The government spending multiplier, uncertainty and sovereign risk

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Abstract

How does the presence of sovereign risk affect the dynamic consequences of government spending shocks? I employ a medium-scale DSGE model with financial frictions and sovereign risk, and solve it using a third-order approximation to equilibrium dynamics. This implies that the degree of uncertainty affects the government spending multiplier. Output effects of government spending shocks are smaller when impulse responses are generated with a third-order approximation compared to its linear counterpart. Furthermore, the government spending multiplier becomes state-dependent. This captures that the increase in uncertainty associated with a higher public debt dampens the expansionary impulse of government spending shocks. When the economy moves closer to the fiscal limit, government spending multipliers can even become negative.

Keywords: government spending multiplier, sovereign risk, fiscal limit, financial frictions, third-order-approximation, time-varying volatility; non-linear impulse response, state-dependent multipliers

JEL Classification: E32, E62, E63, H30, H60

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1. Introduction

The sustainability of public finances is taking center stage in the attempts of policy makers to stabilize the euro area. As one of the key features in many economies in the recent years was an increase in the level and volatility of the risk premia on government bonds, it is a natural question to ask: How does the presence of sovereign risk affect the size of the government spending multiplier?

This paper investigates on the size of the government spending multiplier in the context of a dynamic stochastic general equilibrium model with financial frictions and sovereign risk. The model is a variation of the framework developed by Gertler and Karadi (2011), augmented by government bonds and sovereign risk in the spirit of van der Kwaak and van Wijnbergen (2014). The probability of a sovereign default is modeled in the form of a fiscal limit function as discussed by Leeper and Walker (2011) and used in similar variations by various authors.[1] The model is solved employing a third-order approximation to the equilibrium dynamics using the non-linear moving average approach by Lan and Meyer-Gohde (2013b). The introduction of a fiscal limit into a DSGE model directly implies the presence of an endogenous time-varying volatility, which affects the dynamic consequences of shocks. As I will show, this effect is quantitatively relevant, and thus justifies the use of a third-order approximation of equilibrium dynamics. As a consequence, time-varying risk and the precautionary motive of agents is accounted for. Second, the dynamic consequences of shocks are state dependent. In the context of this paper this has the consequence that government spending shocks, which take place in regions of the state space that are associated with higher sovereign default probabilities, have other effects than government spending shocks of the same size and sign, which hit the economy when it is at its steady state. Following Sims and Wolff (2013) I conduct simulations of the equilibrium dynamics, and compare the effects of government spending shocks at different regions in the state space. In my case, I compare the effects of government spending shocks at the steady state, where risk premia are near-zero, and in the region of the state space, where the economy is close to the fiscal limit, and annualized risk premia are higher than 300 bp.
This allows me to answer two more specific versions of the research question stated above: "How does the anticipation of higher sovereign risk, affect the size of the government spending multiplier?", and "How does the presence of sovereign risk affect the government spending multiplier, when the economy is close to its fiscal limit?"

My findings suggest the following: First, the impact of uncertainty on the transmission of government spending shocks is quantitatively relevant, and thus justify the use of the third-order approximation. Secondly, accounting for uncertainty lowers the output effects of government spending shocks. The financial accelerator embedded in the model, and the risk of a sovereign default, are key to generating a relevant effect of risk on the equilibrium dynamics. Uncertainty affects the size of the government spending multiplier mainly through the reaction of the credit supply, and investment. After a positive government spending shock, investment is crowded out, prices of capital assets and government bonds fall, and reducing the net worth of risk averse financial intermediaries. When the risk of future, potentially further detrimental shocks is accounted for, the intermediaries reduce their exposure to the risky asset to a larger degree. Credit supply and investment fall by more, and output increases by less, than in a linear world, where risk has no impact on the dynamics of the model. At the steady state, where risk premia are relatively small and the sensitivity of the default probability to fundamentals is relatively small, the output effects of government spending shocks are smaller, when they are generated with a third-order approximation which entails an adjustment for the risk of future shocks, compared to the linear

[1] Examples include: Bi and Traum (2012a), Bi and Traum (2012b), Corsetti, Kuester, Meier, and Müller (2013), van der Kwaak and van Wijnbergen (2013), van der Kwaak and van Wijnbergen (2014), Bi, Leeper, and Leith (2014).

counterpart. At regions of the state space, where the probability of a default on government debt is high, the long-run government spending multipliers can even become negative. The main difference Overall, this results result is in line with the notion that in times of increased sovereign risk, an increase in government spending may hurt the economy, and even have a recessionary impact, while, on the other hand, fiscal retrenchment may stabilize the economy and might even have expansionary effects on output.[2]

This study builds on the work by van der Kwaak and van Wijnbergen (2014). They employ the model framework by Gertler and Karadi (2011) and augment it by risky long-term government debt. They find that due to the leverage constraint on financial intermediaries, deficit financed government spending crowds out loans to private firms, which decreases the government spending multiplier. Furthermore, a longer duration of government bonds and the presence of default risk decreases the effectiveness of government spending. While I verify their findings that are obtained by solving the model with a first order approximation to equilibrium dynamics, I demonstrate the additional effects of uncertainty on the size of the multiplier, that are implied by the presence of a fiscal limit. Furthermore, the third-order approximation allows me to analyze state-dependent mutipliers. Another recent theoretical paper on governement spending and the presence of sovereign risk is Corsetti et al. (2013). They find that in the presence of sovereign risk, fiscal retrenchment, insofar as it stabilizes public finances, may also enhance the stability of the macroeconomy. In the extreme case this may even imply a negative government spending multiplier. The possibility of a expansionary fiscal contraction was originally proposed by Giavazzi and Pagano (1990). Bertola and Drazen (1993) build a model in which a fiscal retrenchment improves the expectations of households on the future growth path and increases consumption. Alesina and Perotti (1997) add the lower risk premia as a channel through which fiscal contraction may stimulate investment and economic growth. [3]

Recently, Born, Müller, and Pfeifer (2015) and Strobel (2015) employ regime-switching SVARs to analyze the effects of fiscal shocks, when risk premia on government bonds are high. Strobel (2015) finds that in the case of Italy, the output response to a fiscal contraction in the form of a decrease in the growth rate of public debt, is weaker in a regime with higher sovereign risk.[4] In my model, I focus on the effect of government spending shocks, and calibrate the parameters to Italian data. Here, the presence of sovereign risk dampens the output effects of government spending shocks, which is qualitatively in line with the empirical result in Strobel (2015). Born et al. (2015) analyze a panel of countries and find that in periods of higher risk premia on government bonds, fiscal multiplier are higher than in periods with lower risk premia. This is at odds with the theoretical arguments above. . Analyzing panel data, and using public debt measures as indicators for the sustainability of public finances, Corsetti, Meier, and Müller (2012) find no significant differences of fiscal multipliers across countries with higher and lower debt, whereas Perotti (1999) finds evidence of smaller government spending multipliers in countries with high public debt levels or deficits, and Ilzetzki, Mendoza, and Végh (2013) find surprisingly large, negative government spending multipliers for countries with high public debt.

[2]While, in principle, the non-linear approximation does introduce an asymmetry in the responses to positive and negative shocks, this asymmetry is very small in the context of the model that I employ.

[3]Other examples of papers that discuss explain the possibility of expansionary fiscal contraction are: Blanchard (1990) and Sutherland (1997). For emipirical evidence in support of the this hypothesis, see: e.g., Giavazzi and Pagano (1990), Alesina and Ardagna (2010) and Bergman and Hutchison (2010). Studies who cast doubt on the empirical evidence for the expansionary contraction hypothesis are Perotti (2011) and Guajardo, Leigh, and Pescatori (2011).

[4]Due to data restriction, this paper analyzes the effects of shocks to the growth rate of public debt, instead of government spending shocks. This allows to capture the full expansionary or contractionary stance of fiscal policy, in the face of lacking data on tax revenue for a sufficiently long time horizon.

In the model that I employ, government spending affects output through various channels. My model features preferences that allow for a wealth effect on the labor supply as discussed in detail by, e.g. Christiano and Eichenbaum (1992) and Baxter and King (1993). As the model features nominal rigidities, the size of the government spending multiplier depends on the degree of price stickyness, as pointed out by Monacelli and Perotti (2008). Basu and Kimball (2003) highlight the effects of capital and investment adjustment costs on the transmission of government spending shocks. Coenen, Erceg, Friedman, Furceri, Kumhof, Lalonde, Laxton, Lindé, Mourougane, Muir, Mursula, de Resende, Roberts, Roeger, Snudden, Trabandt, and in't Veld (2012) provide a general discussion of the effects of government spending shocks in medium scale and large scale DSGE models. The government spending multiplier in a model with financial frictions a la Gertler and Kiyotaki (2010) together with risky government bond debt are discussed in van der Kwaak and van Wijnbergen (2014). Section 4.1. analyzes the effect of government spending shocks in the model framework by Gertler and Karadi (2011), which contains various of these channels, before the effects of sovereign risk and uncertainty are discussed separately in the following parts of section 4.

An important feature of the model is a fiscal limit a la Leeper and Walker (2011) that determines the probability of a sovereign default. A number of panel data studies investigate on the drivers of sovereign risk. Examples include Alesina, de Broeck, Prati, and Tabellini (1992), Bernoth, von Hagen, and Schuhknecht (2003), Manganelli and Wolswijk (2009), di Cesare, Grande, Manna, and Taboga (2012), Beirne and Fratzscher (2013) and others. Generally, these studies find that higher public debt and lower growth raise sovereign risk in the same period. Beirne and Fratzscher (2013) suggest that there was a contagion effect spilling over from Greece. Among others, they argue that there has been a "fundamental contagion" which caused the markets to become more aware of the economic fundamentals in periphery countries. As only the periphery countries with high public debt and low GDP growth were affected by these spells, the results of this literature points towards an increase of the sensitivity of sovereign risk to economic fundamentals in crisis time. The shape of the fiscal limit function allows for capturing the non-linear relationship between fundamentals and sovereign risk spreads, by modeling the probability of the sovereign default as a logistic distribution function, which depends on the debt-to-GDP ratio.

The remainder of the paper is structured as follows: Section two gives an overview of the model. Section three discusses the calibration and the solution method. The fourth section provides an anatomy of the dynamic consequences of government spending shocks, and an analysis of the dynamic effects of government spending shocks for the model with sovereign risk. It compares linear impulse response functions with their non-linear counterpart, and the illustrate teh state dependence of the the multiplier by comparing the consequences of shocks that hit the economy at its steady state and of shocks that hit the economy when the default probability is high. Section five provides a sensitivity analysis, and section six concludes.

2. The model

The environment

The model is a variation of the framework used by Gertler and Karadi (2011), augmented by a second asset on the bank's balance sheet, namely government bonds, and a meaningful role for the fiscal sector. Time is discrete, and one period in the model represents one quarter. The firm sector consists of intermediate good producers, capital good producers, and retailers. The modeling of the two last types of firms allows to isolate the dynamic investment and pricing decisions, and to simplify the solution. The public sector consists of a fiscal authority, and a separate monetary authority. The model includes habit formation in consumption, convex investment adjustment costs, variable capital utilization, and price indexation to enhance the empirical plausibility of the model dynamics, and to facilitate the comparability of my results with the results by other authors which have used this framework.

Households

There is a continuum of households with a unit mass. As in Gertler and Karadi (2011) a constant fraction f of the household works as banker, whereas the other fraction $(1-f)$ of the household supplies its labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods, and contribute to the households income only when they exit the banking sector, bringing home the accumulated profits. To ensure that both fractions of the household face the same consumption stream, perfect consumption insurance within the household is assumed. The household's expected lifetime utility is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t [ln(C_t - hC_{t-1}) - \frac{\chi}{1+\phi} L_t^{1+\phi}],$$

where C_t is consumption and L_t is labor. β is the discount factor, h is the parameter of the habit formation, ϕ is the inverse of the Frisch elasticity, and χ scales the weight of the disutility from labor in the preferences. Households can save via a one period bank deposit, which earns the risk less interest rate, R_t. The income stream of the household is thus composed of the wage income $W_t L_t$, banker's profits Υ_t^b, firm profits, Υ_t^f net the payment of lump sum taxes T_t. It uses this income to purchase consumption goods or to renew its deposits. The budget constraint thus reads:

$$C_t + D_{t+1} = W_t L_t + R_t D_t + \Upsilon_t^b + \Upsilon_t^f - T_t.$$

Maximizing life-time utility with respect to consumption, labor and deposit holdings subject to the budget constraint yields the first order conditions of the household.

$$W_t = \frac{\chi L_t^{\phi}}{U_{c,t}} \tag{1}$$

$$U_{c,t} = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1} \tag{2}$$

$$\Lambda_{t,t+1} = \frac{U_{c,t+1}}{U_{c,t}} \tag{3}$$

$$1 = E_t \beta \Lambda_{t,t+1} R_{t+1} \tag{4}$$

Firm sectors

The model contains three types of firms. Intermediate goods are produced by perfectly competitive firms, which use capital and labor as inputs for production. Monopolistically competitive retailers buy a continuum of intermediate goods, and assemble them into a final good. Nominal frictions as in Calvo (1983) make the retailers optimization problem dynamic. Additionally, a capital producing sector buys up capital from the intermediate good producer, repairs it, and builds new capital, which it sells to the intermediate good sector again. Investment in new capital is subject to investment adjustment costs.

Intermediate Good Producers

The setup with three types of producers allows us to isolate intermediate good firms from nominal rigidities and investment adjustment costs. The optimization of the intermediate good producer can be treated as a sequence of static problems. Their production function takes a standard Cobb Douglas form, given by:

$$Y_{mt} = A_t(\xi_t U_t K_{t-1})^{\alpha} L_t^{1-\alpha}, \tag{5}$$

where $0 < \alpha < 1$, U_t is the variable utilization rate of the installed capital. K_{t-1} is the capital purchased and installed in period $t-1$, which becomes productive in period t, and ξ_t is a shock to the quality of capital which can be interpreted as obsolesce of the employed capital. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) use this capital quality shock to simulate the recent banking crisis.

At the end of each period the intermediate good producer sells the capital stock that it used for production to the capital producer which refurbishes the capital, and purchases the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the price Q_t per unit, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. The interest rate the firm has to pay on the loan from the bank is $R_{k,t}$. Since the assumption is that the competitive firms make zero profits, the interest rate on their debt will just equal the realized ex-post return on capital. The resale value of the capital used in production depends on the realization of the capital quality shock, and the depreciation rate which in turn depends on the capital utilization rate in the following way:

$$\delta(U_t) = \delta_c + \frac{b}{1+\zeta} * U_t^{1+\zeta}, \tag{6}$$

where δ_c and b are constants and ζ is the elasticity of the depreciation rate with respect to the utilization rate. Furthermore, capital evolves according to the following law of motion:

$$K_t = (1-\delta(U_t))\xi_t K_{t-1} + I_t \tag{7}$$

Hence each period the firm in its investment decision maximizes

$$E_t[\beta\Lambda_{t,t+1}(-R_{k,t+1}Q_t K_t + P_{m,t+1}Y_{m,t+1} - W_{t+1}L_{t+1} + (1-\delta(U_{t+1}))Q_{t+1}K_t\xi_{t+1})]$$

with respect to K_t. In optimum the ex-post return then is as follows:

$$R_{k,t+1} = \frac{P_{m,t+1}\alpha\frac{Y_{m,t+1}}{K_t} + (1-\delta(U_{t+1}))Q_{t+1}\xi_{t+1}}{Q_t} \tag{8}$$

Additionally, the optimal choices of labor input and the capital utilization rate yield the first order conditions

$$W_t = P_{mt}(1-\alpha)\frac{Y_{mt}}{L_t} \tag{9}$$

$$\delta'(U_t)\xi_t Q_t K_{t-1} = P_{mt}\alpha\frac{Y_{mt}}{U_t}. \tag{10}$$

Capital Good Producers

The capital good producer's role in the model is to isolate the investment decision that becomes dynamic through the introduction of convex investment adjustment costs, which is a necessary feature to generate variation in the price of capital. Capital good producers buy the used capital, restore it and produce new capital goods. Since capital producers buy and sell at the same price, the profit they make is determined by the difference between the quantities sold and bought, i.e. investment. Thus they choose the optimal amount of investment to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left\{ [(Q_t - 1) I_t - f\left(\frac{I_t}{I_{t-1}}\right) I_t \right\}.$$

The first order condition of the capital producer reads:

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \beta \Lambda_{t,t+1} \left(\frac{I_t}{I_{t-1}}\right)^2 f'\left(\frac{I_t}{I_{t-1}}\right), \tag{11}$$

where the functional form of the investment adjustment costs is:

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

Retailers

Retailers produce differentiated goods by re-packaging the intermediate goods. They operate under monopolistic competition and face nominal rigidities à la Calvo (1983). As an additional element to smooth the equilibrium dynamics of inflation, it is assumed that in each period the fraction of firms that cannot choose its optimal price, γ, indexes its price to the inflation of the foregoing period. The parameter of price indexation is γ_p.

Aggregate final output, Y_t, is described by a CES aggregator of the individual retailers' final goods, Y_{ft}:

$$Y_t = \left(\int_0^1 Y_{ft}^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where $\epsilon > 1$ is the elasticity of substitution between different varieties of final goods. Thus the demand for its final goods that the retailer faces is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t,$$

where P_{ft} is the price chosen by retailer f. The aggregate price index is:

$$P_t = \left(\int_0^1 P_{ft}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}},$$

which due to the specific assumptions on the nominal rigidity can be written as:

$$\Pi_t^{1-\epsilon} = (1-\gamma)(\Pi_t^*)^{1-\epsilon} + \gamma \Pi_{t-1}^{\gamma_p(1-\epsilon)} \tag{12}$$

where $\Pi_t := \frac{P_t}{P_{t-1}}$, and $\Pi_t^* := \frac{P_t^*}{P_{t-1}}$. As the retailers' only input is the intermediate good which is sold by competitive producers, the marginal cost of the retailers equals the price of the intermediate good. Hence, each retailer chooses its optimal price to maximize the sum of its expected discounted profits:

$$E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right\} Y_{f,t+i},$$

subject to the demand constraint.
The first order condition for optimal price setting is:

$$E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left\{ \frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - \frac{\epsilon-1}{\epsilon} P_{m,t+i} \right\} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\epsilon} Y_{t+i} = 0,$$

Accordingly, the optimal choice of the price implies:

$$\Pi_t^* = \frac{\epsilon}{\epsilon-1} \frac{F_t}{Z_t} \Pi_t \tag{13}$$

where F_t and Z_t are defined recursively as:

$$F_t = Y_t P_{mt} + \beta \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} \Pi_t^{-\gamma_p \epsilon} F_{t+1} \tag{14}$$

$$Z_t = Y_t + \beta \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} \Pi_t^{-\gamma_p(\epsilon-1)} Z_{t+1}. \tag{15}$$

Equations (13)-(16) constitute the equilibrium conditions which in a linearized form are equivalent to a New Keynesian Phillips Curve with price indexation. Aggregate output of final goods, Y_t, is related to the aggregate intermediate output, Y_{mt}, in the following way,

$$Y_{mt} = \Delta_{p,t} Y_t, \tag{16}$$

where Δ_t is the dispersion of individual prices, which evolves according to the law of motion:

$$\Delta_{p,t} = \gamma \Delta_{p,t-1} \Pi_t^{\epsilon} \Pi_{t-1}^{-\gamma_p \epsilon} + (1-\gamma) \left(\frac{1 - \gamma \Pi_t^{\epsilon-1} \Pi_{t-1}^{-\gamma_p(\epsilon-1)}}{1-\gamma} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{17}$$

The markup X_t of the monopolistic retailers is the inverse of their marginal costs, which is equivalent to the price of the intermediate good.

$$X_t = \frac{1}{P_{mt}}. \tag{18}$$

Banks

The sector for financial intermediation is a slight variation of the framework used by van der Kwaak and van Wijnbergen (2013) and van der Kwaak and van Wijnbergen (2014). Banks finance their operations by creating deposits, D_t, which are held by households, and by their net worth, N_t. They use their funds to extend loans to intermediate good producers for acquiring capital, K_t, and for the purchases of government bonds, B_t at their market price Q_t^b. The balance sheet of bank j is given by:

$$Q_t K_{jt} + Q_t^b B_{jt} = N_{jt} + D_{jt}. \tag{19}$$

The banks retain the earnings, generated by the return on their assets purchased in the previous period, and add it to their current net worth. Thus, the law of motion for the net worth of a bank is given by:

$$N_{jt} = R_{kt} Q_{t-1} K_{j,t-1} + R_{bt} Q_{t-1}^b B_{j,t-1} - R_{t-1} D_{j,t-1}. \tag{20}$$

Note that while the interest rate on deposits raised in period $t-1$, is determined in the same period, the return of the risky capital assets and risky government bonds purchased in period $t-1$ is determined only after the realization of shocks at the beginning of period t. Substituting the balance sheet into the law of motion for net worth yields:

$$N_{jt} = (R_{kt} - R_{t-1}) Q_{t-1} K_{j,t-1} + (R_{bt} - R_{t-1}) Q_{t-1}^b B_{j,t-1} + R_{t-1} N_{j,t-1}. \tag{21}$$

Bankers continue accumulating their net worth, until they exit the business. Each period, each banker faces a lottery, which determines, regardless of the history of the banker, whether he exits his business or stays in the sector. Bankers exit the business with an exogenous probability $1-\theta$, or continue their operations with probability θ. The draws of this lottery are iid. When a banker leaves the sector, it adds his terminal wealth V_t to the wealth of its household. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth:

$$\begin{aligned} V_{jt} &= \max E_t \sum_{i=0}^{\infty}(1-\theta)\theta^i \beta^{i+1}\Lambda_{t,t+1+i}N_{j,t+1+i} \\ &= \max E_t[\beta\Lambda_{t,t+1}(1-\theta)N_{j,t+1}+\theta V_{j,t+1}]. \end{aligned}$$

As banks operate under perfect competition, with perfect capital markets the risk adjusted return on loans and government bonds would equal the return on deposits. However, bankers face an endogenous limit on the amount of funds that households are willing to supply as deposits. Following Gertler and Karadi (2011), I assume that bankers can divert a fraction of their assets and transfer it to their respective households. However, if they do so, their depositors will choose to withdraw their remaining funds and force the bank into bankruptcy. To avoid this scenario, households will keep their deposits at a bank only as long as the bank's continuation value is higher or equal to the amount that the bank can divert.
I make the assumption that the fractions loans that bankers can divert, λ, is smaller than the fraction of government bonds it can divert, λ_b. This is motivated by the fact, that, in general, the collateral value of government bonds is higher than that of loans.[5] The reason is that loans to private firms are less standardized than government bonds contracts. Additionally, information on the credit-worthiness of the government is publicly available, while often the credit-worthiness of private firms is only known to the bank and the firm, and not easy to assess for depositors, making it easier for banks to divert a fraction of their value. Formally, the incentive constraint of the bank reads:

$$V_{jt} \geq \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt} \tag{22}$$

Shifting part of the portfolio from loans to bonds relaxes the incentive constraint, and vice versa. As discussed below, this requires the return on capital to be higher than the return on bonds in steady state.

The initial guess for the form of the value function is:

$$V_{jt} = \nu_{kjt}Q_t K_{jt} + \nu_{bjt}Q_t^b B_{jt} + \nu_{njt}N_{jt}, \tag{23}$$

where ν_{kjt}, ν_{bjt} and ν_{djt} are time varying coefficients. Maximizing (23) with respect to loans and bonds, subject to (22) yields the following first order conditions for loan holdings and deposits: Hence, the first order conditions for loans, bonds, and the Lagrangian multiplier, μ_t, are:

$$\nu_{kjt} = \lambda \frac{\mu_{jt}}{1+\mu_{jt}} \tag{24}$$

$$\nu_{bjt} = \lambda_b \frac{\mu_{jt}}{1+\mu_{jt}} \tag{25}$$

$$\nu_{kjt}Q_t K_{jt} + \nu_{bjt}Q_t^b B_{jt} + \nu_{njt}N_{jt} = \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt} \tag{26}$$

[5]This is in the vein of Meeks et al. (2014), who use the same approach to distinguish between the collateral values of loans and asset backed securities.

where μ_{jt} is the Lagrangian multiplier on the incentive constraint. Given that this constraint holds, a bank's supply of loans can be written as:

$$Q_t K_{jt} = \frac{\nu_{bjt} - \lambda_b}{\lambda - \nu_{kjt}} Q_t^b B_{jt} + \frac{\nu_{njt}}{\lambda - \nu_{kjt}} N_{jt} \tag{27}$$

As (27) shows, the supply of loans decreases with an increase in λ, which regulates the tightness of the incentive constraint with respect to capital, and increases with an increase in λ_b, which makes the holding of bonds more costly in terms of a tighter constraint. Plugging the demand for loans into (23), and combining the result with (24) and (25) one can write the terminal value of the banker as a function of its net worth[6]:

$$V_{jt} = (1 + \mu_{jt})\nu_{njt} N_t. \tag{28}$$

A higher continuation value, V_{jt} is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive constraint. Defining the stochastic discount factor of the bank to be:

$$\Omega_{j,t} \equiv \Lambda_{t-1,t}((1-\theta) + \theta(1 + \mu_{jt})\nu_{njt}), \tag{29}$$

plugging (28) into the Bellman equation and using the law of motion for net worth, one can then write the value function as:

$$\begin{aligned} V_{jt} &= E_t[\beta \Lambda_{t,t+1}(1-\theta) N_{j,t+1} + \theta V_{j,t+1}] \\ &= E_t[\beta \Omega_{j,t+1}((R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t-1})], \end{aligned}$$

and verify the initial guess for the value function as:

$$\nu_{kjt} = \beta E_t \Omega_{jt+1}(R_{k,t+1} - R_t) \tag{30}$$
$$\nu_{bjt} = \beta E_t \Omega_{jt+1}(R_{b,t+1} - R_t) \tag{31}$$
$$\nu_{njt} = \beta E_t \Omega_{jt+1} R_t. \tag{32}$$

Aggregation of financial variables

To facilitate aggregation of financial variables, I assume that banks share the same structure to the extent that they derive the same respective values from holding loans and bonds, and from raising deposits (i.e., $\forall j : \nu_{kjt} = \nu_{kt}, \nu_{bjt} = \nu_{bt}, \nu_{njt} = \nu_{nt}$). Furthermore, I assume that all banks have the same ration of capital assets to government bonds, $\varsigma_t \equiv \frac{Q_t K_t}{Q_t^b B_t}$, in their portfolio. As a direct implication, the leverage ratio of banks does not depend on the conditions that are specific to individual institutes, and all banks share the same leverage ratio:[7]

$$\phi_t \equiv \frac{\nu_{nt}(1 + \varsigma_t)}{(\lambda - \nu_{kt})(1 + \frac{\lambda_b}{\lambda}\varsigma_t)} = \frac{Q_t K_t + Q_t^b B_t}{N_t}.^{8} \tag{33}$$

Note that the lower divertability of government bonds relative to capital assets, allows the bank to increase its leverage ratio, compared to a scenario in which banks only hold capital assets. The aggregate balance sheet constraint reads:

$$Q_t K_t + Q_t^b B_t = D_t + N_t. \tag{34}$$

[6]Detailed derivations are delegated to the appendix.

[7]Details are delegated to the appendix.

[8]Note that if the collateral values of capital assets and lambda were the same ($\lambda = \lambda_b$), the leverage ratio would take the same form as in Gertler and Karadi (2011), or in van der Kwaak and van Wijnbergen (2014)

The net worth of the fraction of bankers that survive period $t-1$ and continue operating in the banking sector, θ, can be written as:

$$N_{ot} = \theta \left[R_{kt} Q_{t-1} K_{t-1} + R_{bt} Q^b_{t-1} B_{t-1} - R_{t-1} D_{t-1} \right]. \tag{35}$$

A fraction $(1-\theta)$ of bankers leaves the business. There is a continuum of bankers, and the draws out of the lottery, which determines whether a banker stays in business or exits the sector, are iid. Hence, by the law of large numbers, it follows that the share of assets that leaves the sector is as well $(1-\theta)$ of the total assets. At the same time, new bankers enter the sector. New bankers are endowed with "start-up funding" by their households. The initial endowment of the new bankers is proportionate to the assets that leave the sector. The net worth of the new bankers, N_{nt}, can be written as:

$$N_{nt} = \omega \left[Q_{t-1} K_{t-1} + Q^b_{t-1} B_{t-1} \right], \tag{36}$$

where ω is calibrated to ensure that the size of the banking sector is independent of the turnover of bankers. Aggregate net worth, N_t, is then the sum of the net worth of old and new bankers:

$$N_t = N_{ot} + N_{nt}. \tag{37}$$

Fiscal Policy

The fiscal sector follows closely the structure in van der Kwaak and van Wijnbergen (2013) and van der Kwaak and van Wijnbergen (2014). The government finances its expenditures, G_t, by issuing government bonds, which are bought by banks, and by raising lump sum taxes, T_t. Government spending is exogenous and follows an AR(1) process:

$$G_t = G e^{g_t} \tag{38}$$

$$\text{and} \quad g_t = \rho_g g_{t-1} + \epsilon^g_t, \tag{39}$$

where G is the steady state government consumption, ρ_g is the autocorrelation of government consumption, and ϵ^g_t is a shock to government spending. Taxes follow a simple feedback rule, such that they are sensitive to the level of debt and to changes in government expenditures:

$$T_t = T + \kappa_b (B_{t-1} - B) + \kappa_g (G_t - G), \tag{40}$$

where T and B are the steady state levels of tax revenue and government debt, respectively. κ_b is set to ensure that the real value of debt grows a rate smaller than the gross real rate on government debt. As shown by Bohn (1998), this rule is a sufficient condition to guarantee the solvency of the government. If κ_g is set to zero, increases in government expenditures are entirely debt-financed. In turn, when $\kappa_g = 1$, changes in government spending are tracked one-to-one by changes in taxes. To allow for the calibration of a realistic average maturity of government debt, bonds are modeled as consols with geometrically decaying coupon payments, as in Woodford (1998) and Woodford (2001). A bond issued in period t at the price of Q^b_t, pays out a coupon of r_c in period $t+1$, a coupon of $\rho_c r_c$ in period $t+2$, a coupon of $\rho_c^2 r_c$ in $t+2$, and so on. Setting the decay factor ρ_c equal to zero captures the case of a one-period bond, in which the entire payoff of the bond is due in period $t+1$. Setting $\rho_c = 1$ delivers the case of a perpetual bond. The average maturity of a bond of this type is $1/(1-\beta\rho_c)$. For investors, this payoff structure is equivalent to receiving the coupon r_c and a fraction, ρ_c, of a similarly structured bond in period $t+1$. The beginning-of-period debt of the government can thus be summarized as $(r_c + \rho_c Q^b_t) B_{t-1}$.
At the beginning of each period, the government has the option to default and write of a fraction of its debt, $D \in (0,1)$. Investors take this into account, and demand a higher return on government bonds,

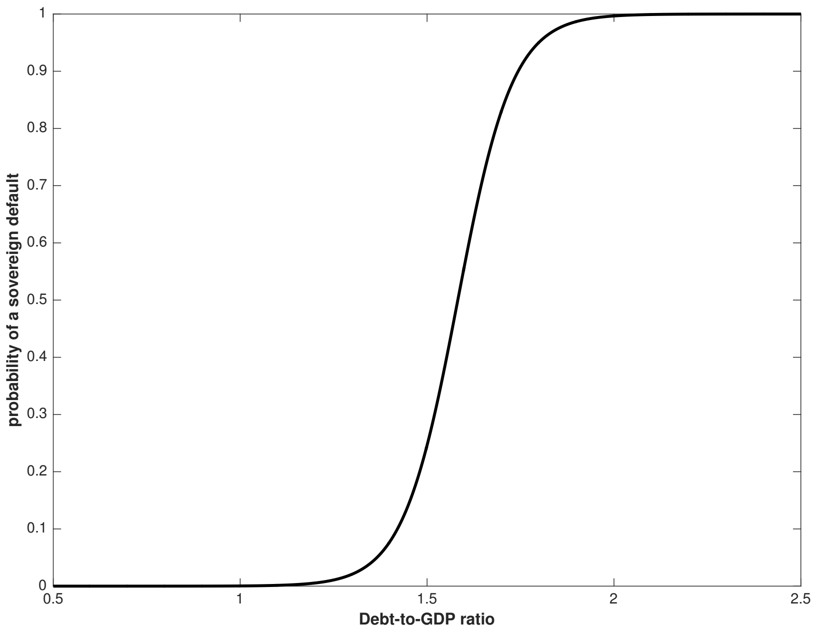

Figure 1: Default indicator, Δ_t^d

when the expected probability of a sovereign default, Δ_{t+1}^d, increases. The return to government bonds, adjusted for default risk, can thus be written as:

$$R_{b,t} = (1 - \Delta_t^d * D)\left[\frac{r_c + \rho_c Q_t^b}{Q_{t-1}^b}\right]. \tag{41}$$

The flow budget constraint of the government reads:

$$G_t + Q_t^b B_t = R_{bt} Q_{t-1}^b B_{t-1} + T_t, \tag{42}$$

$$\text{or:} \quad G_t + Q_t^b B_t = (1 - \Delta_t^d * D)\left[\frac{r_c + \rho_c Q_t^b}{Q_{t-1}^b}\right] Q_{t-1}^b B_{t-1} + T_t. \tag{43}$$

There are several ways to motivate the risk of a sovereign default. The standard approach in the literature is to link the default probability to the level of public debt or the debt-to-GDP ratio (see, e.g., Eaton and Gersovitz (1981), Arellano (2008) or Leeper and Walker (2011). A higher level of public debt implies a higher debt service, and, in turn, requires higher tax revenues to service the interest rate payments. As tax increases are not popular and only up to a maximum level politically feasible, it is plausible to posit a maximum capacity of levying taxes, or fiscal limit. With an increasing public debt, the economy moves closer to the fiscal limit.
The probability of a sovereign default is described by the logistical distribution function:

$$\Delta_t^d = \frac{exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t})\right)}{1 + exp\left(\eta_1 + \eta_2 \frac{B_t}{4Y_t}\right)}, \tag{44}$$

which depends on the debt-to-GDP ratio, and is depicted in figure (1). The fiscal limit function is uniquely pinned down by the parameters η_1 and η_2. I use the results of the structural estimation by Bi and Traum (2012a) to calibrate these parameters.

Monetary Policy and Good Market Clearing

The policy tool of the central bank in this economy is the nominal interest rate, i_t, which is set to the Taylor-type rule:

$$i_t = \rho_i i_{t-1} + (1-\rho)(i + \kappa_\pi \pi_t + \kappa_y \hat{mc}_t) + \epsilon_t^i, \tag{45}$$

where the smoothing parameter ρ_i lies between zero and one, ϵ_t^i is a monetary policy shock, and $\kappa_\pi > 1$ to satisfy the Taylor principle and guarantee the determinacy of the rational expectation equilibrium. The real interest rate on deposits and the nominal policy rate of the central banked are linked via the Fisher equation:

$$1 + i_t = R_{t+1} \frac{E_t P_{t+1}}{P_t}. \tag{46}$$

Finally, the good market clears.

$$Y_t = C_t + I_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + G_t \tag{47}$$

3. Calibration and Solution Method

Solution method

The model is solved using an third-order approximation to equilibrium dynamics. I employ the algorithm developed by Lan and Meyer-Gohde (2013b). Their solution methods solves for the policy functions in the form of nonlinear moving averages. Let the nonlinear DSGE model be:

$$E_t f(y_{t+1}, y_t, y_{t-1}, \epsilon_t) = 0, \tag{48}$$

where y_t and ϵ_t represent the vectors of endogenous variables and the vector of exogenous shocks, respectively. Then the solution to this model can be written as a system of policy functions of the form:

$$y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \ldots), \tag{49}$$

where σ scales the uncertainty of the model.[9] Under the assumption of normally distributed shocks (i.e., with zero skewness), the third-order Taylor approximation of the policy function takes the form:

$$y_t = \overline{y} + \frac{1}{2} y_{\sigma^2}\sigma^2 + \frac{1}{2}\sum_{i=0}^{\infty}\left(y_i + y_{\sigma_i^2}\sigma^2\right)\epsilon_{t-i} + \frac{1}{2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} y_{i,j}(\epsilon_{t-i}\otimes\epsilon_{t-j}) \tag{50}$$
$$+\frac{1}{6}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} y_{i,j,k}(\epsilon_{t-i}\otimes\epsilon_{t-j}\otimes\epsilon_{t-k}),$$

where $\overline{y}$ denotes the vector of deterministic steady state values of the respective endogenous variables, and the partial derivatives $y_i, y_{i,j}, y_{i,j,k}, y_{\sigma^2}$ and $y_{\sigma_i^2}$ are evaluated at the deterministic steady state. Up to the first order approximation the policy function is independent of the degree of uncertainty in the model. However, when the model is approximated with a third-order approximation, two terms enter the policy function and adjust it for future risk. While y_{σ^2} is constant, $y_{\sigma_i^2}$ varies over time and interacts the linear impulse responses to realized shocks with the conditional volatility of future risk.

In the case at hand, this becomes particularly relevant due to the shape of the fiscal limit function.

[9] $\sigma = 0$ corresponds to the non-stochastic model, whereas $\sigma = 1$ corresponds to the model with the originally assigned distribution of shocks.

As the economy moves closer to the fiscal limit, the default probability increases and additionally becomes more sensitive to the fluctuations in the debt-to-GDP ratio. This corresponds to the empirical finding of a higher sensitivity of default risk premia to fundamentals in a sovereign risk crisis as found by Beirne and Fratzscher (2013). [10] The introduction of risky government bonds on the balance sheets of financial intermediaries thus directly implies the presence of an endogenously time-varying volatility, which, as I will show, affects the dynamic consequences of shocks in a quantitatively relevant way, and thus requires a third-order approximation of equilibrium dynamics.[11]

Lan and Meyer-Gohde (2013b) show that an advantage of the nonlinear moving average method is that, if the first-order solution is stationary and saddle-stable, these properties carry over to higher-order approximation. Thus a "pruning out" of unstable higher-order terms as suggested by Kim, Kim, Schaumburg, and Sims (2008) becomes unnecessary.[12]

I compare the dynamic consequences of government shocks that occur at the steady state with government spending shocks of the same size and sign that hit the economy when it is in a region of the state space where souvereign risk premia are above 300bp. Impulse responses at the steady state are directly generated by the toolkit by Lan and Meyer-Gohde (2013b). In the generation of impulse response functions in the risky region of the steady state, I follow the steps proposed by Sims and Wolff (2013). In the first step, I conduct an simulation of the economy over 20.000 periods and average over realizations of the state, in which the annualized risk premia are higher than 300 bp. For this state I simulate two paths for output: one without any additional shock, and one with an additional government spending shock. The difference of these two paths over an horizon of 20 periods, adjusted by the steady state government spending ratio, constitutes the cumulative government spending multiplier. I repeat this procedure 150 times to minimize the sampling bias.

Calibration

The analysis of sovereign risk in a closed economy model with financial frictions is motivated by the case of Italy, as a large, relatively closed economy with high public debt, and recurring periods of high interest rate spreads, but without a sovereign default in the last decades. As the data on Italy is not sufficiently rich to perform an estimation of the parameters of such a large model, I follow Bocola (2015) and Bi and Traum (2012a), who obtain some of the relevant parameters estimates in estimations of structural models on Italian data. In the calibration of parameters that do no appear in their models, I build on the model by Gertler and Karadi (2011), which is the fundament for the structure of the framework at hand. Those parameters, for which there is no source given in Table 1, follow out of the computation of the deterministic steady state. For the steady state spread of the return on capital over deposits I use the same target as in Gertler and Karadi (2011). For the steady state spread of government bonds over deposits I take the estimate by Bocola as a guideline. The difference of the steady state spreads of the two assets is reflected by the values of the respective divertability parameters, λ and λ_b. In steady state these two parameters are linked by the relation:

$$\frac{\lambda}{\lambda_b} = \frac{R_k - R}{R_b - R}.$$

[10] Beirne and Fratzscher (2013) label this as an increase in "fundamental awareness", a notion which, however, is at odds with the concept of rational expectations.

[11] Note, that while in principle, time varying volatility also affects the dynamic consequences of level shocks in a basic RBC or NK model, the risk-adjustment components are negligible, as those models do not generate a sufficient amplification of shocks.

[12] For a detailed discussion see: Lan and Meyer-Gohde (2013a).

β	discount factor	0.99	Bi and Traum (2012)
σ	CRRA coefficient	1	Bi and Traum (2012)
h	habit formation	0.14	Bi and Traum (2012)
χ	weight for disutility of labor	4.7125	
φ	inverse of Frisch elasticity	0.276	Gertler and Karadi (2011)
α	capital share	0.33	Bi and Traum (2012)
ζ	cap. util. param.	7.2	Gertler and Karadi (2011)
b	cap. util. param.	0.03760101	Gertler and Karadi (2011)
δ_c	depreciation param.	0.020414511	Gertler and Karadi (2011)
η_i	invest. adjust. param.	1.728	Gertler and Karadi (2011)
ϵ	elasticity of substitution	4,167	Gertler and Karadi (2011)
γ	Calvo param.	0.779	Gertler and Karadi (2011)
γ_p	price indexation param.	0.241	Gertler and Karadi (2011)
$Rk-R$	steady state spread	0.01	Gertler and Karadi (2011)
$Rb-R$	steady state spread	0.005	Bocola (2015)
λ	divertibility of capital assets	0.4479	
λ_b	divertibility of bonds	0.2239	
θ	survival probability of banker	0.975	Gertler and Karadi (2011)
ω	transfer to new bankers	0.0018	
κ_{pi}	interest rate rule	1.5	Gertler and Karadi (2011)
κ_y	interest rate rule	-0.125	Gertler and Karadi (2011)
G/Y	share of gov. spending	0.1966	Bi and Traum (2012)
$B/4Y$	debt-to-GDP ratio.	1.19	Bi and Traum (2012)
κ_b	tax rule param.	0.3	Bi and Traum (2012)
κ_g	tax rule param.	0.53	Bi and Traum (2012)
η_1	fiscal limit parameter	-21.5285	Bi and Traum (2012)
η_2	fiscal limit parameter	3.4015	Bi and Traum (2012)
D	haircut	0.378	Bi and Traum (2012)
ρ_c	coupon rate	0.04	v.d.Kwaak and v. Wijnbergen (2014)
ρ_i	persistence of i-shock	0.0	
ρ_ξ	persistence of ξ-shock	0.66	Gertler and Karadi (2011)
ρ_a	persistence of a-shock	0.96	Bi and Traum (2012)
ρ_g	persistence of g-shock	0.84	Bi and Traum (2012)
ρ_s	persistence of s-shock	0.84	Bocola (2015)
σ_i	std. of i-shock	0.01	Gertler and Karadi (2011)
σ_ξ	std. of ξ-shock	0.01	Gertler and Karadi (2011)
σ_g	std. of g-shock	0.01	Bi and Traum (2012)
σ_a	std. of a-shock	0.01	Bi and Traum (2012)
σ_s	std. of s-shock	0.01	Bocola (2015)

Table 1: Calibration of Parameters The debt-to-GDP ratio is set to 1.19, which is the average

Italian debt-to-GDP ratio between 1999.Q1 and 2010.Q3. In the calibration of the fiscal limit function I use the values of η_1 and η_2 implied by Bi and Traum (2012a) for in their estimation of the fiscal limit under the assumption of an annualized default rate of 0.019, given a haircut of 0.378 on government bonds in the case of default. The coupon rate on the long-term government bond is set to 0.04 as in van der Kwaak and van Wijnbergen (2014). The persistence and variance parameters of the shocks processes are taken from the structural estimations by Bocola (2015) and Bi and Traum (2012a). The standard deviation of the monetary policy shock is set to 0.1, and the calibration of the capital quality shock is taken from Gertler and Karadi (2011), as to my knowledge there exists no structural estimation of a model with nominal rigidities and fluctuations in the quality of capital on Italian data.

4. Dynamic Analysis of government spending shocks

This section analyses the effects of a shock in government spending of the size of 1 percent of steady state GDP. The model contains various channels through which government spending shocks affect output. Thus, before turning to the role of sovereign risk and uncertainty, this section starts with reviewing the effects of the government spending shock in the context of the model by a model Gertler and Karadi (2011) without sovereign risk (section 4.1.). In this model, government bonds are riskless and held by households. In all other aspects, the model is identical to the framework described in section 2. I then compare the effects of the same shock in the two frameworks, using a linear impulse response analysis (section 4.2.). compare linear and non-linear impulse responses in the model with sovereign risk (section 4.3.), and lastly show the state dependence of the government spending multiplier in the model with sovereign risk (section 4.4).

4.1. The model without risky government bonds

A positive government spending shock of 1 percent of GDP in the model as in Gertler and Karadi (2011) raises output by 0.75 percent (see figure (2)). Since households are Ricardian, and preferences are additive separable in consumption and leisure, the increase in government spending induces a wealth effect on the labor supply. As in many models, consumption falls and the labor supply increases after the shock, leading to increasing equilibrium labor hours and a decline in the real wage. With the higher labor input the marginal product of labor decreases. In the presence of sticky prices real marginal cost increase and the markup declines with the increase in production. As a result, the demand for labor increases for any given real wage, contributing to the increase in equilibrium labor hours and output. As highlighted by Basu and Kimball (2003), the increase in labor leads to an increase in the marginal product of capital and thus to a higher real return on capital. This has two effects in this model. First, investment becomes less attractive, leading to a decline in the capital stock as well as in the price of capital. The firms' demand for loans from financial intermediates contracts. Secondly, the fall in the price of capital reduces the value of the banks' assets. In reaction to these losses, balance sheet constrained banks have to reduce the supply of loans, which increases expected future returns on loans. Hence, the spread of the return on capital assets (loans from the financial intermediaries to the firms), over the return of deposits increases. The fall in asset prices and the increase in the credit spread come along with a decline in the net worth of banks and a tightening of their leverage constraint.[13] This financial accelerator thus amplifies the fall in capital. The first effect contracts the firms' demand for loans from the financial intermediaries. The second effect additionally contracts the supply of loans that the intermediaries can provide.[14] Lastly, the increase in the real marginal cost induces an increase in inflation. In

[13] see: van der Kwaak and van Wijnbergen (2014)

[14] Note that in this model the market for capital and for loans is identical, since the physical capital stock used in production is re-financed fully every period, and loans by the banks are the only funds that are used for this purpose.

response, the monetary authority follows the Taylor rule and increases its nominal interest rate more than one to one with the increase in inflation, leading to a rise in the riskless real return on bank deposits.
Note, that as government bonds are riskless in this model as well, their return is identical to the return on deposits, and their price is time-invariant.

4.2. Introducing risky government bonds and comparing linear impulse responses across models

As discussed in the introduction, most of the literature on the interaction of fiscal multipliers and sovereign risk, theoretical as well as empirical, focusses on the possibility that higher sovereign risk decreases the impact of government spending shocks. Theoretical studies that focus on the effect of fiscal retrenchment suggest that, for instance, an improvement of the expectations of future growth (Bertola and Drazen (1993)), or a decrease in risk premia on government bonds, enhanced financial intermediation and increasing investment (e.g. Alesina and Perotti (1997), Corsetti et al. (2013) or van der Kwaak and van Wijnbergen (2013)) counteract the otherwise detrimental effects of fiscal retrenchment on output growth. Here, I follow the authors who focus in their argument on the channel of financial intermediation and changes in risk premia.
Figure (3) illustrates the comparison of the linear impulse responses in the model without risky government bonds (dashed line) and the model with risky bonds on the banks' balance sheets. Both hit the economy at its steady state.
When risky government bonds are introduced in the model as a second asset of the balance sheet of the bank, additional effects that are not present in the model in section 4.1. form the output reaction to government spending shocks. Now, the increase in government spending, increases public debt and the number of bonds held by the banks. As banks are balance sheet constrained, this leads to a crowding out of loan on the asset side. Whereas the contraction of the loan supply is relatively small in the model without risky government bonds, now the impact of the government shock on the balance sheet of banks is more direct. The fall in capital prices and the increase in the credit spread are far more pronounced. Investment falls up to 2 percent below its steady state value, more than twice as deep as in the model with riskless government bonds. The difference in the response of investment is the main driver for the difference in the output responses. Now the impact multiplier of government spending on output is 0.601.
In line with the steep decline in investment, the fall in the capital stock is roughly twice as deep as in the model without risky government bonds. Due to the arbitrage of bonds and loans, the price of bonds falls as well, and the risk-adjusted bond spread increases, exerting further pressure on the balance sheet of the bank. The fall in the net worth of banks, and the increase in the leverage ratio are roughly 5 times as strong as before. Note that the risk-adjusted return on bonds increases, but the default probability decreases. The fall in the default probability after a fiscal expansion is in line with empirical findings by Born et al. (2015) and Strobel (2015) who find that in periods of a higher of sovereign risk, risk spreads increase on impact after contractionary fiscal shocks. In the context of this model, the default probability is a simple function of the debt-to-GDP ratio. As the increase in GDP is larger in percentage terms than the increase in public debt, the debt-to-GDP ratio actually decreases for the first two quarters. Thereafter it is higher than in the steady state, since the increase in debt is more persistent than the output response.
The deeper decline of the capital stock leads to a lower marginal product of labor and a lower real wage and a smaller increase of equilibrium hours worked. As labor and output increase by less, the increase in marginal cost and inflation is smaller. Hence, the increase in the nominal interest rate implied by the Taylor rule is smaller and the decline in consumption is smaller in the model with sovereign risk. However, although, consumption is crowded out by less in this model, the stronger crowding-out of investment in this model is the dominating factor for the output reaction.

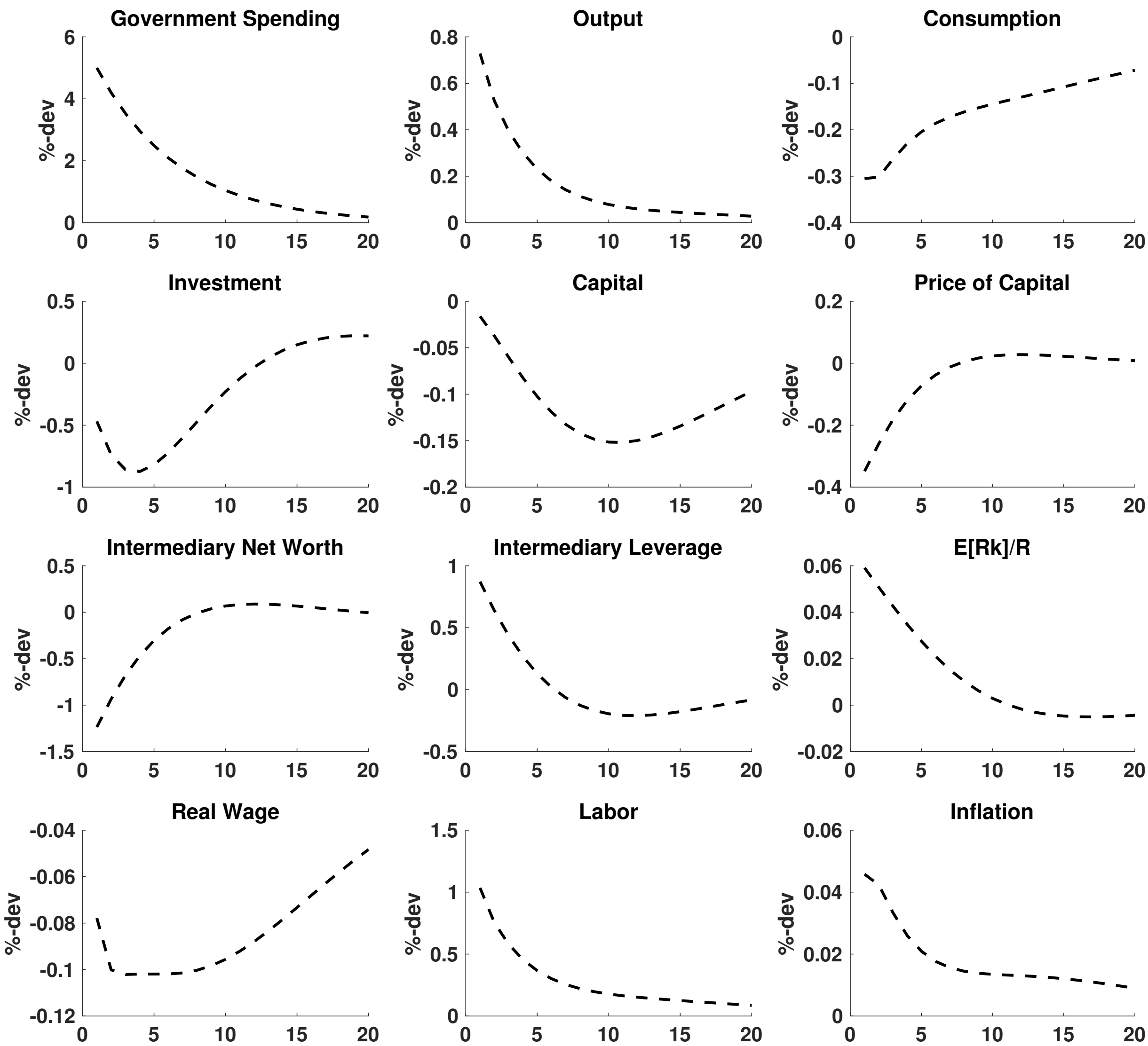

Figure 2: Dynamic consequences of a shock in government spending of 1 percent. The shock hits the economy at its steady state. The blue lines depict the impulse responses obtained from a first-order approximation to equilibrium dynamics.The black line depicts the impulse responses obtained from a third-order approximation to equilibrium dynamics.

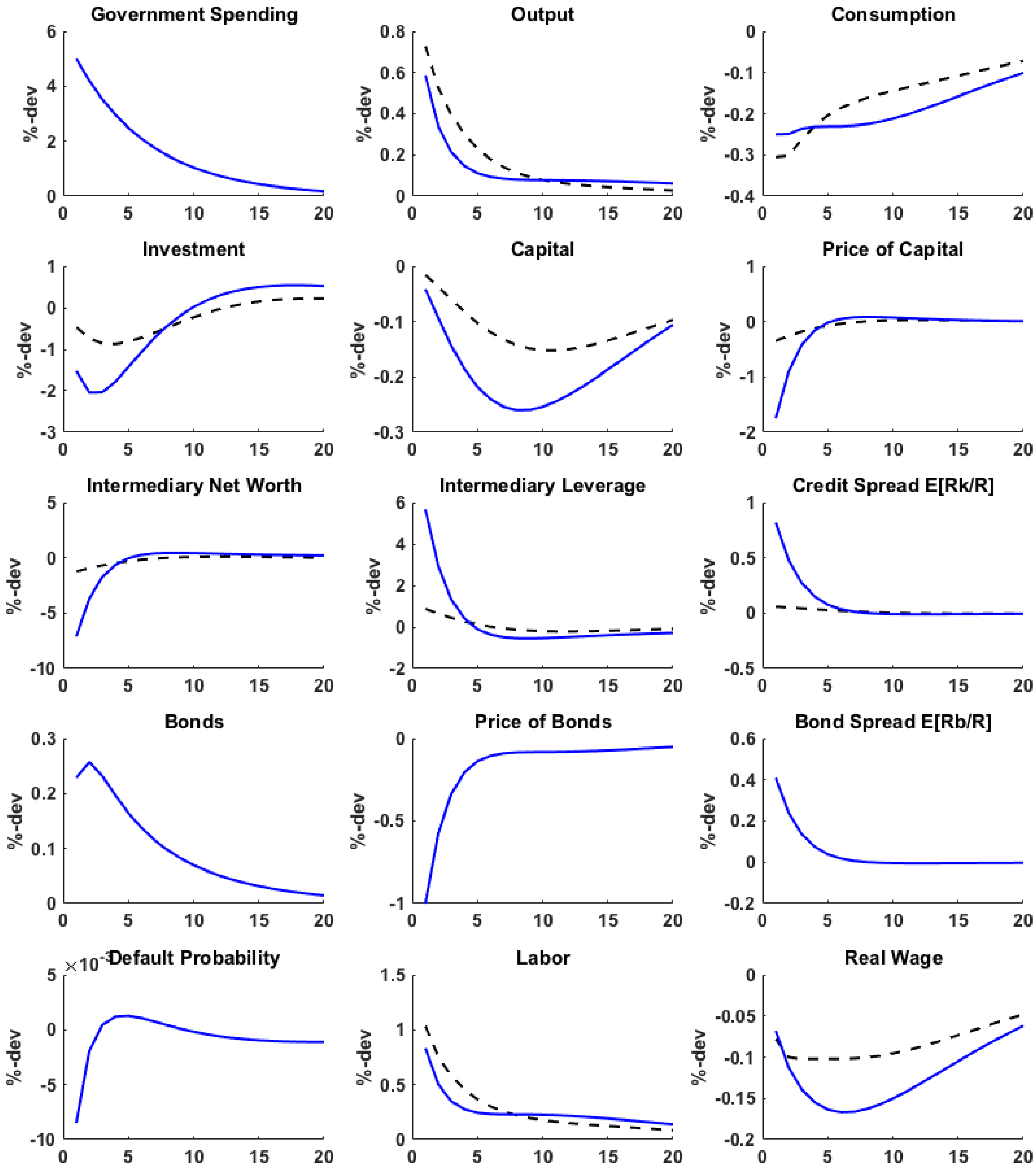

Figure 3: Dynamic consequences of a shock in government spending of 1 percent. The shock hits the economy at its steady state. The dashed line depicts the linear impulse responses obtained in the GK-model. from a first-order approximation to equilibrium dynamics.The blue line depicts the linear impulse responses obtained in the model with sovereign risk.

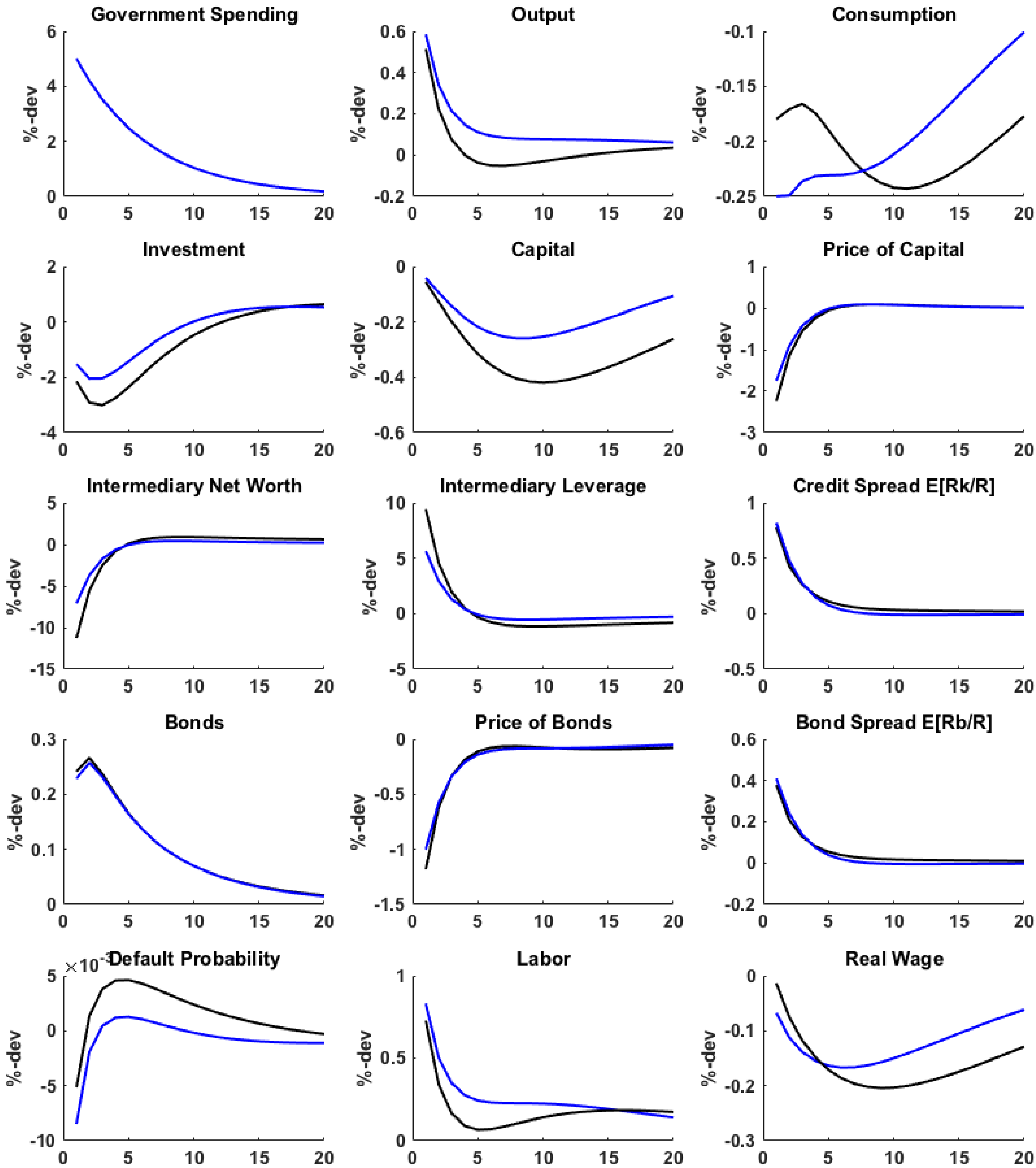

Figure 4: Dynamic consequences of a shock in government spending of 1 percent in the model with sovereign risk. The shock hits the economy at its steady state. The blue lines depict the impulse responses obtained from a first-order approximation to equilibrium dynamics.The black line depicts the impulse responses obtained from a third-order approximation to equilibrium dynamics.

4.3. The role of uncertainty in the model with risky government bonds

Lets now turn to the impact of uncertainty on the transmission of government spending shocks. Figure (4) compares the linear impulse responses to the government spending shock in the model with sovereign risk (blue line) with the impulse responses to the same shock, obtained by a third-order approximation to equilibrium dynamics (black line). In both cases, the shock hits the economy, when it is at its steady state. As mentioned above, the main difference in the two sets of impulse responses lies with the adjustment of the impulse responses to realized shocks for the time-varying conditional volatility of future risk.[15]

In the model there are two types of agents that care about risk, financial intermediaries and households. Both are risk averse. Already in the linear case, the increase in government spending implies decreasing asset prices and fire sales by the intermediaries, who incur losses. Now that the intermediaries take into account the risk of further, potentially detrimental, shocks in the future, they reduce their exposure to the risky assets to an even larger degree. On impact this leads to a further fall in asset prices, and to a stronger contraction of the supply of loans. Hence, investment and the capital stock fall by more than in the linear case.

Again, the reaction of investment drives the differences in the output responses. The impact multiplier on output falls to 0.5057. Even more notable is the decreasing persistence of the stimulus to output. After 4 quarters output has declined below the initial level, causing a mild recession. Now, as the response of public debt to the shock is hardly altered, and the increase in output is small and short-lived, the fall in the debt-to GDP ratio, and by direct implication, the probability of a sovereign default, become smaller and short-lived as well. Already in the second quarter after the government spending increase, the probability of a default is above the initial level, and stays positive thereafter. Due to the non-linear shape of the fiscal limit function, the increase in the probability of default implies a higher sensitivity of interest rates to economic fundamentals. This raises the conditional volatility associated with future shocks. While the current increase in the probability of default amplifies current losses, the outlook of a stronger response of the economy to future shocks increases the motive of the banks to reduce their exposure to risky assets, contributing to stronger fire sales, and the stronger reduction in investment and output.

The second type of agents who cares about risk is the household. The impact of uncertainty on the households responses to shocks is to strenghten the motive of consumption and labor smoothing. Following the shock, they reduce their consumption by less, and due to the smaller wealth effect, increase labor supply by less. However, when the more pronounced decline in capital becomes effective over time, the non-linear response of aggregate consumption falls below its linear counterpart.

4.4. The state-dependence of the fiscal multiplier

In the context of non-linear approximation to the equilibrium dynamics, the effects of shocks, and here the government spending multiplier becomes state-dependent. The literature on state dependent government multipliers has rapidly increased in the past few years. The empirical literature, started by Auerbach and Gorodnichenko (2012) finds that government spending multipliers are far larger in recessions than in expansions.[16] Sims and Wolff (2013) attempt to replicate this empirical finding in a model of the type as in Smets and Wouters (2007) using a third-order approximation to equilibrium dynamics, and simulating shocks at the steady state, as well as in regions of a simulated state space where output is high (expansions), and where output is

[15]In the context of the third-order approximation to the policy functions, shown above, the difference between the two sets of impulse responses is almost entirely driven by the risk correction term $y_{\sigma_i^2}$. The effects by the second-order and third-order terms, describing actually realized fluctuations are negligible.

[16]Other examples contain e.g., Baum, Polawski-Ribeiro, and Weber (2012) and Batini, Callegari, and Melina (2012)

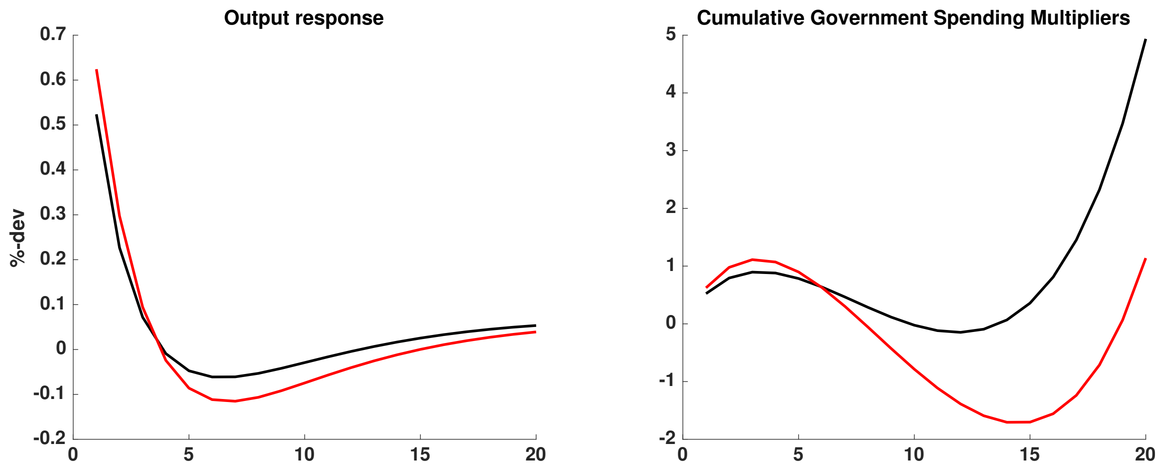

Figure 5: To the left: the dynamic consequence of a shock in government spending of 1 percent of steady state GDP. To the right: the associated cumulative government spending multipliers. Black line: The shock hits the economy at its steady state. Red line: The shock hits the economy at the average state vector in the region of the state space with a annualized default probability higher than 300bp.

low (recessions). They find relatively small differences in the government spending multipliers across states in their model. In contrast to their approach, I focus on the impact of sovereign risk on the size of the government spending multiplier. In the context of my model, the financial accelerator, and the presence of sovereign risk, enhance the role of uncertainty for the effects of shocks, and therefore also imply larger differences in government spending multipliers across the regions of the state space.

The effects that drive the response of the variables to government spending shocks are the same as the ones described in section 4.3. Figure (5) compares the effects of the government spending shock, when it hits the economy at its stochastic steady state (black line), with the effect of the same shock, when it hits the economy at the average simulated state vector with an annualized default probability higher than 300 bp (red line). In the risky region, the default indicator is more sensitive to the reactions of output and debt to the shock. The initially stronger increase of output, relative to public debt leads to an increase in the debt-to-GDP ratio, which initially decreases the default probability to a larger extent in the risky region of the state space. The stronger initial decrease in sovereign risk, causes the output response to the government spending shock to be stronger on impact. When, as in the previous section, after some periods, the debt to GDP ratio is increased by the shock, the increase in the default probability is more pronounced in the risky region of the state space leading to a stronger decline in output, and a deeper and more persistent recessionary impact of the increase in government spending. The implied cumulative government spending multipliers for both regimes are depicted in the right hand panel of figure (5). While for the short run, the difference in the cumulative fiscal multipliers is rather small, being larger at the steady state, for the medium run, cumulative multipliers in the risky part of the state space become are noticably smaller than at the steady state, and become negative a time horizon of 8 quarters and remain negative within a time-horizon of 16 quarters (see Figure 4).[17]. Given the baseline calibration, when sovereign risk is high, the through of the medium cumulative multipliers is at -1.71.

[17]The same effects, namely a higher impact multiplier, lower persistence and more pronounced overshooting compared to the steady-state case, can be observed to differing degree for different definitions of of the risky state, and qualitatively do not depend on the choice of the threshold of 300bp.

The result of negative medium-run multipliers is in line with empirical results by Ilzetzki et al. (2013) who link find negative long-run multipliers for countries with a high debt-to-GDP ratio, and the negative cumulative multipliers for the medium run, in the presence of higher sovereign risk in Italy by Strobel (2015). Additionally, the more general finding of smaller multipliers in the presence of sovereign risk is supported by Perotti (1999), who defines a bad state of fiscal finances in a panel of countries through a higher debt-to-GDP ratio.
This experiment illustrates the dependence of government spending multiplier on the degree of sovereign risk. Additionally, it illustrates the gain of using a third-order approximation to equilibrium dynamics, and taking the effect of time-varying volatility into account, as this facilitates an analysis of the state-dependence of the government spending multiplier.

4.4. Sensitivity analysis

This section contains a sensitivity analysis with the respect to the steady state debt-to-GDP ratio, $B/(4Y)$, the sensitivity of the sovereign default indicator to the debt-to-GDP ratio governed by the parameter η_2 in the default function, and the degree of uncertainty in the model. The key takeaway is that while the responses to government spending shocks that are obtained with a first-order approximation are hardly affected by changes in the degree of sovereign risk, and do not depend on the degree of uncertainty, in the case of a third-order approximation, where risk matters for the equilibrium dynamics, the degree of sovereign risk as well as the degree of uncertainty, have an economically important effect on the transmission of government spending shocks.

Figure (6) depicts the output responses for the first-oder approximation (left column) and the third-order approximation (right column). Lets first consider the response of output to the shock for different values of the debt-to-GDP ratio. Firstly, the size of the debt-to-GDP ratio affects the extent to which the asset side of the financial intermediaries is affected by variations in the price of government bonds, and hence the extent of deleveraging in the face of shocks that decrease asset prices. Secondly, the size of the steady state debt-to-GDP ratio has an influence on the sensitivity of the default probability and the fluctuations of interest rates after the government spending shock. In the context of a linear impulse response function (top left panel), decreasing or increasing the steady state value of $(B/(4Y))$ by ten percent relative to the baseline calibration, hardly affects the output response. In contrast to that, the top right panel shows that in the case of the third-order approximation, the difference in the debt-to-GDP ratio, implies strongly different output responses to the government spending shock. The higher $(B/(4Y))$, the smaller the impact multiplier and the stronger the decline in output in the medium run. This illustrates that taking into account the impact of risk on the equilibrium dynamics strongly amplifies the effect of the higher sovereign risk associated with the higher debt-to-GDP ratio on the real economy, which depresses the government spending multiplier.

The middle row of figure (6) depicts output responses to the shock for different values of η_2. Again, the linear responses of output are hardly affected by changes in the sensitivity of fundamentals to sovereign risk (middle left panel). In contrast to that, differences in the sensitivity of the sovereign default probability to fundamentals affect the output dynamics in a qualitatively similar way as changes in the debt-to-GDP ratio. However, the effect of changes in eta_2 on the output response is smaller, as the second effect associated with changes in the debt-to-GDP ratio is captured, namely the changed sensitivity of sovereign risk, and hence interest rates, with respect to the state of fiscal finances. Lastly, consider an experiment, in which the standard deviation of all shocks in the model, except the government spending shock are scaled up or down, by multiplying them with $k \in \{0; 0.5; 1; 1.5\}$. The bottom row of figure (6)) depicts the output responses for the

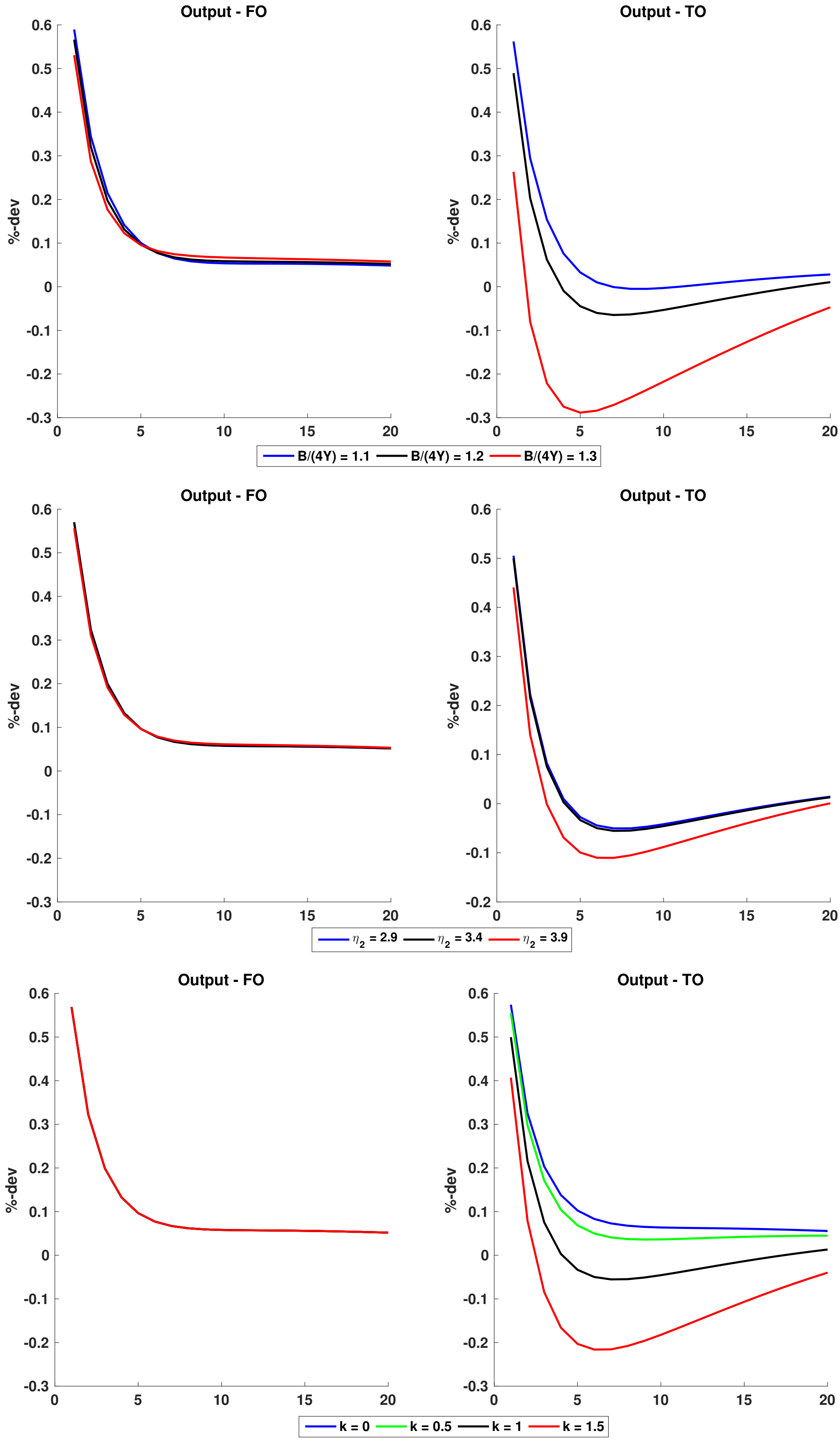

Figure 6: To the left: the dynamic consequence of a shock in government spending of 1 percent of steady state GDP obtained by a first-order approximation to equilibrium dynamics. To the right: the dynamic consequence of the same shock, but obtained by a third-order approximation to equilibrium dynamics. In both cases, the shock hits the economy at its steady state.

different degrees of risk. For $k = 1$, we obtain our baseline results. The impulse responses on the bottom left panel are unaltered by the degree of risk. The panel on the bottom right panel shows, that, for $k = 0$, when the government spending shock is the only source of uncertainty in the model, the dynamic response of output is roughly the same as in the case of the first-order condition. The risk adjustment term as in the policy function (51) becomes very small. [18] When the other sources of shocks are switched on, the degree of uncertainty, reduces the output response to the shock, and decreases the government spending multiplier. When the scale of uncertainty is scaled up by $k = 1.5$. The output response to the shock quickly becomes negative. This experiment underscores that for assessing correctly the quantitative effect of government spending shocks in the presence of risk, one needs precise estimates of all standard deviations of shocks. Secondly, it highlights that the introduction of further sources of uncertainty (e.g., a preference shock, an investment specific technology shock, or a direct shock to the default risk as in Bocola (2015)) will have a potentially important impact on the size of the government spending multiplier.

5. Conclusion

This paper analyzes the effects of uncertainty and sovereign default risk on the transmission of government spending shocks in the context of a DSGE model with financial friction and sovereign risk. The main findings are the following: First, the presence of risky government bonds decreases the size of the government spending multiplier by increasing the extent of crowding out of investment after a positive shock to government spending. Secondly, this effect is stronger, when impulse responses are obtained with a third-order approximation to equilibrium dynamics. The precautionary behaviour by banks, leads to a stronger decrease in asset holdings than in the linear case, following the shock. Taking the effect of uncertainty into account contributes to generating a quantitatively important effect of the presence of sovereign risk on the multiplier. Thirdly, accounting for risk, allows for the analysis of state-dependent multipliers. When the economy moves to the region of the state space with a higher default probability, the sensitivity of the default probability to movement in the debt-to-GDP ratio is increased, and the higher sovereign risk implies smaller government spending multipliers than at the steady state. In the medium run cumulative multipliers can become negative, when the economy moves to the region with increased sovereign risk. As mentioned above, this finding is in line with several empirical results on the impact of sovereign risk or public debt on the size of fiscal multipliers. Fourth, increasing the effects of sovereign risk through an increase in the steady state debt-to GDP ratio or the shape of the default function, as well as increasing the degree of uncertainty in the model further decrease the government spending multiplier.

There are two caveats that come with these results. First, while the results hold up qualitatively, the size of the effect is more sensitive to the correct calibration in the case of a third-order approximation than in the case of a linear model. This highlights the necessity for a careful calibration for a correct quantitative assessment of the effect of sovereign risk on the size of the government spending multiplier. Secondly, this paper does not attempt a full-fledged analysis of a typical sovereign default crisis. Often such episodes come along with steep recessions, featuring high unemployment or liquidity problems of the agents in the economy. Furthermore, increases in government spending are often financed through distortionary taxes. These aspects together with open economy issues, and the political economy of such episodes are omitted from the analysis to focus on the contribution of sovereign risk and uncertainty to the fiscal multiplier. The many possible extensions of the analysis yield an interesting field of future research.

[18] The second and third order terms associated with the realized shocks, are negligible throughout the analysis.

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A Appendix

Bank's optimization problem

The bank maximizes its value function subject to a balance sheet constraint and an incentive constraint:

$$V_{jt} = \max_{\{K_j t\},\{B_j t\},\{D_j t\}} \quad E_t \Lambda_{t,t+1}[(1-\theta)N_{jt} + \theta V_{j,t+1}]$$

$$\begin{aligned} s.t. \qquad Q_t K_{jt} + Q_t^b B_{jt} &= D_{jt} + N_{jt} \\ V_{jt} &\geq \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt} \end{aligned}$$

In the following, I make the assumption that the incentive constraint is always binding. Additionally, the law of motion of net worth is assumed to be:

$$N_{jt} = (R_{kt} - R_{t-1})Q_{t-1}K_{j,t-1} + (R_{bt} - R_{t-1})Q_{t-1}^b B_{j,t-1} + R_t N_{j,t-1}.$$

Guess that the value function is linear in loans, government bonds and net worth

$$V_{jt} = \nu_{kjt} Q_t K_{jt} + \nu_{bjt} Q_t^b B_{jt} + \nu_{njt} N_{jt}.$$

The Lagrangian function for the optimization problem of the bank reads:

$$\mathscr{L} = (1+\mu_{jt})(\nu_{kjt} Q_t K_{jt} + \nu_{bjt} Q_t^b B_{jt} + \nu_{njt} N_{jt}) - \mu_{jt}(\lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt})$$

Hence, the first order conditions for loans, bonds, and the Lagrangian multiplier, μ_t, are:

$$\nu_{kjt} = \lambda \frac{\mu_{jt}}{1+\mu_{jt}}$$

$$\nu_{bjt} = \lambda_b \frac{\mu_{jt}}{1+\mu_{jt}}$$

$$\nu_{kjt} Q_t K_{jt} + \nu_{bjt} Q_t^b B_{jt} + \nu_{njt} N_{jt} = \lambda Q_t K_{jt} + \lambda_b Q_t^b B_{jt}$$

The supply of loans can be obtained by rearranging the incentive constraint:

$$Q_t K_{jt} = \frac{\nu_{bjt} - \lambda_b}{\lambda - \nu_{kjt}} Q_t^b B_{jt} + \frac{\nu_{njt}}{\lambda - \nu_{kjt}} N_{jt}$$

The Value function can be written solely as a function of N_{jt}. by substituting out private assets using the foregoing equation:

$$
\begin{aligned}
V_{jt} &= \nu_{kjt} Q_t K_{jt} + \nu_{bjt} Q_t^b B_{jt} + \nu_{njt} N_{jt} \\
\Leftrightarrow V_{jt} &= \left[\nu_{kjt} \frac{\nu_{bjt} - \lambda_b}{\lambda - \nu_{kjt}} + \nu_{bjt}\right] Q_t^b B_{jt} + \left[\nu_{kt} \frac{\nu_{njt}}{\lambda - \nu_{kjt}} + \nu_{njt}\right] N_{jt} \\
\Leftrightarrow V_{jt} &= \left[\nu_{kt} \frac{\nu_{njt}}{\lambda - \nu_{kjt}} + \nu_{njt}\right] N_{jt} \\
\Leftrightarrow V_{jt} &= \left[\frac{\lambda \nu_{njt}}{\lambda - \nu_{kjt}}\right] N_{jt} \\
\Leftrightarrow V_{jt} &= \left[\frac{\lambda \nu_{njt}}{\lambda - \lambda \frac{\mu_{jt}}{1+\mu_{jt}}}\right] N_{jt} \\
\Leftrightarrow V_{jt} &= (\nu_{njt}(1+\mu_{jt})) N_{jt}
\end{aligned}
$$

Defining: $\Omega_{j,t} \equiv \Lambda_{t-1,t}((1-\theta) + \theta(1+\mu_{jt})\nu_{njt})$, plugging this expression of the value function into the Bellman equation, and using the law of motion of net worth, yields:

$$
\begin{aligned}
V_{jt} &= \nu_{kjt} Q_t K_{jt} + \nu_{bjt} Q_t^b B_{jt} + \nu_{njt} N_{jt} \\
&= \beta E_t \Lambda_{t,t+1}[(1-\theta) N_{jt} + \theta V_{j,t+1}] \\
&= \beta E_t \Omega_{t+1}((R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t}).
\end{aligned}
$$

One can then solve for the coefficients of the value function:

$$
\begin{aligned}
\nu_{kjt} &= \beta E_t \Omega_{jt+1}(R_{k,t+1} - R_t), && \text{(A-1)} \\
\nu_{bjt} &= \beta E_t \Omega_{jt+1}(R_{b,t+1} - R_t), && \text{(A-2)} \\
\nu_{djt} &= \beta E_t \Omega_{jt+1} R_t. && \text{(A-3)}
\end{aligned}
$$

The leverage ratio, ϕ_t, is obtained by defining the ratio of bonds over loans in the portfolio of the bank, $\varsigma_{jt} = \frac{Q_t^b B_t}{Q_t K_t}$ and starting from the incentive constraint:

$$
\begin{aligned}
& (\lambda - \nu_{kjt}) Q_t K_{jt} + (\lambda_b - \nu_{bjt}) Q_t^b B_{jt} &&= \nu_{njt} N_{jt} \\
\Rightarrow\ & (\lambda - \nu_{kjt}) Q_t K_{jt} + \left(\frac{\lambda \nu_{bjt} - \nu_{kjt} \nu_{bjt}}{\nu_{kjt}}\right) Q_t^b B_{jt} &&= \nu_{njt} N_{jt} \\
\Rightarrow\ & (\lambda - \nu_{kjt}) Q_t K_{jt} + \left((\lambda - \nu_{kjt}) \frac{\lambda_b}{\lambda}\right) Q_t^b B_{jt} &&= \nu_{njt} N_{jt} \\
\Rightarrow\ & (\lambda - \nu_{kjt}) Q_t K_{jt} + (\lambda - \nu_{kjt}) \frac{\lambda_b}{\lambda} \varsigma_{jt} Q_t K_{jt} &&= \nu_{njt} N_{jt} \\
\Rightarrow\ & Q_t K_{jt} &&= \frac{\nu_{njt}}{(\lambda - \nu_{kjt})(1 + \frac{\lambda_b}{\lambda} \varsigma_{jt})} N_{jt} \\
\Rightarrow\ & Q_t K_{jt} + Q_t^b B_{jt} &&= \frac{\nu_{njt}(1 + \varsigma_{jt})}{(\lambda - \nu_{kjt})(1 + \frac{\lambda_b}{\lambda} \varsigma_{jt})} N_{jt} = \phi_t N_t
\end{aligned}
$$