minimising the objective (59). The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3. The grey shaded area and the black solid lines show the panel VAR median impulse response along with the 90% confidence bands. The red lines show the fitted model impulse responses.

Figure 6: Restricted estimation of the AOB model



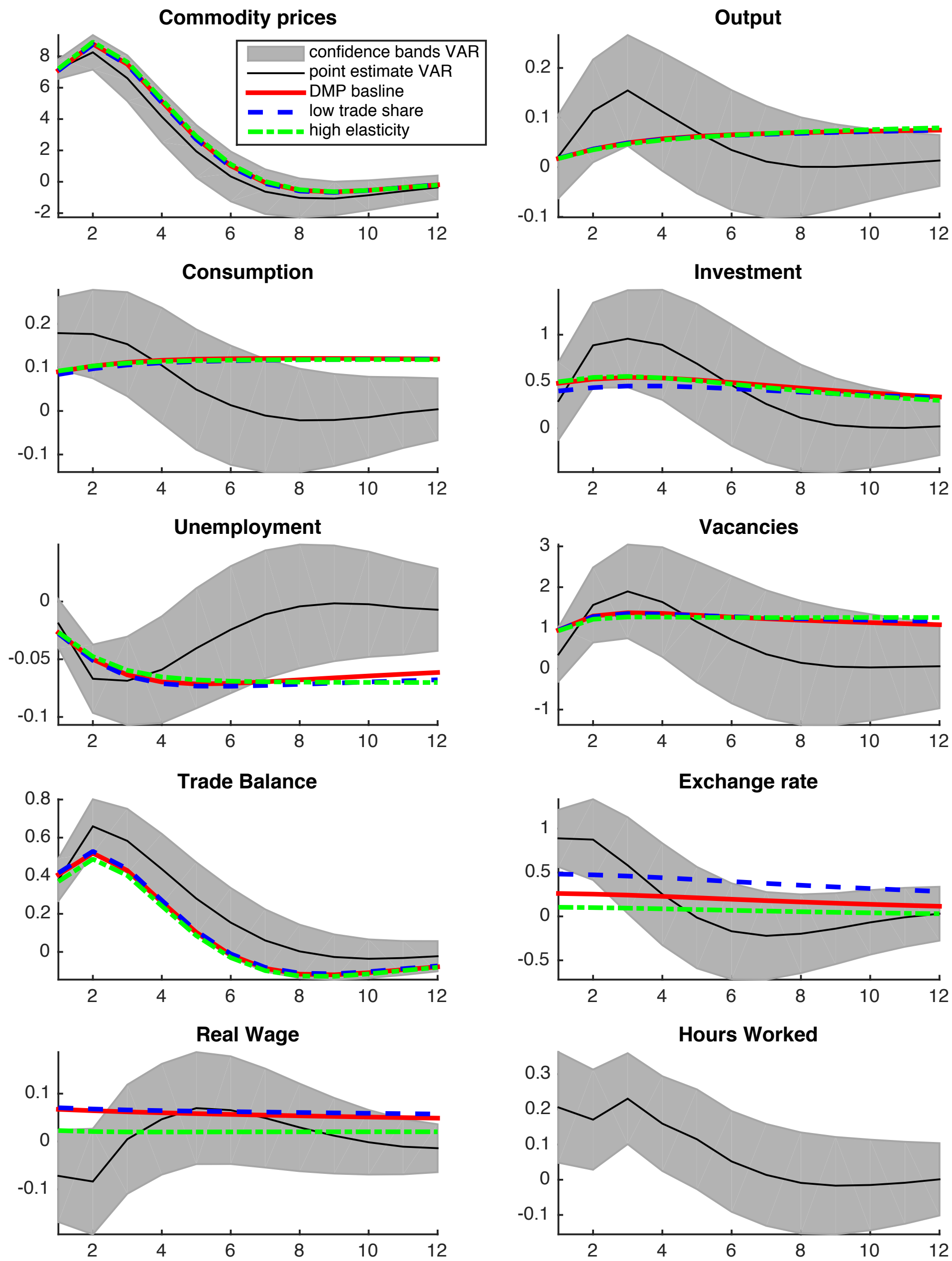
Note: Fitted impulse responses of the DMP model, the Alternating Offer Bargaining model, and the two restricted versions of the AOB model. All variables plotted are considered in minimising the objective (59). The estimated parameter values are provided in Table 4; the values of the remaining model parameters are shown in Table 3.

Figure 7: Financial openness



Note: Financial autarky refers to a calibration of the baseline model where the bond holding cost parameter, ϕ^b is set very high. Inelastic interest rate refers to a calibration of the baseline model where $\phi^b \approx 0$.

Figure 8: Trade openness



Note: Low trade share refers to a calibration where trade constitutes 10% of GDP, as opposed to 30% in the baseline calibration. High trade elasticity refers to a calibration where the trade elasticity, $\theta = 2$ as opposed to 0.8 in the baseline calibration.

Table 1: Share of top three commodity exports

	% of country net exports	% of world net exports
Australia		
Iron ores and concentrates	33	50
Coal; briquettes	18	35
Petroleum gases	8	6
Canada		
Petroleum oils, crude	30	5
Petroleum gases	4	3
Wheat and meslin	4	16
New Zealand		
Milk & cream concentrated	24	37
Lamb meat	8	43
Butter	7	44
Norway		
Petroleum oils, crude	43	3
Petroleum gases	24	8
Fish, excluding fillets	7	62

Notes: Source: Harvard Atlas of Economic Complexity, 2014 data.

Table 2: Volatilities with respect to output

Variable	Australia	Canada	Norway	NZ	Average [Panel]
	Unconditional moments in data [Conditional moments in VAR]				
Price of Commodities	20.9 [65.9]	10.9 [65.7]	15.6 [67.7]	5.1 [73.1]	
Consumption	1.67 [1.12]	0.68 [1.14]	1.22 [1.31]	1.25 [1.27]	
Investment	7.05 [8.29]	3.82 [8.11]	6.01 [7.74]	5.00 [8.97]	
Unemployment rate	0.51 [0.64]	0.41 [0.62]	0.41 [0.61]	0.56 [0.68]	
Vacancies posted	14.1 [18.1]	10.9 [18.6]	16.9 [17.5]	11.6 [20.9]	
Net trade	1.49 [5.69]	0.87 [5.68]	2.10 [5.26]	0.93 [6.21]	
Real Exchange Rate	8.97 [5.65]	3.94 [5.93]	2.07 [6.97]	5.61 [6.33]	
Real wage	2.01 [0.67]	0.91 [0.89]	0.72 [0.98]	0.93 [0.97]	
Total hours worked	1.89 [1.90]	0.57 [1.88]	1.06 [1.80]	1.23 [2.13]	

Notes: Unconditional and [conditional on commodity price shocks] standard deviations relative to GDP.

Table 3: Calibrated parameters

Parameter	Description	DMP	DMP(l)	Models AOB	RBC	RBC(rr)
β	Discount factor			0.99 (all models)		
σ	Curvature of utility function			1.10 (all models)		
θ	Trade elasticity			0.8 (all models)		
α	Share of capital			0.33 (all models)		
δ	Depreciation rate			0.025 (all models)		
v	Home-bias parameter			0.73 (all models)		
$\frac{ex^h_{ss}}{y^h_{ss}}$	Steady state export output ratio			0.2 (all models)		
$\frac{y^c_{ss}}{y^h_{ss}}$	Steady state commodity output ratio			0.075 (all models)		
n_{ss}	Steady state employment	0.95	0.95	0.95	–	–
q_{ss}	Steady state prob. of filling vacancy	0.7	0.7	0.7	–	–
$\frac{\kappa^v v}{y^h}$	Share of vacancy cost in GDP	0.005	0.005	0.005	–	–
ρ	Probability that match breaks up	0.1	0.1	0.1	–	–
b^u/w	Replacement ratio	0.4	0.4	0.4	–	–
χ	Scale parameter in matching function	0.67	0.67	0.67	–	–
ξ	Household's bargaining weight	0.4 [implied]	0.30 [implied]	–	–	–

Notes: DMP = baseline model, DMP(l) = baseline model with elastic labour supply, AOB = alternating offer bargaining model, RBC = Real business cycle model with Walrasian labour markets, RBC(rr) = RBC model with real wage rigidities.

Table 4: Estimated parameters

Parameter	Description	DMP	DMP(l)	Models AOB	RBC	RBC(rr)
ϕ^b	Bond holding cost (× 100)	0.78 [0.27]	0.94 [0.33]	0.82 [0.36]	0.72 [0.25]	0.65 [0.29]
ϕ^v	Cost of vacancies	1.38 [1.46]	1.34 [1.52]	1.02 [3.38]	– –	– –
ϕ	Investment adjustment costs	0 [set]	0.04 [0.15]	0.11 [0.30]	0.03 [0.13]	0.08 [0.17]
ζ	Matching function parameter	0.69 [0.06]	0.73 [0.06]	0.68 [0.06]	– –	– –
δ^{aob}	AOB parameter (× 100)	– –	– –	0.01 [2.68]	– –	– –
c^u/c^w	Consumption share of the unemployed	0.54 [0.01]	0.53 [0.01]	– –	– –	– –
$\bar{\kappa}$	Fixed cost of bargaining	0.01 [1.89]	0.01 [1.92]	0.57 [3.05]	– –	– –
ϕ	Inverse of labour supply elasticity	– –	0.74 [implied]	– –	0.26 [0.14]	0.51 [0.42]
η	Real wage rigidity	– –	– –	– –	– –	0.80 [0.24]

Notes: DMP = baseline model, DMP(l) = baseline model with elastic labour supply, AOB = alternating offer bargaining model, RBC = Real business cycle model with Walrasian labour markets, RBC(rr) = RBC model with real wage rigidities. Numbers in brackets are standard errors.

A Additional details for econometric analysis

A.1 Data description and definitions

Table 5 lists the raw data used in the VARs. All series are taken from Haver Analytics, and the data set is available by request from the authors. For each country in the panel, ten time series are used, which are transformations of the raw data:

$$\begin{aligned}
pcor &= \ln\left(\frac{COM}{\text{US CPI}}\right) && \text{(A.1)}\\
y &= \ln\left(\frac{GDP}{POP}\right) && \text{(A.2)}\\
c &= \ln\left(\frac{C}{POP}\right) && \text{(A.3)}\\
i &= \ln\left(\frac{I}{POP}\right) && \text{(A.4)}\\
nxm &= \left(\frac{NXE}{GDP(L)}\right) && \text{(A.5)}\\
urate &= \left(\frac{UNE}{LAF}\right) && \text{(A.6)}\\
vac &= \ln(VAC) && \text{(A.7)}\\
fxr &= \ln(REER) && \text{(A.8)}\\
wprr &= \ln\left(\frac{WAGE}{CPI}\right) && \text{(A.9)}\\
thp &= \ln\left(\frac{\text{Total HRS}}{POP}\right) && \text{(A.10)}
\end{aligned}$$

Table 5: Data definitions

	Data sources			
Raw Data	Australia	Canada	New Zealand	Norway
GDP	GDP (SA, Mil.Chn.A$)	GDP (SA, Mil.Chn. C$)	GDP (SA, Mil.Chn.NZ$)	GDP (SA, Mil.Chn.NOK)
GDP(L)	GDP (SA, Mil.A$)	GDP (SA, Mil.C$)	GDP (SA, Mil.NZ$)	GDP(SA, Mil.Kroner)
C	HHs Cons. Exp. (SA, Mil.Chn.A$)	Personal Cons. Exp. (SA, Mil.Chn.C$)	Priv. Cons. Exp. (SA, Mil.Chn.NZ$)	Priv. Cons. Exp. (SA, Mil.Chn.NOK)
I	Priv Fixed Cap. Form. (SA, Mil.Chn.A$)	Bus. Fixed Inv. (SA, Mil.Chn.C$)	Gross Fixed Cap. Form. (SA, Mil.Chn.NZ$)	Priv. Fixed Cap. Form. (SA, Mil.Chn.NOK)
NEX	Net Exports of Goods and Serv. (SA, Mil.A$)	Net Exports of Goods and Serv. (SA, Mil.C$)	Net Exports of Goods and Serv. (SA, Mil.NZ$)	Net Exports of Goods and Serv. (SA, Mil.Kroner)
UNE	Unemp.: All Pers. 15 and Over (SA, 1000s)	All Pers. 15 and Over (SA, 1000s)	All Pers. 15 and Over (SA, 1000s)	Registered Unemp. (SA, EOP, Number)
LAF	Labor Force: All Pers 15 and Over (SA, 1000s)	Labor Force: All Pers 15 and Over (SA, 1000s)	Labor Force: All Pers 15 and Over (SA, 1000s)	Labor Force (SA, 1000s)
VAC	Unfilled Job Vacancies (SA, Number)	Vacancies (A. Tasci) (SA)	ANZ Job Ads (SA)	Unfilled Job Vacancies (SA, Number)
REER	Real Effective Exchange Rate	Real Effective Exchange Rate	Real Effective Exchange Rate	Real Effective Exchange Rate
WAGE	Early Est Labor Comp	Same as Australia	Same as Australia	Same as Australia
	Per Unit Labor Input: Tot Econ (SA)			
Total HRS	Total hours (SA)	Total hours(SA)	Total hours (SA)	Total hours (SA)
COM	RBA Commodity Prices: All Items: US$ (NSA)	BoC Commodity Price Index	ANZ World Commodity Price Index (US$, NSA)	Brent Crude Oil (US$ / Barrel)
POP	Population Persons (SA)	Population Persons (SA)	Population Persons (SA)	Population Persons (SA)
CPI	CPI: all items exlc Food & Energy (NSA)	Consumer Price Index (NSA)	Consumer Price Index (NSA)	European Harmonized CPI (NSA)
US CPI	US CPI (SA)	US CPI (SA)	US CPI (SA)	US CPI (SA)

Notes: Source: All data are taken from Haver Analytics.

B Model equations

We display the model equations and steady state relationships for the search and matching model with fixed labour supply. We also discuss the relevant changes to obtain a model with elastic labour supply both in a search and matching framework and a Walrasian labour market.

B.1 Model equations in levels

Collecting relevant equations for numerical implementation:

- households:

$$
\begin{aligned}
U_c(c_t^w) &= \lambda_t && \text{(B.1)}\\
U_c(c_t^u) &= \left(1+\frac{1}{1-n_t}g(n_t,\omega)\right)U_c(c_t^w) && \text{(B.2)}\\
c_t &= n_t c_t^w + (1-n_t)c_t^u && \text{(B.3)}\\
H_t &= \frac{U(c_t^w)-U(c_t^u)}{\lambda_t} + \left(1-\frac{n_t\frac{\partial g(n_t,\omega)}{\partial n_t}}{1+g(n_t,\omega)}\right)[(w_t-b^u)-(c_t^w-c_t^u)]\\
&+ (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right]. && \text{(B.4)}
\end{aligned}
$$

The function $g(n_t,\omega)$ measures the costs of transferring resources from the employed to the unemployed household members. If risk sharing is complete $g(n_t,\omega)=0$ for all n_t; otherwise if $g(n_t,\omega)>0$, risk sharing is incomplete.

We consider a flexible formulation of individual preferences of the form $U(c^i)=\frac{\left(c^i-\frac{\phi_0}{1+\phi}\left(h^i\right)^{1+\phi}\right)^{1-\sigma}}{1-\sigma}$ with $h^w=1$ and $h^u=0$, and $\Phi=\frac{\phi_0}{1+\phi}\left(h^w\right)^{1+\phi}$ similar to Greenwood et al. (1988).[28] By setting $\phi_0=0$, i.e. $\Phi=0$, this specification reduces to CRRA preferences. Employing these assumptions, equations (B.1) - (B.4) imply:

1. the first order conditions for consumption imply

$$
\frac{c_t^w}{c_t^u} = \left(1+\frac{g(n_t,\omega)}{1-n_t}\right)^{\frac{1}{\sigma}} + \left(n_t\left(\frac{c_t^w}{c_t^u}\right)+(1-n_t)\right)\frac{\Phi}{c_t} \tag{B.5}
$$

2. the marginal utility of wealth follows

$$
\lambda_t = c_t^{-\sigma}\frac{\left(n_t\left(\frac{c_t^w}{c_t^u}\right)+(1-n_t)\right)^{\sigma}}{\left(1+\frac{1}{1-n_t}g(n_t,\omega)\right)} \tag{B.6}
$$

3. since the consumption difference is given by

$$
c_t^w - c_t^u = \frac{\left(\frac{c_t^w}{c_t^u}\right)-1}{n_t\left(\frac{c_t^w}{c_t^u}\right)+(1-n_t)}c_t
$$

[28] Albeit using a difference specification of preferences, Hall and Milgrom (2008) also assume that the marginal utility of consumption depends on hours worked, where hours worked are fixed.

and the utility difference follows

$$\frac{U(c_t^w)-U(c_t^u)}{\lambda_t} = -\frac{1}{1-\sigma}\Phi + \frac{1}{1-\sigma}\left(\frac{c_t^w}{c_t^u} - \left(1+\frac{g(n_t,\omega)}{1-n_t}\right)\right)c_t^u.$$

the marginal value of employment can be written as

$$\begin{aligned} H_t &= F\left(\frac{c_t^w}{c_t^u}, n_t, c_t; \omega, \Phi\right) + \left(1 - \frac{n_t \frac{\partial g(n_t,\omega)}{\partial n_t}}{1+g(n_t,\omega)}\right)(w_t - b^u) \\ &+ (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right] \qquad \text{(B.7)} \\ &= F\left(\frac{c_t^w}{c_t^u}, n_t, c_t; \omega, \Phi\right) - \left(\frac{n_t \frac{\partial g(n_t,\omega)}{\partial n_t}}{1+g(n_t,\omega)}\right)(w_t - b^u) \\ &+ (w_t - b^u) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right] \qquad \text{(B.8)} \end{aligned}$$

where

$$\begin{aligned} & F\left(\frac{c_t^w}{c_t^u}, n_t, c_t; \omega, \Phi\right) = -\frac{\Phi}{1-\sigma} \\ & + \frac{\frac{1}{1-\sigma}\left(\frac{c_t^w}{c_t^u} - \left(1+\frac{g(n_t,\omega)}{1-n_t}\right)\right) - \left(1 - \frac{n_t\frac{\partial g(n_t,\omega)}{\partial n_t}}{1+g(n_t,\omega)}\right)\left(\frac{c_t^w}{c_t^u} - 1\right)}{n_t\frac{c_t^w}{c_t^u} + 1 - n_t} c_t \end{aligned} \qquad \text{(B.9)}$$

Under complete risk sharing – $g(n_t,\omega) = 0$ and $\frac{\partial g(n_t,\omega)}{\partial n_t} = 0$ for all n_t – even simpler expressions prevail:

1. CRRA utility:
   (a) $c_t = c_t^w = c_t^u$
   (b) $\lambda_t = c_t^{-\sigma}$
   (c) $H_t = (w_t - b^u) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right]$.
2. GHH utility:
   (a) $c_t^w - c_t^u = \Phi = \Delta\bar{c}$
   (b) $\lambda_t = (c_t - n_t\Delta\bar{c})^{-\sigma}$
   (c) $H_t = -\Delta\bar{c} + (w_t - b^u) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right]$.

   GHH preferences consumption inequality emerges even if risk sharing is complete.

- firms:

$$\begin{aligned}
y_t^h &= a_t k_{t-1}^{\alpha} n_t^{1-\alpha} && \text{(B.10)}\\
k_t &= (1-\delta)k_{t-1} + \iota(x_t, x_{t-1}) && \text{(B.11)}\\
tq_t &= E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\alpha\frac{p_{t+1}^h y_{t+1}^h}{k_t} + (1-\delta)tq_{t+1}\right)\right] && \text{(B.12)}\\
1 &= tq_t\frac{\partial\iota(x_t, x_{t-1})}{\partial x_t} + E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}tq_{t+1}\frac{\partial\iota(x_{t+1}, x_t)}{\partial x_t}\right] && \text{(B.13)}\\
q_t\left(J_t - \bar{\kappa}\right) &= \frac{\partial\kappa(v_t, v_{t-1})}{\partial v_t} + E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\kappa(v_{t+1}, v_t)}{\partial v_t}\right] && \text{(B.14)}\\
J_t &= \left((1-\alpha)\frac{p_t^h y_t^h}{n_t} - w_t\right) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}J_{t+1}\right] && \text{(B.15)}\\
\iota(x_t, x_t - 1) &= \kappa^x x_t\left(1 - \frac{\phi^x}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right) && \text{(B.16)}\\
\frac{\partial\iota(x_t, x_{t-1})}{\partial x_t} &= \frac{\iota(x_t, x_{t-1})}{x_t} - \kappa^x\phi^x\left(\frac{x_t}{x_{t-1}} - 1\right)\frac{x_t}{x_{t-1}} && \text{(B.17)}\\
\frac{\partial\iota(x_t, x_{t-1})}{\partial x_{t-1}} &= \kappa^x\phi^x\left(\frac{x_t}{x_{t-1}} - 1\right)\left(\frac{x_t}{x_{t-1}}\right)^2 && \text{(B.18)}\\
\kappa(v_t, v_{t-1}) &= \kappa^v v_t\left(1 + \frac{\phi^v}{2}\left(\frac{v_t}{v_{t-1}} - 1\right)^2\right) && \text{(B.19)}\\
\frac{\partial\kappa(v_t, v_{t-1})}{\partial v_t} &= \frac{\kappa(v_t, v_{t-1})}{v_t} + \kappa^v\phi^v\left(\frac{v_t}{v_{t-1}} - 1\right)\frac{v_t}{v_{t-1}} && \text{(B.20)}\\
\frac{\partial\kappa(v_t, v_{t-1})}{\partial v_{t-1}} &= -\kappa^v\phi^v\left(\frac{v_t}{v_{t-1}} - 1\right)\left(\frac{v_t}{v_{t-1}}\right)^2 && \text{(B.21)}
\end{aligned}$$

If $\bar{\kappa} = 0$ and $\phi^v = 0$, the equations reduce to their standard formulation.

- trade in goods and assets:

$$\begin{aligned}
\frac{1}{1+r_t} &= E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{rer_{t+1}}{rer_t}\right] && \text{(B.22)}\\
v\left(p_t^h\right)^{1-\theta} &= 1 - (1-v)\left(rer_t\right)^{1-\theta} && \text{(B.23)}\\
y_t^h &= v\left(p_t^h\right)^{-\theta}\left(c_t + x_t + \bar{\kappa}q_t + \kappa\left(v_t, v_{t-1}\right)\right) + ex_t^h && \text{(B.24)}\\
ex_t^h &= v^*\left(\frac{rer_t}{p_t^h}\right)^{\theta^*} y_t^* && \text{(B.25)}\\
tbal_t &= 1 - \frac{c_t + x_t + \bar{\kappa}q_t + \kappa(v_t, v_{t-1})}{gdp_t}\\
&- \frac{1}{gdp_t}\frac{n_t g(n_t, \omega)}{1 + g(n_t, \omega)}\left(w_t - b^u - \frac{\left(\frac{c_t^w}{c_t^u}\right) - 1}{n_t\left(\frac{c_t^w}{c_t^u}\right) + (1-n_t)}c_t\right) && \text{(B.26)}\\
\tilde{b}_t &= (1 + r_{t-1})\frac{gdp_{t-1}}{gdp_t}\frac{rer_t}{rer_{t-1}}\tilde{b}_{t-1} + tbal_t && \text{(B.27)}\\
1 + r_t &= (1 + r_t^*)\, e^{-\phi^b\left(\tilde{b}_t - \bar{b}\right)} && \text{(B.28)}\\
gdp_t &= rer_t p_t^{c*} y_t^c + p_t^h y_t^h. && \text{(B.29)}
\end{aligned}$$

where $tbal_t$ is the trade balance to GDP ratio. Net foreign assets relative to GDP, $\tilde{b}_t$, are given by $\tilde{b}_t = \frac{rer_t b_t}{gdp_t}$.

- search and matching:

$$
\begin{aligned}
m_t &= \chi u_t^{\zeta} v_t^{1-\zeta} && \text{(B.30)}\\
n_t &= (1-\rho)n_{t-1} + m_t && \text{(B.31)}\\
u_t &= 1-(1-\rho)n_{t-1} && \text{(B.32)}\\
\tilde{u}_t &= 1-n_t && \text{(B.33)}\\
\theta_t &= \frac{v_t}{u_t} && \text{(B.34)}\\
s_t &= \chi\theta_t^{1-\zeta} && \text{(B.35)}\\
q_t &= \chi\theta_t^{-\zeta} && \text{(B.36)}
\end{aligned}
$$

- exogenous variables: $a_t, p_t^{c*}, y_t^c, r_t^*, y_t^*$.

The final equation missing from this set of equations is the surplus sharing rule. Under Nash bargaining, the surplus is shared according to:

$$J_t = \frac{1-\xi}{\xi} H_t. \tag{B.37}$$

Under alternating offer bargaining

$$J_t = \beta_1 H_t - \beta_2 \gamma^{aob} + \beta_3 \left((1-\alpha)\frac{p_t^h y_t^h}{n_t} - b^u \right). \tag{B.38}$$

In the case of complete international financial markets, the last two equations in the trade bloc are replaced by

$$\lambda_t = \left[\frac{rer_0 \lambda_0}{(y_0^*)^{-\sigma^*}} \right] \frac{1}{rer_t} (y_t^*)^{-\sigma^*}. \tag{B.39}$$

$\tilde{b}_t$ is removed from the model.

B.2 Steady state and calibration

In our calibration strategy, we fix targets for the steady state values of some of the endogenous variables: q_{ss}, $n_{ss}, \frac{y_{ss}^c}{y_{ss}^h}$, $\frac{ex_{ss}^h}{gdp_{ss}}, \frac{b^u}{w_{ss}}$, $\kappa^{comp} = \frac{\bar{\kappa}q_{ss}+\kappa^v v_{ss}}{gdp_{ss}}, \overline{\frac{\bar{\kappa}q_{ss}}{\kappa^{comp} y_{ss}^h}}$, a_{ss}, rer_{ss}, p_{ss}^h, y_{ss}^*, $p_s^{c*}s$, $\frac{c_{ss}^u}{c_{ss}^w}$, ω.

In terms of parameters, we choose the values of the following ones: β, δ, α, σ, ρ, ζ, κ^x, $\bar{b}$, θ, θ^*, ϕ^x, ϕ^y, ϕ^b. The remaining parameters are determined by the steady state relationships: χ, v, v^*, b^u, ξ, Φ. In the alternating offer model, we add M^{aob}, δ^{aob} to the set of externally specified parameters. γ^{aob} is determined by the surplus sharing rule (instead of ξ).

The following relationships are obtained. From the cost functions:

$$\iota(x_{ss}, x_{ss}) = \kappa^x x_{ss} \tag{B.40}$$

$$\frac{\partial \iota(x_t, x_{t-1})}{\partial x_t}|_{ss} = \kappa^x \tag{B.41}$$

$$\frac{\partial \iota(x_t, x_{t-1})}{\partial x_{t-1}}|_{ss} = 0 \tag{B.42}$$

$$\kappa(v_{ss}, v_{ss}) = \kappa^v v_{ss} \tag{B.43}$$

$$\frac{\partial \kappa(v_t, v_{t-1})}{\partial v_t}|_{ss} = \kappa^v \tag{B.44}$$

$$\frac{\partial \kappa(v_t, v_{t-1})}{\partial v_{t-1}}|_{ss} = 0. \tag{B.45}$$

From the labour market conditions:

$$n_{ss} = n_{ss} \tag{B.46}$$

$$q_{ss} = q_{ss} \tag{B.47}$$

$$u_{ss} = 1 - (1-\rho)n_{ss} \tag{B.48}$$

$$\tilde{u}_{ss} = 1 - n_{ss} \tag{B.49}$$

$$m_{ss} = n_{ss} - (1-\rho)n_{ss} \tag{B.50}$$

$$v_{ss} = \frac{m_{ss}}{q_{ss}} \tag{B.51}$$

$$s_{ss} = \frac{m_{ss}}{u_{ss}} \tag{B.52}$$

$$\theta_{ss} = \frac{v_{ss}}{u_{ss}} \tag{B.53}$$

$$\chi = \frac{m_{ss}}{u_{ss}^{\zeta} v_{ss}^{1-\zeta}}. \tag{B.54}$$

From the firms' side:

$$
\begin{aligned}
a_{ss} &= 1 && \text{(B.55)} \\
tq_{ss} &= \frac{1}{\kappa^x} && \text{(B.56)} \\
y^h_{ss} &= \left(\left(\frac{1}{\beta} - (1-\delta) \right) \frac{tq_{ss}}{\alpha} \right)^{-\frac{\alpha}{1-\alpha}} n_{ss} && \text{(B.57)} \\
k_{ss} &= \frac{y^h_{ss}}{\left(\frac{1}{\beta} - (1-\delta) \right) \frac{tq_{ss}}{\alpha}} && \text{(B.58)} \\
x_{ss} &= \frac{\delta}{\kappa^x} k_{ss} && \text{(B.59)} \\
\bar{\kappa} &= \overline{\frac{\bar{\kappa} q_{ss}}{\kappa^{comp} y^h_{ss}}} \frac{\kappa^{comp} y^h_{ss}}{q_{ss}} && \text{(B.60)} \\
\kappa^v &= \frac{\kappa^{comp} y^h_{ss} - \bar{\kappa} q_{ss}}{v_{ss}} && \text{(B.61)} \\
J_{ss} &= \frac{\kappa^v}{q_{ss}} + \bar{\kappa} && \text{(B.62)} \\
w_{ss} &= (1-\alpha) \frac{y^h_{ss}}{n_{ss}} - (1 - (1-\rho)\beta) J_{ss}. && \text{(B.63)}
\end{aligned}
$$

From the international trade relations:

$$
\begin{aligned}
r_{ss} &= \frac{1}{\beta} - 1 && \text{(B.64)} \\
rer_{ss} &= 1 && \text{(B.65)} \\
p^h_{ss} &= 1 && \text{(B.66)} \\
r^*_{ss} &= r_{ss} && \text{(B.67)} \\
\tilde{b}_{ss} &= \bar{b} && \text{(B.68)} \\
tbal_{ss} &= \tilde{b}_{ss} - (1 + r_{ss}) \tilde{b}_{ss} && \text{(B.69)} \\
p^{c*}_{ss} &= 1 && \text{(B.70)} \\
gdp_{ss} &= \left(\frac{y^c_{ss}}{y^h_{ss}} + 1 \right) y^h_{ss} && \text{(B.71)} \\
v^* &= \frac{ex^h_{ss}}{y^*_{ss}}. && \text{(B.72)}
\end{aligned}
$$

From the household side:

$$
\begin{aligned}
c_{ss} &= \left[(1-tbal_{ss})\,gdp_{ss} - (x_{ss} + \bar{\kappa} q_{ss} + \kappa^v v_{ss}) - \frac{n_{ss} g(n_{ss},\omega)}{1+g(n_{ss},\omega)}(w_{ss} - b^u)\right] \\
&\quad \left[1 - \left(\frac{n_{ss} g(n_{ss},\omega)}{1+g(n_{ss},\omega)} \frac{\left(\frac{c^w_{ss}}{c^u_{ss}}\right) - 1}{n_{ss}\left(\frac{c^w_{ss}}{c^u_{ss}}\right) + (1-n_{ss})}\right)\right]^{-1} && \text{(B.73)} \\
v &= \frac{y^h_{ss} - ex^h_{ss}}{c_{ss} + x_{ss} + \bar{\kappa} q_{ss} + \kappa^v v_{ss}} && \text{(B.74)} \\
\lambda_{ss} &= c_{ss}^{-\sigma} \frac{\left(n_{ss}\left(\frac{c^w_{ss}}{c^u_{ss}}\right) + (1-n_{ss})\right)^{\sigma}}{\left(1 + \frac{1}{1-n_{ss}} g(n_{ss},\omega)\right)} && \text{(B.75)} \\
F_{ss} &= F\left(\frac{c^w_{ss}}{c^u_{ss}}, n_{ss}, c_{ss}; \omega, \Phi\right) && \text{(B.76)} \\
H_{ss} &= \frac{F_{ss} - \left(\frac{n_{ss}\frac{\partial g(n_{ss},\omega)}{\partial n_{ss}}}{1+g(n_{ss},\omega)}\right)(w_{ss} - b^u) + (w_{ss} - b^u)}{1 - (1-\rho)\beta(1-s_{ss})}. && \text{(B.77)}
\end{aligned}
$$

The bargaining sharing rule determines the bargaining weight:

$$\xi = \frac{H_{ss}}{J_{ss} + H_{ss}}. \tag{B.78}$$

An alternative approach is to fix the bargaining weight and then compute the replacement ratio. We use the bargaining sharing rule to solve for H_{ss} and then use the equation defining H_{ss} to solve for b^u.

Finally, the calibration of Φ and ω are linked given the choice of $\frac{c^w_{ss}}{c^u_{ss}}$. If $\Phi = 0$:

$$\frac{g(n_{ss},\omega)}{1-n_{ss}} = \left(\frac{c^w_{ss}}{c^u_{ss}}\right)^{\sigma} - 1 \tag{B.79}$$

and

$$\Phi = \frac{\frac{c^w_{ss}}{c^u_{ss}} - \left(1 + \frac{g(n_{ss},\omega)}{1-n_{ss}}\right)^{\frac{1}{\sigma}}}{n_{ss}\left(\frac{c^w_{ss}}{c^u_{ss}}\right) + (1-n_{ss})} c_{ss} \tag{B.80}$$

alternatively.

Under alternating offer bargaining the additional restrictions are needed:

$$\begin{aligned}
\alpha_1 &= 1-\delta^{aob}+(1-\delta^{aob})^{M^{aob}} && \text{(B.81)}\\
\alpha_2 &= 1-(1-\delta^{aob})^{M^{aob}} && \text{(B.82)}\\
\alpha_3 &= \alpha_2\frac{1-\delta^{aob}}{\delta^{aob}}-\alpha_1 && \text{(B.83)}\\
\alpha_4 &= \frac{1-\delta^{aob}}{2-\delta^{aob}}\frac{\alpha_2}{M^{aob}}+1-\alpha_2 && \text{(B.84)}\\
\beta_1 &= \alpha_2\alpha_1 && \text{(B.85)}\\
\beta_2 &= \alpha_3\alpha_1 && \text{(B.86)}\\
\beta_3 &= \alpha_4\alpha_1 && \text{(B.87)}\\
\gamma^{aob} &= \frac{1}{\beta_2}\left(\beta_1 H_{ss}-J_{ss}+\beta_3\left((1-\alpha)\frac{y^h_{ss}}{n_{ss}}-b^u\right)\right). && \text{(B.88)}
\end{aligned}$$

B.3 Search and matching model with elastic labour supply

The overwhelming share of the labour search literature assumes that labour is supplied inelastically. By contrast models without search and matching frictions always model the labour supply as elastic. Here we consider the case of elastic labour supply in the search and matching framework under the assumption of complete risk sharing among household members.

If the labour supply is elastic, the marginal value of employment to the household is given by:

$$H_t = \frac{U(c^w_t)-U(c^u_t)}{\lambda_t}+w_t h^w_t-b^u-(c^w_t-c^u_t)+(1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}H_{t+1}(1-s_{t+1})\right]. \tag{B.89}$$

We replace the marginal value of employment to the firms by:

$$J_t = \left((1-\alpha)\frac{p^h_t y^h_t}{n_t h^w_t}h^w_t - w_t h^w_t\right)+(1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}J_{t+1}\right] \tag{B.90}$$

and the production function by:

$$y^h_t = a_t k^{\alpha}_{t-1}(n_t h^w_t)^{1-\alpha}. \tag{B.91}$$

Nash bargaining occurs over wages and hours worked, yielding the conditions:

$$\begin{aligned}
J_t &= \frac{1-\xi}{\xi}H_t && \text{(B.92)}\\
-\frac{U^w_h(c^w_t,h^w_t)}{\lambda_t} &= (1-\alpha)\frac{p^h_t y^h_t}{n_t h^w_t}. && \text{(B.93)}
\end{aligned}$$

In deriving equation (B.93), we assume that the household member and the firm take λ_t and the marginal product of the firm $\frac{p^h_t y^h_t}{n_t h^w_t}$ as given in negotiating over hours worked as in Cheron and Langot (2004). Under this assumption the marginal rate of substitution between labour and consumption equals the marginal product of labour.

Under the assumption of GHH preferences and no labour effort by unemployed households, full risk sharing implies:

$$\begin{aligned} c_t^w - c_t^u &= \frac{\phi_0}{1+\phi} (h_t^w)^{1+\phi} \\ \frac{U(c_t^w) - U(c_t^u)}{\lambda_t} &= \frac{1}{1-\sigma} \left(c_t^w - c_t^u - \frac{\phi_0}{1+\phi} (h_t^w)^{1+\phi} \right) = 0. \end{aligned}$$

Consequently, the following conditions prevail:

$$\frac{c_t^w}{c_t^u} = 1 + \left(n_t \left(\frac{c_t^w}{c_t^u} \right) + (1-n_t) \right) \frac{\frac{\phi_0}{1+\phi} (h_t^w)^{1+\phi}}{c_t} \tag{B.94}$$

$$\lambda_t = c_t^{-\sigma} \left(n_t \left(\frac{c_t^w}{c_t^u} \right) + (1-n_t) \right)^{\sigma} \tag{B.95}$$

$$H_t = w_t h_t^w - b^u - \frac{\phi_0}{1+\phi} (h_t^w)^{1+\phi} + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} H_{t+1} (1 - s_{t+1}) \right]$$

$$\phi_0 (h_t^w)^{\phi} = (1-\alpha) \frac{p_t^h y_t^h}{n_t h_t^w} \tag{B.96}$$

C Simple analytics

C.1 DMP model

This appendix provides background on the simple analytical expressions underlying our discussion in Section 6.2. For simplicity, we abstract from several features:

- the vacancy adjustment costs are set to zero, i.e., $\phi^v = 0$ and thus $\kappa(v_t, v_{t-1}) = \kappa^v v_t$,
- the share of capital in production is zero, i.e., $\alpha = 0$,

Noting that under Nash bargaining the surplus is shared according to $J_t = \frac{1-\xi}{\xi} H_t$, equation (21) can be written as:

$$\xi J_t = (1-\xi)(w_t - b^u - \Phi) + \xi (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} (1 - s_{t+1}) \right] \tag{C.1}$$

where Φ measures the (constant) difference in consumption between employed and unemployed household members $\Phi = c_t^w - c_t^u$. Combining the above expression with condition (34) to eliminate wages:

$$J_t + (1-\xi)(b^u + \Phi) = (1-\xi) mpl_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi s_{t+1}) J_{t+1} \right]. \tag{C.2}$$

Applying the definitions for s_t and q_t in equations (7) and (8), and noting that absent vacancy adjustment costs equation (33) yields the following relationship between J_t and labour market tightness, θ_t:

$$J_t = \frac{\kappa^v}{\chi} \theta_t^{\zeta} + \bar{\kappa} \tag{C.3}$$

equation $(C.2)$ can be written as:

$$\frac{\kappa^v}{\chi}\theta_t^\zeta + [(1-\xi)(b^u+\Phi)+\bar{\kappa}] = (1-\xi)mpl_t + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-\xi\chi\theta_{t+1}^{1-\zeta}\right)\left(\frac{\kappa^v}{\chi}\theta_{t+1}^\zeta+\bar{\kappa}\right)\right]. \tag{C.4}$$

It is immediately apparent from equation (C.4) that unemployment benefits, b^u, and the consumption differential, Φ, enter the model in the same manner. The fixed costs of starting the bargaining process, $\bar{\kappa}$, act similarly, but given its presence in the forward-looking term we have to push the analytics a little bit further to gain a clearer image.

As the marginal product of labour is given by:

$$mpl_t = (1-\alpha)\frac{p_t^h y_t^h}{n_t} = p_t^h a_t \tag{C.5}$$

and from equation (40) the relative price p_t^h relates to the real exchange rate as:

$$1 \quad = \quad v\left(p_t^h\right)^{1-\theta} + (1-v)\left(rer_t\right)^{1-\theta} \tag{C.6}$$

changes in commodity prices affect the marginal product of labour in the open economy through variations in the real exchange rate.

Log-linearizing expression (C.4) around the steady state, delivers

$$\begin{aligned} &\zeta\frac{\kappa^v}{\chi}\theta_{ss}^\zeta\hat{\theta}_t - (1-\rho)\beta\left(\frac{\zeta\kappa^v}{\chi}\theta_{ss}^\zeta - \xi\kappa^v\theta_{ss} - \bar{\kappa}\xi\chi(1-\zeta)\theta_{ss}^{1-\zeta}\right)E_t\hat{\theta}_{t+1} \\ = \quad &(1-\xi)mpl_{ss}\widehat{mpl_t} + (1-\rho)\beta\left(\left(1-\xi\chi\theta_{ss}^{1-\zeta}\right)\left(\frac{\kappa^v}{\chi}\theta_{ss}^\zeta+\bar{\kappa}\right)\right)E_t\left(\hat{\lambda}_{t+1}-\hat{\lambda}_t\right). \end{aligned} \tag{C.7}$$

Given our strategy of keeping the steady state value of the job filling probability identical across calibrations, we make use of the fact $q_{ss} = \chi\theta_{ss}^{-\zeta}$, and we apply the definition of the marginal product of labour in our simplified environment to arrive at:

$$\begin{aligned} &\zeta\frac{\kappa^v}{q_{ss}}\left(\hat{\theta}_t - E_t\hat{\theta}_{t+1}\right) + \left[\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)[\zeta-(1-\rho)\beta(\zeta-q_{ss}\xi\theta_{ss})] - \zeta\bar{\kappa}[1-(1-\rho)\beta(1-q_{ss}\xi\theta_{ss})]\right]E_t\hat{\theta}_{t+1} \\ = \quad &(1-\xi)mpl_{ss}\left(\hat{a}_t - \frac{1-\nu}{\nu}\widehat{rer}_t\right) + (1-\rho)\beta\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)(1-\xi q_{ss}\theta_{ss})E_t\left(\hat{\lambda}_{t+1}-\hat{\lambda}_t\right). \end{aligned} \tag{C.8}$$

In the steady state, equation (C.4) implies:

$$\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)[1-(1-\rho)\beta(1-q_{ss}\xi\theta_{ss})] = (1-\xi)[mpl_{ss}-(b^u+\Phi)]. \tag{C.9}$$

Our strategy of parameterizing the DMP model implies assigning a value to the replacement ratio, r^u, rather than unemployment benefits, b^u. The definition of the replacement ratio and the condition (34) yield the following unemployment benefits given the replacement ratio:

$$b^u = r^u w_{ss} = r^u\left(mpl_{ss} - (1-(1-\rho)\beta)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\right). \tag{C.10}$$

After substituting (C.10) into equation (C.11), we obtain the household bargaining weight, ξ, as an implicit function of the replacement ratio, r^u, and the consumption difference between employed and unemployed, Φ:

$$\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\left[1-(1-\rho)\beta\left(1-q_{ss}\xi\theta_{ss}\right)\right]=(1-\xi)\left[(1-r^u)mpl_{ss}+r^u\left(1-(1-\rho)\beta\right)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)-\Phi\right] \quad \text{(C.11)}$$

or solved for ξ:

$$\xi=\frac{1}{1+\frac{(1-(1-\rho)\beta(1-q_{ss}\theta_{ss}))\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)}{(1-r^u)\left(mpl_{ss}-(1-(1-\rho)\beta)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\right)-\Phi}}. \quad \text{(C.12)}$$

ξ is decreasing in r^u and Φ.[29]

For convenience, we define the coefficient Υ as:

$$\begin{aligned}\Upsilon &= \frac{\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\left[\zeta-(1-\rho)\beta\left(\zeta-q_{ss}\xi\theta_{ss}\right)\right]-\zeta\bar{\kappa}\left[1-(1-\rho)\beta\left(1-q_{ss}\xi\theta_{ss}\right)\right]}{\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)\left[1-(1-\rho)\beta\left(1-q_{ss}\xi\theta_{ss}\right)\right]} \\ &= \zeta+(1-\zeta)\frac{(1-\rho)\beta q_{ss}\xi\theta_{ss}}{1-(1-\rho)\beta\left(1-q_{ss}\xi\theta_{ss}\right)}-\zeta\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}. \end{aligned} \quad \text{(C.13)}$$

Only the second term in Υ depends on ξ; therefore Υ is bounded by $\zeta-\zeta\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ (for $\xi=0$) and $1-\zeta\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$.

With these expressions in hand, we can approximate the response of labour market tightness in response to a commodity price shock. To avoid complications from transition dynamics, we focus on the "medium run". The commodity price shock induces highly persistent movements in the endogenous variables, thus implying little change in the endogenous variables from one period to another the initial transition. Applying this reasoning to equation (C.7), the following relationship between labour market tightness and the real exchange rate emerges:

$$\hat{\theta}=\hat{v}-\hat{u}\approx-\frac{1}{\Upsilon}\frac{mpl_{ss}}{(1-r^u)mpl_{ss}+r^u\left(1-(1-\rho)\beta\right)\left(\frac{\kappa^v}{q_{ss}}+\bar{\kappa}\right)-\Phi}\frac{1-\nu}{\nu}\widehat{rer}. \quad \text{(C.14)}$$

The impact of a given change in the real exchange rate onto labour market tightness is determined by the interplay of the parameters r^u and Φ.[30] We abstract for now from the coefficients $\bar{\kappa}$ and κ^v. Although Υ is a function of ξ and therefore depends on the choices of r^u and Φ as is apparent from equation (C.12), the role of this coefficient in delivering big amplification of the exchange rate movement is limited since Υ is bounded. For $\bar{\kappa}=0$, $\Upsilon\in[\zeta,1]$. Much more powerful in delivering amplification are therefore the parameters r^u and Φ. An increase in the replacement ratio or in the consumption gap between employed and unemployed household members reduces the denominator of the second coefficient and results in bigger movements of labour market tightness for a given movement of the real exchange rate. It is for this interplay of the parameters r^u and Φ that our model can match the responses in unemployment and vacancies even for a moderate value of the replacement ration of $r^u=0.4$.

The choice of the parameter $\bar{\kappa}$ also influences the transmission of the shock to labour market tightness. Our calibration strategy imposes a tight relationship with κ^v as we target a specific value of the total (expected) costs

[29] It is worth pointing out that for common calibration choice of the other parameters it is much easier to obtain an implied bargaining weight around 0.5 at a moderate replacement ratio if $\Phi>0$.

[30] Recall, that u_t measure the pool of job-seekers at the beginning of the period and not the unemployment rate which is measured by $\tilde{u}_t$. In the medium run, the two concepts are related via $\hat{u}=\frac{1-(1-\rho)n_{ss}-\rho}{1-(1-\rho)n_{ss}}\hat{\tilde{u}}$.

of vacancy posting, $\kappa^v v_{ss} + q_{ss}\bar{\kappa}$, relative to output in the steady state. Thus the term $\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ equals $\frac{s^{fixed}}{\frac{1-s^{fixed}}{v_{ss}}+s^{fixed}}$, where s^{fixed} is the share of vacancy posting costs that falls onto $q_{ss}\bar{\kappa}$. By increasing s^{fixed}, the term $\frac{\bar{\kappa}}{\frac{\kappa^v}{q_{ss}}+\bar{\kappa}}$ in (C.13) increases and implies a *lower* value of Υ. However, it can also be shown that raising s^{fixed} implies a lower value of $\frac{\kappa^v}{q_{ss}} + \bar{\kappa}$ and a higher value of the Nash bargaining weight ξ. This effect in turn, raises the value of the second term in equation (C.13) and *raises* the value of Υ. Numerically, raising s^{fixed} fails to deliver sufficient amplification.

The most effective way of raising the impact of given change in the real exchange rate on labour market tightness is therefore to allow for preferences that are non-separable in consumption and leisure to justify a consumption gap between the employed and unemployed household members, i.e., $\Phi > 0$. This approach avoids having to set the replacement ratio to unrealistically high levels and is by far more effective than raising the share of the fixed costs of bargaining in the overall vacancy posting costs, s^{fixed}.

C.2 AOB model

As in the DMP model, we can derive an equation describing the evolution of labour market tightness. The bargaining sharing rule implies:

$$\begin{aligned} J_t &= \frac{\beta_1+\beta_3}{1+\beta_1}\left(mpl_t - b^u - \frac{\beta_2}{\beta_1+\beta_3}\gamma^{aob}\right) + (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-\frac{1}{1+\beta_1}s_{t+1}\right)J_{t+1}\right] \\ &\quad -(1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-s_{t+1}\right)\frac{\beta_3}{1+\beta_1}\left(mpl_{t+1} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}\right)\right]. \end{aligned} \tag{C.15}$$

We proceed as under the DMP model to obtain an idea how a change in the real exchange rate is amplified. In the steady state, equation (C.15) implies:

$$\begin{aligned} &\left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa}\right)\left[1-(1-\rho)\beta\left(1-\frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right] - \frac{\beta_1}{1+\beta_1}\frac{\beta_2}{\beta_3}\gamma^{aob} \\ = &\left(\frac{\beta_1}{1+\beta_1} + \frac{\beta_3}{1+\beta1}\left[1-(1-\rho)\beta\left(1-q_{ss}\theta_{ss}\right)\right]\right)\left(mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}\right). \end{aligned} \tag{C.16}$$

Given values of δ^{aob} and M^{aob}, the parameters β_1, β_2, β_3 are determined as demonstrated in Section 5.2. Furthermore, our calibration strategy of targeting q_{ss} and n_{ss} (and therefore θ_{ss}) implies that γ^{aob} is a function of κ^v and $\bar{\kappa}$. The parameter γ^{aob} is not a free parameter.

We approximate the impact of a given change in the real exchange rate on labour market tightness over the medium term as:

$$\hat{\theta} = \frac{\Omega_2 - \frac{\beta_1}{1+\beta_1}\frac{\beta_2}{\beta_3}\gamma^{aob}}{\Omega_1 - (1-\rho)\beta q_{ss}\theta_{ss}(1-\zeta)\frac{\beta_3}{1+\beta_1}\left(mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}\right)}\frac{mpl_{ss}}{mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob}}\frac{1-\nu}{\nu}\widehat{rer} \tag{C.17}$$

where

$$\Omega_1 = \left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa}\right)\left[\zeta - (1-\rho)\beta\left(\zeta - \frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right] - \zeta\bar{\kappa}\left[1-(1-\rho)\beta\left(1-\frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right] \tag{C.18}$$

$$\Omega_2 = \left(\frac{\kappa^v}{q_s s} + \bar{\kappa}\right)\left[1-(1-\rho)\beta\left(1-\frac{1}{1+\beta_1}q_{ss}\theta_{ss}\right)\right]. \tag{C.19}$$

As under the DMP model, the impact of a given change in the real exchange rate on labour market tightness is bigger for higher values of the replacement ratio:

$$b^u = r^u w_{ss} = r^u \left(mpl_{ss} - (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa} \right) \right). \tag{C.20}$$

The important question is, however, to what extent can the model amplify the response of labour market tightness for a moderate value of the replacement ratio? The free parameters in the alternating offer bargaining model are: δ^{aob}, M^{aob}, and s^{fixed}.[31]

To maximize the impact of the shock, it is advisable to:

- maximize the term $\frac{\beta_2}{\beta_3}\gamma^{aob}$ to raise $\frac{mpl_{ss}}{mpl_{ss} - b^u - \frac{\beta_2}{,}\beta_3\gamma^{aob}}$
- minimize β_1 to increase the term $\Omega_2 - \frac{\beta_1}{1+\beta_1}\frac{\beta_2}{\beta_3}\gamma^{aob}$,
- and maximize β_3 to reduce the term $\Omega_1 - (1-\rho)\beta q_{ss}\theta_{ss}(1-\zeta)\frac{\beta_3}{1+\beta_1}\left(mpl_{ss} - b^u - \frac{\beta_2}{\beta_3}\gamma^{aob} \right)$.

Setting a low probability of breakdown in bargaining, δ^{aob} achieves the second and third point. With regard to the first point, note from equation (C.16), the term $\frac{\beta_2}{\beta_3}\gamma^{aob}$ increases in the value of s^{fixed} for given M^{aob} and δ^{aob}.

Our estimation results confirm the conjecture that β_1 is ideally set close to zero given the calibration of the total cost of vacancy posting. Across different calibrations, our point estimate for the probability δ^{aob} is always close to zero.[32] If $\delta^{aob} \approx 0$, the relationship between the real exchange rate and labour market tightness reduces to:

$$J_t \approx -\beta_2\gamma^{aob} + \beta_3(mpl_t - b^u) \tag{C.21}$$

or log-linearized form:

$$\hat{\theta}_t = \frac{\beta_3 mpl_{ss}}{\zeta\frac{\kappa^v}{q_{ss}}}\frac{1-\nu}{\nu}\widehat{rer}_t. \tag{C.22}$$

In this case, the wage sharing rule is (close to) independent of the marginal value of employment to the household, H_t. sEquation (C.22) reveals the importance of s^{fixed} in the alternating offer bargaining model. For a higher value of s^{fixed}, we obtain that the share of total vacancy posting costs accounted for by the variable component $\kappa^v v_s s$ is smaller. The resulting value of κ^v will also be smaller. For s^{fixed} sufficiently large, the model can deliver the desired amplification of the labour market tightness response of a given real exchange rate movement. Note, that restricting s^{fixed} to zero very much restricts the empirical performance of the model.

It is noteworthy, that had we assumed a much lower value of the total vacancy posting costs relative to output, $\frac{\kappa^v v_{ss} + q_{ss}\bar{\kappa}}{y^h_{ss}}$, the estimated value δ^{aob} would have been further away from zero, but s^{fixed} would have been close to zero. In the case of low vacancy posting costs, the implied value of κ^v might be so small — even for $s^{fixed} = 0$ — that the amplification of labour market tightness is too big relative to the data. In this case, the coefficient $\frac{\beta_3 mpl_{ss}}{\zeta\frac{\kappa^v}{q_{ss}}}$ needs to be lowered by reducing β_3, i.e., allowing for a larger value of δ^{aob}. Although in this case, H_t

[31] Recall that s^{fixed} measures the share of the overall vacancy posting costs apportioned to $q_s s\bar{\kappa}$ in our calibration.

[32] Using a different procedure, Christiano et al. (2013) also estimate the probability of breakdown in bargaining to be close to zero.

reenters the analysis and the simplified formula in equation (C.22) no longer applies directly, the intuition just laid out applies.

The sensitivity of the parameter estimates to the calibration of the total vacancy posting costs relative to output, $\frac{\kappa^v v_{ss}+q_{ss}\bar{\kappa}}{y^h_{ss}}$, is unique to the alternating offer bargaining model. In the DMP model a drastic reduction in these costs barely influences the parameter estimates; in particular the DMP model always favours s^{fixed} close to zero.