Prices, Debt and Market Structure in an Agent Based Model of the Financial Market

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Abstract

We develop an agent based model in which heterogenous and boundedly rational agents interact by trading a risky asset at an endogenously set price. Agents are endowed with balance sheets comprising the risky asset as well as cash on the asset side and equity capital as well as debt on the liabilities side. The introduction of balance sheets and debt into an agent based setup is relatively new to the literature and allows us to tackle several research questions that are mostly inaccessible with conventional methodology, especially representative agent models. A number of findings emerge when simulating the model. We find that the empirically observable log-normal distribution of bank balance sheet size naturally emerges and that higher levels of leverage lead to higher inequality among agents. When further analyzing the relationship between leverage and balance sheets, we observe that decreasing credit frictions result in an increasingly procyclical behavior of leverage, which is typical for investment banks. We show how decreasing credit frictions increases volatility but decreases the number of occurring bankruptcies. Furthermore we study price efficiency and trading volume in the context of rationality and disagreement.

JEL classification: C63 – D53 – D84 – G24
Keywords: agent based model – financial markets – financial stability – balance sheet effects – procyclical leverage – size distribution - credit frictions - trading volume

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1 Introduction

The past few years have indicated that the understanding of the dynamics in financial markets is far from satisfactory. Economists and regulators often seem to rely on intuition rather than model-guided comprehension when pondering and designing new rules for the financial system. As a result, most of the suggested financial market regulations are impeded by controversy about the efficiency and uncertainty about their impact. One reason why financial market dynamics prove so difficult to grasp and model is that they are driven by heterogeneous market participants’ actions and interactions that feed back into the financial system.

We present an uncalibrated agent based model (ABM) that includes debt in order to facilitate an analysis of the dynamics ensued by agents’ capital structure. Each agent in our model is therefore endowed with a highly stylized balance sheet containing a tradable risky asset and cash on the asset side and equity capital and debt on the liabilities side. Agents trade according to their price expectations, which they form through either fundamental value considerations (fundamentalists) or technical analysis (chartists). The price of the risky asset depends on agents’ transactions and therefore evolves endogenously. Leverage can generally be managed by agents but is constrained by the debt supply of a risk managing financier. Simulations are conducted to demonstrate the general working of our model as well as some of the new possibilities of analysis the model permits which are unfeasible with either standard representative agent models or existing agent based financial market models that focus predominantly on price dynamics. We can report several findings. Specifically we show how credit frictions\footnote{We define credit frictions as the latency with which agents can acquire and dispose of debt.} can change the relationship between leverage and assets and thereby account for the differences observed for commercial and investment banks in this context: for investment banks leverage is procyclical, while no such relation can be observed for commercial banks. By looking at the emergent market structure of the model, we find that balance sheet size is approximately lognormally distributed and that there is a natural tendency for inequality to increase over time. Higher leverage seems to intensify the evolution towards higher inequality between agents. Furthermore we study the concepts of rationality and disagreement and their impact on price efficiency and trading volume. Here our results are ambivalent: in line with the literature, higher rationality increases price efficiency but reduces trading volume, while higher agreement among fundamentalists increases price efficiency and unexpectedly also in-
creases trading volume. Where possible, we compare the outcomes of the simulations with balance sheet data from a sample of international banks and make reference to the views expressed in the relevant literature. Policy implications, especially with regard to financial stability, are given where appropriate.

The majority of agent based financial market models focus on price dynamics, which emerge through the interaction of heterogeneous agents. Such models have been quite successful in replicating and explaining some intriguing features of the financial market such as endogenous bubbles and crashes as well as stylized facts of return time series including fat tails and clustered volatility. Compelling reviews of the literature can e.g. be found in LeBaron (2006), Chiarella et al. (2009), Hommes and Wagener (2009), and Lux (2009). Incorporating balance sheets containing debt and equity into financial market ABMs is a sensible extension to established models and is mostly novel. Notable exceptions include Raberto et al. (2011) and Thurner et al. (2010). While the model introduced in Raberto et al. (2011) takes a macroeconomic perspective and mainly focuses on the lending channel of banks, the model presented in Thurner et al. (2010) is closer to our approach. However Thurner et al. (2010) are less interested in balance sheet dynamics and rather focus on the effects of leverage on returns, which they find to produce fat tails and clustered volatility. Agents in their model borrow if their own funds are insufficient to carry out a desired investment. Rich agents are therefore less leveraged than poorer ones. To better suit our purposes we, in turn, separate the investment decisions of agents from their leverage strategy and thereby avoid this negative relationship between wealth and leverage.

Although the study of leverage and balance sheet dynamics is novel in the context of agent based models, the issue has been addressed by prominent researchers in other contexts. Early work emphasizing the role of leverage and balance sheets can be found in the debt deflation theory of Fisher (1933), and in Minsky’s financial instability hypothesis (see Minsky, 1986). In Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) leverage acts as a financial accelerator for non-financial borrowers. The resurfacing of research on leverage and balance sheet dynamics in the aftermath of the recent financial crisis suggests its importance for understanding the workings of the financial system and the events precipitating the crisis. Adrian et al. (2010) argue that there is an important relation between financial intermediaries’ balance sheet dynamics and real economic activity. The dynamics of market and funding liquidity which reinforce each other and can lead to destabilizing effects on financial markets are analyzed theoretically by Brunnermeier and Pedersen (2009), while Geanakoplos (2009) shows how
changes to leverage can cause wild fluctuations in asset prices. More generally, the inclusion of the financial sector into new DSGE models (see e.g. [Curdia and Woodford, 2009; Gertler and Kiyotaki, 2010]) is a further sign for the increasing importance of financial markets for economic theory.

In our opinion agent based models constitute a promising methodology for advancing the understanding of the financial system’s underlying dynamics. In the following we briefly emphasize some key concepts clarifying why in the context of our own model we believe there exists a comparative advantage of the agent based methodology over the rational representative agent paradigm.\footnote{More profound and comprehensive criticisms of mainstream economic models can e.g. be found in [Leijonhufvud, 2009; Colander et al., 2009; Kirman, 2010; Stiglitz, 2011]. Comparisons between agent based models and DSGE models can e.g. be found in [Farmer and Geanakoplos, 2009; Fagiolo and Roventini, 2012].} In contrast to the top-down approach of representative agent models, agent based models take a bottom-up modeling approach. Thereby they account for the fallacy of composition (see e.g. [Caballero, 1991; Kirman, 1992]), i.e. ABMs follow the assumption that the aggregate behavior of interacting agents does not have to coincide with the behavior of the individual. By substituting aggregate individual behavior with the behavior of one, mostly rational, representative agent, mainstream macroeconomic models are kept analytically tractable while satisfying the Lucas Critique which demands that models are microfounded (see [Lucas, 1976]). This aggregation approach seems appealing not only to economists, but as [Kirman, 2010] remarks, also to politicians and commentators, who, when speaking of financial markets, often refer to "the market" as if it were an individual. Such oversimplifying assumptions often suppress interesting and important details.\footnote{There are also serious theoretical reservations that adhere to this aggregation approach. See e.g. [Stoker, 1993] for a thorough discussion of the issue.} The event of a bankruptcy is e.g. not feasible within a representative agent framework. The fact that the possibility of default is highly overlooked in theoretical models (cp. [Goodhart and Tsomocos, 2011]) is unfortunate, not least when considering the devastating effects of the Lehman Brothers default in 2008. Furthermore there is no trading in a representative agent model, as there is no counterparty to a trade. However, trading is essential for a liquid market and trading volume may reveal interesting information about the market’s condition and possible asset mispricings (cp. [Hong and Stein, 2007]). When allowing for more than one agent, heterogeneity enters a model. Heterogeneity leads to interactions which lead to endogenous developments. Prices e.g. evolve endogenously and bubbles, crashes or return time series stylized facts emerge. [Stiglitz, 2011] writes:
Standard Models focused on the wrong questions. They focused on explaining the small "normal" variations in the economy - which don’t matter much - and ignored the large variations which matter a great deal. They asked how the economy responded to exogenous shocks, while some of the most important disturbances - the bubbles that periodically occur, and then break - are clearly endogenous.

When simulating our model we find that it exhibits strong path dependence. Within the same parameter constellation repeated simulations with differing error terms (random numbers drawn from the same distribution) display highly variant outcomes ranging from relative efficient and tranquil markets to the collapse of the system with all agents defaulting. This emergent property of agent based models makes them seem arbitrary at times, whereas the existence of countable solutions (unique equilibrium or multiple equilibria) in most mainstream models seems to tell a clearer story. At best, however, stable and empirically testable patterns and distributions emerge in simulations of ABMs. The factors that lead to a certain pattern or distribution can then be analyzed and valuable insights about the workings of the financial system (in our case) may be disclosed. The emergent property inherent to the agent based methodology can thus help to advance our knowledge of the dynamics of the financial system.

The remainder of the paper is organized as follows. Section 2 presents the model, while Section 3 provides simulation results. Thereby we start by showing some basic dynamics of an exemplary simulation in Section 3.1. We proceed, in Section 3.2, by looking at the distribution of agents’ balance sheet size and the effects of leverage on balance sheet evolution. The role credit frictions play in our model market is analyzed in Section 3.3, while Section 3.4 discusses the concepts of rationality and disagreement and their impact on price efficiency and trading volume. Section 4 will conclude.

2 The Model

The model described in the following can be classified as a "few type" agent based financial market model. While agents cannot produce entirely new trading strategies, as is possible through evolutionary learning algorithms in some so-called "many type" models, they can choose from a set of predefined trading rules, the rule that seems most profitable to them under the limitations imposed on their rationality. Specifically, in our model agents can select either a strategy based on fundamentals or a chartists’ strategy based on technical analysis. The implied assumption that real
traders do choose and switch between these two strategies finds strong support in the literature (see e.g. [Menkhoff and Taylor, 2007] and chartist-fundamentalist-approaches figure among the most common agent based financial market models (see e.g. [Lux and Marchesi, 2000], [Farmer and Joshi, 2002] and [Westerhoff and Dieci, 2006]). Heterogeneity enters the model not only through the differing strategy types, but also through departing configurations within the strategies. Disagreement may prevail on the true fundamental value of an asset and there may be different methods and time frames considered by chartists to extrapolate future price movements from historic ones. In general, heterogeneity is key to any agent based model. It is the emergent aggregate behavior ensuing from interacting heterogeneous agents which lies at the focus of agent based analysis and embodies a salient distinction between ABMs and models with a representative agent. In the context of agent based financial market models, emergent aggregate behavior e.g. encompasses stylized facts such as fat tails and clustered volatility, asset bubbles and crashes. Our model furthermore allows for the analysis of emergent leverage and balance sheet dynamics, distributional effects as well as market structure.

2.1 Model Structure

While the replication of financial market stylized facts has constituted the aim of many ABMs, much less attention has been directed at emergent behavior on the balance sheet dimension of financial markets. For this reason, we endow each agent \( j \) in our model with the following schematic balance sheet at time \( t \):

\[
\begin{array}{c|c}
\text{Assets} & \text{Liabilities} \\
Q_{j,t}P_t & E_{j,t} \\
C_{j,t} & O_{j,t}
\end{array}
\]

The assets side of the balance sheet comprises the quantity \( Q_{j,t} \) of a risky asset with price \( P_t \) as well as cash \( C_{j,t} \) which can be held without risk. It will often be useful to consider logarithmic prices, which will be denoted in lowercase (i.e. \( p_t = \log(P_t) \)). On the liabilities side, each agent is endowed

\[4\]In the following we will make use of lowercase letters for logarithmic values and uppercase letters for real values. The main rationale for using log prices \( p_t \) is to ensure that real prices \( P_t \) remain non-negative in the price formation process.
with equity capital $E_{j,t}$ and outside capital (debt) $O_{j,t}$. The balance sheet total $B_{j,t}$ is given by:

$$B_{j,t} = Q_{j,t}P_t + C_{j,t} = E_{j,t} + O_{j,t}$$  \hspace{1cm} (1)

From the start of period $t$ to the start of period $t+1$ balance sheets evolve as sketched below:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(Q_{j,t} + D_{j,t}) \exp(p_t + r_{t+1})$</td>
<td>$E_{j,t} + \Delta E_{j,t+1}$</td>
</tr>
<tr>
<td>$C_t + \Delta C_{t+1}$</td>
<td>$O_{j,t} + \Delta O_{j,t}$</td>
</tr>
</tbody>
</table>

where $D_{j,t}$ is the demand of agent $j$ for the asset in period $t$ and $r_{t+1}$ is the logarithmic return, with $r_{t+1} = p_{t+1} - p_t$. The debt level $O_{j,t+1}$ in period $t+1$ consists of the debt level from the start of period $t$, i.e. $O_{j,t}$, and a change to outside capital $\Delta O_{j,t}$, which depends on the agent’s strategic demand for debt as well as the available supply of debt. As indicated by the time index, the change in outside capital $\Delta O_{j,t}$ already takes place before the end of period $t$, so that agents can use the newly acquired debt for trading in period $t$. The timing of the model schematized in Figure 1. Equity capital grows with the returns $R_{t+1}$ and $R_C$ on the risky and risk free (cash) asset respectively and decreases with the interest $i$ paid on debt. Both the risk free rate and the interest rate on debt are exogenous in our model. In a frictionless market we would assume $R_C = i$. The equity capital (equivalent to an agent’s net worth) evolves endogenously:

$$\Delta E_{j,t+1} = (Q_{j,t}P_t)R_{t+1} + C_{j,t}R_C - iO_{j,t+1}.$$  \hspace{1cm} (2)

We thereby assume that new assets (i.e. $D_{j,t}$) are bought and sold at the price $P_{t+1} = P_t(1 + R_{t+1})$. With the balance sheet equality from Equation (1), the change in cash amounts to

$$\Delta C_{j,t+1} = -D_{j,t}P_{t+1} + C_{j,t}R_C - iO_{j,t+1} + \Delta O_{j,t}.$$  \hspace{1cm} (3)

We model the agent’s portfolio choice (i.e. the proportion $A_{j,t+1}$ of the balance sheet he wants to hold in the risky asset in the upcoming period $t+1$) in dependence of the agent’s forecast of log excess return and his confidence

\footnote{After the initial endowment we assume that agents cannot issue new equity (for instance in the form of a seasoned equity offering), equity capital therefore evolves as the difference between the balance sheet total and debt. Strategic changes to the liabilities side of the balance sheet can therefore only be incurred by changes to the debt level.}

\footnote{The relation between logarithmic ($r$) and real ($R$) returns is defined as $r = \log(1+R)$.}
in this forecast which is modeled with a measure of historic forecast errors \( \sigma_{FE}^{j,t} \):

\[
A_{j,t+1} = \frac{E_{j,t}[r_{t+1}] - r_{C}}{\gamma \sigma_{FE}^{j,t}}
\]  

(4)

Generally we denote the forecast of agent \( j \) made in period \( t \) for the variable \( x \) in period \( t + 1 \) as \( E_{j,t}[x_{t+1}] \). The parameter \( \gamma > 0 \) can be viewed as a risk aversion parameter. The forecast error is modeled as the square root of an exponentially weighted moving average (EWMA) of squared differences between return expectations and return realizations:

\[
\sigma_{FE}^{j,t} = \sqrt{\theta_{FE} \left( E_{j,t} - 1 \right)^2 + (1 - \theta_{FE}) \left( \sigma_{FE}^{j,t-1} \right)^2},
\]  

(5)

with \( \theta_{FE} \in [0, 1] \) being a memory parameter defining how much weight should be assigned to the most recent forecast error.

Note that Equation (4) is of a similar structure as classical myopic portfolio choice models with CARA utility functions or models that maximize a linear combination of return mean and variance (see e.g. Campbell and Viceira, 2002). The essential difference is that here the portfolio choice variable \( A_{j,t+1} \) represents the ratio of risky assets to balance sheet total rather than the ratio of risky assets to net worth. Thus, to implement the portfolio choice from Equation (4), an agent \( j \) must act so that on the balance sheet dimension the following relation is satisfied:

\[
A_{j,t+1} = \frac{E_{j,t}[P_{t+1}](Q_{j,t} + D_{j,t})}{E_{j,t}[B_{j,t+1}]}
\]  

(6)

The proportion of an agent’s balance sheet held in the risky asset is bounded by \([-1, 0] \leq A_{j,t+1} \leq 1\). The upper bound 1 is due to an agent’s budget constraint, while the lower bound can take a value between 0 and \(-1\), depending on the constraints imposed on short selling. The closer to 0 the lower bound is set, the higher the barriers for going short. By varying the lower bound we can thus study how short selling constraints of different intensities affect the financial market. 7

The approach detailed in Equations (4) and (6) allows us to separate an agent’s leverage strategy from his portfolio choice. In classical myopic portfolio choice models leverage is linked to investment opportunities - only when large returns are expected does leverage enter the model (i.e. when

7 While a lower bound of 0 implies that an agent can sell only as many assets as he owns (i.e. \( D_{j,t} = -Q_{j,t} \)), a lower bound of \(-1\) implies that an agent cannot go short in more assets than he has means for repurchasing those assets at any given point in time.
$A_{j,t} > 1$). Here, on the other hand, the agent’s debt choice enters the demand function, which can be obtained by rearranging Equation (6):

$$D_{j,t} = \frac{A_{j,t+1} E_{j,t}[B_{j,t+1}]}{E_{j,t}[P_{t+1}]} - Q_{j,t}$$

(7)

with

$$E_{j,t}[B_{j,t+1}] = E_{j,t}[P_{t+1}](Q_{j,t} + D_{j,t}) + E_{j,t}[\Delta C_{j,t+1}]$$

$$= E_{j,t}[P_{t+1}]Q_{j,t} + C_{j,t}(1 + R_C) - i \tilde{O}_{j,t+1} + \Delta \tilde{O}_{j,t}$$

(8)

The amount of debt $\tilde{O}_{j,t+1}$ held by the agent in the upcoming period is subject to negotiations (indicated by the tilde) between agent and financier. It depends on an agent’s demand for debt and the financier’s willingness to supply the desired debt. If both negotiating parties do not wish to make any changes to the debt level, i.e. $\Delta \tilde{O}_{j,t} = 0$, the absolute debt volume $O_{j,t}$ will need to be rolled over at the interest rate $i$.

### 2.2 Fundamental, Chartist and Debt Strategies

Agents can choose between a fundamental and a chartist strategy when forming expectations of future returns. When following a fundamental strategy, agents (i.e. $j \in \mathcal{F}$) believe that prices will revert to fundamental value. They therefore compare their perception of fundamental value $E_{j,t}[f_{t+1}]$ with the current price in order to obtain a forecast of future returns:

$$E_{j,t}[r_{t+1}] = \alpha^F(E_{j,t}[f_{t+1}] - p_t), \quad \forall j \in \mathcal{F}$$

(9)

with $\alpha^F > 0$ being the speed at which the fundamentalist believes prices converge to fundamental value. The fundamentalist updates his perception of fundamental value by evaluating relevant fundamental news $\Delta f_t$ and by identifying and correcting past valuation errors:

$$E_{j,t}[f_{t+1}] = E_{j,t-1}[f_t] + \frac{(\Delta f_t + \epsilon_{j,t})}{\text{past valuation}} + \frac{\theta^F(f_t - E_{j,t-1}[f_t])}{\text{evaluation of news}} + \frac{\theta^E(f_t - E_{j,t-1}[f_t])}{\text{past error correction}}$$

(10)

The error term $\epsilon_{j,t} \sim \mathcal{N}(0, \sigma^2_f)$ accounts for fundamentalists’ imperfect information and limited cognition and implies disagreement about the true value $f_t$ of the risky asset. In the model we assume that disagreement on fundamental value may persist for some time, but agents will eventually become
aware of erroneous evaluations and correct for them. The speed of this error correction is thereby given by \( \theta^F \in [0, 1] \).

In order to obtain a forecast of future returns, chartists \((j \in C)\), in a first step, extrapolate a buy or sell signal. They do so by employing moving average (MA) rules which are among the simplest and most popular with practicing technical analysts.\(^8\) The signal is generated by comparing a short term MA of prices to a long term MA of prices. Specifically, the chartist identifies an emerging upward trend and a buy signal \((S_{j,t} = +1)\) is generated when the short term MA is higher than the long term MA, and vice versa for a downward trend and sell signal \((S_{j,t} = -1)\):

\[
S_{j,t} = \text{sgn}\left( \frac{1}{s_{j,t}} \sum_{u=0}^{s_{j,t}-1} P_{t-u} - \frac{1}{l_{j,t}} \sum_{v=0}^{l_{j,t}-1} P_{t-v} \right), \quad \forall j \in C \tag{11}
\]

The maximum number of lags \(s_{j,t}\) and \(l_{j,t}\) may differ from agent to agent as well as throughout time. Note that in order to allow for additional heterogeneity within the chartist strategy we do not specify \(s_{j,t} < l_{j,t}\). A chartist \(j\) will thus follow a contrarian strategy whenever \(s_{j,t} > l_{j,t}\). The forecast of future returns then depends on the direction in which the extrapolated signal is pointing, the aggressiveness of the chartist denoted by \(\alpha^C\) and a positive random component \(\rho_{j,t} \sim \lvert N(0, \varsigma_{j,t}^2) \rvert\):

\[
E_{j,t}[r_{t+1}] = \alpha^C S_{j,t} \rho_{j,t} \tag{12}
\]

The random component is necessary because the signal \(S_{j,t}\) extrapolated by chartists does not imply a specific return expectation. We assume that while the moving average rule indicates the direction of the expected return, chartists choose randomly an absolute value of the expected return which is scaled with the perceived price variability which is calculated as an exponentially weighted moving average:

\[
\varsigma_{j,t}^2 = \theta^S (r_t - r_{t-1})^2 + (1 - \theta^S)\varsigma_{j,t-1}^2, \tag{13}
\]

with \(\theta^S\) being a memory parameter specifying how much weight is given to the most recent log return movement. The chartist thus adapts his return expectation to the prevailing price volatility. Chartists can therefore also be viewed as volatility traders that take strong positions in time of high volatility and vice versa.

\(^8\)Brock et al. (1992) provide evidence for the MA rule’s capability to predict stock returns; in an agent based context Chiarella et al. (2006) analyze the ensuing price dynamics when agents employ MA rules.
Generally, we define the change in exposure to outside capital as

$$\Delta \tilde{O}_{j,t} = \mu^O \left( \tilde{O}_{j,t+1} - O_{j,t} \right), \quad (14)$$

with $\tilde{O}_{j,t+1}$ being the targeted debt volume after negotiation with the financier. Because neither the agent nor the financier can force the respective other party to supply or demand more debt than willing, the debt volume will be set to the lower value of the financier’s supply $O_{j,t+1}^S$ and the agent’s demand $O_{j,t+1}^D$:

$$\tilde{O}_{j,t+1} := \min \{O_{j,t+1}^D, O_{j,t+1}^S\} \quad (15)$$

The parameter $\mu^O \in [0, 1]$ in Equation (14) introduces credit friction into the debt market. When $\mu^O < 1$ the targeted changes to the volume of debt take place slower than desired by either agent or financier. In case the financier is delimitating the debt demand of the agent (i.e. $O_{j,t+1}^D > O_{j,t+1}^S$) the friction may e.g. be interpreted as credit maturity hindering the financier to withdraw his funds at once; in case the financier is willing to provide the agent’s full debt demand (i.e. $O_{j,t+1}^D \leq O_{j,t+1}^S$) the friction may e.g. be interpreted as delays in raising funds from different investors. Furthermore, a very low value for $\mu^O$ could be interpreted as limited institutional space to actively manage debt levels. Customer deposits held by commercial banks e.g. constitute such a limitation: while to a certain extent a commercial bank can invest customer deposits, it cannot directly increase or decrease them at will.

The structure of our model allows for the integration of arbitrary debt demand and supply functions. A simple debt strategy for an agent could be to aim for a constant leverage ratio:

$$\lambda_{\text{fix}} = \frac{O_{j,t+1}^D}{E_{j,t}[E_{j,t+1}]} = \frac{O_{j,t+1}^D}{E_{j,t}[B_{j,t+1}] - O_{j,t+1}^D} \quad (16)$$

Note that agents are forward looking, i.e. their desired debt level depends on their expectation of the size of their future balance sheet. Following from the previous equation debt demand can be derived:

$$O_{j,t+1}^D = \lambda_{\text{fix}} E_{j,t}[B_{j,t+1}] \frac{1}{1 + \lambda_{\text{fix}}} \quad (17)$$

With Equation (8) it can be algebraically deduced that for the period $t + 1$ agent $j$ demands:

$$O_{j,t+1}^D = E_{j,t}[P_{j,t+1}]Q_{j,t} + C_{j,t}(1 + R_{C,t}) - O_{j,t} \frac{1}{i + \lambda_{\text{fix}}} \quad (18)$$

---

9 We define leverage as the quotient of debt to equity capital (net worth): $\lambda = O/E$. 

10
For the financier we assume that they do not form expectations about future price movements, but rather try to assess the risk of supplying debt to individual agents. Due to the seniority of debt over equity the financier focuses on the risk that incurred losses in the subsequent periods fully deplete an agent’s equity capital (i.e. the agent goes bankrupt). Specifically the financier is willing to supply debt $O_{j,t+1}^S$ if the probability of default over the next $M$ periods is lower than $\omega$:

$$\Pr \left\{ (E_{j,t} + O_{j,t+1}^S)(1 + R_{j,t}^B)^M \leq O_{j,t+1}^S(1 + i)^M \right\} \leq \omega \quad (19)$$

Because the financier does not have the expertise to assess an agent’s strategy, he must solely rely on the agent’s past performance (i.e debt adjusted balance sheet growth $r_{j,t}^B$), which is modeled as a lognormal random variable with $\log(1 + R_{j,t}^B) = r_{j,t}^B \sim N(\mu_{j,t}^B, z_{j,t}^2)$. Mean and variance are estimated by the financier as exponentially weighted moving averages:

$$\mu_{j,t}^B = \theta_{\text{fin}} \left( \log(B_{j,t} + iO_{j,t}) - \log(B_{j,t-1} + \Delta O_{j,t}) \right) + (1 - \theta_{\text{fin}}) \mu_{j,t-1}^{B}$$

$$z_{j,t}^2 = \theta_{\text{fin}}(r_{j,t}^B - r_{j,t-1}^B)^2 + (1 - \theta_{\text{fin}})z_{j,t-1}^2 \quad (20)$$

$\theta_{\text{fin}}$ thereby defines how much weight is given to the respective last observation. With the risk constraint in Equation (19) and with $H^{-1}(\cdot)$ being the inverse cumulative distribution function of the random variable $r_{j,t}^B$, the maximum amount of debt the financier is willing to supply to agent $j$ can be derived:

$$O_{j,t+1}^S = \frac{E_{j,t} \exp \left( MH^{-1}(\omega) \right)}{(1 + i)^M - \exp \left( MH^{-1}(\omega) \right)}. \quad (21)$$

### 2.3 Choosing a Strategy

Agents in the model try to adapt to the prevailing situation by updating their trading strategy if it seems to be underperforming. For this purpose each agent revises his strategy every $\tau_j$ periods. In order to avoid a synchronized change in strategy, $1 < \tau_j < n$ is a random number drawn from a discrete uniform distribution with $n$ being the maximum number of periods before an agent revises his strategy. Formally agent $j$ revises his strategy at time $t \in K_j := \{ t | t \mod \tau_j = 0 \}^{10}$ When deciding on whether to keep or change strategy, each agent compares a measure of the profit $\Pi_{j,t}$ his strategy has earned to a benchmark $\Pi_t$. This comparison is modeled by a discrete choice

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10The modulo operator ensures that each agent only trades in a period $t$ which is a multiple of his trading frequency $\tau_j$.  

11
model pioneered by Manski and McFadden (1981) and popularized in the context of agent based models by Brock and Hommes (1998). Specifically, when agent $j$ revises his current strategy he will stick to it with probability

\[ W_{j,t}^F = \frac{\exp(\eta \Pi_{j,t})}{\exp(\eta \Pi_{j,t}) + \exp(\eta \Pi_t)} \quad \forall t \in K_j, \tag{22} \]

whereby $\eta > 0$ can be understood as a (bounded) rationality parameter. It limits agents’ abilities to identify whether their strategies are performing well or poorly in comparison to the benchmark. Low values for $\eta$ imply poor identification ability and vice versa.

The profitability measure is computed as an exponentially weighted average of the most recent growth in an agent’s equity capital and past equity growth:

\[
\Pi_{j,t} = \begin{cases} 
\bar{\Pi}_t & \text{if the strategy in } t \text{ does not equal the strategy in } t - 1 \\
\theta^\Pi (\log(E_{j,t}) - \log(E_{j,t-1})) + (1 - \theta^\Pi) \Pi_{j,t-1} & \text{else}
\end{cases}
\tag{23}
\]

with $\theta^\Pi \in [0,1]$ being a memory parameter assigning how much weight is given to the most recent equity growth. Note from Equation (23) that the profitability measure for agent $j$ is set to the benchmark when he changes his strategy. Thereby $\bar{\Pi}_t$ is simply the average of all agents profitability measures, i.e.:

\[ \bar{\Pi}_t = \frac{1}{J} \sum_{j=1}^{J} \Pi_{j,t} \tag{24} \]

Upon opting for a chartist strategy an agent must choose the specifications for the moving average rule, i.e. he must determine the maximum lags in Equation (11). In period $t \in K_j$ agent $j \in C$ draws $s_{j,t}$ and $l_{j,t}$ randomly from a triangular distribution with respective lower limits $s_{\text{low}}$ and $l_{\text{low}}$, respective upper limits $s_{\text{up}}$ and $l_{\text{up}}$ and respective modes $c_{s,j,t}$ and $c_{l,j,t}$, with $s_{\text{low}} \leq c_{s,j,t} \leq s_{\text{up}}$ and $l_{\text{low}} \leq c_{l,j,t} \leq l_{\text{up}}$. The purpose of employing a triangular distribution with a variable mode, is to ensure that chartists gravitate to the specifications of successful moving average rules. Specifically the modes are chosen so that the expected value of the triangular distribution equals to the expected value for the lag parameters $\hat{s}_{j,t}$ and $\hat{l}_{j,t}$ computed from a prob-

\[ ^{11} \text{Given the lower and upper limit, the relation between the mode and the expected value of a triangular distribution amounts to } c = 3\mu - (x_{\text{up}} + x_{\text{low}}) \text{ (Evans et al., 2000).} \]
ability mass function where the respective lags for each chartist is weighted by its relative profitability:

\[
\hat{s}_{j,t} = \sum_{j \in C} s_{j,t} \frac{\exp(\eta \Pi_{j,t})}{\sum_{j \in C} \exp(\eta \Pi_{j,t})}
\]  

(25)

\[
\hat{l}_{j,t} = \sum_{j \in C} l_{j,t} \frac{\exp(\eta \Pi_{j,t})}{\sum_{j \in C} \exp(\eta \Pi_{j,t})}
\]  

(26)

Note that the choice of memory parameter is also dependent on the rationality \( \eta \) of agents.

2.4 Timing, Price Discovery and Bankruptcies

Each period \( t \) in the model represents a trading day in which all agents forecast the return of the following period \( t + 1 \), make a decision on how much debt they want to hold and ultimately trade. Some agents furthermore revise or change their trading strategy. The order in which the mentioned actions take place is fixed and schematized in Figure 1. For simplicity we choose a price discovery process that can be described as Walrasian tâtonnement where all agents trade at the market clearing price. We compute this price numerically by solving for the price \( p_t \) in Equation (7) for which

\[
\sum_{j=1}^{J} D_{j,t}(p_t) = 0.
\]  

(27)

In a model with leverage there is always the possibility of bankruptcy, i.e. the equity capital \( E_{j,t} \) of an agent becomes smaller or equal to zero. This possibility needs to be taken into account by introducing a resolution procedure for bankrupt agents. We force bankrupt agents to liquidize all the assets they hold on their balance sheet upon bankruptcy. Technically the demand function from Equation (7) changes to \( D_{j,t} = -Q_{j,t} \) when \( E_{j,t} \leq 0 \), i.e. all assets of a bankrupt agent are thrown on the market regardless of the execution price. Bankruptcies can thereby impose a fire sale externality on the market. The bankrupt agent then disappears from the market and all losses are borne by the financier.
3 Simulations

For simulating the model described in the previous section, we have to define parameter values and initial conditions. Quite a few parameters including rationality and memory relate to behavioral aspects of market participants and are therefore not directly observable. Since we mainly aim at deriving qualitative results and calibration of complex agent based models pose a considerable challenge (cp. Winker et al. (2007)), we refrain from trying to estimate the behavioral parameters for our model. The choices for parameter values are therefore often without deeper economic meaning. In the exemplary simulation presented in the following subsection, we introduce some of the dynamics the model features with the parameters and initial conditions documented in the appendix, in Tables 2 and 1 respectively. For simulations contained in Sections 3.2-3.4 we make changes to selected parameters in order to analyze their qualitative (ceteris paribus) effect on the model economy. Because our model incorporates random terms at several instances, each simulation result is unique. In fact, simulation outcomes display strong path dependence. In order to make sure that the patterns that emerge in

--12Specifically, this includes the noise $\epsilon_t$ in the expectation process of the fundamental traders. The exact values for the moving average lags are randomly drawn for each agent from a specific distribution. The same holds true for the value $\rho_{j,t}$ determining the absolute value of chartists’ return expectations. Moreover, when agents decide whether to change or stick to their strategy their profitability determines the probability for a change, which of course implies some randomness. Last but not least, the frequency $\tau_j$ with which agents revise their strategy is assigned randomly at the beginning of each simulation.
our simulations are not caused by coincidence, we run 40 simulations\textsuperscript{13} for each parameter value and plot the median result if not stated otherwise.

### 3.1 Exemplary Simulation

We define the process of fundamental value evolution as a noise process with a trend and - in order to emulate upswings and downturns - mean reversion.\textsuperscript{14} The initial endowment of all \( N = 500 \) agents is the same: the balance sheet total of each agent amounts to \( B_{j,0} = 2/N \), and each agent holds the amount of risky assets that leads to an optimal portfolio when expecting the return to be equal to the trend of the fundamental value process. Agents target a leverage ratio of \( \lambda = 25 \), which is broadly consistent with empirical data for Investment Banks. At \( t = 0 \) the passive side of the balance sheet is constructed in order to satisfy a leverage of \( \lambda = 25 \). The constraints imposed by the credit supply of the financier, however, causes the leverage of agents to drop substantially in the first period. We make the simplifying assumption that \( i = R_C = 0 \) and thereby completely abstract from the effect of interest rates in this paper. Initially, chartists and fundamentalists account for 50\% of traders respectively. The specific frequency \( \tau_j \) with which each agent revises his strategy is initially drawn from a uniform distribution with limits of 1 and 250, which means that agents revise their strategy at least once every trading year (one simulation period represents one trading day) and at the most every trading day. For the chartist strategy the boundaries of the moving average lags \( l_{j,t} \) and \( s_{j,t} \), which are initially drawn from a uniform distribution, are set to 1 and 200, which are common values in business practice (see e.g. Lo et al., 2000). In the benchmark simulation, we set the credit friction parameter to its maximum, i.e. \( \mu^O = 1 \), allowing agents and financiers to make immediate changes to the amount of debt they hold on their balance sheet or provide as credit respectively.

Figure 3.1 shows some results of an exemplary benchmark simulation. It can be observed that the price diverges from fundamental value on a regular basis\textsuperscript{15} Nevertheless, the model is stable with only a single default occurring.

\textsuperscript{13}Simulations where all agents become bankrupt are repeated. It is thus possible that the results documented in following subsections contain a sort of survivorship bias.

\textsuperscript{14}Formally, this is modeled by an Ornstein-Uhlenbeck process, where we set the daily expected return to \( \mu^O = 0.05 \), the volatility to \( \sigma^2_f = 0.01 \), and mean reversion speed to \( \theta = 0.1 \). The model is initialized by setting \( p_0 = f_0 = 0 \).

\textsuperscript{15}A shortcoming of the model is the apparent smoothing of the price resulting in first-order autocorrelation. This problem could be tackled by introducing a short term arbitrageur specialized on trading on this anomaly (LeBaron, 2010). However, since the analysis of return time series is not a focus of our model, we abstain from introducing further agent types, which would increase the complexity of the model.
Periods of strong mismeasurement seem to go along with a larger proportion of chartist traders in the market and more specifically with a larger proportion of trend-followers, which we can measure by \( \hat{l}_{j,t} - \hat{s}_{j,t} > 0 \). On the other hand, when contrarians \( \hat{l}_{j,t} - \hat{s}_{j,t} < 0 \) dominate the population of chartists the price is closer to the fundamental value. This is the case because a trend following strategy amplifies the prevalent trend, while the contrarian strategy elicits a negative feedback. Trading volume, which will play a prominent role in Section 3.4, fluctuates strongly in the exemplary simulation but also seems to contain a persistent component. Time periods of relatively high volume alternate with periods of relatively little trading.

\[ \text{Note that the unusually high trading volume in the first 200 periods can be attributed to the fact that chartist traders do not have enough memory to correctly employ their moving average method with maximum time horizon of 200 periods.} \]
Figure 3 shows the dynamics of the mean balance sheet total as well as mean leverage. As stated before, all agents are initially equal. However, the initial homogeneity changes quickly as simulation time progresses. As the plotted quantiles illustrate, substantial differences between agents develop. The nature of how these differences evolve in terms of balance sheet size will be addressed in the upcoming section. Noteworthy is also the apparent comovement of mean leverage and mean balance sheet total, which will be addressed in Section 3.3.

![Figure 3](image)

(a) Balance sheet total  (b) Leverage

Figure 3: Mean and quantiles of agents’ balance sheet total and leverage for the exemplary simulation

### 3.2 Distribution and Leverage

Distributions of e.g. wealth, income or output constitute an emergent property of the economy and can reveal valuable information about its state. Because redistribution is often an explicit goal of economic policy it is important to understand the process which leads to the observable distribution. Agent based models can be helpful in this regard. In the following we will take a look at the distribution of agents’ balance sheet size, which endogenously evolves when simulating our model.

As stated, we initially assume that all agents are of equal size and thereby homogeneous. In the simulation however the distribution converges to a stable log-normal distribution. This result is presented in Figure 4 showing that the Jarque-Bera statistic (testing for the normality of logarithmic balance sheet size) converges to a value lower than the critical value given a
5% significance level. The most convincing argument for the emergence

Figure 4: Jarque-Bera test statistics for the log-balance sheet size distribution in the dynamic simulation

of a log-normal distribution for balance sheet size in our model is given by Gibrat’s law, which states that convergence to lognormality occurs when balance sheet growth is normally distributed and independent of size. Figure 5(a) shows the emerging distribution for the exemplary simulation of the previous subsection. For comparison, Figure 5(b) shows the distribution of an international sample of investment banks. The distributions qualitatively resemble each other as is also confirmed by the Jarque-Bera test for log-normality. The test statistics are provided in the appendix in Table 3.

\footnote{Note that we suppressed the first 200 periods due to the fact that initially we assume that all agents are equal leading to extremely high test statistics. Furthermore, we take the median of the simulation results to control for extreme outliers.}

\footnote{If we assume \( x_t - x_{t-1} = g_t x_{t-1} \) for small values for the growth rate \( g_t \), the function converges to \( \log x_t = \log x_0 + g_1 + g_2 + \cdots + g_t \) implying a log-normal distribution (Sutton, 1997).}

\footnote{Here we use annual balance sheet data of international investment banks from the Bankscope database.}

\footnote{As presented in Janicki and Prescott (2006), this result does not hold for commercial banks, which can rather be described by a Pareto distribution. A theoretical rationale can be found in their business model and in a product differentiation argument: regional banks provide credit to regional small and medium sized enterprises. The non-log-normal distribution of non-financial firms (cp. Axtell, 2001) therefore is also reflected in the distribution of commercial banks (Ennis, 2001).}
When looking at the average evolution of balance sheets throughout simulations (see Figure 5), we observe a decreasing trend of mean (log) balance sheet size while the variance steadily increases. Furthermore, the size dispersion of balance sheets, which we measure with the coefficient of variation (i.e., $\sigma/\mu$), increases, which is indicative of an endogenous increase of inequality with progressing simulation time. Effectively, our model suggests that the financial system naturally generates a large number of small institutions and small number of large institutions. Through time, the small get smaller and the large get larger. There is thus a natural tendency for the system to produce institutions that are too big to fail.

---

21 The coefficient of variation provides an inequality measure insensitive to changes in the mean (Cowell, 2000).
Leverage seems to play an interesting role in the evolution of balance sheet distribution. In order to analyze this role we replace the debt supply function of the risk managing financier with unlimited debt supply, while agents keep aiming for a constant leverage $\lambda$. By varying the target leverage for all agents from $\lambda = 0-15$, we can now control for the overall leverage in the model economy. Note that we only provide simulations up to a leverage target of $\lambda = 15$ rather than the more realistic target value of $\lambda = 25$ in the benchmark simulation. In the framework without the stabilizing financier the model market becomes highly fragile for large values of $\lambda$.

As shown in Figure 7(a), our model displays a positive relationship between leverage and size dispersion. Theoretical studies discussing the effects of distribution functions frequently argue with the entry and exit mechanisms in markets. In our simple model, we do not account for entries and exits are only possible through bankruptcies (as opposed to mergers or voluntary liquidation). In this regard, the extreme ascent of inequality observed in our model for values of $\lambda > 13$ may be attributed to the effect of bankruptcies. The linear trend for low values of $\lambda$ (see Figure 7(b)) is followed by a strong non-linear behavior, especially for $\lambda > 13$. Possibly, this is brought about
by defaulting agents and the associated complex market dynamics resulting from fire sales. As shown in Figure 10(c), bankruptcies strongly increase for values of $\lambda > 13$.

Despite the irregularities observed for high values of $\lambda$ in the model, the quintessence of Figure 7(a) is that leverage seems to foster the natural evolution towards higher inequality described above. This conclusion may be of importance for policy makers. In this context, the introduction of a leverage ratio into financial market regulation may not only help to stabilize the financial system in a more traditional sense (lower leverage decreases the probability of default), but could also decrease the speed with which inequality increases. Lower size dispersion arguably generates less institutions that classify as too big to fail.

A first glance at our sample of international investment banks seems to support the notion that high leverage increases inequality. In Figure 8 we plot the average leverage of investment banks from the end of the Dot-Com crisis in 2002 up to 2009. The average increase of leverage between 2002 and 2008 is accompanied by an increase in size dispersion, as predicted by our model. The significant drop in average leverage from 2008 to 2009, on the other hand, is reflected by a sharp decline in size dispersion. Note, however, that with the data available to us we cannot make inferences about causality of the observed relationship.
Although there exists some empirical evidence suggesting that banks, as the agents in our model, do target a constant leverage (see e.g. Gropp and Heider, 2010), an unlimited supply of debt is certainly not a realistic assumption. When looking at the following effects of leverage on our model financial market, it should be kept in mind that a constrained debt supply may lead to less clear or even different results. Nevertheless, we briefly want to show some interesting patterns emerging in simulations in the context of varying leverage targets. As most of these patterns are empirically untested, further research is needed before meaningful conclusions can be reached.

Figure 9(a) shows an emerging positive relationship between leverage and trading volume. Here leverage acts as a multiplier to trades: A higher leverage target causes agents to acquire or dispose of larger sums of nominal debt in order to meet their target as the value of the risky asset on their balance sheet rises or falls respectively. Because debt is obtained and repaid in cash, any changes of agents’ nominal debt also changes the composition of agents’ balance sheets. The rebalancing of portfolios generates trading volume which therefore increases as the leverage target is raised.

Increased trading activity translates into a higher return volatility as can be observed in Figure 9(b). Somewhat of a surprise, however, is that the increased volatility goes along with higher price efficiency (Figure 10(a)) - meaning that prices are more closely connected to their underlying fundamentals.\textsuperscript{22} The reason for this counterintuitive link is depicted in Figure

\textsuperscript{22}As proposed in Westerhoff (2008), we measure inefficiency as the median absolute difference between log-fundamental value and price: $ME = \text{median}(|f_t - p_t|)$. In a first-order approximation this can be interpreted as the percentage point deviation from fundamental value.
higher leverage leads to a greater average proportion of fundamental traders in the model market. Higher leverage means that agents operate with less relative equity capital, which is quickly depleted in downturns. In order to survive, it becomes increasingly important for agents to anticipate price movements. Here fundamentalists have the advantage. Figure 10(c) shows the number of bankruptcies for fundamentalists and chartists respectively. The number of defaulting chartists is always higher than the number of defaulting fundamentalists. The losses incurred by chartists have a larger impact with increasing leverage. Leverage, in our model, may thus help to stabilize the market. This emergent behavior of the model is reminiscent of the classical argument for the existence of efficient markets. Friedman already argued that in the long run speculative trading is not profitable and therefore eventually disappears. On the other hand, the observed efficiency gain is deceptive. Leverage strongly increases the risk of a breakdown of the entire model market. When too many agents default or experience losses at the same time, the fire sale of assets can lead to a positive feedback process triggering a debt deflation spiral as it was first described by Fisher. In essence, falling prices calls for agents to deleverage, which further suppresses prices and eventually leads to the collapse of the market. When conducting simulations we observe an increase in systemic risk with increasing leverage through the rising frequency of model-breakdowns due to the default of all agents.

More precisely the agents that form expectations using technical analysis prior to defaulting.
Figure 10: Market efficiency, composition, and stability for variation of target leverage ratio $\lambda$

### 3.3 Credit Frictions

Agents and financiers in our model actively manage their demand and supply of debt respectively. The immediacy with which desired changes to debt can occur is constraint by the credit-friction parameter $\mu^O$ in Equation (14), with a low value for $\mu^O$ implying high friction and vice versa. Frictions e.g. arise from the maturity structure of debt or from institutional characteristics of different bank types, which both restrict deliberate and immediate changes to the capital structure of agents. Credit-frictions thus have the potential of affecting the behavior of the financial system as a whole. To analyze the effects of credit frictions we first show how they affect the relationship between leverage and balance sheet size. Following the method of [Adrian and Shin (2010)](#), we scatter-plot the logarithmic changes of leverage against the loga-
rithmic changes of balance sheet size. Setting $\mu^O = 0$ means that agents and financiers have passive leverage strategies. The nominal debt agents are endowed with at the beginning of a simulation stays on their balance sheets while changes to the value of agents’ assets lead to a negative relation between leverage and total assets. This negative relationship, as plotted in Figure 11, can typically be observed in data from households (see Adrian and Shin, 2010). It seems however very unlikely that (professional) financial market participants would follow a completely passive leverage strategy. If we allow for slight leverage adjustment, the relationship between leverage and balance sheet size changes. A low value for $\mu^O$ implies that adjustments to agents’ debt levels are constraint and take time. Commercial banks e.g. face such constraints as customer deposits, which they cannot raise nor reduce at will, figure prominently on the liabilities side of their balance sheets. Figure 12(a) shows the leverage-balance-sheet-relationship for $\mu^O = 0.01$ while Figure 12(b) shows the relationship for commercial banks in EU27-states. Both graphs lack a clear positive or negative relationship. On the other hand, Adrian and Shin (2010) show that the relationship between leverage and balance sheet growth is positive, i.e. leverage is procyclical, for investment banks (see Figure 13(b)). Figure 13(a) exemplarily shows that our model produces a clearly positive relationship for values for $\mu^O$ between approximately 0.1

More precisely, logarithmic changes of balance sheet size are changes in logarithmic balance sheet size after 50 periods, i.e. $\log(B_{j,t}) - \log(B_{j,t-50})$. The same applies for leverage.

$\frac{\partial \lambda}{\partial B} = -\frac{O}{(B-O)^2} < 0$, with $O$ being the constant nominal value of debt, $B$ the balance sheet total and leverage being defined as $\lambda = \frac{O}{B-O}$. We use quarterly data between 1996 and 2009 from the Bankscope database.
and 1. In these cases there are little constraints on the adjustment of leverage. Investment banks tend to have very short term debt (e.g. Repos) on their balance sheets, which allow them or their financiers to quickly adjust the leverage to values they deem appropriate. This characteristic is reflected in the high values for $\mu$. The procyclical nature of leverage that enters the model with low credit frictions can be explained by the debt supply function of the financier: a persistent positive development of an agent’s balance sheet suggests to the financier that the agent is well informed, thus he perceives a lower risk level and is willing to supply more debt. On the other hand, when losses reduce the size of balance sheets, the financier will be more concerned about the safety of credit supplied to agents and will consequentially reduce his supply of debt.

Figure 12: The relationship between balance sheet total and leverage with high credit frictions

Figure 13: The relationship between balance sheet total and leverage with little credit frictions
The procyclicality of leverage induced by low credit frictions and the financier’s risk management affects the price behavior of the risky asset. Figure 14(a) shows a negative relationship between credit frictions and average return volatility. An increase in friction reduces the procyclicality of leverage and yields a lower average volatility. Empirical evidence supports this relationship: Adrian and Shin (2010) find that the growth rate of short term debt (Repos) on dealers balance sheets significantly forecasts changes in the Chicago Board Options Exchange Market Volatility Index (VIX). Our model suggests that return volatility could be reduced through an increase of credit frictions e.g. by curtailing the short term debt supply to agents. Short term borrowing has often been cited as a key contributor to financial instability and its curtailment has been called for on various occasions (for a treatment see e.g. Diamond and Rajan, 2001). However, as the next graph indicates, such regulation could result to be counterproductive. Figure 14(b) plots the average number of bankruptcies occurring over the 1000 periods of a simulation run for different values for the credit friction parameter $\mu^O$. The trend indicates that the average number of bankruptcies increases with increasing credit frictions. Thus, the active management of debt levels, which can be accomplished best in an environment with few credit frictions, is essential for the stability of our model financial market.

We conclude that through changes to the credit friction parameter $\mu^O$ our model is able to reproduce the empirically observed relationship between leverage and balance sheet size of different bank types. Similar to the behavior observed for investment banks, when credit frictions in our model

27To show a clearer graph, the first observation for $\mu^O = 0$ has been omitted. At $\mu^O = 0$ the average number of bankruptcies amounts to 280 agents, which is more than half of all agents.
are small, leverage becomes procyclical. Supported by empirical evidence, procyclical leverage increases the return volatility of the model’s risky asset. When judging the stability of the model’s financial system through return volatility, credit frictions are beneficial to stability, whereas the opposite must be concluded when model bankruptcies are observed.

3.4 Rationality, Disagreement and Trading Volume

Models that operate under the representative rational agent paradigm are unable to take into account issues related to trading volume. It has however been advocated that when searching for the origins of mispricings, looking at the joint behavior of prices and trading volume can yield interesting insights (cp. Hong and Stein 2007). In the following we consider how changes to rationality and disagreement in our model affect price efficiency and trading volume.

In our model the (bounded) rationality parameter $\eta$ controls agents’ aptness to identify the most successful trading strategy, which focuses on either the risky asset’s fundamental value or on technical analysis. Figure 15(a) shows that the average proportion of fundamentalists in the market increases for higher values of $\eta$. A large proportion of fundamentalists in the market leads to an improved alignment of the price with fundamental value, i.e. the market becomes more efficient (see Figure 15(b)). This, in turn, reduces fundamentalists’ average forecast error (see Figure 15(c)), which makes them more confident of their return expectations. A market with confident fundamentalists can withstand the price pressure emanating from the "irrational" demand or supply of chartists, making the market persistently more efficient. This is the classical story of efficient markets, where the sophisticated trader instantaneously corrects any mispricings (cp. e.g. Fama 1970). The chartists on the other hand are loosing confidence, as they are no longer able to drive the market into their forecasted direction. As the proportion of fundamentalists increases and the confidence-wedge between chartists and fundamentalists becomes larger, the market seems more homogenous. This is also reflected by a decrease in average trading volume for increasing values for $\eta$ as depicted in Figure 15(d). There is a large body of theoretical and empirical work supporting the relationship between heterogeneous believes and trading volume (see e.g. Ziebart 1990, Ajinkya et al. 1991, Harris and Raviv 1993).

While increasing "rationality" in our model leads to a more homogenous market through an increasing number of fundamentalists and their higher average confidence, we can also analyze the impact of (dis)agreement within the fundamentalist group. This is done by varying the parameter $\theta^F$, which in Equation (10) defines the speed at which fundamentalists correct past
evaluation errors. The higher the speed the weaker will the respective perceptions of fundamental value deviate from each other, i.e. the stronger will be the agreement among fundamentalists.

Somewhat surprising, higher agreement on average leads to a slight reduction of the proportion of fundamentalists in the market, as seen in Figure 16(a). However this reduction stays within approximately half a percentage point. Nevertheless, price inefficiencies are significantly reduced as perceptions of fundamental value converge (see Figure 16(b)). Consequentially fundamentalists’ confidence in their forecasts rises as indicated by the fall in forecast errors displayed in Figure 16(c). On the other hand chartists’ confidence remains approximately constant. A remarkable effect of higher agreement among fundamentalists is the relationship between agreement and trading volume shown in Figure 16(d). The positive correlation between trading volume and agreement among fundamentalists seems to contradict the above cited theoretical and empirical work on the issue. The reason behind this relationship in our model can be explained as follows: for low values of
the disagreement between perceptions of fundamental value is more persistent. Past valuation errors can e.g. lead an agent to persistently believe that an asset is overvalued, which gives the agent little reason to make fast changes to his portfolio. If, on the other hand, fundamentalists agree that the current price of an asset is close to its fundamental value, each new piece of information can tip the asset from being overvalued to being undervalued and vice versa. Such changes in valuation requires the agent to rebalance his portfolio. The frequent rebalancing of portfolios as well as the high confidence level of the average fundamentalist generates growing trading volume through increased agreement. It is however unclear if the portrayed relationship between trading volume and (dis)agreement bears some semblance to reality or simply constitutes an artifact of the assumptions in our model.

Figure 16: The effects of agreement on the model market
4 Conclusion

In this paper we develop an agent based model of the financial market where agents are endowed with balance sheets that contain equity capital as well as debt. Via simulations we are able to analyze several aspects of the financial system that are mostly inaccessible with conventional economic models. Several results are reported: we show that the distribution of agents balance sheet size evolves endogenously to an approximately log-normal distribution, which is typically observed for banks and especially for investment banks. With progressing simulation time our model inherently produces higher inequality, i.e. an increasing proportion of agents are becoming smaller, while a small number of agents exhibit extraordinarily large balance sheets. Leverage results to have a substantial effect on the evolution of balance sheets, with high leverage leading to high inequality and vice versa. The relationship between leverage and balance sheet size can exert a significant influence on the dynamics of financial markets. In our model the nature of this relationship depends on credit frictions, which we define as the latency with which agents can acquire and dispose of debt. When agents follow a passive debt strategy the relationship between leverage and balance sheets is negative, as observed for households. On the other hand, low credit frictions cause leverage to display the procyclical behavior observed for investment banks. In accordance with empirical findings, our model shows that procyclical leverage increases return volatility. At a first glance our model therefore suggests that higher credit frictions could help tame financial markets. However, the average number of defaults increases with higher frictions, which is certainly not a sign of financial stability. When looking at the level of rationality and (dis)agreement inscribed into our model, we can detect seemingly contradictory effects on trading volume. Both increasing rationality and agreement among fundamentalists on the valuation of an asset lead to a more efficient market. Nevertheless, while the relationship between rationality and trading volume is negative, it is positive for agreement and trading volume. The positive relationship is caused by a frequent rebalancing of portfolios as fundamentalists’ perceptions of an asset’s fundamental value increasingly coincide with its market price. The probability thereby increases that any new piece of information can lead to a change in an asset’s assessment from previously being overvalued to currently being undervalued and vice versa. This, in turn, necessitates a rebalancing of portfolios which generates trading volume.

The investigations conducted in this paper represent just a small subset of feasible investigations. The effects of agents’ memory or the impact of short selling constraints are among the aspects that have not been considered. The propagation of external shocks is another issue that could be assessed
with the model framework presented in this paper. However, the potential usefulness of the model is constrained most notably by a lacking calibration and insufficient validation. While a more thorough validation of the model’s qualitative predictions would clarify where the model can reveal valuable insights, a calibration of the model would make it more attractive for policy considerations. Future work in this regard could prove to be exceptionally beneficial.

References


## Appendix

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<td>$\theta_{FE}$</td>
<td>Memory for forecast error</td>
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<td>$\theta^F$</td>
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Table 1: Benchmark simulation parameters
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financier</td>
<td>$\mu_{j,0}$ Initial estimator for adjusted balance sheet growth</td>
<td>$r_{j,t}^B$</td>
</tr>
<tr>
<td></td>
<td>$z_{j,t}^2$ Initial estimator for volatility of adjusted balance sheet growth</td>
<td>$(\sigma_{j,0}^{FE})^2 = 0.05$</td>
</tr>
<tr>
<td>Portfolio composition</td>
<td>$(\sigma_{j,0}^{FE})^2$ Initial forecast error for all agents</td>
<td>$0.05 \left( \frac{\Delta}{5} \cdot \sigma_f^2 \right)$ ( \overset{\Delta}{=} 5 \cdot \text{Var}(\Delta f) )</td>
</tr>
<tr>
<td>Chartist trading</td>
<td>$\zeta_{j,0}^2$ Price volatility estimator of chartists</td>
<td>$0.01 \overset{\Delta}{=} \sigma_f^2$</td>
</tr>
<tr>
<td></td>
<td>$s_{j,0}/l_{j,0}$ Length of short and long moving average</td>
<td>Drawn from uniform distribution with the limits of 1 and 200</td>
</tr>
</tbody>
</table>

Table 2: Benchmark simulation initial conditions

<table>
<thead>
<tr>
<th>Year</th>
<th>Jarque-Bera test statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>5.06</td>
<td>11.53</td>
</tr>
<tr>
<td>1997</td>
<td>9.99</td>
<td>11.16</td>
</tr>
<tr>
<td>1998</td>
<td>4.74</td>
<td>10.90</td>
</tr>
<tr>
<td>1999</td>
<td>5.66</td>
<td>10.85</td>
</tr>
<tr>
<td>2000</td>
<td>5.84</td>
<td>10.86</td>
</tr>
<tr>
<td>2001</td>
<td>4.45</td>
<td>10.85</td>
</tr>
<tr>
<td>2002</td>
<td>2.44</td>
<td>10.99</td>
</tr>
<tr>
<td>2003</td>
<td>2.85</td>
<td>11.05</td>
</tr>
<tr>
<td>2004</td>
<td>2.34</td>
<td>11.10</td>
</tr>
<tr>
<td>2005</td>
<td>0.74</td>
<td>11.17</td>
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<tr>
<td>2006</td>
<td>0.27</td>
<td>11.20</td>
</tr>
<tr>
<td>2007</td>
<td>0.03</td>
<td>11.27</td>
</tr>
<tr>
<td>2008</td>
<td>0.79</td>
<td>11.42</td>
</tr>
<tr>
<td>2009</td>
<td>0.14</td>
<td>11.35</td>
</tr>
</tbody>
</table>

Table 3: Jarque-Bera test statistics computed for the log-balance sheet size of OECD investment banks (Bankscope data).