

# Coordination on bubbles in large-group asset pricing experiments

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## Abstract

We present a large-group asset pricing experiment with the design of Hommes et al. (2008). Participants are asked to predict the price of a risky asset, whose realization depends on an aggregation of all individual forecasts. The asset markets consist of 21 to 32 participants, a group size larger than in most experiments. Multiple large price bubbles are observed in six out of seven markets. The bubbles emerge even faster and more frequently than in the smaller markets of Hommes et al. (2008). Individual forecast errors do not cancel out at the aggregate level and expectations cannot be called rational in the sense of Muth. Participants tend to coordinate on a trend-following prediction strategy that gives rise to the large bubbles. The price patterns observed in the experiment can be captured by a behavioral heuristics switching model. Heterogeneity in expectations seems crucial to explain the market dynamics.

**JEL codes:** D84, C91, C92, G12

**Keywords:** Experimental economics, Asset price bubbles, Heterogeneous expectations, Heuristics switching

# 1 Introduction

Expectations are a crucial part of many economic systems. Asset markets are an example of positive feedback systems: when many traders expect the price of an asset to rise, the demand for the asset will increase, leading to a higher market price. This price rise in turn affects expectations. If traders extrapolate a trend in the asset price, the positive feedback can give rise to a bubble, which occurs when the market price becomes significantly higher than the fundamental value of the asset. Such expectations-driven bubbles were observed in several asset pricing experiments (e.g. Hommes et al. (2005a) and Hommes et al. (2008)).

The main goal of this paper is to study the effect of group size on the (in)stability of experimental asset markets: will bubbles arise in large groups? It is important to know whether the results of small-scale asset market experiments can be generalized to settings with larger groups. Nevertheless, surprisingly little work focusing on group size in experiments has been done, perhaps because larger experiments are costly. We conduct an asset pricing experiment with larger groups than usual and analyze both individual expectations and aggregate outcomes to empirically test whether the same type of expectations-driven bubbles also occurs in larger markets.

Any dynamic model of an asset market strongly depends on the underlying expectations hypothesis. It is therefore essential to develop a theory about how people form expectations and how they learn and adapt their forecasting strategies over time. Over the past decades, there has been a gradual shift in the expectations hypotheses in economics from using perfectly rational expectations to assuming boundedly rational agents who use simple heuristics or “rule of thumb” strategies. In the behavioral finance literature, many patterns in the behavior of individual traders are identified, such as trend extrapolation and positive feedback or momentum trading, contrarian trading, under- and overreaction and overconfidence (see e.g. Hirshleifer (2001), Barberis and Thaler (2003) and Barber and Odean (2013) for extensive reviews). Expectations are closely linked to these types of boundedly

rational behavior. Moreover, investors might not all have the same expectations. This heterogeneity may also play an essential role in market dynamics.

Laboratory experiments are well-suited to study individual expectation formation and possible heterogeneity in expectations. Since the experimenter can control the underlying economic fundamentals, it is possible to obtain explicit observations on expectations, investigate how individual behavior shapes market outcomes and study whether aggregate outcomes deviate from fundamentals. The experimental data can be used to empirically validate different expectations hypotheses. It can provide insight in which forecasting strategies are more likely to be used by human subjects, so that the “wilderness of bounded rationality” can be disciplined.

Learning-to-forecast experiments (LtFEs) are especially useful to study expectations in feedback systems. This type of experiment was introduced by Marimon and Sunder (1993, 1994). In LtFEs, the participants’ only task is to submit forecasts in a particular economic setting. All other actions are computerized. Because LtFEs separate expectation formation from other choices, such as trading decisions, they provide “clean” data on expectations. In the asset pricing LtFE of Hommes, Sonnemans, Tuinstra and Van de Velden (2008) (henceforth HSTV08), participants had to predict the price of a risky asset for 50 periods. Each experimental asset market consisted of six participants. In every period, the market price was derived from mean-variance optimization and depended on the average price forecast. Participants only had qualitative information about the asset market, but they knew the risk free interest rate and the mean dividend of the risky asset. This is enough information to be able to calculate the fundamental value of the asset. Nevertheless, large price bubbles occurred, with predictions hitting the upper bound of 1000 and prices rising up to fifteen times the fundamental value. Analysis of the individual expectations revealed that participants within the same market coordinated on a common trend-following prediction strategy. A similar asset pricing LtFE was conducted by Hommes et al. (2005a), although the design of that experiment inhibited the formation of large bubbles because of the presence of fundamental

robot traders and an upper bound on prices of 100. Hommes (2011) provides a review of LtFEs in different economic settings.

In macroeconomic and financial market experiments, group size is an important issue. The scale of many macroeconomic situations can often not be replicated in experiments because of the limited lab size and financial constraints. Yet, the use of macroeconomic models with microfoundations makes it possible to test the assumptions on individual behavior in small-scale lab experiments. Many examples of macroeconomic experiments are discussed by Duffy (2008, 2015) and Ricciuti (2008), who argue that lab experiments are useful to complement the theoretical and empirical research in macroeconomics. However, the question remains whether the results of such small-scale experiments are robust to increases in group size. The answer to this question is critical if the results from the experiment are compared to real macroeconomic situations with many interacting agents.

Experiments with large groups are rare. One of the few topics in the literature where large-group experiments are conducted is the provision of public goods. Several papers examine the effects of group sizes ranging from 4 to 100 participants (Isaac and Walker, 1988; Isaac et al., 1994; Weimann et al., 2012; Xu et al., 2013; Diederich, 2013). Two examples of market experiments with large groups are Williams and Walker (1993) and Bossaerts and Plott (2004). But the vast majority of experiments work with small groups, often consisting of less than ten participants. Strategic interaction between participants already becomes more difficult in a group of e.g. six participants than in a group of two or three participants. Still, this does not directly imply that behavior in small-scale experiments is representative of behavior in experiments with larger groups.

In this paper, we address the issue of group size by analyzing an asset pricing experiment with the same design as HSTV08, but with larger markets. The experiment consists of seven sessions with 21 to 32 participants per market. This is a group size that fits in most labs for economic experiments. Although the groups in this experiment are still small compared to the number of traders in a real asset

market, the increase in market size compared to HSTV08 can shed light on the differences that group size could make in both individual expectations and market outcomes. With more participants in a market, individual predictions have less influence on the market price and coordination on a common prediction strategy is likely to be harder. Therefore, it might be that individual forecast errors cancel out at the aggregate level, which would be consistent with the formulation of the rational expectations hypothesis by Muth (1961). In this case, the market could stabilize quickly. On the other hand, participants might see a rising price as a trend and adopt a trend-following strategy, just as in the experiment of HSTV08. This behavior may cause big bubbles in a larger market as well. Only a new experiment with larger group sizes can clarify which effect will dominate.

Large-scale asset market experiments based on the classical design of Smith et al. (1988) have shown that bubbles and crashes also occur in markets with 244, 304 and 310 participants (Williams and Walker, 1993; Williams, 2008). In their experiment, participants buy and sell a risky asset in a double auction market. The fundamental value of the finitely lived asset is monotonically decreasing over time. Despite the fact that the fundamental value is explicitly given to all participants in each trading round, a large price bubble forms in all markets, followed by a crash. The authors observe tremendous heterogeneity in bid and ask prices and conclude that common information is clearly not sufficient to induce common (rational) expectations. The main difference with our experiment is that Williams and Walker (1993) only consider trading decisions, while our LtFE specifically focusses on expectation formation. Furthermore, the fundamental value of the risky asset in our experiment is constant instead of declining, and not explicitly given. Lastly, participants in the experiment of Williams and Walker (1993) only received extra credits for a microeconomics course, whereas we use monetary incentives.

The results of our LtFE show that six out of seven markets exhibit large price bubbles. In fact, these bubbles emerge even faster and occur more frequently than in the smaller markets of HSTV08. The typical price pattern shows multiple large

bubbles with decreasing amplitude. Due to the high instability of the market, the forecasting performance of most participants is poor, resulting in very low earnings. Participants are able to coordinate on similar prediction strategies, but there is also evidence of heterogeneity in expectations and strategy switching. Estimation of individual prediction strategies shows that the behavior of many participants can be captured by simple linear forecasting rules. Coordination on trend extrapolation seems to cause the formation of large bubbles in the larger groups.

The individual prediction rules are further investigated with a behavioral heuristics switching model (HSM). This model takes account of heterogeneity in expectations and evolutionary selection among different forecasting heuristics. We use the same benchmark HSM as in Anufriev and Hommes (2012b), which is an extension of the model of Brock and Hommes (1997). The results of one-period-ahead simulations show that the bubbles are amplified by the use of a strong trend-following prediction rule. The use of a more flexible anchoring and adjustment heuristic can explain the persistent price oscillations. The fit of the benchmark model is compared with rational expectations, five homogeneous prediction rules, a HSM with fixed fractions and a HSM with parameters that are fitted to the experimental data. The flexibility of the HSM substantially improves the model fit. 50-period-ahead simulations of the HSM demonstrate that the typical price pattern of large bubbles can be replicated without the use of experimental data. Again, the importance of strong trend extrapolation for the formation of large bubbles is highlighted.

The remainder of this paper is organized as follows. Section 2 briefly reviews the experiment of HSTV08 and the HSM of Anufriev and Hommes (2012b). In Section 3, the design of our experiment with larger markets is explained in detail. The results of the experiment are given in Section 4. In Section 5, we estimate simple linear forecasting rules for each participant. Section 6 examines the formation of the first bubble in our large groups as compared to the small groups of HSTV08. The specification of the HSM is described in Section 7 and the results of the model are presented in Section 8. Section 9 summarizes and concludes.

## 2 Expectations and bubbles in HSTV08

### 2.1 Learning-to-forecast experiment of HSTV08

The framework of the LtFE of HSTV08 is a simple asset market with a risk free asset paying a fixed interest rate, and a risky asset paying an uncertain dividend every period. Participants in the experiment have the role of an advisor to a large pension fund. Their only task is to give two-period-ahead predictions of the price of the risky asset for 50 periods. Earnings were based on prediction accuracy. Given their forecast, a computerized trader calculates optimal demand for the risky asset, derived from mean-variance optimization. The realized market price then depends on the average price forecast of all subjects in the market (see Equation (5), Section 3). The participants do not know the underlying market equilibrium equation, but they are told that the higher their price forecast, the larger the demand for the risky asset. Hence they could infer that there is positive feedback. The interest rate  $r = 5\%$  and the mean dividend  $\bar{y} = 3$  are given, so subjects have enough information to calculate the fundamental price of the risky asset,  $p^f = \bar{y}/r = 60$ . For practical reasons, there is an upper bound on predictions of 1000. Participants will only learn about this upper bound when they try to enter a prediction that is larger than 1000.

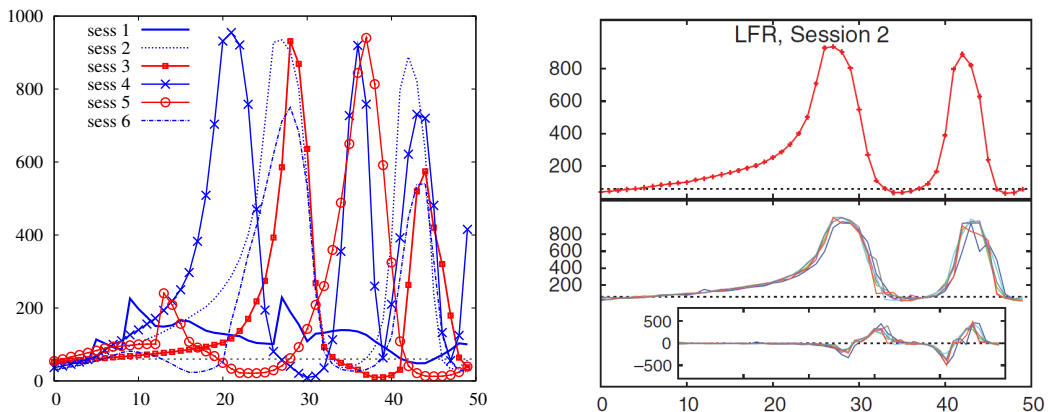


Figure 1: Experimental results of HSTV08.

*Left:* market prices in all sessions. The dashed line indicates the fundamental value of 60. *Right:* prices (top panel), individual predictions (middle panel) and prediction errors (bottom panel) in session 2.

HSTV08 conducted six sessions of this experiment with six participants in each session. The left panel in Figure 1 shows the realized market prices in all sessions. The markets are highly unstable, with large price bubbles occurring in five out of six sessions. Once predictions reach the upper bound of 1000, the trend reverses and the market crashes rapidly. There is no convergence to the fundamental value of 60.<sup>1</sup> Analysis of the individual predictions shows that expectations could not be described as rational. Participants made structural forecast errors and were alike in the way they made these errors. The right panel in Figure 1 illustrates this coordination of expectations in a typical session: the differences in individual predictions are very small. The coordination on a common prediction strategy is observed in all sessions. This result is remarkable, since participants did not observe the forecasts of the other subjects, so they could only coordinate through the realized market prices.

## 2.2 A model of heterogeneous expectations

Anufriev and Hommes (2012b) further examined the results of HSTV08 with a heuristics switching model (HSM), extending the switching model of Brock and Hommes (1997). A HSM is a simple learning model of heterogeneous expectations. The idea is that participants make a choice between several simple forecasting rules in every period. They tend to switch to rules that performed better in recent periods. Anufriev and Hommes (2012b) discuss a HSM with four heuristics: adaptive expectations (ADA), a weak trend-following rule (WTR), a strong trend-following rule (STR) and a learning anchoring and adjustment heuristic (LAA) (see Section 7 for a detailed specification of the HSM). They use the model for one-period-ahead simulations of the market price, using experimental prices as inputs. The HSM thus uses the same information as the participants in the experiment.

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<sup>1</sup>Heemeijer et al. (2009) show that the positive feedback drives the non-convergence. In another treatment with negative feedback, typical for supply-driven commodity markets, prices are stable and quickly converge to the fundamental value of 60. In particular, a positive feedback system with a near unit root leads to coordination on bubbles (Sonnemans and Tuinstra, 2010; Bao and Hommes, 2015).



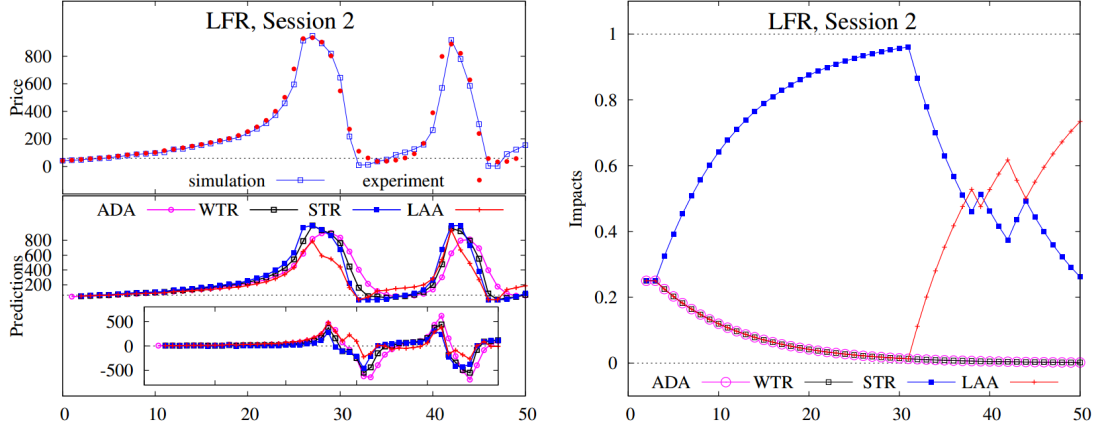


Figure 2: Results of the HSM for session 2 of HSTV08  
*Left:* experimental prices and simulated prices (top panel), forecasts of the four heuristics (middle panel) and corresponding forecast errors (bottom panel). The dashed line in the top and middle panel indicates the fundamental value of 60. *Right:* impacts of the four heuristics.

Figure 2 illustrates the one-period-ahead simulations of the HSM for session 2 of HSTV08. The top left panel displays the experimental prices and the simulated prices of the HSM. The predictions of the four heuristics and the corresponding forecast errors are shown in the bottom left panels. The right panel shows the impacts of the four heuristics over time. The impact of the strong trend-following rule increases quickly to more than 90% after 25 periods. This suggests that the large bubble is amplified by coordination on trend extrapolation. After the crash, the impact of the LAA rule starts increasing, since this heuristic is more flexible and better able to explain trend reversals and price oscillations. Comparing to several homogeneous expectations models, the HSM better predicts the observed price patterns in the experiments. Thus despite the coordination, heterogeneity in expectations and switching between forecasting rules seem crucial to explain the results of the asset pricing LtFEs.

### 3 Asset pricing experiment

The LtFE of HSTV08 shows that large expectations-driven bubbles occur in markets with six participants. Is this result robust to increases in group size? To study this important question empirically, we conducted a new asset pricing LtFE with

the same design as HSTV08, but with larger markets. The markets consist of 21 to 32 participants, a group size that typically fits into experimental labs.

### 3.1 Asset pricing model with heterogeneous expectations

The experiment is based on a standard asset pricing model with heterogeneous beliefs, as in Campbell et al. (1997) and Brock and Hommes (1998). The asset market consists of  $I$  traders. Each trader can invest in a risk free asset or a risky asset. The risk free asset (e.g. a savings account) pays a fixed interest rate  $r$ . Each share of the infinitely lived risky asset pays an uncertain dividend  $y_t$  in every period  $t$ . The dividends are independently and identically distributed with mean  $\bar{y}$ . The fundamental value of the risky asset is thus given by  $p^f = \bar{y}/r$ . The price of the risky asset in period  $t$  is denoted by  $p_t$  and the number of shares purchased by trader  $i$  in period  $t$  is denoted by  $z_{it}$ . Note that a negative value of  $z_{it}$  means that the trader sells a number of shares of the risky asset. The realized wealth of the trader in the next period is given by

$$W_{i,t+1} = RW_{i,t} + (p_{t+1} + y_{t+1} - Rp_t)z_{it}, \quad (1)$$

where  $R = 1 + r$  is the gross rate of return of the risk free asset.

Traders differ in their beliefs about the conditional mean  $E_{it}$  and the conditional variance  $V_{it}$  of the evolution of wealth. Traders are myopic mean-variance optimizers, so the demand for shares  $z_{it}$  corresponds to the solution of

$$\begin{aligned} \max_{z_{it}} \left\{ E_{it}(W_{t+1}) - \frac{1}{2}aV_{it}(W_{t+1}) \right\} = \\ \max_{z_{it}} \left\{ z_{it}E_{it}(p_{t+1} + y_{t+1} - Rp_t) - \frac{1}{2}az_{it}^2V_{it}(p_{t+1} + y_{t+1} - Rp_t) \right\}, \end{aligned} \quad (2)$$

where  $a$  measures the degree of risk aversion. It is assumed that traders believe that the conditional variance of excess returns is constant:  $V_{it}(p_{t+1} + y_{t+1} - Rp_t) = \sigma^2$  for all  $i$ . This assumption can be made because the experiment only asks for point

predictions, so traders beliefs about the distribution of returns are not considered. The solution of the maximization problem is then

$$z_{it} = \frac{E_{it}(p_{t+1} + y_{t+1} - Rp_t)}{a\sigma^2}. \quad (3)$$

An increase in the expected price of the risky asset in period  $t + 1$  thus leads to an increase in demand in period  $t$ .

Assume that the outside supply of shares  $z^s$  is zero, which means that some traders buy the shares that other traders sell, so that  $z_{it}$  can be interpreted as excess demand. This is equivalent to assuming risk neutral traders ( $a = 0$ ). The market equilibrium condition is then given by

$$\sum_{i=1}^I z_{it} = \frac{1}{a\sigma^2} \sum_{i=1}^I E_{it}(p_{t+1} + y_{t+1} - Rp_t) = z^s = 0. \quad (4)$$

Using that the mean dividend  $\bar{y}$  is common knowledge so that  $E_{it}(y_{t+1}) = \bar{y}$  for all  $i$  and all  $t$ , we can solve the equation for the market equilibrium price:

$$p_t = \frac{1}{1+r} \left[ \frac{1}{I} \sum_{i=1}^I p_{i,t+1}^e + \bar{y} \right], \quad (5)$$

where  $E_{it}(p_{t+1}) = p_{i,t+1}^e$  denotes the prediction in period  $t$  for the price in period  $t + 1$ . Note that the price in each period depends on the price predictions for the next period. When traders are in period  $t$  and they have to forecast the price for period  $t + 1$ , they do not know  $p_t$  yet and thus can only use the past prices up to period  $t - 1$ . Hence, they make a two-period-ahead price prediction.

Equation (5) shows that there is positive feedback: if the average price prediction is high, than the realized market price will also be high, and vice versa. This is an important characteristic of asset markets. The level of the interest rate  $r$  determines the strength of feedback, given by  $\lambda = \frac{1}{1+r}$ . The fundamental price  $p^f = \bar{y}/r$  gets a weight of  $1 - \lambda = \frac{r}{1+r}$  in the pricing mechanism. So the higher the interest rate, the weaker the expectations feedback and the stronger the market

price is pushed towards the fundamental value.

### 3.2 Benchmark expectation rules

To get a feeling for the dynamics of the asset pricing model, we simulate the evolution of the market price under homogeneous expectations, i.e. when  $p_{i,t+1}^e = p_{t+1}^e$  for all  $i$ . We consider some well-known heuristics: rational expectations, adaptive and naive expectations, trend-following rules and anchoring and adjustment rules. Figure 3 shows the price dynamics for these benchmark expectation rules. Anufriev and Hommes (2012b) provide formal proofs of the existence and stability of steady states of the asset pricing model under homogeneous expectations.

In a rational expectations equilibrium of the model, the realized price in period  $t$  coincides with the prediction for that period. The unique rational expectations equilibrium steady state is where all traders predict the fundamental price of the asset:

$$p_{t+1}^e = p^f = \bar{y}/r. \tag{6}$$

This is called fundamental expectations. This solution satisfies a no-bubbles condition, which ensures that predictions and prices do not explode. However, as explained in Hommes et al. (2008), there is a continuum of other solutions of the form  $p_{t+1}^e = p^f + R^{t+1}c$  with  $c > 0$  that do not satisfy this condition. These so-called rational bubbles grow at rate  $R$ . Figure 3a shows the fundamental price and a rational bubble for  $c = 15$ .

The rational expectations hypothesis is quite demanding. It requires all traders to be able to calculate the fundamental price and coordinate on the rational expectations equilibrium steady state. If traders instead use simple forecasting heuristics, it could be that predictions and prices converge to the fundamental value because of

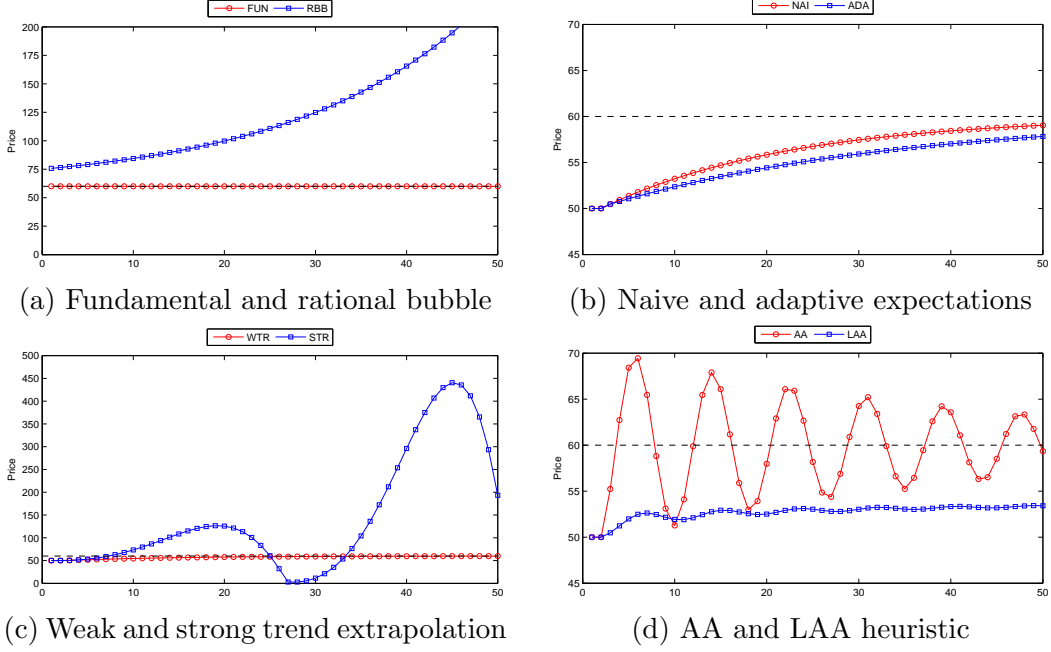


Figure 3: Simulations with homogeneous expectations  
Initialization of the simulations:  $p_1 = p_2 = 50$  and  $p_3^e = 50$ . The dashed line indicates the fundamental value of 60. Note that the scale of the vertical axis differs in the four figures.

individual learning. This is the case when all traders form expectations adaptively:

$$p_{t+1}^e = wp_{t-1} + (1-w)p_t^e = p_t^e + w(p_{t-1} - p_t^e) \quad (7)$$

with weight  $w \in [0, 1]$ . The forecast is formed by adapting the previous prediction in the direction of the last observed price. Naive expectations are a special case of this rule, obtained for  $w = 1$ . The prediction is then simply the last observed price. The asset pricing model is stable under adaptive or naive expectations. Prices converge slowly but monotonically to the fundamental value. This is illustrated by Figure 3b, which shows prices for naive expectations and adaptive expectations with  $w = 0.65$ .

A trend-following rule bases the prediction on the last observed price and adjusts in the direction of the last observed price change:

$$p_{t+1}^e = p_{t-1} + \gamma(p_{t-1} - p_{t-2}) \quad (8)$$

with  $\gamma > 0$ . The strength of adjustment is determined by the extrapolation coefficient  $\gamma$ . The trend-following rule is called weak if  $\gamma < 1$  and strong if  $\gamma > 1$ . The price dynamics depend on the extrapolation coefficient. For weak trend extrapolation with a small value of  $\gamma$ , there is monotonic convergence. This is illustrated for  $\gamma = 0.4$  in Figure 3c. For  $\gamma$  close to but smaller than 1, convergence becomes oscillatory. For strong trend extrapolation, the oscillations increase in amplitude and prices diverge. Figure 3c shows an example of this with  $\gamma = 1.3$ .

The anchoring and adjustment (AA) rule is based on Tversky and Kahneman (1974) and has a clear behavioral interpretation. The average of the fundamental price and the last observed price serves as an anchor for the next prediction. The prediction is adjusted in the direction of the last observed price change:

$$p_{t+1}^e = 0.5(p^f + p_{t-1}) + (p_{t-1} - p_{t-2}). \quad (9)$$

A variant of this heuristic is the learning anchoring and adjustment (LAA) rule, where traders do not have to know the fundamental value, but they might learn it through the sample average of past prices. For this rule,  $p^f$  is replaced by  $p_{t-1}^{av} = \frac{1}{t-1} \sum_{j=1}^{t-1} p_j$ . Under the AA rule, prices converge to the fundamental value, but convergence is slow and oscillatory. There is also convergence under the LAA rule, but even slower and with less pronounced oscillations. This can be seen from Figure 3d.

The evolution of prices can thus vary greatly with the prediction rule that is used. When expectations are heterogeneous instead of homogeneous, almost any type of price dynamics can occur. Brock and Hommes (1998) consider an evolutionary competition between simple heuristics, such as fundamentalists versus trend followers. They find price oscillations, bubble and crash dynamics and chaotic behavior of the model.

### 3.3 Experimental design

The experiment is aimed at investigating expectation formation in a positive feedback environment. Therefore, the participants only have one task: predicting the price of the risky asset. Trading is computerized in the sense that the computer calculates the optimal demand for shares of each participant according to the asset pricing model. Given the forecasts of all participants in the market, the realized market price is determined by Equation (5).

In the instructions (see Appendix A), the participants are told that they are a financial advisor to a large pension fund. It is explained that the pension fund can invest in a risk free asset and a risky asset. Participants have to forecast the price of the risky asset so that the pension fund (i.e. the computer) can calculate the optimal demand for shares. It is a two-period-ahead prediction: in period  $t$ , the available information for forecasting the price in period  $t + 1$  consists of past prices up to period  $t - 1$  and past predictions up to period  $t$ . After all participants have entered their predictions for period  $t + 1$ , the realized market price in period  $t$  is revealed. This process continues for 51 periods. Participants do not have any information about past prices in the first two periods, but they are told that it is very likely that the price will be between 0 and 100 in those periods.

Participants receive only qualitative information about the asset market. They are informed that the risk free interest rate is  $r = 5\%$  and the mean dividend of the risky asset is  $\bar{y} = 3$ . In principle, they could calculate the fundamental value of the risky asset:  $p^f = \bar{y}/r = 60$ . Participants are told that the higher their price forecast, the larger the demand for shares by the pension fund. They are also informed that supply is fixed and that the price is determined by the equilibrium between demand and supply. Hence, they could infer that there is positive feedback. However, they do not know the underlying market equilibrium equation. They also do not know the exact number of pension funds in the market or the identity of the other participants in their group.

Because of their role of advisor to a pension fund, it is reasonable to reward

participants based on their prediction accuracy: the better their price forecast, the better the investment decision that the pension fund can make.<sup>2</sup> The number of points  $e_{it}$  earned by participant  $i$  in period  $t$  is determined by the quadratic scoring rule

$$e_{it} = \max \left\{ 1300 - \frac{1300}{49}(p_t - p_{it}^e)^2, 0 \right\}. \quad (10)$$

Note that participants could not lose money, because earnings are simply zero when the forecast error  $|p_t - p_{it}^e|$  is larger than 7. The instructions include a table where participants could find the number of points they would earn for each value of the forecast error. At the end of the experiment, the participants receive 0.5 euro per 1300 points that they have earned.

To keep the experiment comparable with HSTV08, an upper bound on predictions of 1000 is imposed. The instructions do not mention this upper bound. If a participant tries to enter a forecast higher than 1000, the computer screen shows a message that predictions above 1000 are not accepted and that the participant has to submit a new and lower prediction.

At the beginning of the experiment, the participants receive instructions on paper. They could consult these instructions at any time. After reading the instructions, participants answer five control questions to check if they understand the experiment. The computer screen during the 51 periods shows a table and a graph of all past prices and predictions that are available in that period, earnings in the previous period, total earnings, the interest rate and the mean dividend. After the last period, participants fill in a short questionnaire, with a single open question about the strategy they used in the experiment.

The main motivation for our experiment is to test whether the results of the

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<sup>2</sup>As explained in Hommes (2001), using the quadratic forecast error as a performance measure is equivalent to using realized risk-adjusted utility from wealth or profits. Moreover, Bao et al. (2014) run an experiment with a learning-to-optimize treatment, where participants trade and are paid by realized profits. They show that bubbles form even more easily than in the learning-to-forecast treatment.



LtFE of HSTV08 are robust to an increase in the number of participants in the market. However, the group size was limited to the number of computers in the laboratory. The aim was to run sessions with groups of around 30 participants. To avoid having to cancel a session because not enough participants showed up, the group size was flexible. A session would start if 20 or more participants arrived at the lab on time. Seven sessions were run, with respectively 26, 26, 24, 32, 21, 21 and 32 participants in session 1–7. No sessions were cancelled.

The experiment was conducted with student subjects. It took place in the CREED laboratory of the University of Amsterdam in April 2014. Each session lasted for about 1.5–2 hours. As discussed in Section 4, the number of points that participants earned during the experiment was very low in all sessions. To reward them for their time, it was announced after the session that a lump sum payment of €15 was added to their earnings. Including this additional payment, average earnings were €17.27.

## 4 Experimental results

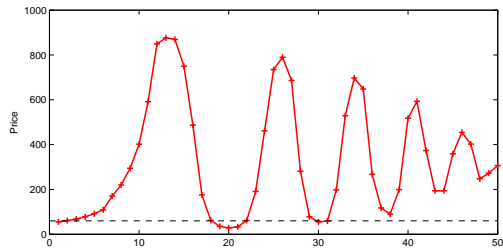
### 4.1 Aggregate outcomes

Figure 4 shows the realized market prices in the seven sessions of the experiment. Summary statistics can be found in Table 7 in Appendix B. It is immediately clear that the large price bubbles that were observed in HSTV08 also occur in this experiment. In six out of seven sessions, the markets are highly unstable and the price approaches the upper bound of 1000.<sup>3</sup> In these sessions, the fundamental value of 60 does not seem to play a role in the evolution of the market price at all.

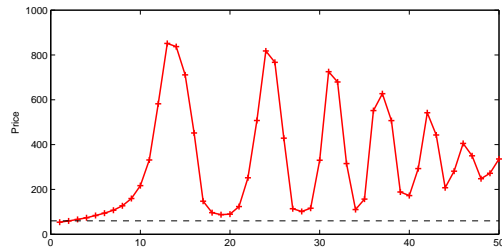
In all sessions, the market price starts rising right from the beginning. The reason for this might be twofold. First, many participants make an initial prediction of around 50. This is probably because it is mentioned in the instructions that the

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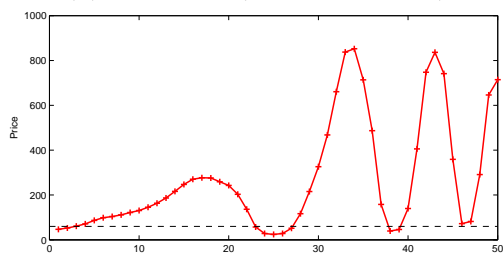
<sup>3</sup>The maximum price that can be attained is actually  $(1000 + 3)/(1 + 0.05) = 955.24$ . The realized prices in session 1, 2, 3, 4, 6 and 7 come close to this maximum price.



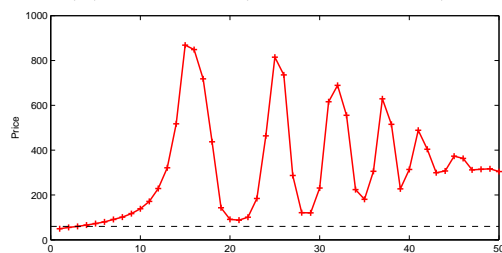
(a) Session 1 (26 participants)



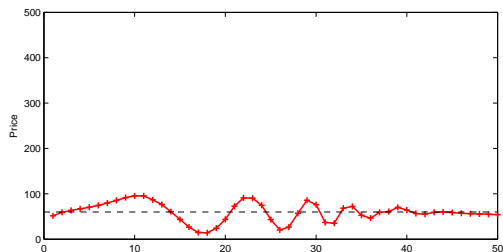
(b) Session 2 (26 participants)



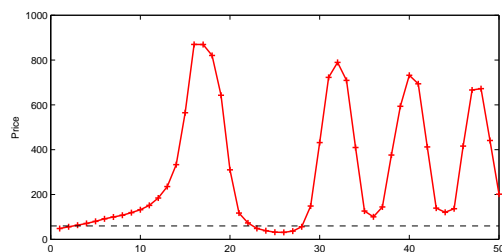
(c) Session 3 (24 participants)



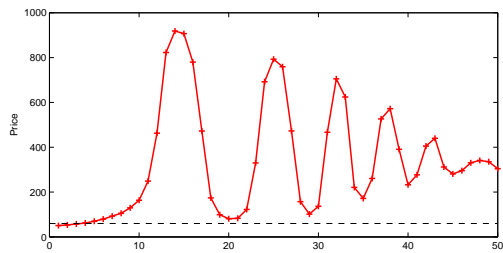
(d) Session 4 (32 participants)



(e) Session 5 (21 participants)



(f) Session 6 (21 participants)



(g) Session 7 (32 participants)

Figure 4: Realized market prices

The dashed line indicates the fundamental value of 60. Note that the vertical axis for session 5 runs up to 500 instead of 1000.

price in the first two periods is likely to be between 0 and 100, and participants have no information about past prices in the first two periods. Since the fundamental price is 60, the initial market price turns out to be a little higher than the average prediction in the first two periods. Therefore, many participants increase their predictions after they learn the first market price, resulting in an upward trend. This might evoke trend extrapolating behavior, leading to a bubble. Second, it is notable that 132 out of 182 participants make a prediction for period 2 that is higher than their prediction for period 1.<sup>4</sup> They do this without having any information about past prices. This suggests that a majority of participants expects an upward trend from the beginning.

Session 1, 2, 4, 6 and 7 display the same general pattern. A large price bubble forms immediately. As the price approaches 1000 after about fifteen periods, most participants discover the upper bound and lower their predictions.<sup>5</sup> The trend reverses and the market rapidly crashes back to a price below 100. After that, a new bubble quickly forms again and the process repeats itself, although the amplitude of the bubbles is decreasing. The price at which the bubble peaks becomes lower each time, and the price at which the bubble starts to grow again becomes higher. This could be because participants learn to anticipate a new crash (bubble), and therefore decrease (increase) their forecasts earlier each time. There seems to be some kind of convergence to a price of around 300 in these sessions, except in session 6. But a price of 300 is five times the fundamental value, so prices are still far above the equilibrium.

It is remarkable that the bubbles in these sessions arise even faster than in the small-group experiment of HSTV08. In the large groups, the bubble already peaks between period 13 and 16. It takes more time before the trend really takes off in the small groups: there is only a slow upward trend in the first fifteen to twenty

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<sup>4</sup>Furthermore, 30 participants submit equal predictions for the first two periods. Only 20 participants predict a lower price for period 2 than for period 1.

<sup>5</sup>In session 1, 21 out of 26 participants entered a prediction of 999 or 1000 in one or more periods around the time of the first peak. For session 2, 4, 6 and 7, these numbers were 16/26, 20/32, 14/21, and 23/32, respectively. This suggests that more than 60% of the participants found out about the upper bound.

periods, after which the price starts increasing rapidly. The bubble takes between 22 and 38 periods to reach its peak (see Table 8 in Appendix B). In the typical large-group sessions, a total of four to six repeated bubbles and crashes occur. Only two or three bubbles per session are observed in the small groups of HSTV08. This difference also reflects that bubbles arise faster in our large-group experiment.

Session 3 and session 5 are the exceptions to the general price pattern of multiple large bubbles. Session 3 starts with a small bubble that forms slowly. The upward trend reverses when two participants predict 0 or 1 for two or three periods. After the first small bubble, the pattern in session 3 is similar to the sessions discussed above. The price starts rising quickly and a large bubble occurs, followed by two more bubbles that are only slightly smaller.

Large bubbles did not occur in session 5. In this session, the price stays between 0 and 100. This price range is mentioned in the instructions and might therefore serve as an anchor, causing some participants to lower their predictions as they approach 100.<sup>6</sup> Some participants submit a forecast higher than 100 in some periods, but these forecasts do not become higher than 131.<sup>7</sup> The price oscillates, but the amplitude of the oscillations decreases and the price converges to around 55, which is fairly close to the fundamental value of 60.

The bubbles in each session can be quantified and compared by the Relative Absolute Deviation (RAD) and Relative Deviation (RD), as proposed by Stöckl

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<sup>6</sup>Ten participants (48%) do not predict a price higher than 100 and lower their predictions in the two periods before they observe the price peak. They might think that the price can or will not become higher than 100 in all periods, not just the first two. In the other sessions, this behavior occurs less often, so that the upward trend just continues. There are at most four participants in each of the other sessions that lower their predictions in the periods before the price actually becomes higher than 100, after which they adjust their expectations upwards.

<sup>7</sup>The maximum prediction is actually 440 in period 40, but that is most likely a typing error. The last observed market price at that time was 60.81, and the participant predicted 50 in both period 39 and period 41.

et al. (2010). For our experiment, these measures are defined as follows:

$$\text{RAD} = \frac{1}{50} \sum_{t=1}^{50} \frac{|p_t - p^f|}{p^f} \quad (11)$$

$$\text{RD} = \frac{1}{50} \sum_{t=1}^{50} \frac{p_t - p^f}{p^f} \quad (12)$$

The RAD measures the average level of mispricing in the market: a value of e.g. 0.10 means that on average the market price differs 10% from the fundamental. The RD measures the average level of overvaluation. Equal values of RAD and RD indicate that the asset is always overvalued, while differences imply that the market also has periods where the asset is undervalued.

Table 1 shows the bubble measures for both our large groups and the small groups of HSTV08. As expected, the bubble measures indicate that the deviations from the fundamental price are large in the markets where large bubbles occurred. In the large groups, RAD is between 3.66 and 4.67. In the small groups, this range is wider: RAD is between 2.15 and 4.27. Small-group sessions 2 and 4 exhibited two bubbles where the price almost reached the upper bound. This results in a larger RAD than for sessions 3, 5 and 6, in which at least one of the bubbles stayed smaller. RAD is considerably lower in the markets without large bubbles, namely 0.26 in large-group session 5 and 0.98 in small-group session 1. In all sessions, RD is only slightly smaller than RAD, meaning that the asset is mostly overvalued. RD is virtually equal to zero in large-group session 5, indicating that the price

Table 1: Relative Absolute Deviation (RAD) and Relative Deviation (RD)

	Large groups		Small groups	
	RAD	RD	RAD	RD
Session 1	4.52	4.46	0.98	0.93
Session 2	4.40	4.40	3.67	3.56
Session 3	3.66	3.55	2.17	1.96
Session 4	4.36	4.35	4.27	4.10
Session 5	0.26	0.00	2.15	1.78
Session 6	4.25	4.16	2.40	2.05
Session 7	4.67	4.66		
Average	3.73	3.65	2.60	2.40

oscillated around the fundamental value. On average, RAD and RD are higher in the large groups, confirming that the large markets are even more unstable than the small markets.

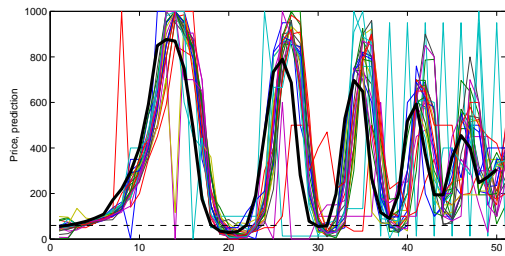
## 4.2 Individual expectations

Figure 5 shows the individual predictions of all participants in each session in color, together with the realized market price in black. After a few periods, it seems that the participants coordinate on a common trend-following prediction strategy, leading to the first bubble. The differences in predictions within markets appear to be somewhat larger than in the experiment of HSTV08. This is probably simply because it is harder to achieve strong coordination in a larger group. Although most participants submit similar forecasts, there are also some participants that predict very different prices. The discovery of the upper bound of 1000 seems to break the coordination to a large extent. In the typical sessions 1, 2, 4, 6 and 7, the differences in predictions become larger after the first two bubbles, and forecasts vary a lot towards the end of the experiment.

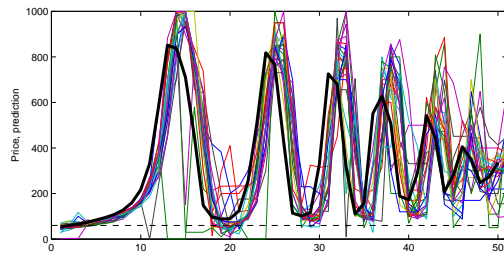
A way to investigate the importance of coordination quantitatively is to compute the average individual quadratic forecast error. This is the individual quadratic forecast error averaged over the last 45 periods and over the  $I$  participants in a session. The first five periods of the experiment are omitted in the computation to allow for a short learning phase. The average individual quadratic forecast error can be broken up in two terms, the average dispersion error and the average common error:

$$\frac{1}{45I} \sum_{i=1}^I \sum_{t=6}^{50} (p_{it}^e - p_t)^2 = \frac{1}{45I} \sum_{i=1}^I \sum_{t=6}^{50} (p_{it}^e - \bar{p}_t^e)^2 + \frac{1}{45} \sum_{t=6}^{50} (\bar{p}_t^e - p_t)^2, \quad (13)$$

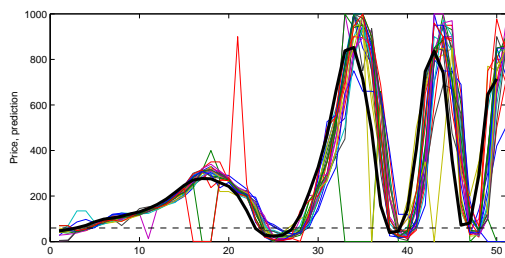
where  $\bar{p}_t^e = \frac{1}{I} \sum_{i=1}^I p_{it}^e$  is the average price prediction for period  $t$ . The average dispersion error measures the squared distance between the individual prediction  $p_{it}^e$  and the average prediction  $\bar{p}_t^e$ , averaged over time and participants. This term is



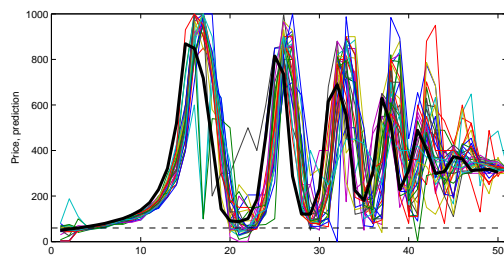
(a) Session 1 (26 participants)



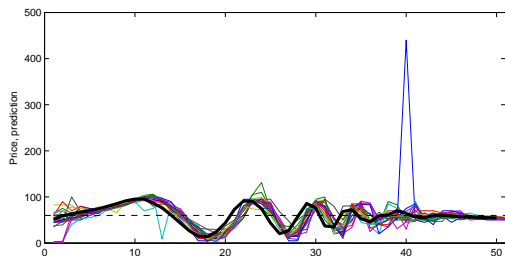
(b) Session 2 (26 participants)



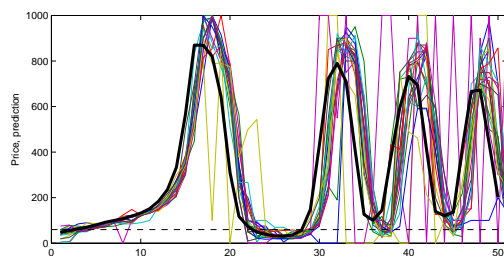
(c) Session 3 (24 participants)



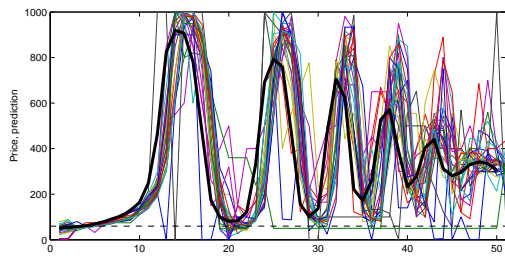
(d) Session 4 (32 participants)



(e) Session 5 (21 participants)



(f) Session 6 (21 participants)



(g) Session 7 (32 participants)

Figure 5: Individual predictions

The colored lines indicate the individual predictions, the thick black line indicates the market price. The dashed line indicates the fundamental value of 60. Note that the vertical axis for session 5 runs up to 500 instead of 1000.

Table 2: Average individual quadratic forecast error

Session	Av. individual error	Av. dispersion error	(%)	Av. common error	(%)
1	54,804	24,210	44%	30,594	56%
2	53,749	16,917	31%	36,832	69%
3	33,225	11,432	34%	21,793	66%
4	46,764	14,326	31%	32,438	69%
5	478	242	51%	236	49%
6	50,127	21,095	42%	29,033	58%
7	52,813	22,459	43%	30,354	57%

zero if all participants use exactly the same prediction strategy. Hence, a relatively small average dispersion error indicates that there is coordination on a common strategy. The average common error measures the squared distance between the average prediction  $\bar{p}_t^e$  and the realized price  $p_t$ , averaged over time. Muth's (1961) formulation of the rational expectations hypothesis implies that individual expectations may be wrong, but expectations should be approximately correct in the aggregate. If this is the case, then the average common error is relatively small.

Table 2 presents the value of the average individual quadratic forecast error in each session, together with the values and percentages of the average dispersion error and the average common error. In all sessions with large bubbles, the average dispersion error is between 31% and 44%. This indicates that predictions are similar, but coordination is not perfect. The relatively large average common error (between 56% and 69%) implies that individual prediction errors do not cancel out in the aggregate. Even in the stable session 5, the average common error is still 49%. It suggests that expectations in this experiment cannot be called rational. Compared to HSTV08, the size and proportions of the average dispersion error and the average common error are roughly the same. Apparently, the difference in coordination between the two experiments is not that large.

Clearly, the average individual quadratic forecast errors are very large, especially in the sessions with large bubbles. This is also reflected in the low earnings of the participants during the experiment (see Table 3). In the sessions with large bubbles, earnings are between €0.00 and €3.60, while the maximum possible earn-



Table 3: Earnings (excluding the lump-sum payment)

Session	Average	Minimum	Maximum
1	0.77	0.0	1.8
2	1.30	0.2	3.4
3	2.08	0.8	3.6
4	1.57	0.1	3.2
5	7.75	4.2	12.8
6	2.14	0.4	3.6
7	1.61	0.0	3.2

ings are €25.00. During a bubble, participants barely earn anything. Most points are earned in the first ten periods or just after a crash, when the price starts going up again. In the relatively stable session 5, earnings were substantially higher, but still not maximal. In this session, participants also earn points in the beginning, but most points are earned in the last ten periods, when the price converges.

Figure 5 already indicates that there is heterogeneity in expectations. Of course, participants do not have to use the same prediction strategy during the whole experiment. As they learn more about the market, they might switch to a different strategy. A graphical inspection of individual predictions shows that this indeed occurs. Figure 6 gives four examples of participants who switch between forecasting rules. The figure shows for each period  $t$  the lagged market price  $p_{t-2}$  and the individual prediction  $p_{i,t}^e$ . This thus illustrates how the last observed market price is used to form the two-period-ahead price forecast. For example, if the two time series coincide, the participant is using a naive prediction strategy.

In session 3, participant 2 starts with a strategy close to naive expectations (see Figure 6a). After some periods, she begins to extrapolate the trend in the market price. In period 31, she realizes that she is not making any money during a bubble, so she attempts to bring the price down by predicting zero. In some periods, she tries to make an accurate prediction by extrapolating the trend.

Participant 32 in session 4 (Figure 6b) uses a trend-following strategy in the first half of the experiment. He then switches to a strategy that looks like an anchoring and adjustment rule, with an anchor of around 300 and a relatively

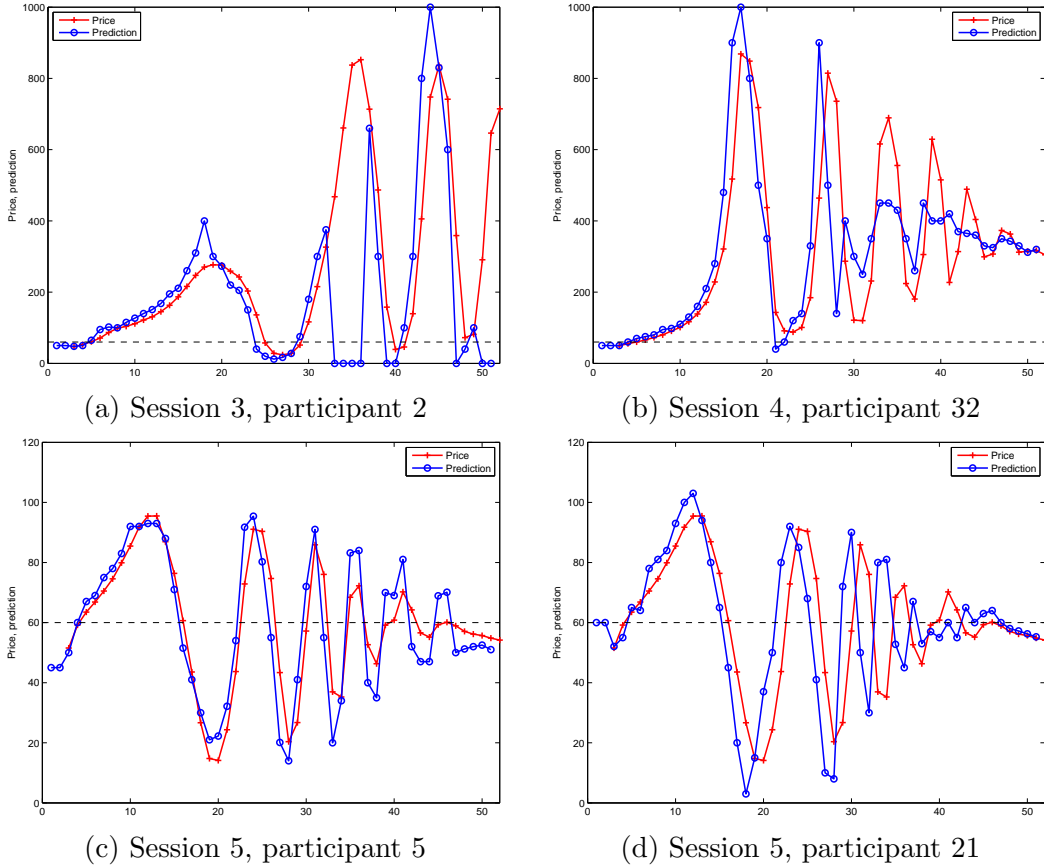


Figure 6: Examples of strategy switching  
The red line indicates the market price lagged for two periods, the blue line indicates the individual prediction. The dashed line indicates the fundamental value of 60.

small adjustment in the direction of the last price change.

Figure 6c shows that in session 5, a weak form of trend extrapolation is used by participant 5 until period 30. After that, she switches to a much stronger trend-following rule, despite the large forecast errors of this strategy. In the last five periods, she uses an adaptive forecasting rule, predicting a price between the last realized price and her own last prediction.

In the same session, participant 21 realizes that the fundamental price of the asset is 60 and predicts this price in the first two periods (Figure 6d). However, the market price turns out to be lower than 60, so she lowers her prediction. She uses an almost naive strategy for some periods and then begins to extrapolate the price trends. After two bubbles, she tries to anticipate the trend reversals, in which she is quite successful. When the price converges in the last five periods, she goes back

to a strategy close to naive.

## 5 Estimating individual prediction strategies

### 5.1 General linear forecasting rules

It seems that the prediction strategies of participants could be described by fairly simple heuristics. Estimating some simple linear specifications of forecasting rules can give insight in the way participants form expectations.<sup>8</sup> Analyses similar to the one presented in this section are done in Hommes et al. (2005a), Heemeijer et al. (2009) and Massaro (2012). Starting with a general specification, a simple linear rule of the form

$$p_{i,t+1}^e = \alpha + \sum_{k=1}^4 \beta_k p_{t-k} + \sum_{l=0}^3 \gamma_l p_{i,t-l}^e + u_t \quad (14)$$

is estimated for each participant  $i$ .<sup>9</sup> This rule uses the last four observations of the market price and the last four own predictions to form the new prediction. The forecasting rule is estimated from period  $t = 5$ , which is the first period that all regressors are available. The first five periods can also be considered as a learning phase. Insignificant regressors are removed one by one, highest  $p$ -value first, until all remaining regressors are significant at a 5% level. The final prediction rule is tested for autocorrelation (Breusch-Godfrey test, two lags), heteroskedasticity (White test, no cross terms) and misspecification (Ramsey RESET test, one fitted term).

Table 10 in Appendix B presents the estimation results. It seems that the general linear specification can describe the forecasting rules fairly well for most participants. 139 out of the 182 estimated rules (76%) pass all three diagnostic

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<sup>8</sup>An alternative way to find out which forecasting rules are used is to do a strategy experiment as in Hommes et al. (2005b). As participants formulate a complete prediction strategy, there is no need to estimate their strategies from the observed predictions.

<sup>9</sup>The forecasting rule is estimated after removing outliers, i.e. predictions that differ substantially from what would be expected from the general pattern of the other observations. A total of 24 outliers for 19 participants were removed by linear interpolation.

tests. In all treatments, the most popular regressors are the last two observations of the market price:  $p_{t-1}$  is used by 89% of the participants,  $p_{t-2}$  by 71%. For 20% of the participants, these are the only two significant regressors. In all these cases, the coefficient of  $p_{t-2}$  is negative, so that it is a trend-following rule. The last own prediction  $p_t^e$  is significant in 36% of the estimated rules. 11% of the participants use a form of anchoring and adjustment, basing their predictions on  $p_{t-1}$ ,  $p_{t-2}$  and  $p_t^e$ .

Although these are the most commonly used regressors, a majority of participants (69%) also use higher lags of the market price and their own predictions in their forecasting rules. The use of more past observations indicates learning behavior by participants. Particularly in session 5,  $p_{t-3}$  and  $p_{t-4}$  are significant in more than half of the estimated rules. These third and fourth lags of the market price are also used more often in session 3 than in the other sessions. Furthermore, the estimation results show 22 different combinations of significant regressors across all sessions. Within each session, ten to fourteen different forecasting rules are estimated.

We also estimated these forecasting rules for the experiment of HSTV08. Relatively less rules pass all three diagnostic tests, namely 15 out of 36 estimated rules (42%). Again, the last two observations of the market price are the most widely used:  $p_{t-1}$  is used by 97% of the participants,  $p_{t-2}$  by 75%. For the 11% of the participants that only used these two regressors, the coefficient of  $p_{t-2}$  is always negative, so that it is a trend-following rule. In 33% of the estimated rules, the last own prediction  $p_t^e$  is significant. 8% of the participants base their predictions on  $p_{t-1}$ ,  $p_{t-2}$  and  $p_t^e$  and thus use a form of anchoring and adjustment. Just as in the large groups, a majority of participants (75%) also base their forecasts on higher lags of the market price and their own predictions. Sixteen different combinations of significant regressors across all sessions are found in total, with four to five different rules within each session. All in all, the results in the small groups of HSTV08 are remarkably similar to the results in our large-group experiment.

Hommes et al. (2005a) estimated the same specification for the individual predictions in their LtFE and found different results. In their experiment,  $p_{t-1}$  and  $p_{t-2}$  are the only two significant regressors for a majority of 63%. For 30% of the participants, the last own prediction  $p_t^e$  is significant as well, but only 2% use a form of anchoring and adjustment. This is because almost all participants that base their forecast on  $p_{t-1}$ ,  $p_{t-2}$  and  $p_t^e$  also use higher lags of prices and predictions in their forecasting rule. In total, 23% of the participants makes use of higher lags. In their experiment, fourteen different combinations of regressors are observed in total, with two to five different rules per session. Comparing the results of Hommes et al. (2005a) to ours, we see that the prediction rules that we observe in our experiment are more diverse and use more information on past prices and predictions. This could be due to the larger instability of the markets in our experiment, which calls for more complicated forecasting rules. Of course, we should keep in mind the differences in group size: Hommes et al. (2005a) only have six participants per session and sixty participants in total. Therefore, we cannot directly conclude that the larger variety in forecasting heuristics implies that there is more heterogeneity in expectations in our LtFE with larger groups.

## 5.2 First-order heuristics

Since the last two observed market prices and the last own prediction are the most popular regressors, it is interesting to consider a simplified forecasting rule based on these variables:

$$p_{i,t+1}^e = \alpha_1 p_{t-1} + \alpha_2 p_{i,t}^e + (1 - \alpha_1 - \alpha_2) \bar{p} + \beta(p_{t-1} - p_{t-2}) + v_t, \quad (15)$$

where  $\bar{p} = \frac{1}{50} \sum_{t=1}^{50} p_t$  is the average market price in the session. This rule is called a first-order heuristic. It is a more general form of the anchoring and adjustment rule. The anchor is now a weighted average of the last observed market price, the

last own prediction and the sample average price.<sup>10</sup> The adjustment is based on the difference between the last two observed prices.

The first-order heuristic has a couple of benchmark expectation rules as special cases. For  $\alpha_1 = 1, \alpha_2 = \beta = 0$ , the rule reduces to naive expectations. If  $\alpha_1 + \alpha_2 = 1, \beta = 0$ , expectations are adaptive. In case  $\alpha_1 = \alpha_2 = \beta = 0$ , the forecast is simply the sample average price. This case is referred to as fundamental expectations, although the average price is not near the fundamental price of 60 in this experiment, except in session 5. The sample average price just serves as a proxy for the long-run price level, even though it is not close to the steady state level in all but one session. Finally, when  $\beta > 0$ , the adjustment is called trend-following.

The first-order heuristic is estimated for those participants whose general forecasting rule of type (14) can be restricted to a heuristic of type (15). This is verified by an  $F$ -test on joint parameter restrictions.<sup>11</sup> The first-order heuristics are again estimated from period  $t = 5$ , to allow for some learning. The regressor with the highest  $p$ -value is removed until there are no more insignificant regressors.  $F$ -tests are used to see if the first-order heuristic can be restricted to one of the benchmark expectation rules.

The estimation results can be found in Table 11 in Appendix B. For 58 out of 182 participants (32%), the estimated general forecasting rule can be successfully restricted to a first-order heuristic. In session 5, there is only one general rule that can be restricted, since many participants in that session used higher lags of the market price and their own predictions. For the same reason, only three first-order

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<sup>10</sup>The sample average price of the entire session is of course not available to the participants at the moment they form their predictions. It is included in the estimation as a proxy for the equilibrium price level that they are trying to learn. In the HSM of Section 7, the LAA heuristic uses the sample average of past prices available at each point in time. This generally converges quickly to the sample average price.

<sup>11</sup>Note that separate  $t$ -tests were used to remove insignificant regressors from the general forecasting rules of type (14), while the  $F$ -test used here is a joint significance test. The two ways of testing are not equivalent. It is perfectly possible that the individual  $t$ -tests indicate that all regressors are significant, while the  $F$ -test cannot reject the null hypothesis that a number of these coefficients are jointly zero. Hence, the number of rules that can be restricted to a heuristic of type (15) is not the same as the number of rules of type (14) that only have  $p_{t-1}$ ,  $p_{t-2}$  and  $p_t^e$  as significant regressors.

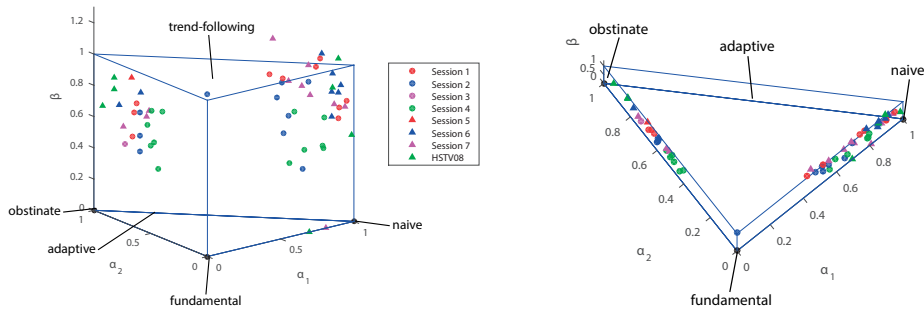


Figure 7: Estimated coefficient vectors of first-order heuristics (excluding outliers)

heuristics are estimated in session 3.

The last two columns of the tables indicate if the anchor and the adjustment can be restricted to a benchmark case. Eight heuristics are classified with a naive anchor, one of those heuristics is purely naive ( $\beta = 0$ ). One heuristic has a fundamental anchor. For the other heuristics, the sample average price generally gets little weight:  $1 - \alpha_1 - \alpha_2$  ranges from 0.01 to 0.6 and is on average 0.3. The anchors of 49 heuristics are not equivalent to a benchmark and are therefore classified as “mixed”. The adjustment is trend-following ( $\beta > 0$ ) for all but one first-order heuristic. The nonzero values of  $\beta$  range from 0.4 to 1.3 and the average is 0.8.<sup>12</sup> For ten heuristics,  $\beta$  is larger than 1, indicating a strong trend-following adjustment.

Figure 7 illustrates the estimated coefficients of the first-order heuristics in the three-dimensional space  $(\alpha_1, \alpha_2, \beta)$ . Each dot in the prism represents one of the estimated heuristics.<sup>13</sup> Even though most first-order heuristics cannot be fully restricted to a benchmark rule, the prism shows a clear clustering of heuristics. A majority of 33 out of 58 heuristics (57%) has a relatively high value of  $\alpha_1$  and a low value of  $\alpha_2$ . These anchors are close to naive, giving a high weight to the last observed market price and some weight to the sample average price. Combined

<sup>12</sup>Remarkably, 0.4 and 1.3 are exactly the values of the extrapolation coefficient that are used for the two trend-following rules in HSM of Section 7. The rules in this model were based on estimates of forecasting behavior in Hommes et al. (2005a).

<sup>13</sup>Five coefficient vectors with a negative estimate for  $\alpha_2$  fall outside the prism. They are excluded from the plot to make it easier to distinguish the dots for the coefficient vectors that lie within the prism.

with a positive value for  $\beta$ , these rules are almost pure trend-following. A group of 19 heuristics (33%) shows the reverse pattern: a low value of  $\alpha_1$  and a relatively high value of  $\alpha_2$ . These participants base their anchor on their last own prediction and the sample average price, ignoring the last observed market price. Adaptive expectations are not observed, probably because this forecasting rule does not work well when markets are as unstable as in the experiment. There are no clear differences between sessions in the estimated first-order heuristics.

The first-order heuristics that we estimated for the individual predictions in HSTV08 are also largely comparable to the heuristics in our large-group experiment. The estimated general forecasting rule can be successfully restricted to a first-order heuristic for 10 out of 36 rules (28%). Two first-order heuristics were estimated in session 1, 2, 5 and 6, and one first-order heuristic was estimated in session 3 and 4. Two heuristics are classified with a naive anchor ( $\alpha_1 = 1$ ). One heuristic has an obstinate anchor ( $\alpha_2 = 1$ ), meaning that the participant only considered his own past prediction and not the last observed market price or the sample average price. The other seven heuristics have a mixed anchor. Again, the weight of the sample average price is generally low, with an average of 0.15. For eight out of ten first-order heuristics, the adjustment is trend-following. The nonzero values of  $\beta$  range from 0.6 to 1.2 and the average is 0.7.  $\beta$  is larger than 1 in two cases. The prism in Figure 7 also shows the estimated coefficients of the ten first-order heuristics of HSTV08.<sup>14</sup> Three heuristics are part of the cluster of almost pure trend-following rules (a relatively high value of  $\alpha_1$ , a low value of  $\alpha_2$  and a positive  $\beta$ ). Three heuristics are part of the other cluster with a low value of  $\alpha_1$  and a relatively high value of  $\alpha_2$ . Adaptive expectations are not found, just as in the large groups.

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<sup>14</sup>Three coefficient vectors with an estimate of  $\alpha_1 > 1$  fall outside the prism and are excluded from the plot.



## 6 Formation of the first bubble

In our large-group experiment, bubbles emerge faster than in the smaller markets of HSTV08. The bubbles in large markets already peak after about 15 periods, while the bubbles in small markets take around 30 periods to reach the highest price. Still, we did not find substantial differences in the estimated heuristics for both experiments. This could be due to the fact that these estimates are based on the last 45 periods. The price oscillations that occur after the first bubble bursts are a large part of that. These oscillations call for a different prediction strategy than the trend-following behavior that causes the first bubble. Therefore, we now examine the formation of the first bubble in more detail.

### 6.1 Growth rates

If prices evolve according to a rational bubble,  $p_t = p^f + R^t c$ , so the growth rate of prices must be  $R = 1.05$ . Defining  $q_t = \ln(p_t - p^f)$ , we must have that  $q_{t+1} - q_t = \ln(R)$ . We calculate  $q_{t+1} - q_t$  and its mean  $\bar{q}$  for the first large bubble<sup>15</sup>. The implied growth rate is then  $\hat{R} = e^{\bar{q}}$ . Note that  $q_t$  only exists when the price is above the fundamental value. Moreover, a price between  $p^f = 60$  and 61 gives a negative value of  $q^t$ . This leads to a relatively large value of  $q_{t+1} - q_t$ , causing  $\bar{q}$  and  $\hat{R}$  to be too high. Therefore, the first observation that we take into account satisfies  $p_t > (p^f + 1)$ . Because the upper bound on predictions flattens the last part of the bubble, the last observation that we take into account satisfies  $p_t - p_{t-1} > p_{t-1} - p_{t-2}$ .

Table 4 shows the results of this calculation. The implied growth rates  $\hat{R}$  differ significantly from  $R = 1.05$  at the 5% level, so the bubbles cannot be called rational. Furthermore, the implied growth rates in the large groups are larger than in the small groups, except small-group session 5. The implied growth rate in that session is relatively high because the first half of the experiment is ignored in the calculation. The small bubble in large-group session 5 grows slower than the large

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<sup>15</sup>Large-group session 5 does not exhibit large bubbles, so we simply take the first (small) bubble. We disregard small-group session 1, because there are no clear bubbles in that session.

Table 4: Implied growth rates of first (large) bubbles

	$\bar{q}$	$\hat{R}$	$p$ -value	Sample
Large groups				
Session 1	0.659	1.932	0.004	2-12
Session 2	0.486	1.626	0.000	3-13
Session 3	0.524	1.690	0.022	28-33
Session 4	0.449	1.567	0.000	4-15
Session 5	0.331	1.392	0.006	3-10
Session 6	0.422	1.525	0.000	3-16
Session 7	0.657	1.928	0.001	4-13
Small groups (HSTV08)				
Session 2	0.225	1.253	0.000	6-27
Session 3	0.292	1.339	0.000	6-29
Session 4	0.310	1.363	0.001	6-21
Session 5	0.718	2.049	0.045	29-37
Session 6	0.307	1.359	0.013	22-28

*Note:*  $\bar{q}$  is the mean of  $q_{t+1} - q_t$  for the given sample, and  $\hat{R} = e^{\bar{q}}$  is the implied growth rate. The  $p$ -value corresponds to a  $t$ -test of whether  $\hat{R}$  differs significantly from  $R = 1.05$ .

bubbles in the other large groups. However, the implied growth rate is similar to the large bubbles in the small groups, which take longer to form. Because of these two exceptions, the difference in means is not statistically significant ( $p$ -value = 0.226), even though it seems that bubbles grow faster in large groups than in small groups.

## 6.2 Initial predictions and prices

It is interesting to look at the two initial predictions  $p_1^e$  and  $p_2^e$ , because they are made without any information about past prices. Therefore, they may be informative about the participants initial expectations of the price dynamics. Furthermore, the third prediction  $p_3^e$  is submitted after the first price  $p_1$  becomes known. Note that the first two prices  $p_1$  and  $p_2$  actually result from the predictions  $p_2^e$  and  $p_3^e$ . These predictions determine the first price increase and could thus initiate an upward trend. Tables 7 and 8 in Appendix B give the average initial predictions, their standard deviations, and the initial prices for our large-group experiment and

the small-group experiment of HSTV08.

On average, the initial predictions and prices are higher in the large groups than in the small groups. However, the difference is not observed in all sessions. In the large groups,  $p_1$  is around 50 and  $p_2$  is around 55. This is comparable to session 1, 3 and 5 of the small groups. Initial prices are lower in the other small groups (session 2, 4 and 6):  $p_1$  is around 35 and  $p_2$  is around 40. This difference does not seem to be the explanation for the faster formation of bubbles in the larger groups.

Many participants seem to expect an upward trend from the beginning. In the large groups,  $p_2^e$  is higher than  $p_1^e$  for 71% of the participants, in the small groups this holds for 69%. There is however no clear difference between sessions where the bubble forms fast and sessions where it does not. For example, in HSTV08 session 6, all participants predicted a higher price for the second period than for the first period, but the large bubble does not form until after twenty periods.

The expectation of an upward trend is immediately fulfilled in all large groups. On average, predictions increase in the first three periods:  $\bar{p}_1^e < \bar{p}_2^e < \bar{p}_3^e$ . Moreover, the increase from  $\bar{p}_2^e$  to  $\bar{p}_3^e$  is larger than the increase from  $\bar{p}_1^e$  to  $\bar{p}_2^e$ . This indicates that the trend takes off right away. The same pattern is not observed in the small groups:  $\bar{p}_1^e < \bar{p}_2^e < \bar{p}_3^e$  does not hold in session 1 and 2 and  $\bar{p}_3^e - \bar{p}_2^e$  is not larger than  $\bar{p}_2^e - \bar{p}_1^e$  in session 1, 5 and 6. As a result, the initial price increase  $p_2 - p_1$  is bigger in the large groups than in the small groups, with the exception of large-group session 7. It would be an intuitive idea that a large bubble can form faster if the first price increase is bigger, but this is not completely in line with the results of the experiments. For instance, large-group session 5 has a relatively large first price increase, but the bubbles remain small throughout the whole session. In contrast, the first price increase in large-group session 7 is relatively small, but a large bubble forms quickly.

It appears that once the first price becomes known, it is used as a coordination device. In all sessions, the individual predictions for the first two periods are much more dispersed than the predictions for the third period. This is also reflected in the

standard deviation of the predictions, which is generally lower for the third period than for the first and second period. Many participants adjust their expectations for the third period upwards when they learn that the price in the first period was higher than they expected, and vice versa.

It is noteworthy that there are always some participants in the large groups (between one and six per session) that start out with two very low predictions. These predictions are anchored around the mean dividend,  $\bar{y} = 3$ , so the participants probably misunderstand the value of the asset. Once  $p_1$  turns out to be much higher, they have a large upward adjustment in their prediction for the third period. This seems to make the trend in the large groups stronger from the beginning. This behavior is not observed in the small groups.

### 6.3 Trend-following rules

Looking at individual predictions, it seems that the first bubble is caused by trend extrapolation. For each participant, we thus estimate the trend extrapolation coefficient  $\gamma$  in the following trend-following heuristic:

$$p_{i,t+1}^e = p_{t-1} + \gamma(p_{t-1} - p_{t-2}) + \varepsilon_t. \quad (16)$$

All regressors are defined from period  $t = 3$ . The last period we take into account is the peak of the first large bubble. We focus on sessions where a large bubble is formed immediately: session 1, 2, 4, 6 and 7 of our large-group experiment and session 2, 3, 4 and 6 of HSTV08. The estimation results are presented in Table 9 in Appendix B.

On average, the estimated trend extrapolation coefficient  $\gamma$  is higher in the large groups than in the small groups. However, the difference in means is not statistically significant ( $p$ -value = 0.616). The result does not hold true for each session individually. For instance, the average  $\gamma$  in session 3 of HSTV08 is larger than in all large groups except session 4.

Furthermore, simulations with the estimated trend-following rules do not always correspond to the price patterns in the experiment. Taking the initial prices and predictions from the large-group experiment,  $\gamma$  needs to be about 1.45–1.5 to produce a bubble that is similar to the typical first bubble in the experiment. For session 1 and 7, the average estimated  $\gamma$  is smaller than this, while the average estimated  $\gamma$  for session 4 is larger. For the small-group experiment,  $\gamma$  should be around 1.43. This is larger than the average estimated  $\gamma$  in session 2, 4 and 6, but smaller than in session 3. There is no clear explanation for these differences. For example, the estimated  $\gamma$ 's in large-group session 1 and 7 are almost all lower than in session 2, 4 and 6. Still, a large bubble formed quickly in all these sessions.

All in all, the results do not fully correspond to the intuition that a larger trend extrapolation coefficient leads to larger and faster bubbles. Hence, this trend-following rule alone cannot explain the faster formation of the first bubbles that we observed in the experiments.

## 7 Heuristics switching model

### 7.1 A learning model with evolutionary selection

The previous sections discussed some important observations about the individual forecasting behavior in the experiment. The prediction strategies of many participants can be described by a simple linear forecasting rule. However, as participants learn during the experiment, they sometimes switch to a different strategy. Moreover, there is heterogeneity in prediction strategies, both between and within sessions. A heterogeneous expectations model such as the HSM is based on simple heuristics, but takes adaptive learning and strategy switching into account. Therefore, the HSM is a very suitable model to analyze the experimental data.

The main idea of the HSM is that participants can choose between a number of simple heuristics to make a price prediction. In each period, participants evaluate the past performance of all heuristics, measured by the quadratic forecast error.

Then evolutionary selection takes place, meaning that participants tend to switch to better performing rules. Hence, the impact that the different rules have on the market price changes over time. Of course, the realized market price in turn affects the impacts of the heuristics. The HSM is thus a dynamic model that exhibits path dependence, since a different pattern of the market price or a different distribution of impacts in the first couple of periods leads to a different course of both prices and impacts in later periods. This property makes it possible for the HSM to explain very different market outcomes in very similar market settings.

We use the same model setup as in Anufriev and Hommes (2012b). In their paper, the HSM is used for one-period-ahead simulations of the asset pricing LtFEs of Hommes et al. (2005a) and HSTV08. In Anufriev and Hommes (2012a), they reproduce the price patterns in those experiments with 50-period-ahead simulations of the HSM. Slightly different versions of the HSM are used to analyze LtFEs in different settings, i.e. positive versus negative expectations feedback (Bao et al., 2012; Anufriev et al., 2013) and a New-Keynesian macro-model (Massaro, 2012). There are large differences in the observed behavior and the resulting market outcomes in all these experiments. Still, the HSM is able to capture these different patterns and fit the experimental data quite well. In particular, the HSM outperforms models with homogeneous expectations. This indicates that the flexibility of a heterogeneous expectations model such as the HSM is an important advantage. By using the same specification of the HSM as in Anufriev and Hommes (2012b), it is tested if this model is also suitable for the asset pricing experiment with larger markets.

## 7.2 Model specification

Participants can choose among  $H$  heuristics that give a two-period-ahead price prediction. The prediction for time  $t + 1$  of each heuristic  $h$  is a function of the

available information at time  $t$ :

$$p_{h,t+1}^e = f_h(p_{t-1}, p_{t-2}, \dots; p_{h,t}^e, p_{h,t-1}^e, \dots). \quad (17)$$

The realized market price depends on the population-weighted average  $\bar{p}_{t+1}^e = \sum_{h=1}^H n_{h,t} p_{h,t+1}^e$  of the heuristics. The weight  $n_{h,t}$  is called the impact of heuristic  $h$ , which indicates the percentage of participants using this heuristic at time  $t$ . The market price is then determined in the same way as in the experiment:

$$p_t = \frac{1}{1+r} (\bar{p}_{t+1}^e + \bar{y}). \quad (18)$$

The impact of each heuristic depends on its past performance. The performance measure is based on the quadratic forecast error. Note that the quadratic forecast error was also used to determine the earnings in the experiment, so that the performance measure is in line with the incentive structure in the experiment. The performance measure up to time  $t-1$  is defined as

$$U_{h,t-1} = -(p_{t-1} - p_{h,t-1}^e)^2 + \eta U_{h,t-2}. \quad (19)$$

The parameter  $\eta \in [0, 1]$  can be interpreted as the memory. For  $\eta = 0$ , the performance is only based on the most recent forecast error. If  $\eta > 0$ , all past forecast errors affect the performance, with exponentially decaying weights.

The impact of heuristic  $h$  at time  $t$  is determined by a discrete choice model with asynchronous updating:<sup>16</sup>

$$n_{h,t} = \delta n_{h,t-1} + (1 - \delta) \frac{\exp(\beta U_{h,t-1})}{\sum_{h=1}^H \exp(\beta U_{h,t-1})}. \quad (20)$$

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<sup>16</sup>In practice, the performance measures can become very negative, so that the normalization factor  $\sum_{h=1}^H \exp(\beta U_{h,t-1})$  leads to a problem if it becomes too close to zero. This problem can be solved by defining a slightly adjusted performance measure  $\bar{U}_{h,t-1} = U_{h,t-1} - \max_{h \in H} U_{h,t-1}$ , and using this measure to calculate the impacts  $n_{h,t}$ . Since the same number is subtracted from the performance of each heuristic in a certain period, this does not affect the discrete choice model.

The parameter  $\delta \in [0, 1]$  gives some inertia in the evolution of the impacts. This reflects the observation in the experiment that not all participants change their prediction strategy in every period or at the same time.  $\delta$  represents the percentage of participants that stick to their previous strategy, regardless of the performance. If  $\delta = 1$ , the initial impacts of the heuristics never change. If  $\delta = 0$ , all participants consider to change their strategy based on past performance. In general,  $1 - \delta$  is the fraction of participants that update their strategy according to the discrete choice model. Note that this does not necessarily mean that they all switch to the most successful strategy. This depends on how sensitive participants are to differences in performance, measured by the intensity of choice  $\beta \geq 0$ . The higher the intensity of choice, the faster they switch to more successful heuristics. If  $\beta = 0$ , the impacts converge to an equal distribution. In the extreme case that  $\beta = \infty$ , all participants that update their strategy switch to the best performing heuristic.

### 7.3 Implementing the model

Before the HSM can be used, the  $H$  different heuristics should be chosen, the parameters  $\beta$ ,  $\eta$  and  $\delta$  should be fixed, and the model should be initialized. Anufriev and Hommes (2012b) based their selection of heuristics on the forecasting rules that were observed in the experimental data. To keep the model relatively simple, they include only four heuristics: adaptive expectations (ADA), a weak trend-following (WTR) and a strong trend-following (STR) rule, and a learning anchoring and adjustment (LAA) heuristic. The heuristics are defined as follows:

$$\text{ADA: } p_{1,t+1}^e = 0.65p_{t-1} + 0.35p_{1,t}^e, \quad (21a)$$

$$\text{WTR: } p_{2,t+1}^e = p_{t-1} + 0.4(p_{t-1} - p_{t-2}), \quad (21b)$$

$$\text{STR: } p_{3,t+1}^e = p_{t-1} + 1.3(p_{t-1} - p_{t-2}), \quad (21c)$$

$$\text{LAA: } p_{4,t+1}^e = 0.5(p_{t-1}^{av} + p_{t-1}) + (p_{t-1} - p_{t-2}), \quad (21d)$$



where  $p_{t-1}^{av}$  is the sample average of all past prices up to (and including) time  $t-1$ .<sup>17</sup>

The parameter values for the benchmark HSM are  $\beta = 0.4$ ,  $\eta = 0.7$  and  $\delta = 0.9$ . Four periods are needed to initialize the model. Given two initial prices  $p_1$  and  $p_2$ , the four heuristics generate predictions for period 4. The initial prediction of the ADA heuristic for period 3 is the fundamental price,  $p^f = 60$ . The initial distribution of impacts in period 3 is equal for all heuristics:  $n_{h,3} = 0.25$  for  $h = \{1, 2, 3, 4\}$ . The performance measure for period 3 is zero. The predictions for period 4 and the impacts in period 3 are then used to calculate the price in period 3. In period 4, the same initial impacts are used, since past performance is not well defined yet. After these four periods, the model is fully initialized. In every period  $t \geq 5$ , first the performance measure  $U_{h,t-1}$  is updated, then the new impacts  $n_{h,t}$  are determined, the predictions  $p_{h,t+1}^e$  are computed, and the new market price  $p_t$  is realized.

## 8 Results of the heuristics switching model

### 8.1 One-period-ahead simulations

To get insight in the adaptive learning and strategy switching, we use the experimental data for one-period-ahead simulations of the HSM. For each session, the two initial prices are chosen to correspond to the first two prices in that session. The prices in the experiment are also used to compute the performance measure and the predictions of the four heuristics. Each period, the model thus simulates a price for period  $t$ , but this price is not used in period  $t+1$  to simulate the next price. By using the experimental prices instead, the model uses exactly the same information about past prices that was available to the participants in the experiment.

Figure 8 visualizes the results of the HSM. For each session, the figures show the experimental and simulated prices, the forecasts and the forecast errors of the four

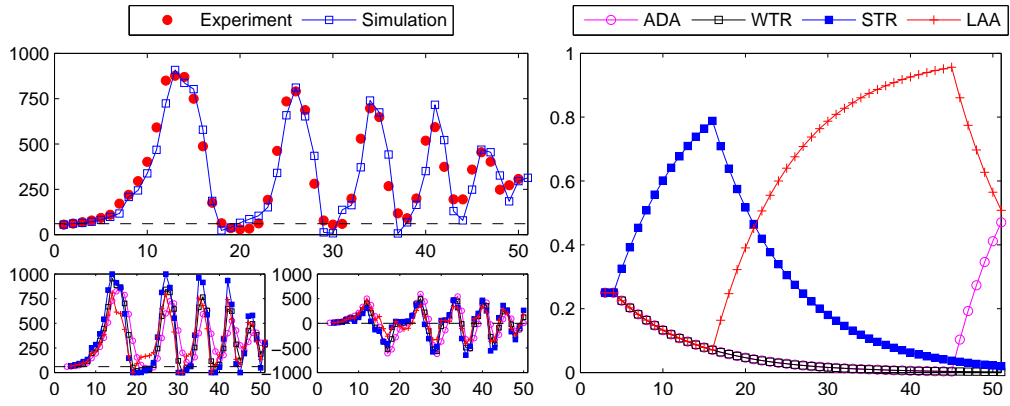
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<sup>17</sup>Just as in the experiment, the predictions should be between 0 and 1000. If heuristic  $h$  leads to a prediction lower than 0,  $p_{h,t+1}^e$  is set to 0. Similarly, if the heuristic leads to a prediction higher than 1000,  $p_{h,t+1}^e$  is set to 1000.

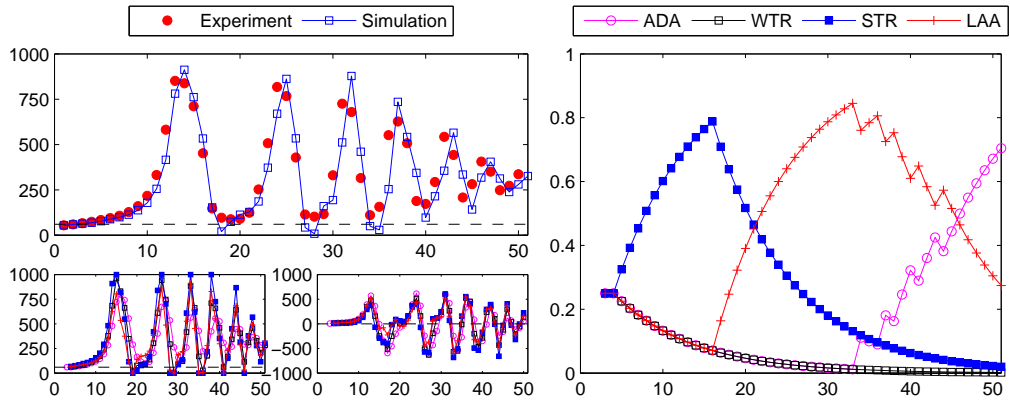
heuristics, and the evolution of impacts. The figures suggest that the fit of the HSM is reasonable. Not surprisingly, the simulated prices follow the same bubble-and-crash pattern as in the experiment. However, when the price rises rapidly during a bubble, the simulated price is in most cases lower than the experimental price, and vice versa during a crash. It seems that the four heuristics are not able to capture the extremely large price changes. The model fit is better when prices are relatively stable, mainly in the beginning of a session and if there is some convergence in the last periods. The simulated prices are then quite close to the realized prices in the experiment.

In all sessions, we see that the four heuristics make similar predictions. This is in line with coordination of expectations. Forecast errors are defined as the experimental price minus the predicted price, so that a positive error means that the prediction is too low. The plots of the forecast errors also indicate that the heuristics produce forecasts that are generally too low during a bubble and too high during a crash. The forecast errors are correlated: all heuristics either overpredict or underpredict at the same time. The order of magnitude of the errors is in line with the forecast errors of the participants in the experiment.

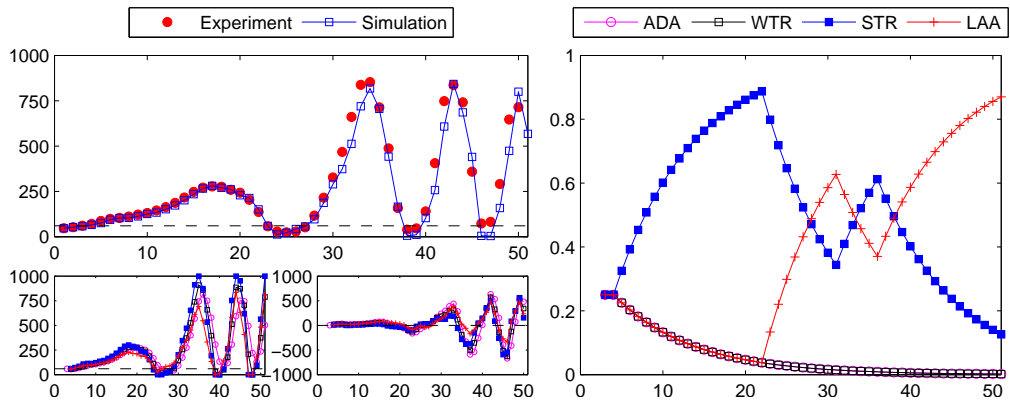
The evolution of impacts in the six sessions with large bubbles is similar. The impact of the STR rule immediately starts increasing from 25% up to 80% or more, while the impacts of the other heuristics gradually drop. The dominance of the STR rule reflects the strong upward trend in prices. But this heuristic misses the trend reversals when the market crashes and when a new bubble starts to form. The LAA rule is more flexible and better at predicting trend reversals and price oscillations. Hence, the impact of STR decreases after the first crash and the LAA rule gains impact, rising to more than 80%. In session 1, 2, 4 and 7, the amplitude of the bubbles decreases and prices slowly seem to converge. The impact of the ADA heuristic increases at the expense of LAA, explaining the price stabilization. There is no convergence in session 3 and 6, where the impact of the LAA rule keeps increasing and enforces persistent price oscillations.



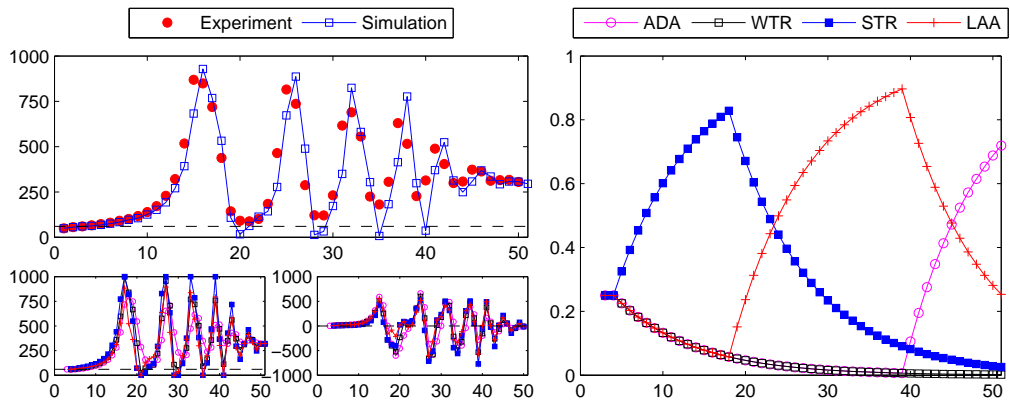
(a) Session 1



(b) Session 2

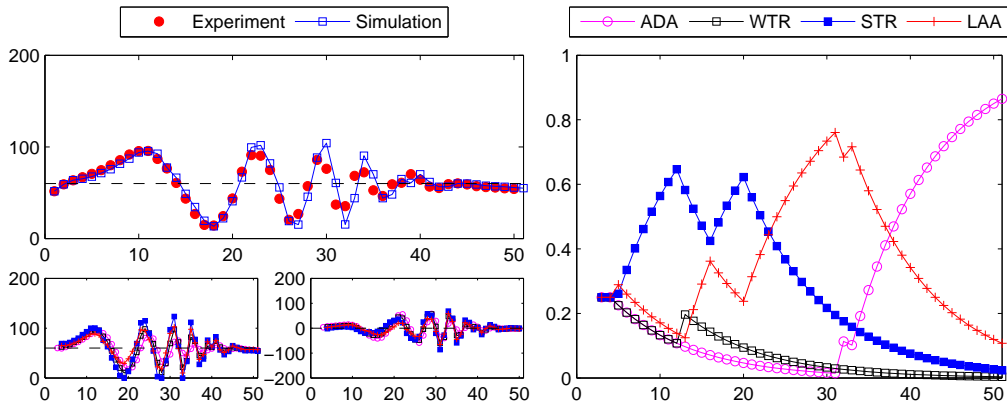


(c) Session 3

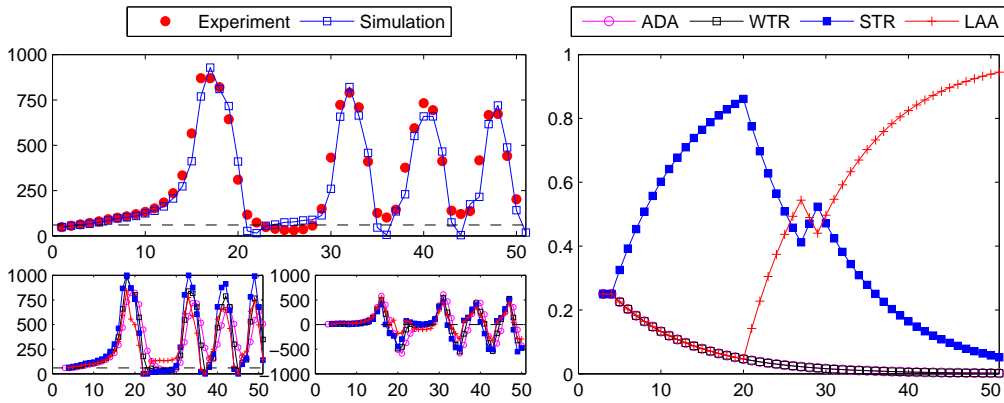


(d) Session 4

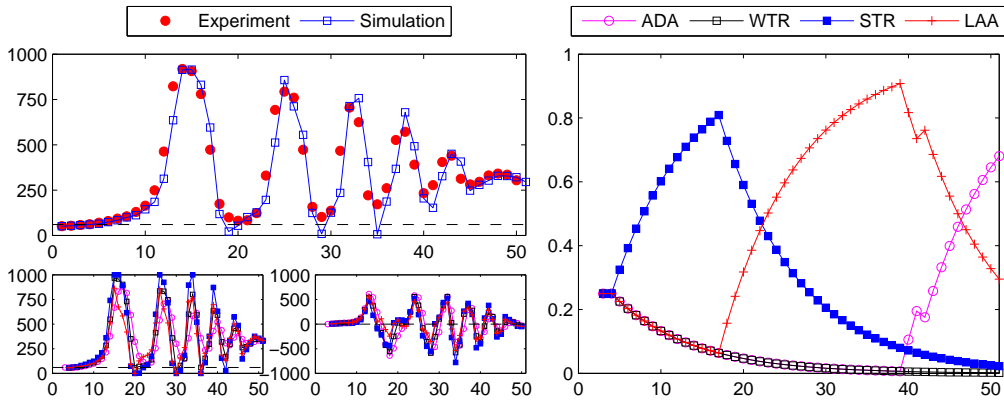
Figure 8: One-period-ahead simulations of the HSM (session 1-4)



(a) Session 5



(b) Session 6



(c) Session 7

Figure 8: One-period-ahead simulations of the HSM (session 5-7)

*Left:* experimental prices and simulated prices (top panel), forecasts of the four heuristics (bottom left panel) and corresponding forecast errors (bottom right panel). The dashed line in the top and bottom left panel indicates the fundamental value of 60. Note that the vertical axis for session 5 runs up to 200 instead of 1000. *Right:* impacts of the four heuristics.

In session 5, the general pattern is comparable to the sessions with large bubbles. The STR rule dominates in the beginning, but is taken over by the LAA heuristic when the price starts oscillating. The impacts grow to a maximum of around 65% for STR and 75% for LAA. These impacts are not as large as in the other sessions, because the bubbles are smaller and the price stabilizes more quickly. This also causes the impact of the ADA rule to rise up to 85% and dominate the other rules in the last ten periods, reinforcing price stabilization.

All these results are in line with Anufriev and Hommes (2012b). Session 5 is similar to the oscillating markets of HSTV05, and the other sessions are comparable to the markets with large bubbles of HSTV08.

## 8.2 Forecasting performance

The model fit can be evaluated by the mean squared error (MSE). We compare six homogeneous expectation rules and three versions of the HSM. The homogeneous rules are fundamental expectations, naive expectations, and the four heuristics of the HSM. The versions of the HSM are the benchmark model, a HSM with fixed fractions ( $\delta = 1$ ), and a HSM that is fitted to the experimental data.

The fitted HSM is found by using a grid search to minimize the MSE over the parameters  $\beta$ ,  $\eta$  and  $\delta$ . The optimal parameter values can be found in Table 5. In most sessions, the optimal value of  $\beta$  is close to or slightly higher than the benchmark value of 0.4. Recall that a larger  $\beta$  means that participants switch faster to more successful rules. Still, the difference with the benchmark case is small even for the highest value of  $\beta = 1.72$ , bearing in mind that the grid search considered values for  $\beta$  in the interval  $[0, 10]$ .<sup>18</sup> The optimal value of  $\eta$  is lower than the benchmark value of 0.7 in all but one session. This indicates that the memory of the performance measure is slightly shorter, so that more distant prediction errors

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<sup>18</sup>Anufriev and Hommes (2012b) found that  $\beta = 10$  is the optimal value for four of the fourteen sessions of Hommes et al. (2005a) and for one of the six sessions of HSTV08. In addition, they found a relatively high value of  $\beta = 5.93$  for another session of HSTV08. The other values of  $\beta$  and the optimal values of  $\eta$  and  $\delta$  that they found for the sessions of HSTV08 are comparable to our results.

Table 5: MSE and parameters fitted HSM

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Session 7
Homogeneous models							
FUN	150,194	136,948	117,585	131,006	456	147,455	152,423
NAI	27,275	32,837	20,217	28,928	209	25,825	27,067
ADA	42,515	45,084	33,072	38,881	295	41,589	40,376
WTR	18,184	25,325	11,907	23,311	157	15,917	19,006
STR	17,878	23,630	<b>6,182</b>	22,199	238	<b>11,721</b>	18,518
LAA	<b>12,732</b>	<b>16,613</b>	9,140	<b>17,209</b>	<b>119</b>	11,994	<b>13,425</b>
Heuristics switching models							
Fixed frac.	13,475	18,254	8,344	17,559	120	10,789	13,978
Benchmark	<b>6,673</b>	<b>11,530</b>	<b>3,781</b>	<b>13,028</b>	<b>93</b>	<b>5,144</b>	<b>8,352</b>
Fitted	<i>5,735</i>	<i>10,851</i>	<i>3,055</i>	<i>9,688</i>	<i>60</i>	<i>4,874</i>	<i>6,869</i>
Optimal parameter values of the fitted HSM							
$\beta \in [0, 10]$	0.49	0.59	0.38	1.05	0.37	0.86	1.72
$\eta \in [0, 1]$	0.39	0.69	0.93	0.56	0.36	0.38	0.57
$\delta \in [0, 1]$	0.77	0.81	0.87	0.48	0.66	0.78	0.72

*Note:* For each session, the lowest MSE is shown in italic and it is always the MSE of the fitted HSM. The benchmark HSM has the second lowest MSE, which is shown in bold. The MSE of the best performing homogeneous model is also shown in bold for comparison.

get less weight. In all sessions, the optimal value of  $\delta$  is lower than the benchmark value of 0.9, suggesting that the inertia is somewhat smaller and a larger fraction of participants considers to update their rule in each period.

For each model, the MSE is calculated for  $t = 5, \dots, 50$ . This is because the HSM takes four periods to fully initialize. It can also be seen as a learning phase. Table 5 shows the MSE of the nine models in every session. The fundamental prediction performs much worse than all other models, especially in the sessions with large bubbles where prices deviate far from the fundamental. The best performing homogeneous model in the sessions where prices converge in the last periods (session 1, 2, 4, 5 and 7) is the LAA heuristic. In the sessions without convergence (session 3 and 6), the STR rule performs better.

The fitted HSM has the minimal MSE of the HSMs by definition. Ignoring the fitted model, the benchmark HSM always has the lowest MSE. The differences of the benchmark HSM compared to the homogeneous models and the fixed fractions HSM are substantial. In most sessions, the improvement of fit of the fitted HSM compared to the benchmark HSM is not as large as when the benchmark HSM is compared to the best homogeneous model. This suggests that the results of the

HSM are not very sensitive to the parameters. Nonetheless, the HSM with fixed fractions has a higher MSE than the best performing homogeneous model, except in session 6. It seems that the model fit significantly improves due to the flexibility of the strategy switching.

Compared to the other sessions, the MSE of all models is much lower in session 5. This illustrates that it is easier to predict prices in a relatively stable market. The very high MSEs in the other sessions show that it is extremely hard to forecast the large bubbles and crashes.

### 8.3 50-period-ahead simulations

Next, we consider 50-period-ahead simulations of the HSM. The goal is to qualitatively replicate the price patterns in the experiment as accurately as possible. These simulations do not use experimental data. We only specify values for  $\beta$ ,  $\eta$  and  $\delta$ , initial prices for period 1 and 2, and initial impacts for period 3 and 4. These parameters and initial values are found by trial and error.

Figure 9 shows 50-period-ahead simulations of the typical price pattern with large bubbles (similar to session 1, 2, 4, 6 and 7) and the price pattern with small bubbles (similar to session 5). The initializations that were used for these simulations can be found in Table 6.

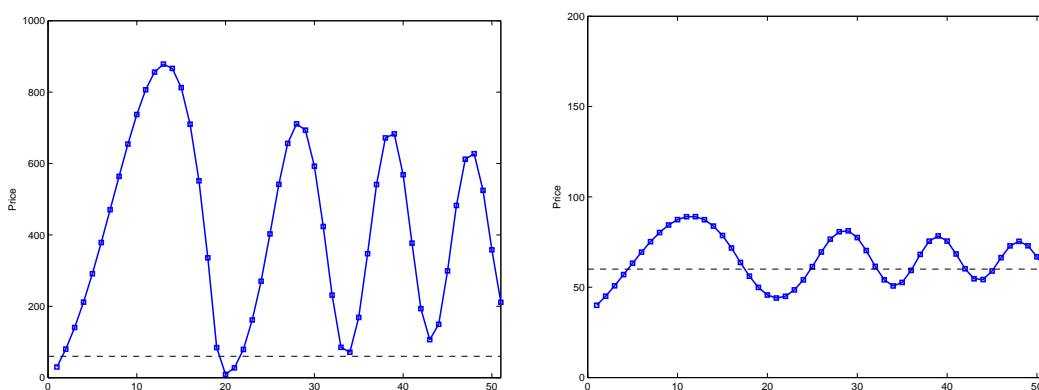


Figure 9: 50-period-ahead simulations of the HSM  
*Left:* typical price pattern with large bubbles (similar to session 1, 2, 4, 6 and 7). *Right:* price pattern with small bubbles (similar to session 5). The dashed line indicates the fundamental value of 60.

Table 6: Parameters and initial values for 50-period-ahead simulations

	Parameters			Prices		Impacts ( $t = 3, 4$ )			
	$\beta$	$\eta$	$\delta$	$p_1$	$p_2$	$n_{1,t}$	$n_{2,t}$	$n_{3,t}$	$n_{4,t}$
Large bubbles	0.4	0.7	0.9	30	80	0	0	0.99	0.01
Small bubbles	0.4	0.5	0.9	40	45	0	0.2	0.7	0.1

The formation of large bubbles strongly depends on the initial prices and impacts. Choosing two prices that are relatively far apart gives a strong upward trend and triggers a quick formation of a large bubble. The initial impacts of the heuristics determine the course of the market price. A very large impact of the STR rule reinforces the strong trends and leads to large bubbles. Starting with a small impact of the LAA heuristic allows this rule to gain impact over time, leading to persistent oscillations with decreasing amplitude. We simply used the benchmark parameter values to simulate the large bubbles. Changing  $\beta$  barely has any effect. Slightly lowering  $\eta$  and  $\delta$  makes the bubbles form a bit faster.

To simulate the small bubbles, the initial prices are relatively low and close to each other, to give rise to a weak upward trend. The STR rule still dominates in the beginning so that a bubble forms, but now the WTR rule and the LAA heuristic have larger initial impacts to make the oscillations smaller and converging. We used the benchmark values for  $\beta$  and  $\delta$ , but a lower value of  $\eta$ . The shorter memory makes sure that the bubbles remain small.

The HSM is thus able to qualitatively match the typical price patterns. However, it is difficult to perfectly replicate the frequency and amplitude of the oscillations. Unfortunately, we were not able to replicate the atypical price pattern of session 3. With this specification of the HSM, it seems impossible to simulate a market that starts with a small bubble and then suddenly forms several large bubbles.<sup>19</sup>

<sup>19</sup>Recall that the use of a strong trend-following rule leads to increasing bubbles (see Figure 3c). It is possible to simulate a price pattern that is qualitatively similar to session 3 by using a trend-following rule with a slightly larger trend extrapolation coefficient as in the HSM, i.e.  $\gamma = 1.4$  instead of  $\gamma = 1.3$ .



## 9 Conclusion

Previous studies have shown that large bubbles caused by trend extrapolation often occur in asset pricing experiments with small groups of ten participants or less. Will these bubbles also occur in larger groups? We increase the market size in the learning-to-forecast experiment of HSTV08 to the size of a typical computer lab. The experiment consists of seven sessions with 21 to 32 participants per market, a group size larger than in most laboratory experiments.

Our paper shows that the answer to the above question is yes: large bubbles arise in six out of seven markets. Prices increase rapidly up to fifteen times the fundamental value. The bubbles occur even faster and more frequently than in the smaller markets of HSTV08. As in their experiment, participants seem to coordinate on a trend-following prediction strategy, causing the first large bubble. We also observe heterogeneity in expectations and switching between prediction strategies. Simple linear forecasting rules provide a good description of the behavior of many participants. The strategies resemble simple benchmark heuristics, such as trend-following rules and anchoring and adjustment.

Heuristics switching models are suitable to analyze the experimental data, because these heterogeneous expectations models incorporate adaptive learning and switching between simple strategies. We illustrate with both one-period-ahead and 50-period-ahead simulations that the HSM is able to capture the price patterns observed in the experiment. The evolution of the impacts of the four heuristics highlights that strong trend extrapolation is important for the formation of a large bubble, that the flexible anchoring and adjustment rule promotes persistent oscillations, and that adaptive expectations lead to price stabilization and convergence. The HSM has better forecasting performance than several benchmark homogeneous expectation rules. It appears that allowing for learning and strategy switching substantially improves the model fit. Heterogeneity in expectations seems crucial to explain the price dynamics.

Coordination, heterogeneity in expectations, the use of simple forecasting heuristics and strategy switching are all observed in multiple LtFEs (see e.g. Hommes et al., 2005a, 2008; Heemeijer et al., 2009; Massaro, 2012; Anufriev and Hommes, 2012b). The larger markets in our experiment did not result in individual forecast errors cancelling out at the aggregate level. Hence, expectations cannot be called rational in the sense of Muth (1961). The results of our large-group experiment are thus largely comparable to the results of HSTV08 and other related experiments with small groups.

## References

- Anufriev, M. and Hommes, C. (2012a). Evolution of market heuristics. *The Knowledge Engineering Review*, 27(2):255–271.
- Anufriev, M. and Hommes, C. (2012b). Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments. *American Economic Journal: Microeconomics*, 4(4):35–64.
- Anufriev, M., Hommes, C., and Philipse, R. (2013). Evolutionary selection of expectations in positive and negative feedback markets. *Journal Of Evolutionary Economics*, 23(3):663–688.
- Bao, T. and Hommes, C. (2015). When speculators meet constructors: Positive and negative feedback in experimental housing markets. CeNDEF Working Paper 2015-10, University of Amsterdam.
- Bao, T., Hommes, C., and Makarewicz, T. (2014). Bubble formation and (in)efficient markets in learning-to-forecast and -optimize experiments. CeNDEF Working Paper 2014-01, University of Amsterdam.
- Bao, T., Hommes, C., Sonnemans, J., and Tuinstra, J. (2012). Individual expectations, limited rationality and aggregate outcomes. *Journal of Economic Dynamics and Control*, 36(8):1101–1120.
- Barber, B. and Odean, T. (2013). The behavior of individual investors. In Constantinides, G., Harris, M., and Stulz, R., editors, *Handbook of the Economics of Finance*. Amsterdam: Elsevier.
- Barberis, N. and Thaler, R. (2003). A survey of behavioral finance. In Constantinides, G., Harris, M., and Stulz, R., editors, *Handbook of the Economics of Finance*. Amsterdam: Elsevier.
- Bossaerts, P. and Plott, C. (2004). Basic principles of asset pricing theory: Evidence from large-scale experimental financial markets. *Review of Finance*, 8:135–169.

- Brock, W. and Hommes, C. (1997). A rational route to randomness. *Econometrica*, 65(5):1059–1095.
- Brock, W. and Hommes, C. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22(4):1235–1274.
- Campbell, J., Lo, A., and MacKinlay, A. (1997). *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.
- Diederich, J. (2013). *Determinants of the Voluntary Provision of Public Goods: Experimental Investigations*. PhD thesis, Heidelberg University.
- Duffy, J. (2008). Experimental macroeconomics. In Durlauf, S. and Blume, L., editors, *New Palgrave Dictionary of Economics*. New York, NY: Palgrave Macmillan.
- Duffy, J. (2015). Macroeconomics: a survey of laboratory research. In Kagel, J. and Roth, A., editors, *Handbook of Experimental Economics Vol. 2 (forthcoming)*. Princeton, NJ: Princeton University Press.
- Heemeijer, P., Hommes, C., Sonnemans, J., and Tuinstra, J. (2009). Price stability and volatility in markets with positive and negative expectations feedback: An experimental investigation. *Journal of Economic Dynamics and Control*, 33:1052–1072.
- Hirshleifer, D. (2001). Investor psychology and asset pricing. *The Journal of Finance*, 56(4):1533–1597.
- Hommes, C. (2001). Financial markets as nonlinear adaptive evolutionary systems. *Quantitative Finance*, 1(1):149–167.
- Hommes, C. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35:1–24.

- Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2005a). Coordination of expectations in asset pricing experiments. *Review of Financial Studies*, 18(3):955–980.
- Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2005b). A strategy experiment in dynamic asset pricing. *Journal of Economic Dynamics and Control*, 29:823–843.
- Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2008). Expectations and bubbles in asset pricing experiments. *Journal of Economic Behavior and Organization*, 67:116–133.
- Isaac, R. and Walker, J. (1988). Group size effects in public goods provision: The voluntary contributions mechanism. *The Quarterly Journal of Economics*, 103(1):179–199.
- Isaac, R., Walker, J., and Williams, A. (1994). Group size and the voluntary provision of public goods: Experimental evidence utilizing large groups. *Journal of Public Economics*, 54:1–36.
- Marimon, R. and Sunder, S. (1993). Indeterminacy of equilibria in a hyperinflationary world: Experimental evidence. *Econometrica*, 61(5):1073–1107.
- Marimon, R. and Sunder, S. (1994). Expectations and learning under alternative monetary regimes: An experimental approach. *Economic Theory*, 4:131–162.
- Massaro, D. (2012). *Bounded rationality and heterogeneous expectations in macroeconomics*. PhD thesis, University of Amsterdam.
- Muth, J. (1961). Rational expectations and the theory of price movements. *Econometrica*, 29(3):315–335.
- Ricciuti, R. (2008). Bringing macroeconomics into the lab. *Journal of Macroeconomics*, 30:216–237.

- Smith, V., Suchanek, G., and Williams, A. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica*, 56(5):1119–1151.
- Sonnemans, J. and Tuinstra, J. (2010). Positive expectations feedback experiments and number guessing games as models of financial markets. *Journal of Economic Psychology*, 31:964–984.
- Stöckl, T., Huber, J., and Kirchler, M. (2010). Bubble measures in experimental asset markets. *Experimental Economics*, 13:284–298.
- Tversky, A. and Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*, 185:1124–1130.
- Weimann, J., Brosig-Koch, J., Hennig-Schmidt, H., Keser, C., and Stahr, C. (2012). Public-good experiments with large groups. FEMM Working Paper 120009, Otto-von-Guericke University Magdeburg.
- Williams, A. (2008). Price bubbles in large financial asset markets. In C.R., P. and Smith, V., editors, *Handbook of Experimental Economics Results*. Amsterdam: Elsevier.
- Williams, A. and Walker, J. (1993). Computerized laboratory exercises for microeconomics education: Three applications motivated by experimental economics. *The Journal of Economic Education*, 24(4):291–315.
- Xu, B., Cadsby, C., Fan, L., and Song, F. (2013). Group size, coordination, and the effectiveness of punishment in the voluntary contributions mechanism: An experimental investigation. *Games*, 4:89–105.

# Appendix

## A Experimental instructions

### General information

You are a **financial advisor** to a pension fund that wants to optimally invest a large amount of money. The pension fund has two investment options: a risk free investment and a risky investment. The risk free investment is putting all money on a bank account paying a fixed and known interest rate. The alternative risky investment is an investment in the stock market with uncertain return. In each time period the pension fund has to decide which fraction of its money to put on the bank account and which fraction of the money to spend on buying stocks. In order to make an optimal investment decision the pension fund needs an accurate prediction of the price of the stock. As their financial advisor, you have to predict the stock market price during 51 subsequent time periods. Your earnings during the experiment depend upon your forecasting accuracy. The smaller your forecasting errors in each period, the higher your total earnings.

### Forecasting task of the financial advisor

The only task of the financial advisors in this experiment is to forecast the stock market index in each time period as accurate as possible. The stock price has to be predicted **two** time periods ahead. At the beginning of the experiment begins, you have to predict the stock price in the first **two** periods. It is very likely that the stock price will be between 0 and 100 in the first two periods. After all participants have given their predictions for the first two periods, the stock market price for the first period will be revealed and based upon your forecasting error your earnings for period 1 will be given. After that you have to give your prediction for the stock market index in the third period. After all participants have given their predictions for period 3, the stock market index in the second period will be revealed and, based upon your forecasting error your earnings for period 2 will be given. This process continues for 51 time periods.

The available information for forecasting the stock price in period  $t$  consists of

- all past prices up to period  $t - 2$ , and
- past predictions up to period  $t - 1$ , and
- total earnings up to period  $t - 2$

## **Information about the stock market**

The stock market price is determined by equilibrium between demand and supply of stocks. The stock market price in period  $t$  will be that price for which aggregate demand equals supply. The supply of stocks is fixed during the experiment. The demand for stocks is determined by the aggregate demand of a number of large pension funds active. Each pension fund is advised by a participant of the experiment.

## **Information about the investment strategies of the pension funds**

The precise investment strategy of the pension fund that you are advising and the investment strategies of the other pension funds are unknown. The bank account of the risk free investment pays a fixed interest rate of 5% per time period. The holder of the stock receives a dividend payment in each time period. These dividend payments are uncertain however and vary over time. Economic experts of the pension funds have computed that the average dividend payments are 3 euro per time period. The return of the stock market per time period is uncertain and depends upon (unknown) dividend payments as well as upon price changes of the stock. As the financial advisor of a pension fund you are **not** asked to forecast dividends, but you are only asked to forecast the price of the stock in each time period. Based upon your stock market price forecast, your pension fund will make an optimal investment decision. The higher your price forecast the larger will be the fraction of money invested by your pension fund in the stock market, so the larger will be their demand for stocks.



## Earnings

Earnings will depend upon forecasting accuracy only. The better you predict the stock market price in each period, the higher your aggregate earnings. Your earnings in each period will be determined by the following scoring rule:

$$e_t = \max\left\{1300 - \frac{1300}{49}(p_t - p_t^e)^2, 0\right\},$$

where  $e_t$  is the number of points you earn in period  $t$ ,  $p_t$  is the realized stock price in period  $t$ , and  $p_t^e$  is your prediction of the stock price in period  $t$ . The earnings table below gives the number of points corresponding to each value of the forecasting error. At the end of the experiment, your total earnings in points will be converted into euros, at an exchange rate of **0.5 euro for 1300 points**.

Payoff Table									
1300 points equal 0.5 euro									
error	points	error	points	error	points	error	points	error	points
0.1	1300	1.5	1240	2.9	1077	4.3	809	5.7	438
0.15	1299	1.55	1236	2.95	1069	4.35	798	5.75	423
0.2	1299	1.6	1232	3	1061	4.4	786	5.8	408
0.25	1298	1.65	1228	3.05	1053	4.45	775	5.85	392
0.3	1298	1.7	1223	3.1	1045	4.5	763	5.9	376
0.35	1297	1.75	1219	3.15	1037	4.55	751	5.95	361
0.4	1296	1.8	1214	3.2	1028	4.6	739	6	345
0.45	1295	1.85	1209	3.25	1020	4.65	726	6.05	329
0.5	1293	1.9	1204	3.3	1011	4.7	714	6.1	313
0.55	1292	1.95	1199	3.35	1002	4.75	701	6.15	297
0.6	1290	2	1194	3.4	993	4.8	689	6.2	280
0.65	1289	2.05	1189	3.45	984	4.85	676	6.25	264
0.7	1287	2.1	1183	3.5	975	4.9	663	6.3	247
0.75	1285	2.15	1177	3.55	966	4.95	650	6.35	230
0.8	1283	2.2	1172	3.6	956	5	637	6.4	213
0.85	1281	2.25	1166	3.65	947	5.05	623	6.45	196
0.9	1279	2.3	1160	3.7	937	5.1	610	6.5	179
0.95	1276	2.35	1153	3.75	927	5.15	596	6.55	162
1	1273	2.4	1147	3.8	917	5.2	583	6.6	144
1.05	1271	2.45	1141	3.85	907	5.25	569	6.65	127
1.1	1268	2.5	1134	3.9	896	5.3	555	6.7	109
1.15	1265	2.55	1127	3.95	886	5.35	541	6.75	91
1.2	1262	2.6	1121	4	876	5.4	526	6.8	73
1.25	1259	2.65	1114	4.05	865	5.45	512	6.85	55
1.3	1255	2.7	1107	4.1	854	5.5	497	6.9	37
1.35	1252	2.75	1099	4.15	843	5.55	483	6.95	19
1.4	1248	2.8	1092	4.2	832	5.6	468	error $\geq 7$	0
1.45	1244	2.85	1085	4.25	821	5.65	453		

## B Tables

### B.1 Experimental results

Table 7: Summary statistics large-group experiment

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Session 7
Average initial pred.							
Period 1	50.87	48.00	42.24	44.36	46.80	40.91	42.33
Period 2	55.19	53.57	46.21	48.87	51.28	47.94	50.08
Period 3	61.27	59.64	52.40	55.40	59.28	56.04	53.45
St. dev. initial pred.							
Period 1	21.26	12.91	17.79	24.06	21.42	20.21	21.13
Period 2	23.02	14.35	18.24	34.19	24.05	22.63	24.57
Period 3	16.26	19.21	19.70	14.18	13.58	10.00	6.17
Initial prices							
Period 1	55.28	53.74	46.73	49.26	51.55	48.38	50.41
Period 2	61.09	59.54	52.65	55.50	59.19	56.11	53.64
Average price	327.32	323.98	273.05	321.28	59.96	309.42	339.48
Minimum price	28.59	53.74	24.17	49.26	14.12	31.02	50.41
Period min. price	20	1	25	1	18	26	1
Maximum price	876.29	851.28	852.56	868.59	95.48	869.72	918.55
Period max. price	13	13	34	15	11	16	14

Table 8: Summary statistics small-group experiment (HSTV08)

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6
Average initial pred.						
Period 1	42.50	41.83	51.58	33.43	47.58	24.17
Period 2	51.67	40.61	52.17	35.54	54.25	29.01
Period 3	51.00	44.62	54.42	40.13	59.33	33.25
St. dev. initial pred.						
Period 1	14.05	14.63	15.49	15.33	6.17	20.41
Period 2	14.38	13.07	10.46	15.68	5.69	18.76
Period 3	6.79	3.68	1.28	6.22	6.56	3.01
Initial prices						
Period 1	52.06	41.53	52.54	36.71	54.52	30.49
Period 2	51.43	45.35	54.68	41.08	59.37	34.52
Average price	115.52	273.45	177.45	306.25	167.07	182.98
Minimum price	48.4	32.86	9.68	9.21	12.56	23.78
Period min. price	44	48	39	31	46	18
Maximum price	228.70	934.54	931.11	954.75	940.16	749.62
Period max. price	28	28	29	22	38	29

## B.2 Estimated forecasting rules

Table 9: Estimated trend extrapolation coefficient  $\gamma$

P	Large-group experiment					Small-group experiment (HSTV08)			
	Session 1	Session 2	Session 4	Session 6	Session 7	Session 2	Session 3	Session 4	Session 6
1	1.603	1.774	2.201	1.357	0.578	1.077	2.113	0.868	0.647
2	0.587	1.789	2.380	1.920	0.178	0.527	2.148	0.991	1.094
3	0.642	1.800	1.779	1.638	1.112	1.097	1.656	1.138	1.020
4	1.227	1.769	0.517	2.235	0.888	1.147	2.078	1.119	1.355
5	0.512	0.826	1.512	1.259	0.904	0.891	2.006	0.866	2.058
6	0.323	1.348	1.886	1.403	1.010	1.066	1.795	1.081	0.508
7	1.372	1.346	2.448	2.232	0.924				
8	1.287	1.039	1.577	0.957	0.799				
9	0.916	-2.056	1.673	2.037	1.082				
10	0.053	2.103	1.287	1.911	1.072				
11	1.226	0.854	2.461	1.898	1.113				
12	0.989	1.825	2.390	1.867	1.276				
13	1.084	1.763	2.106	1.361	0.931				
14	1.120	0.617	2.473	1.741	1.249				
15	1.275	2.244	2.579	2.033	0.456				
16	1.189	1.934	2.829	1.718	0.710				
17	1.200	1.840	1.913	1.256	0.737				
18	1.278	1.874	2.027	1.863	1.127				
19	1.108	1.075	2.067	1.316	-0.188				
20	1.103	1.593	2.310	1.552	0.743				
21	0.918	1.672	2.392	1.754	1.117				
22	1.000	1.422	0.866		1.061				
23	1.411	2.098	1.711		1.023				
24	0.985	2.106	2.393		1.158				
25	1.424	1.639	2.838		1.181				
26	1.182	2.232	1.983		0.999				
27			2.114		1.114				
28			2.367		1.486				
29			1.492		1.135				
30			1.688		0.934				
31			3.156		0.638				
32			1.807		0.556				
$\bar{\gamma}$	1.039	1.482	2.038	1.681	0.909	0.968	1.966	1.01	1.114

*Note:* ‘P’ indicates the number of the participant. The last row gives the average estimated trend extrapolation coefficient  $\bar{\gamma}$  for each session.

Table 10: General linear forecasting rules

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
1	1	<b>141.0</b>	2.198	-2.359	0	0	0	0	0.765	0	0.716	0.098	0.059	<b>0.027</b>
1	2	32.9	1.422	0	0	0	0	-0.476	0	0	0.952	0.821	0.557	0.070
1	3	<b>40.8</b>	1.727	-0.780	0	0	0	0	0	0	0.959	0.859	0.118	0.352
1	4	<b>188.5</b>	0	0	0	-0.392	0	0.868	0	0	0.669	0.412	<b>0.009</b>	0.642
1	5	45.5	0.701	-0.439	0	0	0.479	0	0	0	0.555	0.725	<b>0.014</b>	0.410
1	6	<b>73.5</b>	1.568	-0.680	0	0	0	0	0	0	0.910	0.516	0.701	0.672
1	7	69.2	1.901	-1.459	0	0	0	0	0.349	0	0.782	0.988	<b>0.006</b>	0.181
1	8	70.2	2.829	-1.046	0	0	-0.880	0	0	0	0.853	0.599	<b>0.046</b>	0.458
1	9	<b>137.8</b>	0.836	0	0	0	0.720	-0.913	0	0	0.789	0.622	0.424	0.197
1	10	6.2	0.505	0	-0.169	0	0.490	0	0	0	0.828	0.387	0.057	<b>0.006</b>
1	11	<b>180.1</b>	1.535	-1.024	0	0	0	0	0	0	0.772	0.757	0.504	<b>0.022</b>
1	12	<b>81.1</b>	1.255	-1.871	1.115	-0.413	0.699	0	0	0	0.892	0.917	0.081	0.334
1	13	<b>128.8</b>	0.913	-0.795	0	0	0.522	0	0	0	0.762	0.503	0.210	0.272
1	14	<b>126.5</b>	1.781	-1.048	0	0	0	0	0	0	0.854	0.706	0.219	0.268
1	15	<b>153.3</b>	0	0	0	0	1.115	-0.510	0	0	0.667	0.457	0.210	0.819
1	16	47.2	2.155	-1.004	0.499	0	0	-0.696	0	0	0.810	0.875	0.679	0.912
1	17	<b>119.0</b>	1.702	-1.033	0	-0.363	0	0	0.435	0	0.861	0.410	0.718	0.555
1	18	<b>162.3</b>	0	0	0	0	1.222	-0.653	0	0	0.744	0.730	0.100	0.643
1	19	<b>142.0</b>	0	0	0	0	1.254	-0.625	0	0	0.757	0.238	0.426	0.277
1	20	<b>119.5</b>	1.606	0	0	0	0	-0.932	0	0	0.636	0.077	0.171	0.349
1	21	47.3	1.895	-0.786	-0.489	0	0	0	0	0.355	0.923	0.653	0.568	0.332
1	22	<b>178.9</b>	1.637	0	0	0	0	-0.920	0	0	0.748	0.908	<b>0.032</b>	0.254
1	23	<b>118.0</b>	0.875	0	0	0	0.554	-0.632	0	0	0.790	0.507	0.112	0.487
1	24	<b>190.0</b>	1.366	-0.717	0	0	0	0	0	0	0.717	0.092	0.145	0.055
1	25	<b>248.5</b>	1.478	-1.068	0	0	0	0	0	0	0.475	0.557	0.192	0.443
1	26	<b>100.9</b>	1.855	-1.091	0	0	0	0	0	0	0.864	0.556	0.134	0.110

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
2	1	<b>127.1</b>	1.082	-1.293	0	0	0.525	0	0.319	0	0.817	0.382	0.071	0.052
2	2	<b>190.6</b>	1.500	-0.998	0	0	0	0	0	0	0.644	0.676	<b>0.009</b>	0.981
2	3	<b>105.2</b>	1.502	-0.925	0	0.292	0	0	0	0	0.776	0.505	<b>0.009</b>	0.136
2	4	<b>136.2</b>	1.593	-1.285	0	0	0	0	0.343	0	0.688	0.319	<b>0.007</b>	0.824
2	5	<b>171.7</b>	0	-0.438	0	0	0.954	0	0	0	0.594	0.384	0.479	0.299
2	6	<b>159.3</b>	1.643	-1.371	1.433	-0.871	0	0	-0.954	0.679	0.732	<b>0.046</b>	0.297	0.491
2	7	<b>167.4</b>	1.184	-0.546	0	0	0	0	0	0	0.747	<b>0.077</b>	0.145	0.920
2	8	72.7	1.321	0	0	0	0	-0.649	0	0.261	0.745	0.139	0.198	0.656
2	9	<b>171.4</b>	0	0	0	0	0.496	0	-0.502	0.346	0.378	0.732	0.504	0.092
2	10	<b>157.1</b>	1.584	-1.397	0.494	0	0	0	0	0	0.705	0.116	0.203	0.995
2	11	<b>169.9</b>	1.187	-0.673	0	0	0	0	0	0	0.665	0.713	0.681	0.924
2	12	<b>195.3</b>	1.378	-0.910	0	0	0	0	0	0	0.666	0.824	0.254	0.399
2	13	<b>169.5</b>	0	0	0	0.644	0.905	-0.415	0	-0.624	0.628	0.129	0.085	0.358
2	14	<b>116.9</b>	1.087	0	0	0	0	-0.314	0	0	0.663	0.948	0.135	0.967
2	15	<b>146.1</b>	0.563	-0.607	0	0	0.621	0	0	0	0.637	0.378	<b>0.013</b>	0.415
2	16	<b>141.3</b>	1.643	-0.962	0	0	0	0	0	0	0.799	0.762	0.124	0.628
2	17	<b>136.2</b>	1.519	-1.100	0	-0.581	0	0	0	0.806	0.650	0.161	0.673	0.981
2	18	<b>130.3</b>	1.672	-1.477	0.477	0	0	0	0	0	0.666	0.661	<b>0.016</b>	0.833
2	19	<b>84.6</b>	1.528	-0.972	0	0	0	0.324	0	0	0.822	0.595	0.371	<b>0.019</b>
2	20	88.3	1.632	-1.376	0.501	0	0	0	0	0	0.607	0.722	0.081	0.230
2	21	<b>141.2</b>	1.328	-0.775	0	0	0	0	0	0	0.679	0.160	0.139	0.076
2	22	<b>119.5</b>	1.382	-0.756	0	-0.829	0	0	0	0.855	0.779	<b>0.044</b>	0.477	0.435
2	23	<b>306.7</b>	0	-0.524	0	0	0.610	0	0	0	0.441	0.869	<b>0.019</b>	<b>0.033</b>
2	24	<b>187.5</b>	0	0	0	0	1.190	-0.701	0	0	0.745	0.536	0.457	0.228
2	25	<b>136.2</b>	1.060	-0.415	0	0	0	0	0	0	0.558	0.761	<b>0.006</b>	0.095
2	26	71.7	1.907	-1.489	0.435	0	0	0	0	0	0.769	0.856	0.091	0.512

Note: ‘S’ indicates the number of the session, ‘P’ indicates the number of the participant. A bold value of  $\alpha$  indicates that the constant is significant at the 5% level. The columns ‘BG’, ‘W’ and ‘RR’ report  $p$ -values of the Breusch-Godfrey test, White test and Ramsey RESET test. Bold values indicate significance at the 5% level.

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
3	1	<b>47.1</b>	1.358	0	0	0.823	0	-0.664	0	-0.564	0.940	0.346	<b>0.001</b>	0.824
3	2	46.0	0.542	-0.484	0	0	0.658	0	0	0	0.548	0.280	<b>0.002</b>	<b>0.035</b>
3	3	-11.4	2.836	0	0	0.224	-0.696	-1.138	0	0	0.940	0.280	<b>0.004</b>	0.794
3	4	26.1	2.702	-2.809	1.999	0	0	0	-0.787	0	0.902	0.398	<b>0.001</b>	0.616
3	5	32.3	1.531	0	-1.211	0	0	0	0	0.599	0.911	0.419	0.475	0.103
3	6	29.9	2.486	-2.307	0	0	0	0	0	0.769	0.898	0.556	0.157	0.945
3	7	<b>31.4</b>	2.376	-0.604	0	0	-0.815	0	0	0	0.955	0.772	<b>0.018</b>	0.098
3	8	55.2	2.383	-2.3	0.793	0	0	0	0	0	0.864	0.205	<b>0.000</b>	0.651
3	9	<b>57.0</b>	2.324	0	-3.640	0.675	-0.576	0	1.982	0	0.964	0.155	<b>0.044</b>	0.956
3	10	9.1	1.647	-0.902	0.514	-0.584	0	0	0	0.340	0.982	0.137	0.086	0.660
3	11	42.3	1.889	-1.117	0	1.540	0	0	0	-1.243	0.898	0.101	0.605	0.094
3	12	-16.8	3.340	0	0	0.644	-0.946	-1.041	-0.637	0	0.915	0.338	<b>0.021</b>	0.325
3	13	<b>58.6</b>	0	-0.790	0	0.301	1.284	0	0	0	0.874	0.270	0.050	0.407
3	14	23.1	2.876	-1.571	0.851	0	0	-1.168	0	0	0.880	0.733	0.053	0.222
3	15	48.7	1.310	-2.436	2.117	-1.002	0.773	0	0	0	0.845	0.273	<b>0.027</b>	0.091
3	16	3.5	2.375	-0.896	0	0.388	0	-0.739	0	0	0.906	<b>0.017</b>	0.244	0.255
3	17	-5.0	2.242	0	-1.068	0.959	0	-0.932	0	0	0.877	0.230	<b>0.021</b>	0.185
3	18	-5.4	1.958	-0.385	0	-0.762	-0.480	0	0	0.711	0.984	0.430	<b>0.000</b>	0.231
3	19	28.8	2.756	-0.692	0	0	-1.038	0	0	0	0.963	0.863	0.330	0.768
3	20	54.0	1.008	0	-0.846	0.610	0	0	0	0	0.531	0.930	<b>0.013</b>	0.875
3	21	<b>46.1</b>	1.332	-1.353	0	0	0.436	0.437	0	0	0.949	0.128	<b>0.009</b>	0.723
3	22	56.3	2.734	-1.979	0	0	0	-0.676	0.773	0	0.889	0.544	0.066	0.678
3	23	10.5	1.399	-1.029	0	0.877	1.018	-1.225	0.820	-0.780	0.936	<b>0.018</b>	<b>0.002</b>	0.282
3	24	5.5	1.962	-1.068	0	0.533	0.506	-0.829	0	0	0.901	0.163	<b>0.009</b>	<b>0.026</b>

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
4	1	<b>123.9</b>	1.602	-0.983	0	0.205	0	0	0	0	0.755	0.362	0.559	0.473
4	2	<b>113.4</b>	1.780	-1.440	0	0	0	0	0.394	0	0.788	0.612	0.558	0.772
4	3	<b>132.3</b>	1.524	-1.208	0.392	0	0	0	0	0	0.657	0.447	0.220	0.306
4	4	<b>98.4</b>	1.307	-0.531	0	0	0	0	0	0	0.797	0.757	<b>0.022</b>	<b>0.036</b>
4	5	<b>146.4</b>	1.201	0	0	0	0	-0.581	0	0	0.652	0.890	0.455	0.257
4	6	<b>183.8</b>	1.075	-0.617	0	0	0	0	0	0	0.516	0.086	<b>0.022</b>	0.799
4	7	<b>139.2</b>	0.739	-0.939	0	0	0.538	0	0	0.304	0.699	0.879	0.821	0.340
4	8	<b>123.7</b>	1.169	-0.609	0	0	0	0	0	0	0.699	0.335	<b>0.001</b>	0.265
4	9	<b>76.9</b>	1.305	-0.516	0	0	0	0	0	0	0.851	0.809	0.152	0.586
4	10	37.1	2.106	0	0	0	-0.879	-0.449	0	0	0.800	0.338	0.077	0.776
4	11	<b>130.0</b>	1.606	-1.503	0.946	0	0	0	0	-0.381	0.694	0.908	0.596	0.824
4	12	<b>186.2</b>	0	0	0	0	0.949	-0.461	0	0	0.549	0.620	0.768	0.151
4	13	<b>110.5</b>	1.494	-0.709	0	0	0	0	0	0	0.774	0.231	0.641	0.922
4	14	<b>195.7</b>	1.330	-0.871	0	0	0	0	0	0	0.572	0.322	<b>0.036</b>	0.854
4	15	<b>171.7</b>	0	0	0	0	1.005	-0.489	0	0	0.589	0.537	0.078	0.446
4	16	<b>153.9</b>	1.410	0	0	0	0	-0.715	0	0	0.548	0.536	0.292	0.864
4	17	<b>189.4</b>	0	0	0	0	0.777	-0.315	0	0	0.417	0.887	0.124	0.302
4	18	<b>156.2</b>	1.378	-0.793	0	0	0	0	0	0	0.653	0.344	0.055	0.846
4	19	<b>162.0</b>	1.022	-0.469	0	0	0	0	0	0	0.485	0.556	0.055	0.765
4	20	<b>144.3</b>	1.855	-1.554	0	0	0	0	0	0.351	0.722	0.193	<b>0.049</b>	0.375
4	21	<b>124.0</b>	1.506	-1.576	0.639	0	0	0	0	0	0.617	<b>0.019</b>	0.648	0.361
4	22	32.6	1.677	0	0.822	0	-0.613	-0.594	-0.484	0	0.899	0.450	0.842	0.733
4	23	<b>92.8</b>	1.205	-0.537	0	0	0	0	0	0	0.699	0.882	0.144	0.118
4	24	<b>101.4</b>	1.733	-1.098	0	0	0	0	0	0.172	0.775	0.919	0.252	0.882
4	25	<b>178.9</b>	1.346	-0.811	0	0	0	0	0	0	0.643	<b>0.004</b>	0.541	0.479
4	26	<b>195.8</b>	0	0	0	0	0.918	-0.447	0	0	0.526	0.733	0.053	0.107
4	27	<b>193.2</b>	0.899	-0.524	0	0	0	0	0	0	0.376	0.088	<b>0.007</b>	0.718
4	28	<b>174.5</b>	1.262	0	0	0	0	-0.642	0	0	0.623	0.294	0.216	0.502
4	29	52.2	1.701	-0.867	0	0	0	0	0.393	-0.276	0.797	0.971	0.418	0.282
4	30	<b>122.5</b>	1.580	-1.337	0.470	0	0	0	0	0	0.730	0.239	0.647	0.902
4	31	<b>202.5</b>	0	-0.485	0	0	0.903	0	0	0	0.591	0.396	0.265	0.256
4	32	95.8	1.063	0	0	0.742	0	-0.509	0	-0.568	0.642	<b>0.048</b>	<b>0.011</b>	0.624

Note: 'S' indicates the number of the session, 'P' indicates the number of the participant. A bold value of  $\alpha$  indicates that the constant is significant at the 5% level. The columns 'BG', 'W' and 'RR' report  $p$ -values of the Breusch-Godfrey test, White test and Ramsey RESET test. Bold values indicate significance at the 5% level.

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
5	1	<b>25.5</b>	0.806	-1.357	0	0	0.806	0	0.331	0	0.799	0.106	0.227	0.732
5	2	7.6	2.668	-1.806	0	0	-0.469	0	0.517	0	0.908	0.151	0.065	0.075
5	3	<b>17.1</b>	1.123	-0.865	0	0	0.464	0	0	0	0.881	0.347	0.230	0.565
5	4	4.8	2.333	-2.526	1.471	-0.881	0	0	0	0.524	0.879	0.105	0.240	0.467
5	5	3.1	1.396	-0.480	0	0	0	0	-0.182	0.213	0.958	0.452	0.686	0.287
5	6	<b>20.2</b>	1.994	-2.466	1.813	-0.694	0	0	0	0	0.729	0.400	0.748	0.343
5	7	<b>14.8</b>	1.353	-1.503	1.243	-0.686	0.358	0	0	0	0.883	0.211	0.399	0.354
5	8	10.4	1.935	-2.097	1.400	-0.446	0	0	0	0	0.756	0.224	0.181	0.815
5	9	<b>20.2</b>	1.657	-2.091	1.848	-0.771	0.526	-0.468	0	0	0.825	0.053	<b>0.017</b>	0.137
5	10	3.3	1.993	-1.826	0.751	0	0	0	0	0	0.824	0.170	0.997	0.643
5	11	<b>14.6</b>	1.661	-1.656	1.096	-0.421	0	0	0	0	0.763	0.359	0.215	0.114
5	12	4.0	2.452	-2.719	2.111	-0.688	0	-0.238	0	0	0.928	0.082	0.409	0.109
5	13	<b>21.5</b>	2.238	-2.690	1.417	-0.781	0	0	0.456	0	0.835	0.841	0.914	0.660
5	14	<b>13.1</b>	2.010	-1.982	1.191	-0.465	0	0	0	0	0.860	0.944	0.156	0.290
5	15	9.7	1.926	-1.447	0.352	0	0	0	0	0	0.833	0.960	0.730	0.300
5	16	10.0	1.344	-2.077	1.086	0	0.409	0.358	0	-0.291	0.829	0.459	0.838	<b>0.000</b>
5	17	<b>24.8</b>	2.050	-2.571	1.895	-0.783	0	0	0	0	0.764	0.205	0.652	0.262
5	18	<b>23.8</b>	1.291	-2.337	1.942	-0.865	0.564	0	0	0	0.749	0.080	0.093	0.581
5	19	7.1	1.704	-1.082	0.485	0	0	0	0	-0.235	0.845	0.663	0.106	0.227
5	20	<b>24.0</b>	0	-0.993	0.956	-0.514	1.147	0	0	0	0.765	0.565	0.382	0.319
5	21	13.5	2.492	-2.195	1.522	-0.713	0	-0.977	0.658	0	0.729	0.063	<b>0.047</b>	<b>0.048</b>

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
6	1	39.9	2.117	-0.776	0	0	-0.356	0	0	0	0.940	0.651	0.269	0.729
6	2	40.8	1.552	-0.704	0	0	0	0	0	0	0.836	0.311	<b>0.008</b>	0.883
6	3	43.0	2.336	-0.899	0	0	-0.491	0	0	0	0.936	0.327	0.394	0.847
6	4	59.3	1.831	0	-0.866	0.573	0	-0.591	0	0	0.822	0.217	0.058	0.970
6	5	14.5	2.129	-1.703	0.610	0	0	0	0	0	0.887	0.076	<b>0.010</b>	0.172
6	6	<b>155.4</b>	1.179	0	-0.492	0	0	-0.479	0	0	0.471	0.069	0.152	<b>0.018</b>
6	7	56.2	2.237	-1.858	0.484	0	0	0	0	0	0.865	0.797	<b>0.043</b>	0.682
6	8	47.4	1.992	-1.280	0	0.221	0	0	0	0	0.894	0.236	0.134	0.960
6	9	36.2	1.931	-0.851	0	0.657	0	0	-0.707	0	0.876	0.732	<b>0.006</b>	0.579
6	10	<b>148.7</b>	1.666	-1.057	0	0	0	0	0	0	0.745	0.144	0.873	0.727
6	11	54.5	2.530	-1.199	0	0.198	-0.644	0	0	0	0.909	0.803	0.344	0.877
6	12	<b>299.1</b>	0.866	0	0	0	0	0	-0.555	0	0.283	0.676	0.548	0.893
6	13	<b>82.6</b>	1.941	-1.350	0	0.218	0	0	0	0	0.855	0.196	0.071	0.982
6	14	<b>85.7</b>	2.665	-1.146	0	0	-0.628	0	0	0	0.899	0.685	<b>0.006</b>	0.210
6	15	26.8	0.858	-0.646	0	0	0.601	0	0	0	0.765	0.083	<b>0.002</b>	0.547
6	16	38.6	2.030	-1.685	1.281	0	0	0	-0.614	0	0.910	0.071	0.994	0.708
6	17	<b>75.8</b>	1.699	-0.857	0	0	0	0	0	0	0.887	0.559	0.416	0.623
6	18	<b>69.7</b>	1.728	-0.844	0	0	0	0	0	0	0.893	0.198	<b>0.023</b>	0.090
6	19	23.5	2.135	-0.450	0	0	-0.686	0	0	0	0.970	0.504	0.368	0.288
6	20	62.9	1.605	-1.207	0	0.431	0	0	0	0	0.612	0.579	0.068	0.772
6	21	<b>105.4</b>	0	0	-0.894	0.510	1.059	0	0	0	0.770	0.545	0.513	0.106

Note: 'S' indicates the number of the session, 'P' indicates the number of the participant. A bold value of  $\alpha$  indicates that the constant is significant at the 5% level. The columns 'BG', 'W' and 'RR' report  $p$ -values of the Breusch-Godfrey test, White test and Ramsey RESET test. Bold values indicate significance at the 5% level.

S	P	$\alpha$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$R^2$	BG	W	RR
7	1	19.3	1.689	-0.748	0	0	0	0	0	0	0.868	0.758	0.395	0.075
7	2	22.1	1.393	-0.585	0	0	0	0	-0.299	0.508	0.828	0.573	0.138	0.877
7	3	<b>152.5</b>	0	0	0	0	1.072	-0.470	0	0	0.641	0.790	0.555	0.702
7	4	82.9	1.681	-1.328	0.452	0	0	0	0	0	0.707	0.841	0.397	0.407
7	5	<b>127.5</b>	0	0	-0.449	0	0.772	0	0	0.327	0.549	0.645	0.067	0.886
7	6	<b>117.8</b>	0.796	0	0	0	0	0	0	0	0.524	0.689	0.425	0.683
7	7	<b>96.1</b>	0.561	-0.633	0	0	0.790	0	0	0	0.768	0.289	<b>0.019</b>	0.199
7	8	<b>128.8</b>	1.463	-1.501	0.633	0	0	0	0	0	0.462	0.074	0.204	0.830
7	9	<b>68.1</b>	2.355	-0.887	0	0	-0.576	0	0	0	0.885	0.505	0.542	0.164
7	10	<b>168.6</b>	0.876	0	0	0	0.506	-0.725	0	0	0.634	0.789	0.857	0.485
7	11	<b>143.9</b>	1.583	-0.944	0	0	0	0	0	0	0.786	0.734	0.412	0.125
7	12	<b>220.9</b>	0	-0.501	0	0	0.909	0	0	0	0.588	0.842	0.452	0.066
7	13	<b>83.6</b>	1.633	-0.775	0	0	0	0	0	0	0.906	0.931	0.442	0.830
7	14	<b>138.7</b>	1.524	0	0	0	0	-0.789	0	0	0.634	0.238	0.528	0.861
7	15	<b>50.1</b>	1.404	0	0	0	-0.206	-0.260	0	0	0.952	0.831	<b>0.019</b>	0.720
7	16	<b>67.4</b>	2.690	0	0	0	-0.885	-1.028	0	0	0.848	0.842	0.551	0.857
7	17	<b>52.8</b>	1.994	0	0	0.365	-0.505	-0.563	-0.275	0	0.845	0.883	0.108	0.091
7	18	<b>139.6</b>	1.420	0	-1.121	0	0	0	0	0.416	0.878	0.191	0.470	0.998
7	19	<b>163.2</b>	1.134	0	-0.813	0	0	-0.418	0	0.731	0.593	0.293	0.937	0.827
7	20	52.3	1.540	0.992	0	0.462	0	-1.357	-0.800	0	0.797	0.336	0.517	0.805
7	21	<b>146.3</b>	1.746	-1.069	0	0	0	0	0	0	0.752	0.731	0.701	0.620
7	22	91.2	1.397	0	0	0	0	-0.629	0	0	0.642	0.334	<b>0.019</b>	0.944
7	23	<b>113.8</b>	1.562	0	0.868	0	0	-0.814	-0.687	0	0.771	0.220	0.999	0.940
7	24	<b>154.9</b>	1.563	-0.874	0	0	0	0	0	0	0.758	0.555	0.501	0.531
7	25	<b>184.2</b>	1.544	-0.999	0	0	0	0	0	0	0.692	0.862	0.798	0.105
7	26	<b>118.7</b>	1.282	-1.086	0	0	0.335	0	0	0.232	0.741	0.571	0.053	0.103
7	27	<b>172.0</b>	0.864	0	0	0	0.632	-0.883	0	0	0.695	0.175	0.832	0.792
7	28	<b>181.0</b>	0.996	-0.734	0	0	0.361	0	0	0	0.452	0.467	0.777	0.306
7	29	<b>163.2</b>	1.752	-1.299	0	0	0	0	0	0	0.651	0.866	<b>0.024</b>	<b>0.005</b>
7	30	<b>96.4</b>	0	0	0	-0.203	0.869	0	0	0	0.752	0.531	<b>0.000</b>	0.168
7	31	30.8	1.464	0	-1.159	0.685	0	0	0	0	0.884	0.500	0.374	0.342
7	32	17.0	1.390	0	0.308	0	0	-0.670	0	0	0.904	0.557	0.139	0.233

Note: 'S' indicates the number of the session, 'P' indicates the number of the participant. A bold value of  $\alpha$  indicates that the constant is significant at the 5% level. The columns 'BG', 'W' and 'RR' report  $p$ -values of the Breusch-Godfrey test, White test and Ramsey RESET test. Bold values indicate significance at the 5% level.

Table 11: First-order heuristics

S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
1	3	0.953	0	0.047	0.781	naive	trend-following
1	5	0	0.662	0.338	0.574	mixed	trend-following
1	6	0.897	0	0.103	0.682	naive	trend-following
1	8	1.694	-0.798	0.104	1.043	mixed	trend-following
1	11	0.516	0	0.484	1.025	mixed	trend-following
1	13	0	0.620	0.380	0.799	mixed	trend-following
1	14	0.742	0	0.258	1.050	mixed	trend-following
1	17	0.898	0	0.102	0.750	naive	trend-following
1	24	0	0.645	0.355	0.730	mixed	trend-following
1	25	0.424	0	0.576	1.070	mixed	trend-following
1	26	0.769	0	0.231	1.092	mixed	trend-following
S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
2	2	0.511	0	0.489	0.998	mixed	trend-following
2	5	0	0.597	0.403	0.496	mixed	trend-following
2	7	0	0.596	0.404	0.746	mixed	trend-following
2	11	0.517	0	0.483	0.673	mixed	trend-following
2	12	0.474	0	0.526	0.91	mixed	trend-following
2	15	0	0.591	0.409	0.598	mixed	trend-following
2	16	0.692	0	0.308	0.962	mixed	trend-following
2	21	0.551	0	0.449	0.776	mixed	trend-following
2	23	0	0	1	1.041	fundamental	trend-following
2	25	0.651	0	0.349	0.415	mixed	trend-following
S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
3	2	0	0.723	0.277	0.505	mixed	trend-following
3	7	1.730	-0.778	0.048	0.627	mixed	trend-following
3	19	1.781	-0.794	0.014	0.761	mixed	trend-following
S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
4	4	0.785	0	0.215	0.531	mixed	trend-following
4	6	0	0.473	0.527	0.589	mixed	trend-following
4	8	0	0.529	0.471	0.686	mixed	trend-following
4	9	0.793	0	0.207	0.516	mixed	trend-following
4	13	0.798	0	0.202	0.710	naive	trend-following
4	14	0	0.400	0.600	0.810	mixed	trend-following
4	18	0.592	0	0.408	0.794	mixed	trend-following
4	19	0.558	0	0.442	0.470	mixed	trend-following
4	23	0.663	0	0.337	0.536	mixed	trend-following
4	25	0	0.489	0.511	0.785	mixed	trend-following
4	27	0	0.433	0.567	0.433	mixed	trend-following
4	31	0	0.500	0.500	0.561	mixed	trend-following
S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
5	3	0	0.668	0.332	0.949	mixed	trend-following

Note: 'S' indicates the number of the session, 'P' indicates the number of the participant.



S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
6	1	0.912	0	0.088	0.892	naive	trend-following
6	2	0.846	0	0.154	0.703	mixed	trend-following
6	3	0.853	0	0.147	0.980	mixed	trend-following
6	10	0	0.584	0.416	0.874	mixed	trend-following
6	14	0.781	0	0.219	1.122	mixed	trend-following
6	15	0	0.774	0.226	0.743	mixed	trend-following
6	17	0.848	0	0.152	0.863	mixed	trend-following
6	18	0.891	0	0.109	0.852	naive	trend-following
6	19	1.536	-0.553	0.017	0.511	mixed	trend-following

S	P	$\alpha_1$	$\alpha_2$	$1 - \alpha_1 - \alpha_2$	$\beta$	anchor	adjustment
7	1	0.941	0	0.059	0.748	naive	trend-following
7	6	0.811	0	0.189	0	naive	none
7	7	0	0.741	0.259	0.611	mixed	trend-following
7	9	1.374	-0.493	0.118	0.899	mixed	trend-following
7	11	0.645	0	0.355	0.945	mixed	trend-following
7	12	0	0.533	0.467	0.739	mixed	trend-following
7	13	0.868	0	0.132	0.778	mixed	trend-following
7	21	0.688	0	0.312	1.072	mixed	trend-following
7	24	0.703	0	0.297	0.878	mixed	trend-following
7	25	0.554	0	0.446	1.002	mixed	trend-following
7	28	0	0.512	0.488	0.780	mixed	trend-following
7	29	0.446	0	0.554	1.297	mixed	trend-following

Note: 'S' indicates the number of the session, 'P' indicates the number of the participant.