Decoupling Nominal and Real Rigidities

Philipp J. König[a], Alexander Meyer-Gohde[b,c]

[a]German Institute for Economic Research, Mohrenstraße 58, 10117 Berlin, Germany
[b]Humboldt-Universität zu Berlin, Sonderforschungsbereich 649, Spandauer Straße 1, 10178 Berlin, Germany
[c]Universität Hamburg, Professur für VWL insb. Wachstum und Konjunktur, Von-Melle-Park 5, 20146 Hamburg, Germany

Abstract

We analyze a canonical model of price setting with menu costs that exhibits multiple equilibria for nominal aggregate demand shocks of intermediate size. Introducing strategic uncertainty into the price setting decision yields a unique symmetric threshold equilibrium where agents adjust prices whenever the demand shock falls outside the thresholds. We find that the endogenous adjustment to shocks of agents' expectations regarding the aggregate price level weakens the link between real and nominal rigidities.

1. Introduction

Price setting with fixed costs to adjustment (i.e., menu costs) and strategic complementarities in firms' pricing decisions may give rise to multiple equilibria. The canonical models by Blanchard and Kiyotaki (1987), and Ball and Romer (1990, 1991) show that monopolistically competitive firms can be more inclined to incur menu costs and adjust their prices in response to monetary shocks if other firms do too. Two equilibria sustained by self-fulfilling beliefs emerge. In one, all firms adjust prices, whereas in the other, prices are rigid. In the present paper we address this equilibrium multiplicity by introducing strategic uncertainty following the literature on global games (for an overview, see Morris and Shin (2003)). We derive a unique threshold equilibrium such that firms pay the menu cost and adjust their price if and only if the magnitude of the shock exceeds a certain threshold. This complements Ball and Romer (1990) (henceforth BR), who derive upper (lower) bounds such that shocks outside (inside) the bounds price adjustment (rigidity) is the only equilibrium; for shocks of intermediate magnitude multiple equilibria emerge. In our version, the unique equilibrium threshold equals the midpoint of BR's bounds.

Our equilibrium alone strengthens the conclusion that the existence of monetary non-neutralities and price stickiness in a standard representative agent model with menu costs requires implausible parameter values (BR). However, our equilibrium assigns a qualitatively different role to real rigidities (forces that render price setters reluctant to fully adjust their relative prices to a change in nominal aggregate demand) in explaining monetary non-neutralities than the standard result from the literature. BR, for example, examine only the comparative statics of the upper bound on the region of multiplicity, thereby neglecting how price setters' beliefs about the behavior of other price setters adjust in equilibrium. Their conclusion, widely echoed in the literature (e.g., Kimball (1995); Woodford (2003); Mankiw and Reis (2010)), leads them to conclude that real and nominal rigidities move in concert. In contrast, we show that the link between real and nominal rigidities becomes tenuous when the endogenous response of price setters' expectations (or beliefs) about the behavior of other agents is taken into account.

As a consequence, our comparative statics differ qualitatively from those in BR's original analysis. To illustrate this, consider a situation where a price setter's optimality condition becomes less sensitive to changes in

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both relative prices and aggregate demand, but where the decrease in sensitivity to relative prices is stronger than the decrease in sensitivity to aggregate demand. This relative change in sensitivity amounts to a decrease in real rigidities. In BR's analysis, an agent will focus more than before on her response to aggregate demand given that her beliefs regarding the aggregate price level remain unchanged, rendering her more inclined to adjust her price. Thus, the decrease in real rigidities implies a decrease in nominal rigidities. This reasoning is, however, incomplete as it neglects the endogenous adjustment of her expectations regarding the aggregate price level. If real rigidities decrease, price setters are not only more inclined to adjust their prices in response to large shocks even if others do not, but they are also more inclined to tolerate other agents' price adjustments in response to smaller shocks. Due to the initial decrease in real rigidities, agents' optimality conditions are now relatively less sensitive to relative prices than to aggregate demand, implying that agents are more strongly inclined to tolerate price adjustments by others for a shock of a given magnitude and may be more reluctant to adjust their prices. Whenever this reluctance dominates, a reduction in real rigidities is associated with a higher degree of nominal rigidities.

Multiplicity of equilibrium in BR's analysis is driven by the indeterminacy of price setters' expectations, i.e., by self-fulfilling beliefs regarding the behavior of others. Such an indeterminacy results from agents sharing common knowledge about the model's fundamentals and each being perfectly aware of what the others do in equilibrium (Morris and Shin, 2003). This implies that agents' actions and beliefs are perfectly coordinated in equilibrium thus potentially leading to equilibrium multiplicity (Morris and Shin, 2001). To eliminate such multiplicity, the global games approach, pioneered by Carlsson and van Damme (1993), abandons the assumption of common knowledge regarding the economy's fundamental and instead considers an environment of incomplete information. Essentially, the original (non-stochastic) setting is embedded in a larger world of similar settings differing stochastically with respect to payoff-relevant parameters (the economy's fundamentals or state—in our case, as discussed below, a nominal aggregate demand shock). The resulting equilibrium of the global game can be characterized by a unique threshold that partitions the parameter space into regions with different equilibrium outcomes, the uniqueness of the threshold remains in the limit as agents' information becomes perfectly precise.

In the present paper, we adopt a similar approach to derive a unique threshold equilibrium in BR's model by using a global game information structure to derive a unique equilibrium in threshold strategies in contrast to BR's ad hoc equilibrium selection.[1] When uncertainty regarding the nominal shock vanishes, agents' beliefs remain subject to strategic uncertainty (i.e., uncertainty about the price setting behavior of other agents), breaking the perfect coordination of actions and beliefs and thereby eliminating the equilibrium multiplicity of BR's original analysis.

In what follows, Section 2 introduces the canonical price-setting model with multiple equilibria. Section 3 presents our unique threshold equilibrium. Section 4 gives our general comparative statics result. Section 5 provides a numerical illustration in BR's baseline model of monopolistic competition. Section 6 concludes.

2. Multiple Equilibria

We consider a model of price-setting in the spirit of Blanchard and Kiyotaki (1987) and Ball and Romer (1990, 1991)[2] with a representative price setter who produces a differentiated good facing fixed costs to price adjustment. Agent i's utility is given by

$$U_i = W\left(Y, \frac{P_i}{P}\right) - zD_i \tag{1}$$

where Y is real aggregate expenditures, $\frac{P_i}{P}$ is the agent's relative price and z is the menu cost—a small resource cost of changing a nominal price; D_i is equal to one if agent i changes her price and zero otherwise.

[1] For settings with additional public information, the unique global game selection requires that private information is sufficiently more precise than public information, see Hellwig (2002). Due to the peculiar nature of the strategies in our model, we rely on a similar condition, i.e. that private signal noise vanishes, even without the presence of public signals.

[2] For clarity of exposition, we adopt the notation of BR.

Assuming a quantity theory approach to expenditures, we have $Y = M/P$, where M is the nominal money supply.[3] Hence, in equilibrium

$$U_i = W\left(\frac{M}{P}, \frac{P_i}{P}\right) - zD_i \tag{2}$$

We assume $W_2(1,1) = 0$, $W_{22}(1,1) < 0$, and $W_{12}(1,1) > 0$, implying that the optimal relative price is normalized to unity, that the second order condition is satisfied at this price, and that the symmetric equilibrium is stable.

The optimal price is governed by the first-order condition

$$W_2\left(\frac{M}{P}, \frac{P_i}{P}\right) = 0 \tag{3}$$

A first order expansion in log deviations from a symmetric equilibrium $\overline{P_i} = \overline{P} = \overline{M}$ yields

$$W_2\left(e^{m-p}, e^{p_i - p}\right) \approx W_{21}(1,1)(m-p) + W_{22}(1,1)(p_i - p) = 0 \tag{4}$$

where lowercase m, p, and p_i all represent log deviations of their uppercase counterparts from the symmetric equilibrium. Agent i's optimal deviation from the symmetric equilibrium becomes

$$p_i^* = \beta m + (1-\beta)p \tag{5}$$

where $\beta \equiv -\frac{W_{21}(1,1)}{W_{22}(1,1)}$ measures the responsiveness of the optimal price to changes in aggregate demand. Changes in the aggregate price level p induce two competing effects on agent i's price (Blanchard and Kiyotaki, 1987). First, agents want to maintain stable relative prices, i.e. p_i is raised proportionately to increases in p; second, since an increase in p is associated with a reduction in aggregate demand (by lowering real balances), agents are inclined to lower their relative price in response to an increase in p. The overall effect of changes in p depends on the responsiveness coefficient β. If $\beta > 1$, the latter effect dominates, implying that price setting decisions are strategic substitutes; conversely, if $\beta < 1$, the former effect is stronger and pricing decisions are strategic complements. Since the prevalence of strategic complementarities is a necessary precondition for the existence of multiple equilibria, we henceforth assume $\beta \in (0,1)$ in line with Blanchard and Kiyotaki (1987) and Ball and Romer (1990). Note further that β is a measure of the degree of real rigidities (Ball and Romer, 1990), where a low (high) value of β implies that real rigidities are high (low).[4]

Whenever an agent changes her price, she will do so optimally and choose $p_i = p_i^*$. In deciding *whether* to change her price, she compares the payoff difference between setting the optimal price, $p_i = p_i^*$, and maintaining her old price, $p_i = 0$, to the menu costs z. This payoff difference can be expanded to second order in terms of log deviations from the symmetric equilibrium $\overline{P_i} = \overline{P} = \overline{M}$ as[5]

$$PC(m, p, p_i^*) \doteq W\left(e^{m-p}, e^{p_i^* - p}\right) - W\left(e^{m-p}, e^{-p}\right) \approx -W_{22}(1,1)\left(p_i^*\right)^2 \tag{6}$$

Now consider the range of monetary deviations, m, for which rigidity, $p_i = p = 0$, and adjustment, $p_i = p = p_i^*$, are Nash equilibrium. Suppose agent i believes that no other agents will adjust their prices, $p = 0$. Agent i will adjust her price nevertheless if

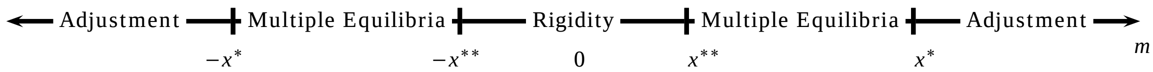
$$PC(m, 0, p_i^*) > z \Leftrightarrow -\frac{1}{2} W_{22}(1,1)\left(p_i^*\right)^2 > z$$

From (5), agent i's optimal price is $p_i^* = \beta m$ and together with the latter inequality yields the threshold

$$x^* = \frac{1}{\beta}\sqrt{\frac{2z}{-W_{22}(1,1)}} \tag{7}$$

[3] Following BR, we assume without loss of generality that aggregate demand fluctuations arise from the nominal money supply.
[4] See also the discussions in Mankiw and Reis (2010) and Woodford (2003, pp. 158–173).
[5] See König and Meyer-Gohde (2014) for a detailed derivation.

Figure 1: Dominance regions and multiple equilibria under common knowledge of m.



hence adjustment is the dominant choice for *all* agents whenever $|m| > x^*$ and is, consequently, the only equilibrium.

Conversely, suppose that agent i believes that all agents adjust their prices, impling her optimal price is $p^* = m$. Agent i will nonetheless not adjust her price if

$$PC(m, m, p_i^*) < z \tag{8}$$

This yields another threshold value

$$x^{**} = \sqrt{\frac{2z}{-W_{22}(1,1)}} \tag{9}$$

implying that if $|m| < x^{**}$, not adjusting is the dominant choice for *all* agents and rigidity is the only equilibrium.

The assumption that $\beta \in (0,1)$ implies $x^{**} < x^*$ and thus multiple pure strategy Nash equilibria exist for shocks of intermediate size. These equilibria are sustained by self-fulfilling beliefs about other price setters' behavior: in one equilibrium agents believe that everyone else adjust prices and therefore adjusting becomes agents' best response; in the other equilibrium agents believe that no one else adjusts and not adjusting becomes the best response, see figure 1.

Proposition 1. *Multiple Equilibria*
For large nominal aggregate demand shocks, $|m| > x^*$, all agents consider it strictly dominant to adjust prices. For small shocks, $|m| < x^{**}$, all agents consider it strictly dominant not to adjust prices. For shocks of intermediate size $x^{**} < |m| < x^*$ both, adjustment and rigidity can be sustained as pure strategy Nash equilibria.

Proof. See the preceding exposition. □

3. Unique Threshold Equilibrium

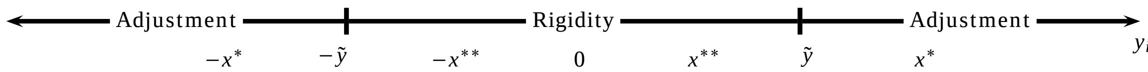
Echoing previous analyses, Morris and Shin (2001, p. 139) emphasize that "comparative-statics analyses [...] are only as secure as the equilibrium," and a multiplicity of equilibria prohibits the derivation of general comparative statics. The multiplicity here is due to the indeterminacy of price setters' beliefs that follows from agents' common knowledge regarding nominal aggregate demand shocks. Hence, agents' beliefs and their price setting behavior are perfectly coordinated in equilibrium (Morris and Shin, 2001). The price-setting literature has largely dealt with this problem by arbitrarily selecting a particular equilibrium. For example, BR focus on the x^* threshold and examine its comparative statics properties, thereby ignoring the effects of multiplicity. They justify this on grounds that x^* is associated with the largest range of price rigidities.

In contrast, we introduce strategic uncertainty into the price setting decision to obtain a unique equilibrium and valid set of comparative statics. Strategic uncertainty breaks the coordination of beliefs while maintaining the underlying fundamental certainty of the original model. We proceed by assuming that the realization of m is not common knowledge. Agents can only observe private signals concerning its realization.[6] These signals are of the form

$$y_i = m + \sigma\epsilon_i, \quad \text{with} \quad \epsilon_i \sim \mathcal{N}(0,1), \text{ i.i.d.} \tag{10}$$

[6] We take an "improper prior" approach often made in the literature and assume that m is drawn from a uniform distribution over the real line. This is innocuous from a technical perspective since we confine our attention to posterior conditional distributions which are still well-defined, see Morris and Shin (2003, Section 2.1).

Figure 2: Threshold Strategy



The parameter σ captures the degree of fundamental uncertainty about the nominal demand shock: If σ is small, there is little uncertainty as agents' information about m is relatively precise; conversely, if σ is large, an agent places more weight on shocks far away from her signal y_i when forming her expectations. In our analysis, we will focus entirely on the limiting case of $\sigma \to 0$, recovering all but an infinitesimal amount of fundamental certainty regarding the demand shock, which suffices to maintain strategic uncertainty regarding the beliefs and actions of others.

A strategy for an agent is a mapping from her signal into her action space consisting of adjusting to the optimal price (conditioned on her signal) or not adjusting her price at all. We restrict the analysis to symmetric threshold strategies, which in this model are described by values $\tilde{y}_i$ such that agent i adjusts if and only if $|y_i| > \tilde{y}_i$. A symmetric threshold strategy is simply described by a common threshold for all agents, $\tilde{y}_i = \tilde{y}$, see figure 2. In order to derive the equilibrium threshold of the model, we exploit the indifference of an agent between adjusting and not adjusting when the magnitude of her signal observation y_i is just equal to the threshold $\tilde{y}$ and she believes that other agents also use a strategy around $\tilde{y}$.

However, we are interested in studying the implications not of imperfect information in price setting itself (the interested reader is directed to Mankiw and Reis (2010)), but rather of the unique equilibrium delivered by deviated as minimally as possible from perfect information. Accordingly, we let $\sigma \to 0$, which eliminates all but an infinitesimal amount of uncertainty, which itself still suffices to resolve the equilibrium multiplicity. Thus we approach fundamental certainty as in the original model, but introduce strategic uncertainty into the price setting problem, as the infinitesimal uncertainty regarding the fundamental cascades through agents' higher order beliefs. The resulting equilibrium is summarized in the following proposition.

Proposition 2. *Unique Threshold Equilibrium*
As fundamental uncertainty regarding the size of the monetary shocks vanishes, $\sigma \to 0$, there exists a unique threshold, $\tilde{y}$, such that all agents adjust if and only if $|y_i| < \tilde{y}$ and refrain from adjusting otherwise. The threshold is given by

$$\tilde{y} = \frac{x^* + x^{**}}{2} \tag{11}$$

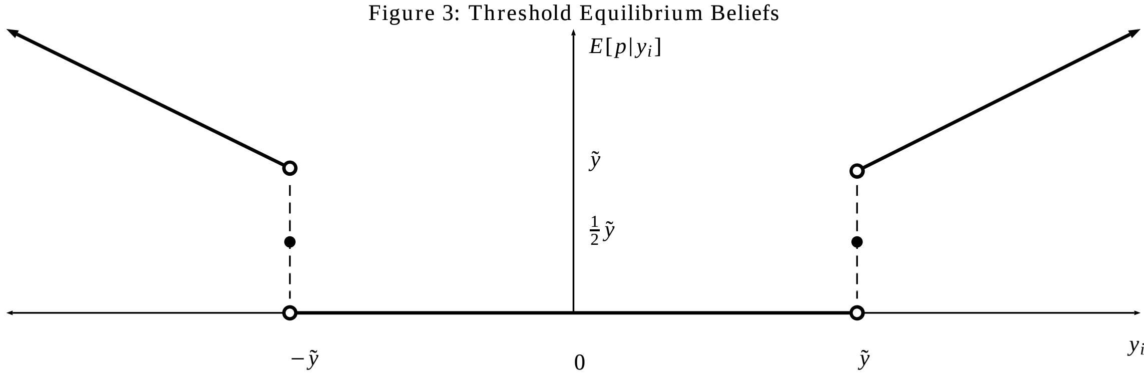
Proof. See the appendix. □

When σ goes to zero, fundamental uncertainty vanishes and an agent's subjective distribution about other signals collapses. She expects that others receive almost exactly the same signal as she does. Away from the threshold she would expect the aggregate price level to equal her signal, consistent with the symmetric adjustment equilibrium from section 2. However, whenever she receives a signal such that $|y_i| = \tilde{y}$, her subjective distribution is split into two halves, one to the left and one to the right of the threshold, see figure 3. In other words for a signal equal in magnitude to the threshold, the belief distribution collapses to a point mass divided evenly between rigidity and adjustment. The resulting aggregate price level then equals the average of what it would be with an infinitesimally larger or smaller shock and splits beliefs equally between adjustment and rigidity, leaving the agent indifferent to both choices.

4. Decoupling Real and Nominal Rigidities

The threshold $\tilde{y}$ is our measure for the degree of nominal rigidities. A larger threshold means there is a larger range of monetary shocks for which nominal prices remain unchanged. Two aspects of the threshold equilibrium are worth noting. First, a direct consequence of propositions 1 and 2 is $\tilde{y} < x^*$ so that the unique threshold equilibrium is associated with less nominal rigidity than the x^* threshold examined by BR would

Figure 3: Threshold Equilibrium Beliefs



imply. This is not surprising as BR themselves point out that they examine the region with the largest possible nominal rigidities. Hence, our result strengthens conclusions by BR or Blanchard and Kiyotaki (1987) that menu costs are unlikely to serve as a good explanation of large monetary non-neutralities. Second, and more importantly, the unique threshold equilibrium allows us to examine whether Ball and Romer's (1990, p. 184) conclusion that "[t]he degree of nominal rigidity [...] is increasing in the degree of real rigidities" is still valid. As we will show, our answer is negative. The link between nominal and real rigidities becomes tenuous once we take into account that beliefs regarding other price setters' behavior respond endogenously.

In the x^* equilibrium, the agent takes as given that no other agent adjusts, whereas under our threshold, $\tilde{y}$, she takes an adjustment of the aggregate price level into account through her belief that other agents use the same threshold strategy as she does. Under both thresholds, indifference between adjusting and not adjusting entails that the agent's payoff difference equals the menu cost z. If parameter variations increase one of these payoff differences and decrease the other, the former threshold will fall and the latter will rise. Formally, we differentiatw the payoff difference to obtain (which is valid for either threshold),[7]

$$\mathrm{d}PC = p_i^* \left(2(-W_{22})\,\mathrm{d}p_i^* - p_i^*\,\mathrm{d}W_{22}\right) \tag{12}$$

The signs of the differentials (12) under the two different prices differ if and only if

$$2(-W_{22})\,\mathrm{d}p_i^* - p_i^*\,\mathrm{d}W_{22} \tag{13}$$

have different signs under the two thresholds. Except for $\mathrm{d}p^*$ all terms in the last equation are independent of the thresholds. Hence, the important question is how the optimal prices $p_i^*\big|_{\mathrm{BR}}$ and $p_i^*\big|_{\tilde{y}}$ change.

This can be ascertained from the optimal pricing equation (5) for a given signal y_i,

$$\mathrm{d}p_i^* = y_i\,\mathrm{d}\beta - E\left[p|y_i\right]\mathrm{d}\beta + \left(1-\beta\right)\mathrm{d}E\left[p|y_i\right] \tag{14}$$

It is in the last two terms, the expectations regarding the price level and the response thereof, that our comparative statics will differ from those of BR. Evaluated at the threshold x^*, given BR's assumption that $E\left[p|y_i\right] = \mathrm{d}E\left[p|y_i\right] = 0$,

$$\mathrm{d}p_i^*\big|_{\mathrm{BR}} = x^*\,\mathrm{d}\beta \tag{15}$$

Yet, when evaluated at our threshold $\tilde{y}$, using $\tilde{y} = (1+\beta)x^*/2$ and the fact that $E\left[p|\tilde{y}\right] = p_i^*/2$,

$$\mathrm{d}\ p_i^*\big|_{\tilde{y}} = (\tilde{y} - E[p|\tilde{y}])\,\mathrm{d}\beta + \left(1-\beta\right)\mathrm{d}E[p|\tilde{y}] = \left(\tilde{y} - \frac{p_i^*}{2}\right)\mathrm{d}\beta + \left(1-\beta\right)\frac{1}{2}\mathrm{d}p_i^* \Leftrightarrow \mathrm{d}\ p_i^*\big|_{\tilde{y}} = \frac{x^*}{1+\beta}\mathrm{d}\beta \tag{16}$$

[7] From (6) follows that the optimal prices under both thresholds are identical since, by definition of the thresholds, the optimal price equates the expected payoff difference with the menu cost z: $p_i^*\Big|_{\mathrm{BR}} = \beta x^* = \frac{2\beta}{1+\beta}\tilde{y} = p_i^*\Big|_{\tilde{y}}$. The first equality follows as x^* is derived under the assumption that nobody else adjusts, the second equality can be obtained from (11), and the last equality follows because the expected aggregate price level under our unique threshold equilibrium is is given by $\beta\tilde{y}/(1+\beta)$ (see (A7) in the appendix).

From (15) and (16) follows $|\mathrm{d}\, p_i^*|_{\tilde{y}}| < |\mathrm{d}\, p_i^*|_{\mathrm{BR}}|$, implying there exists values for $\mathrm{d}\beta$ such that

$$2(-W_{22})\mathrm{d}\, p_i^*\big|_{\mathrm{BR}} - p_i^* \mathrm{d}W_{22} > 0 \quad \text{and} \quad 2(-W_{22})\mathrm{d}\, p_i^*\big|_{\tilde{y}} - p_i^* \mathrm{d}W_{22} < 0.$$

Moreover, by using the differential $\mathrm{d}\beta = \frac{\beta}{W_{21}}\left(\mathrm{d}W_{21} + \beta \mathrm{d}W_{22}\right)$, it is straightforward to break down the parameter restrictions under which the above two inequalities hold simultaneously to restrictions on changes to W_{21} and W_{22}, the sensitivity of the price setting optimality condition to aggregate demand and relative prices. That is, the composition of changes to β (i.e., through W_{21} and W_{22}) and not changes in β per se are decisive in determining the difference between BR's threshold and ours.

The following proposition provides a general formalization of the necessary and sufficient conditions under which the comparative statics of the threshold x^* differ from those of $\tilde{y}$.

Proposition 3. *Comparative Statics*
Assume $W(\cdot,\cdot)$ depends on a finite number of parameters denoted by α_k, $k = 1,\ldots,I$. For marginal changes in the parameters the thresholds $\tilde{y}$ and x^* are moving in opposite directions if and only if

$$\left|\beta \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i\right| > \left|\frac{2}{\beta} \sum_{i=1}^{I} \frac{\partial W_{21}}{\partial \alpha_i} \mathrm{d}\alpha_i + \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} \mathrm{d}\alpha_i\right| > 0 \tag{17}$$

Proof. See the appendix. □

BR's analysis focuses on the x^* equilibrium, which neglects how the expected aggregate price level adjusts to changes in parameters fixing its value at zero. In our equilibrium, the expected aggregate price level is not fixed at zero and agents take into account how it is affected by parameter variations. This can be seen from expression (14), where the second term in brackets measures the effect of parameter changes on an agent's response to her beliefs regarding the aggregate response of agents and the third component measures how her beliefs themselves respond to these parameter changes. In contrast to BR's analysis, this means that we can find combinations of parameter changes that lead to real and nominal rigidities moving in opposite directions. For example, whenever parameter variations reduce agents' sensitivity towards the shock by less than their sensitivity towards changes in relative prices (decrease in real rigidities), agents may accept variations in relative prices for a larger range of monetary shocks. Since this holds true for all agents, the aggregate price becomes smaller, thus further reducing agents' incentives to adjust their price. As a consequence, the change in any agent's optimal price is smaller and the gains from adjustment can thus fall short of the menu costs. In the next section we provide an example of this result based on BR's baseline monopolistically competitive yeoman farmer model with differentiated goods which allows to proceed from a standard utility specification to derive the function $W(\cdot,\cdot)$.

5. Numercial Illustration

5.1. Model of Monopolistic Competition

Suppose the economy is populated by a unit mass of *ex ante* identical agents. Agent i produces her good, Y_i, with the production function $Y_i = L_i$ using her own labor, L_i, to maximize her utility, given by

$$U_i = \left[\int_0^1 C_{ij}^{(\epsilon-1)/\epsilon} dj\right]^{\epsilon/(\epsilon-1)} - \frac{\epsilon-1}{\gamma\epsilon} L_i^{\gamma} - zD_i \tag{18}$$

where C_{ij} is agent i's consumption of good j. The elasticity of substitution between goods, $\epsilon > 1$, and the degree of marginal disutility to labor, $\gamma > 1$, constitute the two-dimensional parameter vector ($I = 2$).

With $Y = M/P$, where aggregate production is given by $Y = \int_0^1 Y_i di$ and the aggregate price level, given by $P = \left[\int_0^1 P_i^{1-\epsilon} dj\right]^{1/(1-\epsilon)}$, the agent's utility function (18) can be written in the form of (2)

$$U_i = \left(\frac{M}{P}\right)\left(\frac{P_i}{P}\right)^{1-\epsilon} - \frac{\epsilon-1}{\gamma\epsilon}\left(\frac{M}{P}\right)^{\gamma}\left(\frac{P_i}{P}\right)^{-\gamma\epsilon} - zD_i \tag{19}$$

Table 1: Cost of Adjustment, 5% Monetary Shock, Baseline Model

Labor Supply Elasticity		Markup 5%	15%	50%	100%
	$x^* = 5\%$	2.38	2.16	1.64	1.22
0.05	$\tilde{y} = 5\%$	8.66	6.77	3.72	2.20
		(3.65)	(3.13)	(2.27)	(1.81)
	$x^* = 5\%$	0.79	0.71	0.53	0.39
0.15	$\tilde{y} = 5\%$	2.87	2.23	1.22	0.72
		(3.65)	(3.14)	(2.30)	(1.86)
	$x^* = 5\%$	0.23	0.20	0.14	0.10
0.5	$\tilde{y} = 5\%$	0.85	0.65	0.35	0.20
		(3.65)	(3.17)	(2.42)	(2.04)
	$x^* = 5\%$	0.11	0.10	0.06	0.04
1	$\tilde{y} = 5\%$	0.42	0.31	0.16	0.09
		(3.66)	(3.22)	(2.56)	(2.25)

The entries give the menu costs z necessary to leave the agent indifferent between adjustment and non-adjustment as measured in percent of flexible price revenue. The entries in parentheses give the ratio of the entries in the $\tilde{y} = 5\%$ rows to those in the $x^* = 5\%$ rows.

where we have made use of the agent's budget constraint, $PC_i \doteq P\left[\int_0^1 C_{ij}^{(\epsilon-1)/\epsilon} dj\right]^{\epsilon/(\epsilon-1)} = P_i Y_i$, and the demand function for agent i's product, $Y_i^D = Y(P_i/P)^{-\epsilon}$.

Using (7), (11), and (19), it is straightforward to express the thresholds x^* and $\tilde{y}$ in terms of the underlying structural parameters ϵ and γ as[8]

$$x^* = \left(\frac{\epsilon(\gamma-1)+1}{\gamma-1}\right)\sqrt{\frac{2z}{(\epsilon-1)(\epsilon(\gamma-1)+1)}} \quad \text{and} \quad \tilde{y} = \left(\frac{(\epsilon+1)(\gamma-1)+1}{2(\gamma-1)}\right)\sqrt{\frac{2z}{(\epsilon-1)(\epsilon(\gamma-1)+1)}}$$

5.2. Numerical Results

Consider a numerical experiment along the lines of BR, where we ask how big the menu costs would need to be to just render rigidity an equilibrium given that a 5% monetary shock occurs. Setting $x^* = 0.05$ and solving for the associated menu cost z from the indifference condition (9) yields the values reported by Ball and Romer's (1990, p. 151, Table 1). For comparison, we then set $\tilde{y} = 0.05$ to recover the menu costs that correspond to our threshold equilibrium. Table 1 here compares the different menu costs for different values of the markup implied by the substitution elasticity, ϵ, and the labor supply elasticity implied by γ; reporting both values for the menu costs as well as their ratio.

The results in table 1 would appear to indicate that increases in the markup and/or the elasticity of labor supply increase nominal rigidity associated with both the x^* threshold and the $\tilde{y}$ threshold. This is misleading: A simultaneous increase in the one and decrease in the other parameter of appropriate magnitude will move the two thresholds in opposite directions. Compare, for example, the entries at a labor supply elasticity of 0.15 and a 5% markup with those at an elasticity of 0.05 and a 100% markup: under x^* nominal rigidities have fallen, whereas under $\tilde{y}$ they have risen.

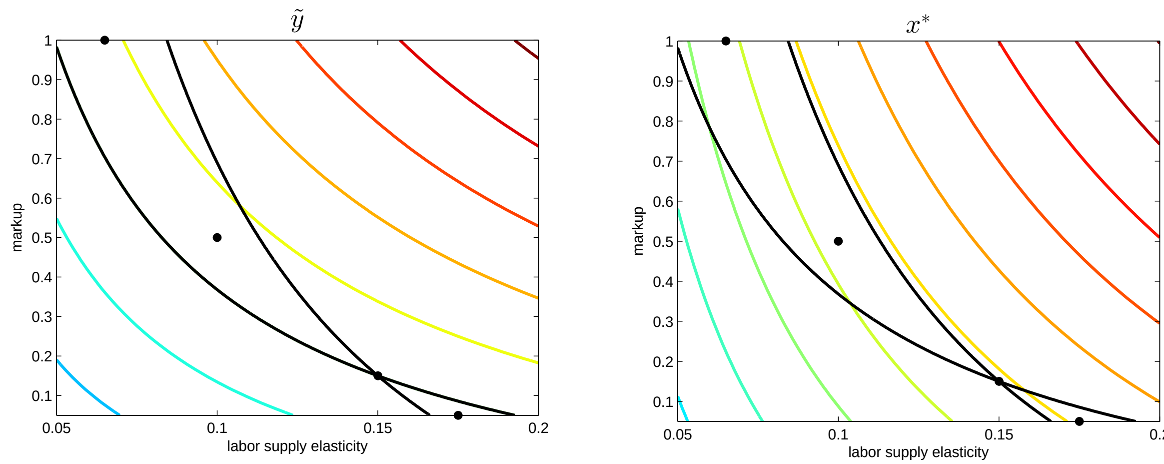
Graphically, this can be seen in figure 4. By tracing contours in the thresholds, the figure outlines the nonlinear region of parameter changes, starting from a 15% markup, labor supply elasticity of 0.15 and z such that $\tilde{y} = 0.05$, associated with the two thresholds moving in opposite directions. Movements towards the northeast imply increases in both thresholds, holding the menu costs fixed. The bounds in proposition 3 give the slopes of these contours at the point associated with the 15% markup and labor supply elasticity of 0.15 and the points in the figure correspond to the pairs of markups and labor supply elasticities considered in table 2. Note that the points are located by construction between the black curves that trace out isolines in the thresholds, such that they are associated with the thresholds moving in opposite direction.

For example, increasing the markup from 5% to 100% while simultaneously decreasing the elasticity of labor supply from 0.175 to 0.065 raises the menu cost required to leave an agent indifferent between adjusting

[8] Note that $W_{12}(1,1) = W_{21}(1,1) = (\epsilon-1)(\gamma-1)$, $W_{22}(1,1) = (\epsilon-1)(\epsilon(1-\gamma)-1)$, and $\beta = 1/\left(\epsilon + \frac{1}{\gamma-1}\right)$.

Figure 4: Threshold Contour Plot

(a) $\tilde{y}$-equilibrium (b) x^*-equilibrium



and not adjusting in response to a 5% monetary shock. The higher menu cost implies that the x^* threshold decreases and nominal rigidity (as measured by x^*) is decreasing. In contrast, $\tilde{y}$ rises as the menu costs are falling, implying that nominal rigidity is actually increasing. Notice that β, inversely related to the degree of real rigidity or strategic complementarities, is increasing here. Thus, the x^* threshold has nominal and real rigidities moving in concert, whereas our $\tilde{y}$ threshold has the two rigidities moving in opposite directions.

The reason behind this difference in the movements of x^* and $\tilde{y}$ can be understood as follows. A large increase in the markup and small decrease in the elasticity of labor supply leads to a substantial decrease in the concavity of the agent's utility function with respect to her relative price, $-W_{22}$, but only a mild decrease in the sensitivity of the optimality condition to a change in real balances, W_{21}. With agents' beliefs fixed on rigidity under the x^* threshold, they are relatively more inclined to offset the change in real balances, thus requiring a larger menu cost to leave them indifferent between adjusting and not adjusting. This same motive, however, implies a stronger reaction of the price level under the $\tilde{y}$ threshold. This reduces the change in real balances and along with the reduced concavity, given by the fall in $-W_{22}$, this renders agents' utility less sensitive to monetary shocks. As shown in table 2, this translates to a reduction in the menu costs required for indifference with a 5% shock to the money supply.

5.3. Discussion

Our numerical comparative statics results are based on BR's stylized yeoman farmer model. This model is essentially a simplified version of Blanchard and Kiyotaki (1987) where the labor market is suppressed. Given that BR preserve the main structural features of Blanchard and Kiyotaki's (1987) model, it stands to reason that our approach to deriving a unique equilibrium can *mutatis mutandis* be also applied to derive a unique equilibrium in their more general framework. Some comments, however, are in order with respect to the assumptions underlying this specific model.

Utility depends linearly on consumption. While this assumption has been quite common in the early New Keynesian macroeconomics literature, it has frequently been viewed as being implausible Woodford (2003,

Table 2: Cost of Adjustment, 5% Monetary Shock

Markup	5%	15%	50%	100%
Labor Supply Elasticity	.175	.15	.1	.065
$x^* = 5\%$	0.67	0.71	0.81	0.93
$\tilde{y} = 5\%$	2.46	2.23	1.84	1.69
β	0.0472	0.1279	0.3226	0.4843

The entries give the size of the menu costs z necessary to leave the agent indifferent between adjustment and non-adjustment as measured in percent of flexible price revenue.

p. 165). In itself, however, linearity is not of critical importance for generating the complementarities in price setting. For example, it is straightforward to show that complementarities still prevail when the utility from consumption is CRRA utility. In this case, β depends on the coefficient of relative risk aversion $\eta > 0$; its explicit closed form expression being $\beta = 1/\left(\epsilon + \frac{1-\eta}{\gamma-(1-\eta)}\right)$. As this depends positively on η, a higher degree of relative risk aversion reduces the degree of strategic complementarities. It can further be shown that the threshold $\tilde{y}$ is decreasing in η, thus, in line with the literature (e.g., Woodford (2003)), leading to a decline in nominal rigidities. However, strategic complementarities and the potential for multiple equilibria persist for standard utility specifications; strategic complementarities give way to strategic substitutes in price setting only when η becomes sufficiently greater than unity.

Moreover, extending the set-up to a richer framework, as done by BR themselves, also implies that conclusions based on comparative statics of the x^* threshold are not generally robust compared to the conclusions derived from a unique equilibrium. As an example, consider BR's customer markets model where consumers are subject to a "home seller" bias and can only observe price changes of their home seller. In such a set-up, demand becomes relatively more sensitive to price increases compared to price decreases. A conclusion derived just from the analysis of the x^* threshold is that customer markets can constitute an important source of nominal rigidities, as strategic complementarities in price setting rise with the relative sensitivity of price increases. However, our equilibrium implies the opposite conclusion in the customer markets model, as a higher sensitivity of demand increases the range of nominal demand shocks where adjustment is an equilibrium (as measured by the bound x^{**}).

6. Conclusion

Menu costs are often invoked to explain the infrequency of price adjustments and nominal rigidities (Golosov and Lucas (2007); Nakamura and Steinsson (2009) provide more recent examples). The existence of strategic complementarities in price setting of agents operating under imperfect competition, as emphasized in Blanchard and Kiyotaki (1987), Ball and Romer (1990, 1991), or Cooper (1999), is a necessary condition for such models to be plagued by multiple equilibria (Cooper and John, 1988). This issue is as relevant for dynamic models as static models (Caballero and Engel, 1993; Dotsey and King, 2005). By and large, the literature has dealt with this problem by focusing on parameter spaces where equilibrium multiplicity does not pose a problem and by deriving the respective parameter conditions that rule multiplicity out. For example, John and Wolman (2008), investigating the uniqueness of equilibria for dynamic state-dependent pricing models in the spirit of Dotsey, King, and Wolman (1999), conclude that the issue of multiplicity is of lesser importance once agents' discount factor is sufficiently close to unity; similarly, Nakamura and Steinsson (2009) assume idiosyncratic shocks of sufficient magnitude to reduce the potential for multiplicity as in Caballero and Engel (1993); likewise Dotsey and King (2005), stress that multiplicity may not always arise, although they point out that the issue is intriguingly complex due to the discontinuity of the equilibrium correspondence. These studies notwithstanding, the fundamental source of equilibrium multiplicity, namely that agents' beliefs are indeterminate, has not yet been addressed in either static or dynamic models. Mankiw and Reis (2010) recognize the potential for applying equilibrium selection methods to models with menu cost, and our analysis is a first attempt in this direction. We employ ideas from the literature on global games that have been successfully applied in other fields of economics where multiple equilibrium problems were prevalent, e.g. currency crises (Morris and Shin, 1998; Heinemann and Illing, 2002) or bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005). Our model thereby belongs to the broader research agenda that seeks a better understanding of implications of multiple equilibria in macroeconomic models (Morris and Shin, 2001).

The main conclusion we derive from the resulting equilibrium is at odds with conventional wisdom that nominal and real rigidities always move in lock-step. We demonstrate that an endogenous response of agents' beliefs can turn the BR result on its head. Thus, our analysis shows that the notion often resounded in the New Keynesian literature that higher degrees of strategic complementarities / real rigidities induce more nominal rigidity and stronger real effects of nominal shocks (Kimball, 1995; Woodford, 2003; Mankiw and Reis, 2010) should be viewed cautiously, especially when fixed costs in price adjustment are present.

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Appendix

The proof of proposition 2 which follows thereafter rests on the following claim which provides an expression of the expected aggregate price level in the limit as $\sigma \to 0$, given the assumption of a symmetric threshold strategy around k.

Claim A1. *Let $E[p|y_i,k] \doteq h(y_i,k,\sigma)$ denote aggregate price level expectations of an agent who observes signal y_i and believes other agents to follow a symmetric threshold strategy around k. Then,*

$$\lim_{\sigma\to 0} h(y_i,k,\sigma) = h(y_i,k,0) = \begin{cases} y_i & \text{if } y_i < -k \\ -\frac{\beta k}{1+\beta} & \text{if } y_i = -k \\ 0 & \text{if } y_i \in (-k,k) \\ \frac{\beta k}{1+\beta} & \text{if } y_i = k \\ y_i & \text{if } y_i > k \end{cases}$$

Proof of Claim A1. Suppose that all other agents use a threshold strategy around some value k, i.e., they adjust if and only if the magnitude of their signal, $|y_j|$ exceeds the value k. Consider agent i who observes signal y_i. Her optimal price is given by

$$E[p_i^*|y_i,k] = \beta y_i + \left(1-\beta\right) E[p|y_i,k] \tag{A1}$$

To calculate the conditionally expected price level $h(y_i,k,\sigma)$, we exploit the symmetry of the price setting problem and use the fact that agent $j \neq i$ with signal y_j uses an optimal price setting rule of the form given by equation (A1) and expects the price level $h(y_j,k,\sigma)$.

Conditional on the threshold strategy around k, agent i calculates,

$$h(y_i,k,\sigma) = (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(y_j,k,\sigma) f(y_j|m)\mathrm{d}y_j\, f(m|y_i)\mathrm{d}m \tag{A2}$$

$$-(1-\beta)\int_{\mathbb{R}}\int_{-k}^{k} h(y_j,k,\sigma) f(y_j|m)\mathrm{d}y_j\, f(m|y_i)\mathrm{d}m \tag{A3}$$

$$+\beta\int_{\mathbb{R}}\int_{\mathbb{R}} y_j f(y_j|m)\mathrm{d}y_j\, f(m|y_i)\mathrm{d}m \tag{A4}$$

$$-\beta\int_{\mathbb{R}}\int_{-k}^{k} y_j f(y_j|m)\mathrm{d}y_j\, f(m|y_i)\mathrm{d}m \tag{A5}$$

where (A3) and (A5) reflect the definition of the threshold strategy, whereby no agent adjusts her price for signals in the interval $[-k,k]$.

With signal noise terms distributed normally, the distributions of y_j conditional on m and vice versa are

$$f(y_j|m) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(y_j-m)^2}{2\sigma^2}\right), \quad f(m|y_i) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-y_i)^2}{2\sigma^2}\right)$$

We now rewrite parts (A2) - (A5) of $h(y_i,k,\sigma)$ to obtain expressions which allow us to study the case where $\sigma \to 0$.

Define $\mu = \left(y_j - m\right)/\sigma$ and perform the change of variables, $d\mu = dy_j/\sigma$. The inner integral of (A2) can be rewritten as

$$\int_{\mathbb{R}} h(y_j,k,\sigma) f(y_j|m)\mathrm{d}y_j = \int_{\mathbb{R}} h(y_j,k,\sigma)\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(y_j-m)^2}{2\sigma^2}\right)\mathrm{d}y_j = \int_{\mathbb{R}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu$$

Inserting this into (A2) and using $\gamma = \left(m-y_i\right)/\sigma$ and another change of variables, $d\gamma = dm/\sigma$, part (A2) becomes

$$\begin{aligned}(1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(y_j,k,\sigma) f(y_j,m)\mathrm{d}y_j\, f(m|y_i)\mathrm{d}m &= (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-y_i)^2}{2\sigma^2}\right)\mathrm{d}m \\ &= (1-\beta)\int_{\mathbb{R}}\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+y_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\end{aligned}$$

Next, we rewrite the inner integral of (A3) as

$$\int_{-k}^{k} h(y_j,k,\sigma) f(y_j|m)dy_j = \int_{-k}^{k} h(y_j,k)\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(y_j-m)^2}{2\sigma^2}\right)\mathrm{d}y_j = \int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu$$

where we have used the change of variables from before. Using $\mathrm{d}\gamma = \mathrm{d}m/\sigma$ again, part (A3) then becomes

$$\begin{aligned}&= -(1-\beta)\int_{\mathbb{R}}\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} h(\sigma\mu+m,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-y_i)^2}{2\sigma^2}\right)\mathrm{d}m \\ &= -(1-\beta)\int_{\mathbb{R}}\int_{\frac{-k-y_i}{\sigma}-\gamma}^{\frac{k-y_i}{\sigma}-\gamma} h(\sigma\mu+\sigma\gamma+y_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\end{aligned}$$

Note further that signals are unbiased, i.e. $\int_{\mathbb{R}} y_j f(y_j|m)\mathrm{d}y_j = m$ and $\int_{\mathbb{R}} m f(m|y_i)\mathrm{d}m = y_i$. Hence, part (A4) simplifies to βy_i.

Finally, we rewrite the inner integral of part (A5) as

$$\int_{-k}^{k} y_j f(y_j|m)\mathrm{d}y_j = \int_{-k}^{k}(y_j - m)f(y_j|m)\mathrm{d}y_j + m\int_{-k}^{k} f(y_j|m)\mathrm{d}y_j$$
$$= \int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \sigma\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{\mu^2}{2}\right)\sigma\mathrm{d}\mu + m\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{\mu^2}{2}\right)\sigma\mathrm{d}\mu$$
$$= \sigma\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu + m\left[\Phi\left(\frac{k-m}{\sigma}\right) - \Phi\left(\frac{-k-m}{\sigma}\right)\right]$$

changing variables using $\mu = (y_j - m)/\sigma$ as before. Substituting this for the inner integral, we can express (A5) as

$$-\beta\left\{\sigma\int_{\mathbb{R}}\int_{\frac{-k-m}{\sigma}}^{\frac{k-m}{\sigma}} \mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-y_i)^2}{2\sigma^2}\right)\mathrm{d}m + \int_{\mathbb{R}} m\left[\Phi\left(\frac{k-m}{\sigma}\right) - \Phi\left(\frac{-k-m}{\sigma}\right)\right]\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-\frac{(m-y_i)^2}{2\sigma^2}\right)\mathrm{d}m\right\}$$

Changing variables using $\gamma = (m - y_i)/\sigma$ as above, (A5) becomes

$$= -\beta\left\{\sigma\int_{\mathbb{R}}\int_{\frac{-k-y_i}{\sigma}-\gamma}^{\frac{k-y_i}{\sigma}-\gamma}\frac{1}{\sqrt{2\pi}}\mu\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma + \int_{\mathbb{R}}(\sigma\gamma + y_i)\left[\Phi\left(\frac{k-y_i}{\sigma}-\gamma\right) - \Phi\left(\frac{-k-y_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right\}$$
$$= -\sigma\beta\int_{\mathbb{R}}\left\{\int_{\frac{-k-y_i}{\sigma}-\gamma}^{\frac{k-y_i}{\sigma}-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu + \gamma\left[\Phi\left(\frac{k-y_i}{\sigma}-\gamma\right) - \Phi\left(\frac{-k-y_i}{\sigma}-\gamma\right)\right]\right\}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$
$$-\beta y_i\int_{\mathbb{R}}\left[\Phi\left(\frac{k-y_i}{\sigma}-\gamma\right) - \Phi\left(\frac{-k-y_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$

Putting all the pieces together, the expectation of the price level conditional on the signal y_i and the threshold strategy around k is

$$h(y_i,k,\sigma) = (1-\beta)\int_{\mathbb{R}}\left[\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+y_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu - \int_{\frac{-k-y_i}{\sigma}-\gamma}^{\frac{k-y_i}{\sigma}-\gamma} h(\sigma\mu+\sigma\gamma+y_i,k,\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma$$
$$+\beta y_i\left(1 - \int_{\mathbb{R}}\left[\Phi\left(\frac{k-y_i}{\sigma}-\gamma\right) - \Phi\left(\frac{-k-y_i}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma\right)$$
$$-\sigma\beta\int_{\mathbb{R}}\left\{\gamma\left[\Phi\left(\frac{k-y_i}{\sigma}-\gamma\right) - \Phi\left(\frac{-k-y_i}{\sigma}-\gamma\right)\right] + \int_{\frac{-k-y_i}{\sigma}-\gamma}^{\frac{k-y_i}{\sigma}-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)d\mu\right\}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)d\gamma$$

Now consider the limit as $\sigma \to 0$. We then have for $y_i \neq k$,

$$h(y_i,k,0) = \begin{cases} y_i & \text{if } y_i < -k \\ 0 & \text{if } y_i \in (-k,k) \\ y_i & \text{if } y_i > k \end{cases}$$

Moreover for $y_i = k$, we calculate

$$h(k,k,\sigma) = (1-\beta)\int_{\mathbb{R}}\left[\int_{\mathbb{R}} h(\sigma\mu+\sigma\gamma+k,k)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu - \int_{-\frac{2k}{\sigma}-\gamma}^{-\gamma} h(\sigma\mu+\sigma\gamma+k,k)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$
$$+\beta k\left(1 - \int_{\mathbb{R}}\left[\Phi(-\gamma) - \Phi\left(-\frac{2k}{\sigma}-\gamma\right)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right)$$
$$-\sigma\beta\int_{\mathbb{R}}\left[\gamma\left[\Phi(-\gamma) - \Phi\left(-\frac{2k}{\sigma}-\gamma\right)\right] + \int_{-\frac{2k}{\sigma}-\gamma}^{-\gamma}\mu\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma \tag{A6}$$

And in the limit,

$$\lim_{\sigma\to 0} h(k,k,\sigma) = h(k,k,0) = (1-\beta)\int_{\mathbb{R}}\left[\int_{\mathbb{R}} h(k,k,0)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu - \int_{-\infty}^{-\gamma} h(k,k,0)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu^2}{2}\right)\mathrm{d}\mu\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma$$
$$+\beta k\left(1 - \int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp(-\frac{\gamma^2}{2})\mathrm{d}\gamma\right)$$

Where the last line in (A6) is zero. Rewriting the right hand side of the above yields

$$= (1-\beta)h(k,k,0)\int_{\mathbb{R}}\left[1-\Phi(-\gamma)\right]\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma + \beta k\left(1 - \int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right)$$
$$= h(k,k,0)(1-\beta)\left(1 - \int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right) + \beta k\left(1 - \int_{\mathbb{R}}\Phi(-\gamma)\frac{1}{\sqrt{2\pi}}\left(-\frac{\gamma^2}{2}\right)\mathrm{d}\gamma\right)$$

Using the fact that $\int_{\mathbb{R}} \Phi(-\gamma) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\gamma^2}{2}\right) d\gamma = \frac{1}{2}$,[9] and we can solve for the conditionally expected price level

$$\lim_{\sigma \to 0} h(k,k,\sigma) = h(k,k,0) = h(k,k,0)(1-\beta)\frac{1}{2} + \beta k \frac{1}{2} \Rightarrow h(k,k,0) = \frac{\beta}{1+\beta} k \tag{A7}$$

□

Proof of Proposition 2. We can now use the previous claim to prove the proposition. Note that an agent's payoff difference between adjusting and not adjusting conditional on her signal y_i and given her assumption that others follow the joint strategy around k is

$$E[PC(m,p,p_i^1,p_i^2)|y_i,k] \approx W_{22}(1,1)\left(\frac{p_i^1 + p_i^2}{2} - E[p_i^*|y_i,k]\right)\left(p_i^1 - p_i^2\right) \tag{A8}$$

as we assume the prices set by each agent are known to her, implying that the expression above is linear in p_i^*, which allows us to pass the expectations operator through. Evaluating the latter at $p_i^1 = E[p_i^*|y_i,k]$, $p_i^2 = 0$ yields

$$E[PC(m,p,E[p_i^*|y_i,k],0)|y_i,k] \approx -W_{22}(1,1)\left(E[p_i^*|y_i,k]\right)^2 \tag{A9}$$

To simplify notation, we abbreviate

$$E[PC(m,p,E[p_i^*|y_i,k],0)|y_i,k] \doteq PC_\sigma(y_i,k)$$

Now note that for k to constitute a threshold equilibrium, an agent with the marginal signal has to be just indifferent between adjusting and not adjusting. That is, the optimal price conditional on observing $y_i = k$ must be such that $PC_0(k,k) = z$. Using the expected aggregate price level provided in claim A1, the optimal price for this case can be written as

$$p_i^* = \beta k + \beta \frac{1-\beta}{1+\beta} k = 2\frac{\beta}{1+\beta} k$$

which, upon inserting this into the payoff difference yields

$$PC_0(k,k) = -W_{22}\left(2\frac{\beta}{1+\beta}k\right)^2 \tag{A10}$$

Equating the latter with the menu cost z, yields

$$\sqrt{\frac{2z}{-W_{22}(1,1)}} = 2\frac{\beta}{1+\beta} k \tag{A11}$$

Solving for k yields the critical signal

$$\tilde{y} \doteq k = \frac{1}{2}\frac{1+\beta}{\beta}\sqrt{\frac{2z}{-W_{22}(1,1)}}$$

which can be written in terms of the thresholds under perfect information, (7) and (9), as expressed in the proposition.

That $\tilde{y}$ constitutes a threshold equilibrium follows by observing that for $|y_i| > \tilde{y}$, $|h(y_i,\tilde{y},0)| > |h(\tilde{y},\tilde{y},0)|$ and conversely for $|y_i| < \tilde{y}$. This implies $PC_0(|y_i|,\tilde{y}) > PC_0(\tilde{y},\tilde{y}) = z$ for $|y_i| > \tilde{y}$ so that agents adjust their price and conversely for $|y_i| < \tilde{y}$.

Finally, uniqueness of the threshold equilibrium is obvious from the expression for $PC_0(k,k)$ provided in (A10): $PC_0(0,0) - z < 0$ and $PC_0(\infty,\infty) - z > 0$ ($PC_0(-\infty,-\infty) - z > 0$) and since $PC_0(k,k)$ strictly increases (decreases) in k for $k > 0$ ($k < 0$) there exists exactly two values k and $-k$ such that $PC_0(k,k) = z = PC_0(-k,k) = z$.

□

Proof of Proposition 3. Differentiate x^* to yield

$$dx^* = -(2z)^{1/2} W_{21}^{-2} (-W_{22})^{1/2} \sum_{i=1}^{I} \frac{\partial W_{21}}{\partial \alpha_i} d\alpha_i - \frac{1}{2}(2z)^{1/2} W_{21}^{-1} (W_{22})^{-1/2} \sum_{i=1}^{I} \frac{\partial W_{22}}{\partial \alpha_i} d\alpha_i \tag{A12}$$

and differentiate $\tilde{y}$ to deliver

$$d\tilde{y} = \frac{1}{2}(2z)^{1/2}\left(-W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial \alpha_i}d\alpha_i - \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i + \frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i\right) \tag{A13}$$

For $d\tilde{y} > 0$, it must hold that

$$-W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial \alpha_i}d\alpha_i - \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i + \frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i > 0 \tag{A14}$$

which can be rearranged as

$$\frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i > W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial \alpha_i}d\alpha_i + \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i \tag{A15}$$

Note that the right hand side of the inequality is $-dx^*(2z)^{-1/2}$ and so $dx^* < 0$ adds

$$\frac{1}{2}(W_{22})^{-3/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i > W_{21}^{-2}(-W_{22})^{1/2}\sum_{i=1}^{I}\frac{\partial W_{21}}{\partial \alpha_i}d\alpha_i + \frac{1}{2}W_{21}^{-1}(W_{22})^{-1/2}\sum_{i=1}^{I}\frac{\partial W_{22}}{\partial \alpha_i}d\alpha_i > 0 \tag{A16}$$

Multiplying through with $2W_{21}(W_{22})^{1/2}$ and recalling the definition of β delivers the expression in the main text. The inequalities for $d\tilde{y} < 0$ but $dx^* > 0$ follow analogously. □

[9] A simple proof of this result can be found in the working paper version of this paper (König and Meyer-Gohde, 2014).