Latent variables and shocks contribution in DSGE models with occasionally binding constraints

May 29, 2016[1]

Marco Ratto, Massimo Giovannini (European Commission, Joint Research Centre)

We implement an algorithm to estimate latent variables and shocks in DSGE models with occasionally binding constraints. The algorithm also provides an estimate of the sequence of regimes along the historical periods. We use the Occbin solution method developed by Guerrieri and Iacoviello (2015) to treat the occasionally binding constraint via a piecewise linear solution. We discuss how to quantify the *non additive* contribution of the smoothed estimates of historical shocks onto observed variables, namely a non-linear extension of the usual linear/additive historical shock decompositions. We show one illustrative example and finally apply this on an estimated DSGE model for the Euroarea.

1. Estimation of latent variables and shocks under occasionally binding constraints

We use the Occbin solution method developed by Guerrieri and Iacoviello (2015) to treat the occasionally binding constraint via a piecewise linear solution[2]. The algorithm similar to Anzoategui, et al. (mimeo, 2015, Appendix A2) to obtain smoothed estimates of latent variables The algorithm for estimating latent variables is as follows:

1) Guess an initial sequence of regimes for each historical period $R_t^{(0)}$ for $t = 1, \dots T$
2) Given the sequence of regimes, compute the sequence state space matrices $\boldsymbol{\Upsilon}_{\boldsymbol{t}}^{(0)}$ following the piecewise linear solution method of Guerrieri and Iacoviello (2015).
3) For each iteration $j = 1, \dots, n$
    a. feed the state space matrices $\boldsymbol{\Upsilon}_t^{(j-1)}$to a Kalman Filter[3]/ Fixed interval smoothing algorithm to determine initial conditions, smoothed variables $\boldsymbol{y}_t^{(j)}$ and shocks $\boldsymbol{\epsilon}_t^{(j)}$.
    b. given initial conditions and shocks perform Occbin simulations that endogenously determine a new sequence of regimes $R_t^{(j)}$, from which a new sequence of states space matrices is derived $\boldsymbol{\Upsilon}_{\boldsymbol{t}}^{(j)}$

[1] The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement 'Integrated Macro-Financial Modeling for Robust Policy Design' (MACFINROBODS, grant no. 612796)
The views expressed in this paper are those of the authors and should not be attributed to the European Commission.

[2] This solution may also be viewed as an iterative application of the solution of Cagliarini and Kulish (2013) where the iterations are used to find the expected duration, which is consistent with the binding constraint.

[3] Kulish et al. (2014) also apply the piecewise linear solution in the Kalman filter to estimate DSGE models with forward guidance.

4) The algorithm stops when $R_t^{(j)} = R_t^{(j-1)}$ for all $t = 1, \dots, T$.

This algorithm provides initial conditions, smoothed variables and shocks, consistent with the occasionally binding constraint, i.e. it also estimates a sequence of regimes along the historical periods.

2 Estimating contributions of shocks for the piecewise linear solution

One interesting issue is the estimation of the shock contributions to the observed data consistent with the piecewise linear solution, namely the extension of the standard historical shock decompositions to the case of occasionally binding regimes. The contribution of individual smoothed shocks, however, is not the mere additive superposition of each shock propagated by the sequence of state space matrices $\mathbf{\Upsilon}^{(j)}$ estimated with the smoother. The occurrence of a specific regime at time *t*, in fact, is a *non-linear function* of the states in *t-1* $\boldsymbol{y}_{t-1}$ and of the whole set of shocks *simultaneously* affecting the economy, i.e. it is *conditional* on the sequence and combination of shocks simultaneously hitting the economy:

$$\mathbf{\Upsilon}_{\boldsymbol{t}}^{(\epsilon)} = f(\epsilon_{1t}, \dots, \epsilon_{kt}, \boldsymbol{y}_{t-1}), t = 1, \dots, T$$

It is easy to verify that, taking subsets of shocks or individual shocks, the sequence of occasionally binding regimes will change. One way of measuring the effect of shocks in this non-linear context is to consider simulations conditional to given shock patterns, i.e. performing counterfactuals opportunely choosing combinations of shocks and initial conditions.

In particular, we can consider two definitions that generalize the concept of shock contributions to the non-linear case, which degenerate to the standard shock decompositions for the linear case. We define ϵ_{lt} the shock or group of shocks of interest, while $\epsilon_{\sim lt}$ denotes the complementary set of shocks.

1) Compute the contribution of ϵ_{lt} by performing simulations (counterfactuals) using the sequence of smoothed shocks for the shocks within the group *l*, setting to zero all other shocks $\epsilon_{\sim lt}$ and ignoring the initial condition. We denote this contribution as the *conditional contribution*: $\boldsymbol{y}_t(\epsilon_{lt}|\epsilon_{\sim lt} = 0, \boldsymbol{y}_0 = 0)$.
2) Compute the contribution of ϵ_{lt} by setting to zero the shocks ϵ_{lt} and performing simulations (counterfactuals) using the initial condition *and* the sequence of smoothed shocks for the *complementary* set of shocks $\epsilon_{\sim lt}$. We denote this counterfactual as $\boldsymbol{y}_t(\epsilon_{\sim lt}, \boldsymbol{y}_0|\epsilon_{lt} = 0)$. The contribution of the shocks of interest will be the *complement* of this simulation to the smoothed variables $\boldsymbol{y}_t$: $\boldsymbol{y}_t(\epsilon_{lt}|\epsilon_{\sim lt}, \boldsymbol{y}_0) = \boldsymbol{y}_t - \boldsymbol{y}_t(\epsilon_{\sim lt}, \boldsymbol{y}_0|\epsilon_{lt} = 0)$. We call this the *residual contribution*.

Note that each of these simulations provides a different sequence of regimes, which in general will be different from the historical one. The *residual contribution*, in particular, $\boldsymbol{y}_t(\epsilon_{lt}|\epsilon_{\sim lt}, \boldsymbol{y}_0)$, triggers key non-linear features associated to the interaction between shock realization and the occasionally binding constraints.

We use the *residual contribution* to measure the contribution of shocks to observed variables under occasionally binding constraints.

Definitions 1) and 2) above can be further extended in terms of *conditional expectations*, namely:

1a) the *expected conditional contribution*: $E_{\epsilon_{\sim lt},\boldsymbol{y}_0}[\boldsymbol{y}_t(\epsilon_{lt}|\epsilon_{\sim lt},\boldsymbol{y}_0)]$

2a) the *expected residual contribution*: $\boldsymbol{E}_{\epsilon_{lt}}[\boldsymbol{y}_t(\epsilon_{lt}|\epsilon_{\sim lt},\boldsymbol{y}_0) = \boldsymbol{y}_t - \boldsymbol{E}_{\epsilon_{lt}}[\boldsymbol{y}_t(\epsilon_{\sim lt},\boldsymbol{y}_0|\epsilon_{lt})]$.

Both conditional contributions can be computed by performing a set of Monte Carlo counterfactuals drawing respectively ϵ_{lt} or $\epsilon_{\sim lt},\boldsymbol{y}_0$ from the normal distributions

[to be completed]

3 Simple illustrative examples

[to be completed]

4 Fiscal policy under the ZLB constraint in an estimated DSGE model

We perform smoothed estimates of latent variables and shocks in a three-country DSGE model for Euroarea-US-RoW (Kollmann et al. 2016) by enforcing the ZLB constraints in both EA and US, using the estimated parameters in the baseline estimation without ZLB. We use the Occbin solution method developed by Guerrieri and Iacoviello (2015) to treat the occasionally binding constraint via a piecewise linear solution[4]. Finally, we use these smoothed estimates to compute the *residual contribution* of individual shocks [or of groups of shocks] onto GDP, following the methodology described in Section 2.

The sequence of regimes is reported in the next Table 1. It is worth noting that agents in both EA and US anticipated ZLB starting in 2009q1. EA in particular anticipates quite prolonged ZLB, which influences significantly shock contributions in EA in 2009. Moreover, *both* EA and US faced a constrained monetary policy in the second half of 2009. Monetary policy is again constrained for both US and EA since 2013q1.

In Figure 1 we show the historical pattern of the 'shadow' unconstrained nominal interest rate (i_{kt}^{NC}, black dots) versus the actual data of policy rates (i_{kt}, red)[5]. When i_{kt}^{NC} is below the threshold the constraint is binding, otherwise the regime will be either 'normal' or anticipating future binding regimes. This is provided in Table 1 as well.

[4] This solution may also be viewed as an iterative application of the solution of Cagliarini and Kulish (2013) where the iterations are used to find the expected duration, which is consistent with the binding constraint.

[5] Note that, in the estimation, we used the money market rate for US. The latter fell less abruptly in 2008/09 with respect to the Fed Funds rate.

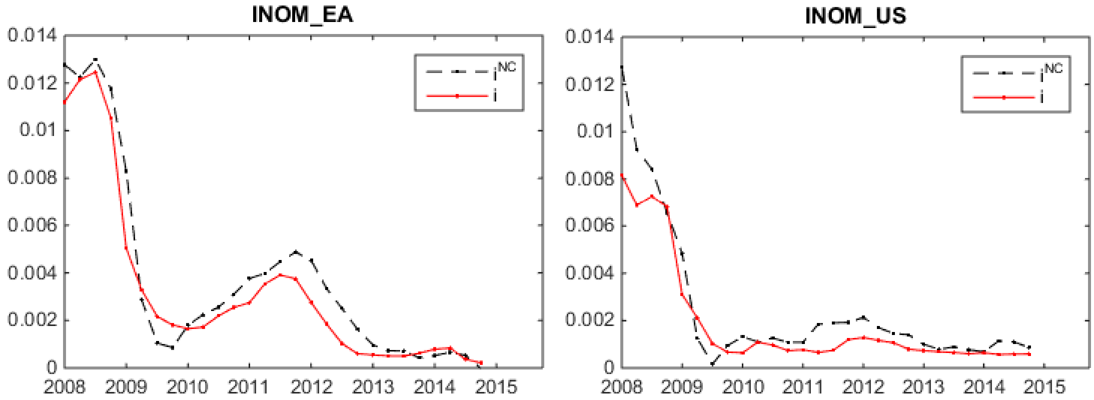


Figure 1. Observed interest rate i_{kt} vs. 'shadow' interest rate i_{kt}^{NC}.

Estimating contributions of shocks for the piecewise linear solution

One interesting issue is the estimation of the shock contributions to the observed data consistent with the piecewise linear solution, namely the extension of the standard historical shock decompositions to the case of occasionally binding regimes.

We use the *residual contribution* to measure the impact of the ZLB on the contribution of shocks to observed variables. In particular, we focus in Figure 2 on the impact of fiscal shocks on yoy GDP in EA and US.

The two major outcomes of this non-linear analysis are:

a) the effect of fiscal shocks in 2009 changes *both in sign and size*, implying a significant *positive contribution* of fiscal measures at the onset of the great recession when ZLB is considered. This is due to the combination of two effects. The fiscal shocks grouped for this study include three supply shocks (govt. consumption and investment, transfers) and one lump sum tax shock. In 2009 there are positive demand shocks, which trigger a positive effect on GDP and quite large *positive* tax shocks, that trigger a decline in GDP. Under unconstrained regimes the fiscal multiplier to government spending is small so that the effect of tax shocks prevails, so the aggregate effect of all fiscal shocks is slightly negative. Under the ZLB, the multiplier almost doubles, implying that the effect of positive fiscal spending shocks dominates against the negative effect of the tax shock. This explains the sign reversal in the shocks contribution.
b) the negative contribution of fiscal shocks in the subsequent slump is magnified by the ZLB. This makes the impact of fiscal policy more visible, although this is still not the main driver of the slump. In EA, in particular, the contribution of fiscal shocks in 2013 is about -0.35% out of a maximum decline of about -2.6% in 2013q1 (i.e. about 15% the decline). In 2014, the fiscal shocks under ZLB still have a negative impact in the first three quarters, by about -0.15% out of an overall GDP decline of -0.55%. The linear shock decomposition, in turn, implies no or slightly positive contribution of the fiscal shocks in 2014 for the EA.

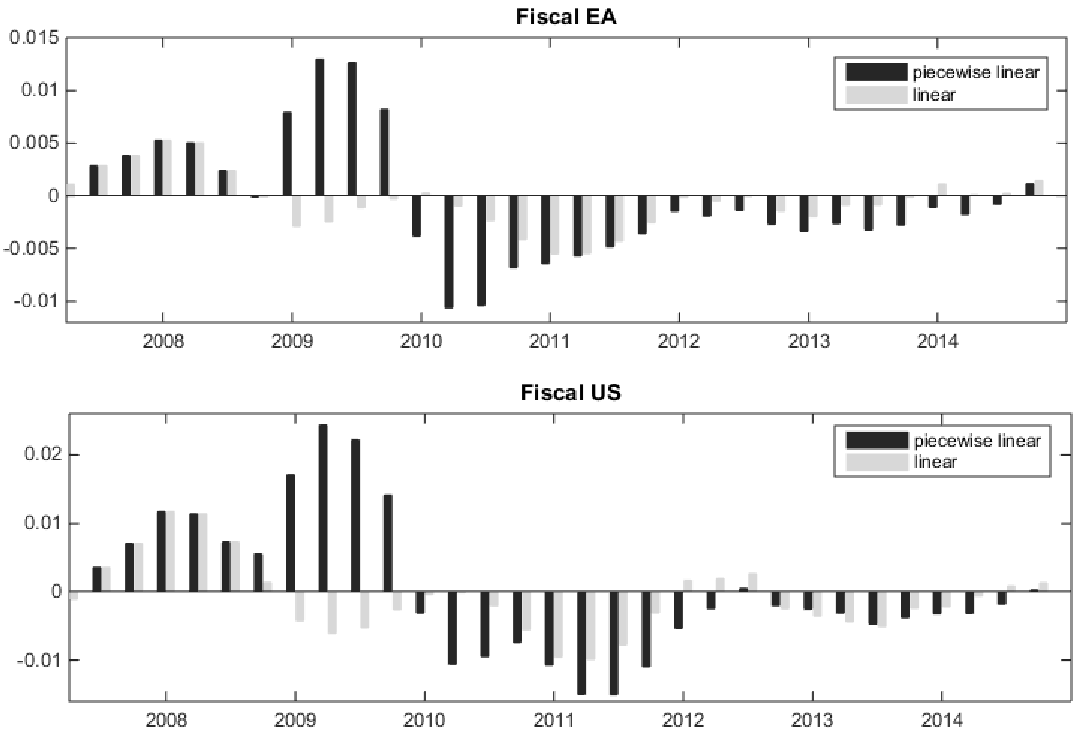


Figure 2. Contribution of fiscal shocks to yoy GDP growth in EA and US. Comparison of linear and piecewise linear solutions

To better understand the interaction between shocks and regime sequences, we report in Table 2 the regimes obtained shutting off the fiscal shocks. This shows that, for both EA and US, without fiscal shocks there would have been more severely binding constrained regimes in 2009. Moreover, in EA, constrained regimes would be less binding in 2013 without fiscal shocks. In US, the absence of fiscal shocks would have implied more prolonged constrained regimes in 2010, 2012 and 2014.

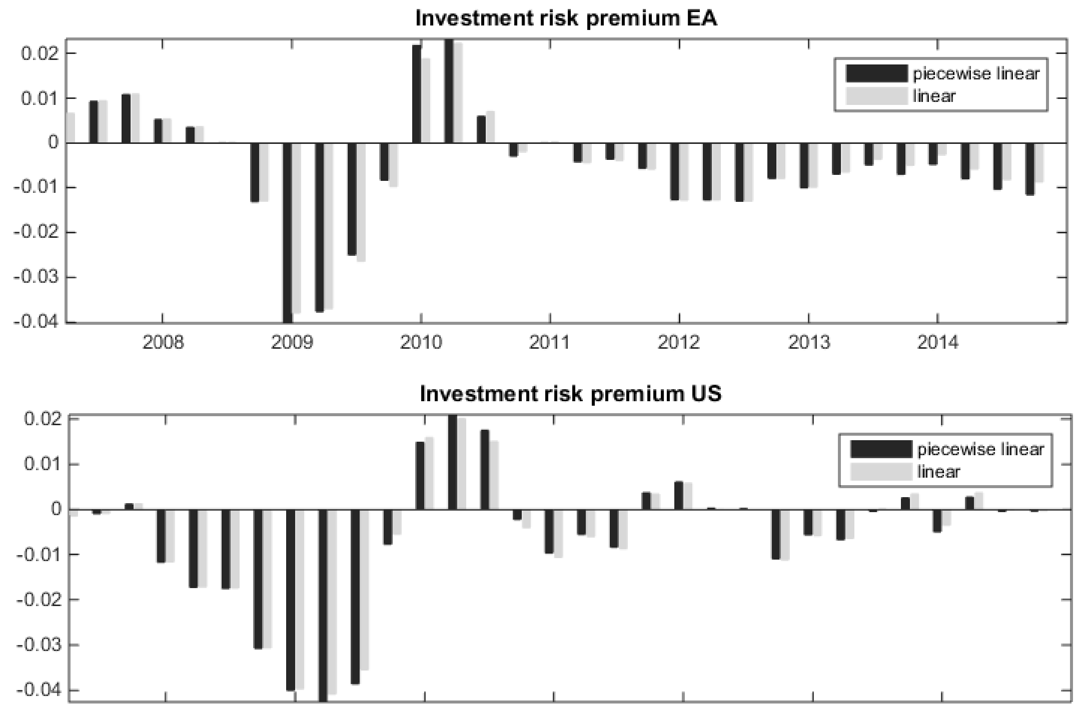


Figure 3. Contribution of investment risk premia shocks to yoy GDP growth in EA and US. Comparison of linear and piecewise linear solution

Fiscal multipliers/IRFs under constrained regimes

To better highlight the change in the transmission mechanism of fiscal shocks for the estimated DSGE model, we perform IRFs with ZLB consistent with the estimated timing and duration of the constrained regimes.

As an example, we perform counterfactual exercises as follows. Using as starting point the smoothed variables in 2008q4, we shut off all fiscal shocks and simulate the model with all other shocks.

We perform another simulation adding a negative government spending shock of -0.25% of quarterly GDP. The difference between the two simulations provides the IRF of a government spending shock under a constrained regime. For both EA and US the multiplier becomes bigger than one and fiscal consolidation generates a *comovement* of consumption and investment with government spending for some periods at the beginning of the simulations.

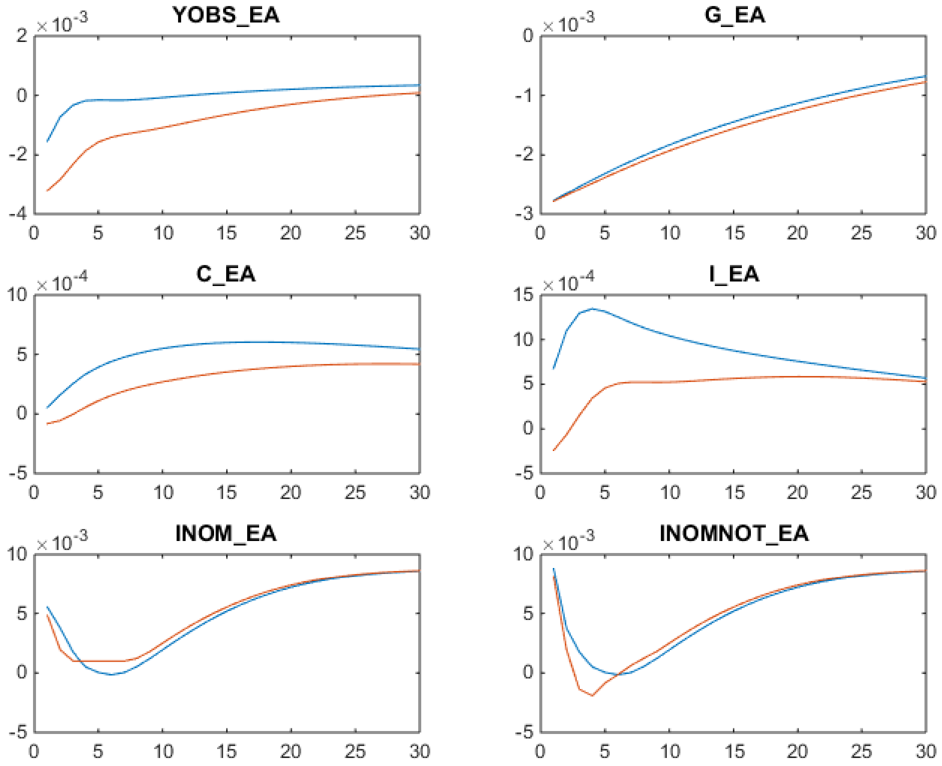
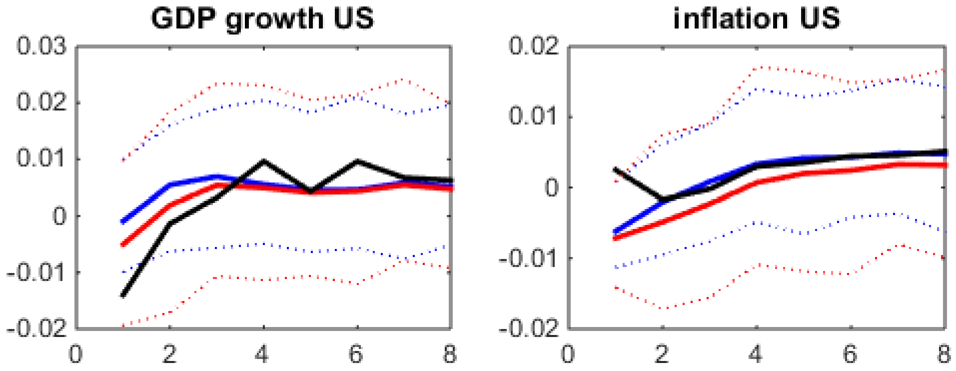


Figure 4. Negative fiscal shock of 0.25% of GDP in EA in 2009q1, on top of all the other historical shocks. Blue is the linear model, red is the piecewise linear one. INOM = nominal int. rate; INOMNOT = shadow int. rate

Forecasting performance with/without occasionally binding constraints

Preliminary results on real-time forecasting starting in 2008Q4.

Blue lines are the results of the linear model, while red lines show stochastic simulations allowing the possibility of constrained monetary policy. It seems that under OBC the data (black).



[to be completed]

Conclusions

[to be completed]

References

Anzoategui D., Comin D., Gertler M., Martinez J., Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence, mimeo, 2015.

Cagliarini A., M. Kulish, Solving Linear Rational Expectations Models with Predictable Structural Changes, 2013, Review of Economics and Statistics, 95(1), pp 328-336

Guerrieri, L. and M. Iacoviello (2015) OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily, Journal of Monetary Economics, 70, 22-38.

Kollmann R., B. Pataracchia R. Raciborski M. Ratto W. Roeger, L. Vogel, The Post-Crisis Slump in the Euro Area and the US: Evidence from an Estimated Three-Region DSGE Model, mimeo, 2016

Kulish M., J. Morley, T. Robinson, Estimating DSGE models with Forward Guidance, mimeo, 2014.

Table 1 Estimation of the historical sequence of occasionally binding regimes

	EA		*US*	
time	regime sequence[6]	starting period of regime[7]	regime sequence	starting period of regime
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 3 8	0 1 0	1 3 7
2009.25	0 1 0	1 2 7	0 1 0	1 2 7
2009.5	0 1 0	1 2 4	1 0	1 3
2009.75	1 0	1 2	1 0	1 2
2010	0	1	0	1
2010.25	0	1	0	1
2010.5	0	1	0	1
2010.75	0	1	0	1
2011	0	1	0	1
2011.25	0	1	0	1
2011.5	0	1	0	1
2011.75	0	1	0	1
2012	0	1	0	1
2012.25	0	1	0	1
2012.5	0	1	0	1
2012.75	0	1	0	1
2013	1 0	1 3	1 0	1 2
2013.25	1 0	1 3	1 0	1 3
2013.5	1 0	1 3	1 0	1 3
2013.75	1 0	1 3	1 0	1 3
2014	1 0	1 3	1 0	1 3
2014.25	1 0	1 3	0	1
2014.5	1 0	1 3	0	1
2014.75	1 0	1 3	1 0	1 3

[6] 0 = unconstrained; 1 = constrained.
[1 0] indicates a constrained regime. [0 1 0] indicates a regime that anticipates FUTURE constraints.
[7] Periods for which the regime starts.
[1 7] indicates a constrained regime for 6 periods. [1 2 7] indicates a regime that anticipates FUTURE constraints starting in period 2 until period 6.

Table 2 Sequence of regimes obtained removing fiscal shocks in EA and US respectively

	EA		*US*	
time	regime sequence[8]	starting period of regime[9]	regime sequence	starting period of regime
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0 1 0	1 3 6
2009	0 1 0	1 2 8	1 0	1 9
2009.25	1 0	1 7	1 0	1 8
2009.5	1 0	1 5	1 0	1 6
2009.75	1 0	1 3	1 0	1 4
2010	1 0	1 3	1 0	1 3
2010.25	0	1	1 0	1 5
2010.5	0	1	1 0	1 4
2010.75	0	1	1 0	1 3
2011	0	1	1 0	1 3
2011.25	0	1	0	1
2011.5	0	1	0	1
2011.75	0	1	0	1
2012	0	1	0 1 0	1 2 3
2012.25	0	1	1 0	1 3
2012.5	0	1	1 0	1 4
2012.75	0	1	1 0	1 5
2013	0	1	1 0	1 3
2013.25	0	1	1 0	1 4
2013.5	0 1 0	1 2 3	1 0	1 3
2013.75	1 0	1 3	1 0	1 3
2014	1 0	1 2	1 0	1 5
2014.25	1 0	1 3	1 0	1 3
2014.5	1 0	1 3	1 0	1 3
2014.75	1 0	1 4	1 0	1 4

[8] 0 = unconstrained; 1 = constrained.
[1 0] indicates a constrained regime. [0 1 0] indicates a regime that anticipates FUTURE constraints.

[9] Periods for which the regime starts.
[1 7] indicates a constrained regime for 6 periods. [1 2 7] indicates a regime that anticipates FUTURE constraints starting in period 2 until period 6.