UNCERTAINTY, FINANCIAL FRICTIONS, AND AGENTS' BELIEFS

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Abstract. Fitting a Markov-switching structural vector autoregression to U.S. data, we show that uncertainty affects real economy differentially depending on the state of financial markets; e.g., an adverse shock that causes a 10 percentage points increase in the VIX index implies a one percent output decline in a regime of financial stress, but effects that are close to zero in tranquil regime. We use this evidence to estimate key parameters of a business cycle model, in which agents are aware of the possibility of regime switches in economic dynamics. We show that the differences in dynamics across regimes do not only result from changes in the degree of financial frictions, but also on agents' beliefs around these changes. Pessimistic beliefs about future financial conditions amplify contractionary effects of uncertainty shocks on aggregate activity.

1. Introduction

Until the onset of the Great Recession, the literature on "business cycles" studies has largely ignored the role of uncertainty as a source of business cycle fluctuations. Total factor productivity, oil, monetary and fiscal factors were considered as traditional sources. The bulk of the evidence in the aftermath of the Lehman crisis has challenged this traditional interpretation of business cycles and thereby stimulated a research agenda on uncertainty fluctuations — notable examples are Bloom (2009) and Stock and Watson (2012).[1] With hindsight, one question is: How could macroeconomists possibly have missed this important source of fluctuations, for over the decades ?

We argue that this lack of attention before the recent crisis results from the fact that uncertainty does not *always* matter. We fit a Markov-switching structural vector autoregression (MS-SVAR) to U.S. data, while maintaining weak identifying assumptions to isolate uncertainty shocks and its effects on economic activity. Our results show changes not only in the variances of structural disturbances over time, but also in the equation coefficients

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[1]See Bloom (2014) for a survey on the research agenda on fluctuations in uncertainty.

that describe the behavior of the economy. In particular, we identify repeated fluctuations in equation coefficients between tranquil and financial stress periods. The latter period was seen in nearly all the years of the recent recession, also sporadically during the 2001-2003 period (the 9/11 terrorist attacks, Dot-com bubble, and corporate scandals), and during episodes of high inflationary pressure in the 1970s. We show that uncertainty affects real economy differentially depending on the state of financial markets. That is, a one standard deviation increase in uncertainty that causes a 25 basis points increase in the VIX index implies a 0.50 percent output decline in the regime of financial stress, but effects that are close to zero in tranquil regime. These amplification effects on output, in the financial stress regime, are also observed in credit spread, the response of which is more than twice as high than its response in tranquil regime.

We use this regime-dependent evidence to estimate the key macroeconomic and financial parameters of a Markov-switching Dynamic Stochastic General Equilibrium (MS-DSGE) model with financial frictions as in Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014). Our empirical approach is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR. Our estimates imply that the differences in impulse responses across regimes result from changes in the degree of financial market frictions. In periods of financial stress, lenders pay monitoring costs — i.e., 12 percent of the realized gross payoff to the firm's capital — much higher than that in tranquil periods — less than 1 percent — to observe an individual borrower's realized return. The implied sensitivity of the elasticity of the external finance premium to the net-worth position becomes, therefore, much higher in crisis state. As a result, after an uncertainty shock that raises the dispersion of entrepreneurs' idiosyncratic productivity, more entrepreneurs draw low-levels of productivity, inducing higher agency costs in the regime of financial stress, and thus making them unable to reimburse their debts. Therefore, credit spreads rise, investment spending falls and output declines. Our estimated model generates macroeconomic dynamics that are consistent with our empirical evidence produced from the SVAR model.

The key insight of our MS-DSGE model is the role of agents' beliefs on the transmission mechanism of uncertainty shocks. Our estimates lies in the fact that agents are aware of the possibility of regime switches in financial markets. That is, our MS-SVAR-based impulse response matching approach takes into account the fact that all agents of the DSGE model know the (smoothed) probabilities assigned by the Markov-switching process of the SVAR model and use them when forming expectations.

Under these circumstances, in any given regime, agents anticipate that uncertainty shocks may lead to switch to the other regime, altering considerably the macroeconomic outcomes. We consider how these expectation effects on a particular regime may affect equilibrium in the other regime. In tranquil periods, characterized by a small degree of agency problems, agents may expect that the economy will move to the financial stress regime. This over-pessimistic behavior, anticipating that the possibility that the agency problems may become more severe in the future, will lead to amplify the small, but notable, contractionary effects of uncertainty shocks on aggregate activity. Conversely, an over-optimistic behavior dampens these negative effects. As a result, agents' beliefs are part of the financial amplification mechanism of uncertainty shocks.

This paper proceeds as follows. Section 2 relates our contributions to the literature. To illustrate the possibility of nonlinearity between uncertainty and the macroeconomy, Section 3 provides some insight into how different the impact of uncertainty on aggregate activity is between financial stress and non-stress periods. Section 4 interprets these differences in terms of an estimated DSGE model with financial frictions, in which agents form expectations on possible changes on the economy, and investigates the expectation effect of regime switching in the degree of financial frictions. Section 5 concludes.

2. Literature review

This paper is related to an increasing literature that examines how uncertainty manifests itself and what their effects are on the rest of the economy.

Focusing on the United States, Bloom (2009), Stock and Watson (2012), Bloom, Floetotto, and Jaimovich (2014), Leduc and Liu (2015), and Basu and Bundick (2015) employ the standard approach (i.e, the "constant-parameters" approach) to quantify the role of uncertainty on business cycle fluctuations. In particular, all studies adopt linear SVARs and find a significant and long-lasting decrease of output after an uncertainty shock. However, their linear VARs rule out, by construction, any time-varying in equation coefficients and shock variances and, therefore, cannot answer directly to the questions that we posed previously, especially the time-varying role of uncertainty as a source of business cycle fluctuations.

Mumtaz and Theodoridis (2016) extend the standard approach by allowing time-varying parameters in SVARs. They emphasize the importance of taking into account shifts in the generation of uncertainty shocks. They show, in particular, the impact of uncertainty shocks on aggregate activity has declined over time. However, the limitation of this paper to study episodes of financial stress, as considered herein, lies in the methodology itself — a model with smooth and drifting coefficients seems to be less suited for capturing rapid shifts in the behavior of the data as observed during the periods of financial stress. Financial crises are well-known for hitting the economy instantaneously, which favors models with abrupt changes like Markov-switching models. Therefore, we follow Sims and Zha (2006) and employ a Markov-switching VARs with Bayesian methods.

Employing an alternative regime-switching method, Alessandri and Mumtaz (2014) show that the real effects of uncertainty shocks strongly depend on the state of financial markets are in when they occur. In particular, they estimate that its impact on output is five times larger in periods of financial stress. Our approach clearly differs since we assign probabilities to events and, therefore, do not make the unrealistic assumption that the probability of a regime switch is either one or zero. Estimating these probabilities is essential to analyse how agents' beliefs on the regime switching impact the transmission mechanism of uncertainty shocks to the aggregate economy.

Our analysis is related to a growing body of evidence which documents the interactions between uncertainty and financial conditions within a equilibrium business cycle framework — notable examples are Arellano, Bai, and Kehoe (2012), Gilchrist, Sim, and Zakrajšek (2014), and Christiano, Motto, and Rostagno (2014). More specifically, our framework closely follows the latter, who investigate the real role of volatility shocks in the context of the financial accelerator model initially developed by Bernanke, Gertler, and Gilchrist (1999). Note, however, that the severity of agency problems (i.e., monitoring costs) remains unchanged over time. Levin, Natalucci, and Zakrajsek (2004) and more recently Fuentes-Albero (2014) and Lindé, Smets, and Wouters (2016) make it time-varying without, however, investigating the macroeconomic implications of uncertainty shocks, and the role of agents' beliefs on agency problems in shaping the macroeconomic outcomes.

Our paper is also related to an increasing literature investigating the role of agents' beliefs in shaping business cycles in a Markov-switching framework. Liu, Waggoner, and Zha (2009) examine the importance of the expectation effect of regime switching in monetary policy in a calibrated New Keynesian model. They show that the possibility of regime shifts in policy can, critically, alter rational agents' expectation formation and, therefore, equilibrium dynamics. Bianchi (2013) have taken a step forward by estimating such a model. He finds that if, in the 1970s, agents had anticipated a more aggressive response to inflation by Federal Reserve, inflation would have been lower. Bianchi and Ilut (2015) extend this approach by allowing monetary/fiscal policy mix changes. More generally, there is a growing literature dealing with DSGE models in which stochastic volatilities and structural parameters are allowed to follow a Markov-switching process. This literature also includes Liu, Waggoner, and Zha (2011), Davig and Doh (2014), Lhuissier and Zabelina (2015), and Lhuissier (forthcoming).

To the best of our knowledge, our paper represents the first attempt to estimate a medium-scale Markov-switching DSGE model by matching the MS-SVAR-implied impulse responses to those produced by the MS-DSGE model. The standard approach for inference of MS-DSGE models — employed by the literature cited above — is to build the state-space representation of the MS-DSGE models adapted from the the standard Kim and Nelson (1999)'s filter. In contrast, our procedure dispense with the standard filter, which mixes both the Hamilton (1989)'s filter and the Kalman filter to build the state-space representation of the model. We believe our MS-SVAR-implied impulse responses approach is a promising tool to infer MS-DSGE models.

3. Evidence of time variation in the effects of uncertainty shocks

This section documents changes in the effects of uncertainty shocks on aggregate activity over time by employing a Markov-switching framework.

3.1. Markov-switching Structural Bayesian VARs.

Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a Markov-switching Bayesian structural VAR model of the following form:

$$y_t' A(s_t) = \sum_{i=1}^{\rho} y_{t-i}' A_i(s_t) + C(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \qquad t = 1, \ldots, T,$$

where y_t is defined as $y_t \equiv [gdp_t, vix_t, sp_t]'$; gdp_t is the logarithm of the interpolated monthly real GDP[2]; vix_t is a proxy for uncertainty; and sp_t is the BAA-AAA credit spread. Data sources are presented in Appendix A. The overall sample period is 1963:M12 to 2013:M12. Based on the monthly VARs literature, we set the lag order to $\rho = 6$.

We assume that ε_t follows the following distribution:

$$E(\varepsilon_t) = \text{normal}(\varepsilon_t | 0_n, I_n), \tag{1}$$

where 0_n denotes an $n \times 1$ vector of zeros, I_n denotes the $n \times n$ identity matrix, and $\text{normal}(x|\mu, \Sigma)$ denotes the multivariate normal distribution of x with mean μ and variance Σ. Finally, T is the sample size; $A(s_t)$ is a n-dimensional invertible matrix under the regime s_t; $A_i(s_t)$ is a n-dimensional matrix that contains the coefficients at the lag i and the regime s_t; $C(s_t)$ contains the constant terms; and $\Xi(s_t)$ is a n-dimensional diagonal matrix.

[2]We employ the Chow and Lin (1971) procedure to interpolate monthly real GDP (gdp_t).

For $1 \leq i, j \leq h$, the discrete and unobserved variable s_t is an exogenous first order Markov process with the transition matrix Q

$$Q = \begin{bmatrix} q_{1,1} & \cdots & q_{1,j} \\ \vdots & \ddots & \vdots \\ q_{i,1} & \cdots & q_{i,j} \end{bmatrix}, \quad (2)$$

where h is the total number of regimes; and $q_{i,j} = \Pr(s_t = i | s_{t-1} = j)$ denote the transition probabilities that s_t is equal to i given that s_{t-1} is equal to j, with $q_{i,j} \geq 0$ and $\sum_{j=1}^{h} q_{i,j} = 1$.

Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior to structural-form parameters. We also introduce dummy observations as a component of the prior in order to favor unit roots and cointegration.[3] For more details, see Doan, Litterman, and Sims (1984) and Sims (1993). Appendix B provides the details techniques for the Sims and Zha (1998) prior.

Finally, the prior duration of each regime is about ten months, meaning that the average probability of staying in the same regime is equal to 0.90. We have also used other prior durations and the main conclusions remain unchanged.

3.2. **Identification.** Identified vector autoregressions decompose the time series variation into mutually independent components. Identification turns out to be extremely important in isolating the effects of a particular shock — uncorrelated to other structural shocks — on the vector of endogenous variables y_t. Since we are studying the macroeconomic effects of uncertainty shocks, particular attention is paid on this structural shock. As the issue of identification is well-known for macroeconomists, we briefly summarize it. Suppose that the VAR process has no constant terms and there is only one regime ($s_t = 1$), such that $\Xi(s_t = 1) = I$. The idea is strictly the same for a MS-VAR model. Using (3.1), the model can be rewritten in a reduced-form VAR as follows:

$$y_t' = x_t' B + \mu_t', \quad (3)$$

with

$$B = F A_0^{-1} \quad \text{and} \quad \mu_t' = \varepsilon_t' A_0^{-1}, \quad (4)$$

where $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' \end{bmatrix}$ and $F = \begin{bmatrix} A_1 & \cdots & A_\rho \end{bmatrix}'$.

[3]Regarding the Sims and Zha (1998) prior, the hyperparameters are defined such that the marginal data density (MDD) of the constant-parameters VAR model is maximized. A grid-search approach is employed to maximize the marginal data density. We obtain the following values: $\mu_1 = 0.57$ (overall tightness of the random walk prior); $\mu_2 = 0.13$ (relative tightness of the random walk prior on the lagged parameters); $\mu_3 = 1.0$ (relative tightness of the random walk prior on the constant term); $\mu_4 = 0.1$ (erratic sampling effects on lag coefficients); $\mu_5 = 5.0$ (belief about unit roots); and $\mu_6 = 5.0$ (belief in cointegration relationships).

The variance–covariance matrix Σ of the reduced-form VAR is a symmetric and positive definite matrix. It defines in the following way:

$$\mathrm{E}[\mu_t \mu_t'] = \Sigma = (A_0 A_0')^{-1}. \tag{5}$$

If there are no identifying restrictions, equations (4) and (5) define a relationship between the structural and reduced-form parameters (B, Σ), which is not unique. One can find two parameter points, (A_0, F) and $(\tilde{A}_0, \tilde{F})$, that are observationally equivalent if, and only if, they imply the same distribution of y_t for $1 \leq t \leq T$. That is, they have the same reduced-form representation (B, Σ) if, and only if, there is a orthonormal matrix P, such that $A_0 = \tilde{A}_0 P$ and $F = \tilde{F} P$.

There is a long tradition in macroeconomics of applying a Cholesky decomposition to the matrix Σ, implying exact linear restrictions on the elements of A_0. This results in a unique solution called a "recursive identification". The recent work by Rubio-Ramírez, Waggoner, and Zha (2010) has shown that the SVAR model is exactly identified. Following this tradition, the contemporaneous matrix A_0 is an upper triangular matrix with the recursive ordering as describe above.

Our identification scheme is as follows. Following previous work by Leeper, Sims, and Zha (1996), we propose that the production sector (output) does not respond contemporaneously to the credit market sector; namely, uncertainty and credit spread. In other words, the credit market sector has only lagged effects on our macroeconomic variable. The argument for this restriction is based on the idea that most firms are subject to planning delays. There are also planning processes involved in changing the prices of labor and manufactured goods.

Finally, the VAR specification assumes that uncertainty and credit spread are ordered second last and last, respectively. This implies that credit spread reacts contemporaneously to every endogenous variable. The justification is not surprising. The financial market-related variables are forward-looking variables, which have a considerable predictive content for economic activity. See, for example, Gilchrist and Zakrajšek (2012). Specifically, our identification scheme is similar to Gilchrist, Sim, and Zakrajšek (2014)

Several approaches to identifying the effects of uncertainty shocks have been proposed in the VAR literature. Bloom (2009), Gilchrist, Sim, and Zakrajšek (2014), Jurado, Ludvigson, and Ng (2015) and Leduc and Liu (2015) identify uncertainty shocks with zero restrictions. Benati (2014) considers both zero and sign restrictions. Caldara, Fuentes-albero, and Gilchrist (2014) identify shocks as innovations explaining the maximum amount of variability in an uncertainty indicator using the penalty function criterion developed by Faust (1998) and Uhlig (2005).

3.3. **Empirical results.** In this section, we report our main empirical results produced by the MS-BVAR model. First, in Section 3.3.1, we estimate and compare various types of models with alternative specifications. Second, we present, in Section 3.3.2, the posterior distribution of the model. We then report, in Section 3.3.3, impulse responses of endogenous variables to our uncertainty shock.

3.3.1. *Model fit.* In order to fit our MS-SVAR model to U.S. data, we estimate and compare various versions of the model with the following specifications:

- $\mathfrak{M}_{\text{constant}}$: Each equation (coefficients and variances) is time-invariant.
- $\mathfrak{M}_{\#\text{v}}$: The variances of all structural disturbances follow the same #-regimes Markov process.
- $\mathfrak{M}_{\#_1\text{c}\#_2\text{v}}$: Each equation allows the coefficients to change under one $\#_1$-regimes Markov process; the variance under another independent $\#_2$-regimes Markov process.
- $\mathfrak{M}_{\#\text{cv}}$: Each equation allows the coefficients and the variance of structural disturbance to change under the same #-regimes Markov process.

The results shown are based on 10 million draws with the Gibbs sampling procedure (see Appendix B for details). We discard the first $1,000,000$ draws as burn-in, then keep every 100th draw. We choose the normalization rule by Waggoner and Zha (2003b) to determine the signs of the columns (or equations) of the matrix A_0 and F. This turns out to be important as it allows us to avoid bimodal distribution in the contemporaneous impulse responses of variables to structural shocks.

The comparison of models is based on marginal data density (also called marginal likelihood), which is a measure of model fit. We employ the Meng and Wong (1996) method of "bridge-sampling" to compute the marginal data density (MDD) for each model [except for the $\mathfrak{M}_{\text{constant}}$ model, for which we employ the Chib (1995) procedure]. Fruhwirth-Schnatter (2004) demonstrates that the bridge method appears to be the most robust method to estimate and compare the marginal likelihood of such mixture models, as Markov-switching models.

Table 1 reports the log-values of MDDs for each model. The constant-parameter model, $\mathfrak{M}_{\text{constant}}$, is clearly rejected. The best-fit model is $\mathfrak{M}_{2\text{c}2\text{v}}$; that is, the model in which the equation coefficients is allowed to change over time independent of time variation in shock variances. The log-values of the MDD associated with this model remain far above the values of the other MDDs mentioned in this paper — a difference of the order of 28 in absolute value compared to the second highest MDD. This is a noticeable difference that dramatically

Table 1. Measure of fit

Model	Specification	Log MDD
$\mathfrak{M}_{\text{constant}}$	Time-invariant model	-1636.70
$\mathfrak{M}_{2v}$	2 synchronized regimes in shock variances	-1349.40
$\mathfrak{M}_{\text{2c2v}}$	2-regimes in all equation coefficients and 2-regimes in shock variances (not synchronized)	-1321.43
$\mathfrak{M}_{\text{2cv}}$	2-regimes synchronized in all equation coefficients and in all shock variances	-1361.43

Note: The method for computing the marginal data densities (MDDs) is the Meng and Wong (1996) method.

supports changes not only in the variance of structural shocks, but also in the systematic component of the behavior of the economy.

These results are robust to alternative methods for computing marginal data densities; namely, the Sims, Waggoner, and Zha (2008) method and the unpublished Muller (2004) method. Although not reported, the MDDs computed from these two methods confirm that the $\mathfrak{M}_{\text{2c2v}}$ model is the one that best fits U.S. data. In the next sections, we will exclusively use this model for our economic implications.

3.3.2. *Posterior distribution.* In this section, we present some key results produced from the best-fit model. Figures 1 and 2 shows the probabilities of a specific regime for each process (s_t^v and s_t^c) over time. The probabilities are smoothed in the sense of Kim (1994); i.e., full sample information is used in getting the regime probabilities at each date.

When looking at the process in which equation coefficients are allowed to change (see s_t^c shown in Figure 1), it is apparent that Regime 1, ($s_t = 1$), was dominant during the age of the 9/11 attacks, Dot-com bubble, and corporate scandals. This regime is also in place during the financial crisis originated by subprime mortgages, as well as during the European debt crisis. We label this regime as the financial stress regime. All of the above-mentioned sub-periods, captured by this regime, contain the same similarities. Regime 1 prevails in periods of major disruption in financial markets. Regime 2 has prevailed for the remaining years of the sample, characterized by episodes of non-stress. This is the tranquil regime.

Regarding the process governing the structural disturbance variances, s_t^v, the model clearly captures two distinct regimes of volatility: a *low-* and a *high-volatility* regimes, as shown in Table 2. Figure 2 displays the (smoothed) probabilities of the *high-volatility* regime. There are repeated fluctuations between the two regimes. While nearing 1 from early 1980s to late 1980s, the probability of the high-volatility regime rapidly falls in early 2004 and remains close to zero until the early 2000s. The high-volatility regime covers sporadically the period

TABLE 2. Relative shock standard deviations across regimes.

	Production gdp	Uncertainty vix	Financial sp
$s_t^v = 1$	1.0000 [1.0000;1.0000]	1.0000 [1.0000;1.0000]	1.0000 [1.0000;1.0000]
$s_t^v = 2$	0.2591 [0.2270;0.2959]	0.2966 [0.2435;0.3538]	0.0868 [0.0755;0.1019]

of 2000-2001 as well as the 2007-2009 recession. These results corroborate with Sims and Zha (2006) and Liu, Waggoner, and Zha (2011).

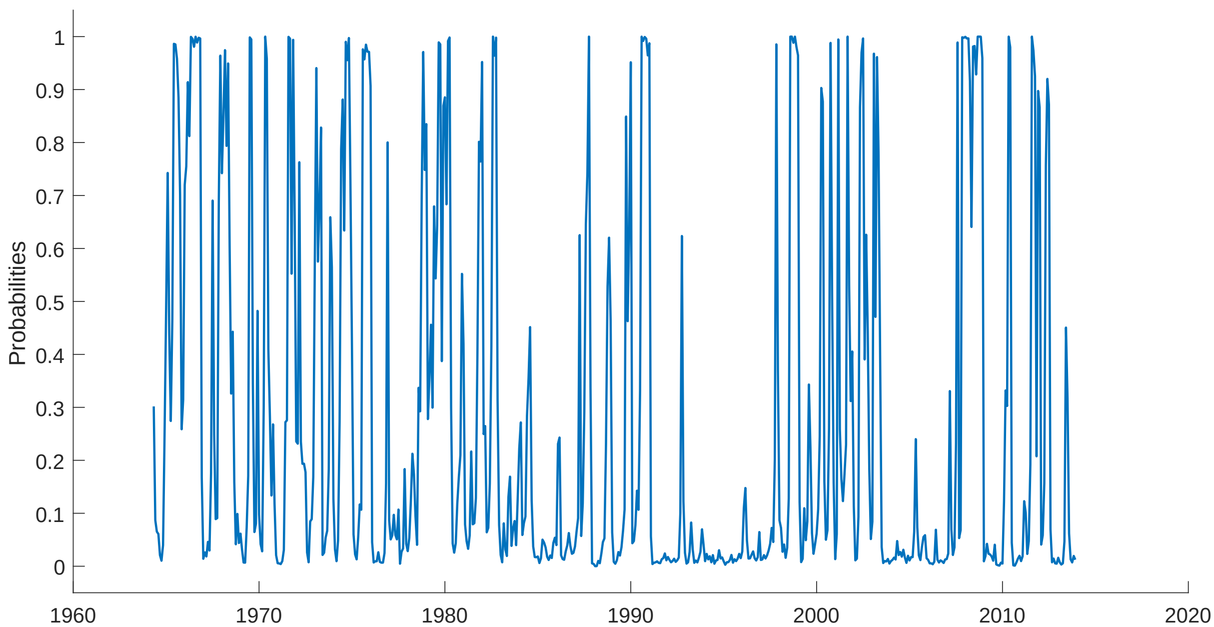

FIGURE 1. Sample period: 1963.M12-2013.M12. Smoothed probabilities of Regime 1 [s_t^c].

The following estimated transition matrices (at the posterior mode) summarize the two Markov-switching processes:

$$Q^{\mathrm{c}} = \begin{bmatrix} \underset{[0.6427;0.7624]}{0.7069} & \underset{[0.0775;0.1259]}{0.1141} \\ \underset{[0.2376;0.3573]}{0.2931} & \underset{[0.8740;0.9225]}{0.8859} \end{bmatrix} \quad \text{and} \quad Q^{\mathrm{v}} = \begin{bmatrix} \underset{[0.8325;0.9151]}{0.8902} & \underset{[0.0323;0.0607]}{0.0366} \\ \underset{[0.0849;0.1672]}{0.1098} & \underset{[0.9393;0.9677]}{0.9634} \end{bmatrix}$$

where Q^{c} denotes the transition matrix governing equation coefficients and Q^{v} the transition matrix for the structural disturbances. The 68% probability intervals are indicated in brackets. Looking at the s_t^c process, the regime of financial stress ($q_{11} = 0.7069$) is much less persistent (an average duration of 3 months) than the tranquil regime ($q_{22} = 0.8859$) which covers most of the sample with an average duration over 8 months. The tight interval probabilities reinforce the estimated mode values.

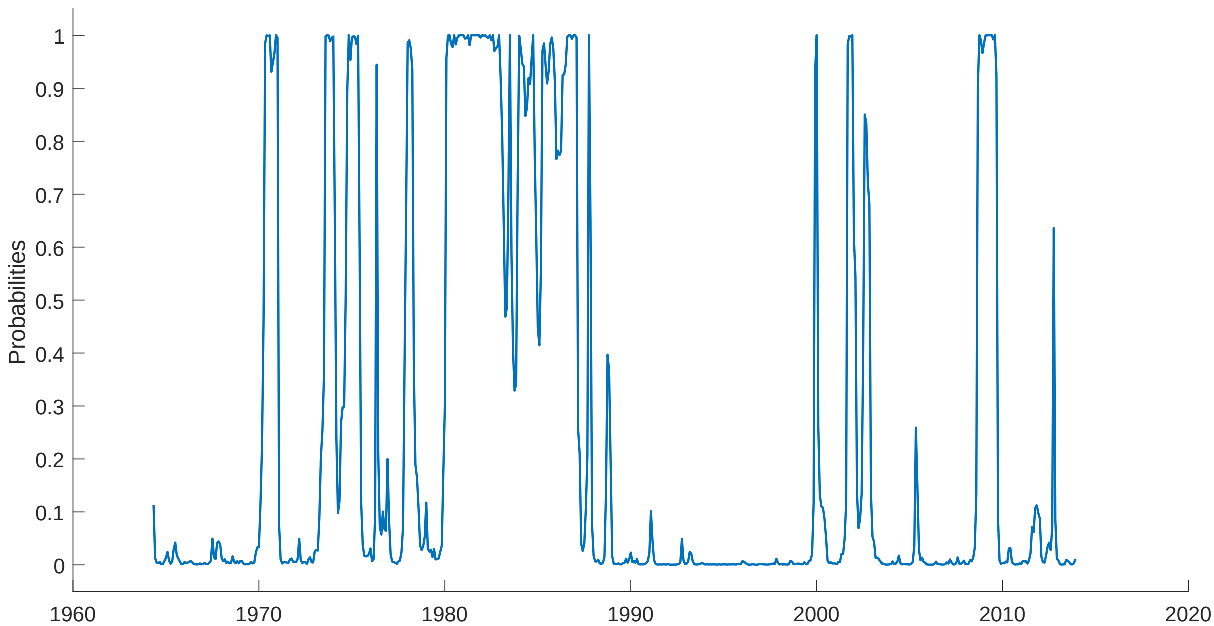

FIGURE 2. Sample period: 1963.M12-2013.M12. Smoothed probabilities of Regime 1 $[s_t^v]$.

In summary, financial stress periods produce shocks whose the size is significantly "larger" than those experienced in non-crisis periods. Furthermore, the behavior of the economy — characterized by the systematic part of the model — in financial stress periods is different from those in tranquil periods.

3.3.3. *Regime-dependent dynamic effects of uncertainty shocks.* As a way to illustrate possible differences in dynamics across the two regimes, we examine the response of the rest of the economy to a disturbance in our uncertainty equation ("one-time uncertainty shock"). Figure 3 report the impulse responses of endogenous variables across the two regimes. The first column shows the responses in the tranquil regime, while the responses in the financial stress regime are displayed in the second column. All of these panels display the deviation in percent for the series entered in log-levels (output), whereas it displays the deviation in percent points (p.p) for other variables (VIX and credit spread). The third column shows the differences between impulse responses of the two regimes. In any column, the solid blue lines represent the median, with the 16th and 84th percentile displayed in dotted black lines. For comparability across regimes, our uncertainty shock is scaled to induce a 25 percentage points immediate increase in the VIX index.

Looking at this figure, the responses of our macroeconomic variable do vary much over time, indicating that the differences among the two regimes in the coefficients of the system of equations are very large. After a positive innovation in our uncertainty measure that

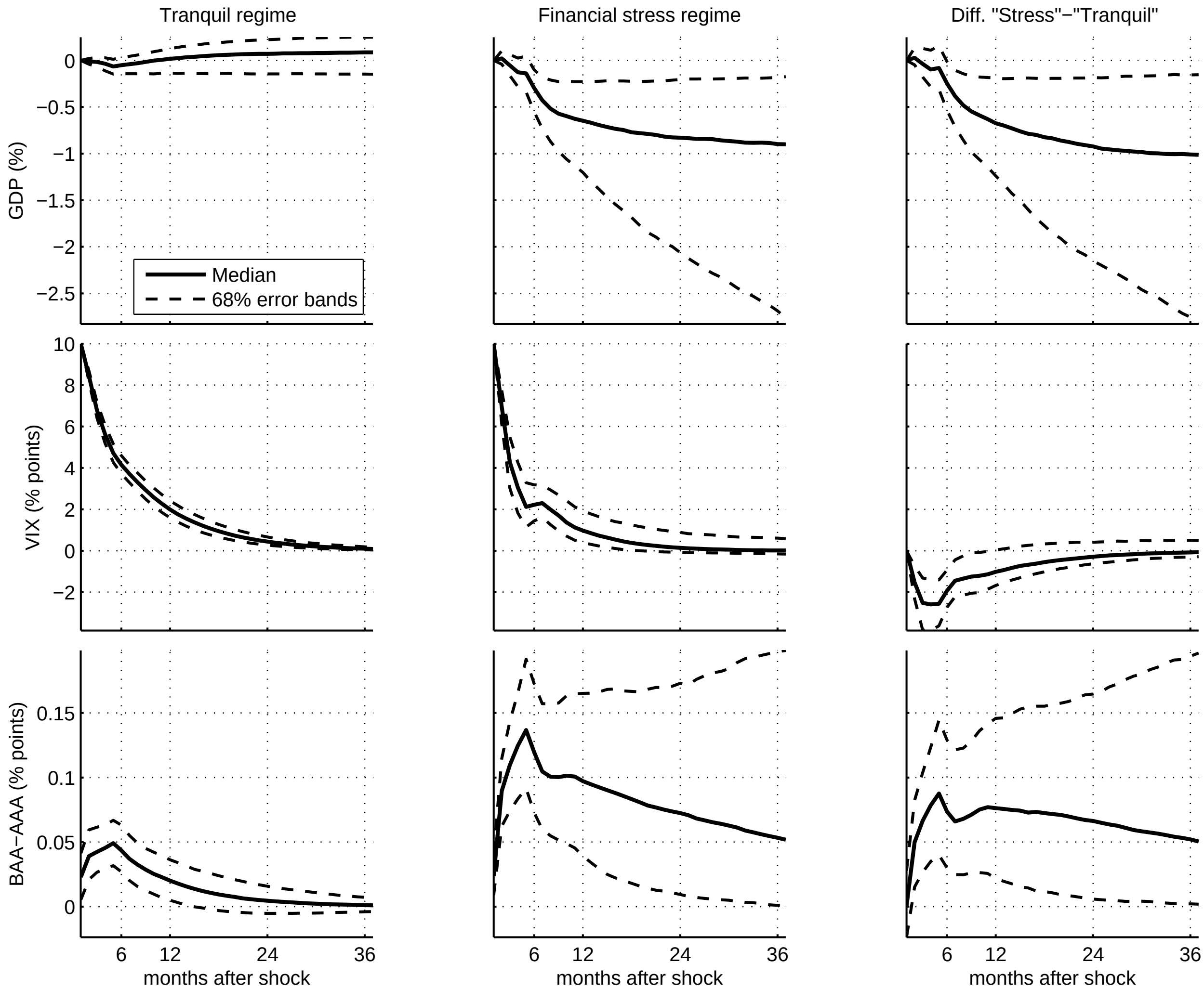

FIGURE 3. Impulse-response functions to uncertainty shock under both regimes obtained from the identified MS-BVAR model. The first and second column report impulse responses of endogenous variables under tranquil and financial stress regimes, respectively. The last column displays the difference between the two regimes. In each case, the median is reported in solid line and the 68% error bands in dotted lines.

causes a 25 percentage points increase in the VIX index, the output remains unchanged in the tranquil regime, but falls quickly and considerably in the financial stress regime, reaches its minimum after 36 months, then begins to recover in a steady manner. These differences seem to be statistically significant when taking into account the 68 percent probability intervals; error bands of the differences lie within the negative region during the entire period.

Interestingly, the response of credit spread is much larger in the financial stress regime, indicating credit costs for firms and households are relatively much tight. Once again, error bands reinforce these results. As a result, we might say that the amplification effects on output occur primarily through changes in credit spreads.

To provide a structural interpretation, we move, in the next section, to inference of a MS-DSGE model by using a our regime-dependent impulse responses obtained from the identified MS-VAR model.

4. A structural interpretation

This section provides a structural interpretation of the empirical results described in section 3. Before discussing the estimation results in Section 4.2, we first present our micro-founded model in Section 4.1.

4.1. **A Markov-switching DSGE Model with financial frictions.**

We develop a DSGE model to interpret our empirical evidence. To account for the regime-dependant effects of uncertainty shocks, our DSGE model should include a source of uncertainty shocks as well as a transmission mechanism to the real economy. We consider a state-of-the-art DSGE model which includes both financial frictions and financial shocks, as defined by Christiano, Motto, and Rostagno (2014). However, we extend their model by allowing regime changes in key macroeconomic and financial parameters in order to capture the regime-dependent evidence, as shown above.

4.1.1. *The SWFF model.* We choose to extend the Smets-Wouters Model with Financial Frictions (henceafter SWFF) model developed by Del Negro, Giannoni, and Schorfheide (2014), which is a log-linearized version of the medium scale DSGE model with real and nominal frictions originally developed by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Sticky nominal prices and wages adjust following a Calvo mechanism with partial indexation. The nominal interest rate is set according to a Taylor rule. The model incorporates a variable capital utilisation and costs of adjusting the capital stock in the production sectors. Households' preferences are characterized by habit formation in consumption. Contrary to Del Negro, Giannoni, and Schorfheide (2014), we consider only one source of business cycles, namely the risk shocks in the entrepreneurial sector recently introduced by Christiano, Motto, and Rostagno (2014), and not other real or nominal shocks. We interpret risk shocks as the theoretical counterparts of the uncertainty shocks identified in the empirical MS-SVAR. The full general equilibrium model is provided in the section C of the Appendix.

4.1.2. *Financial shocks and financial frictions.* Bernanke, Gertler, and Gilchrist (1999) introduce the population of entrepreneurs to determine to equilibrium financial contract between the borrowers and the lenders in the context of a costly-state verification problem as defined by Townsend (1979). At the end of period t, an entrepreneur receives a one-period loan from the lender ans uses it, together with personal wealth, to purchase capital. The purchased capital is then shifted by an idiosyncratic productivity shock, ω, which converts k_{t+1} units of capital into efficiency units ωk_{t+1}. ω follows a cumulative distribution function $F_t(\omega) \equiv F(\omega, \sigma_{\omega,t})$ with a unit-mean and a standard deviation of $\log \omega$ equal to σ_t. The standard deviation $\sigma_{w,t}$ is the result of an exogenous stochastic process defined as "risk shock" by Christiano, Motto, and Rostagno (2014). The debt contract between an entrepreneur and the financial intermediary is based on the costly-state verification framework. The contract is stated in the end of period t, before the realization of the idiosyncratic shock, and is settled in the end of period $t+1$. For every state, defined by the realization of ω, with the associated R^k_{t+1}, a matched entrepreneur has to either (i) pay a state-contingent gross interest rate or (ii) default. If the entrepreneur defaults, the bank seizes all its assets but a fraction μ is used to cover the bankruptcy costs. An entrepreneur pays back the loan if $\omega > \overline{\omega}_{t+1}$, where $\overline{\omega}_{t+1}$ is the productivity threshold. A positive risk shocks corresponds to an increase in $\sigma_{w,t}$ which makes higher the costs of default for lenders. Consequently, the risk premium should be higher to ensure the participation of lenders to the financial contract. The associated rise in the credit spread increases the costs of debt for borrowers and leads to a reduction in the demand for capital responsible a the fall in the price of capital, and then in the investment flow and finally in the level of production.

4.1.3. *A Markov-switching framework.* We proceed in several steps to implement our regime-switching model following Lhuissier and Zabelina (2015). First, because the economy exhibits a trend, we stationarize variables by their corresponding trend.

Second, we compute the steady state of the stationary model and then we log-linearize it around its steady state. It follows that the model can be put in a concise form as follows

$$A f_t = B f_{t-1} + \Psi \varepsilon_t + \Pi \eta_t \tag{6}$$

where f_t is a vector of endogenous components stacking in x_t and a predetermined component consisting of lagged and exogenous variables stacking in z_t. The vector f_t is $f'_t = [x'_t \quad z'_t \quad E_t x'_{t+1}]$. Finally, ε_t is a vector of exogenous shocks and η_t is vector of expectational errors. This represents the `GENSYS` form of the model [see Sims (2001)].

Third, we add an index s_t, corresponding to the regime switches, that governs the time-variation of parameters into the log-linearized model. The model becomes as follows

$$A(s_t) f_t = B(s_t) f_{t-1} + \Psi(s_t) \varepsilon_t + \Pi(s_t) \eta_t, \tag{7}$$

with s_t is an exogenous first-order Markov process, with the same transition probabilities p_{ij} as those described in Section 3.

4.1.4. *Solving MS-DSGE model.* We employ the solution algorithm based on the Mean Square Stable (MSS) concept proposed by Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), and Cho (forthcoming). Such an algorithm allows to take into account the possibility of future regime shifts when forming expectations.

4.2. **Empirical Results.** This section provides the main quantitative results from the estimated MS-DSGE model. First, we present our estimation strategy in Section 4.2.1. Second, we report the estimates of structural parameters in Section 4.2.2. Third, we present, in Section 4.2.3, the impulse responses of major macroeconomic to the risk shock.

4.2.1. *Estimation strategy.* Our estimation strategy is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR[4]. Our empirical analysis matches the estimated impulse responses functions of output and credit spread, but we do not include the VIX index, which is not observable in the theoretical model. To the best of our knowledge, Basu and Bundick (2015) are the first to define the VIX index in a DSGE model, but this requires a third-order approximation to the model policy functions. At this stage, there is no efficient estimation algorithm to allow high-order approximations for MS-DSGE models. Nevertheless, it should be stressed that Foester, Rubio-Ramirez, Waggoner, and Zha (forthcoming) attempt to fill part of this gap using perturbation methods. However, their solution methods is not enough fast and accurate to be used in an estimation algorithm.

Let $\tilde{\xi}$ is a $N \times 1$ vector, which stack the contemporaneous and 15 lagged responses to each of two endogenous variables to the uncertainty shock. The number of elements in $\tilde{\xi}$, N, is, in principle, 2 (i.e., the number of regimes) times 2 (i.e., the number of variables) times 16 (i.e., the number of responses) = 64 elements. Let $\xi(\theta)$ denotes the mapping from θ to the MS-DSGE model impulse response functions, with $\theta = [\theta^1, \theta^2]$ is a $N \times 1$ vector containing all estimated parameters under Regime 1 (θ^1) and under Regime 2 (θ^2). The likelihood function of the data, $\tilde{\xi}$ is defined as as function of θ:

$$f(\tilde{\xi}|\theta, \bar{V}) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |\bar{V}^{-\frac{1}{2}}| \times \left[-\frac{1}{2}(\tilde{\xi} - \xi(\theta)')\bar{V}^{-1}(\tilde{\xi} - \xi(\theta))\right], \tag{8}$$

[4]We thank Christiano, Trabandt, and Walentin (2010) for sharing their code on inference of DSGE models with the standard impulse response matching approach.

where $\bar{V}$ is a diagonal matrix with the sample variances of the $\tilde{\xi}$'s along the diagonal. Conditional on $\tilde{\xi}$ and $\bar{V}$, the Bayesian posterior of θ is as follows:

$$f\left(\theta, \bar{V}\right) \propto f(\tilde{\xi}|\theta, \bar{V}) \times f(\theta), \tag{9}$$

where $f(\theta)$ denotes the priors on θ.

The strategy of estimation begins by maximizing (9) using the `CSMINWEL` program, the optimization routine developed by Christopher A. Sims. Once at the posterior mode, we can start a Markov Chain Monte Carlo method to sample the posterior distribution. More specifically, we employ the Random-walk Metropolis Hasting procedure to generate draws from the joint posterior distribution of the MS-DSGE model. The results shown in the paper is based on 300,000 draws. We discard the first 50,000 draws as burn-in, and every 100th draws is retained.

4.2.2. *Estimates of key parameters.* In order to keep the estimation procedure tractable, we calibrate several parameters. They are set along the line of Del Negro, Giannoni, and Schorfheide (2015), except for p_{11} and p_{22}, which are those obtained, at the mode, from the identified MS-SVAR model. Because our DSGE model is expressed in quarterly frequency, we raise the (monthly) posterior probabilities (p_{11} and p_{22}) to their third power to convert them into quarterly frequency. Table 3 summarizes it.

TABLE 3. Calibration of structural parameters.

α	Capital share	0.1687	π^*	SS quarterly inflation	0.5465
ζ_p	Calvo prices	0.7467	σ_c	elasticity utility	1.5073
ι_p	Price indexation	0.2684	ρ	Taylor rule smoothing	0.8519
Υ	technological progress	1.0000	$F(\omega)$	default rate	0.0300
h	Consumption habit	0.4656	sp^*	SS quarterly spread	1.1791
ν_l	elasticity labor	1.0647	γ^*	survival rate	0.9900
ζ_w	Calvo wages	0.7922	γ	SS quarterly growth rate	0.4010
ι_w	Wage indexation	0.5729	β	Discount factor	0.7420
ψ_1	Taylor rule inflation	1.8678	p_{11}	prob. staying in Regime 1	0.7069^3
ψ_2	Taylor rule output	0.0715	p_{22}	prob. staying in Regime 2	0.8859^3
ψ_3	Taylor rule output growth	0.2131			

Note: Calibration is based on the estimated parameters in Del Negro, Giannoni, and Schorfheide (2015), except for p_{11} and p_{22}, which are those obtained, at the mode, from the identified MS-SVAR model. They are raised to their third power to convert them into quarterly frequency.

Most of them corroborates with those reported in the literature. See, for example, Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005). Regarding the parameters of the financial contract, we set the default rate and the survival rate at 3 percent and 99 percent per annum, respectively. Table 4 reports the specific distribution, the mean and the standard deviation for each estimated parameter. Most of the prior distributions for the parameters follow those in Smets and Wouters (2007).

TABLE 4. Prior and posterior distribution.

		Prior			Posterior		
Coefficient	Description	Density	para(1)	para(2)	Mode	[5;	95]
$\Phi(s_t = 1)$	Fixed costs	N	1.50	0.12	1.4889	1.2847	1.6523
$\Phi(s_t = 2)$	Fixed costs	N	1.50	0.12	1.5036	1.3110	1.7052
$S''(s_t = 1)$	Investment adjustment costs	G	1.00	0.75	0.3825	0.0248	1.6052
$S''(s_t = 2)$	Investment adjustment costs	G	1.00	0.75	0.3930	0.0526	1.9947
$\psi(s_t = 1)$	Elas. capital utilization costs	B	0.50	0.15	0.4823	0.2354	0.7222
$\psi(s_t = 2)$	Elas. capital utilization costs	B	0.50	0.15	0.5096	0.2659	0.7583
$\zeta_{sp}(s_t = 1)$	Elas. financial contract	U	0.00	1.00	0.0375	0.0017	0.3100
$\zeta_{sp}(s_t = 2)$	Elas. financial contract	U	0.00	1.00	0.0000	0.0000	0.0025
$\rho_{\sigma_w}(s_t = 1)$	Persistence risk shock	B	0.75	0.15	0.9527	0.8047	0.9956
$\rho_{\sigma_w}(s_t = 2)$	Persistence risk shock	B	0.75	0.15	0.7334	0.5434	0.7919
σ_{σ_w}	Risk shock	Inv-G	0.05	4.00	0.0417	0.0225	0.0471

Note: N stands for Normal, B Beta, G for Gamma, Inv-G for Inverted-Gamma and U for Uniform distributions. The 5 percent and 95 percent demarcate the bounds of the 90 percent probability interval. Para(1) and Para(2) correspond to the means and standard deviations for the normal, beta and gamma distributions, to ν and s for the inverted-gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, and to the lower and upper bound for the uniform distribution.

A few of them deserve further discussion. In order to capture changes in the way macroeconomic variables respond to risk shock, we allow most of them to have different values between the two regimes. We believe there are two set of candidates for explaining the differences in economic dynamics between both regimes. The first contains three structural parameters which are related to the capital expenditures: fixed costs, adjustment costs of investment and costs of capital utilization. The second comes from the financial frictions in the economy, through the elasticity of credit spread to net worth, ζ_{sp}.

First, we consider the priors for the first set of candidates. The prior for fixed costs, $\Phi(s_t)$, follows a normal distribution, with the mean 1.50 and the standard deviation 0.12. The prior for costs of investment adjustment, $S''(s_t)$, follows a gamma distribution with the mean 1.00

and the standard deviation 0.75. The prior for the costs of capital utilization, $\psi(s_t)$, follows a beta distribution with the mean 0.50 and the standard deviation 0.15.

The prior for the parameter of financial contract, $\zeta_{sp}(s_t)$, is rather dispersed and cover a large parameter space. We employ a uniform distribution defined over $[0;5]$.

The prior distributions of risk shock process is weakly informative. We use a beta distribution for ρ_σ, t, with the mean 0.75 and standard deviation 0.15. Regarding the shock variance, we impose an inverted gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. The hyper-parameters, ν and s, are 0.05 and 4.00, respectively. For obvious reasons, we do not allow the size of the shock to change across the two regimes.

The group of estimated parameters is stacked as follows:

$$\theta = \left[\Phi(k), S''(k), \psi(k), \zeta_{sp}(k), \rho_{\sigma_w}(k), \sigma_{\sigma_w}\right], \quad \text{with} \quad k = \{1, 2\}. \tag{10}$$

The last three columns of Table 4 report the posterior mode with the 90 percent probability interval for each structural parameters. Clearly, none of capital parameters seem to account for the differences in the dynamics between the two regimes. The estimates for $\Phi(s_t)$ and for $\psi(s_t)$ indicates that the posterior mode is closely similar to the mean of the prior, meaning our impulse responses contain little information about fixed and utilization costs of capital in the economy. The estimates for $S''(s_t)$ is about 0.39 in both regimes, much lower than those reported in the literature.

In contrast, the parameter of the financial contract differs considerably between the two regimes. At the posterior mode, its estimates is close to zero in the tranquil regime, but turns out to be relatively high in financial stress regime, for a value of 0.0375. This result implies that, in periods of financial stress, lenders pay monitoring costs — i.e., 12 percent of the realized gross payoff to the firm's capital — much higher than that in tranquil periods — less than 1 percent — to observe an individual borrower's realized return. Clearly, this finding shows that uncertainty shocks only matter for the real economy when the degree of financial frictions is relatively high in the economy.

We can easily recover the values of monitoring costs, denoted $\mu^e(k)$ the deep parameter of the financial accelerator, from $\zeta_{sp}(k)$, the sensitivity of the external finance premium to leverage ratio. For a $\zeta_{sp}(s_t = 1) = 0.0375$, lenders pay monitoring costs which are 12 percent of the realized gross payoff to the firm's capital to observe an individual borrower's realized return. These costs are much higher than those for a $\zeta_{sp}(s_t = 2) = 0.000$ that only implies less than 1 percent of the realized gross payoff to the firm's capital.

Recently, Fuentes-Albero (2014) emphasizes the crucial role played by time-varying monitoring costs in shaping the business cycles. Our approach is, however, substantially different.

Fuentes-Albero (2014) considers, in some way, shocks to the monitoring cost which generates the impulsion at the origin of business cycles, while in our approach, changes in the monitoring cost represents the amplification and propagation mechanisms of risk shocks. In this respect, the Lindé, Smets, and Wouters (2016) specification is closest to our approach; namely they estimate with, full information methods, a DSGE model with financial frictions à la Bernanke, Gertler, and Gilchrist (1999) in which the monitoring cost is allowed to change according to a Markov-switching process. Interestingly, they capture changes in the constraints from the financial frictions, with repeated changes in the monitoring costs between a low (2.90 percent) and high (8.40 percent) value over time. Clearly, these estimated values corroborate with our finding. Finally, Moody's (2016) provides some microeconomic evidence of time-variation in recovery rates, which can be closely compared to our monitoring costs. In particular, this study reports that recovery rates are higher in periods of financial distress.

4.2.3. *Impulse responses.* Figure 4 reports, in red line, the impulse responses of endogenous variables to a risk shock across the two regimes. The first column represents the responses under the tranquil regime, while the second column represents those in the financial stress regime. For comparison purposes, we also present the 68 percent probability intervals of the MS-SVAR model-implied responses.

A number of results are worth emphasizing here. First, the model performs well at accounting for the dynamic responses of the economy to a risk shock. Most of the DSGE model-implied responses lie within the 68 percent probability intervals computed from the MS-VAR model. From a qualitative point of view, the responses of the output and the credit spread in the tranquil regime share some common features with the responses in the financial stress regime. Credit spread and output move in opposite directions; output declines progressively, while credit spread rises immediately and then begins to return its pre-shock level steadily.

The transmission mechanism is straightforward. The risk shock directly alters the degree of risk associated with the asymmetric information between lenders and entrepreneurs who borrow external funds to produce physical capital goods. It moves the dispersion of entrepreneurs' idiosyncratic productivity. With imperfect financial markets, this shock implies higher agency costs since more entrepreneurs draw low levels of productivity and are then unable to reimburse their debts. Then, a positive risk shock increases both the risk of default and the cost of external funds which lead to a fall in the economic activity of entrepreneurs transmitted to the overall economy in general equilibrium trough an increase of the credit

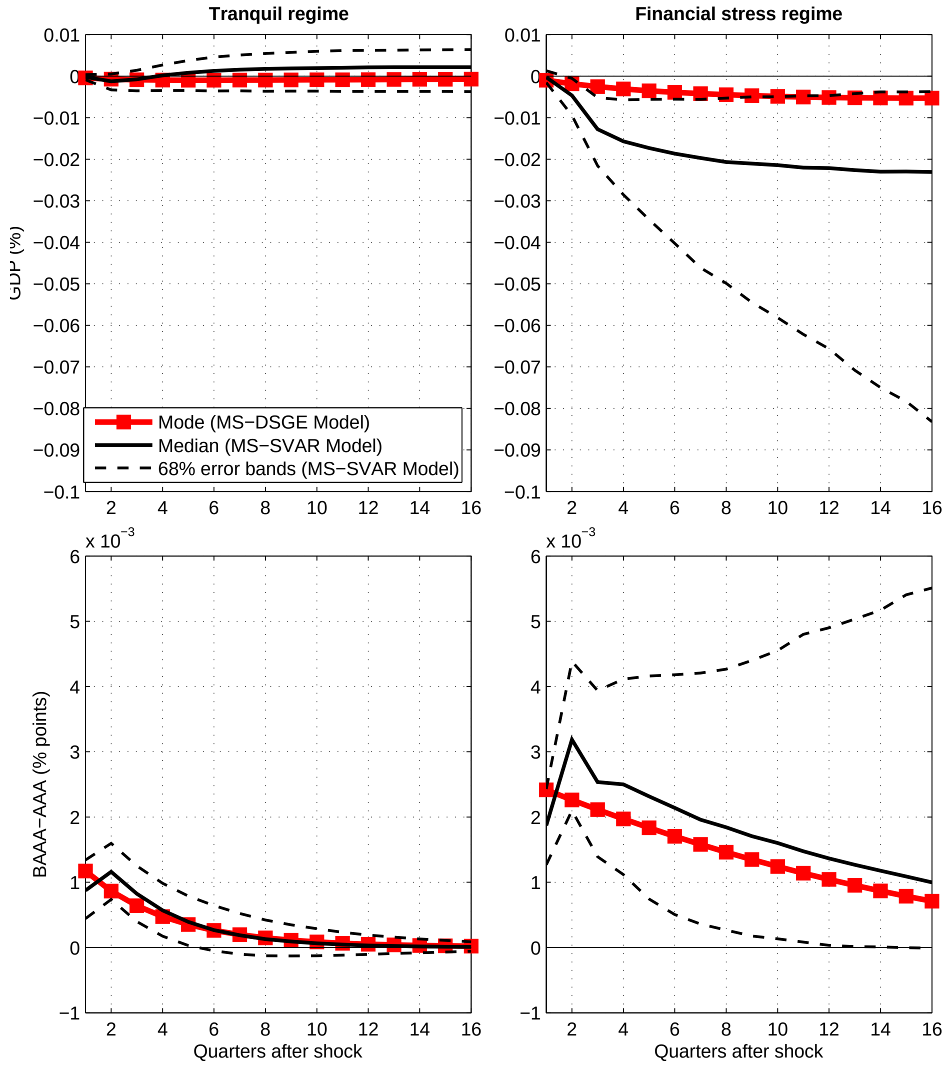

FIGURE 4. Impulse-response functions to a risk (i.e uncertainty) shock. For each regime (i.e., each column), the median responses from the identified MS-BVAR model is reported in solid black line and the 68% error bands in dotted black lines. The red line reports the reponses (at the mode) from the MS-DSGE model with financial frictions.

spread and a fall in investment and production. Say it differently, financial frictions act as the main mechanism through which changes in uncertainty affect macroeconomic variables.

Second, the model succeeds in accounting for the differences in the responses of endogenous variables between the two regimes, expect for the response of output during the 3-6 quarters that follow the shock in financial stress regime. Indeed, there is a notable change in the way both output and credit spread respond to risk shock. Concerning the changes in the impulse responses between the two regimes, the responses under the financial stress regime are remarkably amplified compared to those in the tranquil regime. Under these circumstances, financial frictions act as an amplification mechanism.

This amplification effect can be explained as follows. The elasticity parameter of financial contract, $\zeta_s p$, relates our measure of the external finance premium (i.e., credit spread) to the firm's net worth. Under high stress, $\zeta_{sp}(s_t = 1) = 0.0375$, the premium becomes much more sensitive to a firm's net worth, compared to tranquil periods ($\zeta_{sp}(s_t = 2) = 0.00$). Under these circumstances, a risk shock causes a larger credit spread increases, and therefore, larger and long-lasting negative effects in economic activity. In contrast, when stress is low, the economy is better capable of absorbing the coming economic shocks. As result, the economic effects are less pronounced.

4.3. **The role of agents' beliefs about financial conditions.** In the previous section, we have illustrated the role of financial frictions in propagating risk shocks by comparing economic outcomes of two possible regimes: one regime with a high elasticity of the credit spread to the net worth position, and another regime with a low-degree of financial frictions, i.e., a low elasticity in financial contract. This section gauges the importance of the expectation effect when agents takes into account possible switches between these two financial regimes.

Figure 5 displays the impulse responses of macroeconomic variables following a risk shock when the probability of staying in the same regime varies between 0.00 to 1.00 . Each panels represents the response of a specific variable. When considering $p_{11} = 1$, agents believe that the financial stress regime will last indefinitely while they believe that the tranquil regime will last indefinitely when considering $p_{22} = 1$.

Clearly, the expectation effect plays an important role in shaping the dynamic behavior of macroeconomic variables. As one can see, if agents take into account the effects of possible changes in future financial conditions. The more are agents optimistic about future financial conditions (i.e., gradual moves toward $p_{11} = 1$ and $p_{22} = 0$), the more macroeconomic effects are dampened. Reciprocally, pessimism of agents (i.e., gradual moves toward $p_{11} = 0$ and $p_{22} = 1$) amplify the effects of uncertainty shocks.

The expectation effect embedded in our model shares some features with the anticipation effect described by He and Krishnamurthy (2014) in the context of a model with occasionally binding financial constraints. In the He and Krishnamurthy (2014)'s model, financial constraints have effects on the equilibrium even when they are not binding (which corresponds to the tranquil regime in our model) if agents anticipate that they may bind in the future (which corresponds to the realization of the stress regime in our model). However, their model can be used to compute the conditional probability of falling into a crisis while this probability

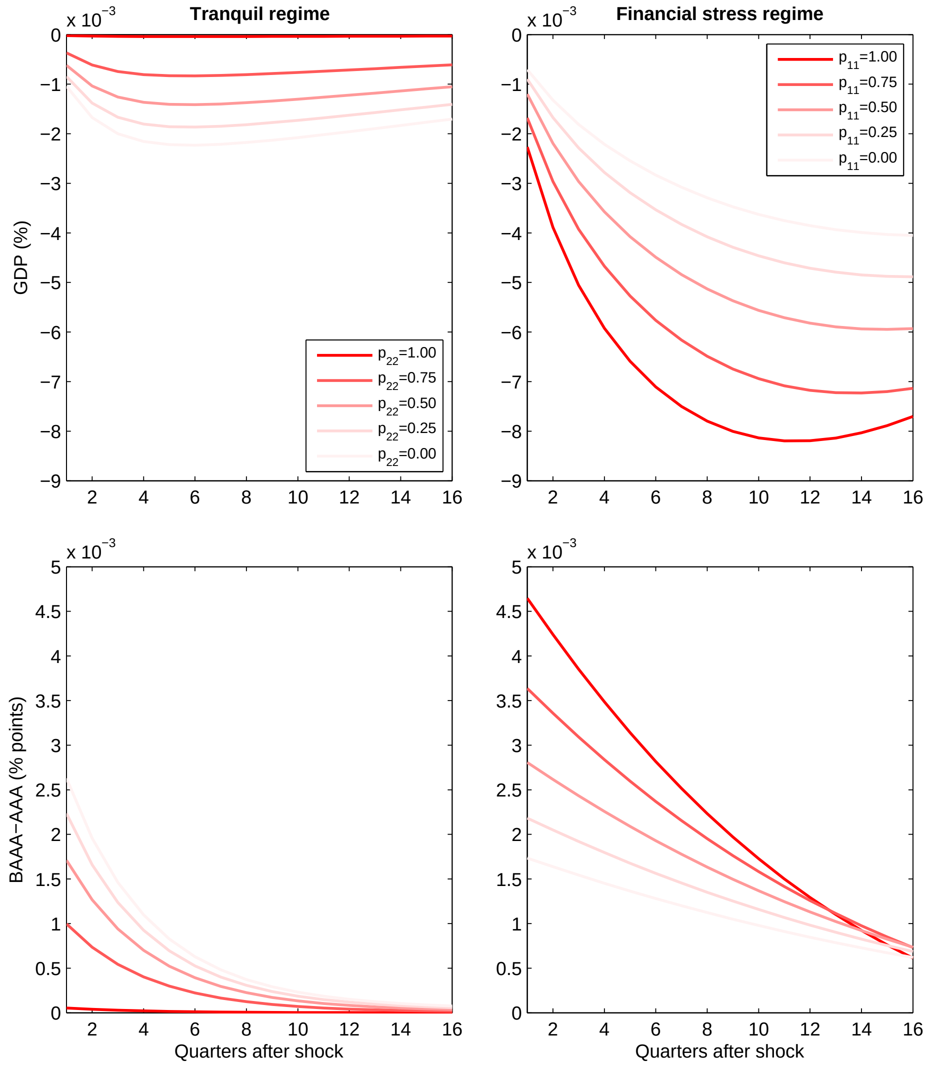

FIGURE 5. Impulse-response functions to uncertainty shock as a function of the probability of staying in the same regime.

is purely exogenous in our model. The expectation effects of exogenous regime shifts on business cycles is emphasized by Liu, Waggoner, and Zha (2009) and Bianchi (2013), but for monetary policy decisions and not for the degree of financial frictions as we do. Nevertheless, we can interpret our results as the outcome of beliefs counterfactuals, using the terminology of Bianchi (2013)[5], but for macroprudential policies instead of monetary policies. Indeed, the bulk of the evidence suggests that macroeconomic policies reduce the frequency and severity of financial crises, see for example BIS (2010), and Benigno, Chen, Otrok, Rebucci, and

[5] In particular, Bianchi (2013) shows that if, in the 1970s, agents had anticipated a more aggressive response to inflation, inflation would have been lower.

Young (2013) and Bianchi and Mendoza (2015) for theoretical models. Hence, if a macro-prudential policymaker is able to manage agents' expectations by limiting the probability of switching from the tranquil to the stress regime, it would be possible to dampen considerably risk shocks in both regimes.

5. Conclusion

Using a Markov-switching Bayesian vector autoregression, this paper has showed that effects of changes in uncertainty on the U.S. economy depend upon the state of financial markets. In a regime of financial stress, the macroeconomic responses are dramatically amplified compared to those in tranquil regime.

Using this regime-dependent evidence, we have estimated a MS-DSGE model, in which agents are aware of the possibility of regime shifts in economic dynamics, to interpret these changes and to explore the role of agents' beliefs. We highlight the importance of expectation effects of regime switching in the degree of financial frictions. Optimistic beliefs about future financial conditions dampen contractionary effects of uncertainty shocks on aggregate activity. Conversely, pessimistic beliefs amplify their effects. Under these circumstances, the ability of a central bank to manage agents' expectations reveals to be crucial in shaping business cycle fluctuations.

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Appendix A. Data

All data are organized monthly from December 1963 to December 2013. Most data comes from Federal Reserve Economic Database (FRED).

- gdp_t: output is the real interpolated GDP (GDPC1). The Chow and Lin (1971) procedure is used to interpolate the real quarterly GDP.
- vix_t: uncertainty is the Chicago Board of Options Exchange Market Volatility Index. From 1963 to 2009, we use the constructed index by Bloom (2009). Then, from 2009, we follow Stock and Watson (2012) and take a monthly average of daily VIX.
- sp_t: credit spread is constructed as the difference between BAA corporate bond yields (BAA) and AAA corporate bond yields (AAA).

For inference, we use the natural log of output. Our spread and uncertainty variables remain unchanged.

Appendix B. Markov-switching Structural Bayesian VAR model

This section provides a detailed description of the Bayesian inference employed in this paper. More specifically, we closely follow Sims, Waggoner, and Zha (2008).

B.1. **The posterior.** Before describing the posterior distribution, we introduce the following notation: θ and q are vectors of parameters where θ contains all the parameters of the model (except those of the transition matrix) and $q = (q_{i,j}) \in \mathbb{R}^{h^2}$. $Y_t = (y_1, \ldots, y_t) \in (\mathbb{R}^n)^t$ are observed data with n denoting the number of endogenous variables and $S_t = (s_0, \dots, s_t) \in H^{t+1}$ with $H \in \{1, \dots, h\}$.

The log-likelihood function, $p(Y_T|\theta, q)$, is combined with the prior density functions, $p(\theta, q)$, to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

B.1.1. *The likelihood.* Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a class of Markov-switching structural VAR models of the following form:

$$y_t' A(s_t) = x_t' F(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \tag{11}$$

with $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' & 1 \end{bmatrix}$ and $F(s_t) = \begin{bmatrix} A_1(s_t) & \cdots & A_\rho(s_t) & C(s_t) \end{bmatrix}'$.

Let $a_j(k)$ be the jth column of $A(k)$, $f_j(k)$ be the jth column of $F(k)$, and $\xi_j(k)$ be the jth diagonal element of $\Xi(k)$. The conditional likelihood function is as follows:

$$p(y_t|s_t, Y_{t-1}) = |A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left(y_t' a_j(s_t) - x_t' f_j(s_t)\right)^2\right). \tag{12}$$

To simplify the Gibbs-sampling procedure described in the next section, it is preferable to rewrite the condition likelihood function with respect to free parameters from matrix $A(s_t)$ and $F(s_t)$:

$$|A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left((y_t' + x_t' W_j) U_j b_j(s_t) - x_t' V_j g_j(s_t)\right)^2\right), \tag{13}$$

where $a_j(s_t) = U_j b_j(k)$ and $f_j(s_t) = V_j g_j - W_j U_j b_j(k)$ is a result from the linear restrictions $R_j \begin{bmatrix} a_j & f_j \end{bmatrix}' = 0$; and U_j and V_j are matrices with orthonormal columns and W_j is a matrix. See Waggoner and Zha (2003a) for further details.

The log likelihood function is given by

$$p(Y_T|\theta, q) = \sum_{t}^{T} \ln\left\{\sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) \Pr\left[s_t|Y_{t-1}\right]\right\}, \tag{14}$$

where

$$\Pr\left[s_t = i|Y_{t-1}\right] = \sum_{j=1}^{h} \Pr\left[s_t = i, s_{t-1} = j|Y_{t-1}\right] \tag{15}$$

$$= \sum_{j=1}^{h} \Pr\left[s_t = i|s_{t-1} = j\right] \Pr\left[s_{t-1} = j|Y_{t-1}\right]. \tag{16}$$

with $q_{i,j} = \Pr[s_t = i | s_{t-1} = j]$ are the transition probabilities from the $h \times h$ matrix Q

$$Q = \begin{bmatrix} q_{1,1} & \cdots & q_{1,j} \\ \vdots & \ddots & \vdots \\ q_{i,1} & \cdots & q_{i,j} \end{bmatrix} \tag{17}$$

The probability terms are updated as follows:

$$\Pr[s_t = j | Y_t] = \Pr[s_t = j | Y_{t-1}, y_t] = \frac{p(s_t = j, y_t | Y_{t-1})}{p(y_t | Y_{t-1})} \tag{18}$$

$$= \frac{p(y_t | s_t = j, Y_{t-1}) \Pr[s_t = j | Y_{t-1}]}{\sum_{j=1}^{h} p(y_t | s_t = j, Y_{t-1}) \Pr[s_t = j | Y_{t-1}]}. \tag{19}$$

B.1.2. *The prior.* Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior from structural-form parameters. Dummy observations are introduced as a component of the prior. The n first dummy observations are the "sums of coefficients" by Doan, Litterman, and Sims (1984); and the last dummy observation is a "dummy initial observation" by Sims (1993). Using linear restrictions, the overall prior, $p(\theta, q)$, is given in the following way:

$$p(b_j(k)) = \text{normal}(b_j(k) | 0, \bar{\Sigma}_{b_j}), \tag{20}$$

$$p(g_j(k)) = \text{normal}(g_j(k) | 0, \bar{\Sigma}_{g_j}), \tag{21}$$

$$p(\xi_j^2(k)) = \text{gamma}(\xi_j^2(k) | \bar{\alpha}_j, \bar{\beta}_j), \tag{22}$$

$$p(q_j) = \text{dirichlet}(q_{i,j} | \alpha_{1,j}, \ldots, \alpha_{k,j}), \tag{23}$$

where $\bar{\Sigma}_{b_j}$, $\bar{\Sigma}_{\psi_j}$, and $\bar{\Sigma}_{\delta_j}$ denotes the prior covariance matrices and $\bar{\alpha}_j$ and $\bar{\beta}_j$ are set to one, allowing the standard deviations of shocks to have large values for some regimes.

The Gamma distribution is defined as follows:

$$\text{gamma}(x | \alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha - 1} e^{-\beta x}. \tag{24}$$

Regarding the transition matrix, Q, suppose that $q_j = [q_{1,j}, \ldots, q_{h,j}]'$. The prior, denoted $p(q_j)$, follows a Dirichlet form as follows:

$$p(q_j) = \left(\frac{\Gamma\left(\sum_{i \in H} \alpha_{i,j}\right)}{\prod_{i \in H} \Gamma(\alpha_{i,j})} \right) \times \prod_{i \in H} (q_{i,j})^{\alpha_{i,j} - 1}, \tag{25}$$

where Γ denotes the standard gamma function.

B.2. **Gibbs-sampling.** Following Kim and Nelson (1999) and Sims, Waggoner, and Zha (2008), a Markov Chain Monte Carlo (MCMC) simulation method is employed to approximate the joint posterior density, $p(\theta, q, S_T|Y_T)$. The advantage of using VARs is that conditional distributions like $p(S_T|Y_T, \theta, q)$, $p(q|Y_T, S_T, \theta)$, and $p(\theta|Y_T, q, S_T)$ can be obtained in order to exploit the idea of Gibbs-sampling by sampling alternatively from these conditional posterior distributions.

B.2.1. *Conditional posterior densities, $p(\theta|Y_T, q, S_T)$.* To simulate draws of $\theta \in \{b_j(k), g_j(k), \xi_j^2\}$ from $p(\theta|Y_T, S_t, q)$, one can start to sample from the conditional posterior

$$p(b_j(k)|y_t, S_t, b_i(k)) = \exp\left(-\frac{1}{2}b_j'(k)\bar{\Sigma}_{b_j}^{-1}b_j(k)\right) \times \prod_{t\in\{t:s_t=k\}}\left[|A(k)|\exp\left(-\frac{\xi^2(s_t)}{2}(y_t'a_j(k) - x_t'f_j(k))^2\right)\right], \tag{26}$$

using the Metropolis-Hastings (MH) algorithm. Then a multivariate normal distribution is employed to draw $g_j(k)$:

$$p(g_j(k)|y_t, S_t) = \text{normal}(g_j(k)|\tilde{\mu}_{g_j(k)}, \tilde{\Sigma}_{g_j(k)}). \tag{27}$$

The computational details of the posterior mean vectors and covariance matrices are given in Sims, Waggoner, and Zha (2008).

Disturbance variances ξ_j^2 are simulated from a gamma distribution

$$p(\xi_j^2(k)|y_t, S_t) = \text{gamma}(\xi_j^2(k)|\tilde{\alpha}_j(k), \tilde{\beta}_j(k)), \tag{28}$$

where $\tilde{\alpha}_j(k) = \bar{\alpha}_j + \frac{T_{2,k}}{2}$ and

$$\tilde{\beta}_j(k) = \bar{\beta}_j + \frac{1}{2}\sum_{t\in\{t:s_{2t}=k\}}(y_t'a_j(s_t) - x_t'f_j(s_t))^2, \tag{29}$$

with $T_{2,k}$ denoting the number of elements in $\{t : s_{2t} = k\}$.

B.2.2. *Conditional posterior densities, $p(S_T|Y_T, \theta, q)$.* A multi-move Gibbs-sampling is employed to simulate $S_t, t = 1, 2, ..., T$. First, draw s_t according to

$$p(s_t|y_t, S_t) = \sum_{s_{t+1}\in H} p(s_t|Y_T, \theta, q, s_{t+1})p(s_{t+1}|Y_T, \theta, q), \tag{30}$$

where

$$p(s_t|Y_t, \theta, q, s_{t+1}) = \frac{q_{s_{t+1},s_t}p(s_t|Y_t, \theta, q)}{p(s_{t+1}|Y_t, \theta, q)}. \tag{31}$$

Then, in order to generate s_t, one can use a uniform distribution between 0 and 1. If the generated number is less than or equal to the calculated value of $p(s_t|y_t, S_t)$, we set $s_t = 1$. Otherwise, s_t is set equal to 0.

B.2.3. *Conditional posterior densities,* $p(q|Y_T, S_T, \theta)$. The conditional posterior distribution of q_j is as follows:

$$p(q_j|Y_t, S_t) = \prod_{i=1}^{h} (q_{i,j})^{n_{i,j}+\beta_{i,j}-1}, \tag{32}$$

where $n_{i,j}$ is the number of transitions from $s_{t-1} = j$ to $s_t = i$.

Appendix C. The Model

The SW Model describes the dynamics of the following set of variables: c_t which stands for consumption, l_t for labor supply, R_t for the nominal interest rate, π_t for inflation, i_t for the level of investment, q_t^k for the value of capital in terms of consumption, r_t^k is the rental rate of capital, u_t for the utilization rate of physical capital, w_t^h for the household's marginal rate of substitution between consumption and labor, y_t for the output, and y_t^f for the output in the flexible price/wage economy.

The log-linearized equilibrium conditions are given for the stationary variables and the symbol $*$ denotes the steady state value of the variable. The structural parameters of the economy impact the equilibrium conditions[6] for the level of consumption

$$\begin{aligned} c_t \quad = \quad & \frac{(1-he^{-\gamma})}{c\,(1+he^{-\gamma})}\left(R_t - E_t\left[\pi_{t+1}\right]\right) + \frac{he^{-\gamma}}{(1+he^{-\gamma})}c_{t-1} \\ & + \frac{1}{(1+he^{-\gamma})}E_t\left[c_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c\,(1+he^{-\gamma})}\frac{w_* l_*}{c_*}\left(l_t - E_t\left[l_{t+1}\right]\right) \end{aligned} \tag{33}$$

for the labor input,

$$w_t^h = \frac{1}{1-he^{-\gamma}}\left(c_t - he^{-\gamma}c_{t-1}\right) + \nu_l l_t \tag{34}$$

for the level of investment,

$$q_t^k = S'' e^{2\gamma}\left(1+\overline{\beta}\right)\left(i_t - \frac{1}{1+\overline{\beta}}i_{t-1} - \frac{\overline{\beta}}{1+\overline{\beta}}E_t\left[i_{t+1}\right]\right) \tag{35}$$

for the utilization rate of physical capital,

$$\frac{1-\psi}{\psi}r_t^k = u_t \tag{36}$$

given the production technology of the final good

$$y_t = \Phi_p\left(\alpha k_t + (1-\alpha)\,l_t\right) \tag{37}$$

[6]See Del Negro, Giannoni, and Schorfheide (2014) for a presentation of the full equilibrium.

In these equations, the parameter σ_c captures the degree of relative risk aversion, h the degree of habit persistence in the utility function, S'' the second derivative of the adjustment cost function, δ for the depreciation rate, $\overline{\beta} = \beta e^{(1-\sigma_c)\gamma}$ the intertemporal discount rate, σ_c the degree of relative risk aversion, ψ the costs of capital utilization, Φ_p the fixed cost of production, α the income share of physical capital in the production function, ν_l the curvature of the disutility of labor, and γ the steady-state growth rate.

The Phillips curves for prices (π_t) and wages (w_t) are, respectively,

$$\begin{aligned} \pi_t &= \frac{\left(1-\zeta_p\overline{\beta}\right)\left(1-\zeta_p\right)}{\left(1+\iota_p\overline{\beta}\right)\zeta_p\left(\left(\Phi_p-1\right)\epsilon_p+1\right)}\left(w_t+\alpha l_t-\alpha k_t\right) \\ &+\frac{\iota_p}{1+\iota_p\overline{\beta}}\pi_{t-1}+\frac{\overline{\beta}}{1+\iota_p\overline{\beta}}E_t\left[\pi_{t+1}\right] \end{aligned} \tag{38}$$

and

$$\begin{aligned} w_t &= \frac{\left(1-\zeta_w\overline{\beta}\right)\left(1-\zeta_w\right)}{\left(1+\overline{\beta}\right)\zeta_w\left(\left(\lambda_w-1\right)\epsilon_w+1\right)}\left(w_t^h-w_t\right)-\frac{1+\iota_w\overline{\beta}}{1+\overline{\beta}}\pi_t \\ &+\frac{1}{1+\overline{\beta}}\left(w_{t-1}+\iota_w\pi_{t-1}\right)+\frac{\overline{\beta}}{1+\overline{\beta}}E_t\left[w_{t+1}+z_{t+1}+\pi_{t+1}\right] \end{aligned} \tag{39}$$

where the parameters ζ_p, ι_p, ϵ_p, and λ_p are the Calvo parameter, the degree of indexation, the curvature parameter in the aggregator for prices, and the mark-up, and ζ_w, ι_w, ϵ_w, and λ_w are the corresponding parameters for wages. The policy rule of the monetary authority for the nominal interest rate is policy rule

$$\begin{aligned} R_t &= \rho_R R_{t-1}+\left(1-\rho_R\right)\left(\psi_1\left(\pi_t-\pi_t^*\right)+\psi_2\left(y_t-y_t^f\right)\right) \\ &+\psi_3\left(\left(y_t-y_t^f\right)-\left(y_{t-1}-y_{t-1}^f\right)\right) \end{aligned} \tag{40}$$

where ρ_R measures the persistence of the policy and the $\psi_.$ parameters the sensitivity of he central bank to the fundamentals. In the SW model, without financial frictions, the arbitage condition makes equal the return to capital and the riskless rate, that is

$$\frac{r_*^k}{r_*^k+(1-\delta)}E_t\left[r_{t+1}^k\right]+\frac{1-\delta}{r_*^k+1-\delta}E_t\left[q_{t+1}^k\right]-q_t^k=R_t-E_t\left[\pi_{t+1}\right] \tag{41}$$

This equation is no longer valid in the SW model with financial frictions, called SWFF, and is replaced by

$$E_t\left[\widetilde{R}_{t+1}^k-R_t\right]=\zeta_{sp,b}\left(q_t^k+\overline{k}_t-n_t\right)+\zeta_{sp,\sigma_w}\sigma_{w,t} \tag{42}$$

where $\widetilde{R}_{t+1}^k$ and R_t are the return for physical capital and the nominal interest rate in the economy, q_t^k the price of capital, n_t the net worth of the entrepreneurs, and $\sigma_{w,t}$ the risk shocks. It is worth mentioning that $\zeta_{sp,b}$ and $\zeta_{sp,\sigma}$ are not a structural parameters, but rather the combination of several structural parameters and steady-state values of endogenous variables. Moreover, the value of $\zeta_{sp,\sigma}$ is directly dependant on the value of $\zeta_{sp,b}$. See Section D for details. Risk shocks evolve according to

$$\log \sigma_{w,t} = (1-\rho_\sigma)\log\sigma_\omega + \rho_\sigma \log \sigma_{\omega,t-1} + \varepsilon_{\omega,t} \tag{43}$$

where ρ_σ is the degree of persistence of risk shocks in the regime s_t and $\varepsilon_{\omega,t}$ the innovations.

Appendix D. Elasticities of the SWFF Model

Let us first introduce the following notations:

$$\varrho = \frac{B}{N} \tag{44}$$

$$\Gamma(\overline{\omega}) = \overline{\omega}\left[1 - F(\overline{\omega})\right] + G(\overline{\omega}) \tag{45}$$

$$G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega dF(\omega) \tag{46}$$

where ϱ is the leverage of entrepreneurs, $[1-\Gamma(\overline{\omega})]$ the share of capital revenues earned by the entrepreneur while the lender gets the share $[\Gamma(\overline{\omega}) - \mu^e G(\overline{\omega})]$, where μ^e measures the monitoring costs.

The log-linearized equations for the financial contract consider the elasticities $\zeta_{sp,b}$ and ζ_{sp,σ_w}, which are defined as follows,

$$\zeta_{sp,b} \equiv -\frac{\frac{\zeta_{b,\overline{\omega}}}{\zeta_{z,\overline{\omega}}}}{1-\frac{\zeta_{b,\overline{\omega}}}{\zeta_{z,\overline{\omega}}}}\frac{1}{\varrho *} \tag{47}$$

$$\zeta_{sp,\sigma_w} \equiv -\frac{\frac{\zeta_{b,\overline{\omega}}}{\zeta_{z,\overline{\omega}}}\zeta_{z,\sigma_\omega} - \zeta_{b,\sigma_\omega}}{1-\frac{\zeta_{b,\overline{\omega}}}{\zeta_{z,\overline{\omega}}}} \tag{48}$$

where

$$\zeta_{z,x} \equiv \frac{\frac{\partial}{\partial x}\left[\Gamma(\overline{\omega}) - \mu^e G(\overline{\omega})\right]}{\left[\Gamma(\overline{\omega}) - \mu^e G(\overline{\omega})\right]}x \tag{49}$$

for $x = \{\overline{\omega}, \sigma_\omega^2\}$, and the elasticities $\zeta_{b,x}$

$$\zeta_{b,x} \equiv \frac{\frac{\partial}{\partial x}\left[\left\{[1-\Gamma(\overline{\omega})] + \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega}) - \mu^e\Gamma'(\overline{\omega})}\left[\Gamma(\overline{\omega}) - \mu^e G(\overline{\omega})\right]\right\}\frac{\widetilde{R}^l_*}{R_*} - \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega}) - \mu^e\Gamma'(\overline{\omega})}\right]}{\left\{[1-\Gamma_*(\overline{\omega}_*)] + \frac{\Gamma'_*(\overline{\omega}_*)}{\Gamma'_*(\overline{\omega}_*) - \mu^e\Gamma'_*(\overline{\omega}_*)}\left[\Gamma_*(\overline{\omega}_*) - \mu^e G_*(\overline{\omega}_*)\right]\right\}\frac{\widetilde{R}^l_*}{R_*}}x \tag{50}$$

for $x = \{\overline{\omega}, \sigma_\omega^2\}$.

The log-linearized equation for the accumulation of net worth is

$$\begin{aligned} n_t &= \zeta_{n,\widetilde{R}^k}\left(\widetilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t\right) + \zeta_{n,qK}\left(q^k_{t-1} + \overline{k}_{t-1}\right) \\ &\quad + \zeta_{n,n} n_{t-1} - \zeta_{n,\sigma_w}\zeta_{sp,\sigma_w}\sigma_{w,t-1} \end{aligned} \tag{51}$$

with the following elasticities

$$\zeta_{n,\widetilde{R}^k} \equiv \gamma_* \frac{\widetilde{R}^k_*}{\pi_* e^{z^*_*}} \left(1+\varrho_*\right) \left[1 - \mu^e_* G_*\left(\omega_*\right)\left(1 - \frac{\zeta_{G,\omega}}{\zeta_{z,\omega}}\right)\right] \tag{52}$$

$$\zeta_{n,R} \equiv \gamma_* \beta^{-1} \left(1+\varrho_*\right) \left[1 - \frac{n_*}{k_*} + \mu^e_* G_*\left(\omega_*\right) \frac{\widetilde{R}^k_*}{R_*} \frac{\zeta_{G,\omega}}{\zeta_{z,\omega}}\right] \tag{53}$$

$$\zeta_{n,qk} \equiv \gamma_* \frac{\widetilde{R}^k_*}{\pi_* e^{z^*_*}} \left(1+\varrho_*\right) \left[1 - \mu^e_* G_*\left(\omega_*\right)\left(1 - \frac{\zeta_{G,\omega}}{\zeta_{z,\omega}}\right)\right] - \gamma_* \beta^{-1}\left(1+\varrho_*\right) \tag{54}$$

$$\zeta_{n,n} \equiv \gamma_* \beta^{-1} + \gamma_* \frac{\widetilde{R}^k_*}{\pi_* e^{z^*_*}} \left(1+\varrho_*\right) \mu^e_* G_*\left(\omega_*\right) \frac{\zeta_{G,\omega}}{\zeta_{z,\omega}\omega_*} \tag{55}$$

$$\zeta_{n,\sigma_\omega} \equiv \gamma_* \mu^e_* G_*\left(\omega_*\right) \frac{\widetilde{R}^k_*}{\pi_* e^{z^*_*}} \left(1+\varrho_*\right) \zeta_{G,\omega} \left(1 - \frac{\zeta_{z,\sigma_\omega}}{\zeta_{z,\omega}}\right) \tag{56}$$