Interest Rate Rules and Inflation Risks in a Macro-Finance Model

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Abstract

Long-term bond yields contain risk-premium as a compensation for consumption and inflation risks. The ability of a DSGE model in generating inflation-risks relies heavily on the specification of the Taylor rule. A simple rule containing contemporaneous inflation implies higher inflation risks than a rule which uses an average of current and past inflation rates. With interest-rate smoothing households enjoy the gradual response of monetary authority to the shocks which they can insure against through labour supply. The definition of the output gap also matters a lot. Inflation risks are lower when output gap is defined as the deviation of sticky-price output from its flexible price version relative to the case when it is defined relative to steady-state output. We found that a time-varying inflation target with learning contributes little to inflation risks.

Keywords: zero-coupon bond, nominal term premium, inflation risk, Taylor rule, New Keynesian

JEL: E13, E31, E43, E44, E62

1 Introduction

The inflation risk premium content in the yields of long-term nominal government bonds (Treasuries in the US or gilts in the UK) can be defined as the difference between the nominal and the real term premium which are the compensation, respectively, for the nominal and real risks that the bond holder has to bear. In other words bond holders expect long-term bonds to pay an extra return called term premium for future uncertainty about consumption and inflation. Hördahl and Tristani (2012) gives an overview of the papers that attempted to measure inflation risks. The latter papers estimate the inflation risk-premium in the range of 10 to 60 basis points depending on the time-period and the countries included in the sample.

This paper uses a simple New Keynesian DSGE model with only price-rigidity and labour in production to generate inflation risks. We discuss how the specification of the interest-rate rule (so called Taylor rule) affect inflation risks. In particular, the timing assumption on inflation in the interest-rate rule and inclusion of interest-rate smoothing matter a lot from the point of view of inflation risks (i). The definition of the output gap and the size of the coefficient on the output-gap also has sizeable impact on the magnitude of inflation risk premia (ii).

We consider i) first. It can be shown that the less focused the central bank is on current inflation either because it aims to stabilise inflation on the medium-run (see expected inflation in the Taylor rule of some medium-size DSGE models like the GIMF of the IMF, see Carabenciov et al. (2008) or the ToTEM model of the Bank of Canada, see Murchison and Rennison (2006)) or its evaluation of current period inflation is based on a moving average of past inflation (see Rudebusch and Swanson (2012), RS henceforth) the more weight is given to real economy considerations (captured by the output-gap in the Taylor rule) and, thus, the lower is the real term premium on long-term bonds. We also demonstrate that monetary policy inertia—interest-rate smoothing in the Taylor rule—substantially contributes to the mitigation of real and nominal risks by lending households time to react to negative shocks through the adjustment of labour supply which would change less with immediate reaction of monetary policy.

Now we turn to elaborate on ii). Inflation risks are also influenced by the size of the coefficients on inflation and output-gap in the Taylor rule. Empirical estimates of the output-gap coefficient are typically close to either zero or one (see table 1 which is based on the estimates by Clarida et al. (1998, 2000) on US data). The higher is the coefficient on the output-gap, ceteris paribus, the lower are real risks and the more elevated are inflation risks as the central bank cares more about stabilisation of the output gap relative to mitigating deviations of inflation from its target. An output-gap coefficient close to one ensures that the real risks are virtually zero as in the context of the RS model.

Literature claims (see e.g. RS) that a time-varying inflation target with learning of the inflation target helps to increase inflation risks. As an extension of their baseline model RS posit a Taylor rule in which the time-varying inflation target is revised upward (downward) if the current inflation target is higher (lower) than the current inflation target. Hence, agents learn inflation target in an adaptive way. However, we find using various specifications of the Taylor rule that long-run time-varying inflation target with learning contribute little to inflation risks. This finding is also consistent with Hördahl et al. (2008) who made use of a model with time-varying inflation target but without learning of the inflation target. Their paper successfully replicates some macro and finance moments of the real term-structure but does not capture inflation risks.

Our paper is closest related to Rudebusch and Swanson (2012) (RS henceforth) who found using the same model as ours that the nominal term premium can be large and volatile. However, RS do not decompose the nominal term premium into real term premium and inflation risk premium which we accomplish in this work. Our paper is also related to Hsu et al. (2015) who also calculate and find positive inflation risk-premium using a different measure of inflation risk-premia and employing a more detailed model including wage rigidity as well. Moreover, Hsu et al. claim that the source of inflation risks are permanent productivity shocks in a model with wage-setting frictions. By contrast, in this paper and in RS inflation risks emerge due to transitory technology shocks leading to negative correlation between consumption growth and inflation. In our model nominal assets are risky in the sense that low consumption growth—due to negative realisations of technology—is associated with high inflation and, thus, low real returns on the bonds. In other words nominal bonds do not provide insurance to the households against negative supply shocks.

2 The model

2.1 Households

Our model is based on the New Keynesian DSGE model of RS. The description of the households' and firms' problem below follows closely RS.

Table 1: Taylor-rule estimates of Clarida et al. (1998, 2000) for the US

	ρ	ϕ_π	ϕ_y
Rule 1 (Clarida et al. 1998) for 1979-1994	0.92	1.79	0.07
Rule 2 (Clarida et al. 2000) for 1983-1996*	0.79	2.16	0.93

Notes: This table is borrowed from Kaszab and Marsal (2015). Clarida et al. (1998, 2000) estimated the following forward-looking Taylor rule: $i_t = \rho i_{t-1} + (1-\rho)[\phi_\pi \bar{\pi}_{t+1} + \phi_y y_t]$. In RS $\bar{\pi}_t$ is used instead of $\bar{\pi}_{t+1}$, although we found similar results for the case of $\bar{\pi}_{t+1}$. *Quite close to the values of RS who utilised the estimates by Rudebusch (2002): $\rho = 0.73$, $\phi_\pi = 2.1$ and $\phi_y = 0.93$ [Remark: in RS inflation is annualised in their Taylor rule and, therefore, $\phi_\pi = 0.53$ is set].

The household maximises the continuation value of its utility (V) which has the Epstein-Zin form and follows the specification of RS:

$$V_t = \begin{cases} U(C_t, L_t) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\ U(C_t, L_t) - \beta \left[E_t (-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0. \end{cases} \tag{1}$$

The households' problem is subject to its flow budget constraint:

$$P_t^b B_t + P_t C_t + T_t = W_t L_t + D_t + P_t^b B_{t-1}. \tag{2}$$

In equation (1) β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure ($1 - L_t$). E_t denotes expectations conditional on information available at time t. As the time frame is normalised to one leisure time ($1-L_t$) is what we are left with after spending some time working (L_t). $W_t L_t$ is labour income, B_t is nominal bond holdings, P_t^b is the price of the bond and D_t is dividend income. T_t are taxes net transfers.

To be consistent with balanced growth RS imposes the following functional form on U:

$$U(C_t, L_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-L_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0. \tag{3}$$

where Z_t is an aggregate productivity trend and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$ and the Frisch labour supply elasticity is given by $(1-\bar{L})/\chi\bar{L}$ where $\bar{L}$ is the steady-state of hours worked.

For the functional form in equation (3) RS derives the following relationship between coefficient of relative risk-aversion ($CRRA$) and the curvature parameter α in the recursive utility (1):

$$CRRA = \frac{\varphi}{1+\frac{\varphi}{\chi}\frac{1-\bar{L}}{\bar{L}}} + \alpha \frac{1-\varphi}{1+\frac{1-\varphi}{1-\chi}\frac{1-\bar{L}}{\bar{L}}}.$$

With time-separable preferences households fear uncertainty about consumption on the short-run. EZ preferences allow for a separation between risk-aversion and the intertemporal elasticity of substitution. With EZ preferences it is possible to increase the risk-aversion of investors who are, therefore, more concerned about the volatility of their consumption path on short-, medium- as well as long-run.

2.2 Firms

There is a perfectly competitive sector that purchases the continuum of intermediary goods and turns them into a single final good using a CES aggregator. Intermediary firms maximise their profits and face price-setting frictions of Calvo style. Accordingly, a $1-\xi$ fraction of firms can set its price optimally in each period. Intermediary firm i produces output ($Y_t(i)$) using the technology:

$$Y_t(i) = A_t[K_t(i)]^{1-\eta}[Z_t L_t(i)]^{\eta} \quad (4)$$

which after substituting for $Y_t(i)$ the demand for product i ($Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} Y_t$) and aggregating across firms gives way to:

$$Y_t = \Delta_t^{-1} A_t [K_t]^{1-\eta}[Z_t L_t]^{\eta}, \;\; 0 < \eta < 1, \quad (5)$$

where $K_t = Z_t \bar{K}$ is the aggregate capital stock ($\bar{K}$ is fixed), η is the share of labour in production, $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} di$ is price dispersion, A_t is a stationary aggregate productivity shock:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

where ε_t^A is an independently and identically distributed (iid) stochastic technology shock with mean zero and variance σ_A^2.

2.3 Monetary and Fiscal policy

The New-Keynesian model is closed by a monetary policy rule (so called Taylor rule). In this paper we consider different Taylor rule specifications which are nested by the form in RS:

$$R_t = \rho_i R_{t-1} + (1-\rho)[R + \mathcal{I}\log\bar{\Pi}_t + g_\pi(\log\bar{\Pi}_t - \log\Pi_t^*) + g_y(Y_t - Y_t^*)/Y_t^*] + \varepsilon_t^i \tag{6}$$

where R_t is the policy rate, $\bar{\Pi}_t$ is a four-quarter moving average of inflation and Y_t^* is the trend level of output yZ_t (where y denotes the steady-state level of Y_t/Z_t). Π_t^* is the target rate of inflation, ε_t^i is an iid shock with mean zero and variance σ_i^2. In the baseline version of the Rudebusch and Swanson (2012) model without long-run inflation risks the inflation target is constant ($\Pi_t^* = \Pi^*$ for all t). The choice of $\mathcal{I} = 1$ delivers the Taylor rule in RS. We also study interest rate rules with $\mathcal{I} = 0$ (in this case the coefficient on the inflation term is $1 + g_\pi$).

The four-quarter moving average of inflation ($\bar{\Pi}_t$) can be approximated by a geometric moving average of inflation:

$$\log\bar{\Pi}_t = \theta_\pi \log\bar{\Pi}_{t-1} + (1-\theta_\pi)\log\Pi_t, \tag{7}$$

where the choice of $\theta_\pi = 0.7$ ensures that the geometric average in equation (7) has an effective duration of about four quarters.

In one version of the RS model the inflation target is time-varying and has three properties: i) the inflation target is inertial ($\rho_{\pi^*}\Pi_{t-1}^*$), households adjust the inflation target upwards (downwards) when current inflation is higher (lower) than the inflation target ($\vartheta_{\pi^*}(\bar{\Pi}_t - \Pi_t^*)$) and is stochastic due to inflation target shocks ($\varepsilon_t^{\pi^*}$):

$$\Pi_t^* = \rho_{\pi^*}\Pi_{t-1}^* + \vartheta_{\pi^*}(\bar{\Pi}_t - \Pi_t^*) + \varepsilon_t^{\pi^*}, \quad \vartheta_{\pi^*} > 0,\ \varepsilon_t^{\pi^*} > 0, \tag{8}$$

where $\varepsilon_t^{\pi^*}$ is an iid inflation target shock with mean zero and variance $\sigma_{\pi^*}^2$. Note that equation (8) can also be described as a learning process.

As RS argues the main source of nominal and real risks are technology shocks and therefore we abstract from government expenditure and monetary policy shocks.

We include default-free zero coupon nominal and real bonds in the model for maturities ranging from one to fourty quarters. Risk-averse investors purchase bonds of a particular maturity and use the nominal (real) stochastic discount factor to price nominal (real) bonds while risk-neutral investors roll over their one-year investments in each year. The latter strategy is consistent with the expectations hypothesis of the term structure.

Nominal (real) term premium (NTP and RTP) for a given maturity is calculated as the difference between the nominal (real) yields expected by the risk-averse and the risk-neutral investors, respectively.

Inflation risk premium (IRP) can be approximated as the difference between NTP and RTP (Andreasen (2012)).

3 Calibration and solution method

Calibration can be found in Table (2) that follow the baseline parameter values of RS. The whole model is approximated to the second order using Dynare (Adjemian et al. 2011).

Table 2: Calibration

γ	1.0025	φ	2	ρ_i	0.73	ρ_A	.95
$\tilde{\beta}$	0.99	χ	3	g_π	0.53	ρ_G	.95
δ	0.02	CRRA	75	g_y	0.93	σ_A^2	0.005^2
$\bar{L}$	1/3	η	2/3	Π^*	1	σ_G^2	0.004^2
K/Y	10	θ	0.2	ρ_{π^*}	0.99	σ_i^2	0.003^2
G/Y	0.17	ξ	0.75	$\sigma_{\pi^*}^2$	$.0005^2$		
ε	6			ϑ_{π^*}	0.01		

Notes: G/Y is the government spending-GDP ratio, K/Y is the share of fixed capital in GDP, δ is the depreciation rate of fixed capital. ε is the elasticity of substitution among intermediary goods implying a net markup of 20% ($\theta = 0.2$). The rest of the parameters are explained above.

4 Results

4.1 Interest rate rule without time-varying inflation target

We explore the effect of the specification of the Taylor rule on inflation risk premia. Table 3 contains the NTP, RTP and IRP for the different versions of the interest-rate rules we consider when coefficient on the output-gap is closer to zero (0.125) rather than to one. In Tables 3-4 there is no time-varying inflation target (see column with $\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$). We also explore the robustness of our results with two different versions of the output gap. Column 1-3 contains a simple version of the output gap defined as the deviation of the sticky-price output from its deterministic steady-state while in columns 4-6 it is used relative to the flexible-price output.

First, we consider the simplest Taylor rule without interest rate smoothing and in which monetary policy reacts to current period inflation (case 1). When we use a simple definition of the output-gap the IRP is higher (42 basis points) than with flexible-price version of the output gap (26 basis points). With the simple output-gap nominal risks are higher but real risks are lower compared to the case of flexible price output gap (the same pattern is true for the remaining cases). The simple output-gap reports overheating of the economy (positive gap) in response to a positive technology shock. Whereas the flexible price version arrives at exactly the opposite conclusion because the flexible price output rises more than the sticky-price output in case of a technology shock leading to a negative output-gap. In the latter case, therefore, both inflation and output are below their reference levels and therefore inflations risks are small even if the Taylor rule precribes a fall in real rates supporting recovery. It can be also recognised that the flexible-price version is directly influenced by the technology shocks making the output gap more volatile lifting real risks.

In case 2a (2b) we replace current inflation with expected inflation, π_{t+1} (a moving average of past inflation, $\bar{\pi}_t$). Case 2a delivers results similar to case 1. In case 2b, however, NTP and RTP as well as IRP are much lower in case 1 and 2a. When agents use a weighted average of past and current inflation instead of just current inflation in the Taylor rule they obtain more accurate picture of the evolution of inflation and hence stabilisation of inflation is more successful resulting in lower inflation risks.

In case 3a (3b) we extend cases 2a (2b) with interest smoothing which helps households reducing both nominal and real risks through the delayed response of monetary policy. To illustrate this let us consider a negative technology shock which leads to rise in the real interest rate and makes the households consume less according to the logic of the Taylor rule. The rise in the real interest rate is gradual with interest-rate smoothing, and thus the household has more time to insure itself from the negative effects of the shock (less consumption) by adjusting its labour supply.

In Table 4 we recalculate the above models with high coefficient on the output gap. The previous patterns are confirmed. Furthermore, we recognise that the higher output-gap coefficient systematically generates higher inflation risks. This happens because there is no divine coincidence in the model. Hence, the standard deviation of inflation and output gap cannot be reduced at the same time. When monetary policy puts more efforts into stabilising fluctuations the output gap inflation risks receive relatively less attention and they rise automatically.

Table 3: Taylor rule specifications when coefficient on the output gap is closer to zero
low coefficient on the output gap.

Case	Meaning	simple output gap			flexible output gap		
		NTP	RTP	IRP	NTP	RTP	IRP
1	Π_t in the Taylor rule	.6130	.1923	.4207	.4796	.2133	.2663
2a	Π_{t+1} in the Taylor rule	.6441	.1865	.4576	.5038	.2099	.2939
2b	$\bar{\Pi}_t$ in the Taylor rule	.4506	.0846	.3660	.3739	.1508	.2231
3a	1 with smoothing	.4778	.0998	.3780	.3735	.1493	.2242
3b	2b with smoothing	.3696	.0341	.3355	.3068	.1126	.1942
4	2b with $\mathcal{I}$ =1	.3061	.0716	.2345	.2894	.0933	.1961

Notes: NTP, RTP and IRP stand for nominal, real and inflation risk premium, respectively. IRTP is calculated as NTP-RTP. ind. means idetermi-nacy. In all cases $\mathcal{I}$ =0 and $g_y = 0.125$ unless indicated otherwise. In this setup there are no long-run inflation risks ($\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$). Case 4 corresponds to the RS model with $g_y = 0.125$. In line with RS we expressed inflation and interested rates in annual terms in case 4.

4.2 Does time-varying inflation target with learning generate inflation risks?

Literature claims that time-varying inflation target is a potential source of inflation risks (see e.g. RS). In this section we revisit the question whether the time-varying inflation target and the learning of the inflation target lead to higher inflation risks. Now we incorporate time-varying interest rate and learning into the Taylor rule ($\vartheta_{\pi^*} > 0$, $\rho_{\pi^*} > 0$, $\varepsilon_t^{\pi^*} > 0$). We conduct this experiment in the case of a high output gap coefficient. When comparing Table 4 and Table 5 it is apparent that long-run inflation risks with learning has marginal effects on inflation risk premia for both versions of the output-gap.

Table 4: Taylor rule specifications when coefficient on the output gap is close to one
High output-gap coefficient (NO inflation risks)

Case	Meaning	simple output-gap			flexible price gap		
		NTP	RTP	IRP	NTP	RTP	IRP
1	Π_t in the Taylor rule	.7846	.1236	.6610	.4059	.2229	.1830
2a	Π_{t+1} in the Taylor rule	.7752	.1152	.6600	.4172	.2215	.1957
2b	$\bar{\Pi}_t$ in the Taylor rule	.6444	-.0557	.7001	.3548	.1938	.1610
3a	1 with smoothing	.6768	-.0250	.7018	.3339	.1803	.1536
3b	2b with smoothing	.5641	-.1420	.7061	.3001	.1625	.1377
4	2b with $\mathcal{I}$ =1	.4008	.0158	.4166	.2928	.1154	.1774

Notes: NTP, RTP and IRP stand for nominal, real and inflation risk premium, respectively. IRP is calculated as NTP-RTP. ind. means idetermi-nacy. In all cases $\mathcal{I}$ =0 unless indicated otherwise. In this setup there are no long-run inflation risks ($\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$). Case 4 corresponds to the RS model who expressed inflation and interested rates in annual terms in case 4.

5 Conclusion

This paper shows that the specification and the size of inflation and output-gap coefficients in the Taylor rule has large impact on the existence of inflation risks obtained from the model. We found that interest-rate smoothing is a powerful tool that lends households time to adjust to negative shocks by raising labour supply before monetary policy takes full action. When output gap is defined as the deviation of the sticky-price output from its flexible-price one nominal risks are lower and real risks are higher.

Table 5: Taylor rule specifications when coefficient on the output gap is close to one

High output gap coefficient and inflation risks.

Case	Meaning	simple output gap			flexible output gap		
		NTP	RTP	IRP	NTP	RTP	IRP
1	Π_t in the Taylor rule	.7724	.1343	.6381	.4304	.2249	.2055
2a	Π_{t+1} in the Taylor rule	.7413	.1288	.6126	.4452	.2240	.2212
2b	$\bar{\Pi}_t$ in the Taylor rule	.6592	-.0589	.7181	.3736	.1934	.1802
3a	1 with smoothing	.6828	-.0326	.7154	.3498	.1793	.1705
3b	2b with smoothing	.5820	-.1557	.7378	.3130	.1606	.1523
4	2b with $\mathcal{I}$ =1	.4129	-.0203	.4333	.2991	.1135	.1856

Notes: NTP, RTP and IRP stand for nominal, real and inflation risk premium, respectively. IRP is calculated as NTP-RTP. ind. means ideterminacy. In all cases $\mathcal{I}$ =0 unless indicated otherwise. In this setup long-run inflation risks are included ($\vartheta_{\pi^*} > 0$, $\rho_{\pi^*} > 0$, $\varepsilon_t^{\pi^*} > 0$). Case 4 corresponds to the RS model with inflation risks. In line with RS we expressed inflation and interested rates in annual terms in case 4.

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