Internal Validation of Temporal Disaggregation: A Cloud Chamber Approach

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Summary

Temporal disaggregation is a recurrent problem in applied econometrics. This paper proposes a novel test approach for checking internal consistency of the disaggregation procedure. This test can serve as a substitute for external validations which deems useful when disaggregating under data poor conditions. The test builds on Chow and Lin's 1971 disaggregation model and rests on the known parameter decay triggered by the temporal aggregation. A simulation study shows that the test indeed provides useful information. Temporal disaggregation of Swiss GDP figures illustrate the approach.

1 Introduction

Quantitative economic analysis very often has to rely on data whose observation frequency is systematically lower than desired. For example, economic activity which is commonly expressed as the flow of value added is generated continuously. However, as it would require enormous resources to actually observe this process, most countries use annual estimates of economic activity as the basis of their statistics. By contrast, many other variables such as money stock and interest rates are available at a far higher frequency (and can often also be observed more accurately). Nevertheless, researchers, policy makers and the public, all have genuine interest in high frequency information about low frequency data for efficient and timely decision making. Therefore, statistical offices all around the world work on providing temporally disaggregated data to serve this aim. Statisticians at the European Commission have even developed a free software tool (ECOTRIM) for conducting temporal disaggregation.

It is evident that the quality of the disaggregation is very important for the users. Unfortunately, an according assessment is in general haunted by the fact that the true high frequency data is unobservable. Observing the true data would, however, be imperative

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for testing the quality of the disaggregation procedure. In many applications external validation procedures serve this aim. For example, a researcher might calculate forecasts of the aggregate, observable data based on disaggregate information. She thereby indistinguishably mixes forecast capability and disaggregation quality, however. Conclusions drawn from such a forecast comparison may therefore not be very informative about the appropriateness of the disaggregation method itself.

This paper proposes a statistical test that checks the internal consistency of a frequently applied disaggregation procedure due to Chow and Lin (1971), henceforth CL.[1] The new test entirely builds on the observable fallout of temporally aggregated, unobservable data and is thus akin to the physicists' approach for inferring the properties of undetectable particles by observing their observable remains after radioactive decay. As the latter are tracked in a "cloud chamber" we call our method cloud chamber approach. This approach can be considered unique in the temporal disaggregation literature.

As a further, albeit admittedly small, advantage, the proposal allows for estimation and testing at the aggregated data level while many CL estimation methods require switching between high and low frequency data.

The next section reviews the disaggregation approach in question, reasons why this model is still very attractive despite its numerous alternatives and sets thus the framework of the analysis. The third section describes the new estimation approach while the fourth outlines the test set-up and provides simulation result. To provide an example of the method, an application of the proposed method to Swiss GDP data is presented in Appendix B which can be downloaded from the journal's website at www.jbnst.de. Finally, conclusions will be drawn.

2 Chow & Lin revisited

2.1 The basic CL model

Aggregate income and industrial production are widely recognised macroeconomic variables which are hence also the subject of intense econometric research. In general, official statistics serve as the benchmark for evaluating econometric models. However, statistical offices typically do not only calculate aggregate income but also its components. The aggregate sum therefore also depends on the quality of the estimates of its parts. In contrast to the total, however, for many GDP components there is only a very limited number of indicators available (if any).[2] Thus, GDP estimation and forecasting as it is discussed in the literature tends to be a very different issue when compared to the actual process which generates the GDP data where "simple" approaches are alive and kicking. In other words, the procedures such as static (Stock/Watson 2002a,b) and dynamic (Mariano/Murasawa 2003; Forni et al. 2004, 2005) factor models, bridge models (possibly with ragged edge) and MIDAS (Ghysels et al. 2006)[3] may turn out very useful for gauging the aggregate value but at the same time they might lend little support to statistical offices when having to calculate this benchmark bottom-up.

[1] The European commission's ECOTRIM software essentially is an implementation of CL's procedure.

[2] As many as 24 indicators are sometimes considered few, see e.g. Caggiano et al. (2011) or Antipa et al. (2012).

[3] See also Kuzin et al. (2011) and Antipa et al. (2012) for comparisons of various approaches.

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A distinct feature of the CL method is its reliance on the knowledge of the structure of the high frequency process. Most alternatives treat this structure to be unknown. Using those alternatives (Wei/Stram 1990; for example) the estimation of the low frequency model based on actual data can therefore be made unrestrictedly. The challenge in turn becomes to identify the structure and the parameters of the underlying high frequency model.

In the CL framework this logic is reversed. CL assume that the high frequency process has a specific structure. Therefore, identification of the low frequency yet not of the high frequency process is an issue. Because the low frequency data usually is available, this identification problem should, in principle, be easier to solve.

Buying the CL assumption might be considered a costly investment. This paper argues that the return on it, however, is the opportunity to test whether or not the CL assumption can be justified. If so, one can take advantage of the arguably simple, straightforward CL disaggregation procedure. Otherwise, one should use one of the alternatives. The remainder of the paper describes a way to facilitate this test.

Let us start with re-stating CL's assumptions about the data generating process for the high frequency observations.

$$\begin{aligned} Y_h &= \beta X_h + U_h \\ U_h &\sim \left(0, \Sigma_h\right) \end{aligned} \tag{1}$$

The endogenous variable, $Y_h = (y_{h,1}, y_{h,2}, \ldots, y_{h,T})'$, is thus assumed to depend on an exogenous variable, or a set of exogenous variables, $X_h = (x_{h,1}, x_{h,2}, \ldots, x_{h,T})'$, and an innovation process, U_h. The subscript h indicates high frequency.

It is important to keep in mind that Y_h cannot be observed by any means. If Y_h is observable, with some delay, for example, it is not worthwhile considering the Chow-Lin approach any further since the well established alternatives discussed earlier may serve the aim of disaggregation better.

The general idea is to use high frequency information on x and an estimate of Σ_h to obtain estimates for $y_{h,t}$ which is not directly observable. In order to estimate $y_{h,t}$, CL suggest a particular structure for Σ_h. Denoting $U_h = (u_{h,1}, u_{h,2}, \ldots, u_{h,T})'$ they suggest to consider the stationary process

$$u_{h,t} = \rho u_{h,t-1} + \epsilon_{h,t} \tag{2}$$

$$|\rho| < 1 \tag{3}$$

$$\epsilon_{h,t} \sim i.i.d.(0, \sigma_h^2).$$

In the CL approach (1) is transformed into a low frequency model with observable data series $y_{l,t}$, $x_{l,t}$ and the error process $\varepsilon_{l,t}$. In the following, the focus will be on the temporal aggregation which is accomplished by pre-multiplying (1) by a matrix C_m of dimension $(T/m \times T)$ where

$$C_m = \begin{pmatrix} 1_{1\times m} & 0_{1\times m} & \ldots & 0_{1\times m} \\ 0_{1\times m} & 1_{1\times m} & \ldots & 0_{1\times m} \\ \vdots & & \ddots & \vdots \\ 0_{1\times m} & \ldots & \ldots & 1_{1\times m} \end{pmatrix}$$

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and m is the number of high frequency observations that are temporally aggregated to yield the low frequency data. For example, if temporal aggregation of quarterly to annual data is considered, $m = 4$ would be chosen. Temporal aggregation of Y_h yields

$$\begin{aligned} Y_l &= \beta X_l + U_l \\ Y_l &= C_m Y_h \\ X_l &= C_m X_h \\ U_l &= C_m U_h \end{aligned} \tag{4}$$

and thus $Y_l = (y_{l,1}, y_{l,2}, \ldots, y_{l,\tau}, \ldots)'$, $X_l = (x_{l,1}, \ldots, x_{l,\tau}, \ldots)'$, and $U_l = (u_{l,1}, \ldots, u_{l,\tau}, \ldots)'$ are temporal aggregates. Therefore, the aggregation only affects the parameters of the error process while the parameter characterising the linear relationship between dependent and independent variable is still completely described by β. The latter is thus independent of C_m for all m.

Temporal aggregation of general, univariate ARIMA processes is discussed in Weiss (1984), Wei and Stram (1990) and Wei (1990), for example, while Marcellino (1999) looks at multivariate autoregressive models. An extension of the CL to multivariate models for disaggregation purposes is offered by Moauro and Savio (2005), for example, while Silvestrini and Veredas (2008) provide a survey of various univariate and multivariate models for temporal aggregation. The CL approach also appears as a special case in the general temporal disaggregation framework due to Dagum and Cholette (2006: chap. 3, 7) in their regression-based additive model.

2.2 Estimation

Chow and Lin (1971) suggest to estimate ρ and β subject to the aggregation constraint (4). They propose the following feasible generalised least squares estimator:

$$\begin{aligned} \hat{\beta}_{CL} &= \left(X_l'\Sigma_l^{-1}X_l\right)^{-1}X_l'\Sigma_l^{-1}Y_l \\ \Sigma_l &= E(U_l U_l') \\ \hat{U}_l &= \left[C_1 - X_l\left(X_l'\Sigma_l^{-1}X_l\right)^{-1}X_l'\Sigma_l^{-1}\right]Y_l \end{aligned} \tag{5}$$

The key element in the estimation is the matrix Σ_l, which can, however, be obtained by considering that

$$\begin{aligned} \Sigma_l &= E(U_l U_l') \\ &= E(C_m U_h U_h' C_m') \\ &= C_m \Sigma_h C_m'. \end{aligned}$$

Since Σ_h is known by assumption and C_m by construction, Σ_l is also identified up to ρ. In general, as Marcellino (1999) has shown, the elements of Σ_l are nonlinear functions of ρ and m. Upon choosing an initial value for ρ estimation of Σ_l can be facilitated by a sequence of linear regressions. Subsequent updates of ρ can be based on a regression of $\hat{u}_{h,t}$ on $\hat{u}_{h,t-1}$ where $\hat{u}_{h,t}$ is the estimated high frequency residual.[4] The resulting estimates for Y_h have desirable properties such as being BLUE (cf. CL).

[4] The details of this calculation are not repeated here. The reader is referred to Chow and Lin (1971: 373) instead.

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This standard CL model can be considered the simplest in the general ARIMA(p,d,q) class considered in Wei and Stram (1990). However, despite its simplicity it appears to be very popular nonetheless. For example, Proietti (2006: 365) claims that the Italian National Statistical Institute bases its temporal disaggregation procedures "primarily, if not exclusively" on this CL procedure.

Recently, Abeysinghe (2000) has pointed out that least squares estimates of ρ will be systematically biased for small values of ρ. This bias vanishes as ρ approaches one. Proietti (2006), by contrast, nests the CL model in a more general state space form and estimates the parameters by maximum likelihood. There is no bias reported in Proietti's 2006 contribution.

3 Restricted arma estimation of the Chow & Lin model

3.1 Alternative estimation model

In the following, an estimation procedure is described which also offers a maximum-likelihood test for the (dis)aggregation model. The key idea of the new approach is to test whether or not the implicit nonlinear restrictions on the parameters of the *low frequency* residual process are statistically acceptable or not. Thus, before turning to the test, the estimation strategy will be sketched.

We start by repeating that the aggregation restriction can alternatively be expressed as a restriction on the error process U_l. Following Wei (1990: 409), temporal aggregation of a covariance stationary autoregressive process of order one results in a new process which can be described as an autoregressive–moving average process $ARMA(p^*, q^*)$ with $p^* = q^*$.[5] Given the fact that β is invariant to C_m the aggregation restriction is thus a restriction on the parameters of the low frequency error process, U_l.

The following lines exemplify the approach for $m = 4$. The basic step is to express ρ^* and ϕ^* in terms of ρ and $m = 4$.[6] Define $z_{h,t} = y_{h,t} - \beta x_{h,t}$ and calculate $z_{l,\tau} = z_{h,t} + z_{h,t-1} + z_{h,t-2} + z_{h,t-3}$. After some algebra, one obtains

$$z_{l,\tau} = \rho^4 z_{l,\tau-1} + u_{l,\tau}, \tag{6}$$

where

$$\begin{aligned} u_{l,\tau} &= \epsilon_{h,t} + (1+\rho)\epsilon_{h,t-1} + (1+\rho+\rho^2)\epsilon_{h,t-2} + (1+\rho+\rho^2+\rho^3)\epsilon_{h,t-3} \\ &\quad + (\rho+\rho^2+\rho^3)\epsilon_{h,t-4} + (\rho^2+\rho^3)\epsilon_{h,t-5} + \rho^3\epsilon_{h,t-6} \end{aligned} \tag{7}$$

and hence $\rho^* = \rho^4$. We now make use of the fact that $u_{l,\tau}$ has an $MA(1)$ structure (Wei 1990: 409)

$$\begin{aligned} u_{l,\tau} &= \phi^* \varepsilon_{l,\tau-1} + \varepsilon_{l,\tau} \\ \varepsilon_{l,\tau} &\sim i.i.d.(0, \sigma_l^2) \end{aligned}$$

[5] See also Abeysinghe (2000: 118) for an application of this conjecture to the CL model.
[6] The online appendix, downloadable from http://www.jbnst.de, derives ρ^* and ϕ^* for general m.

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with

$$E(u_{l,\tau}, u_{l,\tau-s}) = \begin{cases} (1+\phi^{*2})\sigma_l^2 & \text{for } s = 0, \\ \phi^*\sigma_l^2 & \text{for } s \pm 1, \\ 0 & \text{else.} \end{cases} \tag{8}$$

The first and second line can alternatively be expressed as

$$\begin{aligned} (1+\phi^{*2})\sigma_l^2 &= \sigma_b^2 S_0 = \sigma_b^2\big(4 + 6\rho + 8\rho^2 + 8\rho^3 + 8\rho^4 + 6\rho^5 + 4\rho^6\big) \\ \phi^*\sigma_l^2 &= \sigma_b^2 S_1 = \sigma_b^2\big(\rho + 2\rho^2 + 4\rho^3 + 2\rho^4 + \rho^5\big) \end{aligned}$$

where it has been made use of (7) and (8). The solution for ϕ^* can consequently be given as

$$\phi^* = \frac{S_0}{2S_1} \pm \sqrt{\frac{S_0^2}{4S_1^2} - 1}. \tag{9}$$

3.2 Identification and estimation

Wei and Stram (1990) have stressed that serious identification issues arise when trying to derive the disaggregate *ARIMA* model parameters from the aggregate model. For the main purpose of this paper, identification is not really necessary, however. The test of valid aggregation can be carried out for all possible disaggregate models since they will all be equivalent with respect to the properties of their auto-covariance functions. Nevertheless, in order to derive the test statistics some choice will have to be made. The following lines argue in favour of a pragmatic identification strategy.

According to Wei and Stram's 1990 Lemma 2 the identification of the parameters in the CL approach hinges on m only. This is because further complications due to possible complex and multiple real roots are not an issue in CL's disaggregate stationary $AR(1)$ model. In short, applying Wei and Stram's 1990 Lemma 2 we face the following situation. Let r denote the number of AR polynomials in the disaggregate model, then provided that the assumptions of the basic CL model hold, r is found to be

$$r = \begin{cases} 1, \text{ if } m \text{ is odd} \\ 2, \text{ if } m \text{ is even.} \end{cases}$$

Therefore, in the example discussed above two possible solutions arise and hence one more identifying assumption has to be made.

Notice that the square root term in (9) is always positive which can be conjectured from its monotonicity in ρ and looking at the limiting cases of $\rho \to 0$ and $\rho \to \pm 1$ respectively. As it turns out only one of the solutions yields an invertible MA representation. Furthermore, the whole expression will be dominated by the first term since $|\frac{S_0}{2S_1}| > \sqrt{\frac{S_0^2}{4S_1^2} - 1}$ given $\rho \neq 0$. The first term can also be written as $\frac{1}{2\rho}\frac{S_0}{(1+2\rho+4\rho^2+2\rho^3+\rho^4)}$ telling that the sign

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of ρ and the sign of ϕ^* always are the same. The variance of the aggregated error process can be calculated as $\sigma_l^2 = S_0/(1+\phi^{*2})\sigma_h^2$. As expected, the latter implies that ϕ^* can finally be identified as the choice out of the two possible options that always ensures a non degenerate σ_l^2.

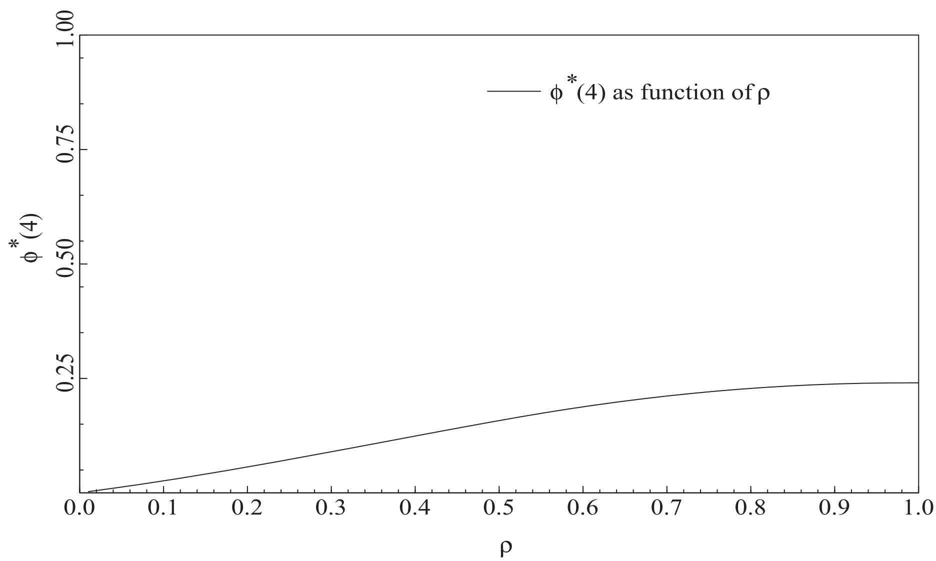
Observing that σ_l^2 approaches zero for $\rho \to 0, \rho > 0$ and $\phi^* = \frac{S_0}{2S_1} + \sqrt{\frac{S_0^2}{4S_1^2} - 1}$, it is reasonable to consider the resulting invertible *MA* coefficient as the true coefficient of the aggregated process. In other words, the additional identifying assumption is to choose the restricted *ARMA* process which yields invertible *MA* coefficients.

This completes the characterisation of the aggregated model. Figure 1 illustrates the results by depicting the relation between ρ and ϕ^* for $m = 4$. The axes span all combinations of non-negative ρ and ϕ. Whenever the aggregation restriction is in place, however, only the points which constitute the solid black line are admissible for the aggregated $(ARMA(1,1))$ process.

As a notational convention we will provide the value of m (in parentheses) if convenient because S_0, S_1, ϕ^*, and ρ^* all depend not only on ρ but also on m.

Finally, (9) implies a restriction of the new approach. It is given by the impossibility to calculate ϕ^* for $\rho = 0$. Special (computational) pre-caution should therefore be taken once values of ρ close to zero are considered.

There are a variety of options available for estimating the model parameters. The model could be cast in state-space form (Durbin/Koopman 2012) or be estimated in its restricted $ARMA(1,1)$ form (Ooms/Doornik 1999; Doornik/Hendry 2001), for example. Actual



Solid black line: The parameter ϕ^* (vertical axis) as a function of ρ (horizontal axis) for $m = 4$.

Figure 1 The aggregation restriction ($m = 4$)

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experience with both approaches led to equally satisfactory results making the final choice a matter of convenience.

Using standard notation, L is the lag operator with $Lx_t = x_{t-1}$, and we set up a model as in (4), but consider the $ARMA(1,1)$ error process

$$(1 - \varrho L) z_{l,\tau} = (1 + \phi L)\varepsilon_{l,\tau} \tag{10}$$

where $\varrho = \rho^*$ and $\phi = \phi^*$ when the aggregation restriction is in place. The corresponding empirical model is denoted $ARMA^*(1,1)$. Furthermore, the parameter vectors $\theta = \begin{pmatrix} \varrho \\ \phi \end{pmatrix}$ and $\theta^*(m) = \begin{pmatrix} \rho^*(m) \\ \phi^*(m) \end{pmatrix}$ provide handy expressions for future use.

4 A parameter decay test of temporal disaggregation

4.1 Temporal aggregation restriction

Once estimated the newly available $ARMA^*(1,1)$ coefficients of the process describing the aggregate data allow us to peek into the characteristics of the otherwise unobservable disaggregate process. In other words, by observing the results of the estimation we can back out the properties of the unobservable, underlying data generating process. Thereby we are able to judge indirectly the appropriateness of the disaggregation procedure itself.

This approach is akin to a widely used method in physics where particles which are beyond our means of recognition leave measurable traces upon decaying into some other sub-particles. Crucially, the sub-particles are visible while their parent particles are not. This is equivalent to the aggregation problem where the unobservable high frequency data leaves its footprint on its observable, aggregate counterpart in the due course of temporal aggregation. Since theory tells physicists the properties of the fallout of the decay depending on the properties of the original particle, a match between theoretical and actual characteristics of the decayed particles lends support to the theory put forth.

In the case of the Chow-Lin process the fallout are the estimated parameters of the restricted $ARMA^*(1,1)$ process for the aggregated data. Starting there it is straightforward to calculate a corresponding maximum likelihood value by filtering the data with the derived $ARMA^*(1,1)$. Based on this value and on the likelihood value of a corresponding unrestricted $ARMA(1,1)$ model, a likelihood-ratio test can be computed that maintains under the H_0 that the restricted $ARMA^*(1,1)$ model is indistinguishable from its unrestricted counterpart.

If the null hypothesis can be maintained, the estimated $ARMA^*(1,1)$ coefficients can be used to identify the coefficients of the underlying disaggregate process. The Figure 2 summarises the underlying concept.

The unobservable domain is populated by the parameter ρ of the underlying disaggregate, high frequency process. In the course of aggregation this parameter decays into the two coefficients of the aggregate $ARMA(1,1)$ process. Quite like decaying particles which are slowed down in a cloud chamber leave characteristic, visible traces the new parameters ρ^* and ϕ^* are observable and should exhibit characteristic properties. These features are subject to scrutiny by standard maximum likelihood tests.

The disaggregation test is hence based on the following pairs of hypotheses

$$H_0 : \theta = \theta^*(m) \text{ vs. } H_1 : \theta \;\; arbitrary, \tag{11}$$

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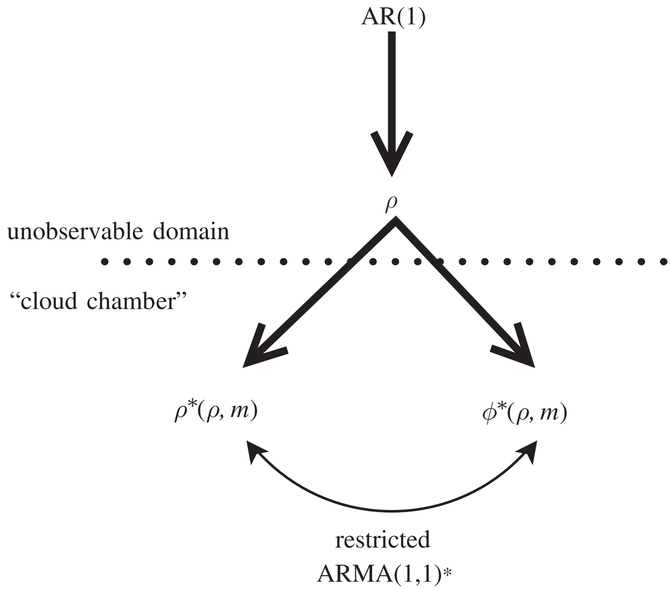

Figure 2 The aggregation-triggered parameter decay in the "cloud chamber"

given $m > 1$. Here, the decision rule is to accept the disaggregation procedure if H_0 cannot be rejected.

More formally, let us denote by ℓ^* the log-likelihood value obtained by estimating (10) and by ℓ the log-likelihood under the alternative. We then suggest to calculate $-2\left(\ell^* - \ell\right)$ and compare this value with the critical value of the χ^2-distribution with one degree of freedom.

4.2 Simulation design

For the current purpose it appears reasonable to test H_0 against some other, general $ARMA(1,1)$ process. In particular, data will be generated by $ARMA(1,1)$ processes (ref. (10)) where either ϱ and ϕ each systematically vary unidirectionally, or where $\theta = \theta^*(m)$ and ρ varies.

This section presents the results of a simulation study that scrutinises the test properties. The artificial data is filtered by the $ARMA^*(1,1)$ and an unrestricted $ARMA(1,1)$ process. A ˜ on top of the estimates indicates the means of the estimators over the random draws. The test results may then be interpreted in the following way. Significance of H_0 indicates that, given m, the $ARMA(1,1)$ error process does not originate in the aggregation of an $AR(1)$ process. In such a case the CL disaggregation procedure should not be applied. On the other hand, if H_0 is not rejected the data can be regarded aggregated and disaggregation by the CL method is appropriate. Besides the test statistics the average coefficient estimates are reported.

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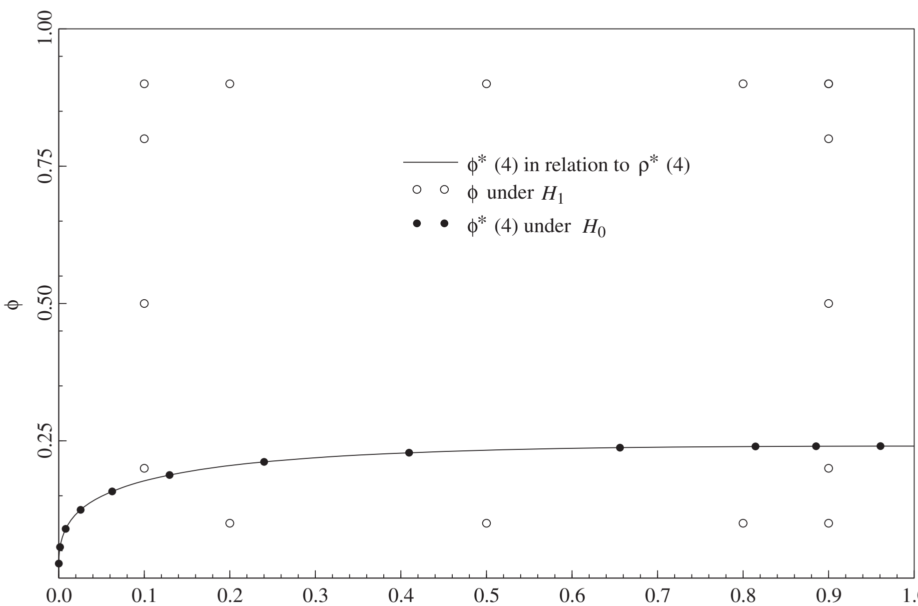
The first simulation regards the size of the test of H_0. Data is generated according to (4) and (10) with $\varrho = \rho^*(4)$, $\phi = \phi^*(4)$, and $x_{l,\tau}$, $\varepsilon_{l,\tau}$ as independent pseudo normal variables. The empirical size for various choices of ρ, and σ_h^2 is compared to its nominal level which is set to .05. In each of the 1000 draws $x_{l,\tau}$ is a zero mean random normal variable with variance σ_h^2. For each draw 1000 (pre-sample) values of $y_{l,t}$ are discarded to reduce the dependency on starting values. The means of testing is a standard likelihood-ratio test being chi-square distributed with one degree of freedom under the null hypothesis and under the assumption of independent tests.

Figure 3 summarises the simulation set-up. The size of the test is investigated along the solid line in Figure 3. This line represents the possible values of $\phi^*(4)$ given ρ and $\rho^*(4)$. The solid circles signify the points at which the size will be investigated. Furthermore, $ARMA(1,1)$ processes are generated under H_1 with choices of ϱ and ϕ represented by the circles off the solid line. Thus, these test results indicate the power of the test against arbitrarily chosen alternatives within the admissible parameter space.

In order to assess the test procedure Tables 2 through 3 give the corresponding rejection frequencies as well as the population means and standard deviations of the estimates for ϱ and β. Note however, for the $ARMA^*(1,1)$ models the reported ϱ correspond to the value in (2).

The calculations were performed using an *Ox3.40* program (Doornik/Ooms 2001) making use of its standard *ARFIMA* package (Ooms/Doornik 1999; Doornik/Hendry 2001:



Horizontal axis: ϱ under H_1, $\rho^*(4)$ under H_0. Vertical axis: ϕ. The circles represent the points at which data is generated under H_1, the solid circles those relating to H_0 in the simulation experiment. The line indicates the possible values of $\phi^*(4)$ given the choice of $\rho^*(4)$.

Figure 3 Simulation design

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chap. 13). It is capable of filtering data by various definitions of ARFIMA models and of calculating the according likelihood function. In combination with a simple hill climbing algorithm estimates for ρ and ϕ^* can be obtained.[7]

4.3 Simulation results

4.3.1 Size of the test

The test level is chosen to be $\alpha = 0.05$ which implies a confidence interval of [3.5%, 6.4%] for the admissible rejection frequency if H_0 holds. We start by reporting the results for large samples ($T/m = 500$) since the test statistic also builds on asymptotic arguments. The simulation results (see Table 1) indicate that the empirical size of the test is well within the expected range. Also for the much smaller sample ($T/m = 100$, see Table 2) the empirical size appears acceptable. There is, however, a tendency to exceed the nominal significance level in this situation. For $\rho > 0.5$ the empirical rejection frequency is slightly above the upper limit (6.5% and 6.8%) of the 95% confidence interval. Overall, the test seems to perform very well with respect to its size.

As regards the choice of the model parameters there does not seem to be a systematic relation to the empirical size. While in the small sample simulation a larger ρ is associated with larger empirical size the opposite is true in the large sample simulation.

Table 1 Parameter estimation and empirical test size in a large sample simulation

$y_{h,t} = 1 + 1.5x_{h,t} + u_{h,t}$, $T/m = 500$
$x_{h,t} \sim N(0,\sigma_h^2)$
$u_{h,t} = \rho u_{h,t-1} + \epsilon_t$
$\epsilon_t \sim N(0,\sigma_h^2)$
$m = 4$, ref.: equation (4)

σ_h^2	ρ	model	H_0	%	$\hat{\rho}$	$\hat{\sigma}_{\hat{\rho}}$	$\hat{\phi}$	$\hat{\sigma}_{\hat{\phi}}$	$\hat{\beta}$	$\hat{\sigma}_{\hat{\beta}}$
1	.5	$ARMA^*(1,1)$	H_0	6.4	.49	.06			1.50	.07
		$ARMA(1,1)$			.13	.61	.17	.22	1.50	.07
	.85	$ARMA^*(1,1)$	H_0	5.2	.85	.02			1.50	.09
		$ARMA(1,1)$			.85	.03	.24	.07	1.50	.09
	.95	$ARMA^*(1,1)$	H_0	5.1	.95	.01			1.50	.09
		$ARMA(1,1)$			.95	.01	.24	.05	1.50	.09

$ARMA^*(1,1)$ denotes the restricted $ARMA(1,1)$ model ($m = 4$). The column headed % signifies the rejection frequency of H_0 in percentage points (if applicable). Columns '$\hat{\rho}$' ('$\hat{\sigma}_{\hat{\rho}}$'), '$\hat{\phi}$' ('$\hat{\sigma}_{\hat{\phi}}$'), '$\hat{\beta}$' ('$\hat{\sigma}_{\hat{\beta}}$') report the average (standard deviation) of the estimated model parameters. Note: Estimates for ρ can be compared to the true parameter only for $ARMA^*(1,1)$. A comparison to $\varrho = \rho^*(4)$ should otherwise be made. The nominal level of significance is 0.05. $R = 1000$ draws.

The accuracy of the point estimates of the β parameter is not affected by the choice of ρ. This can be conjectured from the reported standard deviations. There is no situation where the t-ratio comes close to two, it is always much larger. Quite reasonably, the variance depends on the underlying variance of the innovation process and the variance of $x_{l,\tau}$. In sum, when H_0 is true, the performance of the test appears more or less appropriate.

[7] Proietti (2006) combines grid search with Kalman filtering which would work here as well, but hill climbing turns out faster given the same maximum grid width.

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Table 2 Parameter estimation and empirical test size in a small sample simulation

$y_{h,t} = 1 + 1.5x_{h,t} + u_{h,t}$, $T/m = 100$
$x_{h,t} \sim N(0,\sigma_h^2)$
$u_{h,t} = \rho u_{h,t-1} + \epsilon_t$
$\epsilon_t \sim N(0,\sigma_h^2)$
$m = 4$, ref.: equation (4)

σ_h^2	ρ	model	H_0	%	$\hat{\rho}$	$\hat{\sigma}_{\hat{\rho}}$	$\hat{\phi}$	$\hat{\sigma}_{\hat{\phi}}$	$\hat{\beta}$	$\hat{\sigma}_{\hat{\beta}}$
1	.5	*ARMA**(1,1)	H_0	5.7	.46	.16			1.49	.16
		ARMA(1,1)			.05	.73	.20	.42	1.49	.17
	.85	*ARMA**(1,1)	H_0	6.5	.84	.04			1.49	.21
		ARMA(1,1)			.82	.11	.27	.16	1.49	.21
	.95	*ARMA**(1,1)	H_0	6.8	.94	.02			1.49	.20
		ARMA(1,1)			.94	.02	.27	.12	1.49	.20

*ARMA**(1,1) denotes the restricted *ARMA*(1,1) model ($m = 4$). The column headed % signifies the rejection frequency of H_0 in percentage points (if applicable). Columns '$\hat{\rho}$' ('$\hat{\sigma}_{\hat{\rho}}$'), '$\hat{\phi}$' ('$\hat{\sigma}_{\hat{\phi}}$'), '$\hat{\beta}$' ('$\hat{\sigma}_{\hat{\beta}}$') report the average (standard deviation) of the estimated model parameters. Note: Estimates for ρ can be compared to the true parameter only for *ARMA**(1,1). A comparison to $\varrho = \rho^*(4)$ should otherwise be made. The nominal level of significance is 0.05. $R = 1000$ draws.

4.3.2 Power of the test

When regarding the power of the test against arbitrary alternatives data is generated as an $ARMA(1, 1)$ process with various autoregressive and moving average parameters. The results are collected in Table 3. In general, the difference between the true autoregressive parameter and the supposed $\rho^*(4)$ seems to be less important for obtaining a high rejection frequency than the difference for the moving average component. This can be seen from the first five rows in Table 3 where the power is indeed larger the larger ρ. However, while 100 percent rejection is only approached for ρ close to one, for $\phi > .2$ the empirical power is always 1.

As expected, the point estimates of β are again very accurately estimated. The estimates' variance does hardly vary at all across models and is relatively small. Hence, there is a good chance to correctly determine the related series. This leads to the conclusion that independent of whether or not the true disaggregated model can be identified, forecasting or nowcasting on the basis of the true indicator series is, in principle, feasible.

In the context of parameter estimation for the aggregate model Abeysinghe (2000) has pointed out a possible bias when estimating ρ^* by least squares. Referring to Tables 2 through 3 is becomes evident that this bias is not an issue here. The reason simply is that Abeysinghe's 2000 recommendation of taking care of estimating ϕ^* appropriately has been implemented in the estimation procedure.

4.4 Discussion

The proposed disaggregation test stands out as it is the first one that actually addresses the statistical appropriateness of the disaggregation method by relying on the internal consistency of the properties of the aggregate and disaggregate data. Alternative disaggregation methods use external validation approaches for their justification instead.

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Table 3 Empirical power against arbitrary alternative hypotheses

$y_t = 1 + 1.5x_t + u_t,\ t = 1, 2, \ldots, 100$
$x_t \sim N(0,1)$
$(1 - \varrho L)u_t = (1 + \phi L)\varepsilon_t$
$\varepsilon_t \sim N(0,1)$

ϕ	ϱ	model	H_0	%	$\hat{\varrho}$	$\hat{\sigma}_{\hat{\varrho}}$	$\hat{\phi}$	$\hat{\sigma}_{\hat{\phi}}$	$\hat{\beta}$	$\hat{\sigma}_{\hat{\beta}}$
.1	.2	$ARMA^*(1,1)$	H_0	14.4	.58	.03			1.50	.03
		$ARMA(1,1)$			.61	.22	.11	.11	1.50	.03
	.5	$ARMA^*(1,1)$	H_0	65.7	.79	.01			1.50	.02
		$ARMA(1,1)$			.84	.02	.10	.05	1.50	.03
	.8	$ARMA^*(1,1)$	H_0	97.6	.93	.01			1.50	.02
		$ARMA(1,1)$			.94	.01	.10	.04	1.50	.02
	.9	$ARMA^*(1,1)$	H_0	99.1	.97	.00			1.50	.02
		$ARMA(1,1)$			.97	.00	.10	.03	1.50	.02
.2	.1	$ARMA^*(1,1)$	H_0	6.7	.58	.03			1.50	.03
		$ARMA(1,1)$			.38	.41	.20	.10	1.50	.03
.5		$ARMA^*(1,1)$	H_0	100.0	.76	.01			1.50	.02
		$ARMA(1,1)$			.52	.18	.50	.05	1.50	.02
.8		$ARMA^*(1,1)$	H_0	100.0	.81	.01			1.50	.02
		$ARMA(1,1)$			.55	.07	.80	.02	1.50	.01
.9		$ARMA^*(1,1)$	H_0	100.0	.82	.01			1.50	.02
		$ARMA(1,1)$			.55	.06	.90	.01	1.50	.01
.9	.2	$ARMA^*(1,1)$	H_0	100.0	.84	.01			1.50	.02
		$ARMA(1,1)$			.67	.03	.90	.01	1.50	.01
	.5	$ARMA^*(1,1)$	H_0	100.0	.91	.01			1.50	.02
		$ARMA(1,1)$			.84	.01	.90	.01	1.50	.01
	.8	$ARMA^*(1,1)$	H_0	100.0	.97	.00			1.50	.02
		$ARMA(1,1)$			.94	.01	.90	.01	1.50	.01
	.9	$ARMA^*(1,1)$	H_0	100.0	.98	.00			1.50	.02
		$ARMA(1,1)$			.97	.00	.90	.01	1.50	.01
.2	.9	$ARMA^*(1,1)$	H_0	23.1	.97	.00			1.50	.02
		$ARMA(1,1)$			.97	.00	.20	.03	1.50	.02
.5		$ARMA^*(1,1)$	H_0	100.0	.98	.00			1.50	.02
		$ARMA(1,1)$			.97	.00	.50	.03	1.50	.02
.8		$ARMA^*(1,1)$	H_0	100.0	.98	.00			1.50	.02
		$ARMA(1,1)$			.97	.00	.80	.02	1.50	.01

$ARMA^*(1,1)$ denotes the restricted $ARMA(1,1)$ model ($m = 4$). The column headed % signifies the rejection frequency of H_0 in percentage points (if applicable). Columns '$\hat{\varrho}$' ('$\hat{\sigma}_{\hat{\varrho}}$'), '$\hat{\phi}$' ('$\hat{\sigma}_{\hat{\phi}}$'), '$\hat{\beta}$' ('$\hat{\sigma}_{\hat{\beta}}$') report the average (standard deviation) of the estimated model parameters. The values of ϕ and ϱ are only provided if they change from line to line. The results show that the power of the test rapidly increases with the distance between θ and $\theta^*(4)$. The increase is more rapid in the direction of ϕ. $R = 1000$ draws.

Internal validation is feasible due to the explicit assumptions about the underlying disaggregate data generating process in the CL approach. These assumptions simplify the disaggregation problem considerably which surely contributes to the popularity of the method amongst applied statisticians. Making use of these attractive, simplifying features should be justified by an appropriate statistical test.

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The underlying aggregate data is assumed to follow a $AR(1)$ process. While this assumption may be acceptable in problems where only few observations are available, longer time series may require more sophisticated times series structures. The literature already knows many more general processes upon which to base temporal disaggregation (Dagum/Cholette 2006; for example) for which internal consistency checks could also be considered. As yet, the proposed test cannot be easily applied as it is taylored to CL's $AR(1)$ case. A generalisation also seems desirable to enhance its practical usefulness for statisticians and practitioners.

However, as many more general processes such as $ARMA(p, q)$ can be re-written as $AR(1)$ processes there is reason for optimism that in the future the cloud chamber approach to testing could, in principle, also be generalised to a wider range of models. This generalisation would require an extension of the current univariate to the multivariate case which will certainly prove rather challenging. The biggest challenge for disaggregation, by contrast, is the choice of parameters due to identification issues.

5 Summary and conclusions

This paper suggests a test for internal consistency of temporal disaggregation in the CL framework. This test builds on a novel maximum likelihood estimation of the explicit restricted ARMA representation of the aggregated data generating process. In addition to simplifying the analysis by allowing estimation on the basis of the aggregated data alone, it permits to directly test the aggregation restriction. The test principle rests on checking internal consistency of the disaggregation which is to the best of our knowledge unique in the literature of temporal disaggregation. A simulation study reveals that the test indeed has power against alternative data generating processes and can thus be used to assess appropriateness of the disaggregation method. Future research should be devoted to the generalisation of the approach to higher order and multi-dimensional data generating processes in order to increase its usefulness for statisticians and practitioners.

Appendix

A general formulae

In this Appendix to the article Internal validation of temporal disaggregation: A cloud chamber approach I outline the derivation of ϕ^* as a function of m and ρ given the aggregation problem described in the main text. First, I write (6) for general m and then show how S_0 and S_1 result. The value of ϕ^* then results by applying (9).

The starting point is the notion that any $z_{h,t}$ can be given as

$$z_{h,t} = \rho^n z_{h,t-n} + \rho^{n-1}\epsilon_{h,t-n+1} + \rho^{n-2}\epsilon_{h,t-n+2} + \cdots + \epsilon_{h,t} \tag{A.1}$$

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which implies for temporal aggregation over m periods,

$$
\begin{aligned}
& z_{h,t} + z_{h,t-1} + \cdots + z_{h,t-m+1} \\
& \quad = \rho^m z_{h,t-m} + \rho^{m-1}\epsilon_{h,t-m+1} + \rho^{m-2}\epsilon_{h,t-m+2} + \cdots + \epsilon_{h,t} \\
& \quad + \rho^m z_{h,t-m-1} + \rho^{m-1}\epsilon_{h,t-m} + \rho^{m-2}\epsilon_{h,t-m+1} + \cdots + \epsilon_{h,t-1} \\
& \quad \vdots \\
& \quad + \rho^m z_{h,t-2m+1} + \rho^{m-1}\epsilon_{h,t-2m+2} + \rho^{m-2}\epsilon_{h,t-2m+3} + \ldots \\
& \qquad + \epsilon_{h,t-m+1} \\
& \quad = \rho^m (z_{h,t} + z_{h,t-1} + \cdots + z_{h,t-m+1}) + u_{l,\tau}.
\end{aligned}
$$

$$z_{l,\tau} = \rho^m z_{l,\tau-1} + u_{l,\tau} \tag{A.2}$$

where the error term $u_{l,\tau}$ is the sum of the elements of the $(m \times m)$ matrix Φ_τ:

$$\Phi_\tau = \left[\epsilon_t \, \epsilon_{t-1} \, \ldots \, \epsilon_{t-m+1}\right]' \otimes \left[(\rho L)^{m-1} \, (\rho L)^{m-2} \, \ldots \, (\rho L)^0\right]$$

which makes use of the lag operator, L, with $L^i x_t = x_{t-i}$. It is instructive to expand Φ_τ:

$$\Phi_\tau = \begin{pmatrix} \rho^{m-1}\epsilon_{h,t-m+1} & \rho^{m-2}\epsilon_{h,t-m+2} & \rho^{m-3}\epsilon_{h,t-m+3} & \cdots & \rho^0\epsilon_{h,t} \\ \rho^{m-1}\epsilon_{h,t-m} & \rho^{m-2}\epsilon_{h,t-m+1} & \rho^{m-3}\epsilon_{h,t-m+2} & \cdots & \rho^0\epsilon_{h,t-1} \\ \rho^{m-1}\epsilon_{h,t-m-1} & \rho^{m-2}\epsilon_{h,t-m} & \rho^{m-3}\epsilon_{h,t-m+1} & \cdots & \rho^0\epsilon_{h,t-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{m-1}\epsilon_{h,t-2m+1} & \rho^{m-2}\epsilon_{h,t-2m+2} & \rho^{m-3}\epsilon_{h,t-2m+3} & \cdots & \rho^0\epsilon_{h,t-m+1} \end{pmatrix} \tag{A.3}$$

This matrix has an interesting structure. In particular, the innovations with identical time subscripts are to be found along the diagonals. Thus, the variance of $u_{l,\tau}$ is the sum of the squared sums of the diagonal elements. At the same time the power to which ρ is raised is the same in each column. Therefore, every secondary diagonal can be regarded a truncated version of the main diagonal with respect to the power coefficients. The following auxiliary matrices and operator are useful in finding handy expressions. Let me use the operator *diag* which stacks the main diagonal of a symmetric matrix into a vector. Hence,

$$
\begin{aligned}
\Psi &\equiv 1_{m\times 1}\left[\rho^{m-1} \, \rho^{m-2} \, \ldots \, \rho^0\right] \\
diag(\Psi) &= \left[\rho^{m-1}\rho^{m-2}\rho^{m-3}\cdots\rho^0\right]' \\
&= \psi' \\
H &\equiv \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 1 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 1 & 1 & 1 & \ddots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 1 \end{pmatrix}
\end{aligned}
$$

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where $diag(\Psi)$ and H have dimensions $(m \times 1)$ and $(m \times 2m-1)$ respectively, and $1_{m\times 1}$ is a $(m \times 1)$ vector of ones. Notice that H is essentially a matrix of m rows of a m dimensional column vector of ones within a $(m \times 2m-1)$ matrix of zeros where in each successive row the vector of ones is shifted one column to the right. The product ψH now conveniently collects the $2m-1$ sums of the diagonal elements of Φ_τ in a $(1 \times 2m-1)$ vector omitting for the sake of simplicity the innovation terms. The variance of $u_{l,\tau}$ can now be obtained as

$$\begin{aligned} S_0 &\equiv \psi H H' \psi' \\ E(u_{l,\tau} u_{l,\tau}) &= \sigma_h^2 S_0 \end{aligned}$$

which makes use of the *i.i.d.* property of the ϵ_t.

For deriving S_1, decompose $H = (h_1, 1_{m\times 1}, h_2)$ where h_1 and h_2 are $(m \times m-1)$ matrices collecting the sums of the diagonal elements below and above the main diagonal respectively. Consider now $\Phi_{\tau-1} = L^m \Phi_\tau$ whose sum of elements define $u_{l,\tau-1}$. The value of S_1 is linear in the covariance between $u_{l,\tau}$ and $u_{l,\tau-1}$. Therefore, we need to multiply the sums of the elements above the main diagonal of the matrix $\Phi_{\tau-1}$ with the sums of the elements below the main diagonal of the matrix Φ_τ diagonal by diagonal. With the aid of h_1 and h_2 one can write

$$\begin{aligned} S_1 &= \psi h_1 h_2' \psi' \\ E(u_{l,\tau} u_{l,\tau-1}) &= \sigma_h^2 S_1. \end{aligned}$$

The discussion of the identification issues easily generalises to the case for arbitrary m by noting that again $|\frac{S_0}{2S_1}| > \sqrt{\frac{S_0^2}{4S_1^2} - 1}$ if $\rho \neq 0$ and that $\frac{1}{2\rho}\frac{S_0}{\frac{S_1}{\rho}}$ in general implies identical signs for ρ and ϕ^*. Therefore, in the case of even m one might contemplate choosing the invertible MA coefficient out of the two possible for identifying the disaggregate model. As argued before, identification is ensured for uneven m.

B Application

The following application illustrates the testing approach in a realistic setting. Swiss GDP data is calculated by the Swiss Federal Statistical Office (SFSO). This agency produces data at annual frequency. The latest available data very often is older than one year and substantial revisions occur infrequently. High frequency data of Swiss GDP is provided by the ministry of economic affairs' State Secretariat for Economic Affairs (SECO). This data is derived from the annual aggregate by various indicator estimations and disaggregation procedures. It is important to notice that "genuine" high frequency data for Swiss GDP does not exist. Therefore, it appears worthwhile considering alternative options. The next lines exemplify how the new disaggregation test can be used for identifying potential alternatives.

The aim of the analysis will be framed as follows. Out of a set of potential related series and given the CL model we are interested in the most suitable indicator variable that will be used to facilitate disaggregation.[8] Suitability is measured in terms of testing the

[8] To keep things simple and due to the data poor environment we employ just one such indicator variable.

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disaggregation restriction. If there is more than one appropriate model / indicator variable we also consider the forecast properties.

B1 The regression set-up and the related variables

The disaggregate model (1) is employed with $X_h = (c, x_{h,t}^{(i)}, t)$ representing the vector of exogenous variables. Here, c is a constant term and t is a time trend. The variable $x_{h,t}^{(i)}$, $i = 1, \ldots, 9$ represents the related series of interest. We choose x_i to be the *cumulated sum* of business tendency survey variables calculated with the semantic shock identification method due to Müller-Kademann and Köberl (2008). Upon cumulating we probably obtain a degree-one integrated time series with which GDP cointegrates under the CL assumptions.[9]

More specifically, the data source is a business tendency survey in the manufacturing industry. The two survey questions of interest are related to the firms' capacity utilisation. One asks whether the firm's technical capacities are currently too high, just right, or too low. The other inquires the degree of the capacity utilisation within the past three months in percentage points, where the firms can choose from a range of 50% to 110% in five percentage steps. From the latter we can calculate the percentage change in production capacity from t to $t+1$ and compare this to the judgement about capacity utilisation given by the firm in the previous period, that is in t.

We thus obtain the following combinations of expectations (rows) and corresponding realisations (columns).

Table B1 The basic structure of the data

		realisation		
		+	=	–
judgement	+	pp	pe	pm
	=	ep	ee	em
	–	mp	me	mm

The rows describe the judgement of the firms in t about their current technical capacity; '+' stands for 'too high', '=' for just right, '–' for too low. The columns list the possible outcomes in capacity utilisation changes. A '+' means that the level of capacity utilisation has been augmented between t and $t+1$, a '=' stands for an unchanged level and '–' means a lower level. On the basis of theses expectation–realisation combinations we obtain the nine different qualities, $i = 1, \ldots 9$, of $x_{h,t}^{(i)}$.

The related variables $x_{h,t}^{(i)}$ are eventually obtained as the cumulated sums of the expectation–realisation combinations over time. Let denote by $\chi_{h,t}^{(i)}$ the expectation–realisation combination "pp" as of Table B1 at t for which $i = 1$, say. Then $x_{h,t}^{(1)} = \sum_{j=1}^{t} \chi_{h,j}^{(1)}$. The cumulation will most likely result in a degree-one integrated variable which ensures a balanced equation in model (1) and opens the scope for cointegration. Cointegration

[9] In general, formal tests Johansen (1988, 1992) could be used to justify the cointegration approach.

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corresponds to the assumption $|\rho| < 1$. Cointegration could also be tested formally. Without cointegration forecasts will be very bad which might serve as an informal check in practical applications (see Table B2).

This particular data (without cumulation over time) has been shown to generate very good forecasts for GDP growth in a quarterly time series model (Müller/Köberl 2012). In Müller and Köberl (2012) the empirical model builds, however, on the historical SECO high frequency data and thus blurs the value added by the survey data since the forecasts also depend on the quality of SECO's disaggregation and nowcasting capabilities. The following disaggregation exercise, by contrast, will tell whether or not the survey data can also be used for straightforward disaggregation independent of SECO's input.

B2 Variable selection, estimation and testing

The independent variable is official Swiss real GDP data. Due to changes in the definition of industry classifications in 1998 the survey data is available in a consistent form for 1999 onwards only. The effective sample size is thus 1999 – 2010 as we set aside two observations for (pseudo) ex-ante forecasting. Hence, we are in a data poor environment with little statistical guidance available for choosing the best model. Traditionally, only forecast comparisons, coefficient tests and criteria for model fit might be used. On top of these, we are also able to test for the appropriateness of the disaggregation procedure.

For selecting the most suitable model we estimate the CL model, perform t-test on the related variable, calculate the R^2, mean squared forecast error (MSFE) and the p-value for the new likelihood ratio test statistic. In addition, we also give the estimates of ρ. Values of ρ not too close to one (smaller than 0.9 say) offer some comfort with respect to validity of the implicit covariance-stationarity and cointegration assumptions. The table B2 reports the results.

The estimation results tell a very plausible story. For example, for all "negative" surprises (em, pm, pe) the estimated β coefficient is also negative. In other words, the sequence of cumulated negative impulses reported by the surveyed firms is negatively associated with real GDP. There is one more negative β coefficient (pp) but it is insignificant as is the coefficient for mm which is also a category best described as shock adjustment. All estimated ρ are well below one indicating covariance stationarity. The model fit seems to be very satisfactory with no R^2 being smaller than 92%. However, since there are only eleven observations but three explanatory variables, the large R^2 should not come as a surprise.

When it comes to selecting a model for actual disaggregation, the model fit cannot serve as a way to discriminate between related series. The t-values can be used as arguments for deselecting mm and pp. The t-value for me is just marginally larger than 2, so further information seems desirable. The comparison of forecasting capabilities easily reduces the set of potential related variables further down to me, em and the surprise indicator. Among these three the surprise indicator generates the worst forecasts while at the same time featuring the largest R^2 and t-value on β. The variable em does best in terms of forecasting while me is somewhere in between.

The final decision can now be made on the basis of the disaggregation test. Of these three variables only em results in a CL model that is externally (forecasts, model fit, t-value) as well as internally valid. The latter property can be inferred from accepting the null hypothesis of consistent aggregate and disaggregate data generating processes. The

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Table B2 Disaggregation of annual Swiss GDP to quarterly values

$y_t = c + \beta_i x_{h,t}^{(t)} + t + u_t,$
$u_t = \rho u_{t-1} + \epsilon_t$
$E(\epsilon_t) = 0,\ Var(\epsilon_t) = \sigma^2$
effective sample: 1999q1 – 2012q4
forecasts: 2011q1 – 2012q4

Related variable[a]	$\hat{\rho}$	$\hat{\beta}_i$	t-stat.	R^2	p-value[c]	MSFE
mm	0.79	44.53	0.51	0.92	4.18	1.25
me	0.79	102.5	2.01	0.94	3.80	0.92
mp	0.84	243.7	2.21	0.94	11.59	17780.00
em	0.87	−50.4	−3.24	0.95	6.20	0.63
ee	0.84	50.95	3.84	0.96	9.99	48.76
ep	0.86	60.1	3.75	0.96	5.08	48.47
surprise indicator[b]	0.7	−128.7	−6.61	0.98	2.90	1.07
pe	0.69	−105.3	−3.46	0.96	4.18	29.22
pp	0.79	−36.62	−1.41	0.93	3.69	24.99

[a] The related variables are calculated as percentages of firms which have expected an improvement ("p"), no change ("e"), or a decrease ("m") in their business' performance and experience an improvement ("p"), no change ("e"), or a decrease ("m"). Thus nine combinations of expectations and realisations are observed Kademann and Köberl (2008). All percentages enter the regression as cumulated sums.
[b] The surprise indicator is defined as the combination of positive expectations and negative outcome ("pm").
[c] The p-value (in percent) is given for the $\chi^2(1)$ test statistic on (11), see p. 305.

test statistic implies rejection for me and pm at the five percent level while it cannot be rejected for em. Interestingly, the univariate time series model that is used for the forecast experiment reported in Müller and Köberl (2012) also features the surprise indicator (pm) and em as the only relevant variables out of the nine possible considered in Table B2.

B3 The results

Pictures B1 and B2 on pages 317 and 318 respectively illustrate the outcomes. The first, Figure B1, concerns model selection by showing the different forecast properties of selected potential series. The two top panels compare the forecasts for Swiss GDP for the years 2011 and 2012 by the series ee and em. According to the test statistics the aggregation restriction is valid for both series with p-levels of 9.99% and 6.2% respectively. Therefore, em is preferred over ee. The bottom panel shows the forecast on the basis of the surprise indicator (pm). Overall, the surprise indicator is inferior to em because the aggregation restriction is rejected. In terms of forecasting em and pm are very close.

Comparing the official SECO data to the disaggregation by em reveals that the SECO data appears spikier and somewhat lagging. For example, using the survey data puts the local peak marking the onset of the financial crises to 2008q2 while according to SECO GDP kept expanding one more quarter. Similarly, the consecutive trough came in 2009, first quarter according to the survey data while GDP started to grow again in 2009q2 only if judged by SECO data. Both quarterly data series are disaggregation estimates. The survey data approach, however, can be justified by internal validation on the basis of testing the aggregation restriction.

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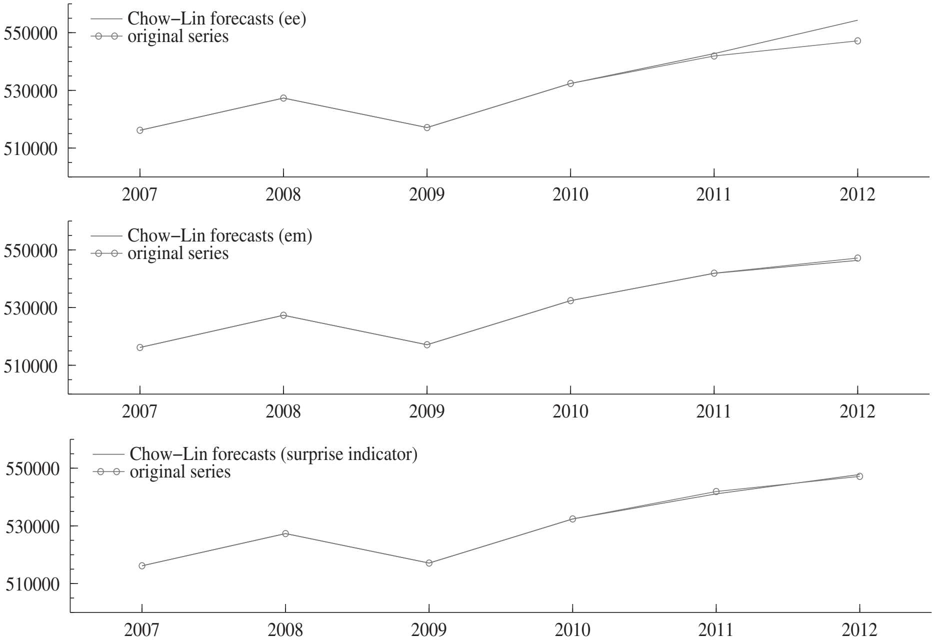

Vertical axis: mill. Swiss Francs, horizontal axis: year. Disaggregation and forecast based on CL model (see equation (1)).

Figure B1 Forecast comparison and temporal disaggregation of Swiss real GDP by the Chow-Lin procedure using survey data

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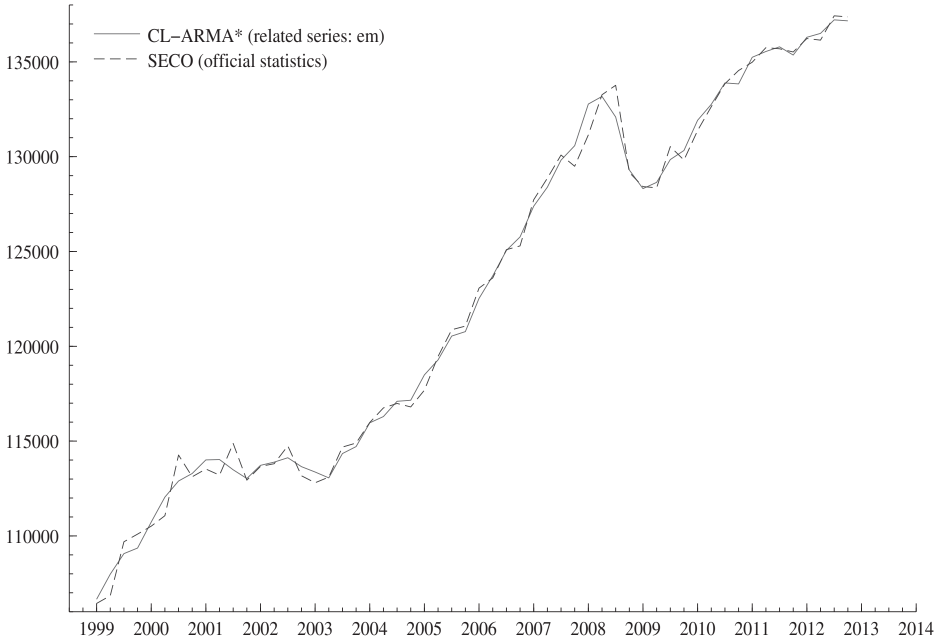

Vertical axis: mill. Swiss Francs, horizontal axis: year and quarter. Disaggregation based on CL model (see (1)) and on SECO methods: http://www.seco.admin.ch/themen/00374/00456/00458/index.html?lang=en.

Figure B2 Temporal disaggregation of Swiss real GDP by the Chow-Lin using survey data and by SECO

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