

# Housing over the Life Cycle and Across Countries: A Structural Analysis

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## Abstract

Micro data document striking differences in household wealth across European countries; for example, the median net wealth in Italy and Spain exceeds more than three times that in Germany. The bulk of this variation in wealth is driven by the home-ownership rate and house prices.

We solve a life-cycle model with a discrete choice between owning and renting a house. Countries differ in the dynamics of house prices and in the institutional set-up of the housing market (maximum loan-to-value ratios, tax deductibility of mortgage payments and costs of buying, selling and maintaining a house). We calibrate the model using country-specific labor income profiles, mortality rates and family sizes over the life cycle.

Through the lens of the model, we study and interpret the substantial differences in home-ownership rates, holdings of wealth and household saving behavior across large European countries.

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**Keywords** Housing, Life-Cycle Model, Structural Estimation, Household Portfolios, Cross-Country Comparisons

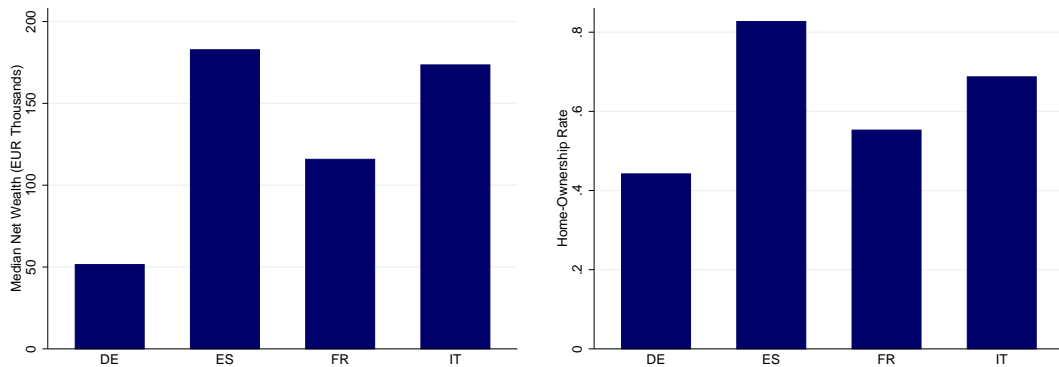
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**Figure 1** Median Net Wealth (Left) and Home-Ownership Rate (Right) Across Countries



Notes: Eurosystem Household Finance and Consumption Survey.

## 1 Introduction

Micro data document striking differences in household wealth across European countries: the median net wealth in Italy and Spain exceeds more than three times that in Germany. The bulk of this variation in wealth is driven by the home-ownership rate and house prices. For example, while more than 80 percent of Spanish households own the house they live in, the median household in Germany, where the home-ownership rate is only 44 percent, is a renter (Figure 1).

To interpret these differences we solve and estimate a life-cycle model with a discrete choice between owning and renting a house à la Li and Yao (2007) and Li et al. (forthcoming). Households face idiosyncratic shocks to income and house prices. They either rent or buy, and save in liquid assets to smooth consumption, to make the down-payment for the house and to bequeath wealth. Housing serves both as a consumption good and as an investment device. Countries differ in the dynamics of house prices and in the institutional set-up of the housing market (maximum loan-to-value ratios, tax deductibility of mortgage payments and costs of buying, selling and maintaining a house). We calibrate the model using country-specific labor income profiles, mortality rates and family sizes over the life cycle.

The key contribution of this paper is that it is the first structural investigation of cross-country differences in household wealth and their determinants: We use our framework to study the determinants of home-ownership rates, holdings of wealth and household saving behavior across large European countries.

The discrete choice between owning or renting a home, which complicates the solution. To speed up our computation, we apply the new Iskhakov et al. (2015) extension of Carroll (2006)'s Endogenous Gridpoints Method to discrete-continuous problems. The DC-EGM method augments the model with taste shocks, which smooth the kinks in the

value functions and discontinuities in the optimal policy functions, and thus make the problem more tractable.<sup>1</sup>

**TO BE FINISHED**

## 2 The Life-Cycle Model

### 2.1 Demographics

Households are born at the age of 20 and live for at most 80 years, with a probability to be alive at age  $j$ ,  $\hat{p}_j$ , conditional on being alive at  $j - 1$ . Households work until the age of  $K = 65$  and then retire.

### 2.2 The Income Process

During working life, households receive exogenous labor income and face idiosyncratic income shocks. Labor income consists of a permanent component and a transitory component (following a large literature; see, e.g., Carroll and Samwick (1997) and Cocco et al. (2005)):

$$Y_{i,t} = P_{i,t}\xi_{i,t}, \quad (1)$$

$$P_{i,t} = \Gamma_{i,t}P_{i,t-1}\psi_{i,t}, \quad (2)$$

where  $\Gamma_{i,t}$  is the growth factor, a deterministic function of age and household characteristics that captures the profile of labor income over the life cycle,  $P_{i,t}$  is the permanent income component, which follows a random walk.  $\xi_{i,t}$  and  $\psi_{i,t}$  are the transitory and the permanent shock to labor income, respectively.<sup>2</sup>

Retirement is deterministic and exogenous. After retirement, households get a constant (risk-free) retirement income, a fraction of the terminal labor income:

$$Y_{i,t} = \tau P_{i,K} \text{ for } t > K.$$

### 2.3 Housing and Mortgages

Households start out each year as renters or owners. The price per unit of housing is  $P_t^H$ , so that the total value of a house is  $P_t^H H_t$ . In addition to housing, households can save in liquid financial assets  $A_t$  with a constant return  $R = (1 + r)$ . Renting a house costs  $\alpha \times P_t^H H_t$  per year. Each year, a renter decides whether to stay a renter or to

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<sup>1</sup>Similar, somewhat simpler model has recently been used by Yao et al. (2015) to analyze how housing and mortgage debt affect households' marginal propensity to consume out of wealth. Their model does not include stochastic house prices, which are important in explaining the differences in holdings of housing wealth.

<sup>2</sup>The shocks  $\psi_{i,t}$  and  $\xi_{i,t}$  are independent and identically distributed. Permanent shocks are log-normally distributed:  $\log \psi_t \sim \mathcal{N}(-\sigma_\psi^2/2, \sigma_\psi^2)$ ; transitory shocks are distributed as follows:

$$\xi_t = \begin{cases} 0 & \text{with probability } \mathcal{U} > 0 \\ \theta_t/\mathcal{U} & \text{with probability } (1 - \mathcal{U}), \text{ where } \log \theta_t \sim \mathcal{N}(-\sigma_\theta^2/2, \sigma_\theta^2), \end{cases}$$

where  $\mathcal{U}$  is the probability of unemployment. These assumptions imply that  $E(\theta_{i,t}) = E(\psi_{i,t}) = 1$ .

buy a house (and become a homeowner). As long as they rent, households can make adjustments to the level of their housing services instantaneously and without any cost.

To buy a house, households pay a down-payment of at least  $\delta \times P_t^H H_t$  and finance the rest through a mortgage, so that  $A_t \geq -(1 - \delta)P_t^H H_t$  (in all  $t$ ). Mortgage-takers pay a constant interest rate  $R^D$  as long as  $A_t \leq 0$ . The mortgage can be adjusted without any cost in each year, and is carried forward to the next year. Borrowing is only allowed through collateralizing the investor's house and getting a mortgage.

Housing is illiquid.<sup>3</sup> Each year, a homeowner either stays in his house or moves. Adjusting the level of housing is costly; selling a house costs  $\phi \times P_t^H H_t$ . To adjust, a homeowner (i) sells his house and moves out to become a renter, or (ii) sells his house and buys another one (with a different size).

## 2.4 Prices and Returns

House prices are stochastic and follow a (geometric) random walk with a drift:

$$P_t^H = P_{t-1}^H \times \tilde{R}_t^H,$$

where  $\tilde{R}_t^H$  is iid  $\mathcal{N}(\mu_H, \sigma_H^2)$ . The return on housing is potentially correlated with labor income growth. Households in the baseline model have an incentive to own homes (rather than rent) because  $\alpha > r + \lambda$  (see Li and Yao (2007)), while  $\mu_h = 0$ .

## 2.5 Preferences and the Bequest Function

Households derive utility from (non-housing) consumption  $C_t$  and housing services  $H_t$ , and from bequeathing terminal wealth. Life-time utility is:

$$\mathbf{E}_t \left\{ \sum_{t=0}^T \beta^t \prod_{s=0}^t p_s (p_t u(C_t, H_t) + (1 - p_t) B(W_t)) \right\}.$$

We assume a CRRA utility function with the Cobb–Douglas consumption aggregate:

$$u(C_t, H_t) = \frac{(C_t^{1-\omega} H_t^\omega)^{1-\gamma}}{1-\gamma}.$$

The parameter  $\omega$  determines the share of housing services in the composite consumption good,  $\gamma$  captures the curvature of the utility function with respect to the composite good (also risk aversion). As long as there are no housing-related transaction costs or borrowing constraints, this functional form implies that the proportion of expenditure on housing is fixed:

$$H_t = \frac{\omega}{\alpha(1-\omega)} \times C_t.$$

(see Yao and Zhang (2005)). Therefore, if renting is optimal, housing is eliminated as a separate choice variable. (However, when households buy, they face housing costs and

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<sup>3</sup>Apart from gaining utility, owning a house enables households to borrow against their home equity, and may imply lower per year costs than renting.

borrowing constraints, and the value of the housing asset becomes a state variable that affects consumption choices.)

Households have a warm-glow bequest motive:

$$B(W_t) = L^\gamma \frac{(W_t(\omega/\alpha P_t^H)^\omega (1-\omega)^{1-\omega})^{1-\gamma}}{1-\gamma},$$

where  $L$  determines the strength of the motive.

## 2.6 Wealth Accumulation and Budget Constraints

Denote  $W_t$  the total spendable resources (cash on hand) that households have at the beginning of each year, conditional on selling their houses and including income. Renter households' wealth at the beginning of  $t + 1$  is:

$$W_{t+1} = RA_t + Y_{t+1}.$$

Owner households' wealth at the beginning of  $t + 1$  is:

$$W_{t+1} = RA_t + Y_{t+1} - RD_t + \tilde{R}_{t+1}^H (1 - \phi) P_t H_t,$$

where the collateral constraint holds:  $A_t \geq -(1 - \delta) P_t^H H_t$ .

The budget constraints, conditional on the housing status are as follows.

- Renter who keeps renting:

$$\begin{aligned} W_t &= C_t + A_t + \alpha H_t P_t \\ W_{t+1} &= R(W_t - C_t - \alpha H_t P_t^H) + Y_{t+1} \end{aligned}$$

- Renter who buys:

$$\begin{aligned} W_t &= C_t + A_t + (\delta + \lambda) H_t P_t^H \\ W_{t+1} &= R(W_t - C_t - (\delta + \lambda) H_t P_t^H) + Y_{t+1} + (1 - \phi) \tilde{R}_{t+1}^H H_t P_t^H \end{aligned}$$

- Owner who keeps the existing house:

$$\begin{aligned} W_t &= C_t + A_t + (\delta + \lambda - \phi) H_{t-1} P_t^H \\ W_{t+1} &= R(W_t - C_t - (\delta + \lambda - \phi) H_{t-1} P_t^H) + Y_{t+1} + (1 - \phi) \tilde{R}_{t+1}^H H_t P_t^H \end{aligned}$$

Note that we need to subtract the selling cost  $\phi$  for the stayer as  $W_t$  is defined as wealth upon home sale.

- Owner who sells and becomes renter:

$$W_t = C_t + A_t + \alpha H_t P_t^H$$

- Owner who sells and buys a new house:

$$W_{t+1} = R(W_t - C_t - (\delta + \lambda) H_t P_t^H) + Y_{t+1} + (1 - \phi) \tilde{R}_{t+1}^H P_t^H H_t$$

## 2.7 The Value Function

The life-cycle optimization problem can be stated recursively. The Bellman equation is:

$$V_t(W_t) = \max_{\{C_t, H_t, A_t\}} \left\{ \widehat{p}_t \left( u(C_t, H_t) + \beta \mathbf{E}_t(V_{t+1}(W_{t+1})) \right) + (1 - \widehat{p}_t) B(W_t) \right\}$$

The vector of endogenous state variables is  $X_t = \{\mathbf{1}_{H>0,t-1}, P_t, P_t^H, H_{t-1}, W_t\}$ ; the vector of choice variables is  $Z_t = \{\mathbf{1}_{H>0,t}, C_t, H_t, A_t\}$

## 2.8 The Normalized Problem

We normalize the household's state and choice variables by the permanent income  $P_t$  and the value function by  $(P_t/(P_t^H)^\omega)^{1-\gamma}$  to reduce the state space.

The normalized decision problem is:

$$v_t(x_t) = \max_{\{c_t, h_t, a_t\}} \left\{ \left( u(c_t, h_t) + \widehat{p}_t \beta \mathbf{E}_t(v_{t+1}(x_{t+1}) \left( \frac{\Gamma_{t+1} \psi_{t+1}}{\tilde{R}_{t+1}^{h\omega}} \right)^{1-\gamma}) \right) + (1 - \widehat{p}_t) B(w_t) \right\}$$

s.t.

$$w_{t+1} = \underbrace{\frac{R}{\Gamma_{t+1} \psi_{t+1}}}_{\equiv \mathcal{R}_{t+1}} a_t + \mathbf{1}_{H>0} \frac{h_t}{\Gamma_{t+1} \psi_{t+1}} \left( \tilde{R}_{t+1}^h (1 - \phi) - R(1 - \delta) \right) + \xi_{t+1},$$

$$h_{t+1} = \underbrace{\frac{\tilde{R}_{t+1}^h}{\Gamma_{t+1} \psi_{t+1}}}_{\equiv \tilde{\mathcal{R}}_{t+1}} h_t,$$

$$w_t = \begin{cases} c_t + a_t + \alpha h_t & \text{renters who keep renting and owners who sell to rent} \\ c_t + a_t + (\delta + \lambda) h_t & \text{renters who buy and owners who adjust} \\ c_t + a_t + (\delta + \lambda - \phi) \bar{h}_t & \text{owners who stay} \end{cases}$$

where  $c_t > 0, h_t > 0, a_t \geq -(1 - \delta)h_t$ .

The vector of endogenous state variables is  $x_t = \{\mathbf{1}_{H>0,t-1}, \bar{h}_t, w_t\}$ ; the vector of endogenous choice variables is  $z_t = \{\mathbf{1}_{H>0,t}, c_t, h_t, a_t\}$ .

## 3 The Solution Method

We solve the model recursively using Iskhakov et al. (2015)'s extension of the endogenous gridpoints method (Carroll (2006)) to discrete-continuous problems. The DC-EGM method augments the model with taste shocks, which smooth the kinks in the value functions and discontinuities in the optimal policy functions, and thus make the problem more tractable.

To solve the model, we divide the computational setup into three parts: the problem of the renter who keeps renting (which is equivalent to the problem of the owner who sells and rents), the problem of the renter who buys (which is equivalent to the owner who adjusts his housing stock), and the problem of the owner who stays.

The renter's problem is solved with the standard EGM. Because adjusting the housing stock for the renter who keeps renting is costless,  $h_t$  is given as a constant fraction of  $c_t$ :  $h_t = \omega/\alpha(1 - \omega) \times c_t$ . Consequently, the problem reduces to a standard consumption-savings model with one control variable,  $c_t$ . In the last period, the value function is determined by the bequest motive.

A homeowner adjusting the housing stock chooses  $c_t$  and  $h_t$  simultaneously. The EGM can still be applied in this two-dimensional set-up, as shown in Hintermaier and Koeniger (2010) and Carroll (2012), section 7.<sup>4</sup>

Homeowners who stay in their homes again only maximize over non-housing consumption as they cannot adjust their housing stock.

In the full model, households maximize over the three choice-specific value functions. While these individual value functions are concave, the combined value, the upper envelope over these functions, is generally not and will have kinks (non-differentiabilities) at the intersection points. As a result, the optimal policy functions are discontinuous and the standard EGM breaks down.<sup>5</sup>

Iskhakov et al. (2015)'s DC-EGM augments the model with additively separable choice-specific random taste shocks  $\varepsilon_t$ , which are iid and have the Extreme Value type I distribution. The problem then becomes more tractable because at least some kinks disappear; if the taste shocks are large enough, the combined value function becomes concave. The taste shocks can be interpreted as a technical device to simplify the problem, or can have a structural interpretation as unobserved state variables.

## 4 The Results

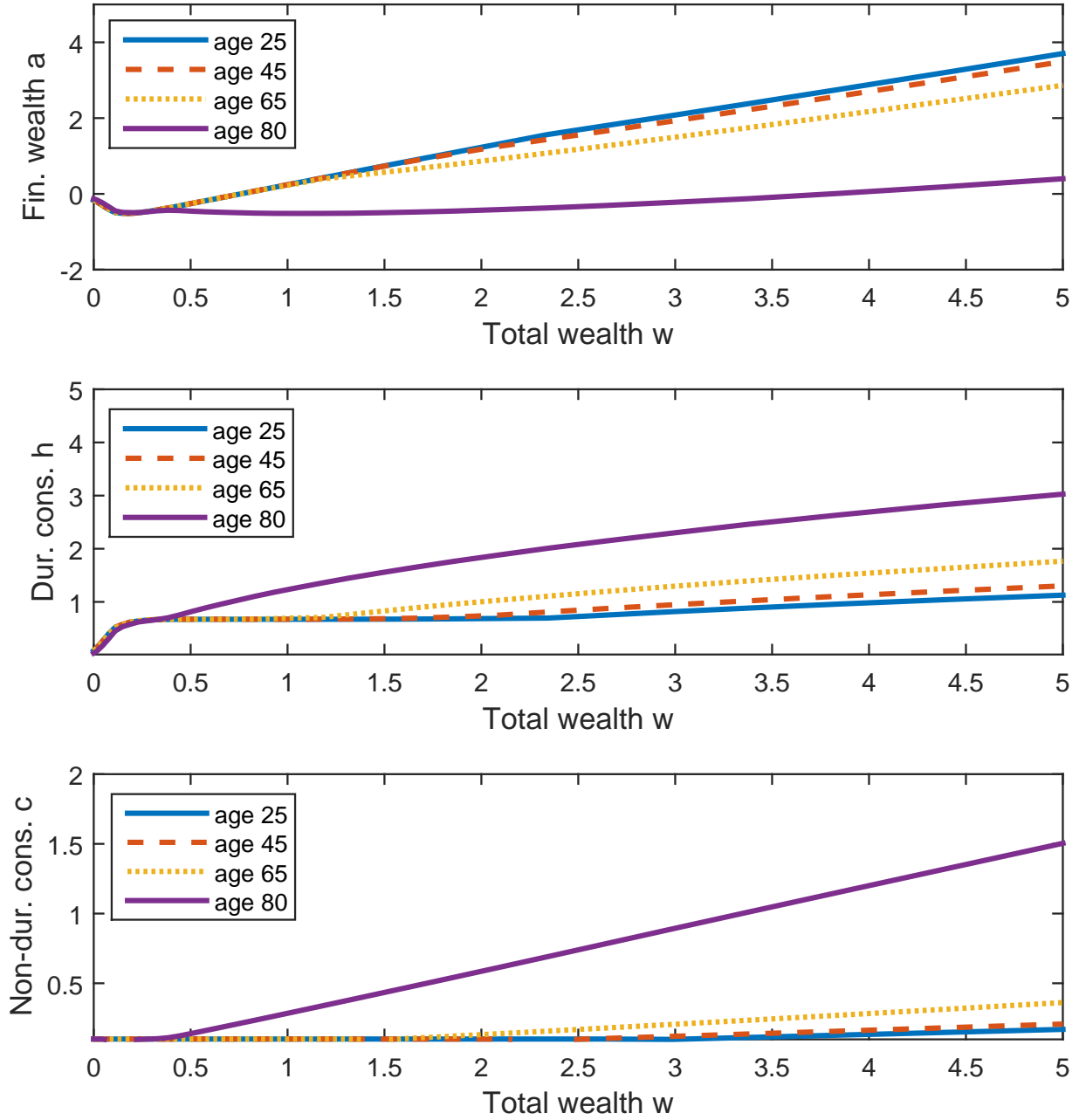
## 5 Data

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<sup>4</sup>We apply the hybrid EGM, see also Ludwig and Schön (2013) and White (2015). Our setup is slightly more complicated than that of Hintermaier and Koeniger (2010) and Carroll (2012) because we have stochastic house prices.

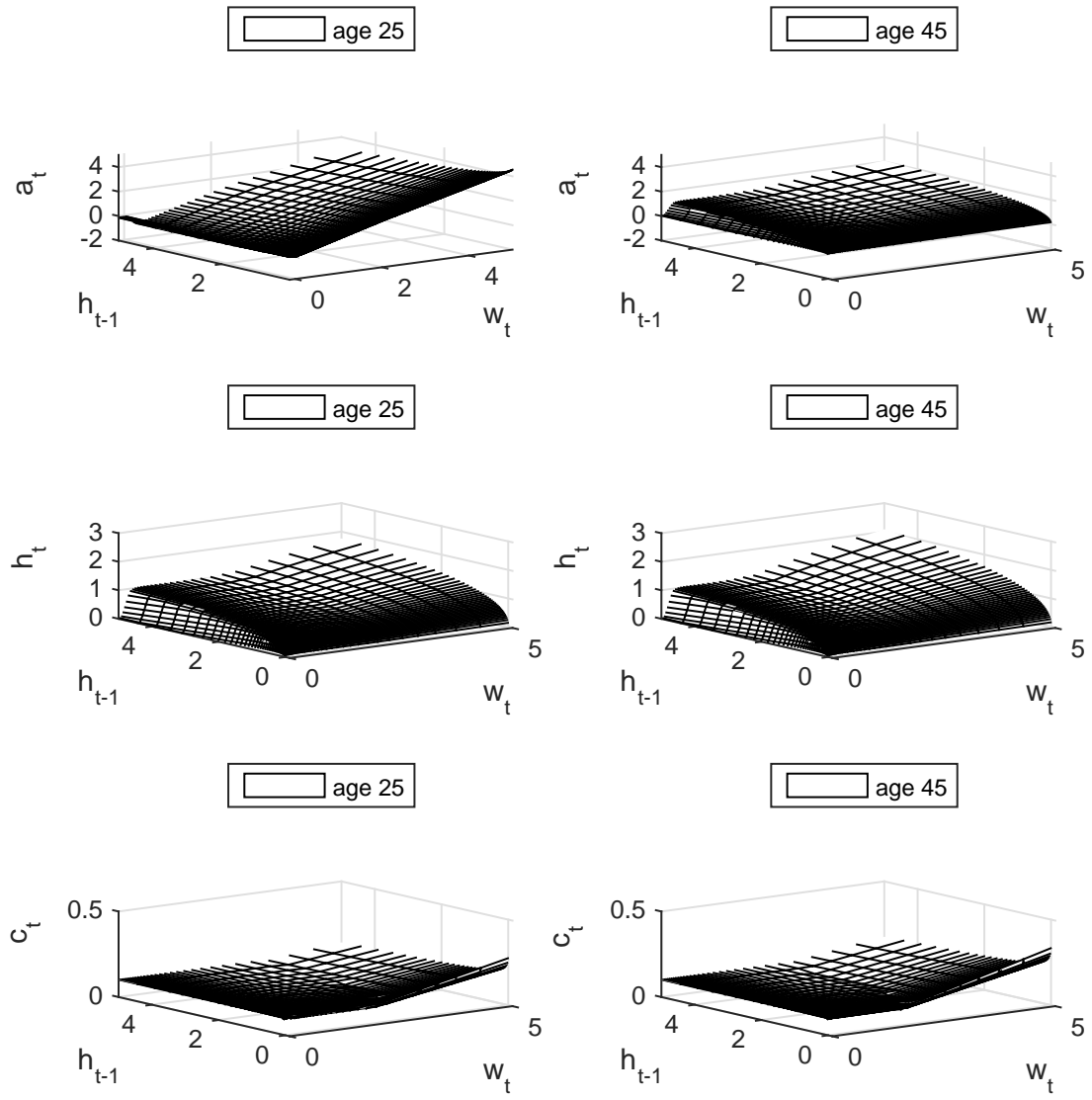
<sup>5</sup>Fella (2014) provides a general method, which, however, is substantially more complicated to implement in our setup than Iskhakov et al. (2015)'s DC-EGM, and is slower.

Figure 2 Wealth Profiles of the Optimal Policy Functions by Age

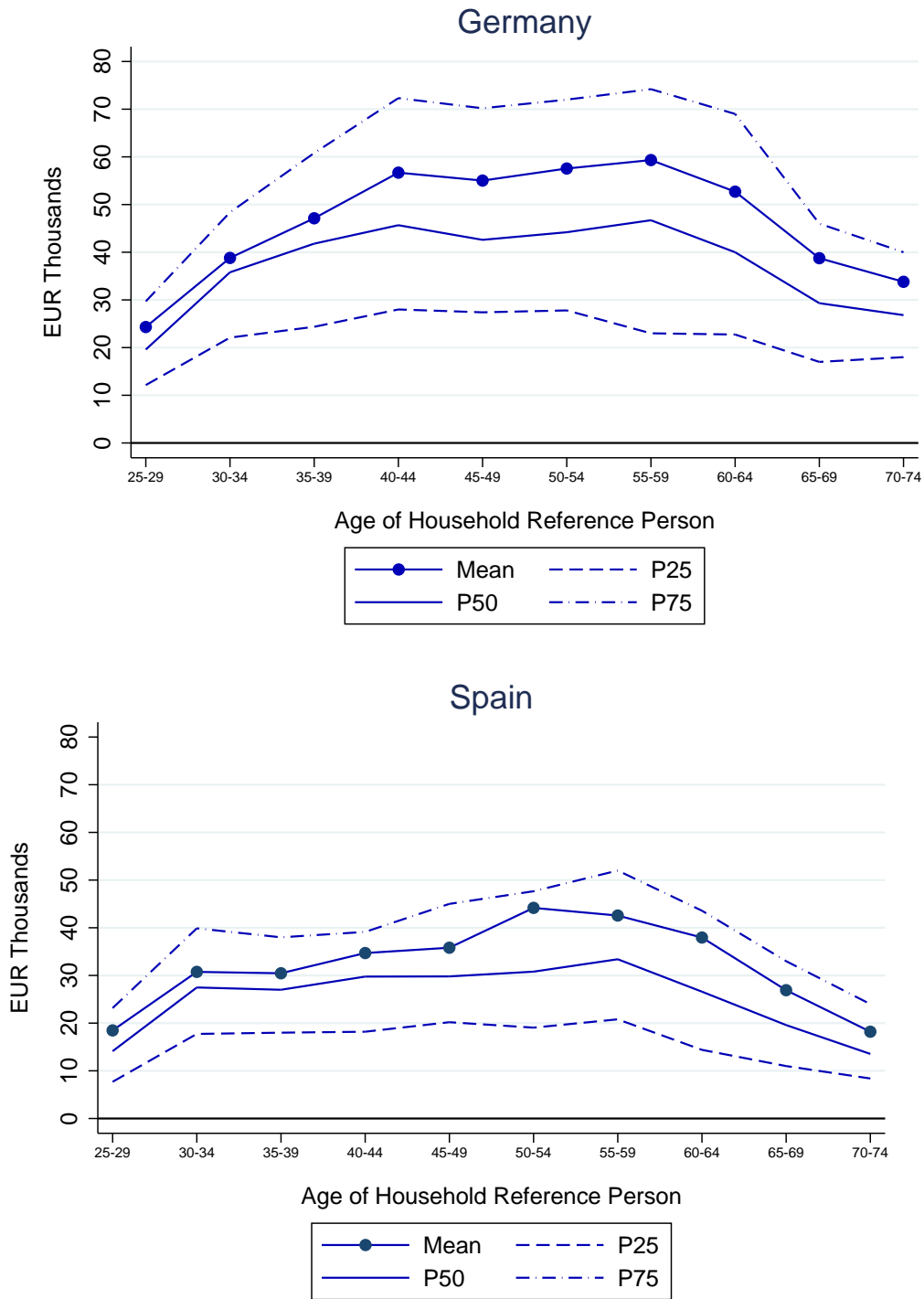




**Figure 3** The Optimal Policy Functions by Age



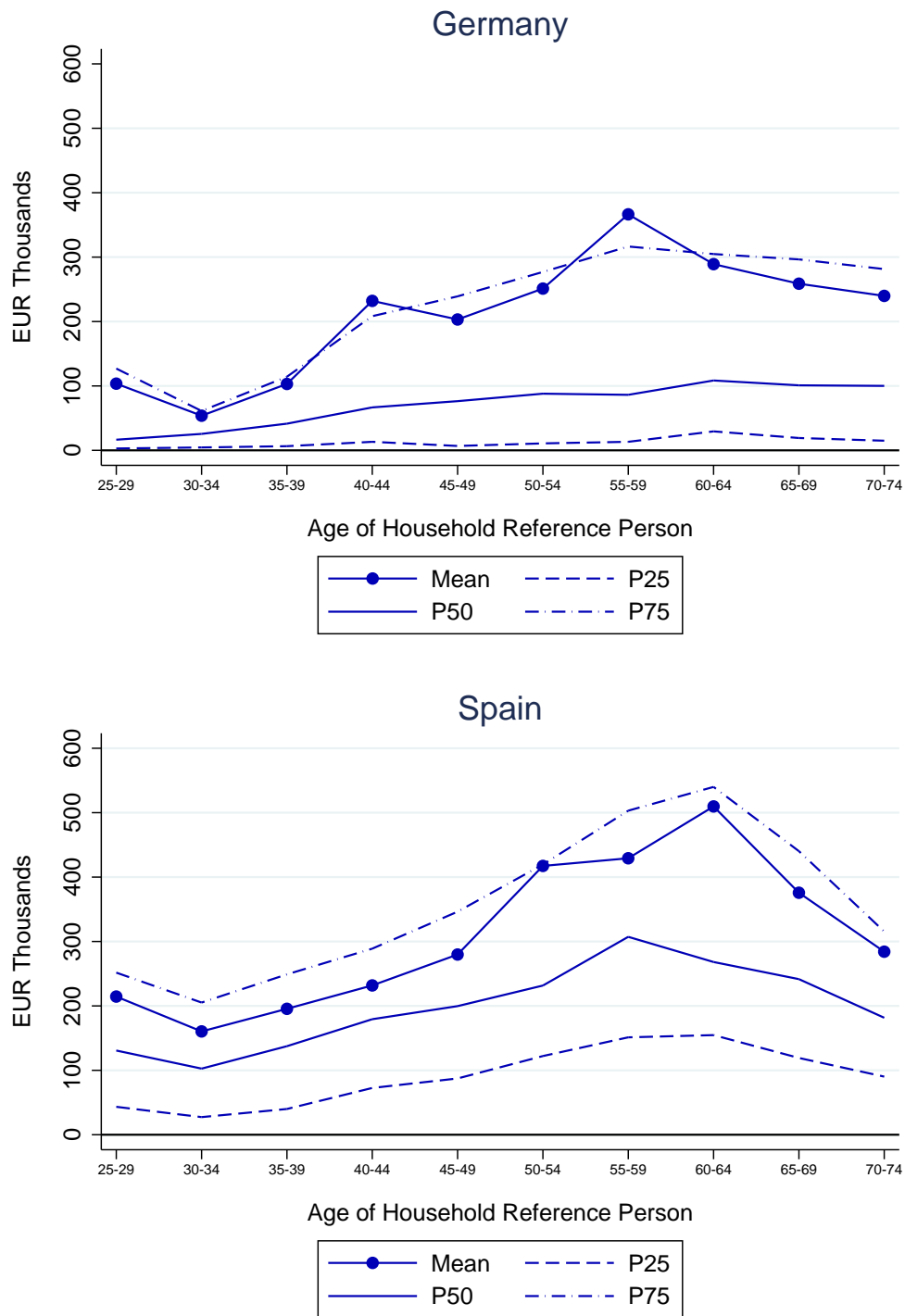
**Figure 4** Distribution of Household Income by Age, Germany and Spain



Notes:

Eurosystem Household Finance and Consumption Survey.

**Figure 5** Distribution of Household Net Wealth by Age, Germany and Spain



Notes: Eurosystem Household Finance and Consumption Survey.

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