Private-Value Auction versus Posted-Price Selling: An Agent-Based Model Approach

Christopher N. Boyer

B. Wade Brorsen

James N. Fain

C. N. Boyer
Department of Agricultural and Resource Economics, University of Tennessee, Knoxville, TN, U.S.A.
Email: cboyer3@utk.edu

B.W. Brorsen
Department of Agricultural Economics, Oklahoma State University, Stillwater, OK 74078-6026
Email: wade.brorsen@okstate.edu, FAX: (405)744-8210

J. N. Fain
Department of Economics and Legal Studies in Business, Oklahoma State University, Stillwater, OK 74078
Email: jim.fain@okstate.edu

Abstract

Research has shown that an auction may or may not be preferred over a posted-price market, depending on the buyers’ valuation of the item being sold. Our objective is to determine if a first-price auction with and without a reserve price or a posted-price market produces the highest expected revenue for a seller when the buyers have private values for the item being sold. An agent-based first-price private-value auction and an agent-based posted-price market are developed to compare these selling methods. If the seller cannot impose a reserve price in the auction, the seller’s expected revenue is highest in the posted-price market, but if the seller can impose a reserve price and has a constant price signal, the seller’s expected revenue is highest in the private-value auction. When the seller’s is uncertain about the value of the item as reflected in a uniformly distributed price signal, the seller is better off selling the item in the auction with or without a reserve price than in the posted-price market. We program these agent-based models in two decidedly different programming environments. The programming environment using a genetic algorithm and Monte Carlo integration solved the agent-based models quicker, and
answers are more precise than the models solved using particle swarm optimization and the
trapezoidal rule.

**Keywords:** agent-based models, genetic algorithm, particle swarm optimization, posted-price
market, private-value auction

1 Introduction

The development of online markets allows individuals to choose between using different selling
methods to sell an item. For instance, individuals can sell an item in an online auction market
with and without a reserve price such as eBay, or can sell an item in an online posted-price
market such as Craigslist. Theoretical and experimental research has shown the seller’s optimal
selling method depends on the buyers’ valuation of the item being sold (Campbell and Levin
2006; Kultti 1999; Wang 1993), but this literature has focused mostly on buyer heterogeneity
with little consideration of seller uncertainty.

Wang (1993) develops a theoretical model to compare a seller’s revenue from selling an
item with a posted-price to selling the same item using an auction with a reserve price (lowest
acceptable bid price) when the buyers have independent private values for the item. When
buyers’ independent private values are widely dispersed, the auction is the optimal selling
method, but when buyers’ independent private values become less dispersed the posted-price
theoretical model to allow buyers’ private values to be correlated, and found similar results to
auction and with a posted price when the buyers’ values for the item are affiliated. Their
theoretical model demonstrates that a seller is not always better off selling an item with an
auction when buyers’ values for the item are affiliated. Kultti (1999) compares sellers’ utility
from selling an item with a posted-price market to selling an item in an auction when the buyers
have a common value for the item. Kultti (1999) uses a model that compares the probability of
buyers and sellers meeting in each market and uses evolutionary game dynamics to solve the model. Kultti (1999) finds that the probability of buyers and sellers meeting in an auction and posted-price market are equivalent; therefore, the seller’s expected utility in the posted-price market and the auction are always equivalent.

Recently, Hammond (2010) sold compact discs in an online ascending-bid second-price auction with a reserve price and an online posted-price market to determine the optimal selling method. Hammond (2010) cannot reject revenue equivalence for the sellers in the auction and posted-price market, and finds the posted-price market has a higher selling price and lower probability of selling the item than the auction. While Hammond (2010) does not know the distribution of buyers’ valuation of the item being sold, this is a unique paper to the literature because it is experimental instead of theoretical.

While auctions have been compared to posted-price markets in the literature, further research is needed. Wang (1993) compares a private-value auction with a reserve price to a posted-price market when the buyers have differing private values, but assumes the seller maintains a constant posted price and reserve price. The literature does not provide any insight into the optimal selling method when the seller is uncertain and must select reserve prices and posted prices based on a price signal that is only statistically affiliated with buyer private values. In an auction setting, a reserve price is a relevant instrument for the seller to reveal information, and in a posted-price market, the posted price is the instrument to reveal information. Cai, Riley, and Ye (2007) assume the seller’s price signal is independent of the buyers’ price signal to demonstrate how seller’s revealing information through a reserve price can influence the market equilibrium. By giving the seller a price signal and allowing the seller to choose the optimal reserve price and posted-price, we can determine how the seller’s price signal affects their
optimal selling method. Furthermore, the literature disagrees about when a seller prefers a 
private-value auction without a reserve price to a posted-price market. A reserve price in a 
private-value auction can increase buyers’ bid prices, decrease buyers’ revenues, increase sellers’ 
revenues, and decrease the probability of the item being sold (Reiley 2006; Riley and Samuelson 
1981). The optimal selling method might be the posted-price market when the seller cannot 
impose a reserve price. Comparing the seller’s revenue for a posted-price market to a private-
value auction without a reserve price would be a unique contribution to the literature.

Solving a private-value auction with the seller choosing a reserve price based on price 
signals would perhaps be intractable to solve analytically, and reproducing a private-value 
auction and a posted-price market in a laboratory or field experiment would likely require many 
rounds and a large sample, which would be expensive to obtain. Agent-based computational 
modeling is an alternative method to compare these two selling methods. Economists use agent-
based models to evaluate economic theories when obtaining data is expensive and the problem is 
intractable (Alkemade et al. 2006; Arifovic 1996; Bonabeua 2002). These models are artificial 
markets where interactive agents simulate economic markets following trading and equilibrium 
rules (Tesfatsion 2001).

Andreoni and Miller (1995) first established an agent-based independent private-value 
auction model. They use a genetic algorithm (GA) to solve the model and find buyers’ bid prices 
and average profits are below the predicted Bertrand-Nash Equilibrium. More recently, Boyer 
(2011) develops an agent-based common-value auction and an agent-based posted-price market 
to determine the optimal selling method for the seller when the buyers’ valuations are common. 
Boyer (2011) uses a particle swarm optimization (PSO) learning algorithm and finds the seller 
prefers selling the item with a posted price when the seller cannot impose a reserve price in the
auction; however, when the seller has a constant price signal and can impose a reserve price, the posted-price market is preferred over the auction. If the seller has a uniformly distributed price signal, the seller’s expected revenue for the auction and the posted-price market are similar. Boyer (2011) shows that a seller’s price signal can impact their optimal selling method when the buyers’ share a common value. Comparing an agent-based private-value auction with and without a reserve price to an agent-based posted-price market when the seller receives a price signal would also be a unique contribution to the agent-based modeling literature. Furthermore, Boyer (2011) used numerical integration to solve the agent-based models, which requires a substantial amount of time. Developing a different approach to solving auction markets when buyers and sellers receive a price signal would be helpful in solving future agent-based auction problems.

We calculate the seller’s expected revenue from selling an item in an agent-based first-price private-value auction with and without a reserve price and the seller’s expected revenue from selling an item in an agent-based posted-price market. We compare the seller’s expected revenue to determine the selling method that provides the seller with the larger expected revenue. We develop an agent-based posted-price market and an agent-based first-price private-value auction with and without a reserve price that includes two buyers and one seller. Two agent-based modeling programming environments and learning algorithms are used. One of the programming environments is written in Mathematica, where the buyers and the seller follow a GA to find their best strategy (bid price, reserve price, willingness-to-pay, and posted price). The other programming environment we use is Eclipse and the buyers and the seller follow a PSO learning algorithm to find their best strategy.
In our agent-based models, we make similar assumptions about the structure of the auction and posted-price market as Wang (1993). We assume buyers in the auction would have the same cost of searching for the same item in other auctions as buyers in the posted-price market searching for the same item in other posted-price markets, and the cost of organizing and establishing a posted-price market is equal to the cost of organizing and establishing an auction. The purpose of making assumptions like Wang (1993) is to illustrate that our agent-based models match theory, which validates the theoretical extensions our model makes to the literature.

2.1 Private-value auction

Buyers receive a uniformly distributed price signal about the value of the item, and have homogeneous preferences for an item based on their price signal. The buyers bid function in the first-price private-value auction is

\[ b_i = u_i - x_i \]  (1)

where \( b_i \) is the bid price for buyer \( i = 1, 2; \) \( x_i \in [-1,1] \) is the choice variable for buyer \( i; \) and \( u_i \sim U[100,101] \) \( \text{cov}(u_i,u_j) = 0 \, \forall \, i \neq j \) is buyer \( i \)'s private-value. A uniform distribution is selected for the private value because the agent-based model can solve with this distributional assumption and this distribution is used in previous studies (Andreoni and Miller 1995; Boyer 2011). We select the range of the uniform distribution mainly to help interpret the results. Any range could be selected, and the results would not change. The agent-based model might handle alternative distributions and selecting an alternative distribution might be an area to consider in future research.
Sellers are assumed to be non-competitive and homogeneous so one representative seller is included in the model. In our agent-based auction model, we allow the seller to choose a reserve price

\[ r = \nu - m \] (2)

where \( m \in [-1, +1] \) is the seller’s choice variable and \( \nu \sim U[100, 101] \) is the seller’s reserve price signal, which is independent of the distribution of buyers’ private values. Wang (1993) assumes the seller imposes a constant reserve price, but we allow the seller to adjust their reserve price based on the signal \( m \). As previously mentioned, a natural way to reveal information about the value of the item in an auction is with a price signal, and a reserve price is an applicable instrument to reveal the seller’s information (Cai et al. 2007). By giving the seller a uniformly distributed reserve price signal, we can demonstrate how seller’s uncertainty about buyers’ bid distributions impacts the seller’s optimal selling method. Results are also presented when the seller’s price signal is constant at the salvage value of $100 (i.e., \( r = 100 - m \)). Furthermore, we include results for an agent-based private-value auction without a reserve price. Buyers must bid greater than the other buyer and greater than or equal to the seller’s reserve price to win the item, and the winning buyer’s expected revenue is

\[ E[R_{1}^{B}] = E[(x_{1})I[b_{1} > b_{2}]I[b_{1} \geq r]] \] (3)

where \( E[R_{1}^{B}] \) is the expected revenue for buyer one; \( I[b_{1} > b_{2}] \) is an indicator function for when buyer one’s bid is greater than buyer two’s bid; and \( I[b_{1} \geq r] \) is an indicator function for when buyer one’s bid price is greater than the reserve price. Since this is a private-value auction, the winning buyer’s revenue is equal to the amount that the buyer can discount the price (i.e., the buyer’s choice variable) from their private value signal and still win the auctions. We allow the choice variable to be both positive and negative because buyers could pay more or less than their
private value. A positive value of $x_i$ means the buyers are shading bids below their private value signal, and a negative value of $x_i$ means the buyers are bidding over their private value. The expected revenue for the losing buyer is zero.

The seller’s expected revenue is the sum of buyer one’s bid price multiplied by the probability of buyer one winning the item and buyer two’s bid price multiplied by the probability of buyer two winning the item. The seller’s expected revenue is

$$E[R^S] = \sum_{i=1}^{2} [E[b_i \mid b_i > b_j, b_i \geq r \ \forall i \neq j]^*]P_i + 100(1-P_1-P_2) \quad (4)$$

where $E[R^S]$ is the expected revenue for the representative seller, and $P_i$ is the probability that buyer $i$ wins the item. If the item is not sold, then the seller maintains a “salvage value of the item” of $100$ (so $100*(1-P_1-P_2)$ is added in Eq. (4) to get the sellers total revenue).

2.2 Posted-price market

The posted price market is based on the same assumptions regarding the seller information and the distribution of buyer values as the private value auction. The primary difference in the posted-price market is the seller imposes a posted-price for the item, and buyers have a willingness-to-pay for the item. Buyer $i$’s willingness-to-pay function is

$$w_i = \theta_i - y_i \quad (5)$$

where $w_i$ is the willingness-to-pay for buyer $i=1,2$; $y_i \in [-1,1]$ is the choice variable in the posted-price market for buyer $i$; and $\theta_i \sim U[100,101]$ \quad \text{cov}(\theta_i, \theta_j) = 0 \ \forall i \neq j$ is buyer $i$’s private value.

The seller sets a posted price that is

$$p = \mu - z \quad (6)$$
where \( z \in [-1,1] \) is the seller’s choice variable in the posted-price market; and \( \mu \sim U[100,101] \) is the posted-price signal that is independent of buyers’ signals. Similar to the auction model, we present results when the seller has a constant price signal (\( p = 100 - z \)). Buyers must be willing to pay a value greater than or equal to the seller’s posted price to purchase the item, but buyers pay the posted price for the item. Buyers arrive in the market at the same time; thus, if both buyers have willingness-to-pay greater than or equal to the posted price, the buyers equally split the item being sold. The expected revenue for buyer one is

\[
E[R^B_1] = E[\theta_1 - p]
\]

where \( E[R^B_1] \) is the expected revenue for buyer one; \( I[w_2 \geq p] \) is an indicator function for when buyer two’s willingness-to-pay is greater than or equal to the posted price; and \( I[w_1 \geq p] \) is an indicator function for when buyer one’s willingness-to-pay is greater than the posted price.

Buyers’ expected revenue is their private value for the item minus the posted price for the item when their willingness-to-pay is greater than or equal to the posted-price. If a buyer’s willingness-to-pay is less than the posted price then their revenue is zero.

The seller’s expected revenue is similar to the seller’s expected revenue in an auction, but the seller only receives the posted price for the item. The seller’s expected revenue is

\[
E[R^S] = p \sum_{i=1}^{2} (0.5)^{I[w_2 \geq p]} I[w_i \geq p]
\]

where \( E[R^S] \) is the expected revenue for the representative seller.

3 Agent-Based Models

The general process of the agent-based auction and posted-price markets starts with the buyers and the seller selecting their choice variable (strategies). The buying and selling agents interact
with each other and track gains and losses from these interactions. From these results, the
expected revenues are calculated for the buyers and the seller at their given choice variable. If
the convergence criterion for equilibrium is not satisfied, the learning algorithm directs the
buyers and the seller to improved strategies in the next iteration, and if the convergence criterion
is met, then the model advances to the next run.

We use two programming environments to find agent-based market equilibriums. In the
First is a PSO learning algorithm. Zhang and Brorsen (2009, 2011) found PSO was faster and
more precise than GA in solving their agent based models. Second is a GA algorithm. The PSO
is programmed in Eclipse and uses numerical integration to calculate expectations at each
iteration. The GA algorithm is programmed in Mathematica and uses Monte Carlo integration to
calculate expectations. We include results from the two programming environments to compare
these programming environments and to offer confirmation of our findings.

3.1 Particle Swarm optimization
The PSO learning algorithm that the buyers and the seller use was developed by Eberhart and
algorithm has $k = 1,...,K$ parallel markets, and in each of the $K$ parallel markets, the buying and
selling agents have a clone participating in each one of the $K$ markets. We have $K$ parallel
auctions and $K$ parallel posted-price markets with each buying and selling agent having a clone
in each of the $K$ parallel auctions and $K$ parallel posted-price market. Thus, the $k$th clone of an
agent selects a bid or reserve price strategy in the $k$th auction. Agents use the parallel auctions to
select the most profitable strategies.
In the PSO, the $i$th agent has a choice variable $x_{i,k,t}$ in parallel market $k$ during iteration $t = 1, \ldots, T$, and an adjustment velocity $v_{i,k,t} \in [-1, +1]$ that directs the agent to a new value of the choice variable. Starting values of the agents’ choice variables are randomly drawn for the clone in each parallel market from a $U[0,1]$. Each velocity change is a function of the local best solution $p_{i,k,t}^l$ where superscript $l$ indicates the best solution in the local market and the agent’s global best solution $p_{i,t}^g$ where superscript $g$ indicates the best global solution. The $i$th agent’s new choice variable in parallel market $k$ is updated by $x_{i,k,t+1} = x_{i,k,t} + v_{i,k,t}$ and the velocity is
\[
v_{i,k,t+1} = w v_{i,k,t} + q_1 u_1 (p_{i,k,t}^l - x_{i,k,t}) + q_2 u_2 (p_{i,t}^g - x_{i,k,t})
\] (9)
where $w$ is the inertia weight factor; $u_1$ and $u_2$ are random variables distributed $U[0,1]$; and $q_1$ and $q_2$ are learning parameters. The learning parameters are $q_1 = q_2 = \frac{\beta_0^{q_1} + \beta_1^{q_1}}{T - t} / T$ where both $\beta_0^{q_1}$ and $\beta_1^{q_1}$ are constants set to one and $T = 500$ is the maximum number of iterations. The selection of an inertia weight $w$ should consider the tradeoff between fully exploring the parameter space and convergence time (Chatterjee and Siarry 2006). A large inertia weight slows convergence, but allows agents to have a larger exploration area, while a small inertia weight increases convergence time, but reduces the agents’ exploration area (Zhang and Brorsen 2009). Zhang and Brorsen (2009) manage this tradeoff by letting $w$ start high and decrease as the model proceeds through the iterations, similar to the learning parameters. Inertia weight expressed in Zhang and Brorsen (2009) as $w_t = \beta_0^w + \beta_1^w (T - t) / T$ where both $\beta_0^w$ and $\beta_1^w$ are constants set at 0.5. We choose the values for the learning and inertia parameters based on what Zhang and Brorsen (2009) found to be the most efficient for their model.

At each iteration, the strategy with the largest revenue for the agent in the $k$th market is chosen as the best local strategy. The agents’ best strategy depends on the other agents’ choice
variables; therefore, the agent’s previous best strategy may not perform well in the next iteration. The other agents’ strategies in the current period are held constant and the past $L$ best locals of each agent are reevaluated in market $k$. The best local solution is expressed as

$$p^l_{i,k,t} = \arg \max \left\{ R_k (p^l_{i,k,t-1}), \ldots, R_k (p^l_{i,k,t-L}), R_k (x_{i,k,t}) \right\}$$

(10)

where $R_k$ is the revenue in parallel market $k$. The best global solution is selected from the best local parameters and is

$$p^e_{i,t} = \arg \max \left\{ R_1 (p^l_{i,1,t}), R_2 (p^l_{i,2,t}), \ldots, R_K (p^l_{i,K,t}) \right\}.$$  

(11)

Table 1 shows the values for the PSO parameters used for the agent-based markets.

Because auctions are winner takes all, we need to calculate expected values at each iteration rather than using the result of a single random auction. Thus, we extend Zhang and Brorsen’s (2009) PSO by using numerical integration to calculate these expectations. Buying and selling agents each choose their strategy and the trapezoidal rule are used to approximate the buying and selling agents’ expected revenues. The trapezoidal rule is efficient enough for the problem being considered and is straightforward to program, but other integration algorithms such as Gaussian quadrature (Miller and Rice 1983) might be used to approximate these integrals. If a large number of buyers were to be considered then Monte Carlo integration or an analytical solution would need to be used or else the algorithm would be too slow to be practical.

The trapezoidal rule uses the area of trapezoids for $n$ subintervals or lengths to approximate the integral. The trapezoidal rule is defined as

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} (y_0 + 2y_1 + \ldots + 2y_{n-1} + y_n)$$

(12)

where $\Delta x = (b-a)/n$, $f(x)$ is the probability density function of winning the item; $y_0, \ldots, y_n$ are the grid points in the integral; $b$ is the upper bound of the integral; and $a$ is the lower bound of the
The probability weights at each point of the integral are $\Delta x$ except at the tails where the probability weight is $\Delta x/2$. With the uniform distribution, $n$ is assigned to be 100, resulting in a $\Delta x = 1/100$, a distance of 1/100 between the points in the integral, and a probability weight of $1/100$ accepts at the tails where the probability weight is $1/200$. Zhang and Brorsen (2011) find that winner-take-all agent-based models do not necessarily converge to theoretical results so a large number of points are used to reduce discreteness that may reduce accuracy in a winner-take-all auction.

A convergence criterion is developed to determine when the buyers and the seller cannot find strategies that improve their previous strategies. The model converges when the total change in the previous 10 strategies for each of the agents is less than 0.0000001 and the total change in the standard deviations of the strategies are less than 0.00000001. The agents have 500 iterations to meet the convergence criterion, and this is repeated 200 times ($E=200$). We present results for the agents’ averages and standard deviations for all 200 runs, which is a similar method to the approach commonly used with stochastic global optimization methods (Hamm, Brorsen, and Hagen 2007). If the convergence criterion is met by all buying and selling agents, the next run begins. If the convergence criterion is not met by all buying and selling agents, the next iteration begins and the PSO learning algorithm guides the agents to improved strategies in the next iteration.

3.2 Genetic Algorithm

In the Mathematica programming environment, we use a GA to solve the agent-based models. We assume there are 200 buyers and 100 sellers interacting in a single market. In each iteration $t = 1,\ldots, T$, the $i$th agent has a choice variable $x_{i,t}$. Starting values are randomly generated for all
the agents from a $U[0,1]$. Buyers and sellers are randomly matched 20 times, attempting to trade each time. At the end of 20 attempted trades, the expected revenues are calculated for each buyer and seller; these determine the “good” and “bad” choice variables for the agents. For the buyers the choice variables that produce the 20 lowest expected revenues are deemed “bad” and are replaced. To create 20 replacement buyers $RepB$, we need 40 buyer parents ($BP$) to breed or reproduce. The buyer parents are selected in a weighted random drawing from the pool of the remaining 180 buyers with good choice variables. The probability of being selected as a buyer parent is weighted by the agent’s expected revenue. The higher the expected revenue the more likely an agent is to be selected as a buyer parent. This is often called roulette wheel selection. The probability of being selected as a parent for the buyers is

$$p^B(x_{i,t}) = \frac{R_{i,t}}{\sum_{i=1}^{B} R_{i,t}}$$ (13)

where $p^B$ is the probability of being selected as a parent for the buyers; $R_{i,t}$ is the revenue gained for buying $x_{i,t}$ agent $i$ during the $t$ iteration; and $B=180$ is the total number of buyers not being replaced. Similarly, the probability of being selected as a parent for the sellers depends on the sellers’ expected revenue obtained from the 20 attempted trades. The higher the expected revenue the more likely a seller is to be selected as a seller parent ($SP$). Since there are 100 total sellers, 10 sellers are deemed “bad” and are replaced ($RepS$). The remaining 90 are deemed to have “good” choice variables. The probability of being selected as a parent for the sellers is

$$p^S(x_{i,t}) = \frac{R_{i,t}}{\sum_{i=1}^{S} R_{i,t}}$$ (14)

where $p^S$ is the probability of being selected as a parent for the sellers; $R_{i,t}$ is the revenue gained for selling agent $i$ during the $t$ iteration; and $S=90$ is the total number of sellers used for mating.
Once the parents are selected, they are then randomly paired. For each pair of parents, a random number between 0.05 and 0.95 is drawn to be a random weight. The new choice variable (to replace one of the bad choice variables) is equal to the choice variable of parent one in the pair multiplied by the random weight plus parent two’s choice variable multiplied by one minus the random weight. This is written mathematically as

$$x_{i,t+1} = wx_{i,j} + (1-w)x_{j,t}$$

(15)

Where $x_{i,t+1}$ is the new choice variable to replace the bad choice variable; $x_{i,j}^{BP}$ is the choice variable for buyer parent $i$ in the pair; $x_{j,t}^{BP}$ is the choice variable for parent $j$ in the pair; and $w$ is the random weight. This is a convex combination of the two parent choice variables rates. In this manner, the worst performing choice variables are replaced with choice variables that have a reasonable chance of being better.

After breeding new choice variables, the choice variable of some surviving agents mutate. It is common in GA that the new choice variables do not mutate since they were just bred. In addition, a certain percentage of the best performers do not mutate to ensure that the best performers remain in the population. We place the top 20 choice variables (in terms of expected revenue) off limits for mutation. Thus, 160 buying agents’ choice variables ($BM$), and 70 selling agents’ choice variables ($SM$), have the possibility of mutating.

We employ two factors that reduce the impact of mutation over as the simulation progresses. First, the percentage of the population subject to mutation decreases as the simulation runs longer. In the first 10 iterations $t$, we allow 12.5% of the available choice variables to mutate for the buyers and the sellers. In iterations 11 to 20, we allow 6.25% of the available choice variables to mutate for the buyers and the sellers. Any iteration above 20, zero mutations are allowed for the buyers and the sellers. The rationale here is to cast the net widely at first, and
allow the mutation rate to fall later to encourage convergence. Secondly, when drawing a
mutated value, we do not use the bounds on the original uniform distribution, which are 0 and 1.
Instead, we take the upper bound on the uniform distribution to be 1.1 and multiple it by the
highest choice variable from among the buyers’ (and the sellers’ respectively) choice variables
that are not replaced by breeding, and the lower bound to be 0.9 and multiple it by the lowest
choice variable from among the buyers’ (and the sellers’ respectively) choice variables that are
not replaced by breeding. This method rules out as mutations some very high and very low
values of the choice variables that have proven to be poor performers. The new choice variables
generated from mating should improve the “gene pool,” but the mutations may not. Table 2
shows the values used in the GA.

A similar convergence criterion is established in the GA models as in the PSO. The
model converges when the mean of the buyers’ and the seller’s strategies changes from one
breeding iteration to the next by less than 0.000001. When the model converges, the average bid
for buyer one (two) is the average bid of buyer one (buyer two) in these 100 auctions, and the
probability of buyer one (buyer two) winning is the number of times buyer one (buyer two) won
one of these 100 auctions divided by 100. The expected revenue for buyer one (buyer two) is the
revenue for the buyer one (buyer two) in these 100 auctions divided by 100. The average
winning bid is the average of the winning bids in these 100 auctions. For the seller, the
transaction probability is the number of sellers that sold an item in these 100 auctions divided by
100, and the seller’s expected revenue is sellers’ total revenue in these 100 auctions divided by
100. Thus, we use Monte Carlo integration in the GA instead of the numerical integration
procedure in the PSO. The model has 200 runs (E=200), and as many breeding iterations are
used as is needed to meet the convergence criterion, and once the convergence criterion is met, the next run begins.

4 Results

We solve the posted-price market and private-value auction under several different scenarios. In all the scenarios the buyers’ private values remain uniformly distributed, and we vary the seller’s price signal. First, we give the seller a constant price signal equal to the salvage value, and allow the seller to choose their optimal reserve price and posted price. This is similar to the Wang (1993) model when the buyers’ values are widely dispersed. We can compare the results from our agent-based models to Wang (1993) to determine if our agent-based models match the theoretical solution. Then, we run our agent-based models with the seller receiving a uniformly distributed price signal. This allows us to determine how a seller’s uncertainty about the selling price affects the seller’s optimal selling method. This is an extension of Wang (1993) and results will provide theoretical insight into the optimal selling method when the seller is uncertain about the optimal reserve price and posted price. The private-value auction model is also solved without the seller imposing a reserve price to compare to the posted-price market. This comparison also has never been made in the literature.

4.1 PSO Models

In the private-value auction without a reserve price, the average winning bid price for the item being sold is $100.11 (Table 3). Since the seller cannot choose to refuse a sale by using a reserve price, the item is always sold and the seller’s expected revenue is equal to the average winning bid price (Table 3). The average winning bid price is below the predicted Bertrand-Nash
equilibrium. Andreoni and Miller (1995) also found the equilibrium price in their agent-based private-value auction to be below the predicted Bertrand-Nash equilibrium. The PSO appears to allow buyers to shade bids and collude. Worasucheep (2012) found PSO can have problems with premature convergence, which might explain why buyers are able to shade their bids more than expected. Further research is needed into calibrating a PSO learning algorithm for agent-based based auction models. When the seller has a constant reserve price signal and can impose a reserve price, the average winning bid price is $100.32 and the seller’s reserve price is $100.02 (Table 3). The reserve price results in buyers increasing their average bid prices, and buyers’ expected revenues decrease. The seller’s expected revenue increases, and the probability of the item selling decreases since the seller will not accept bid prices lower than $100.02 (Table 3). These results suggest that the seller benefits from the reserve price, which matches the reserve price literature (Levin and Smith 1996; Reiley 2006; Riley and Samuelson 1981). Next, the seller selects a reserve price based on a uniformly distributed price signal. The average winning bid price decreases to $100.23, and the seller’s reserve price decreases to an average of $99.73 (Table 3). The buyers’ average bid prices decrease slightly, indicating that buyers might gain market power when the seller’s price signal is uniformly distributed (Table 3). Compared to the auction results when the seller has a constant price signal, the seller is more likely to sell the item and the seller’s expected revenue decreases primarily because the seller will accept lower bid prices (Table 3).

For the posted-price market with the seller having a constant price signal, the average selling posted price is $100.33 (Table 3). The probability of an item selling is 50%, which is lower than the auction market with a reserve price (Table 3). Hammond (2010) finds the posted-price market has a higher selling price and lower probability of selling the item than the auctions
in an experimental study. When the seller’s price signal is constant and the seller can impose a
reserve price, the private-value auction is preferred by the seller (Table 3). This model is similar
to Wang’s (1993) theoretical model when the buyers’ values are widely dispersed and the results
from the agent-based model match the theoretical results. Additionally, if the seller cannot
impose a reserve price in the auction, the seller is better off selling the item in the posted-price
market (Table 3). Going beyond Wang (1993), we give the seller a uniformly distributed price
signal, and the average selling price decreases to $100.19 (Table 3). The buyers’ willingness-to-
pay does not change much from when the seller’s price signal is constant. When the seller has a
uniformly distributed price signal and can impose a reserve price, the auction is preferred by the
seller over the posted-price market (Table 3). Furthermore, the seller would be better off
selling the item in the auction without a reserve price than in the posted-price market when the
seller has a uniformly distributed price signal (Table 3).

4.2 GA Models

Similar results are found with the GA agent-based models; however, the buyers’ bid-shading is
less than in the agent-based models with PSO. In the private-value auction without a reserve
price, the average winning bid price for the item being sold is $100.25 (Table 4). This is the
Bertrand-Nash equilibrium. If the seller has a constant price signal and can impose a reserve
price, the average winning bid price increases to $100.37, and the seller’s reserve price is $99.97
(Table 4). Similarly, the reserve price causes the buyers to increase their average bid price and
causes their expected revenues to decrease. The seller’s expected revenue increases and the
probability of the item selling decreases since the seller will not accept bid prices lower than the
reserve price (Table 4). Giving the seller a uniformly distributed price signal, results in the
average winning bid price decreasing to $100.33, and the reserve price decreasing to $99.70 (Table 4). Buyers slightly decrease their average bid prices, but not as much as in the PSO model (Table 4). Compared to the auction results when the seller has a constant price signal, the seller’s expected revenue decreases, which is due to the seller’s reserve price decreasing. A key difference between the models with PSO and GA is the PSO appears to allow buyers to achieve more market power in all auction scenarios.

In the posted-price market when the seller has a constant price signal, the average selling posted price is $100.42, and the probability of an item selling is 72% (Table 4). Similar to the Eclipse results and Hammond (2010), the probability of the item selling is lower than in the auction scenarios (Table 4). Relative to the private-value auction without a reserve price, the seller’s expected revenue is higher in the posted-price market than in the auction without a reserve price (Table 4). Conversely, if the seller has a constant price signal and can impose a reserve price, the seller’s expected revenue is higher in the private-value auction with a reserve price than in the posted-price market (Table 4). This matches what we found with the PSO and what Wang (1993) observed when the buyers’ values are widely dispersed. When the seller has a uniformly distributed price signal, the average selling price of the item is $100.28 and the probability of the item selling is 66% (Table 4). Similar to the PSO, the buyers’ willingness-to-pay for the item does not change when the seller has a uniformly distributed price signal, indicating buyers do not gain much market power in the posted-price market (Table 4). When the seller has a uniformly distributed price signal, the seller’s expected revenue is larger in the auction than in the posted-price market (Table 4). The seller receives a larger expected revenue by selling the item in the auction without a reserve price than in the posted-price market with a uniformly distributed price signal (Table 4).
5 Discussion and Conclusion

We compare a seller’s expected revenue in a first-price private-value auction with and without a reserve price to a posted-price market when buyers have private values for the item being sold. The purpose is to determine the selling method that provides the seller with the highest expected revenue. We extend Wang (1993) by allowing the seller to choose the optimal reserve price based off a uniformly distributed price signal, and by comparing a posted-price market to a private-value auction without a reserve price. Agent-based models are set up to both match the Wang results and to extend them.

If the seller has a constant price signal and can impose a reserve price, the private-value auction with a reserve price provides the seller with a higher revenue than the posted-price market. However, when the seller has a uniformly distributed price signal, the private-value auction with a reserve price is preferred by the seller over the posted-price market. This matches the results when the seller has a constant reserve price signal, suggesting that the seller’s uncertainty does not impact the optimal selling method. If the seller has a constant price signal and cannot impose a reserve price, the seller’s expected revenue is higher in the posted-price market than in the private-value auction. Interestingly, if the seller has a uniformly distributed price signal and cannot impose a reserve price, the seller is better off selling the item in the auction without a reserve price than in the posted-price market. Thus, when the seller has uncertainty about the value of the item, the seller prefers to use an auction with or without a reserve price over a posted-price market.

In general, both the GA and PSO models favored the same selling method for each scenario. The primary difference is the buyers’ market power or bid-shading was stronger in the
PSO model. The PSO algorithm allows the buyers to bid lower prices on average, which might be due to premature convergence. Moreover, the standard deviations of the GA agent-based models are smaller than in the PSO agent-based models. We selected the most efficient starting values for the PSO found by Zhang and Brorsen (2009), but the randomness of the PSO can cause buyers to make big jumps in bid prices early in the simulation. Zhang and Brorsen (2009, 2011) found the PSO to be faster and more precise than GA; however, their GA produced new values using crossover of binary strings, which is not as efficient as the GA considered here. The Mathematica programming environment that uses GA and Monte Carlo integration solves faster and more precisely than the Eclipse model that is based on PSO and uses numerical integration.
Table 1. Parameter Values for the Particle Swarm Optimization Algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept of inertia weight</td>
<td>$\beta^w_0$</td>
<td>0.5</td>
</tr>
<tr>
<td>Slope of inertia weight</td>
<td>$\beta^w_1$</td>
<td>1</td>
</tr>
<tr>
<td>Learning parameters intercept</td>
<td>$\beta^a_0$</td>
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</tr>
<tr>
<td>Learning parameter slope</td>
<td>$\beta^a_1$</td>
<td>1</td>
</tr>
<tr>
<td>Parallel markets</td>
<td>$K$</td>
<td>20</td>
</tr>
<tr>
<td>Maximum iterations</td>
<td>$T$</td>
<td>500</td>
</tr>
<tr>
<td>Runs</td>
<td>$E$</td>
<td>100</td>
</tr>
<tr>
<td>Memory</td>
<td>$L$</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 2. Parameter Values for the Genetic Algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of buyers</td>
<td>$B$</td>
<td>200</td>
</tr>
<tr>
<td>Number of sellers</td>
<td>$S$</td>
<td>100</td>
</tr>
<tr>
<td>Buyer replacement</td>
<td>$RepB$</td>
<td>20</td>
</tr>
<tr>
<td>Buyer parents</td>
<td>$BP$</td>
<td>40</td>
</tr>
<tr>
<td>Buyers open to mutation</td>
<td>$BM$</td>
<td>160</td>
</tr>
<tr>
<td>Seller replacement</td>
<td>$RepS$</td>
<td>10</td>
</tr>
<tr>
<td>Seller parents</td>
<td>$SP$</td>
<td>20</td>
</tr>
<tr>
<td>Sellers open to mutation</td>
<td>$SM$</td>
<td>70</td>
</tr>
<tr>
<td>Runs</td>
<td>$E$</td>
<td>200</td>
</tr>
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</table>
Table 3. Equilibrium Values for the Agent-Based Private-Value Auctions and Posted Price Markets with PSO

<table>
<thead>
<tr>
<th>Category</th>
<th>Auction</th>
<th>Seller’s reserve price signal is constant</th>
<th>Seller’s reserve price signal is uniformly distributed</th>
<th>Posted Price Market</th>
<th>Seller’s posted price signal is constant</th>
<th>Seller’s posted price signal is uniformly distributed</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Reserve price</td>
<td>100.11 (0.07)</td>
<td>100.32 (0.05)</td>
<td>100.23 (0.07)</td>
<td>100.33 (0.05)</td>
<td>100.19 (0.09)</td>
<td></td>
</tr>
<tr>
<td>Reserve price</td>
<td>100.02 (0.07)</td>
<td>99.73 (0.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buyer 1</td>
<td>Average winning bid price/ selling posted price</td>
<td>99.94 (0.08)</td>
<td>100.05 (0.09)</td>
<td>100.02 (0.08)</td>
<td>100.17 (0.05)</td>
<td>100.16 (0.04)</td>
</tr>
<tr>
<td></td>
<td>Transaction probability</td>
<td>50.0% (0.05)</td>
<td>38.0% (0.09)</td>
<td>44.0% (0.09)</td>
<td>25.0% (0.04)</td>
<td>20.0% (0.05)</td>
</tr>
<tr>
<td></td>
<td>Expected revenue</td>
<td>0.27 (0.03)</td>
<td>100.17 (0.04)</td>
<td>100.20 (0.03)</td>
<td>100.11 (0.03)</td>
<td>100.15 (0.06)</td>
</tr>
<tr>
<td>Buyer 2</td>
<td>Average winning bid price/ willingness-to-pay</td>
<td>99.94 (0.08)</td>
<td>100.05 (0.09)</td>
<td>100.02 (0.08)</td>
<td>100.17 (0.05)</td>
<td>100.16 (0.04)</td>
</tr>
<tr>
<td></td>
<td>Transaction probability</td>
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<td>38.0% (0.09)</td>
<td>44.0% (0.09)</td>
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<td>20.0% (0.05)</td>
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<td>100.17 (0.04)</td>
<td>100.20 (0.03)</td>
<td>100.11 (0.03)</td>
<td>100.15 (0.06)</td>
</tr>
<tr>
<td>Seller</td>
<td>Transaction probability</td>
<td>100.0% (0.00)</td>
<td>76.0% (0.16)</td>
<td>88.0% (0.05)</td>
<td>50.0% (0.08)</td>
<td>40.0% (0.10)</td>
</tr>
<tr>
<td></td>
<td>Expected revenue</td>
<td>100.11 (0.07)</td>
<td>100.24 (0.05)</td>
<td>100.20 (0.06)</td>
<td>100.18 (0.03)</td>
<td>100.07 (0.06)</td>
</tr>
</tbody>
</table>

Note: Standard deviations are presented in parentheses.

a All results, except average winning bid price, are not conditional on winning.

b The seller’s reserve price is not a function of a noisy signal and buyers private values are distributed U[100,101].

c Seller’s signal and buyers’ private values are independently distributed U [100,101].
<table>
<thead>
<tr>
<th>Category</th>
<th>No Reserve price</th>
<th>Seller’s reserve price signal is constant&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Seller’s reserve price signal is uniformly distributed&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Seller’s posted price signal is constant&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Seller’s posted price signal is uniformly distributed&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average winning bid price/selling posted price</strong></td>
<td>100.25 (0.02)</td>
<td>100.37 (0.01)</td>
<td>100.33 (0.02)</td>
<td>100.42 (0.02)</td>
<td>100.28 (0.01)</td>
</tr>
<tr>
<td><strong>Reserve price</strong></td>
<td>99.97 (0.03)</td>
<td>99.70 (0.06)</td>
<td>100.31 (0.01)</td>
<td>100.30 (0.01)</td>
<td>100.18 (0.01)</td>
</tr>
<tr>
<td><strong>Buyer 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average bid price/willingness-to-pay</td>
<td>100.08 (0.02)</td>
<td>100.15 (0.02)</td>
<td>100.39 (0.02)</td>
<td>100.40 (0.01)</td>
<td></td>
</tr>
<tr>
<td>Transaction probability</td>
<td>50.0% (0.01)</td>
<td>45.0% (0.01)</td>
<td>36.0% (0.01)</td>
<td>33.0% (0.01)</td>
<td></td>
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<tr>
<td>Expected revenue</td>
<td>0.21 (0.01)</td>
<td>0.16 (0.01)</td>
<td>0.12 (0.01)</td>
<td>0.14 (0.01)</td>
<td></td>
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<tr>
<td><strong>Buyer 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Average bid price/willingness-to-pay</td>
<td>100.08 (0.02)</td>
<td>100.15 (0.02)</td>
<td>100.39 (0.02)</td>
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<tr>
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<td>50.0% (0.01)</td>
<td>45.0% (0.01)</td>
<td>36.0% (0.01)</td>
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<tr>
<td>Expected revenue</td>
<td>0.21 (0.01)</td>
<td>0.16 (0.01)</td>
<td>0.12 (0.01)</td>
<td>0.14 (0.01)</td>
<td></td>
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<tr>
<td><strong>Seller</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transaction probability</td>
<td>100.0% (0.00)</td>
<td>90.0% (0.02)</td>
<td>94.0% (0.02)</td>
<td>72.0% (0.02)</td>
<td>66.0% (0.02)</td>
</tr>
<tr>
<td>Expected revenue</td>
<td>100.25 (0.02)</td>
<td>100.33 (0.01)</td>
<td>100.31 (0.01)</td>
<td>100.30 (0.01)</td>
<td>100.18 (0.01)</td>
</tr>
</tbody>
</table>

Note: Standard deviations are presented in parentheses.

<sup>a</sup> All results, except average winning bid price, are not conditional on winning.

<sup>b</sup> The seller’s reserve price is not a function of a noisy signal and buyers private values are distributed $U[100,101]$.

<sup>c</sup> Seller’s signal and buyers’ private values are independently distributed $U[100,101]$. 
References


Boyer CN (2011) Agent-based common value auctions. PhD Dissertation. Department of Agricultural Economics, Oklahoma State University


