THE UNDER- AND OVER-REACTION EFFECT OF BEHAVIORAL SENTIMENT IN LIMIT ORDER MARKETS

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ABSTRACT. This paper examines the under/over-reaction effect of behavioral sentiment in an artificial limit order market when agents are risk averse and arrive in the market with different time horizons. Compared to noise bias in belief, we model a type of sentiment bias in belief display under/over-reaction by following a Bayesian learning scheme with a Markov regime switching between conservative bias and representative bias, similar to the one in (Barberis, Shleifer and Vishny 1998, Journal of Financial Economics 49, 307-343). The agent-based simulations show that sentiment and noise belief have very different impact on the limit order market. Sentiment belief gives rise to return predictability, in particular, under/over reaction trading strategies are profitable under sentiment belief, but not under noise belief. In particular, we find that sentiment belief leads to significantly lower volatility, lower bid-ask spread and larger order book depth near the best quotes but lower trading volume when compare to noise belief.

JEL Classification: C63,D84,G12.

Keywords: Sentiment, over-reaction, under-reaction, limit order market, agent-based.

Date: Current version (draft): February 15, 2016.

Acknowledgement: We thank Kristoffer Glover and the participants in the Paul Woolley Conference 2013 in University of Technology, Sydney and the 19th Workshop on Economic Science with Heterogeneous Interacting Agents (WEHIA 2014) in Tianjin University for valuable comments and suggestions. Financial support from the Australian Research Council (ARC) under a Discovery Grant (DP110104487) is gratefully acknowledged.

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1. Introduction

Although it is almost beyond debate that individual investors have behavioral biases, there is less consensus as to whether these biases have a significant impact on the aggregate level (Yan (2010)). Traditional financial theory such as Efficient Market Hypnosis (EMH) considers that the effect of individual's biases should cancel each other out and have little impact on the aggregate level. However, Yan (2010) finds that a modest amount of biases can have a large impact on the equilibrium. Actually, behavioral finance models, such as De Long, Shleifer, Summers and Waldmann (1990) find that noise traders restrict the arbitrage of rational traders thus reducing pricing efficiency. Later, Barberis, Shleifer and Vishny (1998)(hereafter BSV98) propose a type of behavioral sentiment that sentiment traders consider that dividends follow a Markov regime switching between conservative bias and representative bias rather than a random walk thus leading to under- and over-reaction anomies in the long term data (such as monthly and annual data). Recently, empirical studies in both academic (such as Klößner, Becker and Friedmann (2012)) and practice (such as Chekhlov (2010)) find evidences of under/over-reaction in intraday data. Furthermore, Beschwitz, Keim and Massa (2013) report that in recent years, traders have increasingly used high-frequency (can be in seconds) new sources information such as "sentiment" indicator derived from news wire articles. These providers of news analytics (for example," RavenPack") report different sentiment indicators indicating whether the article is good or bad news for the company. If traders use these high-frequency information to forecast the price, they may also lead to under/over-reaction in high frequency data. A consequent question is that *can sentiment in BSV98 still explain under/over reaction in intraday data?*

More importantly, both De Long et al. (1990) and BSV98 employ a representative agent framework so they only study the impact of behavioral bias on the price movement rather than market volatility and liquidity. In nowadays, limit order markets with continuous double auction are dominant in the financial markets. Hence, *what is the impact of these behavioral biases on the limit order markets, in particular on the intraday market volatility, and liquidity including the bid-ask spread, order book depth and trading volume?* This is a very important question but hard to answer by traditional models due to the analytical intractability.

This paper aims to answer this two questions by proposing an agent-based model with behavioral sentiment in a limit order market.[1] In this paper, we assume that the log-fundamental price[2] of the risky asset follows a random walk, therefore the correct expectation of its future value is simply its current value. To setup a benchmark, we first define a noise belief that the expected market price deviates randomly from the current log-fundamental value as in De Long et al. (1990), we call the traders with noise belief as *noise traders.* Then compared to noise traders, we assume sentiment traders follow a Bayesian learning scheme, similar to the one in BSV98, thus sentiment traders are also called *BSV traders*. More specifically, BSV traders think that the expected log fundamental value follows a regime switching model with conservative bias and representative bias rather than a random walk. BSV traders believe that there is a continuation regime as well as a mean-reverting regime, and they use past trends to determine the likelihood of which regime they are currently in. BSV traders' sentiment leads to under/over-reaction to past information, that is, when they believe that the continuation (mean-reverting) regime is more likely, they would over-react (under-react) to the last change in the log-fundamental value.

Traders arrive randomly in the market and can either place market orders or limit orders. Following Chiarella, Iori and Perellò (2009), traders employ a Constant Absolute Risk Aversion (CARA) to maximize their utility , so the order sizes are optimal given their submitted prices. Traders cannot short-sell the risky asset or the risk-free bond. The no-short-sell constraint puts an upper bound on the submission price at which a trader would sell all her current holdings of the risky asset, and also a lower bound at which a trader would use all her cash to purchase shares of the risky asset. Different from Chiarella et al. (2009) who assume the submission price is randomly chosen between the upper and lower bounds, we assume the submission price is either equal to the upper bound or the lower bound, and the probability of buy/sell depends on the distance between the upper/lower bound and the no-trade price. The no-trade price is the price at which it is optimal not to trade at all.

[1]There are agent-based models which examine BSV effect, such as Zhang, Zhang, Xiong and Jin (2006) and Zhang and Zhang (2007), however under a market-maker trading mechanism rather than a limit order book. Chen, Chang and Du (2012) highlight that agent-based models need to employ the limit order book as a realistic trading mechanism to study richer market dynamics in intraday data.

[2]The fundamental value of the risky asset can be interpreted as the present value of its future cashflows.

Moreover, in a related paper of Chiarella, He, Shi and Wei (2014), this model reproduces a number of important stylized facts in limit order markets including fat tail and absence of autocorrelation in returns, volatility clustering, long memory in absolute returns, the bid-ask spread and the trading volume, the hump shape in the mean depth profile closer the best quotes of the order book, an increasing and non-linear relationship between trade imbalance and mid-price return, and also the diagonal effect (event clustering) in submitted order types. In this paper, we focus on the under- and over-reaction effect and the impact of behavioral sentiment on market volatility and liquidity rather than stylized facts.

Our main finding is that BSV belief can have very different impact on the limit order market compare to noise belief. First, BSV belief gives rise to profitable strategies that are based on under/over reaction. In particular, returns after positive mid-price changes are on average higher than those after negative mid-price changes (which is consistent with under-reaction). Moreover, returns after a sequence of consecutive positive mid-price changes are on average lower than those after a sequence of negative mid-price changes (which is consistent with over-reaction). Noise belief does not give rise to such anomalies. Second, we find that under BSV belief, condition on over-reaction periods, trading volume and volatility are significantly higher and order book depth significantly lower than the unconditional periods; whereas under noise belief, there are no significant differences. Last, in general, BSV belief leads to significantly lower volatility, smaller bid-ask spread, larger order book depth near the best quotes and lower trading volume compare to noise belief. Intuitively, given that the true process of the log-fundamental price is a random walk, it is more likely for the BSV traders to under-react than to over-react (because there is less chance to observe a sequence of consecutive increases/decreases in the log-fundamental price), which results in lower volatility and smaller trading volume.

The rest of paper is organized as follows. The agent-based model is outlined in Section 2. Section 3 examine the under/over-reaction effect and the impact of behavioral sentiment on market volatility and liquidity using simulation analysis. Section 4 concludes.

2. THE MODEL

As we mentioned before, the model has been introduced by a related paper of Chiarella et al. (2014) to study the stylized facts in limit order markets. Here we briefly illustrate key setups of the model.

We consider a limit order market with traders who arrive the market and submit orders with different time horizon. Traders do not monitor the market continuously. More explicitly, we assume that the fundamental value F_t of the risky asset is a random walk following a Geometric Brominism Motion with no drift (no dividend), so F_t is given by

$$\ln(F_{t+1}) = \ln(F_t) + \sigma\epsilon_{t+1}, \qquad \epsilon_{t+1} \overset{i.i.d}{\sim} \mathcal{N}(0,1), \tag{2.1}$$

which means that the log fundamental value is a martingale with $\mathbb{E}_t[\ln(F_{t+\tau})] = \ln(F_t)$ for $\tau \geq 1$, where the volatility per period is measured by σ. Trader i with an investment time horizon τ^i enter the market following a Poisson process with the mean of $1/\tau^i$. When agent i enters the market at time t, she knows the fundamental value of the current period v_t, together with historical fundamental values every τ^i periods, so that her information about the fundamental values is given by $I_t^i \equiv \{F_t, F_{t-\tau^i}, \cdots, F_{t-N^i\tau^i}\}$, where N^i measures the length of her observations. We also assume that the risk free cash has no interest in intraday time.

2.1. **Traders' belief.**

Noise belief. To setup a benchmark, we first consider the noise belief as in De Long et al. (1990), that is

$$\mathbb{E}_t^i[\ln(F_{t+\tau^i})] = \ln(F_t) + \tilde{\theta}_t^i \qquad \text{and} \qquad \mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i + (\theta^i)^2 - (\tilde{\theta}_t^i)^2, \tag{2.2}$$

where $\tilde{\theta}_t^i \overset{i.i.d}{\sim}$ Uniform$[-\theta^i, \theta^i]$. This definition of "noisy belief" is similar to De Long et al. (1990) but different from some heterogenous agent models such as Chiarella et al. (2009). We call these traders *noise* traders. When $\theta^i = 0$, noise traders' belief becomes

$$\mathbb{E}_t^i[\ln(F_{t+\tau^i})] = \ln(F_t), \qquad \text{and} \qquad \mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i. \tag{2.3}$$

Sentiment belief. Then we turn to sentiment belief. Following the sprit in Barberis et al. (1998) (hereafter BSV98), we assume that sentiment traders believe that the expected log fundamental price $\ln(F_{t+\tau^i})$ follows

$$\ln(F_{t+\tau^i}) = \ln(F_t) + \theta_{t+\tau^i} + \sigma\epsilon_{t+\tau^i}, \tag{2.4}$$

where $\epsilon_{t+\tau^i} \overset{i.i.d}{\sim} \mathcal{N}(0, \sqrt{\tau^i})$ and the mean growth rate $\theta_{t+\tau^i}$ follows a two-state Markov chain with transition matrix

	$\theta_{t+\tau^i} = \theta^i$	$\theta_{t+\tau^i} = -\theta^i$
$\theta_t = \theta^i$	$\pi_{t+\tau^i}$	$1 - \pi_{t+\tau^i}$
$\theta_t = -\theta^i$	$1 - \pi_{t+\tau^i}$	$\pi_{t+\tau^i}$.

(2.5)

Therefore, trader i believes that there is a good (bad) state in which the mean growth rate of the fundamental price is positive (negative). Given the current state, the probability of staying in the same state is given by $\pi_{t+\tau^i}$. When θ^i is different from zero, the agent exhibits sentiment in the same spirit as BSV98, believing in an underlying structure for the mean growth rate, which does not exist. Furthermore, as in the BSV98 model, agent i believes that the transition probability $\pi_{t+\tau^i}$ also follows a Markov chain with transition matrix,

	$\pi_{t+\tau^i} = \pi_L$	$\pi_{t+\tau^i} = \pi_H$
$\pi_t = \pi_L$	$1 - \lambda_1$	λ_1
$\pi_t = \pi_H$	λ_2	$1 - \lambda_2$.

(2.6)

meaning that, agents believe there is one state $\pi_t = \pi_L$ in which the mean growth rate is more likely to remain the same as the last period and a state ($\pi_t = \pi_H$) in which the mean growth rate is more likely to switch from one state to another, in which λ_1 and λ_2 measure the switching intensities. This Markov regime switching model is motivated by two important psychological biases: conservative bias and representative heuristic bias (see BSV98).

Traders do not observe the mean growth rate θ_t and they update their probability beliefs about θ_t and π_t based on a Bayesian learning scheme(see the learning process in the Appendix). Given a sentiment trader's estimated probabilities $q^i_{\pi,t}$ and $q^i_{\theta,t}$, trader i makes a τ^i-period ahead forecast of the log fundamental price

$$\mathbb{E}_t^i[\ln(F_{t+\tau^i})] = \ln(F_t) + \mathbb{E}_t^i[\theta_{t+\tau^i}], \tag{2.7}$$

The details of the deviation of Equation 2.7 are also documented in the Appendix. The variance of log fundamental price perceived by sentiment trader i is given by

$$\mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i + (\theta^i)^2 - (\mathbb{E}_t^i[\theta_{t+\tau^i}])^2. \tag{2.8}$$

We call these traders with behavioral sentiment *BSV* traders. Note that without sentiment ($\theta^i = 0$), BSV traders' belief also becomes the one in Equation (2.3). Therefore, sentiment is the key ingredient in generating heterogeneity in beliefs across BSV traders with different investment horizons (see the example in the Appendix), and the key difference between noise traders and BSV traders is that BSV traders have a learning scheme under bias in belief while noise traders do not.

2.2. **Traders' optimal demand and order submission.** Following Chiarella et al. (2009), we assume that traders maximize a CARA utility function to optimize their demandin equation (2.9):

$$z_t^{i*} = \frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^i)}{\alpha^i p_t^i \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} - s_t^i. \tag{2.9}$$

where s_t^i is the number of shares of the risky asset and c_t^i is the amount of cash agent i holds at time t. p_t^i is the order price and z_t^i is the order size (quantity) submitted by a noise/BSV trader i at time t.

We assume that trader i uses her belief about the fundamental value to estimate the mean and variance of the future market price,

$$\mathbb{E}_t^i[\ln(p_{t+\tau^i})] = \mathbb{E}_t^i[\ln(F_{t+\tau^i})], \qquad \mathbb{V}_t^i[\ln(p_{t+\tau^i})] = \mathbb{V}_t^i[\ln(F_{t+\tau^i})] \tag{2.10}$$

Now to determine the submission price p_t^i for agent i, we assume agents cannot short sell and nor can they borrow at the risk-free rate, this implies that

$$z_t^{i*} \geq -s_t^i \qquad \text{and} \qquad z_t^{i*} p_t^i \leq c_t^i,$$

from which we obtain the following bounds for the submission price p_t^i of agent i,

$$p_t^{i,m} \leq p_t^i \leq p_t^{i,M},$$

where $p_t^{i,M} = \exp\{\mathbb{E}_t^i[\ln(p_{t+\tau^i})]\}$ and $p_t^{i,m}$ is determined implicitly by

$$\frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^{i,m})}{\alpha^i \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} = c_t^i + s_t^i p_t^{i,m}.$$

Define p_t^{i*} as the *no trade price* for agent i, which solves

$$\frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^{i*})}{\alpha^i p_t^{i*} \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} = s_t^i.$$

We assume agent i trades in the following way. She tries to either sell s_t^i shares of the risky asset at a maximum price of $p_t^{i,M}$ or buy $c_t^i/p_t^{i,m}$ shares of the risky assets at a minimum price of $p_t^{i,m}$. If the best ask $a_t^1 < p_t^{i,m}$ or the best bid $b_t^1 > p_t^{i,M}$, then agent i submits a market buy or a market sell order, otherwise she submits a limit buy or limit sell order. Note that this way of determining the submission price is different from Chiarella et al. (2009), where agents randomly pick a price $p_t^i \in [p_t^{i,m}, p_t^{i,M}]$.

Furthermore, we assume the probability of submitting a buy or sell order is given by

$$P_{buy} \equiv \mathbb{P}(z_t^{i*} = c_t^i/p_t^{i,m}) = \frac{p_t^{i*} - p_t^{i,m}}{p_t^{i,M} - p_t^{i,m}}, \qquad P_{sell} \equiv \mathbb{P}(z_t^{i*} = -s_t^i) = \frac{p_t^{i,M} - p_t^{i*}}{p_t^{i,M} - p_t^{i,m}}. \tag{2.11}$$

Intuitively, the further the no-trading price is away from the minimum price, the higher the probability to buy; the further the no-trading price is away from the maximum price, the higher the probability to sell.

Lastly, we assume agent i's expected return and variance or return over her investment horizon are based on the expected value and variance of the log fundamental price and the submitted price, that is

$$\mathbb{E}_t^i[r_{t+\tau^i}] = \mathbb{E}_t^i[\ln(F_{t+\tau^i})] - \ln(p_t^i) \qquad \text{and} \qquad \mathbb{V}_t^i[r_{t+\tau^i}] = \mathbb{V}_t^i[\ln(F_{t+\tau^i})]. \tag{2.12}$$

Upon entering the market, agent i chooses to either place a market order or a limit order which will be stored in the limit order book. A transaction occurs when a market order hits a quote on the opposite side of the order book. Limit orders are executed using both price and time priorities. At time t, agent i submits a buy or sell order with price level p_t^i and order size z_t^i (z_t^i is the optimal order size based on p_t^i). The order leads to a trade when she submits a buy order and $p_t^i \geq a_t^1$ or when she submits a sell order and $p_t^i \leq b_t^1$, where b_t^1 and a_t^1 are the best bid and ask price respectively. If there is enough depth at the best bid or best ask, then the entire order agent i submits is executed at a_t^1 or b_t^1, otherwise part of the order may be executed at prices further away from the best bid or ask or it may become a limit order with price p_t^i as the new best bid or ask price. Furthermore, there can be multiple agents who arrive at the market at the same time, in which case we assume those agents trade in a randomized order.

	buy/sell		Limit/Market	Volume
$X \leq P_{buy}$	buy	$a_t^1 \leq p_t^{i,m}$	Market order	$c_t^i/p_t^{i,m}$
$X \leq P_{buy}$	buy	$a_t^1 > p_t^{i,m}$	Limit order	$c_t^i/p_t^{i,m}$
$X > P_{buy}$	sell	$b_t^1 \geq p_t^{i,M}$	Market order	s_t^i
$X > P_{buy}$	sell	$b_t^1 < p_t^{i,M}$	Limit order	s_t^i

TABLE 2.1. Summary of submission rules of agent i, $0 \leq X \leq 1$ is drawn from a uniform distribution.

Table 2.1 summarizes the order submission rules of agent i in which X is drawn from a uniform distribution on $[0, 1]$. Note that agent i's submission price is either $p_t^{i,m}$ (for buy orders) or $p_t^{i,M}$ (for sell orders). If the depth at the best bid (ask) is not enough to fully satisfy the order size, the remaining volume of the order is executed against limit orders in the book. The agent thus takes the next best buy (sell) order and repeats this operation as many times as necessary until the order is fully executed. This mechanism applies under the condition that quotes of these orders are above (below) price $p_t^{i,M}$ ($p_t^{i,m}$). If the limit order is still unmatched by the time $t + \tau^i$ it is removed from the book.

2.3. **Simulation design and setting.** We use agent-based computational simulations to solve the model. In the simulations we assume agents' investment horizons τ^i follows a uniform distribution between $\tau(1 - \Delta)$ and $\tau(1 + \Delta)$ where the reference investment horizon $\tau = 60$ (approximate one hour) and the range is specified by $\Delta = 0.5$. Furthermore we restrict the investment horizons to be integers. Agents are initially given $s_0^i = 10$ shares of the risky asset and $c_0^i = s_0^i F_0$ amount of cash, where the initial fundamental price $F_0 = \$50$. At the beginning of each period t, each agent i has a probability $1/\tau^i$ of entering the market. Agents observe the fundamental value F_t after they enter the market before submitting an order. Upon entering the market, agent i cancels any unmatched limit order and submits a new order according the order submission rules in Table 2.1. The volatility of the log fundamental price per period is set to $\sigma = 4$ basis points (bp)[3] and risk aversion is set to $\alpha^i = 0.1$ for all agents following Chiarella et al. (2009). For the BSV traders with behavioral sentiment, we assume $\pi_L = \frac{1}{3}$, $\pi_H = \frac{3}{4}$, $\lambda_1 = 0.1$, $\lambda_2 = 0.3$ following BSV98 and $\theta^i = \sigma\sqrt{\tau^i}$. Upon entering the market, agent i estimates the probabilities $q_t^{i,\pi}$ and $q_t^{i,\theta}$ based on her information $I_t^i = \{F_t, F_{t-\tau^i}, \cdots, F_{t-N^i\tau^i}\}$ with initial priors $q_{t-N^i\tau^i}^{i,\pi} = q_{t-N^i\tau^i}^{i,\theta} = 0.5$. We set $N^i = 60$ for every agent and the number of the agents to 1000.[4] The minimum tick size by which prices can differ is given by $\$0.01$. We assume that the "real" fundamental price process is a geometric random walk.

Apart from noise and BSV traders, we assume there are also liquidity traders. Liquidity traders' investment horizons and arrival rates follow the same uniform distribution as noise/BSV traders. They choose randomly between buy and sell orders with equal probability, after which they also choose randomly between market and limit orders with equal probability. The order size is randomly distributed between 1 and 10. Moreover, their limit orders are always at the best bid or ask price. Therefore, the liquidity traders do not set prices on the order book. They either provide or demand liquidity with equal probability. Given the total number of agents in the market, we assume 90% of them are noise/BSV traders and 10% of them are liquidity traders.

We design two simulation cases. The first one is the "Noisy" case which there are 900 noise traders and 100 liquidity traders, we use this case as a benchmark. Then we consider the other case, "BSV" case which there are 900 BSV traders and 100 liquidity traders.

The results reported are the outcome of 30 simulations of 72,000 periods with the first 60,000 steps used as a burn-in period.[5]

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[3]If each trading period is treated as one minute, then the annualized volatility is approximately 10% p.a.

[4]We did some robustness tests with changing N^i to 90 or 60, and the total number of agents to 2000 or 500, the results do not change significantly.

[5]The results remain similar among different simulations.

3. THE UNDER/OVER-REACTION EFFECT AND THE IMPACT ON VOLATILITY AND LIQUIDITY

In this section, we use the simulation results to examine under/over reaction effect and the impact of behavioral sentiment on market volatility and liquidity. We first examine whether B-SV sentiment can generate over-reaction and under-reaction in intraday data. Then we examine the impact on intraday market volatility, liquidity including the bid-ask spread, the order book depth near the best quotes and the trading volume.

3.1. **Under- and Over-Reaction.** According to BSV98, under-reaction is defined as

$$\mathbb{E}[r_{t+1}|z_t = G] > \mathbb{E}[r_{t+1}|z_t = B], \tag{3.1}$$

that is asset return following a piece of good news is on average higher than that following a piece of bad new. Of course, according to the *efficient market hypothesis* (EMH), any good or bad news should already be incorporated into the current price p_t, and should not affect future returns of the asset. Therefore, under-reaction in (3.1) provides counter-evidence against the EMH.

In the limit order market, we interpret an increase (decrease) in the mid-price $\bar{p}_t = \frac{1}{2}(a_t^1 + b_t^1)$ as good (bad) news. Intuitively, an increase (decrease) in the mid-price suggests that investors in the market are revising their expectation of the fundamental value of the risky asset upward (downward). In order to test for under-reaction specified in (3.1), we design the following *under-reaction* trading strategy. Suppose the changes in the mid-price, $\Delta\bar{p}_{t+1} = \bar{p}_{t+1} - \bar{p}_t$, act as trading signals, the signal is (+) when $\Delta\bar{p}_{t+1} > 0$, and (-) when $\Delta\bar{p}_{t+1} < 0$. Initially, if a trader observes a (+) ((-)) signal, she buys (short-sells) one share of the risky asset. Then, as soon as trading signal switches sign, she closes off the initial position. She repeats the strategy starting from the next period. Figure 3.1 provides a graphic illustration of the under-reaction strategy.

The profitability of the strategy is compared to a buy-and-hold strategy in Table 3.1. Panel A assumes that transactions occur at the mid-price, thus ignores the bid-ask spread. Results in Panel A shows that when the market is populated with BSV traders, the *under-reaction* trading

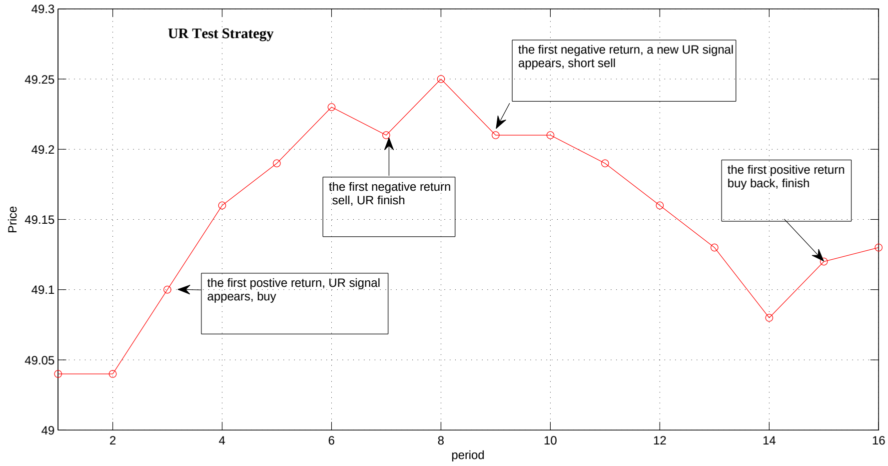

FIGURE 3.1. Graphic illustration of the under-reaction strategy.

strategy is significantly more profitable than the buy-and-hold (B&H) strategy on average, and also outperforms the B&H strategy 100% of the time. In contrast, when the market is populated with noise traders, the under-reaction strategy always under-performs the B&H strategy (win rate is 0%). Moreover, in this the under-reaction strategy on average makes a significant loss rather than a profit, even ignoring any transaction costs in terms of the bid-ask spread. After taking the bid-ask spread into account, Panel B shows that in the market populated by BSV traders, the under-reaction strategy remains more profitable than the B&H strategy thought its profit is halved. However, in the market populated by noise traders, the under-reaction strategy delivers a huge loss. These results confirm that the model generates prices (both trade and mid-prices) that are consistent with the statistical evidence of under-reaction, even after taking into account the realistic features of a limit order market where investors trade by submitting limit and market orders.

Next, we examine whether over-reaction is present in the market. According to BSV98, over-reaction is defined as

$$\mathbb{E}[r_{t+1}|z_t = G, z_{t-1} = G, \cdots, z_{t-j} = G] < \mathbb{E}[r_{t+1}|z_t = B, z_{t-1} = B, \cdots, z_{t-j} = B], \quad (3.2)$$

where j is at least one and probably rather higher. Equation (3.2) suggests that expected return is lower following a sequence of good news than following a sequence of bad news. Again, if

Panel A			
	UR	B&H	win rate
BSV	27.49	-0.50	100%
Noise	-53.34	-0.49	0%
Panel B			
	UR	B&H	win rate
BSV	13.20	-0.52	90%
Noise	-266.45	-0.53	0%

TABLE 3.1. Comparison between the profit/loss of the *under-reaction* (UR) s-trategy and that of the buy-and-hold (B& H) strategy, the win rate is number of simulations where the under-reaction strategy outperformed the B&H strategy as a percentage of the total number of simulations. Panel A assumes that all trans-actions occur at the mid-price, where as Panel B assumes that buy (sell) orders are executed at the best ask (bid).

one treats an increase (decrease) in the mid-price as a (+) ((-)) trading signal, we can construct the following *over-reaction* (OR) trading strategy to exploit any over-reaction present in the market. Initially, after a trader observes j ($j \geq 3$) (-) ((+)) trading signals, she buys (short-sells) one share of the risky asset when the trading signal switches sign. Then, she closes the position after another j trading periods. She repeats the strategy starting from the next period.[6] Figure 3.2 provides a graphic illustration of the over-reaction strategy.

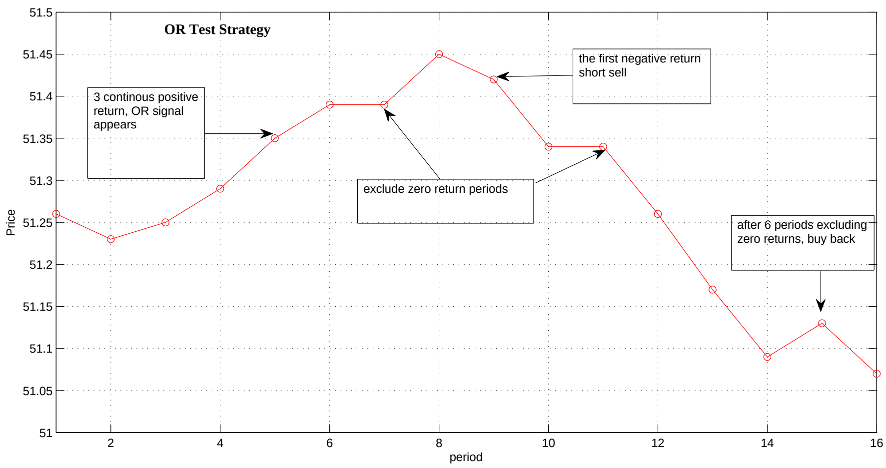

FIGURE 3.2. Graphic illustration of the over-reaction strategy.

[6]We test $j \geq 1, 2, 3, ..., 9$, we find that when $j < 3$, the OR strategy is not profitable, and when $j \geq 3$, the OR strategy is profitable but the opportunities of OR strategies decrease when j increases. Thus we report the result of $j \geq 3$.

Results in Table 3.1 Panel A shows that the over-reaction strategy delivers a profit and is more profitable than the B&H strategy if one ignores any transaction costs arising from the bid-ask spread. Panel B shows that after taking into account the bid-ask spread, the over-reaction strategy is no longer profitable.

Panel A			
	OR	B&H	win rate
BSV	1.73	-0.49	76.7%
Noise	-7.34	-0.49	0%
Panel B			
	OR	B&H	win rate
BSV	-4.53	-0.52	23.3%
Noise	-56.05	-0.53	0%

TABLE 3.2. Comparison between the profit/loss of the *over-reaction* (OR) s-trategy and that of the buy-and-hold (B& H) strategy, the win rate is number of simulations where the under-reaction strategy outperformed the B&H strategy as a percentage of the total number of simulations. Panel A assumes that all transactions occur at the mid-price, where as Panel B assumes that buy (sell) orders are executed at the best ask (bid).

In summary, in a market populated by BSV traders, there is evidence of both under and over-reaction. However, the under-reaction trading strategy is much more profitable compare to the the over-reaction strategy. Intuitively, by the definition according to (3.1) and (3.2), BSV traders under-react to news more often rather than over-react. Therefore, trading on under-reaction would be more profitable.

3.2. **Volatility, Spread, Volume and Order book depth.** We define *volatility* as the sample standard deviation of log-return of the mid-price, that is $\ln(\bar{p}_{t+1}) - \ln(\bar{p}_t)$ per trading period. Moreover, the *bid-ask spread* is measured by the on average number of tick sizes between the best bid price and best ask price, that is $a_t^1 - b_t^1$, per period. Furthermore, *volume* is measured by the average number of shares being transacted per period. We also compute the *average order book depth* near the best quotes. Average order book depth of the best 5 quotes on the ask (bid) side is denoted by *Da5* (*Db5*).

Results in Table 3.2 show that volatility, spread, volume are all significantly smaller in a market populated by BSV trader than in a market populated by noise traders. The intuition is that

	volatilty	spread	volume	Da5	Db5
BSV	1.84[0.23]	2.11 [0.73]	34.38[10.21]	78.35[16.96]	122.36[43.31]
BSV-OR	3.01[0.55]*	2.24[0.64]	56.59[14.94]*	75.33[16.06]*	107.00[28.03]*
Noise	5.33[0.46]	4.19[0.25]	94.83[7.49]	72.00[9.00]	87.25[15.00]
Noise-OR	5.35[0.42]	4.13[0.24]	95.91[7.02]	72.72[8.43]	89.54[9.44]

TABLE 3.3. Volatility, spread, volume and order book depth. Da5 measures the order book depth of the 5 best quotes on the ask side, similarly Db5 measures the order book depth of the 5 best quotes on the bid side. BSV-OR (Noise-OR) corresponds to the over-reaction periods in a market populated by BSV (noise) traders. * indicates that a indicator in BSV(Noise)-OR is significantly different from the corresponding indicator in BSV(Noise) at 1% level. The value in the square brackets is the variance of 30 simulations.

although BSV trader can under-react as well as over-react to news, however they under-react most often. Therefore, BSV traders submit less aggressive orders compare to noise traders, given the same movement in the fundamental value. Results also show that order book depth is larger in a market populated by BSV trader than in market populated by noise traders. Intuition here is that noise traders' expectations deviates randomly from the currently observed log-fundamental value $\ln F_t$ whereas BSV traders' expectations are more centred around $\ln F_t$ because they follow the same learning scheme. Therefore, noise traders are more likely to place limit order further away from the best quotes.

In Table 3.2, we also compute the volatility, spread, volume and order book depth conditioned on the occurrence of over-reaction in the market. Over-reaction is identified when there had been j ($j \geq 3$) consecutive increases (decreases) in the mid-price $\bar{p}_t$, which provides j consecutive (+) ((-)) trading signals. Suppose the first of the j consecutive (+)/(-) trading signals was observed in the trading period $[t, t+1]$, then the over-reaction period is given by $[t, t+2j+1]$ corresponding to the trading horizon of the over-reaction trading strategy. In Table 3.2, BSV-OR (Noise-OR) correspond to the over-reaction periods in a market populated by BSV (noise) traders. Results indicate that in a market populated by BSV traders, volatility, volume increase and order book depth reduces significantly during the over-reaction periods. In contrast, there are no significant differences in any of the observable quantities in a market populated by noise traders. The intuition is clear: only the BSV trader actually over-react during the over-reaction period, during which they trade more aggressively, leading to higher volatility and trading volume, and reduced order book depth. The noise traders, on the other hand, do not over-react, therefore their trading behavior is not affected by the over-reaction periods. Interestingly, the bid-ask spread does not show any significant increases in the over-reaction period even in a market populated by BSV traders, which suggests that even when they over-react, BSV traders do not trade aggressive enough to widen the spread.

4. Conclusion

In this paper, we propose an agent-based model to examine the impact of behavioural sentiment on market volatility and liquidity in limit order markets. Compare to noise traders who do not engage in learning at all and their beliefs deviate randomly from fundamental price, sentiment traders with a Bayesian learning scheme may under-react or over-react to past changes in the fundamental price, thus traders with different investment horizons may have different expectations about the future fundamental price. In an artificial limit order market, traders are allowed to submit market or limit orders, submission price and order size are both determined by CARA utility maximization.

Simulations show that sentiment belief and noise belief have very different impact in a limit order market. Firstly, under-reaction and over-reaction trading strategies are profitable in a market populated by sentiment traders, but not profitable in market populated by noise traders. Moreover, conditional on over-reaction (signal by consecutive positive/negative mid-price returns), sentiment belief leads to significant increases in trading volume and volatility, and reduction in order book depth whereas noise belief does not. Overall, we find that sentiment belief leads to lower volatility, smaller bid-ask spread and larger order book depth near the best quotes and lower trading volume when compared to noise belief.

APPENDIX: THE LEARNING PROCESS OF BSV TRADERS

We assume that BSV traders do not observe the mean growth rate θ_t so they update their probability beliefs about θ_t and π_t based on Bayesian learning process. Let $q^i_{\theta,t} \equiv \mathbb{P}(\theta_t = \theta^i | I^i_t)$ and $q^i_{\pi,t} \equiv \mathbb{P}(\pi_t = \pi_L | I^i_t)$, where $I^i_t \equiv \{F_t, F_{t-\tau^i}, \cdots, F_{t-N\tau^i}\}$. Define $R_{t+\tau^i} \equiv \ln(F_{t+\tau^i}/F_t)$, trader i updates her probabilities after observing $R_{t+\tau^i}$ as follows,

$$
\begin{aligned}
q^i_{\theta,t+\tau^i} &= q^i_{\pi,t}\mathbb{P}(\theta_{t+\tau^i} = \theta^i | \pi_t = \pi_L, R_{t+\tau^i}) + (1 - q^i_{\pi,t})\mathbb{P}(\theta_{t+\tau^i} = \theta^i | \pi_t = \pi_H, R_{t+\tau^i});\\
q^i_{\pi,t+\tau^i} &= q^i_{\theta,t}\mathbb{P}(\pi_{t+\tau^i} = \pi_L | \theta_t = \theta^i, R_{t+\tau^i}) + (1 - q^i_{\theta,t})\mathbb{P}(\pi_{t+\tau^i} = \pi_L | \theta_t = -\theta^i, R_{t+\tau^i}),
\end{aligned}
\tag{A.1}
$$

where

$$
\begin{aligned}
\mathbb{P}(\theta_{t+\tau^i} = \theta^i | \pi_t, R_{t+\tau^i}) &= \frac{\mathbb{P}(R_{t+\tau^i} | \theta_{t+\tau^i} = \theta^i)\mathbb{P}(\theta_{t+\tau^i} = \theta^i | \pi_t)}{\sum_{\theta_{t+\tau^i} \in \{\theta^i, -\theta^i\}} \mathbb{P}(R_{t+\tau^i} | \theta_{t+\tau^i})\mathbb{P}(\theta_{t+\tau^i} = \theta | \pi_t)};\\
\mathbb{P}(\pi_{t+\tau^i} = \pi_L | \theta_t, R_{t+\tau^i}) &= \frac{\mathbb{P}(\pi_{t+\tau^i} = \pi_L)\mathbb{P}(R_{t+\tau^i} | \theta_t, \pi_{t+\tau^i} = \pi_L)}{\sum_{\pi_{t+\tau^i} \in \{\pi_L, \pi_H\}} \mathbb{P}(\pi_{t+\tau^i})\mathbb{P}(R_{t+\tau^i} | \theta_t, \pi_{t+\tau^i})}
\end{aligned}
$$

for $\theta_{t+\tau^i} \in \{-\theta^i, \theta^i\}$ and $\pi_{t+\tau^i} \in \{\pi_L, \pi_H\}$ and

$$
\begin{aligned}
&\mathbb{P}(R_{t+\tau^i} | \theta_{t+\tau^i}) \propto \exp\left(- \frac{(R_{t+\tau^i} - \theta_{t+\tau^i})^2}{\sigma^2 \tau^i} \right);\\
&\mathbb{P}(\theta_{t+\tau^i} = \theta^i | \pi_t) = q^i_{\theta,t}\pi_t + (1 - q^i_{\theta,t})(1 - \pi_t);\\
&\mathbb{P}(\theta_{t+\tau^i} = -\theta^i | \pi_t) = q^i_{\theta,t}(1 - \pi_t) + (1 - q^i_{\theta,t})\pi_t;\\
&\mathbb{P}(\pi_{t+\tau^i} = \pi_L) = q^i_{\pi,t}(1 - \lambda_1) + (1 - q^i_{\pi,t})\lambda_2;\\
&\mathbb{P}(\pi_{t+\tau^i} = \pi_H) = q^i_{\pi,t}\lambda_1 + (1 - q^i_{\pi,t})(1 - \lambda_2);\\
&\mathbb{P}(R_{t+\tau^i} | \theta_t, \pi_{t+\tau^i}) \propto \pi_{t+\tau^i} \exp\left(- \frac{(R_{t+\tau^i} - \theta_t)^2}{\sigma^2 \tau^i} \right)\\
&\quad + (1 - \pi_{t+\tau^i}) \exp\left(- \frac{(R_{t+\tau^i} + \theta_t)^2}{\sigma^2 \tau^i} \right).
\end{aligned}
$$

for $\theta_t \in \{-\theta^i, \theta^i\}$ and $\pi_t \in \{\pi_L, \pi_H\}$.

Given her estimated probabilities $q^i_{\pi,t}$ and $q^i_{\theta,t}$, trader i makes a τ^i-period ahead forecast of the log fundamental price as the one in Equation 2.7:

$$\mathbb{E}^i_t[\ln(F_{t+\tau^i})] = \ln(F_t) + \mathbb{E}^i_t[\theta_{t+\tau^i}],$$

where

$$\begin{aligned}\mathbb{E}^i_t[\theta_{t+\tau^i}] = \mathbb{P}(\pi_{t+\tau^i} = \pi_L|I^i_t) \begin{pmatrix} q^i_{\theta,t} & 1-q^i_{\theta,t} \end{pmatrix} \begin{pmatrix} \pi_L & 1-\pi_L \\ 1-\pi_L & \pi_L \end{pmatrix} \begin{pmatrix} \theta^i \\ -\theta^i \end{pmatrix} \\ + \mathbb{P}(\pi_{t+\tau^i} = \pi_H|I^i_t) \begin{pmatrix} q^i_{\theta,t} & 1-q^i_{\theta,t} \end{pmatrix} \begin{pmatrix} \pi_H & 1-\pi_H \\ 1-\pi_H & \pi_H \end{pmatrix} \begin{pmatrix} \theta^i \\ -\theta^i \end{pmatrix}.\end{aligned}$$

FIGURE A.1. Sample paths of the fundamental value v_t and the market price p_t simulated for 900 periods. The blue solid line is the fundamental value and the red dotted line is the market price.

To illustrate the effect of the time horizon and sentiment, we consider two BSV traders A and B with the same length of observations $N^A = N^B = 10$ (for example), but agent A has an investment horizon of $\tau_A = 30$ and agent B has investment horizon of $\tau_B = 90$. Figure A.1 shows sample paths of the fundamental value v_t and market price p_t for 900 periods. Table A.1 shows the past changes in the fundamental price that BSV trader A and B would use to update

	1	2	3	4	5	6	7	8	9	10
$\tau^A = 30$	-0.54%	1.79%	-0.82%	-1.19%	-1.4%	-2.67%	2.39%	-0.77%	-0.91%	0.23%
$\tau^B = 90$	1.74%	-0.48%	3.18%	-1.79%	-0.15%	2.05%	4.02%	-0.22%	-1.68%	-1.46%

TABLE A.1. Past changes in the fundamental price used by agent A with $\tau^A = 30$ and by agent B with $\tau^B = 90$.

$q_t^{i,\pi}$	1	2	3	4	5	6	7	8	9	10
$\tau^A = 30$	0.60	0.84	0.91	0.71	0.54	0.43	0.77	0.89	0.70	**0.84**
$\tau^B = 90$	0.60	0.81	0.88	0.93	0.81	0.78	0.60	0.75	0.66	**0.51**
$q_t^{i,\theta}$	1	2	3	4	5	6	7	8	9	10
$\tau^A = 30$	0.01	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	**0.88**
$\tau^B = 90$	1.00	0.10	1.00	0.00	0.45	1.00	1.00	0.28	0.00	**0.00**
$\mathbb{E}_t^i[\theta_{t+\tau^i}]$	1	2	3	4	5	6	7	8	9	10
$\tau^A = 30$	-0.01%	0.04%	-0.05%	-0.02%	0.00%	0.01%	0.03%	-0.04%	-0.02%	**0.03%**
$\tau^B = 90$	0.02%	-0.05%	0.07%	-0.08%	-0.01%	0.05%	0.02%	-0.02%	-0.03%	**0.00%**

TABLE A.2. Estimated probabilities ($q_t^{i,\pi}, q_t^{i,\theta}$) and expected growth of log fundamental price $\mathbb{E}_t^i[\theta_{t+\tau^i}]$ by BSV trader A with $\tau^A = 30$ and by BSV trader B with $\tau^B = 90$.

their probabilities ($q_t^{i,\pi}, q_t^{i,\theta}$, $i = A, B$), which show that the two agents use very different data for updating probabilities though the sample path of the fundamental price remains the same.

Table A.2 shows the probabilities ($q_t^{i,\pi}, q_t^{i,\theta}$) of BSV traders $i = A, B$ estimated on their expected growth rate of the future fundamental price, that is $\mathbb{E}_t^i[\ln(F_{t+\tau^i}/v_t)] = \mathbb{E}_t^i[\theta_{t+\tau^i}]$, $i = A, B$. The probabilities and expectations are very different due to the fact that agents use different past information.

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