Abstract
The present study seeks to analyze the effects of QE on the U.S. economy. For this purpose we develop a DSGE model with financial frictions and an explicit specification of the central bank’s balance sheet policy options. Specifically, we model a banking sector which requires liquidity to facilitate financial intermediation. Liquidity is supplied by the central bank in exchange for collateral. We consider multiple assets that differ with respect to their ability to serve as collateral. This leads to a spread between interest rate on non-eligible and eligible assets, a liquidity premium. We estimate the model for the US economy and further simulate a scenario which matches the 2008/2009 crisis to quantify the effectiveness of different balance sheet policy programs.

JEL classification: E4; E5; E32.
Keywords: Monetary policy, collateralized lending, quantitative easing, credit easing, liquidity premium
1 Introduction

In response to the 2008/2009 financial crisis the major central banks around the world have conducted monetary policy operations that go beyond the standard interest rate policies. Besides providing guidance about the likely future path of key interest rates and the set up of new lending facilities, central banks most prominently embarked on quantitative easing programs which are mainly comprised of large-sale asset purchase programs (LSAP). By the means of those unconventional policy measures the Federal Reserve System’s holdings of domestic securities increased to approximately $2.6 trillion (see Figure 1). To a large extent

![Federal Reserve’s asset holdings](image)

Notes: The Federal Reserve System’s holdings of domestic securities.
Source: Federal Reserve Board of Governors.

those increases were caused by the Fed’s LSAP 1 and LSAP 2 quantitative easing programs. Thereby LSAP 1 embodied purchases of Mortgage-backed securities (MBS), Agency securities, and Treasury securities between late 2008 and early 2010. In contrast to that LSAP 2 was primarily established as a purchase program of longer-term Treasury securities. From November 12, 2010, through June 30, 2011, the Federal Reserve’s Open Market Trading Desk conducted $767 billion of purchases. As depicted in Table 1 the Desk distributed its longer-term Treasury purchases across seven maturity sectors covering largely longer-term Treasury

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4 Further, reinvestment of principal payments from agency debt and MBS into longer-term securities took place to keep constant the Fed’s holdings of securities at their current level.

5 After June 2011 principal payments on all domestic securities were reinvested in Treasury securities to maintain the Federal Reserve’s holdings of domestic securities at approximately $2.6 trillion.
Table 1: Distribution of Treasury purchases across maturity

<table>
<thead>
<tr>
<th>Nominal Coupon Securities by Maturity Range</th>
<th>TIPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^{1/2} - 4$</td>
<td>$5^{1/2} - 7$</td>
</tr>
<tr>
<td>Years</td>
<td>Years</td>
</tr>
<tr>
<td>20%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Notes: Beginning with the operations included in the tentative schedule announced on July 13, 2011, the Desk planned to distribute purchases across the following seven maturity sectors based on the approximate weights above. The on-the-run 7-year note will be considered part of the $5^{1/2} - 7$-year sector, and the on-the-run 10-year note will be considered part of the 7- to 10-year sector.

Source: Federal Reserve Bank of New York.

Figure 2 presents the evolution of Fed’s long-term Treasury security holdings. While Treasury Notes and Bonds holdings increase from $300 billion to roughly $600 billion during the LSAP 1 program, by LSAP 2, which we place under the focus of the subsequent analyses, again the Fed’s holdings more than double from $600 billion to more than $1400 billion.

Figure 2: U.S. Treas. Notes and Bonds held by Fed

Notes: Evolution of the amount of U.S. Treasury Notes and U.S. Treasury Bonds held by the Federal Reserve System in Bill. USD.

These measures have been introduced to enhance credit supply by alleviating financial intermediation, and to support the transmission mechanism of monetary policy. Specifically, with the monetary policy rate at the zero lower bound (ZLB), large-scale purchases of longer-term securities are intended to lower their interest rates. Those effects should be transmitted to private borrowing rates, particularly at longer maturities, and finally stimulate economic

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6U.S. Treasuries which mature in more than one to ten years are denoted as Treasury Notes, U.S. Treasuries maturing after more than ten years are denoted as Treasury Bonds.
activity. Several contributions to the empirical literature find evidence that LSAP programs have indeed been effective in reducing long-term Treasury rates. Krishnamurthy and Vissing-Jorgensen (2011) estimate that LSAP 2 reduced the ten-year Treasury Note yield by 33 basis points, likewise D’Amico and King (2011) estimate a reduction of 55 basis points. However, the lack of experience with these balance sheet policies have also raised questions on the effectiveness of these measures and their transmission to the real economy and fueled discussions about possible inflationary effects.

At the same time, there is only a limited amount of academic research providing guidance and assessing the effectiveness of this policy as pointed out by Fed Chairman Ben Bernanke, at Jackson Hole Meeting 2012:

"...we were guided by some general principles and some insightful academic work - but with the important exception of the Japanese case - limited historical experience. As a result, central bankers in the United States, and those in other advanced economies facing similar problems, have been in the process of learning by doing."

One reason for this lack of guidance is due to the property which standard macroeconomic models used for monetary analysis typically assume that central banks directly control the private sector savings rate (i.e. the consumption Euler rate). By assuming the central bank having a direct control on the savings rate only, important aspects of the monetary transmission mechanism are neglected and severe limitations to the analysis emerge once the zero lower bond on interest rates is reached. Further, by the irrelevance result of Wallace (1981), from a theoretical point of view it is generally expected that non-standard open market operations in private assets do not exhibit an effect on real variables. As shown by Eggertsson and Woodford (2003) this result also applies for open market operations conducted in models with nominal frictions, money in the utility, and where the interest rate is at its ZLB. In this regard Walsh (2010) points out that private agents will demand money up to satiation if the ZLB is reached. Together with the assumption that assets are perfect substitutes and that there is no portfolio-balance effect, in case of LSAPs, market participants will take advantage of arbitrage opportunities, which will make decisions about size and composition of the central bank’s balance sheet irrelevant for the real variables’ allocation. This result even applies to New Keynesian models with credit frictions, such as Curdia, and Woodford (2011), as long as assets purchased (short-term bonds) by the central bank will be perceived as equivalent to reserves.

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7 For a survey of estimates regarding the impact of LSAP 1 on the ten-year Treasury yield see Chen, Curdia, and Ferrero (2012).

8 A notable exception is the work by ?
This paper aims at overcoming these limitations and develops a quantitative macroeconomic model which accounts for central bank asset acquisition and its impact on financial intermediation. The first goal of the paper is to assess the explanatory power of a model with a monetary transmission based on financial intermediation and an endogenous pass-through of policy rate changes. The pass-through in the model goes beyond the standard approach of equating the (real) policy rate with the rate of intertemporal substitution. Specifically, to overcome the irrelevance of balance sheet policy measures found in Curdia, and Woodford (2011) we consider multiple assets that differ with respect to their ability to serve as collateral for central bank money. This leads to a spread between interest rates on non-eligible and eligible assets, which reflects basically a liquidity premium. Interest rates on non-eligible assets are positive, even if the policy rate is at the ZLB. Central bank balance sheet policy therefore can be effective by affecting the liquidity premium.

The model is further applied to address the following questions: First, can shocks to financial intermediation and monetary policy contribute to explain the volatility in real activity following the 2008/2009 financial crisis? Second, what are the effects of central bank large-scale long-term Treasury bond purchase programs, focussing on LSAP 2?

Besides the contribution by Curdia, and Woodford (2011) there are several recent studies on unconventional policies under financial market imperfections. However those studies rely on non-monetary model approaches. Chen, Curdia, and Ferrero (2012) assume that investors have heterogeneous preferences for Treasury bonds of different maturities and postulate that bond markets are segmented. Although the importance of the latter assumption for the transmission mechanism of monetary policy is estimated, this approach can be regarded as being completely ad hoc. Del Negro et al. (2011) incorporate credit market frictions a la Kiyotaki and Moore (2012) into a standard DSGE model. Firms facing investment opportunities can issue own debt securities only up to a certain fraction of illiquid assets on its balance sheet in each period. In contrast to that government securities are not subject to such resaleability constraints which gives government bonds the role of liquidity in the model. However this approach is criticized by Shi (??) (...). Gertler and Karadi (2013) incorporate financial intermediaries within an otherwise standard macroeconomic model where the condition of the intermediary’s balance sheet influences the overall flow of credit. Non-conventional monetary policy is introduced in this environment by the central bank acting as an intermediary by borrowing funds from savers an then lending them to investors in times when private sector financial intermediation is interrupted by a financial crisis.

The model we employ mainly differs from standard macroeconomic models by accounting for the specific role of government bonds to provide liquidity services to commercial banks. This is considered by modelling central bank monetary supply by an asset exchange in open market operations, as it is common practice. Money and reserves resp. are demanded by
banks for liquidity management purposes when they provide intermediation between households and firms. Banks further hold government short-term bonds and long-term bonds where only the former in absence of unconventional policy measures serve as collateral in open market operations (i.e. repurchase agreements). LSAP 2 therefore is implemented by declaring a certain fraction of long-term bonds hold by the banks as eligible collateral for repo agreements with the central bank.

Given that the purpose of the analysis is to provide quantitative results, we specify the model in a way that allows for a calibration/estimation for the US economy. This implies that we specify costs of financial intermediation in a stylized way following Curdia, and Woodford (2011), which facilitates the model estimation, but lacks explicit microfoundations.

The first set of results can be summarized as follows: First, the model – though several novel ingredients have been introduced – generates responses to commonly studied economic shocks and policy shocks which are qualitatively consistent with broad empirical evidence. Beyond that, we introduce shocks which directly affect the financial intermediation. Thereby the impact on financial intermediation is particularly relevant for the size and the persistence of shock responses. Second, the central bank can significantly alleviate adverse effects to the economy by easing the supply of money reserves in exchange for log-term government bonds.

The remainder of this study is organized as follows. Section 2 presents the model. Section 3 discusses empirical implementation, estimation results, and model-implied dynamics. Specifically, we discuss the data and priors for estimation and analyze the model’s empirical performance in terms of implied business cycle moments, variance decomposition, and forecasting performance. Furthermore, we analyze the model-implied endogenous responses to economic shocks and policy shocks. In section 4 we conduct a policy experiment simulating the effects of different LSAP 2 scenarios on the estimated model. Section 5 concludes.

2 The model

The macroeconomic model which the present study is build on largely stems from Christoffel and Schabert (2013) and Schabert and Reynard (2009). The model contains five sectors: The household sector and the firm sector are close to the standard formulation of Smets and Wouters (2007), while the financial intermediation sector (banks) as well as the government and the central bank are enriched and modified to allow for the modelling of balance sheet policies. Banks provide financial intermediation between households and borrowing firms. They further hold short-term bonds and long-term bonds issued by the government, whereas the former in the absence of unconventional monetary policy measures serve as only collateral in open market operations. Firms borrow from banks to finance the up-front payment of wages in the production process, are monopolistic competitive suppliers of differentiated goods, face Rotemberg (1982) price adjustment cost, invest into physical capital, face investment
adjustment cost a la Christiano, Eichenbaum, and Evans (2005), and decide on the level of capital utilization. Households consume, are monopolistic suppliers of differentiated labor, face Rotemberg (1982) wage adjustment cost, exchange state contingent contracts in zero-net-supply among themselves, and deposit funds at the financial intermediaries. The government purchases goods, raises lump-sum taxes, and issues short-term bonds and long-term bonds, whereas the latter are modeled as perpetuities. The Central bank sets the main refinancing rate according to a Taylor-type rule, supplies money in exchange for eligible collateral, and decides on size and composition of its balance sheet.

2.1 Households

The household sector of the model economy is assumed to be comprised of a continuum of infinitely lived households, indexed with \( i \in [0, 1] \). These households have identical preferences and are endowed with potentially different asset stocks. Utility increases with households consumption and decreases with working time. Further, it is assumed that beginning-of-period holdings of deposits \( D_{t-1} \) provide utility, which serves as a convenient short-cut for modelling transaction services of deposits (just like the textbook specification of money-in-the-utility function) Household \( i \) maximizes the expected sum of a discounted stream of instantaneous utilities

\[
E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left( c_{i,t}, n_{i,t}, \frac{d_{i,t-1}}{\pi_t} \right), \tag{1}
\]

\( E_0 \) is the expectation operator conditional on the time 0 information set, and \( \beta \in (0, 1) \) is the subjective discount factor. The term \( \xi_t \) is a stochastic preference parameter, which has been used in several studies on policy options at the ZLB (see e.g. Eggertsson and Woodford (2005), or Eggertsson (2011)). We assume that \( \xi_t \) evolves according to an AR(1) process with a mean of unity

\[
\log(\xi_t) = \rho_x \log(\xi_{t-1}) + \varepsilon_{\xi,t}, \quad \varepsilon_{\xi,t} \sim N(0, \sigma_{\xi}) \tag{2}
\]

The household’s real value of bank deposits is denoted as \( d_{i,t} \), with \( d_{i,t} = \frac{D_{i,t}}{P_t} \), and \( P_t \) denoting the price of the wholesale final goods. The instantaneous utility \( u \) is assumed to be increasing in household consumption \( c_{i,t} \) and real deposits \( d_{i,t} \), and decreasing in working time \( n_{i,t} \). Further, the utility function is assumed to be strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions. Household preferences further allow for external habits in consumption. These assumptions lead to the following specification of households’ instantaneous utility function

\[
u(c_{i,t}, n_{i,t}, d_{i,t}) = \frac{1}{1 - \sigma} \left( c_{i,t} - h \cdot c_{t-1} \right)^{1 - \sigma} + \theta \cdot \frac{1}{1 - \varphi} \left( \frac{d_{i,t-1}}{\pi_t} \right)^{1 - \varphi} - \nu \cdot \frac{1}{1 + v} n_{i,t}^{1 + v} \tag{3}
\]
such that

\[ u_{i,ct} = (c_{i,t} - h c_{i,t-1})^{-\sigma} \]  
(4)

\[ u_{i,dt} = g \pi_t^{-1} \left( \frac{d_{i,t-1}}{\pi_t} \right)^{-\varphi} \]  
(5)

\[ u_{i,nt} = -\nu n_{i,t}^{1/2} \]  
(6)

where \( u_{i,ct} \) denotes household \( i \)'s marginal utility of consumption, \( u_{i,dt} \) denotes marginal utility of services gained from deposit holdings, and \( u_{i,nt} \) denotes marginal (dis-)utility from working time. The flow budget constraint for each household reads:

\[
\frac{D_{i,t}}{R^d_t} - D_{i,t-1} + E_t[\varphi_{i,t+1}S_{i,t}] - S_{i,t-1} + P_t c_{i,t} \leq P_t w_{i,t} n_{i,t} - P_t W A C_{i,t} + P_t \Pi_{i,t} + P_t \tau_{i,t} + P_t \tau_{i,t}^{m_t} 
\]  
(7)

Household \( i \) supplies labor against the real wage rate \( w_t \), invests in deposits, and state contingent claims \( S_{i,t} \), where \( R^d_t \) denotes the risk-free rate of return on deposits, and \( \varphi_{i,t+1} \) the stochastic discount factor. Profits from firms/retailer \( \Pi_{i,t} \), are distributed to the households. \( \tau_{i,t} \) is a lump-sum tax, and \( \tau_{i,t}^{m_t} \) are central bank transfers. We assume households to be optimally setting their individual wage rate \( w_{i,t} \), whereas \( W A C_{i,t} \) denotes wage adjustment cost which each household individually faces when adjusting the wage rate. Following Rotemberg (1982) wage adjustment cost are of the form

\[
W A C_{i,t} = \frac{\omega W}{2} \left( \frac{w_{i,t}}{w_{i,t-1} \left( \frac{w_{i,t-1}}{\bar{\pi}_{t-1} \pi_{t-1}} \right)} - 1 \right)^2 y_t 
\]  
(8)

where \( \bar{\pi}_{t-1} = \frac{P_{t-1}}{P_{t-2}} \) denotes the past inflation and \( \bar{\pi} \) denotes the steady state inflation rate.

Household \( i \)'s borrowing is restricted by the following no-Ponzi game condition

\[ \lim_{s \to \infty} E_t [\varphi_{i,t+s}S_{i,t+s}] \geq 0, \]

as well as by the non-negativity constraint for deposit holdings \( D_{i,t} \geq 0 \).

Maximizing the objective (1) subject the budget constraints (7) and the borrowing constraints, for given initial values \( D_{i,-1} > 0, S_{i,-1}, c_{-1} > 0 \), leads to the following first order conditions for consumption, investments in deposits, and contingent claims,

\[ \xi_t u_{c,i,t} = \lambda_{i,t} \]  
(9)

\[ \frac{\lambda_{i,t}}{R^d_t} = \beta E_t \frac{\lambda_{i,t+1}}{\pi_{t+1}} + \beta E_t \frac{u_{d,i,t+1}}{\pi_{t+1}} \]  
(10)

\[ \varphi_{i,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \]  
(11)

and the budget constraint (7) holding with equality, as well as the transversality conditions.

Where \( \lambda_{i,t} \geq 0 \) denotes the multiplier on the budget constraint.
Households are assumed to be monopolistically supplying differentiated labor services $n_{i,t}$. So-called labor packers (or unions) produce the effective labor units $n_t$ by the production function $n_t^{1/\mu_t} = \int_0^1 n_{i,t}^{1/\mu_t} \, di$. Here, the elasticity of substitution between differentiated labor services is given by $\vartheta_t = \frac{\mu_t w_t}{\mu_t w_t - 1}$ and varies exogenously over time. Specifically we assume that $\vartheta_t > 1$, implying that $0 < \mu_w < 1$, where $\mu_w$ evolves according to

$$\left( \frac{\mu_{t+1}}{\mu_t} \right)^{\vartheta_t} = \left( \frac{\mu_{t-1}}{\mu_t} \right)^{\vartheta_t} e^{\varepsilon_{w,t}}, \quad \varepsilon_{w,t} \sim N(0, \sigma_w)$$

(12)

which is expressed in terms of deviations from its steady state value. Labor packers (unions) sell effective units of labor to intermediate goods producing firms at price $w_t$, which denotes the aggregate wage rate. Profit maximization then leads to the following labor demand $^9$,

$$n_{i,t} = \left( \frac{w_{i,t}}{w_t} \right)^{-\vartheta_t} n_t,$$  

(13)

For setting the optimal wage rate $w_{i,t}$ household $i$ does not only face the objective (1) and the flow budget constraint (7), but as well the labor demand function (13). As households face Rotemberg (1982) adjustment cost when setting the individual wage rate, from the respective first order condition we can derive the model’s wage Phillips Curve $^10$

$$y_t = \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}^w}{\pi_t^w} - 1 \right) \frac{\pi_t^w}{\pi_t - 1} y_{t+1} + \frac{n_t}{(\pi_{t-1}^w - 1) \omega_t} (\mu_t^w mrs_t - w_t)$$

(14)

where $\pi_t^w = \frac{\pi_t^w}{\pi_t - 1}$ denotes the wage inflation rate, and $mrs_t$ denotes the representative household’s marginal rate of substitution between consumption and leisure,

$$mrs_t = -\frac{u_{sl}}{\lambda_t}$$

(15)

Here we already took into account that trade in contingent assets implies that the marginal utility of consumption is the same across households, any household who is permitted to optimize chooses the same supply of $n_{i,t}$. Now assume that labor packers (unions) already have determined $w_t$.

$^9$See Appendix 6.3 for derivation of the labor demand function

$^{10}$See Appendix 6.4 for derivation of the wage Phillips Curve
The first order conditions (9) to (11) can then be summarized as

\[
\frac{1}{R^d_t} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left( 1 + \frac{u_{d,t+1}}{u_{c,t+1}} \right) \right] \tag{16}
\]

\[
\frac{1}{R^E_t} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \right] - \left( \frac{\pi_t^w}{\pi_{t-1}^w} - 1 \right) \frac{\pi_t^w}{\pi_{t-1}^w} y_t - \frac{n_t}{(\mu_t^w - 1)} \omega_W (\mu_t^w m r s_t - w_t) \tag{17}
\]

\[
= \beta E_t \left[ \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left( \frac{\pi_t^w}{\pi_{t-1}^w} - 1 \right) \frac{\pi_t^w}{\pi_{t-1}^w} y_t \right] + \frac{1}{R_t} \left( \frac{\pi_t^w}{\pi_{t-1}^w} - 1 \right) \frac{\pi_t^w}{\pi_{t-1}^w} y_t \tag{18}
\]

where the risk free consumption Euler rate $R^E_t$ is defined in the usual way, $R^E_t = \frac{1}{R_t^E} = \frac{1}{E_t \pi_{t-1}^w}$.

### 2.2 Production

The production sector consists of perfectly competitive intermediate good producing firms, monopolistically competitive retailer, and perfectly competitive bundler who supply the final wholesale good.

There is a continuum of perfectly competitive intermediate goods producing firms. Firm $j \in [0, 1]$ produces intermediate goods $y_{j,t}^m$ with labor, which is hired from households, and with their own stock of capital $k_{j,t}$. The production technology is identical for all firms $j$ and exhibits standard neoclassical properties:

\[
y_{j,t}^m = a_t n_{j,t}^\alpha (u_{j,t} k_{j,t-1})^{1-\alpha}, \tag{19}
\]

where $\alpha \in (0, 1)$, and $a_t$ is a random productivity level with mean one, evolving according to the following process

\[
\log (a_t) = \rho_a \log (a_{t-1}) + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, \sigma_a) \tag{20}
\]

A firm $j$ accumulates physical capital $k_{j,t}$, by investing $x_{j,t}$, and subject to adjustment costs $\Gamma_I \left( \frac{x_{j,t}}{x_{j,t-1}} \right)$ associated with the change in investment

\[
k_{j,t} - (1 - \delta)k_{j,t-1} = \epsilon^I_t \left( 1 - \Gamma_I \left( \frac{x_{j,t}}{x_{j,t-1}} \right) \right) x_{j,t}, \tag{21}
\]

where $\Gamma_I \left( \frac{x_{j,t}}{x_{j,t-1}} \right) = \frac{\gamma_I}{2} \left( \frac{x_{j,t}}{x_{j,t-1}} - 1 \right)^2$ with $\gamma_I > 0$, and $\delta \in (0, 1)$ denotes the depreciation rate of investment expenditures. $u_{j,t} \in (0, 1)$ accounts for the firm $j$’s level of capital utilization. Investment-specific technology $\epsilon^I_t$ is assumed to evolve as an AR(1) process

\[
\log (\epsilon^I_t) = \rho_I \log (\epsilon^I_{t-1}) + \varepsilon_{\epsilon,t}, \quad \varepsilon_{\epsilon,t} \sim N(0, \sigma_I) \tag{22}
\]

Demand for external funds is induced in a simple way by assuming that wages have to be
paid on a banking account before goods are sold, such that firms have to borrow in terms of one-period loans $L_{j,t}$ from banks at the price $\frac{1}{R^t_j}$:

$$\frac{L_{j,t}}{R^t_j} \geq P_t w_t n_{j,t}.$$  \hspace{1cm} (23)

Thereby Christo¤el and Schabert (2013) abstract from asymmetric information issues and limited commitment, and assume that firms fully repay one unit of currency per unit of loan in the subsequent period, such that $R^t_j$ denotes a risk-free rate of return on loans. The budget constraint of firm $j$ can then be written as

$$Z_t a_{t} n_{j,t} \cdot (u_{j,t} k_{j,t-1})^{-1} - P_t w_t n_{j,t} + \left( \frac{L_{j,t}}{R^t_j} \right) + P_t x_{j,t} + P_t v_{j,t}^I \geq L_{j,t-1},$$  \hspace{1cm} (24)

where $P_t v_{j,t}^I$ denotes intermediate goods producing firm $j$’s profits and $Z_t$ the price of the intermediate good. Firm $j$ maximizes the present value of profits subject to (23) and (24) and a no-Ponzi game condition:

$$\max_{\{n_{j,t}, I_{j,t}, x_{j,t}, k_{j,t}\}} E_t \sum_{k=0}^{\infty} \phi_{t,t+k} v_{t+k}^I, \text { s.t. (23) and (24),}$$

where $\phi_{t,t+k} = \varphi_{t,t+1} \cdot \varphi_{t+1,t+2} \cdot \cdots \cdot \varphi_{t+k-1,t+k} \pi_{t+k}$ denotes a stochastic discount factor (see 7), and given that $k_{j,-1} > 0$ and $x_{j,-1} > 0$. The first order conditions for labor demand and loan demand are

$$mc_{t+k} \alpha a_{t+k} n_{j,t+k}^{\alpha-1} (u_{j,t+k} k_{j,t+k-1})^{-1} = w_{t+k} \cdot \left( \frac{R^t_{t+k}}{R^t_{t+k}} \right),$$  \hspace{1cm} (25)

$$\frac{L_{j,t}}{R^t_j} \geq w_t n_{j,t},$$  \hspace{1cm} (26)

where we used $\frac{Z_{t+k}}{R^t_{t+k}} = mc_{t+k}$ and (26) is binding if for the respective multiplier holds that $\chi_{t+k} = \left( \frac{R^t_{t+k}}{Z_{t+k}} \right) - 1 > 0$. The first order conditions for investment expenditures and physical capital read

$$1 = q_{j,t+k} \phi_{t+k} \left( 1 - \Gamma_{I} \left( \frac{x_{j,t+k}}{x_{j,t+k-1}} \right) - \Gamma_{I} \left( \frac{x_{j,t+k+1}}{x_{j,t+k}} \right) \right) + E_{t+k} \phi_{t+k+1} q_{j,t+k+1} \Gamma_{I}^I \left( \frac{x_{j,t+k+1}}{x_{j,t+k}} \right)^2$$  \hspace{1cm} (27)

$$q_{j,t+k} = E_{t+k} \phi_{t+k+1} \left[ q_{j,t+k+1} (1 - \delta) + r k_{j,t+k+1} u_{j,t+k+1} - r k (u_{t+k+1} - 1) + \frac{x \cdot r k}{2} (u_{t+k+1} - 1)^2 \right]$$  \hspace{1cm} (28)

where $q_t$ denotes the standard Tobin’s $q$ and we defined the first derivative of (19) w.r.t.
capacity utilization $u_{t+k}$ as

$$rk_{j,t+k} = mc_{t+k}(1 - \alpha)n_{j,t+k}^\alpha (u_{j,t+k} + k_{j,t+k})^{-\alpha}$$  \hspace{1cm} (29)$$

Further $rk$ denotes the steady state value of (29) and $\alpha$ denotes the inverse of the elasticity of $u_{t+k}$ with respect to (29). It should be noted that the firm’s investment decision is distorted if the interest rate on loans differs from the Euler rate $R^E_t \neq R^F_t$.

Monopolistically competitive retailers buy intermediate goods $y_i^n = \int_0^1 y_{i,t}^n dj$ at the real price $mc_t$. A retailer $k \in [0,1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers. Those bundle the goods $y_{k,t}$ to the final consumption good $y_t$ with the technology $y_t = \int_0^1 y_{k,t}^j dk$ where the elasticity of substitution is defined as $\varepsilon = \frac{1}{\mu_{t}^P}$, with $\varepsilon > 1$, implying $0 < \mu_{t}^P < 1$. The price mark-up $\mu_{t}^P$ evolves according to

$$\left( \frac{\mu_{t}^P}{\mu_{t}^p} \right) = \left( \frac{\mu_{t}^P}{\mu_{t}^P} \right) e^{\varepsilon_{p,t}} \varepsilon_{p,t} \sim N(0, \sigma_p)$$  \hspace{1cm} (30)$$

The cost minimizing demand for $y_{k,t}$ is therefore given by\textsuperscript{11}

$$y_{k,t} = \left( \frac{P_{k,t}}{P_t} \right)^{-\varepsilon} y_t$$  \hspace{1cm} (31)$$

Further, retailers face Rotemberg (1982) price adjustment costs $PAC_{k,t}$ which are defined by

$$PAC_{k,t} = \frac{\omega_p}{2} \left( \frac{P_{k,t}}{P_{k,t-1}} \left( \frac{\pi_t}{\pi_t^{1-p}} \pi_t^{1-p}_{t-1} \right) - 1 \right)^2 y_t$$  \hspace{1cm} (32)$$

From the optimal price setting behavior of the retailer we derive the price Phillips Curve:\textsuperscript{12}

$$\beta E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{1-p}} - 1 \right) \frac{\pi_t}{\pi_t^{1-p}} \frac{y_{t+1}}{y_t} \right] + \frac{\mu_{t}^P}{\omega_p (\mu_{t}^P - 1)} \left( mc_t - \frac{1}{\mu_{t}^P} \right)$$  \hspace{1cm} (33)$$

\subsection*{2.3 Financial intermediaries}

The basic role of the financial intermediation sector is to provide loans to firms which are required to pay wages up-front. The underlying model allows for the propagation of monetary policy through a bank balance sheet channel by including a bank’s balance sheet constraint and an agency cost associated with the supply of credit. Following Curdia, and Woodford (2011) we use a real resource cost related to the supply of credit from banks to firms. These credit costs are increasing in the amount of loans and decreasing in the amount of reserves.

\textsuperscript{11}Which is derived analogously to the labor demand function

\textsuperscript{12}See Appendix 6.5 for derivation of the wage Phillips Curve
While lacking an explicit microfoundation this assumption allows to introduce the relation between balance sheet items and the cost of providing credit. In addition to this, same as Curdia, and Woodford (2011) we assume that banks face a balance sheet constraint requiring deposits holdings to equal the expected payoff from the assets on the balance sheet.

There is a continuum of perfectly competitive financial intermediaries, which we call banks. They receive deposits from household $D_t = \int D_{i,t} di$ and invest in loans $L_t = \int L_{j,t} dj$, and reserves $M_t$. Further, there are two types of bonds which are hold by the banks. Short-term government bonds $B^S_t$, are issued at the price $\frac{1}{R^S_t}$ in period $t$ and deliver the payoff one in period $t+1$. Long-term government bonds $B^L_t$ are assumed to be perpetuities. We model the stock of long-term bonds in the same way as Chen, Curdia, and Ferrero (2012). Perpetuities cost $p^L_t$ at time $t$ and pay exponentially decaying coupon $\rho^s_t$ at time $t+s+1$, with $\rho^s \in (0, 1]$. We assume that the coupon rate is time varying, following the law of motion described by

$$\left(\frac{\rho^s_t}{\rho^s}\right) = \left(\frac{\rho^s_{t-1}}{\rho^s}\right)^{\rho^s_t} e^{\varepsilon^s_{t-1}; t} , \varepsilon^s_{t-1}; t \sim N (0, \sigma^s) \quad (34)$$

It can be shown that by assuming long-term bonds to be perpetuities, the price of a long-term bond in period $t$, issued $s$ periods ago, $p^L_{t-s}$, is a function of the coupon and the current price $p^L_t = \rho^s_t p^L_{t-1}$. Further, it can be shown that $YTM_t$, the gross yield to maturity at time $t$ on the long-term bond, is given by

$$YTM_t = \frac{1}{p^L_t} + \rho^s_t \quad (35)$$

Banks buy perpetuities at the price $p^L_t$ which pay off $p^L_{t+1} YTM^L_{t+1}$ in period $t+1$.

In each period the bank’s balance sheet has to be satisfied. It requires the bank to acquire deposits $D_t = \int D_{i,t} di$ in the maximum amount that it can repay in the end of each following period from the expected payoffs from its assets: add a motivation here ...

$$D_t = M_t + B^S_t + E_t \left[ p^L_{t+1} YTM^L_{t+1} B^L_t \right] + L_t \quad (36)$$

Banks can use government bonds as collateral to get additional reserves $I_t$ from the central bank in open market operations:

$$I_t \leq \kappa^S_t B^S_{t-1} + \kappa^L_t p^L_t YTM^L_t B^L_{t-1} + \varepsilon_{t,omo} \quad (37)$$

where the value of the collateral discounted at the main refinancing rate $R^m_t$. The parameters $\kappa^S_t$ and $\kappa^L_t$ are additional monetary policy instruments which allow the central bank to control...
the fraction of short-term and long-term bonds to be eligible for Repo contacts and to control the amount of reserves supplied to banks resp. Further, $\epsilon_{t, lomo}$ denotes a stochastic shock to the collateral constraint with zero mean. Note that we assume that in the absence of LSAPs, the parameter $\kappa^L_t = 0$, implying that only short-term Treasuries are eligible for Repos.

Costs of financial intermediation are specified in an implicit way, which is particularly useful for calibration/estimation purposes. Banks face real resource costs $\Xi_t \geq 0$ when they supply loans to firms. Following Curdia, and Woodford (2011), these costs are convex, increasing in the amount of loans, with $\Xi_{t,t} \geq 0$, and decreasing in the amount of real reserves $M_{t-1} + I_t - \mu_t D_{t-1}$, which are reducing banks’ costs, $\Xi_{m,t} \leq 0$:

$$\Xi_t = \Xi \left( \frac{L_t}{P_t}, \frac{M_{t-1} + I_t - \mu_t D_{t-1}}{P_t} \right),$$

(38)

where we consider that the bank relies on reserves to manage deposits (e.g. non-modelled random withdrawals) in accordance to the relation $\mu_t D_{t-1}$. Thereby $\mu_t$ denotes the time varying minimum reserve requirements which evolve according to

$$\left( \frac{\mu_t}{\bar{\mu}} \right) = \left( \frac{\mu_t-1}{\bar{\mu}} \right)^{\rho_{\mu}} e^{\varepsilon_{\mu,t}}, \varepsilon_{\mu,t} \sim N(0, \sigma_{\mu})$$

(39)

For the quantitative analysis we will employ a specific form for the cost of financial intermediation which is given by

$$\Xi_t = \zeta_t \left( \frac{L_t}{P_t} \right)^{\eta_{re}} \left( \frac{M_{t-1} + I_t - \mu_t D_{t-1}}{P_t} \right)^{-\phi_{re}}$$

(40)

Given that bonds are discounted at the rate $R_t^{m}$ (see 37) acquisition of reserves $I_t$ is associated with costs $I_t (R_t^{m} - 1)$. The real profits of a bank $v^L_t$ are thus given by

$$P_t v^L_t = \frac{D_t}{R_t^{D_t}} - D_{t-1} - \frac{B_t^S}{R_t^{SB_t}} + B_{t-1}^S - p_t^L B_t^L + p_t^L YTM_t B_{t-1}^L$$

$$- \frac{L_t}{R_t^L} + L_{t-1} - M_t - M_{t-1} - I_t (R_t^{M_t} - 1) - P_t \Xi_t$$

(41)

where real values of the variables are defined as

$$m_t = \frac{M_t}{P_t}, i_t = \frac{I_t}{P_t}, d_t = \frac{D_t}{P_t}, b_t^S = \frac{B_t^S}{P_t}, b_t^L = \frac{B_t^L}{P_t}, \text{ and } l_t = \frac{L_t}{P_t}.$$

The banks aim at maximizing the present value of profits, subject to (37) and (41), which can be reduced to

$$P_t v^L_t = \frac{D_t}{R_t^{D_t}} - \frac{B_t^S}{R_t^{SB_t}} - p_t^L B_t^L - \frac{L_t}{R_t^L} - M_t - I_t (R_t^{M_t} - 1) - P_t \Xi_t$$

(42)

by taking the bank balance sheet condition (36) into account. Further, a no-Ponzi game condition $\lim_{s \to \infty} E_t \phi_{t,t+s} D_{t,t+s} \geq 0$, as well as $L_t \geq 0$, $B_t^S \geq 0$, $B_t^L \geq 0$, and $M_t \geq 0$ have to
hold. The banks’ optimization problem then reads

$$\max E_t \sum_{k=0}^{\infty} \phi_{t,t+k} \nu_{t+k}, \text{ s.t. (36), (37) and (41)},$$

where the stochastic discount factor is defined as

$$\phi_{t,t+k} = \varphi_{t,t+1} \pi_{t+1} \varphi_{t+1,t+2} \pi_{t+2} \ldots \varphi_{t+k-1,t+k} \pi_{t+k}$$

The first order conditions with regard to deposits, short-term bonds, long-term bonds, loans, money holdings, and reserves $I_t$ are given by

$$\frac{1}{R^D_{t+k}} = \frac{1}{R^E_{t+k}} E_{t+k} \left[ -\mu_t \Xi_{m,t+k+1} \right] - \Theta_{t+k} \tag{43}$$

$$\frac{1}{R^{SB}_{t+k}} = \frac{1}{R^E_{t+k}} E_{t+k} \left[ \eta_{t+k+1} \kappa^S_{t+k+1} \right] - \Theta_{t+k} \tag{44}$$

$$\frac{1}{R^L_{t+k}} = \frac{1}{R^E_{t+k}} E_{t+k} \left[ R_{t+k+1} YTM_t \eta_{t+k+1}^L \right] - \Theta_{t+k} \tag{45}$$

$$\Xi_{m,t+k} = 1 - \left( 1 + \eta_{t+k} \right) R^M_{t+k} \tag{48}$$

Note that $\varphi_{t+k,t+k+1} = \frac{\phi_{t,t+k+1}}{\phi_{t,t+k}}$ and $\frac{1}{R^E_{t+k}} = E_{t+k} \left[ \varphi_{t,k+1} \right]$. Where $R^{LB}_{t+k+1} = \frac{P_{t+k+1}}{P_{t+k}}$, and $\Theta_{t+k}$ denotes the multiplier on the balance sheet constraint (36) and $\eta_{t+k}$ denotes the multiplier on the collateral constraint (37). Note that real cost of intermediation are specified as

$$\Xi_t = \varsigma_t (l_t)^{\eta_{rc}} \left( \frac{m_{t-1}}{\pi_i} - \mu_t d_{t-1} \frac{1}{\pi_i} + l_t \right)^{-\phi_{rc}} \tag{49}$$

with the marginal cost of loan provision

$$\Xi_{l,t} = \eta_{rc} \Xi_t \frac{l_t}{l_t} \tag{50}$$

and the marginal contribution of reserves to the reduction of intermediation costs

$$\Xi_{m,t} = -\frac{\phi_{rc} \Xi_t}{(m_{t-1} \pi_i^{-1} - \mu_t d_{t-1} \pi_i^{-1} + i_t)} \tag{51}$$
By eliminating Θt the banks’ first order conditions can be reduced to

\[ \frac{1}{R_{t+k}^{1}} = 1 + (\mu_t - 1) \frac{1}{R_{t+k}^{E}} E_{t+k} \left[ -\Xi_{m,t+k+1} \right] \]  
(52)

\[ \frac{1}{R_{t+k}^{NSB}} = 1 + \frac{1}{R_{t+k}^{E}} \left( E_{t+k} \left[ \eta_{t+k+1} R_{t+k+1}^{S} \right] - E_{t+k} \left[ -\Xi_{m,t+k+1} \right] \right) \]  
(53)

\[ 1 = \frac{1}{R_{t+k}^{E}} \left( E_{t+k} \left[ R_{t+k+1}^{LB} YTM_{t+k+1} \eta_{t+k+1} \kappa_{t+k+1}^{L} \right] ight) 
- E_{t+k} \left[ -\Xi_{m,t+k+1} \right] E_{t+k} \left[ R_{t+k+1}^{LB} YTM_{t+k+1} \right] + E_{t+k} \left[ R_{t+k+1}^{LB} YTM_{t+k+1} \right] \]  
(54)

\[ \frac{1}{R_{t+k}^{L}} = 1 - \Xi_{t,t+k} - \frac{1}{R_{t+k}^{E}} E_{t+k} \left[ -\Xi_{m,t+k+1} \right] \]  
(55)

\[ R_{t+k}^{M} = 1 - \Xi_{m,t+k} - \eta_{t+k} R_{t+k}^{M} \]  
(56)

Further, the following complementary slackness conditions have to be satisfied

\[ i_{t+k} \leq \kappa_{t+k}^{S} \frac{b_{t+k-1}^{S}}{\pi_{t+k} R_{t+k}^{m}} + \kappa_{t+k}^{L} \frac{p_{t+k}^{L} YTM_{t+k+1} R_{t+k-1}^{m}}{\pi_{t+k} R_{t+k}} \]  
(57)

\[ \eta_{t+k} \left( \kappa_{t+k}^{S} \frac{b_{t+k-1}^{S}}{\pi_{t+k} R_{t+k}^{m}} + \kappa_{t+k}^{L} \frac{p_{t+k}^{L} YTM_{t+k+1} R_{t+k-1}^{m}}{\pi_{t+k}} - R_{t+k}^{m} i_{t+k} \right) = 0. \]

as well as the balance sheet constraint (36).

Equation (52) relates the rate of return on deposits to the payoff and the expected change in credit costs implied by the holding of the deposit. The return of the short-term government bond (53) is related to the payoff corrected for the expected marginal contribution by the relaxation of the collateral constraint which is induced by holding the bond and for the expected reduction of the credit costs. Note that the second term of the right-hand side of (53) captures the liquidity premium on the short-term bonds rate which is induced by the bond’s feature of being an eligible collateral for reserves acquisition. Equation (54) relates the price of a long-term bond perpetuity to the expected payoff and the expected credit costs. Note that the first term on the right-hand side of (54) denotes the bond’s expected liquidity premium. In absence of unconventional monetary policy measures, which means that the respective policy parameter is set \( \kappa_{t}^{L} = 0 \), this premium will be equal to zero. The return on loans (55) equals the expected payoff corrected for the marginal cost of loan provision and the expected future change of the credit cost function induced by the holding of another unit of loans. Finally, the return on holding reserves (56) is related to the payoff and the marginal reduction of the banking cost, as well as the marginal cost of reserves acquisition borne by another unit of reserves.

2.4 The government

The government raises lump-sum taxes \( \tau_{t} \) and purchases goods \( g_{t} \). It issues short-term bonds, whereas nominal short-term bond supply grows with a constant rate \( B_{t}^{TS} = \Gamma B_{t-1}^{FS} \), where
It further issues nominal long-term debt securities which are modeled as perpetuities with coupons payments that decay exponentially at the rate \( \rho \in [0, 1] \). Since bonds issued in period \( t - s \) are equivalent as \( \rho^s \) bonds issued in \( t \), we assume – without loss of generality – that all long-term debt are of one type (which implies that the government redeems all old bonds) in each period. The price of a perpetuity issued in period \( t \) is \( p_t^L \), while it pays out

\[
1 + \rho p_t^L = p_t^L \left( \frac{1}{p_{t+1}^L} + \rho^s \right) = p_t^L YTM_{t+1}
\]

units of currency in period \( t + 1 \). Real goods purchases and real issuance of long-term bonds follow exogenous AR(1) processes.

\[
\left( \frac{g_t}{g} \right) = \left( \frac{g_{t-1}}{g} \right)^{\rho_g} e^{\varepsilon_{g,t}}, \quad \varepsilon_{g,t} \sim N(0, \sigma_g)
\]

\[
\left( \frac{p_t^L b_t^{TL}}{p^L b^{TL}} \right) = \left( \frac{p_{t-1}^L b_t^{TL}}{p^L b^{TL}} \right)^{\rho_b} e^{\varepsilon_{b,t}}, \quad \varepsilon_{b,t} \sim N(0, \sigma_b)
\]

where \( \rho_b, \rho_g \in (0, 1) \) and \( \varepsilon_{b,t} \sim N(0, \sigma_b) \), and \( \varepsilon_{g,t} \sim N(0, \sigma_g) \) are i.i.d. innovations.

The treasury’s budget constraint reads

\[
\frac{B_t^{TS}}{R_t} + p_t^L B_t^{TL} = B_t^{TS - 1} + p_t^L YTM_t B_t^{TL} + P_t g_t - P_t \tau_t
\]

The left-hand side of (61) is the market value of the total amount of short-term bonds and long-term bonds issued by the treasury at time \( t \), expressed in nominal terms. The right-hand side is the total deficit at time \( t \), which is the cost of servicing bonds maturing in period \( t \) plus government spending net of taxes.

Let \( B_t^{TS} \) denote the total stock of newly issued short-term bonds, which is either held by banks, \( B_t^S \), or the central bank, \( B_t^{CS} \). Analogously let \( B_t^{TL} \) denote the total stock of newly issued long-term bonds, which is either held by banks, \( B_t^L \), or the central bank, \( B_t^{CL} \). Note that \( B_t^{TS} \) summarizes the total supply of short-term government bonds, which are eligible for open market operations in normal times. To avoid effects on the allocation by fiscal policy via distortionary taxation, we assume that the government has access to lump-sum transfers, which can be adjusted to balance the budget.

### 2.5 The central bank

The central bank supplies money in open market operations outright, \( M_t^H = \int_0^1 M_{t,i}^H di \), and via repurchase agreements against short-term bonds, \( M_t^R = \int_0^1 M_{t,i}^R di \). In times of unconventional monetary policy measures the central bank further supplies money via LSAPs, \( M_t^{QE} = \int_0^1 M_{t,i}^{QE} di \). Newly issued money thus sums up to \( I_t = M_t^H - M_{t-1}^H + M_t^R + M_t^{QE} \), for which the central bank receives government bonds. Note that \( M_t^{QE} = 0 \) in the absence
of LSAPs. Hence, in period $t$ the central bank gets $I_t R^m_t$ units of bonds for $I_t$ units of cash, such that its budget constraint reads

$$\frac{B_t^{CS}}{R_t^S} - B_{t-1}^{CS} + p^L_t B_t^{CL} - p^L_t Y_t M_t B_t^{CL} + P_t \tau^m_t$$

$$= \left( M_t^H - M_{t-1}^H \right) R^m_t + \left( M_t^R + M_t^{QE} \right) (R^m_t - 1) \tag{62}$$

Following Schabert and Reynard (2009), we identify seigniorage revenues as interest earnings from issuing money via repos or from holding interest bearing assets:

$$P_t \tau^m_t = B_t^{CS} - \frac{B_t^{CS}}{R_t^S} + p^L_t Y_t M_t B_t^{CL} - p^L_t B_{t-1}^{CL} + \left( M_t^R + M_t^{QE} \right) (R^m_t - 1) \tag{63}$$

We do not consolidate the public sector and assume that the central bank transfers $P_t \tau^m_t$ directly to households. Substituting out central bank transfers, its bond holdings evolve according to

$$B_t^{CS} = B_{t-1}^{CS} + p^L_t Y_t M_t B_{t-1}^{CL} - p^L_t B_{t-1}^{CL} = R^m_t (M_t - M_{t-1}) . \tag{64}$$

For the policy rate $R^m_t$ we follow a conventional specification and apply a simple feedback rule, which describes how the central bank adjusts the policy rate in response to changes in its own lags, in inflation, a measure for real activity output-gap, and the contemporary output growth:

$$R^m_t = (R_{t-1}^m)^{\rho_R} (R^m)^{1-\rho_R} \left( \frac{\pi_t}{\pi} \right)^{\rho_{\pi} (1-\rho_\pi)} \left( \frac{y_t}{y} \right)^{\rho_y (1-\rho_y)} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{dy} (1-\rho_{dy})} \exp \varepsilon_{t, t}, \tag{65}$$

where $R^m > 1$, $\rho_R \geq 0$, $\rho_\pi \geq 0$, $\rho_y \geq 0$, and $\rho_{dy} \geq 0$ and the $\varepsilon_{t, t}$s are normally and i.i.d. with $E_{t-1} \varepsilon_{t, t} = 0$.

The central bank further controls money supply by deciding on the fraction of eligible assets purchased in period $t$, i.e. it sets $\kappa_t^S$ and $\kappa_t^L$. Both affect the size and the composition of the central bank balance sheet. There are many possible ways to set $\kappa_t^S$ and $\kappa_t^L$, for example $\kappa_t^S = 1$ implies full allotment of short-term bonds, and by assuming $\kappa_t^L = 0$ together with setting

$$\kappa_t^S = \frac{R^m_t}{B_t^{CS} - 1} \tag{66}$$

leads to a constant money supply $i_t \leq i$. We will consider different regimes for $\kappa_t^L$, for the quantitative analysis. We finally have to specify how money is supplied either outright or via repos. For this, we assume that the central bank exogenously sets the fraction of repos

$$\Lambda_t > 0 : M_t = \Lambda M_t^R \tag{67}$$
2.6 Equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear. A complete equilibrium definition can be found in appendix 6.1. Here, we describe some particular properties: We will restrict our attention to equilibria where the liquidity constraint (23) and the collateral constraint (37) are binding, which requires \( R_t^L > R_t^E \) \( \Rightarrow \) and \( \eta_t > 0 \) \( \Leftrightarrow \) \( R_t^m - 1 < -\Xi_{m,t} \). Both requirements will be ensured in equilibrium, by an appropriate choice of parameters for the intermediation cost function (38).

We further use that the total stock of short-term bonds and long-term bonds are either held by banks or the central bank, \( B_t^{TS} = B_t^S + B_t^{CS} \), and \( B_t^{TL} = B_t^L + B_t^{CL} \), and (64), to identify how banks’ bond holdings evolve over time:

\[
B_t^S - B_{t-1}^S + p_t^L B_t^L - p_{t-1}^L B_{t-1}^L = B_t^{TS} - B_{t-1}^{TS} + p_t^L B_t^{TL} - p_{t-1}^L B_{t-1}^{TL} - R_t^m (M_t - M_{t-1}) \tag{68}
\]

where

\[
\frac{B_t^S}{R_t^{SB}} = \frac{B_t^{TS}}{R_t^{SB}} - \kappa_t^S B_{t-1}^S \tag{69}
\]

\[
p_t^L B_t^L = p_t^L B_t^{LT} - \kappa_t^L p_t^L Y T M_t B_{t-1}^L \tag{70}
\]

Condition (68) thus describes how private public debt holdings increases with bond supply and decreases with money supply \( R_t^m (M_t - M_{t-1}) \), where a higher the price of money \( R_t^m \) tends to raise the central banks bond holdings (see 64).

Appendix 6.2 examines the steady state of the model for a binding collateral constraint \( (\eta > 0) \) where variables without time indices denote the particular steady state values. It can be shown that the central bank can independently chose the steady state inflation rate and that the government’s financing decision, i.e. the relation between bond issuance and (lump-sum) tax financing, is neutral in the sense that it does not affect the real allocation in the steady state. These properties are summarized in the following proposition.

**Proposition 1** Consider a steady state of the economy under a binding collateral constraint. The central bank can independently chose its long-run inflation target and the steady state real allocation of resources is independent of public financing.

**Proof.** See appendix 6.2

3 Calibration and estimation

3.1 Calibration

In order to derive quantitative results by simulation experiments the model is partly calibrated and estimated with Bayesian techniques. Table 2 summarizes the calibrated parameters and Table 3 summarizes the estimated parameters. Most parameters affecting the steady state
are calibrated while key parameters driving the dynamics of the model are estimated. We try to use standard parameter values as far as possible. The model’s interest rates are calibrated to match the average values of their empirical counterparts which are calculated for the time period covered by the underlying data sample ranging from 1964:Q3 to 2007:Q4. Hence we calibrate the respective steady state values for the quarterly main refinancing rate $R^m = 1.56$, the deposit rate $R^D = 1.59$, the loan rate $R^L = 2.2$, and the yield to maturity $YTM = 1.79$. For the household preferences the inverse of the intertemporal elasticity of substitution equals $\sigma = 1$, indicating a log utility function. Due to the assumption that deposits directly enter households utility we have to define the scaling parameter $\varrho$. We calibrate $\varrho$ (see 128) such that the steady state value of $R^D$ implied by (76) matches its empirical counterpart.

The steady state elasticity of substitution between differentiated labor types is calibrated as $\mu^w = 1.2$. The steady state elasticity of substitution between differentiated goods is calibrated as $\mu^p = 1.2$. The quarterly time discount factor is set to $\delta = 0.03$, the steady state capital utilization rate $\psi = 1$. The quarterly working time is $n = 1/3$ and the fraction of money held outright $\Lambda = 0.1$. The average Duration measure of a 7 year U.S. Treasury is assumed to be of 5.5 years for the time period under consideration, implying a steady state decay factor for the perpetuity’s coupon rate of $\rho = \frac{\text{Duration}}{\text{YTM}} = 0.88$. By the assumed functional form of the financial intermediation cost function given by equation (49) we have to define three parameters. The steady state scaling parameter $\zeta$ is determined by (130) in order to be able to match the steady state value of the spread $R^L - R^d$ which is implied by (55) minus (52) with its empirical counterpart. Further we set the elasticity parameters $\phi_{rc}$ and $\eta_{rc}$ for the steady state calibration to 2. Note that those parameters will along with other parameters be estimated in a next step. The calibration of the cost function implies the following ordering of the model’s interest rates: $R^L > R^{Euler} > R^d > R^b$. Same as Smets and Wouters (2007) we calibrate an exogenous government spending-GDP ratio $G/Y$. For the present data sample $G/Y$ is found to be equal to 0.2.

Monetary policy in the model operates via two margins. The standard monetary policy channel operates via the Taylor type interest rule. Note that we estimate the parameters

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16 In the estimation we make use of output data but refrain from a further decomposition in order to keep the model and the number of structural shocks small.

17 The observed annualized average U.S. inflation rate for the underlying data sample, 1964:Q3 to 2007:Q4, is 3.98 percent.

18 Source (...)

19 For the derivation of the relationship between the bond Duration measure and the decay factor of a perpetuity see Appendix 6.7.
Table 2: Values assigned to the calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9925</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>Steady-state capital depreciation rate</td>
</tr>
<tr>
<td>$u$</td>
<td>1</td>
<td>Steady-state capacity utilization rate</td>
</tr>
<tr>
<td>$\mu^w$</td>
<td>1.2</td>
<td>Steady-state wage mark-up</td>
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<tr>
<td>$\mu^p$</td>
<td>1.2</td>
<td>Steady-state price mark-up</td>
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<tr>
<td>$G/Y$</td>
<td>0.2</td>
<td>Steady-state ratio of government consumption to GDP</td>
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<td>$n$</td>
<td>0.33</td>
<td>Steady-state of hours worked</td>
</tr>
<tr>
<td>$\kappa^S$</td>
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<td>Share of short term bonds</td>
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<tr>
<td>$\kappa^L$</td>
<td>0</td>
<td>Share of long term bonds, in line with bills-only policy</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.0098</td>
<td>Steady state price inflation</td>
</tr>
<tr>
<td>$R^M$</td>
<td>1.0156</td>
<td>Steady state main refinancing rate</td>
</tr>
<tr>
<td>$R^L$</td>
<td>1.022</td>
<td>Steady state loan rate</td>
</tr>
<tr>
<td>$YTM$</td>
<td>1.0173</td>
<td>Steady state yield to maturity</td>
</tr>
<tr>
<td>Duration</td>
<td>5.5</td>
<td>Duration of long-term bonds</td>
</tr>
<tr>
<td>$R^D$</td>
<td>1.0159</td>
<td>Steady state deposit rate</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0.1</td>
<td>Fraction of money held outright</td>
</tr>
</tbody>
</table>

Notes: This table shows the values for the calibrated parameters and the steady state ratios.

governing this policy reaction function. In addition to the setting of the policy interest rate there are further monetary policy parameters which are directly or indirectly affecting the cost of financial intermediation (49). One of the those is the proportion of deposits that the banks have to hold to satisfy the minimum reserve requirements $\mu$ which is estimated as well. The proportion of outright money supplied by the central bank $\Lambda$ is set to 0.1. For the baseline model and later on in the estimation process we set the fraction of short-term bonds eligible for Repo contracts $\kappa^S$ to unity implying that money injections vary with the real value of the respective collateral. The fraction of long-term bonds eligible for Repos $\kappa^L$ captures the model’s channel for unconventional monetary policy measures, namely LSAPs. In absence of such a policy measures the parameter $\kappa^L$ is set equal to zero. For the policy simulation experiment we analyze the model’s implications for different time paths of $\kappa^L$ which take non-zero values as a policy reaction to a severe crisis scenario.

3.2 Data and shocks

For the estimation we use quarterly U.S. data ranging from 1964:Q3 to 2007:Q4, leaving out the impact of the beginning subprime crisis. The macroeconomic time series are taken from the Federal Reserve Economic Database (FRED) which is maintained by the Federal Reserve Bank of St. Louis. We use real GDP which is the empirical counterpart to the model’s variables $c_t + i_t + g_t$, real consumption $c_t$, real investment $i_t$, hours worked $n_t$, real wages $W_t/P_t$, inflation calculated by using the GDP Implicit Price Deflator deflator $\frac{P_t}{P_{t-1}}$, real bank
reserves $M_t$, and real deposits $\frac{D_t}{P_t}$. The interest rate data we use are on the Federal Funds Rate to proxy for $R^m_t$, the 3-Month Certificate of Deposit rate $R^D_t$, the 7-Year Treasury constant maturity rate $YTM_t$, and on Moody’s Baa-rated corporate bond rate $R^L_t$. All time series are detrended, except for the interest rates, hours worked, and real deposits. We use a linear trend, as our model does not explicitly consider growth.

In order to estimate the model we have to introduce as many shocks as observable variables to the model. We start by introducing eight macroeconomic shocks that are used in comparable studies. A time preference shock ($\varepsilon_{\tau,t}$), a total factor productivity shock ($\varepsilon_{a,t}$), a price mark-up shock ($\varepsilon_{p,t}$), a mark-up shock on wages ($\varepsilon_{w,t}$), an interest rate shock ($\varepsilon_{r,t}$), a government spending shock ($\varepsilon_{g,t}$), an investment-specific technology shock ($\varepsilon_{\iota,t}$), and a reserve policy shock ($\varepsilon_{\mu,t}$). Further we introduce a decay factor shock ($\varepsilon_{\sigma,t}$), and a shock to the long-term bond supply ($\varepsilon_{b,t}$). In addition to these shocks we employ a shock that affects the cost of financial intermediation ($\varepsilon_{\zeta,t}$) and a shock driving a wedge between the real value of government debt and the collateral constraint in the provision of reserves via repos ($\varepsilon_{L,omo}$). The latter shock is necessary to reflect the fact that in the data only roughly one third of the used assets for collateral are government bonds and securities. All shocks except the shock to the Taylor rule and to the collateral constraint are modelled as AR(1) shocks.

### 3.3 Estimation

Employing Bayesian inference methods allows formalizing the use of prior information from earlier studies at both the micro and macro level in estimating the parameters of a possibly complex DSGE model. This seems particularly appealing in situations where the sample period of the data is relatively short, as is the case for the present study. From a practical perspective, Bayesian inference may also help to alleviate the inherent numerical difficulties associated with solving the highly non-linear estimation problem.

The log-linearized DSGE model leads to a rational expectations system (see Sims (2002)) which is given by

$$\Gamma_0 (\theta) s_t = \Gamma_1 (\theta) s_{t-1} + \Psi (\theta) \varepsilon_t + \Pi (\theta) \eta_t$$

(71)

where $s_t$ is a vector of model variables, $\varepsilon_t$ is a vector of exogenous shocks, $\eta_t$ is a vector of rational expectations errors within its elements. $\Gamma_0$, $\Gamma_1$, $\Psi$, and $\Pi$ are matrices that are nonlinear functions of structural parameters of the model. Typically a solution is derived by using standard perturbation techniques which lead as a solution to the linear state-space

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20 See Appendix 6.8 for complete data description and the description of the mapping of these variables to the states.
The first equation is the state transition equation. Here, $\phi_1$ and $\phi_\varepsilon$ are functions of the matrices $\Gamma_0$, $\Gamma_1$, and $\Psi$. The second equation is the observation equation with measurement errors being collected in $u_t$, where $Y_t$ corresponds to the vector of observables at time $t$. The matrix $\psi_1$ which contains zeros and ones, relates the model’s definitions with the data. The vector $\psi_0$ is required to match the means of the observed data.

Let $p(\theta|m)$ denote the prior distribution of the parameter vector $\theta \in \Theta$ for some model $m \in M$, and let $L(Y_T|\theta, m)$ denote the likelihood function for the observed data, $Y_T = \{y_t\}_{t=1}^T$, conditional on parameter vector $\theta$ and model $m$. Bayesian estimation employs the Kalman filter to construct the likelihood of the model under consideration. The joint posterior distribution, $p(\theta|Y_T, m)$, conditional on the sample data $Y_T$ and the model $m$, equals the model likelihood, multiplied by the priors on the model parameters, up to a factor of proportionality which is known as Bayes rule

$$p(\theta|Y_T, m) \propto L(Y_T|\theta, m)p(\theta|m) \quad (74)$$

The Kalman filter generates projections of the state of the linear approximate solution of (71) for the model, given an information set of observed macro time series.

The posterior is evaluated by applying the Random Walk Metropolis (RWM) algorithm which belongs to the class of Metropolis-Hastings algorithms (see An, and Schorfheide (2007), Schorfheide (2000)). For that purpose first a numerical optimization routine is used to maximize the log posterior. The RWM algorithm constructs a Gaussian approximation around the mode of the posterior kernel $L(Y_T|\theta, m)p(\theta|m)$ and a scaled version of the asymptotic covariance matrix which is the inverse of the negative Hessian computed numerically at the posterior mode. The latter serves as a covariance matrix for a proposal distribution. By applying a rejection sampling procedure, the algorithm then generates a sequence of dependent draws from the posterior that can be averaged to approximate posterior moments of interest, such as location measures and measures of dispersion.

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21The following description is based on Schorfheide (2000). This algorithm is used by DYNARE.

22The log posterior kernel is maximized using Dynare’s Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm. This algorithm is applied, in the case if derivative based methods, e.g. quasi-Newton BFGS or conjugate gradient, fail due to a rugged search landscape. The CMA-ES is a second order approach estimating a positive definite covariance matrix within an iterative procedure. Specifically, on convex-quadratic objective functions the covariance matrix of the search distribution is set to the inverse Hessian matrix. In contrast to quasi-Newton methods, the CMA-ES does not use or approximate gradients and does not require their existence (See Hansen (2006), Hansen and Ostermeier (1996)).

23Let $\Sigma_m$ denote the inverse of the (negative) Hessian computed at the posterior mode $\tilde{\theta}$. A starting value $\theta^{(0)}$ is drawn from $N\left(\tilde{\theta}, \epsilon_0^2\tilde{\Sigma}_m\right)$. For $s = 1, \ldots, S$: draw $\theta$ from the proposal distribution $N\left(\theta^{(s-1)}, \epsilon_0^2\tilde{\Sigma}_m\right)$. 

22
3.4 Parameter prior distributions

In Table 3 we summarize the prior distributions for the estimated parameters. We follow the common approach to choose the parameters’ prior distributions according to whether their supported intervals follow economic theory’s implications regarding the parameters values ranges. Specifically, we employ a gamma distribution for the parameters that should be positive to constrain their support on the interval $[0, 1]$, we employ a beta distribution for those parameters that span on the unit interval, and we use the inverse-gamma distribution for the standard deviations of shock innovations.

We chose the prior distributions for the parameters which are part of the canonical DSGE model closely following Smets and Wouters (2007). The utility function’s the habit parameter $h$ is assumed to be beta distributed fluctuating around 0.7 with a standard error of 0.1, and the inverse of the Frisch elasticity of labor supply $\nu$ is assumed to follow a gamma distribution with mean 2.0 and standard error of 0.75. The prior on the labor share in production is set to 1.0 with standard error of 0.25. For the priors of the parameters governing the indexation to past inflation in labor and goods markets $\omega_p$, and $\omega_w$, we chose beta distributions with means set to 0.5 and standard errors of 0.15. We chose the priors for the parameters measuring the degree of price and wage stickiness within the Rotemberg (1982) adjustment cost framework $\omega_p$, and $\omega_w$, following XXX. We assume for both parameters means of 60 with standard errors of 20.

Regarding the parameters governing the Taylor rule, we assume that the interest rate smoothing parameter $\rho_r$ is beta distributed with a mean of 0.6 and a standard deviation of 0.15. The parameters capturing the interest rate policy’s response to inflation $\rho_u$, to output level $\rho_y$, and to output gap $\rho_y$ are assumed to be each following a gamma distribution with means of 1.5, 1.25, and 1.25, whereas standard deviations are assumed to be 0.15, 0.05, and 0.05. Those are fairly standard parameter priors as well.

We assume the standard errors of the innovations to follow an inverse-gamma distribution. Further, same as Smets and Wouters (2007), we try to harmonize the priors on the stochastic

The jump from $\theta^{(s-1)}$ is accepted $(\theta^{(s-1)} = \tilde{\theta})$ with probability $\min \{1, r(\theta^{(s-1)}, \tilde{\theta}|Y_T)\}$ and rejected $(\theta^{(s)} = \theta^{(s-1)})$ otherwise. Here

$$r(\theta^{(s-1)}, \tilde{\theta}|Y_T) = \frac{L(Y_T|\tilde{\theta}, m)p(\tilde{\theta}|m)}{L(Y_T|\theta^{(s-1)}, m)p(\theta^{(s-1)}|m)}$$

We use $S = 1000000$ and drop the first 500000 to let the Markov chain produced by the RWM algorithm converge. The scaling factor $c$ is set to 0.3 which is intended to achieve an average acceptance rate per chain of approximately 25% (See An, and Schorfheide (2007), Schorfheide (2000)).

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processes as much as possible. Specifically, we assume the standard deviations of the shock processes known from the canonical DSGE model, which are the standard deviation for the process of production technology innovations $\sigma_\alpha$, preference innovations $\sigma_\xi$, government spending $\sigma_G$, investment specific technology $\sigma_I$, the price mark-up $\sigma_p$, and the wage mark-up $\sigma_w$, with mean of 0.05 and two degrees of freedom. As there is no previous experience regarding the prior choice for the standard deviations of the innovation processes governing the banking cost shock $\sigma_\zeta$, the minimum reserve policy shock $\sigma_\mu$, the coupon decay factor shock $\sigma_{\rho^*}$, the collateral shock $\sigma_{omo}$, and the long-term bond supply shock $\sigma_b$, we adapt the same assumptions regarding distribution, means, and degrees of freedom here. This seems to be an appropriate approach as these priors can be regarded as being loose. The persistence parameters of the AR(1) processes are assumed to be beta distributed with mean of 0.5 and standard deviation 0.2.

Further, the model contains three non-standard parameters which are the deposit elasticity $\varphi_d$ in the utility function, the loan provision elasticity $\eta_{rc}$ in the banking cost function, and the elasticity with respect to reserves and injections $\phi_{rc}$ in the banking cost function. Those parameters are at the heart of the present study. They directly account for the presumed existence of the channels through which the 2008/2009 financial crisis and later the non standard monetary policy measures worked according to our proposed model, and they further govern the latter’s effectivity. We estimate those parameters to avoid biasing the estimations and simulations in either direction. As there is practically no guidance about the prior shapes we decided to assume loose gamma distributed priors.

### 3.5 Estimation results

Table 3 summarizes the posterior means of the estimated parameters, as well as the 90% probability intervals. The estimated structural parameters, which are the habit parameter $h$, inverse of the Frisch elasticity of labor supply $\nu$, labor share in production $\alpha$, investment adjustment cost $\gamma$, inverse elasticity of capacity utilization, Rotemberg (1982) price adjustment cost parameters $\omega_p$ and $\omega_w$, and indexation to past price inflation and wage inflation, $\epsilon_p$ and $\epsilon_w$, are broadly in line with existing estimates for medium scale DSGE models (see Smets and Wouters (2007), Justiniano, Primiceri, and Tambalotti (2010)). For most parameters the mean of the posterior distribution is found to be relatively close to the mean of the prior assumptions. Here, exceptions are $\omega_w$ and $\epsilon_p$. The mean of the parameter measuring the degree of wage stickiness $\omega_w$ is estimated to be far higher than 60. The mean of the degree of price indexation is estimated to be much less than 0.5. Same as Smets and Wouters (2007) we estimate the investment adjustment cost slightly higher than assumed a priori, suggesting an even slower response of investment to the changes in the value of capital. Further, the posterior mean of the consumption habit parameter is higher than expected, indicating an
Note: The acceptance rate of the chain was roughly 25%. The estimation sample is 1964Q3-2007Q4. Computed by creating a sample of 1,000,000 draws with initial burning sample of 500,000 draws. Average

Table 3: Parameter estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Type</th>
<th>Mean</th>
<th>STD</th>
<th>Posterior Mean</th>
<th>5%-95%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor share in production</td>
<td>$\alpha$</td>
<td>$B$</td>
<td>0.7</td>
<td>0.05</td>
<td>0.71</td>
</tr>
<tr>
<td>Investment adj. cost</td>
<td>$\gamma_l$</td>
<td>$G$</td>
<td>4.0</td>
<td>1.5</td>
<td>5.55</td>
</tr>
<tr>
<td>Credit cost</td>
<td>$\eta_{bc}$</td>
<td>$G$</td>
<td>6.0</td>
<td>3.46</td>
<td>0.54</td>
</tr>
<tr>
<td>Reserves and injections</td>
<td>$\phi_{bc}$</td>
<td>$G$</td>
<td>6.0</td>
<td>3.46</td>
<td>0.41</td>
</tr>
<tr>
<td>Consumption habit</td>
<td>$h$</td>
<td>$B$</td>
<td>0.7</td>
<td>0.1</td>
<td>0.90</td>
</tr>
<tr>
<td>Inverse of Frisch elast.</td>
<td>$\nu$</td>
<td>$G$</td>
<td>2.0</td>
<td>0.75</td>
<td>1.91</td>
</tr>
<tr>
<td>Deposit elasticity</td>
<td>$\varphi_d$</td>
<td>$G$</td>
<td>6.0</td>
<td>3.46</td>
<td>5.16</td>
</tr>
<tr>
<td>Inv. elast. of capital util.</td>
<td>$\pi$</td>
<td>$G$</td>
<td>1.0</td>
<td>0.25</td>
<td>1.26</td>
</tr>
<tr>
<td>Calvo parameter prices</td>
<td>$\omega_p$</td>
<td>$G$</td>
<td>60</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Calvo parameter wages</td>
<td>$\omega_w$</td>
<td>$G$</td>
<td>60</td>
<td>20</td>
<td>122</td>
</tr>
<tr>
<td>Price indexation to past infl.</td>
<td>$\iota_p$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>Wage indexation to past infl.</td>
<td>$\iota_w$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.15</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>Taylor Rule Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho_r$</td>
<td>$B$</td>
<td>0.6</td>
<td>0.15</td>
<td>0.76</td>
</tr>
<tr>
<td>Resp. to inflation</td>
<td>$\rho_{\pi}$</td>
<td>$G$</td>
<td>1.5</td>
<td>0.15</td>
<td>1.81</td>
</tr>
<tr>
<td>Resp. to output</td>
<td>$\rho_y$</td>
<td>$G$</td>
<td>0.125</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Resp. to output growth</td>
<td>$\rho_{dy}$</td>
<td>$G$</td>
<td>0.125</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td><strong>Autocorrelation of shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.92</td>
</tr>
<tr>
<td>Preference</td>
<td>$\rho_{\xi}$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.47</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>$\rho_g$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.92</td>
</tr>
<tr>
<td>Invest. spec. tech.</td>
<td>$\rho_e$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.20</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\rho_p$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.04</td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>$\rho_w$</td>
<td>$B$</td>
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<td>0.2</td>
<td>0.55</td>
</tr>
<tr>
<td>Banking cost</td>
<td>$\rho_{\zeta}$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.004</td>
</tr>
<tr>
<td>Minimum reserve policy</td>
<td>$\rho_{\mu}$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.86</td>
</tr>
<tr>
<td>Coupon decay factor</td>
<td>$\rho_{p'}$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.84</td>
</tr>
<tr>
<td>Long term bond supply</td>
<td>$\rho_{b}$</td>
<td>$B$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>Standard deviation of innovations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\sigma_a$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.007</td>
</tr>
<tr>
<td>Preference</td>
<td>$\sigma_{\xi}$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.052</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>$\sigma_G$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.016</td>
</tr>
<tr>
<td>Invest. spec. tech.</td>
<td>$\sigma_I$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.158</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\sigma_p$</td>
<td>$IG$</td>
<td>0.001</td>
<td>2</td>
<td>0.043</td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>$\sigma_w$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.242</td>
</tr>
<tr>
<td>Banking cost</td>
<td>$\sigma_{\zeta}$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>1.249</td>
</tr>
<tr>
<td>Minimum reserve policy</td>
<td>$\sigma_{\mu}$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.072</td>
</tr>
<tr>
<td>Coupon decay factor</td>
<td>$\sigma_{p'}$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.004</td>
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<tr>
<td>Long term bond supply</td>
<td>$\sigma_b$</td>
<td>$IG$</td>
<td>0.005</td>
<td>2</td>
<td>0.017</td>
</tr>
<tr>
<td>Interest rate shock</td>
<td>$\sigma_{r_m}$</td>
<td>$IG$</td>
<td>0.001</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>Collateral shock</td>
<td>$\sigma_{omo}$</td>
<td>$IG$</td>
<td>0.01</td>
<td>2</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Note: $B, G$ and $IG$ correspond to Beta, Gamma and inverse Gamma distributions. Posterior densities were computed by creating a sample of 1,000,000 draws with initial burning sample of 500,000 draws. Average acceptance rate of the chain was roughly 25%. The estimation sample is 1964Q3-2007Q4.
even stronger desire to smooth consumption. Finally, the data are found to be informative on the structural parameters. Most of the parameters can be regarded as being identified, except for $\omega_p$ and $\alpha$, as the posterior and prior distributions are quite similar.

Regarding most of the exogenous disturbances the data appear to be informative. This does not seem to be the case for the standard deviations of the innovation processes for the price mark-up and wage mark-up, $\sigma_p$ and $\sigma_w$. Further, the AR(1) coefficients for the processes governing technology $\rho_a$, government spending $\rho_g$, minimum reserve policy $\rho_{\mu}$, coupon decay factor $\rho_{\rho^*}$, and long-term bond supply $\rho_{b}$, are estimated to be most persistent. In contrast to that, the estimated persistence parameter for the AR(1) process governing the banking cost innovations is estimated close to zero for the quarters between 1964:Q3 to 2007:Q4. Further, we find that the AR(1) coefficient of the wage mark-up process $\rho_w$ is not identified.

The monetary policy reaction function parameters, all seem to be identified. Same as Smets and Wouters (2007), we find the mean of the reaction to inflation is estimated to be relatively high (1.81). The estimate of the interest rate smoothing parameter implies a considerable degree of interest rate smoothing (0.76). However, policy does not seem to react strongly to the output-gap level (0.06), but to the short-run change in output (0.17).

For the estimates of the parameters governing the cost of financial intermediation, namely the loan provision elasticity $\eta_{rc}$, and the elasticity with respect to reserves and injections $\phi_{rc}$, the data appear to be informative. The posterior means for those parameters are estimated to be 0.54 and 0.41 resp., implying that ceteris paribus an increase by one unit of loans supplied increases banking cost stronger than an increase by one unit of reserves hold decreases banking cost. This provides support for a model where financial intermediation activity poses a channel for direct effects of nominal macroeconomic variables on the real economy. However data do not seem to be informative about the deposit elasticity $\varphi_d$.

### 3.6 Business cycle moments

As a first step to assess the model’s qualitative consistently with the pre-2008/2009 crisis period’s empirical evidence we calculate the model-implied moments at the posterior mean conditional on the structural shocks. As we assume that unconventional monetary policy in terms of large-scale long-term Treasury bond purchases did not took place before 2008/2009 we assume the shock governing the measure for the share of long-term bonds eligible for repurchase agreements $\kappa^L_t$ to have no impact on the variables’ dynamics for that period. Here we focus on the model’s business cycle implications in terms of selected moments. Table 4 compares the standard deviations, correlations with output, and autocorrelations of the observed data with the corresponding model-implied moments. Comparing the model-implied standard deviations with the observed standard deviations shows that the model overpre-
Table 4: Selected moments of observed data and model-implied moments

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Std. deviation rel. to output</th>
<th>Correlation with output</th>
<th>Autocorrelation of order 1</th>
<th>Autocorrelation of order 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Output</td>
<td>3.32</td>
<td>4.59</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.23</td>
<td>4.09</td>
<td>0.97</td>
<td>0.89</td>
<td>0.92</td>
</tr>
<tr>
<td>Investment</td>
<td>8.16</td>
<td>14.36</td>
<td>2.46</td>
<td>3.12</td>
<td>0.68</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.59</td>
<td>1.24</td>
<td>-</td>
<td>-</td>
<td>-0.22</td>
</tr>
<tr>
<td>Hours worked</td>
<td>2.77</td>
<td>4.42</td>
<td>0.83</td>
<td>0.96</td>
<td>0.77</td>
</tr>
<tr>
<td>Wages</td>
<td>2.72</td>
<td>3.46</td>
<td>0.82</td>
<td>0.75</td>
<td>0.10</td>
</tr>
<tr>
<td>Reserves</td>
<td>14.12</td>
<td>12.63</td>
<td>4.26</td>
<td>2.74</td>
<td>0.60</td>
</tr>
<tr>
<td>Deposits</td>
<td>7.05</td>
<td>5.37</td>
<td>2.12</td>
<td>1.17</td>
<td>0.19</td>
</tr>
<tr>
<td>Policy Rate</td>
<td>0.76</td>
<td>2.10</td>
<td>-</td>
<td>-</td>
<td>-0.38</td>
</tr>
<tr>
<td>Loan Rate</td>
<td>0.62</td>
<td>1.01</td>
<td>-</td>
<td>-</td>
<td>-0.73</td>
</tr>
<tr>
<td>Deposit Rate</td>
<td>0.73</td>
<td>0.60</td>
<td>-</td>
<td>-</td>
<td>-0.39</td>
</tr>
<tr>
<td>7yr Treas Yld</td>
<td>0.59</td>
<td>0.54</td>
<td>-</td>
<td>-</td>
<td>-0.66</td>
</tr>
</tbody>
</table>

Notes: The model-implied moments are computed from the solution of the model at the posterior mean. The standard deviations of inflation and the interest rates are in annual terms.
dicts the volatility of output, consumption, investment, inflation, hours worked, wages, the policy rate, and the loan rate, whereas it underpredicts the volatility of deposits, reserves, the deposit rate, and the 7 year Treasury yield. Notwithstanding the fact that volatility of investment, inflation, and hours worked implied by the model is compared to the data significantly overpredicted by roughly the factor 2, and volatility of the policy rate is significantly overpredicted by almost the factor 3, the implied standard deviations can be regarded as adequately matching their empirical counterparts. The model-implied standard deviations calculated relative to the output standard deviation are even closer to the respective moments calculated for the pre-crisis data sample. Still the model-implied volatility of investment relative to output volatility is found to be noticeably overpredicted. Furthermore, the volatility of reserves and the volatility of deposits relative to output still are distinctly underpredicted.

We find some significant discrepancies for the comparison of the model-implied correlations with output versus the observed correlations with output. Whereas the correlation of consumption, investment, hours worked, and deposits, matches the observed correlations with output rather well, implied correlations of the models interest rates with output have opposite signs compared to their empirical counterparts. Furthermore the observed correlation between inflation and output is negative while the model-implied correlation coefficient is positive.\textsuperscript{25} The model-implied autocorrelation patterns seem to provide a surprisingly good fit although there are again some exceptions. Notably the implied autocorrelation coefficients for reserves and the loan rate are significantly different from their observed counterparts.

Overall, the model provides a good description of the observed data for the variables usually examined in the canonical business cycle analysis approach. Despite some backdraws regarding the fit of the implied moments for the non-standard variables included in the present study, they do not give reason for general doubt regarding the proposed model specification. Rather, the model generates business cycle moments roughly consistent with their empirical counterparts though several novel ingredients have been introduced.

3.7 Variance decomposition

To assess the relative contributions of the model’s structural shocks to the variation in the observed data we conduct an unconditional posterior variance decomposition. Results reported in Table 5 imply that in terms of driving forces we can distinguish between macroeconomic variables which are usually analyzed within the canonical framework and variables newly introduced for the purpose of the present study. The former group is comprised of output, consumption, investment, hours worked, inflation, and the policy rate. Results show that these variables are mainly driven by economic shocks and policy shocks which are as well commonly employed for the canonical approach. In this regard the economic shocks are

\textsuperscript{25}One has to take into account that it is particularly problematic to match the observed second moments of interest rates and inflation.
Table 5: Variance decomposition of observed variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>preference</th>
<th>technology</th>
<th>interest rate</th>
<th>mark-up prices</th>
<th>mark-up wages</th>
<th>OMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>1.98</td>
<td>38.30</td>
<td>2.22</td>
<td>24.29</td>
<td>24.80</td>
<td>0.01</td>
</tr>
<tr>
<td>consumption</td>
<td>9.92</td>
<td>35.72</td>
<td>1.86</td>
<td>20.21</td>
<td>19.32</td>
<td>0.01</td>
</tr>
<tr>
<td>investment</td>
<td>8.58</td>
<td>30.36</td>
<td>1.85</td>
<td>20.29</td>
<td>22.94</td>
<td>0.01</td>
</tr>
<tr>
<td>hours worked</td>
<td>1.33</td>
<td>8.58</td>
<td>3.48</td>
<td>38.05</td>
<td>35.78</td>
<td>0.05</td>
</tr>
<tr>
<td>wages</td>
<td>2.88</td>
<td>44.55</td>
<td>1.93</td>
<td>24.34</td>
<td>20.71</td>
<td>0.01</td>
</tr>
<tr>
<td>inflation</td>
<td>2.80</td>
<td>8.21</td>
<td>6.30</td>
<td>60.99</td>
<td>10.33</td>
<td>0.02</td>
</tr>
<tr>
<td>reserves</td>
<td>0.06</td>
<td>0.21</td>
<td>0.04</td>
<td>1.04</td>
<td>0.23</td>
<td>98.23</td>
</tr>
<tr>
<td>deposits</td>
<td>0.19</td>
<td>2.22</td>
<td>0.37</td>
<td>4.74</td>
<td>1.62</td>
<td>0.01</td>
</tr>
<tr>
<td>loans</td>
<td>2.03</td>
<td>23.29</td>
<td>3.72</td>
<td>44.27</td>
<td>16.36</td>
<td>0.12</td>
</tr>
<tr>
<td>policy rate</td>
<td>3.46</td>
<td>13.41</td>
<td>2.86</td>
<td>56.01</td>
<td>13.05</td>
<td>0.02</td>
</tr>
<tr>
<td>deposit rate</td>
<td>0.21</td>
<td>1.93</td>
<td>0.27</td>
<td>4.82</td>
<td>1.51</td>
<td>1.38</td>
</tr>
<tr>
<td>loan rate</td>
<td>0.09</td>
<td>0.78</td>
<td>0.11</td>
<td>1.99</td>
<td>0.61</td>
<td>2.08</td>
</tr>
<tr>
<td>7yr Treas. yld.</td>
<td>0.15</td>
<td>1.66</td>
<td>0.16</td>
<td>2.61</td>
<td>1.11</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>credit cost</th>
<th>invest. spec. tech.</th>
<th>gov. spending</th>
<th>min. reserve pol.</th>
<th>decay factor</th>
<th>long-term bond sup.</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>0.71</td>
<td>5.77</td>
<td>0.76</td>
<td>1.03</td>
<td>0.01</td>
<td>0.11</td>
</tr>
<tr>
<td>consumption</td>
<td>0.61</td>
<td>7.68</td>
<td>3.35</td>
<td>1.03</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>investment</td>
<td>0.57</td>
<td>12.46</td>
<td>1.90</td>
<td>0.94</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>hours worked</td>
<td>3.99</td>
<td>5.87</td>
<td>1.21</td>
<td>1.50</td>
<td>0.00</td>
<td>0.16</td>
</tr>
<tr>
<td>wages</td>
<td>0.54</td>
<td>3.32</td>
<td>0.69</td>
<td>0.87</td>
<td>0.00</td>
<td>0.15</td>
</tr>
<tr>
<td>inflation</td>
<td>1.48</td>
<td>4.10</td>
<td>0.74</td>
<td>3.52</td>
<td>0.00</td>
<td>1.51</td>
</tr>
<tr>
<td>reserves</td>
<td>0.03</td>
<td>0.06</td>
<td>0.01</td>
<td>0.07</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>deposits</td>
<td>0.58</td>
<td>0.24</td>
<td>0.05</td>
<td>0.64</td>
<td>0.00</td>
<td>89.33</td>
</tr>
<tr>
<td>loans</td>
<td>5.58</td>
<td>2.96</td>
<td>0.64</td>
<td>0.87</td>
<td>0.00</td>
<td>0.15</td>
</tr>
<tr>
<td>policy rate</td>
<td>1.40</td>
<td>3.31</td>
<td>0.89</td>
<td>3.92</td>
<td>0.00</td>
<td>1.67</td>
</tr>
<tr>
<td>deposit rate</td>
<td>2.98</td>
<td>0.19</td>
<td>0.05</td>
<td>73.17</td>
<td>0.00</td>
<td>13.48</td>
</tr>
<tr>
<td>loan rate</td>
<td>47.98</td>
<td>0.08</td>
<td>0.02</td>
<td>39.67</td>
<td>0.00</td>
<td>6.59</td>
</tr>
<tr>
<td>7yr Treas. yld.</td>
<td>0.09</td>
<td>0.07</td>
<td>0.03</td>
<td>34.21</td>
<td>45.83</td>
<td>14.03</td>
</tr>
</tbody>
</table>

Notes: Values refer to the unconditional variance in percentage terms at the posterior mean.
namely on preferences $\varepsilon_{\xi}$, technology $\varepsilon_{a}$, price mark-up $\varepsilon_{p}$, wage mark-up $\varepsilon_{w}$, and investment specific technology $\varepsilon_{c}$. The policy shocks are on the monetary policy rate $\varepsilon_{r}$, and government spending $\varepsilon_{g}$. Overall, the decomposition of driving factors for the variation of these macroeconomic variables yields results which are widely in line with broad empirical evidence.

Regarding the non-standard variables included in the model we find that the observed variation of reserves is almost completely explained by the variation of the shock on the collateral constraint $\varepsilon_{omo}$. Note that for the present model in absence of changes in the parameter governing the share of short-term bonds eligible for open market operations $\kappa_{S}^{l}$ operations and large-scale asset purchases, extra reserves can only be gained by innovations to $\varepsilon_{omo}$ (see 37). Notably the variation of the observed series on deposit holdings is found to be mainly driven by the shock on the supply of long-term government bonds $\varepsilon_{bl}$ which is mainly due to the close relationship between financial intermediaries’s assets and liabilities captured by the bank balance sheet constraint (36). Further, results show that shocks to technology, the price mark-up, and the wage mark-up, explain roughly 85% of the variation in the loans series. This reflects the central role loans play for production and the firm sector of the model economy. The credit cost shock $\varepsilon_{\zeta}$ contributes about 5.5% to the variation in loan supply and almost 50% to the variation in the loan rate. This can be regarded as a significant shares and therefore provides evidence for the present model’s proposed credit cost channel. The minimum reserve policy shock $\varepsilon_{\mu}$ explains more than 70% of the variation in the deposit rate, about 40% of the variation in the loan rate, and about 35% of the variation of the 7-year Treasury Notes yield. The relative importance of the minimum reserve policy shock is based on the minimum reserve policy parameter $\mu$ directly governing the cost of financial intermediation in (38). The variation in the observed 7-year Treasury Notes yield is to roughly 45% driven by the decay factor shock $\varepsilon_{r}$ which is due to equation (35).

The analysis of the unconditional posterior variance decomposition implies that the extension of the model by the non-standard shocks except for the banking cost shock and the open market operations shock hardly contributes to the explanation of the variation of the standard model variables. However the standard shocks are found to contribute a sizeable share in the variation of the non-standard model variables except for reserves and deposits.

### 3.8 Forecasting performance

In Figure 3 the time series of the observed macroeconomic variables are plotted together with their respective one-step ahead forecasts implied by the model. The computation of the posterior distribution of one-step ahead forecasts, i.e. $E_{t}y_{t+1}$, is done by applying the

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26 Note that we use pre-2008/2009 financial crisis data to estimate the model. Hence we can not claim that this result still would be reproduced when extending the data sample by the 2008 to 2012 period. However, we expect the contribution of a credit cost shock to the variation of loan supply and the loan rate to increase during the financial crises relative to the pre-crisis period.
Figure 3: Observed data and one-step ahead forecasts

Notes: The one-step ahead forecasts for the period 1964:Q3 - 2007:Q4 are estimates implied by the Kalman filter at the posterior mean. Real variables are measured in percentage deviations from their linear trend. Inflation and interest rates are measured in annualized percentage deviations from their sample mean.
Kalman filter at the posterior mean. By visual inspection one can easily assert that the model forecasts output, consumption, investment, inflation, hours worked, wages, deposits, the policy rate, deposit rate, and the 7yr Treasury securities rate fairly well. However the model-implied one-step ahead forecasts for the expected deviations of the loan rate from the long-run mean considerably overpredict and underpredict the actual deviations for most periods. Furthermore expected deviations of reserves holdings from their linear trend hardly match the actual dynamics of reserve holdings at any period. Therefore the model generates long sequences of unexpected positive and negative open market operation shocks in order to match the dynamics of reserve holdings. This however comes at the expense of bad forecast performance in terms of reserves holdings.

We further compute the mean forecast errors (MFEs) as a measure of how accurate the model forecasts the given observed percentage deviations of macro variables from their trends and means resp. Further the MFE is a mean to assess whether a variable is systematically overpredicted or underpredicted. Results which are calculated based on the one-step ahead forecasts are reported in Table 6. The latter indicate that the present model tends to considerably underpredict investment. Further it is implied that output and hours worked are slightly underpredicted.\textsuperscript{27} The mean forecast errors are however close to zero implying generally an accurate forecasting performance of the model. Note that the zero value of the MFE for reserves holdings is by chance. By the construction of the applied measure forecast errors may sum up to zero despite of the fact that forecast errors show a highly autocorrelated pattern (see Figure 3).

### 3.9 Model dynamics

In this section we examine the short-run dynamics of the estimated model. In particular, we analyze the impact of changes in policy variables on the equilibrium allocation and prices. The model is solved by using a first-order local approximation of the model’s equilibrium conditions around the deterministic steady state.\textsuperscript{28} We compute impulse responses to unexpected changes in the model’s stochastic processes for several shocks. The percent deviation of each of the model’s real variables \( z_t \) from its steady state value \( z \) is denoted as \( \hat{z}_t = 100 \cdot [\log (z_t) - \log (z)] \). Nominal interest rates and inflation are given in annualized absolute deviations, \( \hat{R}^n_t = 400 \cdot (\hat{R}^n_t - \hat{R}^n) \), allowing for the interpretation of \( \hat{R}^n_t = 1 \) to be an increase of the respective gross interest rate by 100 basis points. Throughout the analysis we assume that the central bank is able to achieve its targets according to \( \pi > \beta \) and \( R^m \geq 1 \), which implies that the collateral constraint is binding in the steady state (See Proposition 1). We further assume that shocks are sufficiently small for the economy to remain in the neighborhood of its steady state and that the collateral constraint continues to bind.

\textsuperscript{27}Note that for most variables displayed zero MFE values are due to rounding error.

\textsuperscript{28}The full set of (non-linearized) equilibrium conditions can be found in Appendix 6.1.
Table 6: One-step ahead forecast errors

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean forecast error MFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.02</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.00</td>
</tr>
<tr>
<td>Investment</td>
<td>0.08</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.00</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.04</td>
</tr>
<tr>
<td>Wages</td>
<td>0.01</td>
</tr>
<tr>
<td>Reserves</td>
<td>0.00</td>
</tr>
<tr>
<td>Deposits</td>
<td>0.01</td>
</tr>
<tr>
<td>Policy Rate</td>
<td>0.00</td>
</tr>
<tr>
<td>Loan Rate</td>
<td>0.00</td>
</tr>
<tr>
<td>Deposit Rate</td>
<td>0.00</td>
</tr>
<tr>
<td>7yr Treas Yld</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: The forecast errors $F_t$ are computed as the difference between the observed variable $x_t$ and its one-step ahead forecast $x^f_t$ as $F_t = x_t - x^f_t$. The MFE is calculated by $MFE = T^{-1} \sum_{t=1}^{T} F_t$.

Response to a monetary policy shock  Figure 4 shows estimated mean impulse response functions to a monetary policy shock, measured by an innovation to the policy rate (see 65). An increase in the policy rate (i.e. the main financing rate) reduces the amount of real reserves that can be acquired by banks in open market operations (see 37). The monetary tightening tends to raise banks’ cost of loan creation $\Xi_t$, which increases the loan rate on impact by 7 basis points and the deposit rate on impact by 4 basis points. Deposits and the amount of loans respond in a hump-shaped way to alleviate the increase in the cost of financial intermediation by the reduced amount of reserves acquirable in equilibrium. This in turn induces a reduction of wages which have to be paid up front by the firms and leads to a decline in hours worked, and thereby to a decline in output and consumption. Hence, the monetary policy shock exerts a contractionary effect on the allocation of resources. Furthermore the slope of the yield curve which is defined as the difference between the long-term bond rate and the short-term bond rate declines on impact of the shock. This is caused by a positive response of the short-term bond rate and a decline in the long-term bond rate. As short-term bonds are eligible as collateral in open market operations their price bears a liquidity premium compared to the price of long-term bonds. However, this liquidity premium declines as the policy rate shock lowers the short-term bonds’ value as collateral for the real reserves acquisition. The long-term bond rate declines as monetary tightening increases the expected marginal contribution of reserves to a reduction in the cost of loan creation $E_t [\Xi_{m,t+1}]$ (see 54). This reflects the notion that banks as investors value the fact that future expected payoffs from holding long-term bonds might be used to acquire reserves. This implies an increased demand for
log-term bonds which drives up their prices and decreases their yields. In some sense this reflects a shift in investors' portfolios from short-term bonds to long-term bonds. Note that directly after the first period following the innovation, the policy rate decreases with a fast pace, eventually leading to a negative deviation of the policy rate from its steady state value. This is due to the strong decline in inflation and output following the monetary policy rate shock which forces the Taylor rule type of policy reaction function to lower the policy rate. This in turn strongly increases the amount of reserves that can be acquired in open market operations. Therefore it seems obvious that a weaker pace of decline in the monetary policy rate would have induced a more persistent positive reaction of the model’s other interest rates and a more persistent negative reaction of loan provision, deposit holdings, and the slope of the yield curve.

**Response to a banking cost shock** Figure 4 shows impulse responses estimated for a shock to the cost of financial intermediation, measured by an innovation to the scaling factor $\zeta_t$ which governs the cost of loan creation $\Xi_t$ (see 38). The increase in the banks’ cost of financial intermediation induces a decline in loan provision and deposit holdings. Both variables respond in a hump-shaped way similar to the pattern described in the paragraph above on the model’s response to a negative monetary policy shock. However, the negative deviation of loans and deposits from their steady state values is distinctively stronger in the case of a policy rate shock compared to a banking cost shock. E.g. the maximum peak effect of the monetary policy shock on loans is $-0.56$ percent, compared to $-0.27$ percent by the impact of the banking cost shock. Furthermore, impulse responses of output, consumption, investment, hours worked, and wages are qualitatively similar to the respective impulse responses implied by a monetary policy shock. Likewise as for loans and deposits these variables show a weaker response to the banking cost shock compared to the policy rate shock. For instance output declines at peak by $0.14$ percent induced by the banking cost shock and by $0.29$ percent induced by the policy rate shock. The notable difference between the banking cost shock and the monetary policy shock is that the former does not exert a negative impact on the value of the collateral in open market operations. Therefore immediately after the shock to $\zeta_t$ occurs, the financial intermediary can acquire additional real reserves to alleviate the increase in the cost of financial intermediation. Specifically, the banking cost shock induces reserves and injections to respond positively and in a hump-shaped way. This implies that in absence of the reserve acquisition mechanism the banking cost shock might have had a way stronger impact on the real macroeconomic variables. This seems to be even more likely when turning to the impulse response functions for the interest rates. Accordingly, the sudden increase of the banking costs induces on impact a rise of the loan rate by $253$ basis points which might have been even stronger in the absence of open market operations. It is notable that the implied deviation of the loan rate from its steady
Figure 4: Estimated impulse responses to contractionary shocks

Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.
state induced by the banking cost shock almost returns back to the steady state after only two periods. One important reason for this is in addition to the correspondent increase in real reserves a low estimated persistence parameter \( \rho_\zeta \) for the process governing \( \zeta_t \). Further, the banking cost shock induces the slope of the yield curve to increase. Seen against the background of a maximum peak decline in the long-term bond rate of less than 3 basis points induced by the shock the rise in the slope of the yield curve is due to a large negative response of the short-term bond rate. As short-term bonds are eligible as a collateral in open market operations their carried liquidity premium will increase due to the higher demand for reserves induced by a banking cost shock.

Response to a minimum reserve policy shock  Figure 4 shows the impact by an innovation to banks’ minimum reserve requirements, which is captured by the coefficient \( \mu_t \) (see 38). The increase in the minimum reserve requirements raises the cost of financial intermediation and thereby induces a strong increase in the deposit rate and the loan rate. On impact of the shock this further leads to a decrease of loan provision which in turn induces a negative response of output, consumption, investment, hours worked, and wages. The maximum peak response to the minimum reserve requirements shock on the deposit rate is 101 basis points compared to 4 basis points induced by the monetary policy rate shock and 40 basis points induced by the banking cost shock. This strongly stimulates households’ supply of deposits which induces a positive response of the amount of deposits hold by the banks to the shock. By the banks’ balance sheet constraint the increase in the amount of deposits hold and the decrease of loans provision is compensated by increased holdings of short-term bonds and long-term bonds (see 36). As short-term bonds are eligible as a collateral in open market operations the banks might wish to acquire additional reserves to reduce the cost of loan creation. This would however counteract the contractionary policy measure of increasing the minimum reserve requirements. Therefore by (37) inflation has to increase to compensate the increased amount of collateral in equilibrium. This in turn leads to an increase of the policy rate inducing an eventual negative and hump-shaped response of injections and reserves. Note that the strong positive response of the slope of the yield curve is caused by the strong positive response of the long-term bond rate induced by the shock whereas the short-term bond rate only gradually increases but finally outweighs the positive effect of the long-term rate on the spread. The increase in the short-term bond rate is causes by a gradually decreasing liquidity premium which follows the decreasing value of short-term bonds as collateral for open market operations.

Response to a total factor productivity shock  Figure 5 shows impulse responses for a shock to total factor productivity. The unexpected increase in total factor productivity induces a positive response of production, consumption, investment, hours worked, and wages.
Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.
Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.
The increased wage rate and the increase in hours worked require the firms to demand more loans (see 23). Further, an increase in expected production growth leads to a positive response of inflation (see 33). By the central bank’s monetary policy reaction function an increasing inflation induces a rise in the policy rate. This in turn reduces the amount of real reserves that can be acquired by banks in open market operations. A higher provision of loans together with reduced reserve acquisition rises the cost of financial intermediation and induces the loan rate to gradually increase. By this mechanism which explains the hump-shaped path of impulse responses, the loan provision will be dampened eventually forcing production and the other macro variables back to their steady state values. The positive response of the slope of the yield curve basically follows the long-term bond rate as the short-term bond rate is heavily affected by a diminished liquidity premium due to the decreased collateral value of short-term bonds.

**Response to a government consumption shock**  Figure 5 shows estimated mean impulse response functions to a government consumption shock. The increased aggregate demand induces a positive response of output and hours worked to the government consumption shock. However, the well known crowding out effect induces a decrease in consumption, investment, and wages. This in turn leads to a drop in marginal cost of production and eventually a negative expected production growth. By equation (33) this induces a negative response of the inflation rate. Taking this into account monetary policy reacts with a lagged decrease in the policy rate. Therefore reserves provision increases and responds in a hump-shaped way which exhibits an accommodating effect on the real model variables inducing them to return to the steady state.

**Response to an open market operation shock**  Figure 6 shows impulse responses for an open market operation shock, which is measured by an innovation to $\varepsilon_{omo,t}$ (37). An increase in $\varepsilon_{omo,t}$ affects the banks’ cost of loan provision by a direct injection of additional reserves. This is done at no cost for the banks and without requiring collateral. Therefore the shock induces a negative response to the loan rate and the deposit rate which leads to an increase of loan provision. By this, output, consumption, investment, hours worked and wages respond positively to the shock. In turn inflation increases which is followed by a monetary tightening in terms of an increasing policy rate. Further the slope of the yield curve decreases on impact as the open market operation shock decreases the liquidity premium on short-term bonds.

4 Simulating LSAP II

The present study seeks to analyze the effects of LSAP 2. For that purpose we simulate the 2008/2009 financial crisis. Specifically, we assume that the model economy is hit by a banking
cost shock which is calibrated to make the model’s forecasts match the observed dynamics of key macroeconomic variables following the bankruptcy of Lehman Brothers in September 2008. From this starting point we conduct our policy simulation. The unconventional monetary policy measure we simulate embodies large-scale purchases of long-term treasuries. In particular, we calibrate the path of $\kappa^L_t$, which is the share of long-term bonds eligible for Repo contracts, so as to match the observed dynamics of U.S. Fed holdings of long-term treasuries, while keeping $\kappa^S_t$ constant.

Figure 7: Dynamics of key macro variables during crisis

![Figure 7](image)

Notes: Evolution of real GDP, real consumption, real investment, real wages, long-term Treasury rate, and loan rate during the 2008/2009 crisis. Variables are normalized to zero in 2008:Q3.

4.1 Financial crisis scenario

Figure 7 shows the evolvement of real GDP, real consumption, real investment, real wages, the long-term treasury rate, and the loan rate from 2007:Q4 to 2012:Q3, whereas the former four are calculated relative to the end of 2008:Q3, which was the quarter when Lehman bankruptcy occurred. Here we follow the lines of Del Negro et al. (2011) and calculate the deviations of those variables from their long-run trend which is extracted form the pre-crisis data sample ranging from 1964:Q3 to 2007:Q4. Further, variables are normalized so that their 2008:Q3 level is zero. The shock incurred by the financial crisis hits with a first peak effect real GDP by $-5.91\%$, real consumption by $-5.38\%$, real investment by $-26.49\%$, and real wages by $-1.33\%$. Whereas those first peak effects are observed within about a year

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29 As Chen, Curdia, and Ferrero (2012) point out, it is certainly simplistic to assume that only one shock captures the events in late 2008 and 2009. However, we pursue this line as well, to assess how much mileage we can get from this shock and to evaluate the associated non-conventional policy intervention.
Figure 8: Impulse response no QE

Notes: Impulse responses to the calibrated banking cost shock of output, consumption, investment, wages, long-term Treasury rate, and loan rate.

relative to 2008:Q3. Within that time span the long-term treasury rate falls by 160 basis points relative to 2008:Q3 and the loan rate decreases by 88 basis points relative to 2008:Q3.\textsuperscript{30}

The banking cost shock is then calibrated to make the model’s simulated response of the macroeconomic variables match the impact dynamics observed in the data. Figure 8 shows the response of real GDP, real consumption, real investment, real wages, the long-term treasury rate, and the loan rate to a banking cost shock to $\zeta_t$. As Del Negro et al. (2011) point out, it is to note that impulse response functions show an expected path of a given variable from an ex ante perspective, not a particular realized path in the model.\textsuperscript{31} Therefore impulse responses do not have to match the maximum peak effects in the data which are achieved for some variables more than two years after the occurrence of the crisis. Further, this is particularly important for the interpretation of the impulse response for the interest rates. The model predicts a simultaneous drop in real GDP by $-5.67\%$ (versus $-5.91\%$), real consumption by $-4.71\%$ (versus $-5.38\%$), real investment by $-14.88\%$ (versus $-26.49\%$), and real wages by $-3.02\%$ (versus $-1.33\%$). If we compare the movements of the macroeconomic variables simulated by the model following a banking cost shock with the first peak effects observed for

\textsuperscript{30}Over the period 2007:Q4 to 2012:Q4 the maximum peak effect for real GDP is $-7.42\%$, for real consumption $-8.47\%$, for real investment $-26.49\%$, and for real wages $-3.33\%$. The long-term treasury rate falls at maximum by 260 basis points relative to 2008:Q3 and the loan rate decreases at maximum by 234 basis points relative to 2008:Q3.

\textsuperscript{31}Impulse response functions plot the expected path of variables conditional on the shock hitting at the beginning of the first period. Specifically, all possible state contingent paths induced by the Markov process are averaged, whereas using the associated probabilities as weights.
the empirical counterparts, the model performs fairly well.\textsuperscript{32} Note that we exclude durable goods from our consumption measure and instead treat it as a part of investment whereas we follow Smets and Wouters (2007).

4.2 LSAP scenarios

The share of long-term Treasuries eligible for open market operations $\kappa_t^L$ is in the present framework set to zero in the absence of large-scale long-term Treasury bonds purchases. This represents in a sense our baseline scenario. Our experiment is to simulate the effects of LSAP 2 on the U.S. economy if it was conducted right after the break down of Lehman Brothers in 2008:Q3. Note that LSAP 1 started in late 2008 and lasted until 2010 whereas LSAP 2 was launched 2010:Q4 and ended in 2011:Q2. Thus our financial crisis scenario and baseline scenario resp. takes the potential impact of LSAP 1 on the U.S. economy as given without controlling for this policy measure within the model framework. We simulate LSAP 2 as a series of non-zero values assigned to $\kappa_t^L$. Specifically, as $\kappa_t^L$ is the share of long-term bonds eligible for open market trades, we track the share of U.S. Federal Reserve’s holdings of U.S. Treasury Notes and Bonds relative to total Notes and Bonds outstanding.\textsuperscript{33} We calculate this share starting from the first available data on the Fed’s U.S. Treasury Notes and Bonds holdings in 2003:Q1 to 2012:Q3. As the observed share of Fed holdings of Treasury Notes and Bonds relative to Notes and Bonds outstanding is positive over the whole data sample (see Figure 9 left-hand side) we demean the time series and assume that $\kappa_t^L = 0$ previously to 2008:Q3. Note that after the official end of LSAP 2 in 2011:Q2 the Fed's share was still increasing for another quarter and for the last available observation in 2012:Q3 it was still way higher compared to its average pre-crisis values. The present study develops different scenarios about the further development of the Fed holdings of long-term bonds.

**LSAP Scenario 1** For the first LSAP scenario we assume that after the last observed positive value of the share of the Fed’s holdings of U.S. Treasury Notes and Bonds relative to total outstanding in 2012:Q3 the Fed completely sells its long-term Treasury securities positions. The path for the share of long-term Treasuries eligible for open market operations $\kappa_t^L$ which tracks the Fed’s Treasury purchase program under Scenario 1 is depicted on the right-hand sided plot of Figure 9. The complete redemption of the central banks’s market share requires $\kappa_t^L$ to be equal to zero in 2012:Q4 which implies a duration of the program of only eight quarters.

\textsuperscript{32}Del Negro et al. (2011) note that the disparity between model and data regarding investment may be due to the fact that the model does not account for an explicit residential sector.

\textsuperscript{33}We use quarterly time series data on Treasuries outstanding and Treasuries held by the Federal reserve. The series on the amount of Treasury Notes outstanding (Code 440109711) and Treasury Bonds outstanding (Code 440109712) are from Datastream. From the Federal Reserve Bank of St. Louis FRED databank we take the series on U.S. Treasury securities held by the Federal Reserve, maturing in 1 to 5 years (TREAS1T5), maturing in 5 to 10 years (TREASST10), and maturing in over 10 years (TREAS10Y).
LSAP Scenario 2  For the second LSAP scenario we assume that after 2012:Q2 the Fed gradually reduces its share of Treasury Notes and Bonds holdings relative to total outstanding. The Fed would successively sell its long-term Treasury securities positions over the following 3.5 years after the last observed value. The path for the share of long-term Treasuries eligible for open market operations $\kappa^L_t$ which tracks the Fed’s Treasury purchase program under Scenario 2 is depicted on the right-hand sided plot of Figure 9. The complete redemption of the central banks’s market share $\kappa^L_t$ would happen after 2015:Q4 which implies a duration of more than 21 quarters.

LSAP Scenario 3  For the third LSAP scenario we assume that the Fed holds its share of Treasury Notes and Bonds holdings relative to total outstanding constant after 2012:Q2. The Fed would not sell its long-term Treasury securities positions after the last observation of its market share. The path for the share of long-term Treasuries eligible for open market operations $\kappa^L_t$ which tracks the Fed’s Treasury purchase program under Scenario 3 is depicted on the right-hand sided plot of Figure 9. The complete redemption of the central banks’s market share $\kappa^L_t$ would start at some date after 2015:Q4 which in a sense implies an indefinite duration of LSAP2.

4.3 Simulation results

We simulate the effects of the three LSAP program scenarios on the model’s endogenous variables conditional on the model economy being hit by the calibrated banking cost shock. This is done by calculating the trajectories of the stochastic model’s variables starting from the steady state. Figure 10 shows the trajectories of the endogenous variables for all three
Figure 10: Simulated impulse responses to LSAP 2 scenarios

Notes: Simulated impulse responses to LSAP 2 scenarios of output, consumption, investment, inflation hours worked, wages, reserves injections, the long-term Treasury rate, the loan rate, and the spread between the loan rate and the Euler rate.
scenarios and compares them to the model’s impulse responses in the absence of LSAP programs. The deviation of each of the model’s real variables from its steady state value is denoted in percentage terms. Nominal interest rates and inflation are given in annualized absolute deviations.

An increase in the share of long-term Treasuries eligible for open market operations \( \kappa_t \) increases the banks’ amount of acquirable reserves. This directly affects the banks’ cost of loan provision which induces a negative response of the loan rate and the deposit rate, and eventually increases the amount of loan provision. By this, output, consumption, investment, hours worked and wages respond positively. Further, declaring long-term Treasuries as eligible for open market operations induces a liquidity premium on their prices which should c.p. decrease the spread between the short-term bond rate and the long-term bond rate. Note that the positive reaction of real output should increase the monetary policy rate which is assumed to follow a Taylor-rule type of policy reaction function. However, we assume that the central bank holds the policy rate close to the zero lower bound during the simulation experiment.

Under the first scenario the output contraction at its first minimum peak following the simulated crisis event is dampened by 2.62 percent, compared to 3.60 percent under the second scenario, and 3.95 percent under the third scenario.\(^{34}\) These effects on the model’s real variables result from the fact that under the LSAP programs the loan provision does not decrease in the response to the banking cost shock as strong as in the absence of those programs. It is noteworthy that under all LSAP scenarios the model predicts a dampened effect on inflation. This basically is determined by the pace output recovery after the fifth simulation period as the firms’ price setting behavior is strongly driven by output growth (see 33).

The policy intervention also affects interest rates in all of the three scenarios, but interestingly the impact of the crisis on interest rates is fairly strong relative to its impact on the real macro variables. Specifically, in absence of LSAP programs the loan rate increases by 45 percentage points on impact of the shock whereas the long-term Treasury rate increases by 15 percentage points. Under the first scenario the impact on the loan rate is dampened by 538 basis points relative to the case of an absence of LSAP programs, under the second scenario the impact is dampened by 542 basis points, and under the third scenario the impact is dampened by 552 basis points. However, the impact of the asset purchase programs on the long-term Bond rate is sizably stronger. For the first scenario LSAP reduces the long-term bond rate on impact by 726 basis points, for the second scenario by 947 basis points, and

\(^{34}\)We find that under the scenarios 1, 2, and 3 the drop in consumption is dampened by 2.45 percent, 2.72 percent, and 2.77 percent. The decline in investment is decreased by 6.46 percent, 10.23 percent, and 11.79 percent. The drop in hours worked is dampened by 2.70 percent, 4.61 percent, and 5.51 percent. Further the contraction in the wage rate is dampened by 1.35 percent, 2.40 percent, and 2.65 percent.
for the third scenario by 1117 basis points. These numbers can be interpreted as the model-
implied liquidity premia on long-term bonds, measured in the first simulation period. The
high implied values of the level of the loan rate do obviously not match the observed data.
This is due to the fact that the estimated model requires a tremendous increase in the loan
rate to be able to match the impact of the crisis on the key macroeconomic variables. Thereby
it is to note that we actually do not know how the loan rate would have evolved without
any emergency and short-term policy responses\textsuperscript{35} as well as without the (announcement-)
effect of LSAP 1. Short-term and medium-term funding markets were at this time generally
regarded as being under tremendous pressures which might eventually had lead to a complete
dry out of the respective markets. However, we do not model any of the emergency policy
interventions and focus on matching the dynamics of the real macroeconomic variables.

As it can generally be regarded as a complicated task to match interest rate dynamics
by a forecast derived from a DSGE model we turn to the implied dynamics of the spread
between the loan rate and the Euler rate. This spread is the actual firm’s financing cost (see
25). This spread increases on impact of the crisis by 16 percentage points. The presence
of LSAP under scenario 1 would dampen the increase of the spread on impact by 58 basis points,
under scenario 2 by 59 basis points, and under scenario 3 by 60 basis points. Therefore a
relatively small decrease in this spread has sizeable impact on the real economic variables
such as output.

A surprising result is that an unlimited provision of the long-term Treasury purchase
program as captured by scenario 3 would after a certain phase of economic stimulation event-
ually lead to a drop in output after roughly ten quarters. By the duration of the LSAP
program banks begin to substitute long-term Treasuries against loans as the former provide
in addition to an interest payment a mean to gain liquidity. Further, as under the third
scenario the loan rate is for a relatively long period pushed far below its steady state value,
in equilibrium banks will have to decrease their loan supply. With the provision of loans to
the firms becoming an unprofitable business this might pose a certain downward risk to a
recommendation in favour of a prolonged duration of central banks’ unconventional balance
sheet policies.

5 Conclusion

We extended the canonical DSGE model by financial frictions and liquidity premia. A banking
sector requires liquidity to facilitate financial intermediation, whereas liquidity is supplied
by the central bank in exchange for collateral. Multiple assets that differ with respect to
their ability to serve as collateral are considered. By this, a spread between interest rate

\textsuperscript{35}See measures like the Term Auction Facility (TAF) and Term Asset-Backed Securities Loan Facility
(TALF)
on non-eligible and eligible assets is induced, which is displays a liquidity premium. Using Bayesian estimation techniques we find that the model provides a considerably good fit to the data though new model ingredients have been introduced. We show that the extended model generates responses to commonly studied economic shocks and policy shocks which are qualitatively consistent with broad empirical evidence. Beyond that, we introduce shocks which directly affect the financial intermediation. We find that the impact on financial intermediation is particularly relevant for the size and the persistence of shock responses.

We have simulated the impact of the 2008/2009 financial crisis on macroeconomic variables and estimated the potential effects of several LSAP 2 scenarios on the model economy. From the simulation experiments we conclude that LSAP 2 was likely to yield a considerable contribution to dampen the adverse impact of the financial crisis on the macroeconomy. We find that output contraction following the simulated crisis event is dampened by between 2.62 and 3.95 percent. This poses evidence in favour of the notion that asset purchase programs and balance sheet policy operations resp. are effective at stimulating the economy. Further, results support the model’s proposed transmission channel of non-conventional monetary policy which is due to the existence of financial frictions and liquidity premia. The latter induce interest rates on some groups of assets to be positive, even if the policy rate is at ZLB. Specifically, LSAP 2 successfully lowers interest rates of longer-termed Treasury securities which is transmitted to private borrowing rates. Further we find that asset purchase programs become ineffective with an extended duration and eventually negatively affect the economy.
References


49
6 Appendix

6.1 Rational Expectations Equilibrium

Definition 2 A RE equilibrium is given by a set of sequences \( \{c_t, n_t, d_t, \pi_t, \pi_t^w, w_t, m_{ct}, mrs_t, k_t, r_t, x_t, q_t, \pi_t^p, \pi_t^l, \eta_t, \rho_t, \rho_t, \nu_t, YTM_t\} \) satisfying the following conditions summarizing the optimal behavior of households

\[
mrs_t = -\frac{u_{r,t}}{u_{c,t}} = -\frac{\nu m_{rs_t}^w}{(c_{i,t} - h_{c_{i-1}})^{-\sigma}} \quad (75)
\]

\[
\frac{1}{R_t^f} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \xi_{t+1} u_{c,t+1} \left( 1 + \frac{u_{d,t+1}}{u_{c,t+1}} \right) \right] \quad (76)
\]

\[
\frac{1}{R_t^E} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \xi_{t+1} u_{c,t+1} \right] \quad (77)
\]

\[
n_{i,t} = \left( \frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_{r}}{\rho_{r} - 1}} n_t \quad (78)
\]

\[
w_t = w_{t-1} \frac{\pi_t^w}{\pi_t} \quad (79)
\]

\[
= \beta E_t \left[ \frac{\xi_{t+1} u_{c,t+1} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^w} - 1 \right)}{\xi_t u_{c,t}} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^w} - 1 \right) \frac{n_{t+1}^w}{\omega_w (\mu^w - 1)} (\mu^w mrs_t - w_t) \right] \quad (80)
\]

with

\[
u_{ct} = [c_t - h_{c_{t-1}}]^{-\sigma}, \quad u_{dt} = gd_t^{-\varphi}, \quad u_{nt} = -\nu m_{rs_t}^w
\]

of firms

\[
w_t = m_{ct} \alpha a_t n_t^{\alpha - 1} (u_t k_{t-1})^{1-\alpha} \left( \frac{R_t^f}{R_t^E} \right)^{-1} \quad (81)
\]

\[
rm_t = m_{ct} (1-\alpha) a_t n_t^{\alpha} (u_t k_{t-1})^{-\alpha} \quad (82)
\]

\[
l_t \left( \frac{R_t^f}{R_t^E} \right) = w_t n_t \quad (83)
\]

\[
1 = q_t c_t \left( 1 - \frac{\gamma_t}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 - \gamma_t \left( \frac{x_t}{x_{t-1}} - 1 \right) \frac{x_t}{x_{t-1}} \right)
\]

\[
+ \beta E_t \left[ \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} q_{t+1} \left( x_{t+1}^l \frac{x_t}{x_{t-1}} - 1 \right) \frac{x_{t+1}}{x_t} \right] \quad (84)
\]

\[
q_t = \beta E_t \left[ \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} q_{t+1} (1-\delta) + r_{k,t+1} u_{t+1} - rk (u_{t+1} - 1) + \frac{\chi \cdot r}{2} (u_{t+1} - 1)^2 \right] \quad (85)
\]

\[
= \beta E_t \left[ \frac{\pi_{t+1}^w}{\pi_{t+1}^w} - 1 \right] \frac{\pi_t^w}{\pi_t^w} \frac{y_{t+1}}{y_t} \quad (86)
\]
of banks

\[ YTM_t = \frac{1}{p_t^L} + \rho_t^S \]  
\[ \frac{1}{R_t^{LB}} = 1 + (\mu_t - 1) \frac{1}{R_t^E} E_t [-\Xi_{m,t+1}] \]  
\[ \frac{1}{R_t^{SB}} = 1 + \frac{1}{R_t^E} \left( E_t [\eta_{t+1} \kappa_{t+1}^L] - E_t [-\Xi_{m,t+1}] \right) \]  
\[ 1 = \frac{1}{R_t^E} \left( E_t [R_{t+1}^{LB} YTM_{t+1} \eta_{t+1} \kappa_{t+1}^L] - E_t [-\Xi_{m,t+1}] \right) + E_t [R_{t+1}^{LB} YTM_{t+1}] \]  
\[ \frac{1}{R_t^{LT}} = 1 - \Xi_{l,t} - \frac{1}{R_t^E} E_t [-\Xi_{m,t+1}] \]  
\[ R_t^M = 1 - \Xi_{m,t} - \eta_t R_t^M \]  
\[ d_t = m_t + b_t^L + E_t [p_{t+1}^L YTM_{t+1} b_t^L] + l_t \]  
\[ \frac{b_t^S}{R_t^{SB}} = \frac{b_t^{TS}}{R_t^{SB}} - \kappa_t \frac{b_t^S}{\pi_t} \]  
\[ p_t^L b_t^L = p_t^L b_t^{LT} - \kappa_t p_t^L YTM b_{t-1}^L / \pi_t \]

with

\[ \Xi_t(l_t, i_t) = \zeta_t (l_t)^{\eta_c} (m_{t-1} \pi_{t-1} - \mu_t d_{t-1} \pi_{t-1} + i_t)^{-\phi_c} \]  
\[ \Xi_{l,t} = \eta_c \Xi_{t} / l_t \]  
\[ \Xi_{m,t}(l_t, i_t) = -\phi_c \Xi_t (m_{t-1} \pi_{t-1} - \mu_t d_{t-1} \pi_{t-1} + i_t)^{-1} \]

of the central bank

\[ i_t = \kappa_t \frac{b_t^{S-1}}{\pi_t R_t^m} + \kappa_t \frac{p_t^L YTM b_t^L}{\pi_t R_t^m} + \varepsilon_{omo,t} \]  
\[ i_t = m_t - m_{t-1} \pi_{t-1} + \rho_t^L + m_t^Q \]  
\[ m_t = \Lambda m_t^R \]  
\[ R_t^m = (R_{t-1}^m)^{\rho_c} (R_t^m)^{1-\rho_c} \left( \frac{\pi_t}{\pi} \right)^{\rho_c(1-\rho_c)} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{dy}(1-\rho_c)} \exp \varepsilon_{r,t} \]

and of the government

\[ b_t^{TS} = \Gamma \frac{b_t^{S-1}}{\pi_t} \]  
\[ \left( \frac{p_t^L b_t^{TT}}{p_t^L b_t^{TT-1}} \right) = \left( \frac{p_{t-1}^L b_{t-1}^{TT-1}}{p_t^L b_t^{TT}} \right)^{\rho_t} e^{\varepsilon_{bl,t}} \]
and aggregate resources

\[ y_j = a_t n_j^\gamma (u_t k_{t-1})^{1-\alpha} \]
\[ y_t = c_t + x_t + g_t + \Xi_t + \left( r k (u_t - 1) + \frac{x \cdot r k}{2} (u_t - 1)^2 \right) k_{t-1} \]
\[ + \frac{\omega_p}{2} \left( \frac{\pi_t}{\pi^{1-\upsilon} \pi_{t-1}^{1-\upsilon}} - 1 \right)^2 y_t + \frac{\omega_w}{2} \left( \frac{\pi_t^w}{\pi^{1-\upsilon} \pi_{t-1}^{1-\upsilon}} - 1 \right)^2 y_t \]
\[ k_t = (1 - \delta) k_{t-1} + \epsilon_t \left( 1 - \gamma I \frac{\pi_t}{x_{t-1}} - 1 \right)^2 x_t \]

where \( \frac{1}{R_t} = E_t \phi_{t+1} \), \( R_t^{LB} = \frac{\nu_{t+1}}{p_{t+1}} \), as well as the transversality conditions, a monetary policy setting \( \{ R_t^m \geq 1 \}_{t=1}^\infty \) and \( \pi \geq \beta, \) and \( \kappa_S^T \) and \( \kappa_T^T \) and a fiscal policy \( \{ g_t, \tau_t \}_{t=1}^\infty \) satisfying

\[ \tau_t = b_t^{TS} - \frac{b_t^{TS}}{R_t^\gamma} + p_t^{L} YTMb_t^{TL} - p_t^{L} b_t^{TL} + g_t \]
\[ \left( \frac{g_t}{g} \right) = \left( \frac{g_{t-1}}{g} \right) \rho_g e^{g_{t-1}}, \]

for the stochastic processes for the shocks

\[ \log (\xi_t) = \rho_\xi \log (\xi_{t-1}) + \varepsilon_{\xi,t}, \varepsilon_{\xi,t} \sim N (0, \sigma_\xi) \]
\[ \log \left( \frac{\mu_t^w}{\mu^w_{t-1}} \right) = \rho_{w} \log \left( \frac{\mu^w_{t-1}}{\mu^w_{t-1}} \right) + \varepsilon_{w,t}, \varepsilon_{w,t} \sim N (0, \sigma_w) \]
\[ \log (a_t) = \rho_a \log (a_{t-1}) + \varepsilon_{a,t}, \varepsilon_{a,t} \sim N (0, \sigma_a) \]
\[ \log (\epsilon_t) = \rho_c \log (\epsilon_{t-1}) + \varepsilon_{c,t}, \varepsilon_{c,t} \sim N (0, \sigma_c) \]
\[ \log \left( \frac{\rho_t}{\rho_{t-1}} \right) = \rho_{p} \log \left( \frac{\rho_{t-1}}{\rho_{t-1}} \right) + \varepsilon_{p,t}, \varepsilon_{p,t} \sim N (0, \sigma_p) \]
\[ \log \left( \frac{\rho_t^s}{\rho_{t-1}^s} \right) = \rho_{r} \log \left( \frac{\rho_{t-1}^s}{\rho_{t-1}^s} \right) + \varepsilon_{r,t}, \varepsilon_{r,t} \sim N (0, \sigma_r) \]
\[ \log \left( \frac{\mu_t}{\mu_{t-1}} \right) = \rho_{\mu} \log \left( \frac{\mu_{t-1}}{\mu_{t-1}} \right) + \varepsilon_{\mu,t}, \varepsilon_{\mu,t} \sim N (0, \sigma_{\mu}) \]
\[ \log \left( \frac{\xi_t}{\xi_{t-1}} \right) = \rho_{\xi} \log \left( \frac{\xi_{t-1}}{\xi_{t-1}} \right) + \varepsilon_{\xi,t}, \varepsilon_{\xi,t} \sim N (0, \sigma_{\xi}) \]

and a given initial values \( m_{-1} > 0, l_{-1} > 0, b^{LT}_{-1} > 0, b^{ST}_{-1} > 0, k_{-1} > 0, x_{-1} > 0, \pi_{-1} > 0 \)

### 6.2 Appendix to the steady state

In this appendix, we examine the steady state of the economy, where we particularly focus on the irrelevance of public financing for the real allocation. We further show that the central bank can independently chose the inflation target. We therefore start with the assumption that the central bank sets the long-run inflation rate \( \pi \) and then confirm that this is indeed possible. For a given inflation target \( \pi \), the following conditions then determine the steady state values \( \{ R^{Euler}, k/n, x/n, mc, q \} \):

52
Equations (80) and (86) imply for the steady state marginal cost and the steady state marginal rate of substitution

\[ mc = \frac{1}{\mu^c}, mrs = \frac{w}{\mu^w} \]

Combining (88) and (90), dropping time indices, and assuming for the steady state an absence of LSAPs, with \( \kappa^L = 0 \), yields for the steady state minimum fraction of reserve holding

\[ \mu = \left( \frac{1}{R^D} - \frac{1}{YTM} \right) \frac{YTM}{YTM - 1} \]

This, and calibrating the scaling \( \zeta \) and \( g \) below, is required to make the steady state model calibration to match the historically observed means of the equivalents to the model interest rates. Further, the steady state Euler rate is determined by the steady state inflation \( \pi \) and the discount factor \( \beta \), \( R^E = \frac{\pi}{\beta} \), the steady state of Tobin’s \( q \) is equal to one, the steady state investment-to-labor ratio equals the capital-to-labor ratio times the capital depreciation ratio, \( \tilde{z} = \delta \frac{k}{n} \), and the steady state long-term bond price is implied by (87) equal to \( p^L = \frac{1}{YTM - \rho^S} \).

Combining equations (85) and (82) implies for the steady state capital-to-labor ratio

\[ n = \frac{mc(1 - \alpha)}{1 - \beta(1 - \delta)} \]

and hence (102) implies for the steady state level of output

\[ y = n \cdot \left( \frac{k}{n} \right)^{(1-\alpha)} \]

Combining (81) and (83) implies for the steady state amount of loans loan

\[ l = mc \cdot \alpha \cdot n \frac{k^{(1-\alpha)}}{n} R^Euler \]

Further, we calibrate the steady state deposit holdings as

\[ d = DL \cdot l \]

where \( DL \) is the historically measured deposits-to-loans ratio. From (82) and (83) it is now implied that the steady state capital rental rate and the steady state wage rate are

\[ rk = mc(1 - \alpha) n^{\alpha} (uk)^{-\alpha} \]

\[ w = \frac{l}{n} \cdot R^L \]
Now take into account the banking cost function without time indices
\[
\Xi = \zeta \frac{p_{rc}}{(m(1 + \Lambda^{-1}) - \mu d\pi^{-1})^{\phi_{rc}}},
\]
(121)
\[
\Xi_l = \eta_{rc} \frac{\Xi}{l},
\]
(122)
\[
\Xi_m = -\phi_{rc} \frac{\Xi}{(m(1 + \Lambda^{-1}) - \mu d\pi^{-1})},
\]
(123)
where we used that steady state injections are given by
\[
i = m \left(1 - \pi^{-1} + \Lambda^{-1}\right)
\]
which follows from assuming that money supplied outright is a constant fraction of money reserves \(m = \Lambda m^R\). Now drop time indices in (91) and use (122) to substitute out \(\Xi_l\). Then substitute out \(\Xi_m = -\left(1 - YM^{-1}\right) R^F\) which is implied by (90) to get the steady state banking cost
\[
\Xi = \left(\frac{1}{YM} - \frac{1}{R^F}\right) \frac{l}{\eta_{rc}}.
\]
(124)
Then we can derive the steady state value for the steady state marginal contribution of loans to banking cost (122). Further we define \(gdp = y - \Xi\) as steady state GDP. Where \(g\) is calibrated with the historical government spending-to-GDP ratio, \(GY\), implying \(g = GY \cdot y\).

Now we can determine the steady state value of consumption from the steady state aggregate resource constraint
\[
c = y - g - x - \Xi
\]
(125)
Further, dropping time indices for marginal utility of consumption yields \(u_c = ((1 - h) c)^{-\sigma}\). Plug the former into (75) and combine with the steady state marginal rate of substitution, \(mrs = \frac{w}{\mu\pi}\), we can calibrate the Frisch elasticity \(\nu\), and determine the steady state value for marginal utility of labor \(u_n\)
\[
\nu = \frac{u_c v}{\mu w n^\nu},
\]
(126)
\[
u n = -\nu n^\nu.
\]
(127)
Next is to calibrate the scaling factor \(\vartheta\) which governs marginal utility of deposits. For that reason plug (77) into (76) and set this equal to (88). Now substitute out \(\Xi_m\) from (90)
\[
\vartheta = \left(\frac{d}{\pi}\right)^{\varphi} \left(\frac{(1 - h) c}{YM (\mu (1 - YTM^{-1}) + YTM^{-1})} - 1\right)^{-1}
\]
(128)
Hence, we can determine the steady state value of marginal utility of deposits \(u_d = \vartheta \left(\frac{d}{\pi}\right)^{-\varphi}\). To determine the steady state money holdings, solve (123) for \(m\) and substitute out \(\Xi_m\) by
which yields
\[ m = \frac{1}{1 + \Lambda^{-1}} \left( \mu \frac{d}{\pi} + \frac{\phi_{rr}\Xi}{(1 - YTM^{-1}) RE} \right) \]  
(129)
Now we can determine steady state money reserves \( m^R = \frac{n}{\Lambda} \). Further we can now calibrate the steady state value of the scaling parameter within the banking cost function \( \zeta \) from (121)
\[ \zeta = \frac{\Xi}{l_{yrc}} (m (1 + \Lambda^{-1}) - \mu d \pi^{-1})^{\phi_{rc}} \]  
(130)
From (92) the steady state multiplier on the collateral constraint is implied by
\[ \eta = \frac{1 - \Xi_m}{R^M} - 1 \]  
(131)
This can be used to determine the value of the steady state injections \( i \)
\[ i = m (1 - \pi^{-1} + \Lambda^{-1}) \]  
(132)
Next we can solve (89) for \( R^{SB} \). By substituting out \( \Xi_m \) we get the steady state short-term bond rate
\[ R^{SB} = \left( \frac{1}{RE \eta \kappa^S} + \frac{1}{YTM} \right)^{-1} \]  
(133)
Banks’ steady state short-term bond holdings are implied by (96)
\[ b^S = \frac{i \pi R^M}{\kappa^S} \]  
(134)
Banks’ steady state long-term bond holdings are derived from the steady state bank balance sheet condition \( d = m + b^S + p^L YTM b^L + l \)
\[ b^L = \frac{d - m - b^S - l}{p^L YTM} \]  
(135)
For the government’s steady state long-term bond supply we assume that \( b^{TL} = b^L, \) as \( \kappa^L = 0. \)
For the short-term steady state bond supply we sum up the financial intermediaries’ steady state stocks and the central bank’s steady state stocks
\[ b^{TS} = b^S + R^{SB} R^M \frac{\kappa^S b^S}{\pi R^M} \]  
(136)
Further the steady state lump-sum taxes are implied by (105)
\[ \tau_t = b^{TS} \left( \frac{1}{\pi} - \frac{1}{R^S} \right) + p^L b^{TL} \left( \frac{YTM}{\pi} - 1 \right) + g \]  
(137)
**Proof of proposition 1.** Using that aggregate production satisfies \( y = n \cdot \left( \frac{k}{\pi} \right)^{(1-\alpha)} \) and
the resource constraint is \( c = y - g - x - \Xi \), which leads to

\[
c = \left[ \left( \frac{k}{n} \right)^{(1-\alpha)} - \frac{\delta k}{n} \right] n - g - \Xi
\]

plug in (121) for \( \Xi \) gives

\[
c = \left[ \left( \frac{k}{n} \right)^{(1-\alpha)} - \frac{\delta k}{n} \right] n - g - \zeta \frac{\eta \nu}{(m(1 + \Lambda^{-1}) - \mu d\pi^{-1})^{\phi_{rc}}}
\]

(138)

Combine (81) \( w = mc \cdot \alpha \cdot n^{\alpha-1}k^{1-\alpha} \left( \frac{RE}{Re} \right) \) with \( mr \cdot \mu^w = w \) and plug in \( mr \cdot \mu^w = \frac{\nu \nu}{(1-h)c}^{\nu}

\[
\frac{\mu^w \nu n^w}{((1-h)c)^{-\sigma}} = mc \cdot \alpha \cdot n^{\alpha-1}k^{1-\alpha} \left( \frac{RE}{RE} \right)
\]

(140)

From (91) \( \frac{1}{RE} = 1 - \Xi_l - \frac{1}{RE} (-\Xi_m) \Leftrightarrow \frac{RE}{RE} = R^E (1 - \Xi_l) + \Xi_m \) plug this into the former

\[
\frac{\mu^w \nu n^w}{((1-h)c)^{-\sigma}} = mc \cdot \alpha \cdot n^{\alpha-1}k^{1-\alpha} \left( R^E (1 - \Xi_l) + \Xi_m \right)
\]

(141)

To establish the claims made in the proposition we substitute out \( \Xi_l \) and \( \Xi_m \) in (141) and substitute out \( l \) by (117) in (139). Next we substitute out \( \Xi \) in (129) and \( l \) in (118). The former two expressions provide two conditions by which steady state consumption and working time \( \{c, n\} \) can be determined:

\[
c = \left[ \left( \frac{k}{n} \right)^{(1-\alpha)} - \frac{\delta k}{n} \right] n - g - \zeta \frac{mc \cdot \alpha \cdot n^k (1-\alpha) \left( \frac{RE}{RE} \right)^{\eta \nu}}{(m(1 + \Lambda^{-1}) - \mu d\pi^{-1})^{\phi_{rc}}}
\]

(139)

\[
m = \frac{1}{1 + \Lambda^{-1}} \left( \frac{d}{\mu} + \frac{\phi_{rc}}{(1 - YTM^{-1})R^E} \zeta \frac{mc \cdot \alpha \cdot n^k (1-\alpha) \left( \frac{RE}{RE} \right)^{\eta \nu}}{(m(1 + \Lambda^{-1}) - \mu d\pi^{-1})^{\phi_{rc}}})
\]

\[
d = DL \cdot mc \cdot \alpha \cdot n^k \left( \frac{1}{n} \right) R^E
\]

For a given pair \( \{c, n\} \), we can determine the steady state values \( \{m, l, k, y, m^R i, \Xi, \Xi_m, \Xi_l, w\} \) by (129), (118), (117), (115), \( y = n \left( \frac{k}{n} \right)^{1-\alpha} \), \( m^R = \Lambda^{-1} m \), \( i = m \left( 1 - \pi^{-1} + \Lambda^{-1} \right) \), (121)-(123) and (120). As can immediately been seen, all these steady state values are independent of from government borrowing. For a binding collateral constraint, \( i = \kappa_S \frac{b_S}{\pi R^m} \) further holds in

\[
m(1 - \pi^{-1} + \Lambda^{-1}) = \kappa_S \frac{b_S}{\pi R^m}
\]

(142)
which leads to the following condition for the steady state inflation rate

\[ \pi = \frac{1}{1 + \Lambda^{-1}} \left( 1 + \frac{\kappa^S \cdot b^S}{R^m \cdot m} \right) \]  

(143)

For \( \kappa = 1 \), we get \( \pi = \left( 1 + \frac{b^S}{R^m \cdot m} \right) (1 + \Lambda^{-1})^{-1} \), which implies that the central bank can control the steady state inflation rate by adjusting the fraction of money supplied outright \( \Lambda \) given the policy rate, real balances, and real short-term bonds (e.g. increasing \( \Lambda \) raises the long-run inflation rate). Hence, the central bank can also adjust \( \kappa \) to implement its inflation target, which completes the proof of the proposition.

6.3 Derivation of labor demand

So-called labor packers (or unions) produce the effective labor units \( n_t \), by using the production function \( n_t^{1/\mu^w_t} = \int_0^1 n_t^{1/\mu^w_i} di \). They sell effective units of labor to intermediate goods producing firms at price \( w_t \). Profit maximization of labor packers

\[
\max_{n_{i,t}} \left[ w_t \left( \int_0^1 n_{i,t}^{1/\mu^w_i} di \right)^{\mu^w_t} - \int_0^1 w_{i,t} n_{i,t} di \right]
\]

yields the FOC

\[ w_{i,t} = w_t^{\mu^w_t} \left( \int_0^1 n_{i,t}^{1/\mu^w_i} di \right)^{\mu^w_t-1} \frac{1}{\mu^w_i} n_{i,t}^{1/\mu^w_i-1} \]

and the demand function

\[ n_{i,t} = \left( \frac{w_{i,t}}{w_t} \right)^{\mu^w_t - 1} n_t \]

6.4 Wage Phillips Curve

To derive labor supply the household \( i \) maximizes the objective function (1) subject the budget constraints (7) and the labor demand function (13)

\[
E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u \left( c_{i,t}, \frac{w_{i,t}}{w_t} - \frac{\mu^w_t}{\pi_t} n_t, d_{i,t-1}, \frac{d_{i,t-1}}{\pi_t} \right),
\]

... + \( P_t c_{i,t} \left( \frac{w_{i,t}}{w_t} - \frac{\mu^w_t}{\pi_t} n_t - P_t \frac{\omega^W}{2} \left( \frac{w_{i,t}}{w_{i,t-1} \pi_{i,t-1} - 1} \right)^2 y_t + ... \)

Here we plugged in the labor demand function for \( n_{i,t} \) and the wage adjustment cost (8) for \( WAC_t \).

The FOC w.r.t. \( w_{i,t} \) reads
\[ 0 = \beta^t \lambda_{i,t} \left( -\frac{1}{\mu_t - 1} w_{i,t}^{1-\mu_t} - \frac{1}{\mu_t - 1} \frac{w_t^{1-\mu_t}}{w_t - n_t} \right) \]

\[ -\beta^t \lambda_{i,t} \left( \omega_W \frac{w_{i,t}}{w_{i,t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_t \frac{1}{w_{i,t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} \]

\[ +\beta^{t+1} \lambda_{i,t+1} \omega_W \left( \frac{w_{i,t+1}}{w_{i,t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_{t+1} \frac{w_{i,t+1}}{w_{i,t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} \]

\[ +\beta^t u_{i,nt} \left( -\frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \right) w_{i,t} \left( -\frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \right) n_t \]

some rearranging gives

\[ 0 = -\frac{1}{\mu_t - 1} \left( \frac{w_{i,t}}{w_t} - \frac{w_t^{1-\mu_t}}{w_t - n_t} \right) w_{i,t} \]

\[ -\omega_W \left( \frac{w_{i,t}}{w_{i,t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_t \frac{w_{i,t}}{n_t w_{i,t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} \]

\[ +\beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \omega_W \left( \frac{w_{i,t+1}}{w_{i,t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_{t+1} \frac{w_{i,t+1}}{n_t w_{i,t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} \]

\[ -\frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \frac{u_{i,nt}}{\lambda_{i,t}} \frac{w_{i,t}}{w_t} \left( -\frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \right) \]

For \( \omega_W = 0 \)

\[ \frac{1}{\mu_t - 1} w_{i,t} = \frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \left( -\frac{u_{i,nt}}{\lambda_{i,t}} \right) \]

\[ w_{i,t} = \mu_t^{1-\mu_t} \left( -\frac{u_{i,nt}}{\lambda_{i,t}} \right) \]

(144)

For \( \omega_W \neq 0 \), define \( \pi_t^{1-\kappa \pi_t^{1-\mu_t}} = \frac{u_{i,t}}{u_{i,t-1}} \). Here we assume that labor packers (unions) aggregate over all \( w_{i,t} \) to derive \( w_t \)

\[ \omega_W \left( \frac{w_{t}}{w_{t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_t \frac{w_{t}}{n_t w_{t-1} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} \]

\[ = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \omega_W \left( \frac{w_{t+1}}{w_{t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - 1 \right) y_{t+1} \frac{w_{t+1}}{n_t w_{t} \left( \frac{1}{\pi^{1-\kappa \pi_t^{1-\mu_t}}} - 1 \right)} - \frac{1}{\mu_t - 1} w_{t} - \frac{\mu_t^{1-\mu_t}}{\mu_t - 1} \frac{u_{i,nt}}{\lambda_{i,t}} \]
and rearrange

\[
\left( \frac{\pi_{i,t}^{w}}{\pi_{1,t}^{w} \pi_{1,t-1}^{w}} - 1 \right) \frac{\pi_{i,t}^{w}}{\pi_{1,t}^{w} \pi_{1,t-1}^{w}} y_t = \beta \lambda_{i,t+1} \left( \frac{\pi_{i,t}^{w}}{\pi_{1,t}^{w}} - 1 \right) \frac{\pi_{i,t+1}^{w}}{\pi_{1,t+1}^{w}} y_{t+1} + \frac{n_t}{(\mu_t^{w} - 1)} \omega W \left( \mu_t^{w} \left( \frac{u_{i,t+1}}{\lambda_{i,t}} \right) - u_t \right)
\]

6.5 Price Phillips Curve

The retailer maximizes profits by optimally setting the intermediate good’s price \( P_{k,t} \), taking into account cost for acquisition \( mc_t \), price adjustment cost (32), and the market’s demand function for \( y_{k,t} \) (31). The retailer’s problem therefore reads

\[
\max_{\{P_{k,t}\}} \sum_{k=0}^{\infty} \beta^t \left[ \frac{P_{k,t}}{P_t} y_{k,t} - mc_t y_{k,t} - \frac{\omega P}{2} \left( \frac{P_{k,t}}{P_{k,t-1} (\pi_{1,t}^{p} \pi_{1,t-1}^{p})} - 1 \right) y_t \right]
\]

s.t. \( y_{k,t} = \left( \frac{P_{k,t}}{P_t} \right)^{-\epsilon} y_t \)

plugging in for \( y_{k,t} \) yields

\[
\sum_{k=0}^{\infty} \beta^t \left[ \frac{P_{k,t}}{P_t} \left( \frac{P_{k,t}}{P_t} \right)^{-\epsilon} y_t - mc_t \left( \frac{P_{k,t}}{P_t} \right)^{-\epsilon} y_t - \frac{\omega P}{2} \left( \frac{P_{k,t}}{P_{k,t-1} (\pi_{1,t}^{p} \pi_{1,t-1}^{p})} - 1 \right) y_t \right]
\]

FOC w.r.t. \( P_{k,t} \):

\[
0 = -\frac{1}{\mu_t^{w} - 1} P_{k,t} - \frac{\mu_t^{p}}{\mu_t^{p} - 1} \frac{1}{P_t^{\mu_t^{p} - 1}} y_t
\]

\[
+ mc_t \frac{\mu_t^{p}}{\mu_t^{p} - 1} \frac{1}{P_t^{\mu_t^{p} - 1}} y_t
\]

\[
-\omega P \left( \frac{P_{k,t}}{P_{k,t-1} (\pi_{1,t}^{p} \pi_{1,t-1}^{p})} - 1 \right) y_t \frac{1}{P_{k,t-1} (\pi_{1,t}^{p} \pi_{1,t-1}^{p})}
\]

\[
+ \beta \omega P \left( \frac{P_{k,t+1}}{P_{k,t} (\pi_{1,t}^{p} \pi_{1,t}^{p})} - 1 \right) y_{t+1} \frac{P_{k,t+1}}{P_{k,t} (\pi_{1,t}^{p} \pi_{1,t}^{p})}
\]

some rearranging
\[0 = -\frac{1}{\mu_t^p - 1} \left( \frac{P_{k,t}}{P_t} \right)^{\frac{1}{\mu_t^p - 1}} y_t \frac{1}{P_{k,t}} + m\omega \left( \frac{P_{k,t}}{P_t} \right)^{\frac{\mu_t^p}{\mu_t^p - 1}} y_t \frac{1}{P_{k,t}} \]

\[-\omega p \left( \frac{P_{k,t}}{P_{k,t-1} \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} - 1 \right) y_t \frac{1}{P_{k,t-1} \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} \]

\[+ \beta \omega p \left( \frac{P_{k,t+1}}{P_{k,t} \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} - 1 \right) y_{t+1} \frac{1}{P_{k,t} \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} \]

Now aggregate over all retailers prices \( P_t = \int_0^1 P_{k,t} dk \), then

\[0 = -\frac{1}{\mu_t^p - 1} \left( \frac{P_t}{P_t} \right)^{\frac{1}{\mu_t^p - 1}} y_t \]

\[+ m\omega \left( \frac{P_t}{P_t} \right)^{\frac{\mu_t^p}{\mu_t^p - 1}} y_t \frac{1}{P_t} \]

\[-\omega p \left( \frac{P_t}{P_{t-1} \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} - 1 \right) y_t \frac{1}{P_t \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} \]

\[+ \beta \omega p \left( \frac{P_{t+1}}{P_t \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} - 1 \right) y_{t+1} \frac{1}{P_t \left( \frac{1}{\pi_t^{1-\rho} \pi_t^{\rho}} \right)} \]

define \( \pi_t = \frac{P_t}{P_{t-1}} \)

\[0 = m\omega \frac{\mu_t^p}{\mu_t^p - 1} + \frac{1}{\mu_t^p - 1} \]

\[-\omega \left( \frac{\pi_t}{\pi_t^{1-\rho} \pi_t^{\rho}} - 1 \right) \frac{\pi_t}{\pi_t^{1-\rho} \pi_t^{\rho}} + \beta \omega p \left( \frac{\pi_{t+1}}{\pi_t^{1-\rho} \pi_t^{\rho}} - 1 \right) \frac{\pi_{t+1}}{\pi_t^{1-\rho} \pi_t^{\rho}} \frac{y_{t+1}}{y_t} \]

and rearrange

\[\left( \frac{\pi_t}{\pi_t^{1-\rho} \pi_t^{\rho}} - 1 \right) \frac{\pi_t}{\pi_t^{1-\rho} \pi_t^{\rho}} = \beta \omega \left( \frac{\pi_{t+1}}{\pi_t^{1-\rho} \pi_t^{\rho}} - 1 \right) \frac{\pi_{t+1}}{\pi_t^{1-\rho} \pi_t^{\rho}} \frac{y_{t+1}}{y_t} \]

\[+ \frac{\mu_t^p}{\omega p \left( \frac{1}{\mu_t^p} - 1 \right)} \left( m\omega - \frac{1}{\mu_t^p} \right) \]

### 6.6 Long-Term Bond prices

Consider the following investment problem. Following Woodford (2001) we assume that agents can invest in perpetuities \( B_{t-s}^L \) issued in period \( t - s \), which pay \( \rho^s \) units of currency
in each period \( t \) with \( \rho \in [0, 1] \). Hence, coupons that decay exponentially, such that \( \rho \) governs the duration of a bond. Newly issued perpetuities exhibit the price \( p_L^t \)

\[
... + p_L^t B_L^t + P_t c_t + ... \leq ... + \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L + P_t w_t n_t + ...
\]

Define \( B_{t-1}^L \) as the sum of all payments from past bond issuances in period \( t \), like in Arellano, and Ramanarayanan (2012)

\[
B_{t-1}^L = \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L
\]

Given that

\[
B_t^L = \sum_{s=1}^{\infty} \rho^{s-1} B_{t+1-s}^L = B_t^L + \sum_{s=2}^{\infty} \rho^{s-1} B_{t+1-s}^L = B_t^L + \rho \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L
\]

we get the following relation between \( B_t^L \) and \( B_{t-1}^L \) such that

\[
B_t^L = \rho B_{t-1}^L + B_t^L
\]

Substituting out \( B_t^L \), the budget constraint can be rewritten as \( p_t^L (B_t^L - \rho B_{t-1}^L) \leq B_{t-1}^L + P_t (w_t n_t - c_t) + ... \leftrightarrow \)

\[
... + p_t^L B_t^L + P_t c_t + ... \leq ... + (1 + \rho p_t^L) B_{t-1}^L + P_t w_t n_t + ...
\]

Consider now a secondary market for long-term bonds. The period \( t \) price of a bond issued in \( t - s \) is \( p_{t,t-s}^L \)

\[
p_{t,t-s}^L B_{t,t}^L + \sum_{s=1}^{\infty} p_{t,t-s}^L B_{t,t-s}^L \leq \sum_{s=1}^{\infty} \left( p_{t,t-s}^L + \rho^{s-1} \right) B_{t-1,t-s}^L + ...
\]

where \( B_{t-1,t-s}^L \) denotes the beginning-of-period \( t \) holdings of long-term bonds issued in \( t - s \) and \( B_{t,t-s}^L \) its end-of-period \( t \) holdings. Compare two bonds issued in \( t \) and \( t - s \). Period \( t \)

investments in bonds issued in \( t - s \), \( B_{t,t-s}^L \), satisfy

\[
p_{t,t-s}^L = \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t+1} \pi_{t+1}} \left( p_{t+1,t-s}^L + \rho^{s-1+1} \right) \right]
\]

and in period \( t + 1 \), \( B_{t+1,t-s}^L \)

\[
p_{t+1,t-s}^L = \beta E_{t+1} \left[ \frac{u_{c,t+2}}{u_{c,t+2} \pi_{t+2}} \left( p_{t+2,t-s}^L + \rho^{s-1+2} \right) \right].
\]
Iterating forward we get,
\[ p_{L,t-s} = \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t} t_{t+1}} \left( \beta E_{t+1} \left[ \frac{u_{c,t+2}}{u_{c,t+1} t_{t+2}} \left( p_{L,t+2,t-s} + \rho^{s-1+2} \right) + \rho^{s-1+1} \right] \right) \right] \]
\[ = E_t \left[ \beta \frac{u_{c,t+1}}{u_{c,t} t_{t+1}} \rho^{s-1+1} + \beta^2 \frac{u_{c,t+1}}{u_{c,t} t_{t+1}} \frac{u_{c,t+2}}{u_{c,t+1} t_{t+2}} \rho^{s-1+2} + \beta^2 \frac{u_{c,t+1}}{u_{c,t} t_{t+1}} \frac{u_{c,t+2}}{u_{c,t+1} t_{t+2}} p_{L,t+2,t-s} \right] \]
\[ \vdots \]
\[ = E_t \sum_{k=1}^{\infty} \beta^k \frac{u_{c,t+k}}{u_{c,t} t_{t+k}} \rho^{s-1+k} \]

(145)

Applying the same procedure, for a bond issued in \( t \), leads to the first order condition
\[ p_{L,t} = \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t} t_{t+1}} \left( 1 + p_{L,t+1,t} \right) \right], \]
implying that its price \( p_{L,t} \) satisfies
\[ p_{L,t} = E_t \sum_{k=1}^{\infty} \beta^k \frac{u_{c,t+k}}{u_{c,t} t_{t+k}} \rho^{s-1+k} \]

Hence, that secondary market prices satisfy
\[ p_{L,t}^{t-s} = \rho^s p_{L,t} \]  

(146)

Using this price relation, we get
\[ p_{L,t}^{t-s} B_{L,t}^{t-s} + \sum_{s=1}^{\infty} p_{L,t-s} B_{L,t-s} \leq \sum_{s=1}^{\infty} \left( p_{L,t-s} + \rho^{s-1} \right) B_{L,t-1,t-s} + \ldots \]
\[ \Leftrightarrow \left[ p_{L,t}^{t-s} B_{L,t}^{t-s} + \sum_{s=1}^{\infty} \rho^s B_{L,t-s} \right] \leq \sum_{s=1}^{\infty} \left( \rho^s p_{L,t}^{t-s} + \rho^{s-1} \right) B_{L,t-1,t-s} + \ldots \]
\[ \Leftrightarrow p_{L,t}^{t-s} \left[ \sum_{s=0}^{\infty} \rho^s B_{L,t-s} \right] \leq \left( 1 + \rho p_{L,t}^{t-s} \right) \left[ \sum_{s=1}^{\infty} \rho^{s-1} B_{L,t-1,t-s} \right] + \ldots \]

Note that in period \( t \), the sum of all payments from past bond issuances is given by
\[ B_{L,t-1}^{t-s} = \sum_{s=1}^{\infty} \rho^{s-1} B_{L,t-1,t-s} \]
and in period \( t + 1 \) by
\[ B_{L,t}^{t-1} = \sum_{s=1}^{\infty} \rho^{s-1} B_{L,t+1-t-1}^{t-1} = \sum_{s=0}^{\infty} \rho^{s+1-1} B_{L,t+1-(s+1)}^{t-1} = \sum_{s=0}^{\infty} \rho^s B_{L,t,t-s} \]
where \( B_{L,t}^{t} = \rho B_{L,t-1}^{t-1} + B_{L,t}^{t} \). We can write the budget constraint again as above
\[ \ldots + p_{L,t}^{t-s} B_{L}^{t-s} + \ldots \leq \ldots + \left( 1 + \rho p_{L,t}^{t-s} \right) B_{L,t-1}^{t-1} + \ldots \]

We now follow Woodford (2001) and apply a simplification, which hugely reduces the dimensionality of the investor’s problem. Consider the issuer of long-term debt, who satisfies the
Suppose that debt is issued

\[ \ldots + p_{t,t}^L B_{t,t}^L + \sum_{s=1}^{\infty} p_{t,t-s}^L B_{t,t-s}^L + \ldots \leq \sum_{s=1}^{\infty} \left( p_{t,t-s}^L + \rho^{s-1} \right) B_{t-1,t-s}^L + \ldots \]

Recall that bonds issued in period \( t - s \) are equivalent as \( \rho^s \) bonds issued in \( t \) (see 146). We assume that in each period the issuer redeems all previously issued debt by issuing new debt \( B_{t,t}^L \).

Assumption 1 Suppose that debt is issued \( \forall t \geq 0 \) according to \( \sum_{s=1}^{\infty} p_{t,t-s}^L B_{t,t-s}^L = 0 \).

Assumption 1 implies for period \( t - 1 : \sum_{s=1}^{\infty} \left( p_{t-1,t-(s+1)}^L B_{t-1,t-(s+1)}^L \right) = 0 \). Then, the term \( \sum_{s=1}^{\infty} \left( p_{t,t-s}^L + \rho^{s-1} \right) B_{t-1,t-s}^L \) can be rewritten as

\[
\sum_{s=1}^{\infty} \left( p_{t,t-s}^L + \rho^{s-1} \right) B_{t-1,t-s}^L \\
= \left( p_{t,t-1}^L + 1 \right) B_{t-1,t-1}^L + \sum_{s=2}^{\infty} \left( p_{t,t-s}^L + \rho^{s-1} \right) B_{t-1,t-s}^L \\
= \left( p_{t,t-1}^L + 1 \right) B_{t-1,t-1}^L + \sum_{s=1}^{\infty} \left( p_{t-1,t-(s+1)}^L + \rho^{s+1-1} \right) B_{t-1,t-(s+1)}^L \\
= \left( p_{t,t-1}^L + 1 \right) B_{t-1,t-1}^L + \sum_{s=1}^{\infty} p_{t-1,t-(s+1)}^L \left( \frac{p_{t,t-(s+1)}^L}{p_{t-1,t-(s+1)}^L} + \frac{\rho^{s+1-1}}{p_{t-1,t-(s+1)}^L} \right) B_{t-1,t-(s+1)}^L \\
= \left( p_{t,t-1}^L + 1 \right) B_{t-1,t-1}^L + \frac{1 + p p_{t,t-1}^L}{p_{t-1,t-1}^L} \sum_{s=1}^{\infty} p_{t-1,t-(s+1)}^L B_{t-1,t-(s+1)}^L \\
= \left( 1 + \rho p_{t,t-1}^L \right) B_{t-1,t-1}^L \\
= \left( 1 + \rho p_{t,t}^L \right) B_{t-1,t-1}^L
\]

where we used \( p_{t-1,t-(s+1)}^L = \rho^s p_{t-1,t-1}^L, p_{t,t-(s+1)}^L = \rho^{s+1} p_{t,t-1}^L, \) and \( p_{t,t-1}^L = \rho p_{t,t}^L \). Hence, we end up with

\[
\ldots + p_{t,t}^L B_{t,t}^L + \ldots \leq \ldots + \left( 1 + \rho p_{t,t}^L \right) B_{t-1,t-1}^L + \ldots
\]

where we can simplify notation by dropping the double time index, \( B_{t}^L = B_{t,t}^L \) etc. Hence, the investor’s problem can be written as

\[
\ldots + p_{t}^L B_{t}^L + P_{t} c_{t} \ldots \leq \ldots + \left( 1 + \rho p_{t}^L \right) B_{t-1}^L + P_{t} w_{t} n_{t} + \ldots
\]

The first order condition for holdings of long-term debt \( B_{t}^L \) is thus given by

\[
1 = \beta E_{t} \left[ \frac{u_{c,t+1}^L R_{t+1}^L}{u_{c,t}^L \pi_{t+1}^L} \right]
\]
where the one period rate of return on long-term bonds is given by

\[ R_{t+1}^L = \frac{1 + \rho p_{t+1}^L}{p_t^L} \]

which is obviously state contingent. Suppose for a moment that prices are stable \((\pi_t = 1)\) and agents are risk neutral \((u_{c,t} = const.)\). Then, the pricing condition \((145)\) implies \(p_{t,t}^L = \sum_{k=1}^{\infty} \beta^k \rho^{k-1} = \rho^{-1} \sum_{k=1}^{\infty} \beta^k \rho^k = \frac{\beta}{1 - \beta \rho}\) and \(p_{t+1,t}^L = \sum_{k=1}^{\infty} \beta^k \rho^{k-1+1} = \frac{\beta \rho}{1 - \beta \rho}\), such that the one period rate of return \((1 + \rho p_{t+1}^L) / p_{t,t}^L = (1 + p_{t+1,t}^L) / p_{t,t}^L\) is given by \(1 / \beta\), which satisfies arbitrage freeness.

### 6.7 Duration and yield to maturity

The **Yield to Maturity** is the internal rate of return of an investment, taking into consideration all incomes and expenses and their timing. Hence the Yield to Maturity makes all future payments of a perpetuity bond equal to the current market value. For a perpetuity bought in \(t\) an investor would gain the following stream of incomes \(\sum_{s=1}^{\infty} \rho^{s-1}\) discounted by the current Yield to Maturity \(Y_t^L\). Hence, the Yield to Maturity \(Y_t^L\) is simply given by

\[ p_t^L = \sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(Y_t^L)^s} \]

\[ p_t^L = \frac{1}{\rho} \sum_{s=1}^{\infty} \rho^s \left( Y_t^L \right)^s = \frac{1}{\rho} Y_t^L \frac{1 - \rho}{Y_t^L} \]

\[ p_t^L \left( 1 - \frac{\rho}{Y_t^L} \right) = \frac{1}{Y_t^L} \]

\[ Y_t^L = \left( \frac{1}{p_t^L} \right) + \rho \]

The problem of an investor then reads

\[ ... + P_t c_t + p_t^L B_t^L + ... \leq ... + p_t^L Y_t^L B_{t-1}^L + P_t w_t n_t + ... \]

leading to the first order condition for holdings of long-term debt

\[ 1 = \beta E_t \left[ \frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\pi_{t+1}} \frac{p_{t+1,t}^L Y_t^L}{p_t^L} \right] \]

The concept of **Duration** measures the number of periods it takes for the price of a bond to be repaid by its internal cash flows. A bond’s duration is calculated as a weighted average of the time horizons at which the cash flows from a bond are received. Each time horizon’s weight is the percentage of the total present value of the bond (bond price) paid at that time.
For that the bond’s yield to maturity is used to calculate the present values

\[ D_t = \frac{\sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(Y_t^L)^s}}{\sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(Y_t^L)^s}} = \frac{1}{Y_t^L + 2 + \frac{\rho^2}{(Y_t^L)^2} + \cdots} \]

where \( p_t = \sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(Y_t^L)^s} = (Y_t^L - \rho)^{-1} \) is the bond’s present value in \( t \). Note that

\[ \sum_{s=1}^{\infty} s x^s = \frac{x}{(x-1)^2} \quad \text{for} \quad x \in (0, 1). \]

Dividing by \( p_t = (Y_t^L - \rho)^{-1} \) gives

\[ D_t = \frac{\frac{\rho}{Y_t^L}}{\frac{1}{Y_t^L} + \frac{\rho^2}{(Y_t^L)^2}} \]

\[ = \frac{Y_t^L}{Y_t^L - \rho} \]

Alternatively it can be shown that the duration is the elasticity of the bond’s present value with respect to the discount factor \( Y_t^L \): \( D_t = -\frac{dP_t}{dY_t} \frac{Y_t^L}{P_t} \), implying \( D_t = \frac{1}{(Y_t^L - \rho)} \frac{Y_t^L}{Y_t^L - \rho} = \frac{Y_t^L}{Y_t^L - \rho} \).

### 6.8 Data

We use quarterly U.S. data ranging from 1964:Q3 to 2007:Q4. Time series are taken from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. We construct real per capita GDP by dividing the nominal GDP (FRED: GDP) series by population (CNP160V) and the GDP deflator (GDPDEF). Consumption is measured by the sum of private sector nondurable goods consumption (PCND) and private sector services consumption (PCESV). We measure investment by the sum of the series "Gross Private Domestic Investment" (GDPI) and durable goods consumption (PCDG). Bank reserves are proxyed by the Federal Reserve Bank of St. Louis’ measure for monthly Adjusted Reserves (ADJRESSL). We proxy for deposits by the sum of the series "Total Checkable Deposits" (TCDSL), "Small Time Deposits" (STDSL), and "Savings Deposits" (SAVINGSL) (as part of M2). The former four series are then converted to real per capita values in the same way as the GDP series. Inflation is calculated as the gross growth rate of the GDP implicit price deflator (GDPDEF). We define hours worked by multiplying average weekly hours worked in the nonfarm business sector (PR85006023) with the series "Civilian Employment" (CE16OV) and dividing by total population (CNP16OV). Real wages are derived by dividing the time series "Nonfarm Business Sector: Compensation Per Hour" (COMPNFB) by the
GDP deflator. We use the effective Federal Funds rate (FEDFUNDS) as our measure for the model’s money market rate, the 3-month certificate of deposit secondary market rate (CD3M) as measure for the model’s deposit rate, Moody’s Baa corporate bond yield index (BAA) as a measure for the loan rate, and the 7-Year Treasury constant maturity rate (GS7, Datastream) as a measure for the yield to maturity on long-term treasuries. All time series are detrended by using a linear trend, except for the interest rates, hours worked, and real deposits. The interest rates are all demeaned. The mapping of the variables to the states is

\[
\begin{align*}
\dot{y}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{gdp_{t}}{gdp} \right) \\
\hat{c}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{c_{t}}{c} \right) \\
\dot{\nu}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{\nu_{t}}{\nu} \right) \\
\hat{n}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{m_{t}}{m} \right) \\
\hat{w}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{w_{t}}{w} \right) \\
\hat{m}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{m_{t}}{m} \right) \\
\hat{d}_{t}^{\text{obs}} &= 100 \cdot \log \left( \frac{d_{t}}{d} \right) \\
R_{t}^{m,\text{obs}} &= 100 \cdot (R_{t}^{m} - R^{m}) \\
R_{t}^{D,\text{obs}} &= 100 \cdot (R_{t}^{D} - R^{D}) \\
R_{t}^{L,\text{obs}} &= 100 \cdot (R_{t}^{L} - R^{L}) \\
YTM_{t}^{\text{obs}} &= 100 \cdot (YTM_{t} - YTM) \\
\pi_{t}^{\text{obs}} &= 100 \cdot (\pi_{t} - \pi)
\end{align*}
\]

where all state variables are in deviations from their steady-state values. The detrended observable \( \dot{y}_{t}^{\text{obs}} \) corresponds to the first difference in the detrended log real per capita GDP series, multiplied by 100. Analogously we define \( \hat{c}_{t}^{\text{obs}}, \dot{\nu}_{t}^{\text{obs}}, \hat{w}_{t}^{\text{obs}}, \hat{m}_{t}^{\text{obs}}, \hat{d}_{t}^{\text{obs}} \). The demeaned observable \( R_{t}^{m,\text{obs}} \) corresponds to the contemporary deviation of the effective Federal Funds rate from its sample mean. Analogously we define \( R_{t}^{D,\text{obs}}, R_{t}^{L,\text{obs}}, YTM_{t}^{\text{obs}}, \pi_{t}^{\text{obs}} \).