

# Monetary Policy and Asset Prices with Infinite-Horizon Learning

Marco Airaudo\*

This Draft: February 15, 2016

## Abstract

We study the stabilizing properties of interest rate rules that grant an explicit response to stock prices in a New Keynesian framework where imperfect knowledge and incomplete markets generate wealth effects from equity holdings. Boundedly-rational agents need to form infinite-horizon forecast about pay-off relevant variable in order to solve their intertemporal optimization problems, as in Preston (2005, 2006). In this context, we find that, when the central bank adopts a forecast-based Taylor rule, granting an explicit response to stock prices can facilitate the attainment of a determinate rational expectations equilibrium which is also stable under learning dynamics.

**Keywords:** Interest Rate Rules; Multiple Equilibria; Determinacy; Multiple Sectors; Learning; Expectational Stability;

**JEL Classifications:** C62, D83, E32, E52.

## 1 Introduction

Since burst of the dot.com bubble in early 2000 — and even more after the 2007-8 global financial crisis — policy-makers and academic economists have lively debated whether central banks should move the policy rate in response to asset price fluctuations. Advocates of such policy argue that,

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\*Corresponding Author. School of Economics, LeBow College of Business, Drexel University, Gerri C. LeBow Hall, 3220 Market Street, Philadelphia, PA 19104. Email: marco.airaudo@drexel.edu

because of frictions in credit and/or financial markets, asset prices exert distortionary effects on individual decisions.<sup>1</sup> By raising interest rates during a financial booms (and cutting them during a downturn), a learning-against-the-wind policy could smooth those fluctuations and therefore tame their undesired consequences on the macroeconomy. Opponents of it claim instead that the costs might outweigh the gains as i) part of those fluctuations might be efficient reactions to market conditions, and ii) a policy strictly targeting price stability might be sufficient to stabilize also asset prices.<sup>2</sup>

In this paper, we assess the desirability of interest rate rules granting an explicit response to stock prices for what concerns the determinacy of an equilibrium under rational expectations, and its stability under learning dynamics. We consider a small-scale New Keynesian DSGE model where boundedly-rational agents trade risky equity shares - issues by monopolistically competitive profit-maximizing firms - and form expectations about future variables using simple adaptive rules. In this respect, we adopt the *infinite horizon learning* approach proposed by Preston (2005, 2006): agents and firms are internally rational, in the sense that they take optimal decision subject to known individual objectives and constraints, understanding the mapping between own actions and expected pay-offs, given subjective beliefs about aggregate variables beyond their control. However, they do not have any knowledge about other agents' objectives, constraints and beliefs. Following the terminology of Adam and Marcet (2011), in our framework, agents are *internally rational*, but may not be *externally rational*. As in Preston's work, for arbitrary beliefs satisfying standard probability laws, the solutions to intertemporal optimization problems requires agents to make infinite horizon forecasts about their pay-off relevant variables.

We show that infinite-horizon-learning (henceforth, IHL) induces wealth effects of equity shares holdings on consumption via households' Euler equation. We therefore study the determinacy and expectational stability properties of forward-looking Taylor rules granting an explicit response to stock prices. Our main findings are the followings. First, we find that, in the context of our model, forward-looking rules which exclusively respond to inflation and output perform better than in Preston (2006). The latter shows that i) no determinate equilibrium is learnable, and ii) learnability of the fundamental REE requires an implausibly large response coefficient to inflation

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<sup>1</sup>Case, Quigley and Shiller (2005) and Carrol, Otsuka and Slacalek (2011) provide statistical evidence for housing and financial wealth effects on consumption.

<sup>2</sup>The debate between Bernanke and Gertler (1999, 2001) and Cecchetti et al. (2000, 2002) is the most prominent. Dupor (2005) discusses optimal monetary policy in a New-Keynesian model subject to exuberance shocks.

(larger than 100!). A key difference with respect to ours, is that in Preston's set-up there is no financial market for equity shares: dividends are equally distributed across all households, as lump-sum transfers. The key source of instability under learning dynamics lies in the fact that agents have to form infinite horizon forecasts on future nominal interest rates (and inflation). Under learning and without full knowledge of the policy rule adopted by the central bank, those forecasts do not have to be in line with those implied by the underlying policy rule under RE. Namely, agents are not necessarily expecting higher future real interest rates following higher future inflation - which makes the standard aggregate demand channel of monetary policy transmission weaker. By including a market for equity shares, our framework gives rise to a no-arbitrage condition between investing in risky equity and a riskless bond, whose return is tied to the policy rate. We show that by taking advantage of such condition agents do not need to form forecasts about future policy rates. This improves the stability properties of forward-looking rules by lowering the minimum response to inflation leading to E-stability. Moreover, we find that there exists parameterizations for which a determinate equilibrium is also E-stable. Nevertheless, even in the best scenario, this still requires a rather aggressive response to inflation (around 8).

Second, in the context of our model, for what concerns equilibrium determinacy, there should be no explicit response to stock prices. A policy rule that responds exclusively and actively to inflation (the Taylor principle) is necessary and *de facto* sufficient to guarantee a unique REE. Interestingly, we find that an explicit positive (but not excessive) response to stock prices by monetary policy can make the determinate equilibrium stable under learning while keeping the responsiveness to inflation at standard values (below 2). This result contrasts with what one would obtain under the Euler equation learning approach (henceforth, EEL) of Evans and Honkapohja (2001): in the context of our model, a determinate equilibrium is always E-stable under Euler equation learning. Some sensitivity analysis shows that the size of the policy space for which the equilibrium is both determinate and E-stable is increasing with respect to the elasticity of labor supply and the degree of (real) wage rigidity, but decreasing with respect to the degree of price rigidity, risk aversion, and the elasticity of substitution across goods varieties.

Third, we show that the instability due to learning can be eliminated by either one of the following two alternative policies: a transparent forward-looking interest rate rule (i.e., a policy whose details are fully communicated to the public), or a contemporaneous interest rate rule (for which the degree of transparency is irrelevant).

This paper relates to two separate lines of work in the literature. On the one hand, it is related to papers that have studied the determinacy properties of rules responding to stock prices in the baseline New Keynesian models under rational expectations. Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) show that including a positive response to stock prices in interest rate rules restricts the policy space where the rational expectations equilibrium (REE) is determinate and, therefore, can be a source of sunspot-driven self-fulfilling expectations fluctuations.<sup>3</sup> As a result, rules that respond to stock prices also require a *sufficiently* active response to inflation—a reinforced Taylor principle—to ensure determinacy. To some extent, this result is not surprising. The benchmark New Keynesian model, which is typically based on an infinitely-lived representative agent, does not foresee any structural linkage between financial markets and real activity, making stock prices completely redundant for consumption decisions and, consequently, the business cycle. Hence, there is no specific rationale for why the central bank should move the interest rate in response to stock price changes. Airaudo et al. (2015) extend the analysis to a model with a Blanchard-Yaari’s OLG structure. They show that, because of the presence of non-Ricardian households, there are wealth effects coming from holdings of risky equity, such that stock prices are non-redundant for business cycle fluctuations. In particular, the wedge between the current and the expected level of aggregate consumption is driven not only by the ex ante real interest rate, as in the standard New Keynesian model, but also by the market value of financial wealth. Their analysis shows that, responding to stock prices enlarges the determinacy space. They also study the stability under Euler equation learning of both fundamental and sunspot equilibria.

On the other hand, our work clearly builds on the seminal contributions by Preston (2005, 2006), and further extensions/applications. For instance, the wealth effects in our paper are similar to those in Eusepi and Preston (2012) who show that under learning households may treat government bonds as net wealth if they are uncertain about the fiscal policy regime. Absent rational expectations, household could in fact believe that outstanding public debt differs from the present discounted value of expected future surpluses (hence, a failure of Ricardian equivalence). They find that E-stability might require some modification of Leeper (1991) stabilizing conditions for fiscal-monetary interaction. Eusepi and Preston (2010, 2011) extend the analysis to the case of multiple maturities, while Eusepi et al. (2012) highlight the role of no-arbitrage as an instrument

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<sup>3</sup>For a general discussion on determinacy versus indeterminacy in the benchmark New Keynesian model see, for instance, Bullard and Mitra (2002).

to forecast future policy rates. For what concerns agents' perception of asset prices, our framework is clearly related to the recent contribution on imperfect knowledge and asset pricing by Adam and Marcet (2011).

The rest of the paper is organized as follows. Section 2 lays out the model, with a detailed description of the optimization problems faced by households and firms, and the monetary policy rule. Section 3 described the methodology used for equilibrium determinacy and stability under learning. Section 4 provides analytical and numerical results for equilibrium determinacy and stability under EEL. Section 5 analyzes the case of stability under IHL. Section 6 consider alternative policy specifications. Section 7 concludes.

## 2 The Model

### 2.1 Households

The economy is populated by a continuum of infinitely-lived households, indexed by  $i \in [0, 1]$ . Each household seeks to maximize his expected lifetime utility:

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[ \frac{(C_T^i)^{1-\sigma}}{1-\sigma} - \psi \frac{(H_T^i)^{1+\chi}}{1+\chi} \right], \quad (1)$$

where  $\sigma, \psi, \chi > 0$ , and

$$C_t^i \equiv \left[ \int_0^1 (C_{j,t}^i)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad (2)$$

is a Dixit-Stiglitz's consumption aggregate, bundling together a continuum of differentiated final consumption goods, indexed by  $j \in [0, 1]$ , with  $\epsilon > 1$  denoting the intratemporal elasticity of substitution between any two varieties. Given the  $j$ -th variety's price  $P_{j,t}$ , household's expenditure minimization across goods gives standard relative demand schedules,

$$C_{j,t}^i = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} C_t^i, \text{ for } j \in [0, 1], \quad (3)$$

where  $P_t \equiv \left[ \int_0^1 P_{j,t}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$  is the aggregate price index.

The expectation operator  $\tilde{E}_t^i$  denote household  $i$  subjective beliefs about aggregate pay-off

relevant variables that the household cannot control directly with its decisions. As in Preston (2005, 2006), household's beliefs satisfy standard probability laws (such as the law of iterated expectations, for which  $\tilde{E}_t^i \tilde{E}_T^i x_{T+1} = \tilde{E}_t^i x_{T+1}$  for a generic variable  $x$ , and for any  $T \geq t$ ) so that standard solution methods apply here.

The individual household is subject to the following budget constraint:

$$P_t C_t^i + P_{B,t} B_t^i + P_t \int_0^1 Q_{j,t} S_{j,t}^i dj = B_{t-1}^i + P_t \int_0^1 (Q_{j,t} + D_{j,t}) S_{j,t-1}^i dj + W_t H_t^i - T_t^i \quad (4)$$

The household employs its resources by purchasing the consumption bundle  $C_t^i$  (at price  $P_t$ ),  $B_t^i$  units of a risk free bond at price  $P_{B,t}$  (markets are incomplete), and a portfolio of equity shares issued by a continuum of firms, with  $S_{j,t}^i$  denoting the  $j$ -th firm's shares with (real) market price  $Q_{j,t}$ , for  $j \in [0, 1]$ . Resources are given by payments on previously purchased bonds  $B_{t-1}^i$  and previously purchased shares (with  $D_{j,t}$  denoting real dividends per share of the  $j$ -th firm), as well as labor income,  $W_t H_t^i$ , net of taxes,  $T_t^i$ .

First order conditions with respect to  $C_t^i$ ,  $H_t^i$ ,  $B_t^i$  and  $S_{j,t}^i$  for  $j \in [0, 1]$  give, respectively:

$$(C_t^i)^{-\sigma} = \lambda_t^i P_t \quad (5)$$

$$\psi(H_t^i)^\chi = \lambda_t^i W_t \quad (6)$$

$$\lambda_t^i P_{B,t} = \beta \tilde{E}_t^i \lambda_{t+1}^i \quad (7)$$

$$\lambda_t^i P_t Q_{j,t} = \beta \tilde{E}_t^i [\lambda_{t+1}^i P_{t+1} (Q_{j,t+1} + D_{j,t+1})], \text{ for } j \in [0, 1] \quad (8)$$

Let  $\mathcal{F}_{t,T}^i \equiv \beta^{T-t} \left( \frac{C_T^i}{C_t^i} \right)^{-\sigma} \frac{P_t}{P_T}$  denote the  $i$ -th household's stochastic discount factor between period  $t$  and  $T \geq t$ . From equations (5) and (7), the unit price of a riskless bond  $P_{B,t}$  is

$$P_{B,t} = \tilde{E}_t^i \mathcal{F}_{t,t+1}^i, \quad (9)$$

while equations (5) and (8), together with the definition of  $\mathcal{F}_{t,t+1}^i$ , give an expression for the  $j$ -th firm's share price  $Q_{j,t}$ ,

$$Q_{j,t} = \tilde{E}_t^i \left[ \mathcal{F}_{t,t+1}^i \frac{P_{t+1}}{P_t} (Q_{j,t+1} + D_{j,t+1}) \right], \text{ for } j \in [0, 1]. \quad (10)$$

Letting  $R_t \equiv P_{B,t}^{-1}$  and  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  denote, respectively, the gross nominal interest rate and gross price inflation, equations (5) and (7) combined give a Euler equation for individual consumption:

$$(C_t^i)^{-\sigma} = \beta R_t \tilde{E}_t^i \frac{(C_{t+1}^i)^{-\sigma}}{\pi_{t+1}} \quad (11)$$

Finally, the combination of (5) and (6) yields a standard labor supply equation:

$$\psi(C_t^i)^\sigma (H_t^i)^\chi = w_t \quad (12)$$

where  $w_t \equiv \frac{W_t}{P_t}$  is the real wage.

Let  $A_t^i \equiv B_{t-1}^i + P_t \int_0^1 (Q_{j,t} + D_{j,t}) S_{j,t-1}^i dj$  denote beginning-of-period individual nominal wealth.

We can then write end-of-period wealth  $P_{B,t} B_t^i + P_t \int_0^1 Q_{j,t} S_{j,t}^i dj$  as follows:

$$\begin{aligned} P_{B,t} B_t^i + P_t \int_0^1 Q_{j,t} S_{j,t}^i dj &= \tilde{E}_t^i \mathcal{F}_{t,t+1}^i B_t^i + \int_0^1 S_{j,t}^i \tilde{E}_t^i [\mathcal{F}_{t,t+1}^i P_{t+1} (Q_{j,t+1} + D_{j,t+1})] dj \\ &= \tilde{E}_t^i \mathcal{F}_{t,t+1}^i A_{t+1}^i \end{aligned} \quad (13)$$

where the first equality makes use of equations (9) and (10), while the second one obtains from the definition of  $A_t^i$ . We can then write the household's budget constraint (4) more compactly:

$$P_t C_t^i + A_t^i = \tilde{E}_t^i \mathcal{F}_{t,t+1}^i A_{t+1}^i + P_t I_t^i, \quad (14)$$

where  $I_t^i \equiv \frac{W_t}{P_t} H_t^i - \frac{T_t^i}{P_t}$  denotes net real labor income. Forward iteration of equation (14) yields household  $i$ 's intertemporal budget constraint:

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \mathcal{F}_{t,T}^i P_T C_T^i = A_t^i + \tilde{E}_t^i \sum_{T=t}^{\infty} \mathcal{F}_{t,T}^i P_T I_T^i, \quad (15)$$

which states that the expected present discounted value of consumption equals the present discounted value of labor income, plus initial wealth. After dividing both sides of (15) by  $P_t$  and letting  $\Lambda_{t,T}^i \equiv \beta^{T-t} \left( \frac{C_T^i}{C_t^i} \right)^{-\sigma}$ , we obtain

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i C_T^i = a_t^i + \tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i I_T^i \quad (16)$$

where  $a_t^i \equiv \frac{A_t^i}{P_t}$  is real individual wealth at the beginning of period  $t$ .

As shown in Appendix A.2, the log-linearization of (16) around a symmetric steady state (where  $\bar{C}^i = \bar{C}$ ,  $\bar{I}_t^i = \bar{I}_t$  and  $\bar{\Lambda}_{t,T}^i = \beta^{T-t}$ ) gives an infinite-horizon Euler equation for individual consumption:<sup>4</sup>

$$\frac{\bar{C}}{1-\beta} \hat{C}_t^i = \bar{a} \hat{a}_t^i + \bar{I} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i - \frac{\beta}{1-\beta} [\bar{I} - (1-\sigma^{-1}) \bar{C}] \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \quad (17)$$

where  $\hat{r}_t \equiv \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1}$  is the (log) *ex ante* real interest rate at time  $t$ . Equation (17) states that individual optimal consumption depends on the present discounted values of future labor income and real interest rates, as well as current individual real wealth  $\hat{a}_t^i$ . By log-linearizing  $a_t^i \equiv \frac{A_t^i}{P_t} = b_t^i + P_t \int_0^1 (Q_{j,t} + D_{j,t}) S_{j,t-1}^i dj$  (for  $b_t^i \equiv \frac{B_{t-1}^i}{P_t}$ ) around a symmetric steady state equilibrium where  $\bar{a}^i = \bar{a}$  for every  $i$ ,  $\bar{Q}^j = \bar{Q}$  and  $\bar{D}^j = \bar{D}$  for every  $j$ , and  $\bar{S}_j^i = 1$  (normalizing, without loss of generality, to unity the supply of equity share by each firm  $j$ ), the term  $\bar{a} \hat{a}_t^i$  entering (17) is given by the following expression:

$$\bar{a} \hat{a}_t^i = \bar{b} \hat{b}_t^i + \int_0^1 [(\bar{Q} + \bar{D}) \hat{S}_{j,t-1}^i + \bar{Q} \hat{Q}_{j,t} + \bar{D} \hat{D}_{j,t}] dj. \quad (18)$$

## 2.2 Firms

A continuum of monopolistically competitive firms, indexed by  $j \in [0, 1]$ , supplies differentiated final products. Each firm is subject to a Calvo-style nominal rigidities, with  $\vartheta \in (0, 1)$  denoting the per-period probability that the firm will not be able to re-optimize its price. The  $j$ -th firm solves the following profit maximization problem:

$$\max_{P_{j,t}^*} \tilde{E}_t^j \sum_{T=t}^{\infty} (\vartheta \beta)^{T-t} \frac{U_c(Y_T)}{U_c(Y_t)} \frac{P_t}{P_T} [P_{j,t}^* Y_{j,T} - W_T H_{j,T}]$$

subject to  $Y_{j,T} = \left(\frac{P_{j,t}^*}{P_T}\right)^{-\epsilon} Y_T$ , and technology  $Y_{j,T} = z_T H_{j,T}$ , taking as given the aggregate price  $P_T$  and aggregate output  $Y_T$ .<sup>5</sup> Aggregate TFP  $z_t$  is stochastic: its log-deviation from steady state (without loss of generality fixed to unity) follows a standard AR(1) process, i.e.  $\hat{z}_t = \rho \hat{z}_{t-1} + \hat{\varepsilon}_t$  for  $\rho \in (0, 1)$  and  $\hat{\varepsilon}_t$  a zero mean *iid* disturbance.

<sup>4</sup>From now on, hatted variables will denote log-deviations of variables from their respective steady state value (denoted with a bar).

<sup>5</sup>As in Preston (2006), firms are assumed to value future profits using the *aggregate* marginal rate of substitution between current and future income.



Letting  $\mu \equiv \frac{\epsilon}{\epsilon-1} > 1$  be the gross (steady state) price mark-up and  $mc_t \equiv \frac{w_t}{z_t}$  be real marginal costs, after taking first order condition with respect to  $P_{j,t}^*$ , simple algebraic manipulation gives the optimal pricing rule:

$$\frac{P_{j,t}^*}{P_t} = \mu \frac{\tilde{E}_t^j \sum_{T=t}^{\infty} \vartheta^{T-t} \Lambda_{t,T} \left( \frac{P_T}{P_t} \right)^\epsilon Y_T mc_T}{\tilde{E}_t^j \sum_{T=t}^{\infty} \vartheta^{T-t} \Lambda_{t,T} \left( \frac{P_T}{P_t} \right)^{\epsilon-1} Y_T}. \quad (19)$$

Let  $p_{j,t}^* \equiv \frac{P_{j,t}^*}{P_t}$  denote the optimal relative price. The log-linearization of (19) gives:

$$\hat{p}_{j,t}^* = (1 - \beta\vartheta) \tilde{E}_t^j \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} (\widehat{mc}_T + \hat{\pi}_{t,T}) \quad (20)$$

where  $\hat{\pi}_{t,T} \equiv \hat{P}_T - \hat{P}_t$ . Since they face the same decision problem, under the assumption of homogeneous beliefs - i.e.,  $\tilde{E}_t^j = \tilde{E}_t$  - all firms allowed to re-set their price will choose it according to the following price-setting rule:

$$\hat{p}_t^* = (1 - \beta\vartheta) \tilde{E}_t \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} (\widehat{mc}_T + \hat{\pi}_{t,T}) \quad (21)$$

With the aggregate price index  $P_t$  evolving according to  $P_t^{1-\epsilon} = (1 - \vartheta) (P_t^*)^{1-\epsilon} + \vartheta P_{t-1}^{1-\epsilon}$ , such that  $p_t^* \equiv \frac{P_t^*}{P_t} = \left( \frac{1 - \vartheta \pi_t^{\epsilon-1}}{1 - \vartheta} \right)^{\frac{1}{1-\vartheta}}$  and therefore  $\hat{p}_t^* = \frac{\vartheta}{1-\vartheta} \hat{\pi}_t$ , equation (21) implies an infinite-horizon Phillips curve:

$$\hat{\pi}_t = \kappa \widehat{mc}_t + \tilde{E}_t \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} [\kappa \beta \vartheta \widehat{mc}_{T+1} + (1 - \vartheta) \beta \hat{\pi}_{T+1}] \quad (22)$$

where  $\kappa \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}$ .

### 2.3 Monetary Policy

The government/central bank sets the short-term nominal interest rate according to a forward-looking Taylor-type interest rate rule. In log-linearized terms, the rule takes the following form:

$$\hat{R}_t = \phi_\pi \tilde{E}_t^{cb} \hat{\pi}_{t+1} + \phi_y \tilde{E}_t^{cb} \hat{Y}_{t+1} + \phi_q \tilde{E}_t^{cb} \hat{Q}_{t+1}, \quad (23)$$

namely, the central bank responds to expected future log-deviations of inflation, output and the stock price index - defined as  $\hat{Q}_t \equiv \int_0^1 \hat{Q}_{j,t} dj$  - from their respective steady state (target) values. It is assumed that the central bank responds to household's expectations, with the operator  $\tilde{E}_t^{cb}$  defined as the average expectation across households:  $\tilde{E}_t^{cb} \equiv \int_0^1 \tilde{E}_t^j dj$ . There are two main reasons for why our main analysis will focus on an expectation-based policy rule. First, these types of rules seem to find stronger support in the empirical literature, with respect to rules that respond to past or currently observed variables.<sup>6</sup> Second, this will allow us to compare our results to those of Preston (2006) who highlights the instability problems of expectation-based rules in a New Keynesian model without equity trading. Later, we will also consider the case of contemporaneous rules.

A distinguishing element of the analysis under IHL with respect to the case of RE or EEL is that households need to forecast *real* interest rates into the infinite future. In particular, to determine its optimal level of consumption, the household needs to forecast the term  $\tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T$  entering his Euler equation (17), which constitutes the main channel through which monetary policy affects aggregate demand. By the definition of  $\hat{r}_T$  this term can be written as follows:

$$\tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T = \hat{R}_T + \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{R}_{T+1} - \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\pi}_{T+1} \quad (24)$$

Since forecasting future *real* interest rates requires the agents to form expectations about future *nominal* interest rate and future inflation, the degree of knowledge agents possess about the monetary policy rule adopted by the central bank is crucial.

For most of the analysis we will assume that the policy rule is not public information. Agents will have then to forecast future interest rates. This lack of transparency will make private expectations about future monetary policy unanchored as private forecasts of interest rates will not necessarily coincide with those implied by the actual policy rule followed by the central bank. We will then show how full communication of policy can eliminate the instability of equilibria due to learning.<sup>7</sup>

<sup>6</sup>See, for instance, Clarida et al. (2000), or the more recent study by Boivin and Giannoni (2006).

<sup>7</sup>See Eusepi and Preston (2010) on the issue of communication and stability under learning.

### 3 Methodology

As discussed in the Introduction, we are interested in studying the determinacy and E-stability properties of interest rate rules which include an explicit response to stock prices. We proceed with a brief description of our methodology.

#### 3.1 Equilibrium Determinacy

The equilibrium determinacy analysis employs the standard procedure of Blanchard and Khan (1980). Once linearized around the unique non-stochastic steady state, the model's equilibrium dynamics under Rational Expectations (RE) are described by the reduced form system

$$\hat{x}_t = \Omega E_t \hat{x}_{t+1} + \Theta \hat{z}_t, \quad (25)$$

where  $\hat{x}_t \equiv [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t]'$ , and  $\Omega$  and  $\Theta$  are conformable matrices, whose entries depend on structural and policy parameters. Since none of the three endogenous variables is predetermined, the Rational Expectations Equilibrium (henceforth, REE) is locally determinate if and only if all eigenvalues of the Jacobian  $\Omega$  lie inside the unit circle in the complex plane.

#### 3.2 Expectational Stability under Learning Dynamics

There exists two main approaches in the literature to study the stability under learning dynamics (expectational stability, or E-stability) of REE. Probably, the most common is the so called *Euler Equation Learning* (henceforth, EEL) paradigm proposed by Evans and Honkapohja (2001), where the subjective expectation operator defining agents' beliefs,  $\tilde{E}_t$ , replaces the RE operator,  $E_t$ , only in the reduced form linearized system (25). Under this approach, adaptively learning agents do not solve a dynamic optimization problem, but rather use simple decision rules capturing intertemporal trade-offs across two subsequent periods, which are assumed to coincide with the reduced form intertemporal conditions derived under RE. In such set-up, agents are rather myopic as they need to form expectations only about the next period.

Preston (2005, 2006) proposes an alternative approach, which he defines *Infinite Horizon Learning* (henceforth, IHL). In his environment, agents are internally rational, in the sense that they take optimal decision subject to known individual objectives and constraints, understanding the mapping between own actions and expected pay-offs, given subjective beliefs about aggregate variables

beyond their control. However, they do not have any knowledge about other agents' objectives, constraints and beliefs. He shows that, for arbitrary beliefs satisfying standard probability laws, the solutions to intertemporal optimization problems requires agents to make infinite horizon forecasts about their pay-off relevant variables. In particular, he argues that since, in general, long-horizon forecasts cannot be reduced to simple one-period-ahead expectations, EEL can lead to misleading results about the stability of REE.<sup>8</sup>

Despite this fundamental difference, both approaches assume that, when forming expectations, agents use the functional form implied by the minimum state variable representation of the underlying REE (henceforth, MSV-REE). In the context of our model, the MSV-REE solution looks as follows:

$$\hat{x}_t = a^{RE} + b^{RE} \hat{z}_t$$

where  $\hat{x}_t$  is a vector of endogenous variables,  $\hat{z}_t$  is the unique (exogenous) state variable, with vectors of coefficients  $a^{RE}$  and  $b^{RE}$  known to the agents. Under learning, agents instead estimate the linear regression

$$\hat{x}_t = a_t + b_t \hat{z}_t + \hat{u}_t, \tag{26}$$

where  $\hat{u}_t$  is the error term, and  $\{a_t, b_t\}$  are estimated vectors of parameters, which are updated via RLS as new data/information becomes available. More specifically, letting  $\omega_t \equiv \{a'_t, b'_t\}$ , we have that:

$$\omega_t = \omega_{t-1} + t^{-1} \Gamma_t^{-1} (\hat{x}_{t-1} - \omega'_{t-1} \hat{n}_{t-1}) \tag{27}$$

$$\Gamma_t = \Gamma_{t-1} + t^{-1} (\hat{n}_{t-1} \hat{n}'_{t-1} - \Gamma_{t-1}) \tag{28}$$

where (27) describes the updating algorithm for the regressor coefficients, and (28) the law of motion of the matrix of second moments for the stacked regressors  $\hat{n}_t \equiv \{1, \hat{z}_t\}$ .

At any point in time  $t$ , agents compute  $T$  periods ahead forecasts using the perceived law of motion (26), given their knowledge of the stochastic process driving  $\hat{z}_t$ :

$$\tilde{E}_t \hat{x}_T = a_t + \rho^{T-t} b_t \hat{z}_t \tag{29}$$

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<sup>8</sup>See also Honkapohja et al. (2012) for a discussion on the differences between the EEL and the IHL approach.

for  $T \geq t$ . Under EEL, agents are required to forecast only one period ahead, such that  $T = t + 1$  in (29). After substituting the forecast (29) into the reduced form structural equations describing the model, either under EEL or IHL, after collecting terms, one obtains the actual law of motion (ALM)

$$\hat{x}_t = \check{a}_t + \check{b}_t \hat{z}_t \tag{30}$$

where  $\{\check{a}_t, \check{b}_t\}$  are vectors, whose elements are functions of the estimated forecast parameters  $\{a_t, b_t\}$ . This RLS learning procedure defines a  $T$  mapping from estimated (or, perceived) to actual forecast coefficients:<sup>9</sup>

$$T(a_t, b_t) = (\check{a}_t, \check{b}_t).$$

A REE is a fixed point of this mapping, such that  $T(a^{RE}, b^{RE}) = (a^{RE}, b^{RE})$ . Following Evans and Honkapohja (2001), a REE is learnable/E-stable when the differential equation associated with the mapping  $T$ , namely

$$\frac{\partial}{\partial \tau}(a, b) = T(a, b) - (a, b),$$

is locally stable around the REE. This simply requires that all eigenvalues of the Jacobian matrix associated with the first order linear approximation of this differential equation around the REE have all negative real parts.

Most of the analysis to follow will adopt Preston’s approach. We will show that under IHL there exist a direct feedback from households’ financial wealth to real activity through the Euler equation, and therefore some potential benefits from having monetary policy granting an explicit response to stock prices. On the contrary, this feedback is absent under EEL, making such response a potential source of non-fundamental fluctuations and instability under learning. We start by highlighting this result in the next section.

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<sup>9</sup>This notation might be a little misleading, as we have used  $T$  also as an index of time.

## 4 Equilibrium Determinacy and E-Stability under Euler Equation Learning

Under RE, households and firms have complete knowledge of the objectives, constraints, decisions and beliefs by all economic agents in the economy (including themselves), as well as of all market clearing conditions. Hence, they can form expectations according to the true probability distributions prevailing in equilibrium. We can then study under what conditions on the response coefficients entering the policy rule (23) the REE is locally determinate, i.e., the equilibrium dynamics around the unique non-stochastic steady state do not depend on extrinsic uncertainty (e.g. sunspot shocks) but are exclusively driven by fundamentals.

Consider the individual household's Euler equation (17), and let  $\bar{I}_C \equiv \frac{\bar{I}}{\bar{C}}$  and  $\bar{a}_C \equiv \frac{\bar{a}}{\bar{C}}$  denote, respectively, the (steady state) income-to-consumption and wealth-to-consumption ratios. Given that, under full knowledge, households are identical - as they face identical decision problems and share the same beliefs - we can drop the superscript  $i$  from (17) to find an expression for consumption by the representative household:

$$\hat{C}_t = (1 - \beta) \bar{a}_C \hat{a}_t + (1 - \beta) \bar{I}_C E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T - \beta [\bar{I}_C - (1 - \sigma^{-1})] E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T, \quad (31)$$

where  $E_t$  now denotes the RE operator. With bonds in zero net supply and after normalizing to unity the equity shares issued by each firm in every period ( $S_{j,t} = 1$  for  $j \in [0, 1]$ ), the representative household's beginning-of-period nominal wealth is  $A_t = P_t (Q_t + D_t)$ , where  $Q_t \equiv \int_0^1 Q_{j,t} dj$  and  $D_t \equiv \int_0^1 D_{j,t} dj$  are, respectively, the aggregate stock price index and aggregate dividends. Letting  $\bar{Q}_C \equiv \frac{\bar{Q}}{\bar{C}}$  and  $\bar{D}_C \equiv \frac{\bar{D}}{\bar{C}}$  be, respectively, the (steady state) stock price-to-consumption and dividends-to-consumption ratios, the term  $\bar{a}_C \hat{a}_t$  entering (31) is then given by

$$\bar{a}_C \hat{a}_t = \bar{Q}_C \hat{Q}_t + \bar{D}_C \hat{D}_t, \quad (32)$$

Integrating the pricing equation (10) over  $j$  and log-linearizing it around the steady state, we find an expression for the stock price index  $\hat{Q}_t$ :

$$\hat{Q}_t = \beta E_t \hat{Q}_{t+1} + (1 - \beta) E_t \hat{D}_{t+1} - (\hat{R}_t - E_t \hat{\pi}_{t+1}) \quad (33)$$

After iterating this equation forward and imposing transversality,  $\hat{Q}_t$  is determined by the present discounted value of future aggregate dividends and real interest rates:

$$\hat{Q}_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{D}_{T+1} - \hat{r}_T \right], \quad (34)$$

Real dividends generated by the  $j$ -th wholesale firm are  $D_{j,t} = Y_{j,t} (1 - mc_t)$ . Letting  $Y_t \equiv \int_0^1 Y_{j,t} dj$  be aggregate output, we therefore have that  $D_t = Y_t (1 - mc_t)$ , whose log-linearization gives  $\hat{D}_t = \hat{Y}_t - (\mu - 1)^{-1} \widehat{mc}_t$  since in steady state real marginal costs are the inverse of the steady state price mark-up ( $\overline{mc} = \mu^{-1}$ ). This expression for  $\hat{D}_t$  can be then combined with the aggregate production function,  $\hat{Y}_t = \hat{z}_t + \hat{H}_t$ , and real marginal costs,  $\widehat{mc}_t = \hat{w}_t - \hat{z}_t$ , to give an expression for labor income  $\hat{I}_t$ : namely,  $\hat{I}_t = \hat{w}_t + \hat{H}_t = \mu \hat{Y}_t - (\mu - 1) \hat{D}_t$ . Since in equilibrium  $\hat{C}_t = \hat{Y}_t$ , we can then write the infinite-horizon Euler equation (31) as follows:

$$\begin{aligned} \hat{Y}_t &= \frac{\mu - 1}{\mu} \beta \hat{Q}_t + \frac{\mu - 1}{\mu} (1 - \beta) \hat{D}_t + (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T \\ &\quad - \frac{\mu - 1}{\mu} (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{D}_T - \beta [\bar{I}_C - (1 - \sigma^{-1})] E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \\ &= \frac{\mu - 1}{\mu} \beta \left[ \hat{Q}_t - (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{D}_{T+1} + E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \right] \\ &\quad + (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T - \frac{\beta}{\sigma} E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \\ &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{Y}_T - \frac{\beta}{\sigma} \hat{r}_T \right] \end{aligned} \quad (35)$$

where the first equality follows from the fact that  $\bar{Q}_C = \frac{\beta(\mu-1)}{(1-\beta)\mu}$ ,  $\bar{D}_C = \frac{\mu-1}{\mu}$ , and makes use of the equilibrium expression for  $\hat{I}_t$ ; the second equality from a simple re-arrangement of term; and the third equality from the equilibrium condition (34). We can further rearrange the last expression to obtain the following key relationship:

$$\hat{Y}_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{Y}_{T+1} - \sigma^{-1} \hat{r}_T \right] \quad (36)$$

Equation (36) states that current output depends on the expected discounted stream of its future

realizations, and future real interest rates. In particular, notice that households' holdings of equity shares does not generate any direct wealth effect on real activity. As we will show below, this will not be the case under IHL.

The assumption of RE allows us to retrieve from (36) the standard Euler equation appearing in the baseline New Keynesian framework. Leading (36) one period into the future and applying the RE operator  $E_t$ , we have that

$$E_t \hat{Y}_{t+1} = (1 - \beta) E_t \sum_{T=t+1}^{\infty} \beta^{T-t-1} \hat{Y}_{T+1} - \sigma^{-1} E_t \sum_{T=t+1}^{\infty} \beta^{T-t-1} \hat{r}_T,$$

Plugging the latter into (36), after simple algebra we obtain:

$$\begin{aligned} \hat{Y}_t &= E_t \left[ (1 - \beta) \hat{Y}_{t+1} - \sigma^{-1} \hat{r}_t \right] + E_t \sum_{T=t+1}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{Y}_{T+1} - \sigma^{-1} \hat{r}_T \right] \\ &= E_t \left[ (1 - \beta) \hat{Y}_{t+1} - \sigma^{-1} \hat{r}_t \right] + \beta E_t \hat{Y}_{t+1} \\ &= E_t \hat{Y}_{t+1} - \sigma^{-1} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) \end{aligned} \quad (37)$$

Namely, under RE, current economic activity depends solely on household's expectations of next period activity and the ex-ante real interest rate. As the household recognizes the recursive structure of the Euler equation, he does not need to form expectations about the infinite future.

Following a similar procedure, we can use (22) to find the Phillips curve. After replacing  $\tilde{E}_t$  with  $E_t$ , we can lead that expression one period forward, take expectations and plug the result back into (22), to find the standard New Keynesian Phillips curve relating current inflation to its own future expectations and current real marginal costs:

$$\hat{\pi}_t = \kappa \widehat{mc}_t + \beta E_t \hat{\pi}_{T+1} \quad (38)$$

where, from the labor supply equation (12) and the production technology, real marginal costs equal  $\widehat{mc}_t = (\sigma + \chi) \hat{Y}_t - (1 + \varphi) \hat{z}_t$ . Finally, under RE, the policy rule reads

$$\hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_y E_t \hat{Y}_{t+1} + \phi_q E_t \hat{Q}_{t+1}. \quad (39)$$

What are the implications of having monetary policy granting an explicit response to the stock



price index in the Taylor rule, i.e.  $\phi_q > 0$ , for the local determinacy of a REE? Around the steady state, the equilibrium dynamics are described by a linear system made of equations (37)-(39) together with the stock price equation (33) where  $\hat{D}_t = \left(1 - \frac{\sigma + \chi}{\mu - 1}\right) \hat{Y}_t - \frac{\chi}{\mu - 1} \hat{z}_t$ .

**Proposition 1** *Let  $\gamma \equiv (1 - \beta) \left(\frac{\sigma + \chi}{\mu - 1} - 1\right)$ . Moreover, assume that  $\phi_y < \frac{\sigma(1 + \beta)}{\beta}$ . The REE is locally determinate if and only if the following conditions are satisfied:*

$$i) \phi_q < \Phi^* \equiv \frac{(1 + \beta)\sigma}{\sigma + \gamma} - \frac{\beta}{\sigma + \gamma} \phi_y \quad (40)$$

$$ii) \phi_\pi > \Phi^L \equiv 1 + \frac{\gamma}{\kappa(\sigma + \chi)} \phi_q - \frac{(1 - \beta)}{\kappa(\sigma + \chi)} \phi_y \quad (41)$$

$$iii) \phi_\pi < \Phi^H \equiv 1 + \frac{2\sigma(1 + \beta)}{\kappa(\sigma + \chi)} - \frac{2\sigma + \gamma}{\kappa(\sigma + \chi)} \phi_q - \frac{(1 + \beta)}{\kappa(\sigma + \chi)} \phi_y \quad (42)$$

**Proof.** Under RE, the local dynamics around the steady state are described by the linear system  $\hat{x}_t = \Omega E_t \hat{x}_{t+1} + \Theta \hat{z}_t$ , where  $\hat{x}_t \equiv [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t]'$ , with the Jacobian matrix  $\Omega$  defined as

$$\Omega \equiv \begin{bmatrix} 1 - \frac{\phi_y}{\sigma} & \frac{1 - \phi_\pi}{\sigma} & -\frac{\phi_q}{\sigma} \\ \kappa(\sigma + \chi) \left(1 - \frac{\phi_y}{\sigma}\right) & \beta + \kappa(\sigma + \chi) \frac{1 - \phi_\pi}{\sigma} & -\kappa(\sigma + \chi) \frac{\phi_q}{\sigma} \\ -\gamma - \phi_y & 1 - \phi_\pi & \beta - \phi_q \end{bmatrix}, \quad (43)$$

while  $\Theta$  is a conformable matrix whose specification is not needed for the analysis. Since all variables in  $x_t = [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t]'$  are non-predetermined, the REE is locally determinate if and only if all eigenvalues of  $\Omega$  are within the unit circle in the complex plane. The characteristic polynomial of  $\Omega$  is  $P(e) = e^3 - tr(\Omega)e^2 + S_2(\Omega)e - \det(\Omega) = 0$  where  $tr(\Omega)$ ,  $S_2(\Omega)$  and  $\det(\Omega)$  denote, respectively, the trace, the sum of the 2x2 principal minors, and the determinant of matrix  $\Omega$ . By simple algebra we have that:

$$\begin{aligned} tr(\Omega) &= 2\beta + 1 - \frac{\phi_y}{\sigma} + \kappa(\sigma + \chi) \frac{1 - \phi_\pi}{\sigma} - \phi_q \\ S_2(\Omega) &= \beta(1 + \beta) + \beta \left(1 - 2\frac{\phi_y}{\sigma}\right) - \left(1 + \beta + \frac{\gamma}{\sigma}\right) \phi_q + \beta\kappa(\sigma + \chi) \frac{1 - \phi_\pi}{\sigma} \\ \det(\Omega) &= \beta \left[ \beta \left(1 - \frac{\phi_y}{\sigma}\right) - \frac{\phi_q}{\sigma} (\sigma + \gamma) \right] \end{aligned}$$

It is straightforward to check that one root of  $P(e) = 0$  is real and equal to  $\beta \in (0, 1)$ . This allows us to write the characteristic polynomial as  $P(e) = (e - \beta) \tilde{P}(e) = 0$  where  $\tilde{P}(e) \equiv (e^2 + a_1e + a_2) =$

0, with  $a_1 \equiv \phi_q + \frac{\phi_y}{\sigma} - 1 - \beta - \kappa(\sigma + \chi) \frac{1 - \phi_\pi}{\sigma}$  and  $a_2 \equiv \beta \left(1 - \frac{\phi_y}{\sigma}\right) - \frac{\phi_q}{\sigma}(\sigma + \gamma)$ . All roots of  $\tilde{P}(e) = 0$  are then within the unit circle if and only if a)  $\tilde{P}(1) = 1 + a_1 + a_2 > 0$ , b)  $\tilde{P}(-1) = 1 - a_1 + a_2 > 0$  and c)  $|a_2| < 1$ . Simple manipulation shows that these conditions are equivalent to those spelled in equations (40)-(42) in the proposition. The assumption  $\phi_y < \frac{\sigma(1+\beta)}{\beta}$  guarantees that  $\Phi^H > \Phi^L$  for  $\phi_q = 0$ , so that the determinacy space is non-empty. Notice that it also implies that  $\Phi^* > 0$ . ■

According to the conditions stated in Proposition 1, equilibrium determinacy has two key requirements. First, it requires the response coefficient to the stock prices to be below a certain upper bound  $\Phi^*$ . Second, provided  $\phi_q < \Phi^*$  holds, it requires the response coefficient to inflation,  $\phi_\pi$ , to be two-sided constrained, with the lower bound  $\Phi^L$  and the upper bound  $\Phi^H$  being, respectively, strictly increasing and strictly decreasing functions of the response coefficient  $\phi_q$ , for given value of  $\phi_y$ . In particular, notice that for  $\phi_q > 0$ , the lower bound  $\Phi^L$  becomes larger than unity, which is to say that a central bank granting a positive response to stock prices should also guarantee a sufficiently *active* response to inflation (re-inforced Taylor principle). For  $\phi_q = \Phi^*$ , the upper bound  $\Phi^H$  and the lower bound  $\Phi^L$  collide, making the determinacy region empty. The assumption  $\phi_y < \frac{\sigma(1+\beta)}{\beta}$  guarantees that  $\Phi^* > 0$ , such that the determinacy region is non-empty for  $\phi_q < \Phi^*$ .

We then move to the learnability of the MSV representation of a REE according to the EEL criterion proposed by Evans and Honkapohja (2001).<sup>10</sup>

**Proposition 2** *The Minimal State Variable (MSV) representation of a REE is E-stable under EEL if and only if the following condition holds*

$$\phi_\pi > \Phi^L \equiv 1 + \frac{\gamma}{\kappa(\sigma + \chi)}\phi_q - \frac{(1 - \beta)}{\kappa(\sigma + \chi)}\phi_y \quad (44)$$

**Proof.** Consider  $\Omega$  defined in (43). From the discussion in Section 3.2, for the MSV-FE to be E-stable all eigenvalues of the matrix  $M \equiv \Omega - I$  need to have negative real parts. Simple algebra shows that one of the eigenvalues of  $M$  is real and equal to  $\beta - 1 < 0$ . The characteristic polynomial of  $M$  can then be written as  $P(e) = (\beta - 1)\tilde{P}(e) = 0$  where  $\tilde{P}(e) = e^2 + a_1e + a_2 = 0$ , while  $a_1$  and  $a_2$  are again functions of the policy coefficients  $\phi_\pi$ ,  $\phi_y$ , and  $\phi_q$ . Applying standard results from matrix algebra, all roots of  $M$  have then negative real parts if and only if  $a_2 > 0$  and  $a_1 > 0$ . Simple algebra yields that all roots have negative real parts - hence, the MSV-FE is E-stable - if

<sup>10</sup>One could also show that, within the policy space where the equilibrium is indeterminate, there exists Common Factor (CF) representations of sunspot equilibria which are stable under EEL. See Airaudo et al. (2015).

and only if  $1 + \frac{\gamma}{\kappa(\sigma+\chi)}\phi_q - \frac{(1-\beta)}{\kappa(\sigma+\chi)}\phi_y$ . ■

Once we combine the results of Propositions 1 and 2, we can conclude the followings. First, if our expectational stability criterion was EEL, then the policy-maker should just make sure that the adopted policy pins down a locally unique REE, as the latter will always be attainable under learning. In this sense, the learnability requirement based on EEL does not add any additional restriction to those required by determinacy.

Second, since granting an explicit response to stock prices shrinks the region - namely, the range of values for  $\phi_\pi$  - where the REE is both determinate and learnable, the central bank should not respond to them, but rather keep its focus on inflation. Third, since, for realistic calibrations of the model and for  $\phi_q = 0$ , the upper bound  $\Phi^H$  is rather large (indeed much larger than any empirical estimate of  $\phi_\pi$  in the literature), the simple Taylor principle is necessary and *de facto* sufficient for the REE to be both determinate and learnable.

To get a quantitative sense for these analytical results, we parametrize the model as follows. Taking one period in the model to correspond to a quarter, we set the discount factor  $\beta$  equal to 0.99. The risk aversion coefficient  $\sigma$  is set equal to unity. The inverse Frisch labor elasticity,  $\chi$ , is equal to 0.25—i.e., an elasticity equal to 4—as common in the macro-labor literature.<sup>11</sup> Our baseline value for the price rigidity parameter  $\vartheta$  is 0.75, giving an expected price duration of one year. The parameterization of the elasticity of substitution across differentiated goods is based on the micro-evidence by Broda and Weinstein (2006). They report median elasticity values equal to, respectively, 2.5 and 2.1. for their pre-1990 and post-1990 samples on sectoral U.S. data. We therefore set  $\epsilon$  equal to 2.3, their mid-point estimate. This implies that monopolistically competitive firms enjoy a considerable degree of market power, which, in our set-up, is necessary to generate sufficiently large profits/dividends and, as a result, sufficiently large gains from equity holdings. Finally, we allow for a positive response to output in the Taylor rule by setting  $\phi_y$  equal to 0.5, a standard value in the literature.<sup>12</sup>

The results are displayed in Figure 1. Notice that, although from a theoretical perspective an explicit response to stock prices in the policy rule is detrimental for both determinacy and E-stability, in practice, as long as the response to the stock price is not excessive, it is sufficient to set  $\phi_\pi$  to standard values to guarantee a unique REE stable under EEL. For instance, assuming that

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<sup>11</sup>See, for instance, Imai and Keane (2004).

<sup>12</sup>See Sections 5.2 and 5.3 for alternative parameterizations of  $\epsilon$ .

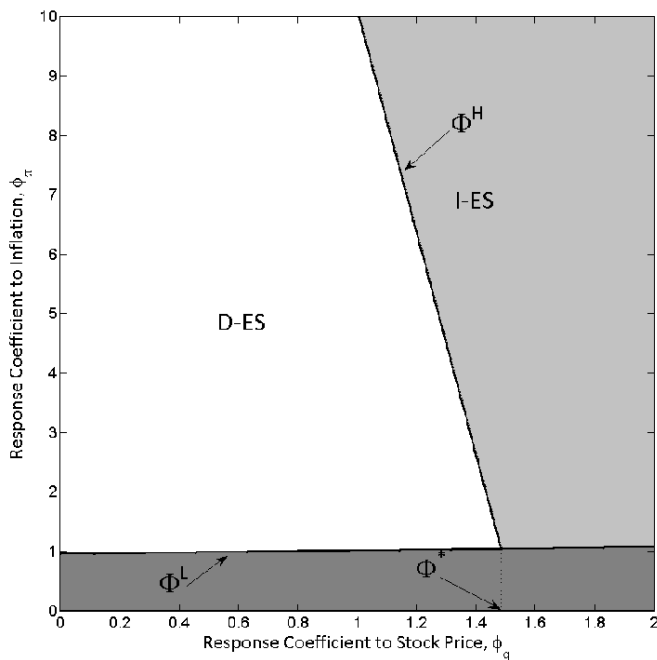


Figure 1: **Equilibrium Determinacy and E-Stability under EEL.**

$\phi_q$  remains below unity, equilibrium determinacy obtains for  $\phi_\pi$  smaller than 10. With the lower bound  $\Phi^L$  essentially flat, the Taylor principle is all that is needed for determinacy and E-stability.

## 5 E-Stability under Infinite-Horizon Learning

As previously discussed, without RE, the infinite-horizon forecasts that households and firms need to form about their pay-off relevant variables cannot be reduced to simple one-period-ahead expectations as obtained in Section 3.1.<sup>13</sup>

Under IHL, firms' pricing behavior is still described by the infinite-horizon Phillips curve (22) derived in Section 2.2:

$$\hat{\pi}_t = \kappa (\hat{w}_t - \hat{z}_t) + \tilde{E}_t \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} [\kappa\beta\vartheta (\hat{w}_{T+1} - \hat{z}_{T+1}) + (1 - \vartheta) \beta\hat{\pi}_{T+1}] \quad (45)$$

For what concerns the households, according to the Euler equation (17), they have to forecast

<sup>13</sup>See Preston (2005) for a detailed discussion on this.

the infinite future streams of own labor income and the ex-ante real interest rate. Assuming no taxes/transfers, from the definition of labor income  $I_t^i$ , we have that:

$$\hat{I}_t^i = \hat{w}_t + \hat{H}_t^i = \frac{1+\chi}{\chi} \hat{w}_t - \frac{\sigma}{\chi} \hat{C}_t^i \quad (46)$$

where the second equality follows from the consumption-leisure trade-off equation (12).<sup>14</sup> This allows us to write the present discount value of the future stream of labor incomes as follows:

$$\begin{aligned} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i &= \frac{1+\chi}{\chi} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_T - \frac{\sigma}{\chi} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i \\ &= \frac{1+\chi}{\chi} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_T - \frac{\sigma}{\chi} \frac{\hat{C}_t^i}{1-\beta} - \frac{\beta}{(1-\beta)\chi} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T, \end{aligned} \quad (47)$$

where the second equality follows from making use of the linearized Euler equation

$$\hat{C}_t^i = \tilde{E}_t^i \hat{C}_{t+1}^i - \frac{1}{\sigma} \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) \quad (48)$$

with  $\hat{r}_T \equiv \hat{R}_T - \tilde{E}_T^i \hat{\pi}_{T+1}$ .

Plugging (47) into the infinite-horizon Euler equation (17), while letting  $\bar{I}_C \equiv \frac{\bar{I}}{\bar{C}}$ ,  $\bar{a}_C \equiv \frac{\bar{a}}{\bar{C}}$ ,  $s_C \equiv 1 + \frac{\sigma}{\chi} \bar{I}_C$ , and  $\xi \equiv \beta \left( \bar{I}_C \frac{1+\chi}{\chi} + \sigma^{-1} - 1 \right)$ , after simple rearrangement of terms we obtain the following expression for individual consumption:

$$\hat{C}_t^i = s_C^{-1} \bar{a}_C (1-\beta) \hat{a}_t^i + s_C^{-1} (1-\beta) \bar{I}_C \frac{1+\chi}{\chi} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_T - s_C^{-1} \xi \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T, \quad (49)$$

Once again, let  $\hat{Q}_t \equiv \int_0^1 \hat{Q}_{j,t} dj$ , and  $\hat{D}_t \equiv \int_0^1 \hat{D}_{j,t} dj$  denote, respectively, the aggregate stock price index and aggregate dividends. Since  $\int_0^1 \hat{b}_t^i di = 0$  (as bonds are in zero net supply) and  $\int_0^1 \int_0^1 \hat{S}_{j,t-1}^i didj = 0$  (as equity shares are in fixed supply), the integration of equation (18) - which defines (log) individual wealth  $\hat{a}_t^i$  - over all households yields an expression for aggregate financial wealth  $\hat{a}_t \equiv \int_0^1 \hat{a}_t^i di$ :

$$\bar{a} \hat{a}_t = \bar{Q} \hat{Q}_t + \bar{D} \hat{D}_t \quad (50)$$

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<sup>14</sup>The individual labor supply equation is part of the household's information set, as it describes how much labor he is willing to supply, at any point in time, for given real wage and given individual consumption.

Using the latter, together with the fact that, in the steady state equilibrium  $\bar{a} = \bar{Q} + \bar{D}$ , and defining the average expectation operator  $\tilde{E}_t \equiv \int_0^1 \tilde{E}_t^i di$ , we can integrate (49) over all households to find aggregate consumption  $\hat{C}_t \equiv \int_0^1 \hat{C}_t^i di$ :

$$\hat{C}_t = s_C^{-1} (1 - \beta) \left( \bar{Q}_C \hat{Q}_t + \bar{D}_C \hat{D}_t \right) + s_C^{-1} (1 - \beta) \bar{I}_C \frac{1 + \chi}{\chi} \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_T - s_C^{-1} \xi \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \quad (51)$$

where, as in the previous section,  $\bar{Q}_C \equiv \frac{\bar{Q}}{C}$  and  $\bar{D}_C \equiv \frac{\bar{D}}{C}$ .

To find an expression for  $\hat{Q}_t$ , we start from the linearization of the stock price equation (10), from which we have:

$$\hat{Q}_{j,t} = \beta \tilde{E}_t^i \hat{Q}_{j,t+1} + (1 - \beta) \tilde{E}_t^i \hat{D}_{j,t+1} - \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right), \text{ for } i, j \in [0, 1]$$

We can then integrate the latter over the  $j$ -indexed firms to obtain the aggregate stock price index:

$$\hat{Q}_t = \beta \tilde{E}_t^i \hat{Q}_{t+1} + (1 - \beta) \tilde{E}_t^i \hat{D}_{t+1} - \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) \quad (52)$$

Notice that equation (52) corresponds to a no-arbitrage condition for the individual agent  $i$  to hold the riskless asset and the equally weighted risky equity portfolio: the expected real return from the riskless bond,  $\hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1}$ , should equal the expected real return from investing in the risky equity portfolio,  $\beta \tilde{E}_t^i \hat{Q}_{t+1} + (1 - \beta) \tilde{E}_t^i \hat{D}_{t+1} - \hat{Q}_t$ .

Despite the fact that (52) has to hold at all times, and for any specification of individual beliefs  $\tilde{E}_t^i$ , in general, it is not possible to iterate it forward, and, after imposing transversality, recover an equation having the stock price index equal to the present discounted value of future aggregate dividends. Following Adam and Marcet (2011), suppose the  $i$ -th household is the marginal agent pricing the risky portfolio at time  $t$ , and let  $m_T \in [0, 1]$  denote the marginal agent in period  $T > t$  (since this will not necessarily be agent  $i$ ). By forward iteration of (52) till period  $T$ , we obtain:

$$\begin{aligned} \hat{Q}_t &= (1 - \beta) \left[ \tilde{E}_t^i \hat{D}_{t+1} + \beta \tilde{E}_t^i \tilde{E}_{t+1}^{m_{t+1}} \hat{D}_{t+2} + \beta^2 \tilde{E}_t^i \tilde{E}_{t+1}^{m_{t+1}} \tilde{E}_{t+2}^{m_{t+2}} \hat{D}_{t+3} + \dots \right] \\ &\quad - \left[ \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} + \beta \tilde{E}_t^i \left( \hat{R}_{t+1} - \tilde{E}_{t+1}^{m_{t+1}} \hat{\pi}_{t+2} \right) + \beta^2 \tilde{E}_t^i \tilde{E}_{t+1}^{m_{t+1}} \left( \hat{R}_{t+2} - \tilde{E}_{t+2}^{m_{t+2}} \hat{\pi}_{t+3} \right) + \dots \right] \\ &\quad + \beta^T \tilde{E}_t^i \tilde{E}_{t+1}^{m_{t+1}} \dots \tilde{E}_{T-1}^{m_{T-1}} \hat{Q}_T \end{aligned}$$

For this equation to collapse to  $\hat{Q}_t = \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \hat{D}_{T+1} - (\hat{R}_T - \hat{\pi}_{T+1}) \right]$  we have to apply the law of iterated expectations to its right hand side, and then impose transversality. While this can certainly be done under RE - as in that case all households are identical, and the representative agent is always the marginal investor - in our learning framework, it would require the  $i$ -th household to believe he will remain forever the marginal investor, or to believe that future marginal investors will have the same beliefs he has. Since we are considering arbitrary beliefs, even if the no arbitrage condition implied by (52) has to hold at all times, there is no reason to expect the stock price index to equal the present discounted value of dividends. We therefore take (52) as the relevant asset pricing equation.

Integrating it over all  $i$ -indexed households, with  $\tilde{E}_t \equiv \int_0^1 \tilde{E}_t^i di$ , yields:

$$\hat{Q}_t = \beta \tilde{E}_t \hat{Q}_{t+1} + (1 - \beta) \tilde{E}_t \hat{D}_{t+1} - \left( \hat{R}_t - \tilde{E}_t \hat{\pi}_{t+1} \right) \quad (53)$$

The stock price index  $\hat{Q}_t$  is then simply a convex combination of its future expectation and future expected dividends, discounted by the ex-ante real interest rate. Although household do not possess any knowledge about the policy rule adopted by the central bank, they can forecast future nominal interest rates by exploiting the non-arbitrage condition (53), which can be rearranged to give an expression for the nominal interest rate  $\hat{R}_t$ :

$$\hat{R}_t = \beta \tilde{E}_t \hat{Q}_{t+1} + (1 - \beta) \tilde{E}_t \hat{D}_{t+1} + \tilde{E}_t \hat{\pi}_{t+1} - \hat{Q}_t \quad (54)$$

Using the latter the present discounted value of expected future nominal interest rates can be written as follows:

$$\begin{aligned} \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{R}_{T+1} &= \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ \beta \hat{Q}_{T+2} + (1 - \beta) \hat{D}_{T+2} + \hat{\pi}_{T+2} - \hat{Q}_{T+1} \right] \\ &= (1 - \beta) \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{D}_{T+2} + \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\pi}_{T+2} - \tilde{E}_t \hat{Q}_{T+1} \end{aligned} \quad (55)$$

After combining equations (24) and (55) with (51), and making use of the market clearing condition  $\hat{C}_t = \hat{Y}_t$ , we obtain a final expression for the infinite-horizon Euler equation (or IS curve) describing aggregate activity:

$$\begin{aligned}
\hat{Y}_t &= s_C^{-1} (1 - \beta) \left( \bar{Q}_C \hat{Q}_t + \bar{D}_C \hat{D}_t \right) + s_C^{-1} (1 - \beta) \bar{I}_C \frac{1 + \chi}{\chi} \hat{w}_t \\
&\quad - s_C^{-1} \xi \left( \hat{R}_t - \tilde{E}_t \hat{\pi}_{t+1} \right) + s_C^{-1} \xi \beta \tilde{E}_t \hat{Q}_{t+1} \\
&\quad + s_C^{-1} \bar{I}_C \frac{1 + \chi}{\chi} \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1 - \beta) \hat{w}_{T+1} - s_C^{-1} \xi \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1 - \beta) \hat{D}_{T+2} \quad (56)
\end{aligned}$$

where  $\bar{Q}_C = \frac{\beta(\mu-1)}{(1-\beta)\mu}$ ,  $\bar{D}_C = \frac{\mu-1}{\mu}$  and  $\bar{I}_C = \mu^{-1}$ , while  $\hat{Q}_t$  is determined by (53), and *current* real dividends  $\hat{D}_t$  and the real wage  $\hat{w}_t$  are given by

$$\hat{w}_t = (\sigma + \chi) \hat{Y}_t - \chi \hat{z}_t \quad (57)$$

$$\hat{D}_t = \left( 1 - \frac{\sigma + \chi}{\mu - 1} \right) \hat{Y}_t + \frac{1 + \chi}{\mu - 1} \hat{z}_t \quad (58)$$

Notice that the Euler equation (56) does not include expectations about future nominal interest rates.<sup>15</sup>

Following the discussion in Section 3.2, households and firms form beliefs about their pay-off relevant variables using simple linear rules, which share the same functional form of the MSV representation of the underlying RE solution. In particular, according to the structural equations describing the behavior of households, firms and the central bank - that is, respectively, equations (56), (45) and (23) - households and firms need to forecast future wages, inflation, output, the stock price index and dividends. They will do that using their PLMs:

$$\hat{x}_t = a + b \hat{z}_t \quad (59)$$

for  $\hat{x}_t \equiv [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t, \hat{w}_t, \hat{D}_t]'$ ,  $a \equiv [a_y, a_\pi, a_q, a_w, a_d]'$  and  $b \equiv [b_y, b_\pi, b_q, b_w, b_d]'$ . In particular, for a generic variable  $\hat{m}_t = a_m + b_m \hat{z}_t$ , for  $m = w, \pi, Y, Q, D$ , and known AR(1) process for  $\hat{z}_t$ , we have that  $\tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{m}_{T+n} = \frac{a_m}{1-\beta} + \frac{\rho^n b_m}{1-\beta\rho} \hat{z}_t$ . Plugging these expectations back into the structural equations (23), (45), and (56)-(58), we have a system of ALMs

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<sup>15</sup>This is an important element of differentiation between our model and Preston (2006), and, as discussed below, a reason why expectation-based rule perform better in our set-up.



$$F_1 \hat{x}_t = F_2 a + (F_3 b + F_4) \hat{z}_t \quad (60)$$

where  $F_i$ , for  $i = 1, \dots, 4$  are 5x5 matrices containing structural parameters.<sup>16</sup> The implied  $T$ -mapping is then  $T(a, b) = (F_1^{-1} F_2 a, F_1^{-1} (F_3 b + F_4))$ , with associated differential equation, in notional time  $\tau$ , given by

$$\begin{aligned} \frac{\partial}{\partial \tau} (a, b) &= T(a, b) - (a, b) \\ &= (F_1^{-1} F_2 a, F_1^{-1} (F_3 b + F_4)) - (a, b) \end{aligned}$$

The related Jacobian matrices with respect to the vector of coefficients  $a$  and  $b$  are, respectively

$$DT_a = F_1^{-1} F_2 - I_5 \quad (61)$$

$$DT_b = F_1^{-1} F_3 - I_5 \quad (62)$$

As discussed in Section 3.2, E-stability for the REE requires all eigenvalues of the matrices  $DT_a$  and  $DT_b$  to have negative real parts.

Because of the dimensions of matrices (61)-(62), it is hard to obtain meaningful analytical conditions for E-stability. We therefore resort to numerical approximation methods under the benchmark parameterization used in the previous section.

Figure 2 displays the results. As we had in Figure (1), under RE, the equilibrium is locally determinate for combinations of  $\phi_\pi$  and  $\phi_q$  falling within the regions labeled with a D (the white and the very dark gray areas), with boundaries given by the straight lines  $\Phi^L$  and  $\Phi^H$ , crossing at the upper-bound  $\Phi^* \approx 1.5$ . As stated in Proposition 2, under EEL, a determinate REE is also E-stable. Suppose the central bank set  $\phi_\pi = 1.5$ , the most common value used in the literature. By the results of Proposition 2, as displayed in the figure, the REE would be both determinate and E-stable as long as the policy response to the stock price was not too large, i.e.,  $\phi_q$  smaller than, roughly, 1.42.

The key result occurring under IHL is that a policy parameterization that leads to equilibrium

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<sup>16</sup>See Appendix A.3 for a detailed derivation.

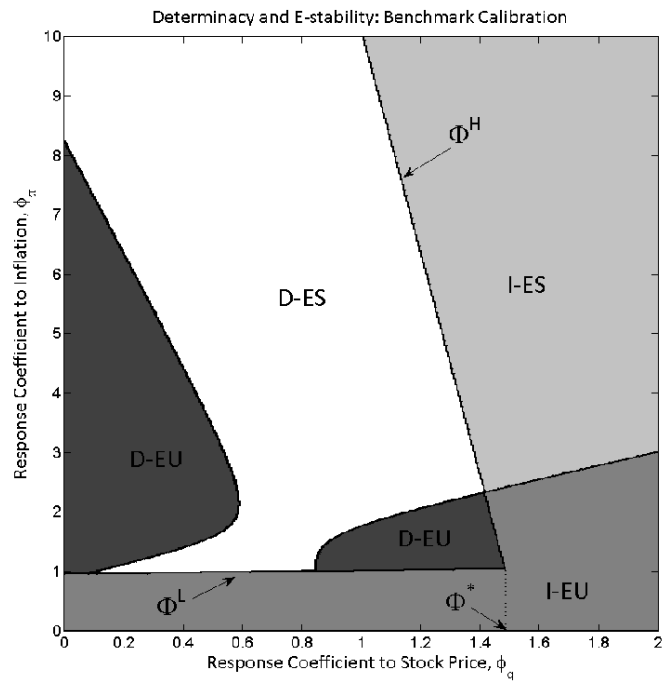


Figure 2: **Equilibrium Determinacy and E-Stability under IHL.** D = determinacy; I = indeterminacy; ES = E-stable; EU = E-unstable.  $\Phi^H$ ,  $\Phi^L$  and  $\Phi^*$  correspond to the definitions in Proposition 1.

determinacy does not necessarily guarantee its learnability. In particular, notice that we now have to distinguish between the case of a determinate REE which is stable under IHL (the white area, labeled D-ES), and one where it is not stable under IHL (the dark gray areas, labeled D-EU). For instance, consider the case of a central bank that does not respond to stock prices,  $\phi_q = 0$ . For the REE to be both determinate and E-stable under IHL, the central bank needs to ensure a very aggressive response to inflation (larger than 8!). This is a policy which a central bank would be very unlikely to commit. How can we restore learnability then? As the figure shows, for given plausible response to inflation, E-stability requires a sufficiently positive (but still not too large) response to stock prices. For instance, for  $\phi_\pi = 2$ , a determinate REE is ensured by setting  $\phi_q$  slightly above 0.6, while having  $\phi_q$  around 0.2 seems enough for  $\phi_\pi = 1.2$ .

## 5.1 Some Intuition

To build some intuition, it is useful to compare our results to those obtained by Preston (2005, 2006) who also considers the stability properties of forward-looking policy rules under IHL. A key difference with respect to ours, is that in his set-up households do not trade in firms' equity shares, so that dividends are equally distributed across all households, as lump-sum transfers. As shown in the Appendix, without equity trading, the infinite-horizon Euler equation (56) is substituted by the following:<sup>17</sup>

$$\begin{aligned}
\hat{Y}_t = & s_C^{-1} (1 - \beta) \frac{\mu - 1}{\mu} \hat{D}_t + s_C^{-1} (1 - \beta) \frac{1 + \chi}{\mu \chi} \hat{w}_T - s_C^{-1} \left( \xi + \beta \frac{\mu - 1}{\mu} \right) \hat{R}_t \\
& + s_C^{-1} (1 - \beta) \frac{1 + \chi}{\mu \chi} \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_{T+1} + s_C^{-1} (1 - \beta) \frac{\mu - 1}{\mu} \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{D}_{T+1} \quad (63) \\
& - s_C^{-1} \left( \xi + \beta \frac{\mu - 1}{\mu} \right) \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{R}_{T+1} + s_C^{-1} \left( \xi + \beta \frac{\mu - 1}{\mu} \right) \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\pi}_{T+1}
\end{aligned}$$

The remaining structural equations are the Phillips curve (22), the wage equation (57), the dividends equation (58), and the policy rule (23), with  $\phi_q = 0$  since, with agents not trading in equity shares, there is no pricing equation for equilibrium stock prices.

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<sup>17</sup>In Preston (2005, 2006), household are assumed to form forecast of total income  $I_t^i$  which now corresponds to wage income,  $\frac{w_t}{P_t} H_t^i$ , plus dividends income,  $D_t$ . We maintain instead the assumption that households use their consumption-leisure trade-off equation (12) to forecast their future labor supply. For what concerns E-stability, our results do not depend on whether households forecast total labor income, or wages and dividends separately.

Consider the Euler equation (63). Notice that, missing the no arbitrage condition (54), agents have to form infinite horizon forecasts not only about future wages and dividends, as in (56), but also about future nominal interest rates and inflation. As Preston (2006) argues, this is the key source of instability under learning dynamics for forward-looking rules. Agents' forecasts of future nominal interest rates do not have to be in line with those implied by the underlying policy rule under RE - that is, agents are not necessarily expecting higher future real interest rates following higher future inflation - which makes the standard aggregate demand channel of monetary policy transmission weaker.

The results displayed in Figure 3 highlight the key role played by equity share trading for the E-stability of the MSV-REE. Under Preston's set-up (panel a), when the central bank adopts a forward-looking rule, we have that 1) a determinate REE is never E-stable (dark gray area), and 2) the MSV-REE can be learned only if the central bank grants an unrealistically high response to inflation, of an order of magnitude around 125 (light gray area).<sup>18</sup> However, in this case, the learnable MSV-REE coexists with a continuum of sunspot-driven equilibria. As the figure shows, the determinacy/indeterminacy and E-stability/instability regions with respect to the policy coefficient  $\phi_\pi$  are insensitive to the intertemporal elasticity of substitution  $\epsilon$ .

Panel b. corresponds to our set-up where equity shares trading is allowed, assuming that the central bank does not grant any explicit response to stock prices, i.e.,  $\phi_q = 0$ . The first thing to notice is that, while the determinacy area still does not depend on  $\epsilon$  (i.e., the REE is locally determinate for values of  $\phi_\pi$  below, about, 28, for any value of  $\epsilon$ , as in Preston's model), the E-stability/instability frontier is strictly increasing in it. In other words, the value range for  $\phi_\pi$  giving E-stability is wider when  $\epsilon$  is lower. For instance, for the benchmark calibration of  $\epsilon = 2.3$ , the minimum response to inflation for E-stability drops from 125 to, about, 8.25. The second (more important) element of differentiation is that, for values of  $\epsilon$  falling between 2 and 4, it is now possible to have a REE which is both determinate and E-stable (white area).

Why is our model more prone to E-stability than Preston's? And why an explicit response to the stock price index can ensure E-stability (and determinacy) without requiring unrealistically large responses to inflation? As argued above, under Preston's set-up, the standard aggregate demand channel of monetary policy transmission is rather weak as agents have to form infinite-horizon forecasts about future inflation and interest rates. If, for whatever reason, agents' expectations de-

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<sup>18</sup>We have fixed the response to output  $\phi_y = 0.5$ , as in our benchmark model.

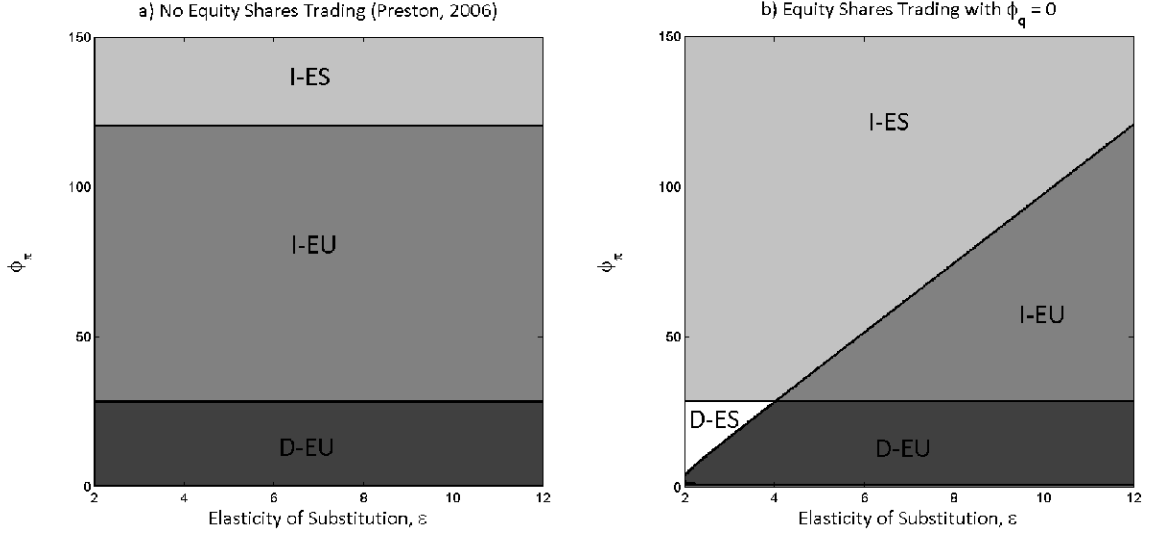


Figure 3: **Equilibrium Determinacy and E-Stability under IHL: Comparison with Preston (2006)**. The results displays in Panel a) are for Preston (2006) set-up without equity share trading. Those in Panel b) are for our set-up. D = determinacy; I = indeterminacy; ES = E-stable; EU = E-unstable.

viate from the RE solution, monetary policy will have a hard time bringing them back, even if it follows the Taylor principle. As we can see by comparing (56) and (63), such source of aggregate instability is less prominent in our set-up since, by taking advantage of the no-arbitrage condition between risky equity and riskless bonds, households do not need to forecast future interest rates.

To better see that, for analytical simplicity, let  $\sigma = 1$ , as in our benchmark parameterization. Combining the Euler equation (56) with the stock price equation (53), the policy rule (23), the real wage (57) and dividends (58), after simple manipulation, current output  $\hat{Y}_t$  is determined by the following expression:

$$\hat{Y}_t = - \underbrace{\left[ (\phi_\pi - 1) \tilde{E}_t \hat{\pi}_{t+1} + (\phi_q - \beta) \tilde{E}_t \hat{Q}_{t+1} \right]}_{\hat{\Delta}_t} + s_C^{-1} \beta \frac{\mu - 1}{\mu} (1 - \beta) \tilde{E}_t \hat{D}_{t+1} + \beta \frac{1 + \chi}{1 + \mu \chi} \Sigma_{w,D} \quad (64)$$

For given expectations on future wages and dividends, contained in the residual term  $\Sigma_{w,D}$ , current

output  $\hat{Y}_t$  depends negatively on the term  $\hat{\Delta}_t \equiv (\phi_\pi - 1)\tilde{E}_t\hat{\pi}_{t+1} + (\phi_q - \beta)\tilde{E}_t\hat{Q}_{t+1}$ . The latter includes the *ex-ante* real interest rate,  $\hat{R}_t - \tilde{E}_t\hat{\pi}_{t+1} = (\phi_\pi - 1)\tilde{E}_t\hat{\pi}_{t+1} + \phi_q\tilde{E}_t\hat{Q}_{t+1}$ , and a term  $-\beta\tilde{E}_t\hat{Q}_{t+1}$  coming from the direct wealth effect of equity holding on consumption. Following an increase in household's expectations of next period inflation and stock prices, a central bank responding actively but exclusively to inflation - i.e.,  $\phi_\pi > 1$  and  $\phi_q = 0$  - will not necessarily put downward pressure on aggregate demand and therefore bring those expectations down (via, respectively, the Phillips curve and the stock price equation) since the increase in  $\tilde{E}_t\hat{Q}_{t+1}$  creates a wealth effect that may counter-act the current increase in the real interest rate  $(\phi_\pi - 1)\tilde{E}_t\hat{\pi}_{t+1}$ , leading to a decrease (rather than an increase) in  $\hat{\Delta}_t$ . To increase the latter, the central bank has to generate a large real interest rate hike by picking a sufficiently large value for  $\phi_\pi$ . This effect is further exacerbated by the fact that current consumption still depends on beyond-next-period future expectations of wages and dividends, on which the central bank has weaker influence. With a positive response to stock prices,  $\phi_q > 0$ , following the initial changes in expectations, the term  $\hat{\Delta}_t$  is more likely to increase without requiring a large response coefficient to inflation.

Equation (64) is also helpful to explain why a lower elasticity of substitution  $\epsilon$  facilitates E-stability. As a lower elasticity  $\epsilon$  implies a larger  $\mu$ , the coefficient multiplying  $\Sigma_{w,D}$  gets smaller, making the infinite-horizon forecasts of wages and dividends less important for the determination of current output. Since those long-term forecast are a source of instability, a lower  $\epsilon$  makes the term  $\hat{\Delta}_t$  relatively more important, thus strengthening the effectiveness of monetary policy.

## 5.2 Sensitivity Analysis

We now consider alternative calibrations for some of the key parameters of the model, such as the degree of nominal price rigidity, the Frisch elasticity of labor supply, risk aversion, and the elasticity of substitution across varieties. We change one parameter at a time, keeping the remaining ones at baseline values. Figure 4 displays the results. As shown by the four panels, the size of the policy space for which the equilibrium is both determinate and E-stable (white area) is inversely related to risk aversion (panel c) and the elasticity of substitution across differentiated goods (panel d), while is positively related to the Frisch elasticity of labor supply (panel b). Its relationship with the degree of price rigidity is more ambiguous (panel a).

Let's start with the latter. An increase in price flexibility (e.g.,  $\vartheta$  goes from 0.75 to 0.66) appears to have two opposite effects. On the one hand, it shrinks the policy space leading to a determinate

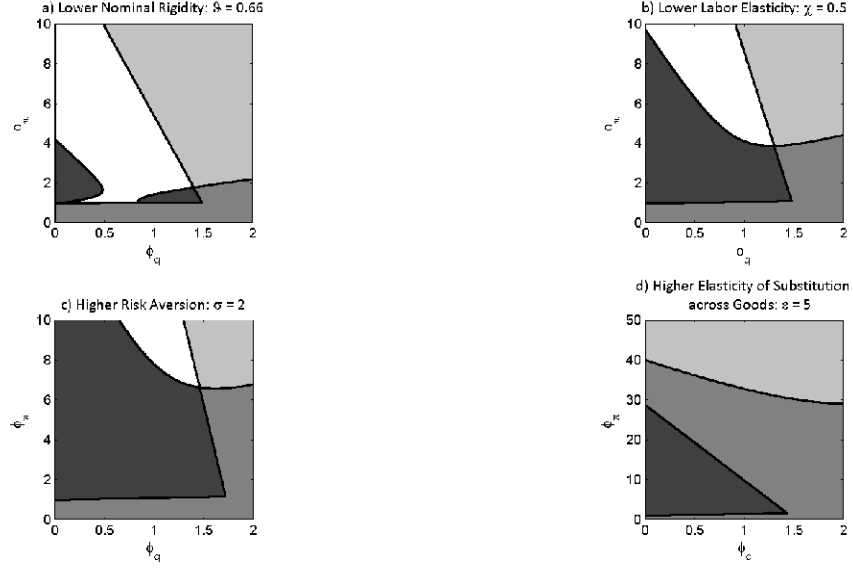


Figure 4: **Equilibrium Determinacy and E-Stability under IHL: Alternative Calibration.**

equilibrium. This is due to the fact that lowering  $\vartheta$  yields a larger  $\kappa$  in the Phillips curve, which in turn diminishes the upper-bound on  $\phi_\pi$  for determinacy (see condition (42) in Proposition 1). On the other hand, it lowers the minimum response to  $\phi_\pi$  for which the fundamental REE is stable (in the specific case, from 8.25 to 4). This is because, with more flexible prices, long-horizon inflation expectations in the Phillips curve become less important (firms' pricing decisions depend more on current marginal costs), which strenghtens the stabilizing role of monetary policy.

A decrease in households' willingness to supply labor (i.e., from 4 to 2, by increasing  $\chi$  from 0.25 to 0.5) affects the household's Euler equation (56) via two different channels.<sup>19</sup> First, it increases  $s_C^{-1}$ , which in turn increases the wealth elasticity of consumption, thus strenghtening the wealth effect due to agents' bounded-rationality. Second, the wage elasticity of consumption is  $s_C^{-1} (1 - \beta) \frac{1+\chi}{\mu\chi} = (1 - \beta) \frac{1+\chi}{1+\mu\chi}$ , which, for  $\mu > 1$ , is a decreasing function of  $\chi$ .<sup>20</sup> Hence, a less elastic labor supply has both destabilizing (via the first channel) and stabilizing (via the second channel) effects. As panel b) shows, the former appears to dominate.

Larger risk aversion (e.g., from 1 to 2) leads to a lower intertemporal elasticity of substitution,

<sup>19</sup>Labor elasticity also affects the slope of the Phillips curve as it determines the responsiveness of the real wage to aggregate activity.

<sup>20</sup>More generally, we have that the wage elasticity of consumption is decreasing (respectively, decreasing) in  $\chi$  for  $\sigma$  larger (respectively, smaller) than  $\mu$ .

which can undermine the demand channel of monetary policy transmission. From the Euler equation (56) again, the (negative) real interest rate elasticity of output is  $s_C^{-1}\xi = \beta \frac{\sigma(1+\chi)+(1-\sigma)\mu\chi}{(\sigma+\mu\chi)\sigma}$ . Simple calculus shows that, for  $\mu < \frac{1+\chi}{\chi}$  (which is the case under our baseline calibration), such elasticity is strictly decreasing in  $\sigma$ . With a weaker transmission channel, monetary policy becomes less effective at controlling expectations, which then leads to instability (see panel c).

Our benchmark parameterization for the elasticity of substitution  $\epsilon$  is rather low if compared to what used in the macro literature, where values typically range between 5 and 11.<sup>21</sup> In panel d), we consider  $\epsilon = 5$ . A higher elasticity of substitution between differentiated products lowers  $\mu$ . From equation (64), we can see that this increases the wage elasticity of output, which, as previously discussed, is a source of instability.<sup>22</sup> In particular, we find that monetary policy can never guarantee determinacy and E-stability at the same time (there is no white area). A determinate equilibrium is always unstable under IHL (dark gray area), while E-stability can be attained with a very aggressive response to inflation. Responding to stock prices does not appear to have significant beneficial effects.

### 5.3 Real Wage Rigidity

One way to allow the model to deliver a determinate and E-stable REE for larger values of  $\epsilon$  is to introduce some sluggish adjustment in wages by dampening their sensitivity to market conditions. The simplest way to do this without a substantial modification of the model is to assume that the *real* wage does not fully respond to labor market conditions, as a result of unmodeled imperfections, similar to Blanchard and Gali (2007). More specifically, let the real wage paid to workers be a weighted average of a *notional wage* (with weight  $1 - \omega$ ) and, using the terminology of Hall (2005), of a *wage norm* (with weight  $\omega$ ). We set the notional wage equal to the real wage occurring in a perfectly flexible labor market, i.e., the marginal rate of substitution between consumption and leisure as given by equation (12). As the wage norm, we consider instead the steady state real wage.<sup>23</sup> The (log) real wage is therefore  $\hat{w}_t = (1 - \omega) \left[ (\sigma + \chi) \hat{Y}_t - \chi \hat{z}_t \right]$ , where the parameter

<sup>21</sup>Such elasticity is often calibrated to give a steady state net price mark-up ranging between 10 and 25%.

<sup>22</sup>A smaller  $\mu$  lowers the coefficient  $s_C^{-1}\beta \frac{\mu-1}{\mu} (1 - \beta)$  multiplying  $\tilde{E}_t \hat{D}_{t+1}$  in (64). However, since  $\beta \approx 1$ , this effect is negligible.

<sup>23</sup>Other formulations of *real* wage rigidities assume the wage norm to be equal to the past wage  $\frac{W_{t-1}}{P_{t-1}}$ , such that the (log) real wage corresponds to an exponentially-decaying weighted average of the infinite stream of past flexible real wages. See, for instance, Uhlig (2007). Our simpler specification retains the same logic—namely, the current real wage does not fully respond to current labor market conditions—without requiring any major modification to the reduced form linear system.



$\omega \in [0, 1]$  indexes the degree of rigidity.<sup>24</sup>

There are two channels through which sticky wages affect our results. Recall the relationship between current output  $\hat{Y}_t$  and the term  $\hat{\Delta}_t$  from equation (64). On the one hand, as wages respond less to aggregate activity, so do agents' expectations about their future dynamics, making the model less susceptible to the instability due to infinite-horizon learning. On the other hand, for  $\sigma = 1$ , simple algebra shows that the temporary elasticity of  $\hat{Y}_t$  to  $\hat{\Delta}_t$  is equal to  $\frac{(1+\chi\mu)\beta}{\beta(1+\chi\mu)+(1-\beta)(1+\chi)\omega}$ . The latter is strictly decreasing in  $\omega$ : i.e., stickier wages weaken the impact of current monetary policy on real activity. While these two channels clearly go in opposite directions, the former dominates since, for  $\beta \approx 1$ , the impact of stickier wages on output elasticity appears negligible.

Figure 5 shows how the (upward-sloping) E-stability/instability frontier, displayed with respect to the elasticity  $\epsilon$  and the policy coefficient  $\phi_\pi$ , shifts rightward as we consider higher degrees of wage rigidity. The case of  $\omega = 0$  corresponds to what depicted in panel b) of Figure 3. As previously argued, with fully flexible wages, the D-ES region is non-empty only for  $\epsilon$  smaller than, about, 4. This upper-bound on  $\epsilon$  is strictly increasing in the degree of wage rigidity. For instance, it becomes 5.5 for  $\omega = 0.5$ , and 8.5 for  $\omega = 0.75$ .<sup>25</sup>

Based on these results, we simulate the model assuming larger elasticities and some rigidity in real wages. Panel a) of Figure 6 has  $\epsilon = 4.3$  and  $\chi = 0.5$ , while panel b) has  $\epsilon = 8.3$  and  $\chi = 0.75$ . The results are comparable to those displayed in Figure 2 for the benchmark calibration with flexible wages.

## 6 Alternative Policy Rules

Our analysis could be summarized as follows. If monetary policy takes the form of an expectation-based interest rate rule (forward-looking Taylor rule) whose functional form is unknown to the public, stability under learning requires the central bank to either a) grant a sufficiently active response to inflation, without responding to stock price, or b) grant a mildly active response to inflation combined with a positive response to stock prices.

Next, we consider two alternative policy specifications which can eliminate the learning-driven

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<sup>24</sup>For  $\omega = 0$ , the real wage is fully flexible, as in the case studied in the previous section. For  $\omega = 1$  instead,  $\hat{w}_t = 0$ , i.e., the real wage is constant, as in the canonical model of Hall (2005).

<sup>25</sup>In their benchmark calibration, Blanchard and Gali (2007) set the wage rigidity parameter (which is their case is the weight on past real wages) equal to 0.9. Using a similar specification, Uhlig (2007) shows that a high degree of real wage rigidity is needed to generate both asset pricing and macroeconomic facts with a baseline real business cycle model.

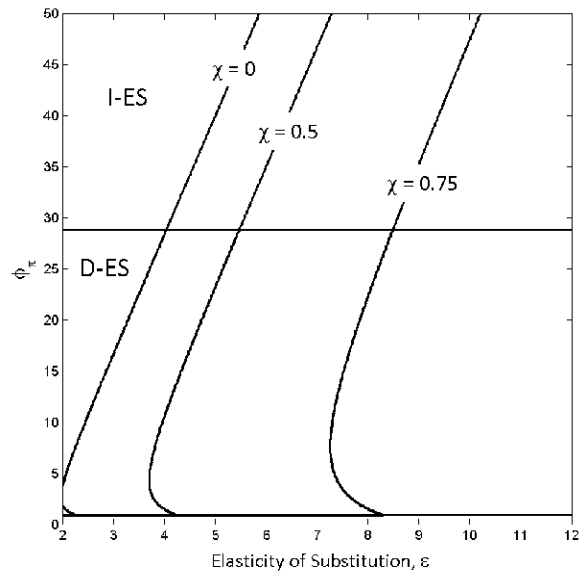


Figure 5: **Equilibrium Determinacy and E-Stability under IHL: Inflation Targeting with Sticky Real Wages.** Plot of E-stability/instability frontier (upward-sloping line) for different degrees of real wage stickiness,  $\chi$ .

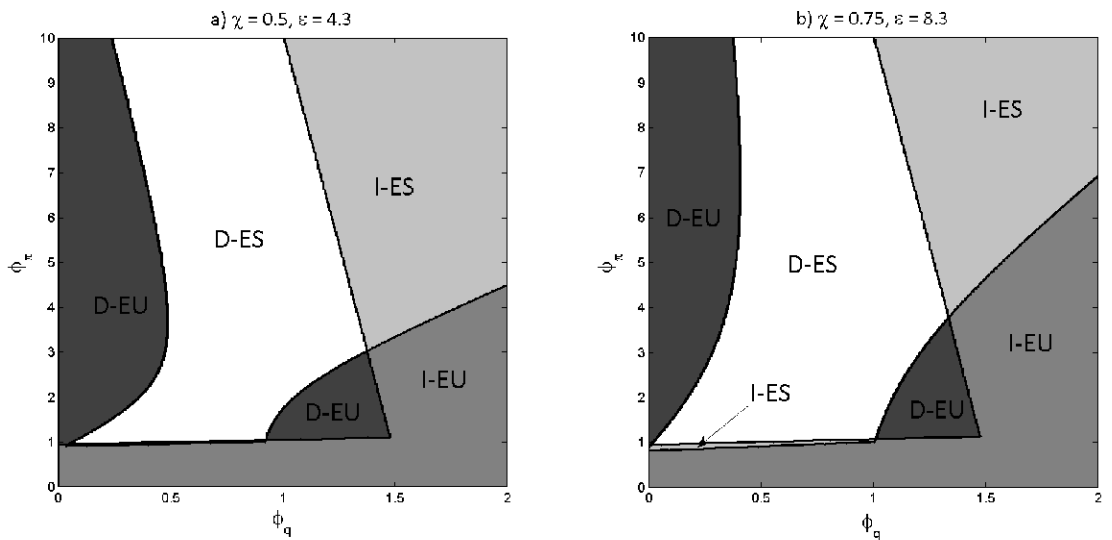


Figure 6: **Equilibrium Determinacy and E-Stability under IHL: Real Wage Rigidity.**

instability. The first one simply requires the central bank to be fully transparent, and communicate to the public all details about the adopted expectation-based rule: namely, its arguments and the related coefficients. The second one requires the central bank to respond to observed, rather than expected, endogenous variables (contemporaneous specification).

## 6.1 Monetary Policy Transparency

Consider the case of a central bank that fully communicates to the public the monetary policy rule (23). With this information available, households' forecast of future interest rates will be in line with what implied by monetary policy, which was not the case in the previous analysis. More specifically, the term  $\tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T$  entering the Euler equation (51) becomes:

$$\tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T = \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (\phi_{\pi} - 1) \hat{\pi}_{T+1} + \phi_y \hat{Y}_{T+1} + \phi_q \hat{Q}_{T+1} \right]$$

The Euler equation (56) is then substituted by the following:

$$\begin{aligned} \hat{Y}_t &= s_C^{-1} (1 - \beta) \left( \bar{Q}_C \hat{Q}_t + \bar{D}_C \hat{D}_t \right) + s_C^{-1} (1 - \beta) \bar{I}_C \frac{1 + \chi}{\chi} \hat{w}_t \\ &+ s_C^{-1} \bar{I}_C \frac{1 + \chi}{\chi} \beta \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (1 - \beta) \hat{w}_{T+1} \\ &- s_C^{-1} \xi \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (\phi_{\pi} - 1) \hat{\pi}_{T+1} + \phi_y \hat{Y}_{T+1} + \phi_q \hat{Q}_{T+1} \right] \end{aligned} \quad (65)$$

Under monetary policy transparency, the structural equations describing the equilibrium under IHL are then (23), (45), (57), (58), and (65). Agents will still need to forecast future wages, inflation, output, the stock price index and dividends, using their PLMs  $\hat{x}_t = a + b\hat{z}_t$ , for  $\hat{x}_t \equiv [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t, \hat{w}_t, \hat{D}_t]'$ .

As shown in panel a) of Figure 7, full policy communication eliminates the instability of the REE due to learning. If determinate, the REE is always E-stable, as for the case of EEL discussed in Section 3.1.

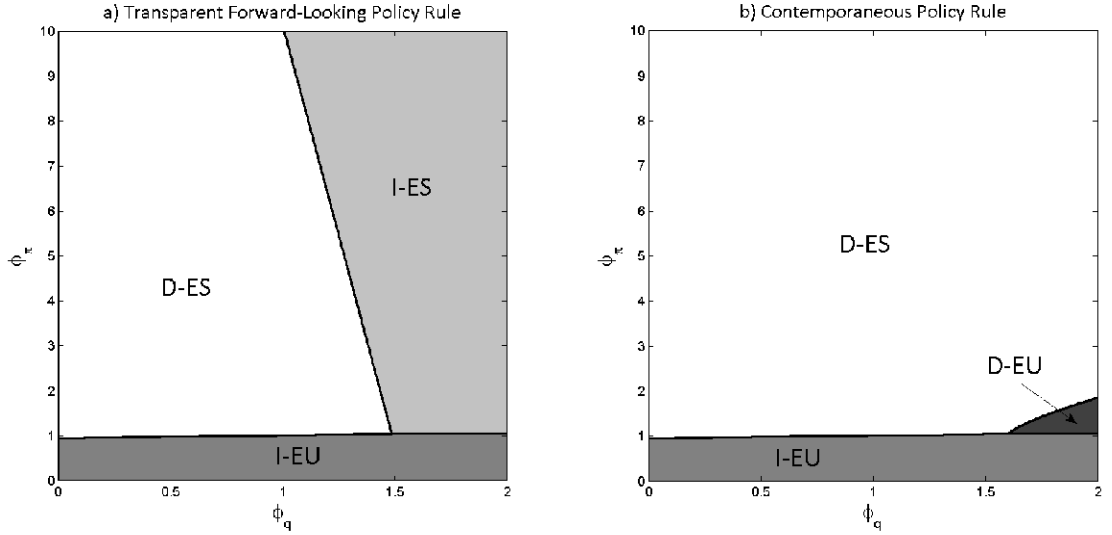


Figure 7: **Equilibrium Determinacy and E-Stability under IHL: Transparency and Contemporaneous Rules**

## 6.2 A Contemporaneous Rule

Suppose now the central bank sets the nominal interest rate in response to changes in *current* inflation, output and the stock price:

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t + \phi_q \hat{Q}_t \quad (66)$$

It is assumed that the policy rule is not communicated to the public, meaning that households would still form expectations about future interest rates making use of the no-arbitrage condition (54). The dynamics under IHL are described by the same structural equations used in Section 5, although now the interest rate  $\hat{R}_t$  entering the Euler equation (56) and the stock price equation (53) is determined by (66) instead of (23). Panel b) in Figure 7 displays the results. As for the case of a transparent forward-looking rule, a determinate equilibrium appears to be always E-stable, unless the response to stock prices is excessively large.

## 7 Conclusions

In the benchmark New-Keynesian model, an explicit response to stock prices in the interest rate rule increases the scope for equilibrium indeterminacy. More specifically, the larger the response to stock prices the larger should be the response to inflation for the equilibrium to be locally unique. This policy trade-off has been clearly highlighted by Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) supporting the conventional wisdom that monetary policy should *not* respond to stock prices. However, the benchmark model does not include any structural linkage between the stock market and real activity, and hence no specific reason for why the central bank should respond to endogenous variables other than inflation and output.

Following Preston (2005, 2006), we have presented a New Keynesian model where agents are internally rational - they take optimal decision subject to known individual objectives and constraints, understanding the mapping between own actions and expected pay-offs, given subjective beliefs about aggregate variables beyond their control - but do not have any knowledge about other agents' objectives, constraints and beliefs. We have shown that this form of bounded rationality generates wealth effects from holding of risky equity shares on consumption.

We have then studied the stability under learning (E-stability) properties of forward-looking interest rate rules that grant an explicit response to stock prices. We have found that a policy rule which exclusively and actively responds to inflation easily guarantees a determinate equilibrium, but is not sufficient to make stable under learning, unless the response is quite large (indeed more than what supported by empirical evidence). This issue is alleviated by responding to stock prices: a mild positive response to them can re-establish the Taylor principle as good policy guidance. All instability issues due to learning disappear if the forward-looking rule is fully communicated to the public, or if the central bank adopts a contemporaneous specification.

## A Appendix

### A.1 Steady State

We consider a zero inflation symmetric steady state. This corresponds to the same steady state for the benchmark representative agent New Keynesian model under RE. In particular, we have that  $\bar{C}^i = \bar{C}$ ,  $\bar{H}^i = \bar{H}$ , and  $\bar{I}^i = \bar{I}$  for all  $i \in [0, 1]$ . Moreover, by market clearing and the linear technology, the following equality holds:  $\bar{C} = \bar{Y} = \bar{H}I$  It therefore follows that  $\bar{\mathcal{F}}_{t,T}^i = \bar{\mathcal{F}}_{t,T} = \beta^{T-t}$

and  $\bar{\Lambda}_{t,T}^i = \bar{\Lambda}_{t,T} = \beta^{T-t}$ , from which the real interest rate is  $\bar{R} = \bar{P}_B^{-1} = \beta^{-1}$ .

Real marginal costs are the inverse of the steady state price mark-up,  $\bar{m}\bar{c} = \mu^{-1}$ . Imposing symmetry also across all firms, real dividends are  $\bar{D}_j = \bar{D} = \bar{Y} (1 - \bar{m}\bar{c}) = \frac{\mu-1}{\mu} \bar{Y}$ . From the stock price equation (10), we have  $\bar{Q} = \beta (\bar{Q} + \bar{D})$ , such that  $\bar{Q} = \frac{\beta}{1-\beta} \frac{\mu-1}{\mu} \bar{Y}$ . Since  $\bar{w} = \bar{m}\bar{c}$ , using the definition of labor income  $\bar{I}$ , we have that  $\bar{I} = \frac{\bar{Y}}{\mu}$ .

## A.2 Derivation of Euler equation

Consider the term on the left hand side of (16). Its log-linear approximation around a symmetric steady state (where  $\bar{C}^i = \bar{C}$  and  $\bar{\Lambda}_{t,T}^i = \beta^{T-t}$ ) gives:

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i C_T^i \approx \bar{C} \hat{C}_t^i + \beta \bar{C} \tilde{E}_t^i \left( \hat{\Lambda}_{t,t+1}^i + \hat{C}_{t+1}^i \right) + \beta^2 \bar{C} \tilde{E}_t^i \left( \hat{\Lambda}_{t,t+2}^i + \hat{C}_{t+2}^i \right) + \dots \quad (\text{A.1})$$

From the definition of  $\Lambda_{t,T}^i$  and the Euler equation (11), we have that:

$$\begin{aligned} \tilde{E}_t^i \hat{\Lambda}_{t,t+1}^i &= \sigma \tilde{E}_t^i \left( \hat{C}_t^i - \hat{C}_{t+1}^i \right) = - \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) = -\hat{r}_t \\ \tilde{E}_t^i \hat{\Lambda}_{t,t+2}^i &= \sigma \tilde{E}_t^i \left( \hat{C}_t^i - \hat{C}_{t+2}^i \right) = - \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) - \tilde{E}_t^i \left( \hat{R}_{t+1} - \tilde{E}_t^i \hat{\pi}_{t+2} \right) = -\hat{r}_t - \tilde{E}_t^i \hat{r}_{t+1} \\ &\dots \end{aligned}$$

where  $\hat{r}_t \equiv \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1}$  is the (log) *ex ante* real interest rate at time  $t$ . After simple algebra, equation (A.1) is equivalent to

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i C_T^i \approx \bar{C} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i - \frac{\beta \bar{C}}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T. \quad (\text{A.2})$$

Following a similar procedure, the log-linearization of  $\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i I_T^i$  on the right hand side of (16) gives:

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i I_T^i \approx \bar{I} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i - \frac{\beta \bar{I}}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \quad (\text{A.3})$$

Next, consider the Euler equation (11), whose log-linearization gives:

$$\hat{C}_t^i = \tilde{E}_t^i \hat{C}_{t+1}^i - \frac{1}{\sigma} \left( \hat{R}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right)$$

Iterating it forward, we obtain an expression for  $\tilde{E}_t^i \hat{C}_T^i$  for  $T \geq t$ :

$$\tilde{E}_t^i \hat{C}_T^i = \hat{C}_t^i + \sigma^{-1} \tilde{E}_t^i \sum_{n=T}^{T-1} \hat{r}_n$$

Using the latter, we can find an expression for the summation  $\tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i$ , entering (A.2):

$$\begin{aligned} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i &= \hat{C}_t^i + \beta \tilde{E}_t^i \hat{C}_{t+1}^i + \beta^2 \tilde{E}_t^i \hat{C}_{t+2}^i + \dots \\ &= \hat{C}_t^i + \beta \left( \hat{C}_t^i + \sigma^{-1} \hat{r}_t \right) + \beta^2 \left[ \hat{C}_t^i + \sigma^{-1} \tilde{E}_t^i (\hat{r}_t + \hat{r}_{t+1}) \right] + \dots \\ &= \frac{\hat{C}_t^i}{1-\beta} + \frac{\sigma^{-1} \beta}{1-\beta} \hat{r}_t + \frac{\sigma^{-1} \beta^2}{1-\beta} \tilde{E}_t^i \hat{r}_{t+1} + \dots \\ &= \frac{\hat{C}_t^i}{1-\beta} + \frac{\sigma^{-1} \beta}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \end{aligned} \tag{A.4}$$

We can then substitute the latter into (A.2), and, together with (A.3), into the log-linearized version of (16) to obtain the following expression:

$$\begin{aligned} &\frac{\bar{C}}{1-\beta} \hat{C}_t^i + \frac{\sigma^{-1} \beta \bar{C}}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T - \frac{\beta \bar{C}}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \\ &= \bar{a} \hat{a}_t^i + \bar{I} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i - \frac{\beta \bar{I}}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \end{aligned}$$

### A.3 Linear System under IHL

Consider the infinite horizon Euler equation (56). Expectations of future variables are computed using the PLMs in equation (59). Namely, we have that

$$\hat{x}_t = a + b \hat{z}_t \tag{A.5}$$

for  $\hat{x}_t \equiv [\hat{Y}_t, \hat{\pi}_t, \hat{Q}_t, \hat{w}_t, \hat{D}_t]'$ ,  $a \equiv [a_y, a_\pi, a_q, a_w, a_d]'$  and  $b \equiv [b_y, b_\pi, b_q, b_w, b_d]'$ . In particular, for a generic variable  $\hat{m}_t = a_m + b_m \hat{z}_t$ , for  $m = w, \pi, Y, Q, D$ , and known AR(1) process for  $\hat{z}_t$ , we have that  $\tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{m}_{T+n} = \frac{a_m}{1-\beta} + \frac{\rho^n b_m}{1-\beta \rho} \hat{z}_t$ .

Making use of the policy rule (23) to substitute for  $\hat{R}_t$ , we can write (56) as follows:

$$\begin{aligned}
& \hat{Y}_t - s_C^{-1} \beta \frac{\mu-1}{\mu} \hat{Q}_t - s_C^{-1} (1-\beta) \frac{\mu-1}{\mu} \hat{D}_t - s_C^{-1} (1-\beta) \frac{1+\chi}{\mu\chi} \hat{w}_t \\
= & s_C^{-1} \frac{1+\chi}{\mu\chi} \beta a_w - s_C^{-1} \xi \phi_y a_y - s_C^{-1} \xi \beta a_d + s_C^{-1} \xi (1-\phi_\pi) a_\pi + s_C^{-1} \xi (\beta - \phi_q) a_q \quad (\text{A.6}) \\
& + s_C^{-1} \frac{1+\chi}{\mu\chi} \beta \frac{(1-\beta)\rho}{1-\beta\rho} b_w \hat{z}_t - s_C^{-1} \xi \rho \phi_y b_y \hat{z}_t - s_C^{-1} \xi \beta \frac{(1-\beta)\rho^2}{1-\beta\rho} b_d \hat{z}_t \\
& + s_C^{-1} \xi \rho (1-\phi_\pi) b_\pi \hat{z}_t + s_C^{-1} \xi \rho (\beta - \phi_q) b_q \hat{z}_t
\end{aligned}$$

Similarly, the Phillips curve (22) becomes:

$$\begin{aligned}
\hat{\pi}_t - \kappa \hat{w}_t = & -\kappa \hat{z}_t + \frac{\kappa\beta\vartheta}{1-\beta\vartheta} a_w + \frac{\beta(1-\vartheta)}{1-\beta\vartheta} a_\pi \\
& + \frac{\kappa\beta\vartheta\rho}{1-\beta\vartheta\rho} b_w \hat{z}_t + \frac{\beta(1-\vartheta)\rho}{1-\beta\vartheta\rho} b_\pi \hat{z}_t - \frac{\kappa}{1-\beta\vartheta\rho} \hat{z}_t \quad (\text{A.7})
\end{aligned}$$

The stock price index equation (53) combined with the policy rule (23) gives instead:

$$\begin{aligned}
\hat{Q}_t = & (1-\phi_\pi) a_\pi - \phi_y a_y + (1-\beta) a_d + (\beta - \phi_q) a_q \\
& + (1-\phi_\pi) \rho b_\pi \hat{z}_t - \phi_y \rho b_y \hat{z}_t + (1-\beta) \rho b_d + (\beta - \phi_q) \rho b_q \hat{z}_t \quad (\text{A.8})
\end{aligned}$$

We have then a system made of equations (A.6), (A.7), (A.8), together with (57) and (58), which we can write compactly as  $F_1 \hat{x}_t = F_2 a + (F_3 b + F_4) \hat{z}_t$ , with matrices  $F_i$ , for  $i = 1, 2, 3, 4$ , defined as follows:

$$F_1 \equiv \begin{bmatrix} 1 & 0 & -s_C^{-1} \beta \frac{\mu-1}{\mu} & -s_C^{-1} (1-\beta) \frac{1+\chi}{\mu\chi} & -s_C^{-1} (1-\beta) \frac{\mu-1}{\mu} \\ 0 & 1 & 0 & -\kappa & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -(\sigma + \chi) & 0 & 0 & 1 & 0 \\ -\left(1 - \frac{\sigma + \chi}{\mu - 1}\right) & 0 & 0 & 0 & 1 \end{bmatrix}$$



$$F_2 \equiv \begin{bmatrix} -s_C^{-1}\xi\phi_y & s_C^{-1}\xi(1-\phi_\pi) & s_C^{-1}\xi(\beta-\phi_q) & s_C^{-1}\frac{1+\chi}{\mu\chi}\beta & -s_C^{-1}\xi\beta \\ 0 & \frac{\beta(1-\vartheta)}{1-\beta\vartheta} & 0 & \frac{\kappa\beta\vartheta}{1-\beta\vartheta} & 0 \\ -\phi_y & 1-\phi_\pi & \beta-\phi_q & 0 & 1-\beta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_3 \equiv \begin{bmatrix} -s_C^{-1}\xi\rho\phi_y & s_C^{-1}\xi\rho(1-\phi_\pi) & s_C^{-1}\xi\rho(\beta-\phi_q) & s_C^{-1}\frac{1+\varphi}{\mu\varphi}\beta\frac{(1-\beta)\rho}{1-\beta\rho} & -s_C^{-1}\xi\beta\frac{(1-\beta)\rho^2}{1-\beta\rho} \\ 0 & \frac{\beta(1-\vartheta)\rho}{1-\beta\vartheta\rho} & 0 & \frac{\kappa\beta\vartheta\rho}{1-\beta\vartheta\rho} & 0 \\ -\phi_y\rho & (1-\phi_\pi)\rho & (\beta-\phi_q)\rho & 0 & (1-\beta)\rho \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_4 \equiv \left[ 0 \quad -\frac{\kappa}{1-\beta\vartheta\rho} \quad 0 \quad -\chi \quad \frac{1+\chi}{\mu-1} \right]'$$

#### A.4 Euler Equation without Equity Trading

Without equity trading, the  $i$ -th household's budget constraint is:

$$P_t C_t^i + P_{B,t} B_t^i = B_{t-1}^i + P_t D_t + W_t H_t^i - T_t^i \quad (\text{A.9})$$

where  $D_t \equiv \int_0^1 D_{j,t} dj$ , as dividends are distributed equally across all households as lump-sum transfers. By forward iteration of (A.9) and the use of first order conditions, it is easy to obtain the following intertemporal budget constraint:

$$\tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i C_T^i = a_t^i + \tilde{E}_t^i \sum_{T=t}^{\infty} \Lambda_{t,T}^i I_T^i \quad (\text{A.10})$$

where now  $a_t^i \equiv \frac{B_{t-1}^i}{P_t}$  and  $I_t^i \equiv \frac{W_t}{P_t} H_t^i + D_t$ . Following the same logic of Section 2.1, the log-linearization of (A.10) yields:

$$\frac{\bar{C}}{1-\beta} \hat{C}_t^i = \bar{a} \hat{a}_t^i + \bar{I} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i - \frac{\beta}{1-\beta} [\bar{I} - (1-\sigma^{-1}) \bar{C}] \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \quad (\text{A.11})$$

with  $\hat{I}_T^i = \frac{\bar{w}\bar{H}}{\bar{Y}} (\hat{w}_t + \hat{H}_t^i) + \frac{\bar{D}}{\bar{Y}} \hat{D}_t$ . Using the fact that, at the steady state,  $\bar{I} = \bar{Y}$ ,  $\bar{w} = \mu^{-1}$  and  $\frac{\bar{D}}{\bar{Y}} = \frac{\mu-1}{\mu}$ , together with the linearized consumption-leisure trade-off equation  $\hat{H}_t^i = \frac{1}{\chi} \hat{w}_t - \frac{\sigma}{\chi} \hat{C}_t^i$ , we have that  $\hat{I}_T^i = \frac{1}{\mu} \left( \frac{1+\chi}{\chi} \hat{w}_t - \frac{\sigma}{\chi} \hat{C}_t^i \right) + \frac{\mu-1}{\mu} \hat{D}_t$ . It therefore follows that

$$\begin{aligned} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{I}_T^i &= \frac{1+\chi}{\mu\chi} \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{w}_T + \frac{\mu-1}{\mu} \tilde{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{D}_T \\ &\quad - \frac{\sigma}{\mu\chi} \frac{1}{1-\beta} \hat{C}_t^i - \frac{\beta}{1-\beta} \tilde{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{r}_T \end{aligned}$$

After combining the latter with (A.11), rearranging terms, noticing that now  $\frac{\bar{I}}{\bar{C}} = 1$ , averaging across households, and imposing the market clearing condition  $\hat{C}_t = \hat{Y}_t$ , it is straightforward to obtain the infinite-horizon IS curve (63).

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