The Impact of Capital Measurement Error Correction on Firm-Level Production Function Estimation

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April 2012

Abstract

Estimation of TFP and returns to scale hinges on the first-stage correct indentification of the underlying production functions. We argue that production function estimation is significantly affected by measurement errors. In particular, we show that measurement error in capital sizeably impacts its coefficient estimate. Using a large panel of Czech manufacturing firms in 2003–2007, we estimate firm-level production functions using the Levinsohn and Petrin (2003) and Wooldridge (2009) approaches, correcting for the measurement error in capital. As our labor and material inputs are measured in physical rather than financial units, we are able to pin down the effect of various measurement issues of capital, including deflator choice. Our results suggest that the capital coefficient estimate approximately doubles (depending on the particular industry) when we control for capital measurement error. Consequently, while the majority of industries exhibit constant or (in)significantly decreasing returns to scale when the standard methods are used, increasing returns cannot be rejected in some industries when the estimation is corrected for capital measurement error.

JEL Codes: C23, C33, D24, C18, O47.

Keywords: Measurement error, capital, firm-level data.

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This research was supported by Czech National Bank Research Project No. D3/09. We thank Adam Geršl, Gábor Kátay, Jan Kmenta, and Jeffrey Wooldridge for comments and the Czech Statistical Office for providing the data. All errors and omissions are ours. The views expressed here are those of the authors and not necessarily those of the Czech National Bank or CERGE-EI.

1. Introduction

Relating aggregate output to productivity (TFP) and production factors is the basis for understanding the sources of economic growth, while estimates of returns to scale across industries have important policy implications.[1] Estimation of TFP and returns to scale hinges on the first-stage correct indentification of the underlying production functions. At the microeconomic level, however, estimating firm-level production functions is a non-trivial exercise owing to simultaneity bias caused by the relationship between unobserved productivity shocks and inputs used in production.

A number of methods have been developed to address the simultaneity bias in production function estimation. While Blundell and Bond (2000) use method of moments techniques, other approaches rely on finding proxy variables for productivity shocks, which are used to invert out productivity from the regression residual in a two-step estimation (Olley and Pakes, 1996; Levinsohn and Petrin, 2003). Wooldridge (2009) proposes a one-step estimation implemented in a generalized method of moments framework.

Another problem in production function estimation is posed by measurement issues. While labor as a measure of production input is available in datasets used in the estimation of production functions, the true stock of capital is difficult to measure.[2] Capital is often recorded in acquisition (book-keeping historical) values that reflect neither the amount of capital used in current production nor its market valuation and capacity utilization. Levinsohn and Petrin (2003) as well as many other researchers use a kind of perpetual investment method where the capital is derived from book-keeping values and depreciation.[3] Another

[1] Returns to scale estimates vary considerably in the literature. In the United States, Basu and Fernald (1997) find constant or slightly decreasing returns to scale in a typical two-digit industry. Altug and Filiztekin (2002) find the existence of increasing returns in durable goods manufacturing industries. Increasing returns to scale in U.S. manufacturing industries are also found in Diewert and Fox (2008). They argue that the U.S. economic growth was driven by increasing returns to scale rather than technological progress in 1950-2000.

[2] Market valuation of capital is available only for publicly traded firms which can severely limit the sample and could be a source of sample bias. In addition, the implied value of capital is not a fixed number but an estimate with a standard error. Values of firms' capital are not constant over time but often quite volatile as the price of the firm changes. These problems are well documented for optimal investment decisions as the optimal investment strategy is often derived based on the equivalence of Tobin's marginal *q* and average *q* (e.g., Hayashi, 1982) which implies the use of yet additional assumptions, especially on the linear homogeneity of the production function. We therefore have to assume that the capital suffers from a measurement error irrespective of the capital recording or estimation method.

[3] The main problem in this approach is that the depreciation rate and the initial stock of capital are unknown; see Hernández and Mauleón (2002, 2005) for suggestions on how to estimate the stock of capital. Furthermore, Hájková (2008) shows that capital services better account for productive capital input in production than the

approach uses real capital as the stock of fixed assets deflated by the average deflator within industries (see, for example, Geršl, Rubene, and Zumer, 2007).[4] However, all these studies treat capital, after these adjustments, as correctly measured and recorded. We argue that capital is measured with an error which should be, and needs to be, addressed in production function estimation similarly as it has been done in labor studies.[5]

Gorodnichenko (2007, in his newer version of 2010) argues that standard inversion-based production function estimators (Olley and Pakes, 1996; Levinsohn and Petrin, 2003) yield inconsistent estimates because the large variation of input prices across firms does not allow for non-stochastic inversion of firm's input choice into firm's unobserved productivity. This type of measurement error due to the variation of input prices thus distorts estimates of productivity, leading to incorrect implications about relative productivity of firms as well as productivity differences across firms. He proposes a simple structural estimator that models the cost and revenue functions simultaneously and treats productivity and input price shocks symmetrically, and demonstrates its properties using Monte Carlo simulations. Using the same Chilean manufacturing data as in Levinsohn and Petrin (2003), Gorodnichenko shows that compared to standard Levinsohn and Petrin estimates, his estimator yields a sizeably higher coefficient estimate of capital and, consequently, increasing returns to scale cannot be rejected.

The literature on the estimation of Czech individual firm-level production functions is scant and the estimations of prodution functions are not the main goal of the research provided but rather a tool in a different analysis. For example, Lizal, Singer, and Baghdasarian (2001) estimate the production functions of Czech industrial firms in the mid-1990s as a by-product of the investment and labor adjustment cost function. They find that Czech industrial firms exhibit decreasing returns to scale.[6] Using the Cobb Douglas production function specification, Hanousek, Kočenda and Mašika (2012) employ a panel version of a stochastic

capital stock net of depreciation and that the net capital stock underestimates the contribution of capital input to production particularly in fast-growing Czech industries.

[4] Ornaghi (2006) shows that the use of common (industry-wide) price deflators leads to misleading results in the estimation of production function parameters.

[5] For example, Bollinger (2003) deals with measurement error in human capital and shows that correctly measured variables are also biased when proxy variables are used. Other studies control for bias caused by measurement errors in reported schooling in the estimation of returns to human capital (Ashenfelter and Zimmerman, 1997; Münich et al., 2005).

[6] Another use of an adjustment costs framework in investment is Lízal and Svejnar (2002). They analyze the investment behavior of firms with various types of ownership and legal status, however without controlling for measurement errors.

production frontier model for medium and large Czech firms in 1996-2007 to estimate the degree of firm efficiency and the effect of ownership structure on the distance from the efficiency frontier. They find that concentration and foreign ownership are positively related to firm efficiency. Their results are consistent with decreasing returns to scale.[7]

Individual production functions are also estimated in Geršl, Rubene, and Zumer (2007), who investigate the inflows of foreign direct investment into Central and Eastern European countries, focusing on the analysis of productivity spillovers. Using firm-level data on manufacturing industries for the period 2000–2005, they estimate the total factor productivity of domestic firms using the Levinsohn and Petrin (2003) approach. Kátay and Wolf (2008) construct a proxy for capacity utilization, allowing them to estimate firm-level total factor productivity that is clean of cyclical capacity utilization, and use these estimates in the decomposition of value added growth in Hungarian manufacturing industries in 1993–2004 into the contributions of primary inputs and total factor productivity growth.

Each production function for an individual firm is an approximation of an underlying production function around the point of current operation. Industries use different technologies and the individual firm technologies may have a different shape than the aggregate overall industry production function.[8]

In this paper, we correct for measurement error in capital in the estimation of production functions. We do so by using appropriate instruments for capital in the Wooldridge (2009) method. We also modify the Levinsohn and Petrin (2003) approach (LP hereafter) to estimating production functions in their TFP procedure, which is implemented in Stata (see Petrin, Poi, and Levinsohn, 2004), correcting for the measurement error in capital. Using a

[7] Returns to scale in individual manufacturing industries in Hungary and Bulgaria in 1995–2001 are estimated in Dobrinsky et al. (2008) and used in the estimation of mark-ups. In particular, constant returns are rejected for most manufacturing industries in Bulgaria in favor of decreasing returns and approximately for a half of industries in Hungary in favor of increasing returns. Dobrinsky et al. (2008) argue that the lower returns to scale in Bulgaria than in Hungary are consistent with the different transition paths of these two economies. They also find that small firms often operate with decreasing returns to scale.

[8] For an illustration of this feature, we refer the reader to Earnhart and Lizal (2006), and mainly Earnhart and Lizal (2011), who examine the link between production and pollution emissions from the perspective of the shape of the relationship and find that certain industries exhibit the commonly assumed linear dependence of emissions on production while other industries show a more complex pattern. In particular, both the metals sector and the energy sector enjoy economies of scale of emissions vis-à-vis production at lower production levels, while facing diseconomies of scale at higher production levels. In contrast, the chemicals sector encounters neither economies nor diseconomies of scale, with an apparent proportional relationship between emissions and production.

two-stage approach, we generate predicted values of capital in the first stage of the LP routine and use these predictions as the capital data input in the LP method together with the prediction error of the capital. We also modify the current LP non-parametric bootstrap used to obtain the standard errors of the coefficient estimates to account for the instrumental variable regression in the first stage. We demonstrate that measurement error correction significantly raises the coefficient estimates of capital, leading to a situation where increasing returns cannot be rejected in some manufacturing industries.

The paper is organized as follows. Section 2 describes the methodology, focusing on the LP and Wooldridge (2009) approaches and describing the correction in measurement error in capital. Section 3 describes the data, while in Section 4 we report the results. Section 5 concludes the paper.

2. Estimation Strategy

To illustrate the identification of production functions, let us consider a standard Cobb-Douglas production function (omitting firm subscripts)

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + \omega_t + \varepsilon_t, \qquad (1)$$

where y_t is the log of real value added (or revenue), k_t is the log of quasi-fixed input (real capital), l_t is the log of freely variable input (labor),[9] and ε_t is an iid error term. The productivity shock ω_t is unobservable to the econometrician but known to the firm, which decides on production and factor utilization. The unobserved productivity shock ω_t is therefore correlated with factor inputs, so that estimating (1) with ordinary least squares without controlling for ω_t yields biased parameter estimates.

The simultaneity problem can be solved using method of moments techniques (Blundell and Bond, 2000), which involve differencing. While differencing removes the unobserved individual productivity shock, it also removes much of the variation in the explanatory variables. In addition, Wooldridge (2009) shows that the instruments are weakly correlated with the differenced explanatory variables, leading to bias in finite samples. Other literature therefore focuses on finding proxy variables for productivity shocks and then uses the

[9] Given these assumptions, one could use the equality of the marginal product of labor and the price of labor (wage) as another identification restriction. However, if wages are set in bargaining process, the equality will not hold. Therefore, we, as the other literature, do not invole such restriction.

information in the proxies to invert out productivity from the residual. For example, Olley and Pakes (1996) use investment as a proxy for the unobserved productivity shock in a two-step estimation of production functions. On the other hand, Levinsohn and Petrin (2003) argue that many firms have zero-investment observations, leading to efficiency loss in the estimation using the Olley and Pakes approach, while non-convex adjustment costs may also affect the responsiveness of investment to the shocks. We also add that the firm may even wish to disinvest and such cases are not directly distinguishable from zero investment observations and one would need to employ an endogenous switching regression framework which would complicate the matters even more. As a solution, Levinsohn and Petrin still rely on a two-step approach, but use intermediate inputs such as materials or energy to invert out the unobserved productivity shock.

In the Levinsohn and Petrin approach, demand for the intermediate input is assumed to depend on the firm's capital k_t and the productivity shock ω_t:

$$m_t = f(k_t, \omega_t). \tag{2}$$

Under mild assumptions about the firm's production technology, Levinsohn and Petrin demonstrate that the intermediate demand function (2) is monotonically increasing in ω_t so that it can be inverted as

$$\omega_t = g(k_t, m_t). \tag{3}$$

The final identification restriction assumes that ω_t follows a first-order Markov process

$$\omega_t = E[\omega_t \mid \omega_{t-1}] + \xi_t, \tag{4}$$

where ξ_t is an innovation to productivity that is uncorrelated with quasi-fixed capital k_t, but not necessarily with labor l_t.

Petrin, Poi, and Levinsohn (2004) implement in Stata the method of Levinsohn and Petrin, based on third-order polynomial approximation of the unknown function in (3). Using (3), equation (1) becomes

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + g(k_t, m_t) + \varepsilon_t \tag{5}$$

or

$$y_t = \beta_1 l_t + \phi(k_t, m_t) + \varepsilon_t, \tag{6}$$

where

$$E(\varepsilon_t \mid l_t, k_t, m_t) = 0 \tag{7}$$

and

$$\phi(k_t, m_t) = \beta_0 + \beta_k k_t + g_t(k_t, m_t). \qquad (8)$$

In (6), a third-order polynomial approximation in k_t and m_t is substituted in place of Φ_t and the parameter β_l is estimated using ordinary least squares. This completes the first stage of the Levinsohn-Petrin routine.

In the second stage, the coefficient β_k is identified. First, estimated values of Φ_t are computed from (6) as

$$\hat{\phi}_t = \hat{y}_t - \hat{\beta}_1 l_t. \qquad (9)$$

Then for a candidate value β_k^* it is possible to calculate (up to a constant) a prediction of ω_t using

$$\hat{\omega}_t = \hat{\phi}_t - \beta_k^* k_t. \qquad (10)$$

A consistent non-parametric approximation to $E[\omega_t \mid \omega_{t-1}]$ is given by the predicted values from the regression

$$\hat{\omega}_t = \gamma_0 + \gamma_1 \omega_{t-1} + \gamma_2 \omega_{t-1}^2 + \gamma_3 \omega_{t-1}^3 + \vartheta_t \qquad (11)$$

which is called $\hat{E}[\omega_t \mid \omega_{t-1}]$. Given $\hat{\beta}_l, \beta_k^*$, and $\hat{E}[\omega_t \mid \omega_{t-1}]$, the estimate of β_k is defined as a solution to the minimization of the squared sample residuals

$$\min_{\beta_k^*} \sum_t \left(y_t - \hat{\beta}_1 l_t - \hat{\beta}_k^* k_t - \hat{E}[\omega_t \mid \omega_{t-1}] \right)^2. \qquad (12)$$

Finally, a bootstrap based on random sampling from observations is used to construct standard errors for the estimates of β_l and β_k.

Levinsohn and Petrin assume that given the quasi-fixed capital, the firm decides on labor and then, given the labor, determines the use of material input. On the other hand, Ackerberg et al. (2006) argue that decisions on labor l_t and intermediate input m_t are taken simultaneously, so that the approach of Levinsohn and Petrin suffers from collinearity problems. Given that (2) holds, labor may also be chosen as $l_t=h(k_t,\omega_t)$. While h is a different function than f, substituting (3) yields $l_t=h(k_t,g(k_t,m_t))=i(k_t,m_t)$. Labor is thus a function of capital and material input, invalidating the identification of the labor coefficient in the first step.[10]

[10] Ackerberg et al. (2006) propose an alternative approach that is still a two-step one, but unlike in Levinsohn and Petrin (2003), the production function parameters are identified in the second step.

Instead of a two-step approach, Wooldridge (2009) proposes to estimate β_l and β_k in one step. Given a production function (1), assume that the error term ε_t is uncorrelated with labor, capital, and material input as in (7), but also with all lags of these:

$$E(\varepsilon_t \mid l_t, k_t, m_t, l_{t-1}, k_{t-1}, m_{t-1}, ..., l_1, k_1, m_1) = 0. \tag{13}$$

Another assumption in Wooldridge (2009) is to restrict the dynamics of unobserved productivity shocks as

$$E(\omega_t \mid k_t, l_{t-1}, k_{t-1}, m_{t-1}, ...) = E(\omega_t \mid \omega_{t-1}) = j(\omega_{t-1}) = j(g(k_{t-1}, m_{t-1})), \tag{14}$$

where $\omega_{t-1}=g(k_{t-1},m_{t-1})$ is used. Now for productivity innovations a_t we can write

$$\omega_t = j(\omega_{t-1}) + a_t, \tag{15}$$

where

$$E(a_t \mid k_t, l_{t-1}, k_{t-1}, m_{t-1}, ..., l_1, k_1, m_1) = 0. \tag{16}$$

Variable inputs l_t and m_t are thus correlated with productivity innovations a_t, but capital k_t and all past values of l_t, m_t, and k_t are uncorrelated with a_t,. Substituting (15) and (14) into (1) yields

$$y_t = \beta_0 + \beta_l l_t + \beta_k k_t + j(g(k_{t-1}, m_{t-1})) + u_t, \tag{17}$$

where $u_t = a_t + \varepsilon_t$ and

$$E(u_t \mid k_t, l_{t-1}, k_{t-1}, m_{t-1}, ..., l_1, k_1, m_1) = 0. \tag{18}$$

To estimate β_l and β_k, we need to specify the functions g and j in (17). Similarly as Levinsohn and Petrin, we may consider low-degree polynomials in the function g of order up to three. In (15), we may assume that the productivity process is a random walk with drift, so that (15) becomes

$$\omega_t = \tau + \omega_{t-1} + a_t. \tag{19}$$

Plugging (19) and $\omega_{t-1}=g(k_{t-1},m_{t-1})$ into (1) yields

$$y_t = (\beta_0 + \tau) + \beta_l l_t + \beta_k k_t + g(k_{t-1}, m_{t-1}) + u_t, \tag{20}$$

where $u_t = a_t + \varepsilon_t$ and (18) holds.

Equation (20) with polynomials in k_{t-1} and m_{t-1} of order up to three approximating for the function g could be estimated using pooled IV, using k_t, l_{t-1}, m_{t-1}, k_{t-1}, and polynomials containing m_{t-1} and k_{t-1} of order up to three as instruments for l_t.[11] Given (16), this approach is robust to the Ackerberg et al. (2006) critique and unlike in Levinsohn and Petrin, bootstrapping is not required to obtain robust standard errors.

While value added, labor, and intermediate input are provided in the data for the identification of production functions, another problem is the measurement error in capital in equation (1), yielding biased production function estimates. In particular, the capital coefficient is attenuated toward zero (see Levinsohn and Petrin, 2003). Hence, we have to acknowledge that capital is measured with an error and one has to use a method that explicitly takes such data properties into account.

To account for the measurement error in capital, we modify the Levinsohn-Petrin routine in the first stage, where we use instrumental variable regression instead of ordinary least squares in (5), employing appropriate instruments for capital. In particular, given the iid measurement error e_t, the true values of capital $\hat{k}_t = k_t - e_t$ are obtained as predicted values from the OLS estimation of

$$k_t = \gamma_0 + \gamma_1 z_{1t} + \ldots + \gamma_N z_{Nt} + e_t, \tag{21}$$

where $z_{1t},\ldots, z_{Nt}$ are determinants (instruments) of capital and γ_0 is a firm-specific fixed effect. Equation (5) then becomes

$$y_t = \beta_0 + \beta_k \hat{k}_t + \beta_l l_t + g\left(\hat{k}_t, m_t\right) + \varepsilon_t, \tag{22}$$

where

$$E(e_t \mid \varepsilon_t) = 0. \tag{23}$$

When higher-order polynomials are used in place of g in (22), the first-step estimates in the Levinsohn and Petrin approach are not consistent.[12] However, this can be solved by using linear approximation of g, which we use in one set of our results.

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[11] This approach is used in Petrin and Levinsohn (2011). In fact, Wooldridge (2009) proposes to estimate equations (5) and (17) in a generalized method of moments framework as a two-equation system with the same dependent variable and with different sets of instruments. He argues that two-step estimators like Levinsohn and Petrin (2003) are inefficient because contemporaneous correlation in the errors across the equations is ignored and because serial correlation and heteroskedasticity are not efficiently controlled for.

[12] To see the point, consider $g = d_1(k_t - e_t) + d_2(k_t - e_t)^2 + d_3(k_t - e_t)^3$. Then $E(k_t - e_t)^2 \neq E^2(k_t - e_t)$ and $E(k_t - e_t)^3 \neq E^3(k_t - e_t)$ so that $\hat{k}_t$ cannot be used instead of k_t in the estimation of (5). With the linear approximation,

In the second stage, we use the predicted values of capital, so that (12) becomes

$$\min_{\beta_k^*} \sum_t \left(y_t - \hat{\beta}_1 l_t - \hat{\beta}_k^* \hat{k}_t - \hat{E}[\omega_t \mid \omega_{t-1}] \right)^2 . \tag{24}$$

Finally, we derive the standard errors of the coefficient estimates using a non-parametric bootstrap. While the Levinsohn-Petrin routine samples with replacement from firms and derives estimates of the standard errors from the variation in the coefficient estimates across the bootstrapped samples, we sample the observations from a distribution that reflects the uncertainty in the capital value. In particular, the capital values for each firm are drawn with 100 replications from a distribution $\hat{k}_t + \eta_t$, where $\hat{k}_t$ is the predicted capital (including the fixed effect) from the regression (21) and $\eta_t \sim N(0,\sigma_k^2)$. The parameter σ_k^2 is the firm-specific variance of predicted capital $\hat{k}_t$ obtained by bootstrap with 1,000 replications.[13]

In the Wooldridge (2009) approach, the correction for measurement error in capital is straightforward. In particular, we have to find appropriate instruments for capital k_t in (20). In the estimation, we use the same instruments for capital as in our modified LP approach.

3. Data Description

We estimate firm-level production functions for 2-digit NACE level manufacturing industries (excluding petroleum and refining) using a large panel of Czech manufacturing firms with 20 or more employees in 2002–2007 containing balance sheet and income statement information gathered by the Czech Statistical Office. While the dataset contains mainly financial variables, we complement the dataset with firm-level information on material consumption in physical units from the Czech Statistical Office. The advantage of our data compared to Levinsohn and Petrin (2003) and Wooldridge (2009) is that all intermediate inputs are

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$E(\hat{k}_t)=E(k_t-e_t)= E(k_t)$ as we assumed that $E(k,e)=0$. In such a case, the Levinsohn-Petrin approach thus may be used with $\hat{k}_t$ instead of k_t in (5).

[13] The sampling is thus performed twice. First, the firm-specific variance of the predicted capital is obtained, and, second, standard LP sampling is done where capital is randomly drawn from the distribution reflecting the firm-specific variance of the predicted capital.

reported in physical units so that there is no (even potential) problem with prices and deflating.[14]

Our sample covers economically active firms with non-zero electricity consumption and non-zero employment in each year and without organizational changes such as mergers and acquisitions. In the dataset, we imputed missing values as averages of adjacent observations.[15] The number of observations across industries and summary statistics are illustrated in Table 1. The real value added growth in manufacturing industries is displayed in Figure 1. It is derived from the sample as the weighted sum of year-on-year growth in firms' real value added.

---<Table 1 about here>---

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4. Estimation Results

When using balance sheets or other data, one has two competing options for calculating value added. The accounting measure is the sum of the firm's sales, stocks, and new investments minus intermediate inputs and sales and services costs. As the balance sheets contain undefined values for some variables, there is a certain portion of missing values. As an alternative, the value added may be defined as an economic proxy utilizing the firm's profit, depreciation, and wage bill. As the results do not differ qualitatively, we further limit ourselves to the precise accounting measure of value added described above. This is accompanied by 2-digit NACE deflators of value added obtained from the Czech Statistical Office.

The main contribution of our paper concerns the issue of capital measurement. Capital is defined as the sum of tangible and intangible assets at the beginning of the period, net of depreciation. In essence, as the capital is measured using historical book value, one has to account for measurement error. As the capital deflator we use the average inflation rate, or,

[14] The dataset used in the estimation is unbalanced, which accounts for firms' death and attrition. As firms' exit depends on their productivity, there is a sample selection bias when using balanced panels. Olley and Pakes (1996) show that using the full sample instead of the balanced panel leads to more plausible production function estimates.

[15] This accounts for about 6% of all the observations. Our results are robust when these observations are dropped from the sample.

alternatively, the interest rate of new borrowing, which reflects the cost of capital, to verify whether the definition of the discount factor matters in the estimation.[16]

As a freely available input factor for production, we use the number of hours worked. As a proxy for unobserved productivity shocks we use the consumption of electricity in physical units (MWh). Depreciation, the full-time equivalent of the average number of employees, and gas consumption in physical units are used as available instruments for capital. Thus, only the left-hand side variable (production or value added) and one explanatory variable (capital) are measured in monetary terms. Measurement error in the left-hand side variable is not a problem (see, e.g. Kmenta, 1997). Capital thus remains the only variable that can be measured with and error either due to deflation or due to record keeeping of capital (book values).

The results by industries in 2003–2007 are summarized in Table 2. The first estimation (column 1) uses the Wooldridge (2009) approach where real capital (deflated by the inflation rate)[17] is instrumented using depreciation, employment, and gas consumption in physical units as instruments. In column 2, the Wooldridge (2009) estimates are reported assuming that real capital is exogenous. Comparing columns 1 and 2, we see that correcting for capital measurement error significantly increases the coefficient estimate of capital as well as standard errors.

In column 3 we show the production function estimates using the LP method as implemented in Stata. In general, except for two industries (rubber and plastic products – NACE 25; other manufacturing – NACE 36–37) we do not observe a significant difference between columns 2 and 3. The estimation using Wooldridge (2009) thus yields similar results to Levinsohn and Petrin (2003), while the Wooldridge (2009) estimates are robust to the Ackerberg et al. (2006) critique. Without assuming the measurement error in capital, both methods thus yield quantitatively similar results.

---<Table 2 about here>---

[16] A significant amount of literature deals with the issue of using the right discount factor for capital; see, for example, Levinsohn and Petrin (2003).
[17] We also used the interest rate of new borrowing as an alternative capital deflator. The results are similar and are available from the authors on request.

In column 4 of Table 2, we use the LP method with correction for the measurement error in real capital. In particular, we estimate (21) using OLS and generate predicted values of capital that are then used as the capital data input to the LP method. The modified non-parametric bootstrap is employed to get corrected standard errors of the coefficients.

As in the Wooldridge (2009) approach (columns 1 and 2), we observe a major difference between columns 4 and 3 in Table 2 in all industries except for manufacture of wood (NACE 20–22); the coefficient associated with real capital often more than double, while the changes in the labor coefficient estimates are minor.[18] Based on our results using the Wooldridge (2009) and Levinsohn and Petrin (2003) approaches we see that measurement error in capital is a substantial problem that affects production function estimates. Not accounting for the measurement error in capital yields an estimate biased toward zero.

As we have shown in Section 2, using predicted values of real capital in the first stage of the LP routine with a higher order polynomial yields inconsistent estimates. We therefore repeat the estimation in columns 3 and 4 in Table 2, assuming linear approximation in place of the function g in equations (5) and (22). The results of this exercise are reported in columns 5 and 6 in Table 2. The difference in the coefficient estimates between columns 3 and 5 and between columns 4 and 6 is small in most industries, suggesting that measurement error in capital affects the estimates more than specific assumptions approximating the unknown function g in equations (5) and (22).

Our finding of a higher coefficient estimate of capital after accounting for the measurement error in capital resembles the results in Gorodnichenko (2010) who replicated estimates of standard production function estimation procedures, including the Levinsohn and Petrin, using the same Chilean dataset as in Levinsohn and Petrin (2003). His structural estimator accounts for measurement errors which may explain why he finds a significantly higher coefficient estimate of capital.

The correction for measurement error of capital affects returns to scale. Table 3 repeats the returns to scale estimates from columns 1–4 in Table 2 for manufacturing industries. While

[18] Similar results are obtained when using gas consumption as a proxy to invert out the unobserved productivity shock in the LP routine and electricity consumption as an instrument for real capital. These alternative results are available from the authors upon request.

most industries using the standard methods of Wooldridge (2009) and Levinsohn and Petrin (2003) exhibit constant or decreasing returns to scale (see columns 2 and 3 in Table 3), we cannot reject the presence of increasing returns in a number of industries when the estimation is corrected for measurement error in capital (columns 1 and 4). The difference in the results hinges on the correction of measurement error in capital, while the degree of the polynomial used in the estimation does not play a crucial role.

5. Conclusions

Based on our results we conclude that the measurement error of capital is a substantial problem that affects production function estimates. The estimated capital coefficient approximately doubles (depending on the particular industry) when we control for capital measurement error. The estimated standard errors of the coefficients naturally also increase when measurement error in capital is assumed, although the difference in the coefficients is so substantial that one can reject the identity of the coefficient with and without measurement error control. Consequently, while the majority of industries using standard Wooldridge (2009) and Levinsohn and Petrin (2003) estimation exhibit constant or (in)significantly decreasing returns to scale, measurement error correction sometimes leads to a situation where even increasing returns to scale cannot be rejected.

To sum up, we conclude that an estimation that ignores possible measurement error in capital might suffer from significant underestimation of the effect of capital on value added formation and that the contribution of capital to value added growth in Czech manufacturing industries was probably higher in 2003–2007 than based on estimates without controlling for measurement error in capital.

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Appendix

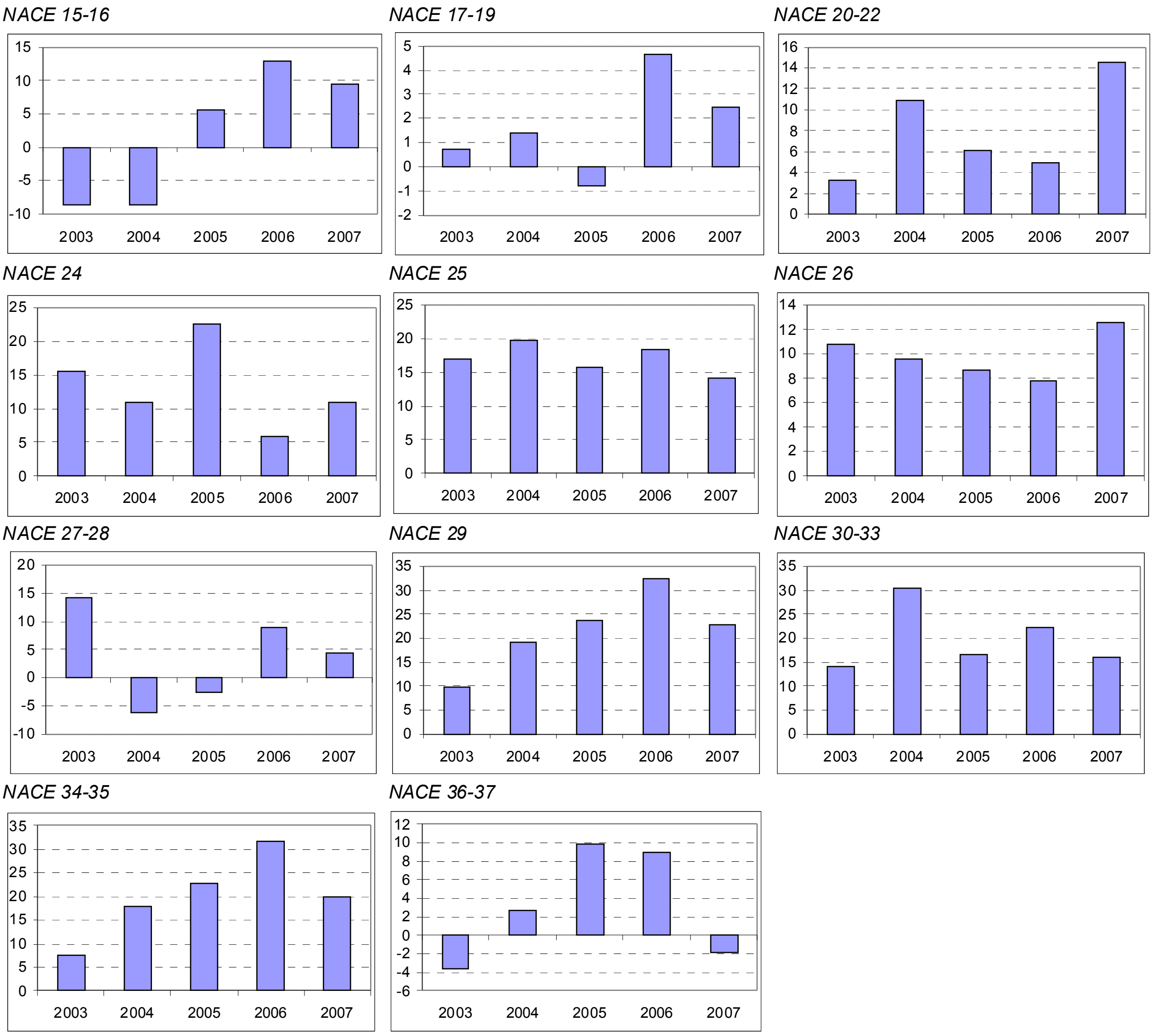
Table 1: Summary statistics

	N	Mean	Std. Dev.
Manufacture of food products, beverages and tobacco products (NACE 15–16)			
Log real value added	1510	10.384	*1.305*
Log hours worked	1510	12.164	*0.983*
Log capital	1510	10.709	*1.671*
Log real capital	1510	10.597	*1.667*
Log electricity consumption	1510	13.912	*1.453*
Log depreciation	1510	8.600	*1.624*
Log employment	1510	4.701	*0.972*
Log gas consumption	1510	12.319	*1.764*
Manufacture of textiles, wearing apparel and leather (NACE 17–19)			
Log real value added	829	10.316	*1.288*
Log hours worked	829	12.056	*1.073*
Log capital	829	9.709	*2.117*
Log real capital	829	9.599	*2.117*
Log electricity consumption	829	13.081	*2.102*
Log depreciation	829	7.638	*1.971*
Log employment	829	4.670	*1.070*
Log gas consumption	829	11.321	*1.865*
Manufacture of wood, pulp and paper, publishing and printing (NACE 20–22)			
Log real value added	620	10.468	*1.444*
Log hours worked	620	12.030	*1.025*
Log capital	620	10.415	*1.932*
Log real capital	620	10.302	*1.930*
Log electricity consumption	620	13.545	*2.107*
Log depreciation	620	8.334	*1.874*
Log employment	620	4.595	*1.021*
Log gas consumption	620	11.288	*2.183*
Manufacture of chemicals (NACE 24)			
Log real value added	444	11.443	*1.372*
Log hours worked	444	12.238	*1.031*
Log capital	444	11.364	*1.792*
Log real capital	444	11.247	*1.793*
Log electricity consumption	444	14.135	*2.355*
Log depreciation	444	9.295	*1.730*
Log employment	444	4.805	*1.043*
Log gas consumption	444	12.555	*2.273*
Manufacture of rubber and plastic products (NACE 25)			
Log real value added	613	11.192	*1.248*
Log hours worked	613	12.338	*1.029*
Log capital	613	10.885	*1.560*
Log real capital	613	10.771	*1.555*
Log electricity consumption	613	14.174	*1.690*
Log depreciation	613	8.924	*1.537*
Log employment	613	4.902	*1.030*
Log gas consumption	613	11.240	*1.737*

Table 1 (continued)

Manufacture of other non-metallic mineral products (NACE 26)			
Log real value added	728	11.197	*1.443*
Log hours worked	728	12.381	*1.094*
Log capital	728	11.183	*1.844*
Log real capital	728	11.068	*1.844*
Log electricity consumption	728	14.522	*1.900*
Log depreciation	728	9.053	*1.866*
Log employment	728	4.948	*1.091*
Log gas consumption	728	12.917	*2.420*
Manufacture of metals (NACE 27–28)			
Log real value added	1673	10.491	*1.240*
Log hours worked	1673	12.188	*1.056*
Log capital	1673	10.390	*1.813*
Log real capital	1673	10.278	*1.810*
Log electricity consumption	1673	13.928	*1.887*
Log depreciation	1673	8.363	*1.718*
Log employment	1673	4.754	*1.059*
Log gas consumption	1673	11.837	*1.855*
Manufacture of machinery and other equipment (NACE 29)			
Log real value added	1510	10.826	*1.231*
Log hours worked	1510	12.221	*1.044*
Log capital	1510	10.280	*1.732*
Log real capital	1510	10.167	*1.729*
Log electricity consumption	1510	13.335	*1.714*
Log depreciation	1510	8.299	*1.646*
Log employment	1510	4.771	*1.049*
Log gas consumption	1510	11.261	*1.712*
Manufacture of electrical and optical machinery and equipment (NACE 30–33)			
Log real value added	1250	11.012	*1.407*
Log hours worked	1250	12.310	*1.202*
Log capital	1250	10.213	*1.871*
Log real capital	1250	10.099	*1.870*
Log electricity consumption	1250	12.966	*1.944*
Log depreciation	1250	8.206	*1.902*
Log employment	1250	4.876	*1.213*
Log gas consumption	1250	10.889	*1.748*
Manufacture of motor vehicles and other transport equipment (NACE 34–35)			
Log real value added	669	11.584	*1.613*
Log hours worked	669	12.850	*1.265*
Log capital	669	11.459	*2.066*
Log real capital	669	11.342	*2.065*
Log electricity consumption	669	14.298	*1.956*
Log depreciation	669	9.493	*2.110*
Log employment	669	5.416	*1.269*
Log gas consumption	669	12.263	*1.792*
Manufacture of furniture, other manufacturing, recycling (NACE 36–37)			
Log real value added	622	10.152	*1.308*
Log hours worked	622	12.055	*0.958*
Log capital	622	10.175	*1.533*
Log real capital	622	10.064	*1.531*
Log electricity consumption	622	12.992	*1.525*
Log depreciation	622	8.007	*1.465*
Log employment	622	4.638	*0.977*
Log gas consumption	622	10.940	*1.647*

Figure 1: Real value added growth in manufacturing industries



Note: Weighted sum of 100*log(y(t)/y(t–1)), where weights are based on nominal value added within industries in a given year.

Table 2: Production function estimates in 2003–2007

	(1)	(2)	(3)	(4)	(5)	(6)
	Man. of food (NACE 15–16)					
Log hours	0.636***	0.686***	0.700***	0.690***	0.700***	0.687***
	[0.0403]	[0.0372]	[0.0348]	[0.0347]	[0.0323]	[0.0383]
Log real capital	0.578***	0.282***	0.301***	0.581***	0.348***	0.541***
	[0.122]	[0.0362]	[0.0721]	[0.103]	[0.0519]	[0.0994]
Observations	1510	1510	1510	1510	1510	1510
Firms	467	467	467	467	467	467
Returns to scale	1.214*	0.968	1.001	1.271**	1.048	1.228**
	Man. of textiles (NACE 17–19)					
Log hours	0.675***	0.553***	0.587***	0.586***	0.609***	0.607***
	[0.0576]	[0.0866]	[0.0885]	[0.0851]	[0.0958]	[0.0881]
Log real capital	0.609***	0.165***	0.156*	0.305***	0.264***	0.298***
	[0.185]	[0.0487]	[0.0796]	[0.101]	[0.0946]	[0.0967]
Observations	829	829	829	829	829	829
Firms	279	279	279	279	279	279
Returns to scale	1.284	0.718***	0.744**	0.891	0.872	0.904
	Man. of wood (NACE 20–22)					
Log hours	0.580***	0.606***	0.657***	0.640***	0.654***	0.639***
	[0.0737]	[0.0908]	[0.0971]	[0.0830]	[0.0900]	[0.0758]
Log real capital	0.697***	0.254***	0.260**	0.326***	0.315***	0.458***
	[0.144]	[0.0588]	[0.111]	[0.118]	[0.110]	[0.153]
Observations	620	620	620	620	620	620
Firms	201	201	201	201	201	201
Returns to scale	1.277*	0.859	0.917	0.965	0.969	1.097
	Man. of chemicals (NACE 24)					
Log hours	0.624***	0.574***	0.610***	0.608***	0.629***	0.619***
	[0.100]	[0.140]	[0.129]	[0.147]	[0.115]	[0.115]
Log real capital	1.997***	0.374***	0.465***	1.204***	0.424**	1.206***
	[0.561]	[0.0993]	[0.146]	[0.197]	[0.185]	[0.213]
Observations	444	444	444	444	444	444
Firms	120	120	120	120	120	120
Returns to scale	2.621***	0.948	1.075	1.812***	1.052	1.825***
	Man. of rubber (NACE 25)					
Log hours	0.548***	0.618***	0.642***	0.629***	0.644***	0.623***
	[0.0705]	[0.0671]	[0.0727]	[0.0701]	[0.0723]	[0.0584]
Log real capital	0.733***	0.290***	0.464***	0.601***	0.451***	0.610***
	[0.136]	[0.0798]	[0.0792]	[0.165]	[0.0805]	[0.152]
Observations	613	613	613	613	613	613
Firms	216	216	216	216	216	216
Returns to scale	1.281**	0.908	1.106	1.229	1.096	1.233
	Man. of other mineral products (NACE 26)					
Log hours	0.345***	0.392***	0.430***	0.421***	0.436***	0.425***
	[0.0514]	[0.0644]	[0.0606]	[0.0637]	[0.0601]	[0.0692]
Log real capital	0.803***	0.328***	0.265**	0.392***	0.297***	0.482***
	[0.191]	[0.0796]	[0.115]	[0.132]	[0.0948]	[0.143]
Observations	728	728	728	728	728	728
Firms	200	200	200	200	200	200
Returns to scale	1.148	0.72***	0.695**	0.814	0.733**	0.907

Table 2 (continued)

	(1)	(2)	(3)	(4)	(5)	(6)
	Man. of metals (NACE 27–28)					
Log hours	0.638***	0.664***	0.684***	0.680***	0.705***	0.700***
	[0.0398]	[0.0445]	[0.0430]	[0.0379]	[0.0404]	[0.0438]
Log real capital	0.575***	0.243***	0.247***	0.371***	0.228***	0.339***
	[0.104]	[0.0365]	[0.0551]	[0.0912]	[0.0596]	[0.0721]
Observations	1673	1673	1673	1673	1673	1673
Firms	592	592	592	592	592	592
Returns to scale	1.213**	0.906*	0.931	1.052	0.934	1.039
	Man. of machinery (NACE 29)					
Log hours	0.711***	0.812***	0.857***	0.849***	0.883***	0.874***
	[0.0452]	[0.0426]	[0.0517]	[0.0452]	[0.0438]	[0.0416]
Log real capital	0.633***	0.171***	0.185***	0.406***	0.193***	0.405***
	[0.108]	[0.0350]	[0.0363]	[0.0753]	[0.0422]	[0.0852]
Observations	1510	1510	1510	1510	1510	1510
Firms	502	502	502	502	502	502
Returns to scale	1.344***	0.983	1.041	1.255***	1.076	1.279***
	Man. of electrical and optical machinery (NACE 30–33)					
Log hours	0.728***	0.820***	0.845***	0.843***	0.868***	0.862***
	[0.0392]	[0.0485]	[0.0493]	[0.0453]	[0.0402]	[0.0396]
Log real capital	0.747***	0.172***	0.204**	0.336***	0.162*	0.344***
	[0.122]	[0.0437]	[0.0837]	[0.115]	[0.0886]	[0.0975]
Observations	1250	1250	1250	1250	1250	1250
Firms	367	367	367	367	367	367
Returns to scale	1.475***	0.993	1.049	1.179	1.03	1.206**
	Man. of motor vehicles (NACE 34–35)					
Log hours	0.642***	0.647***	0.719***	0.685***	0.717***	0.690***
	[0.0861]	[0.0812]	[0.0911]	[0.0794]	[0.0868]	[0.0788]
Log real capital	0.597***	0.13	0.174	0.576***	0.171	0.623***
	[0.176]	[0.0923]	[0.115]	[0.145]	[0.107]	[0.136]
Observations	669	669	669	669	669	669
Firms	192	192	192	192	192	192
Returns to scale	1.239	0.777**	0.894	1.261	0.888	1.314**
	Man. other (NACE 36–37)					
Log hours	1.112***	1.055***	1.093***	1.089***	1.101***	1.101***
	[0.0971]	[0.135]	[0.137]	[0.125]	[0.119]	[0.138]
Log real capital	0.758	0.140*	0.270**	0.752**	0.247**	0.837**
	[0.485]	[0.0783]	[0.135]	[0.321]	[0.118]	[0.363]
Observations	622	622	622	622	622	622
Firms	206	206	206	206	206	206
Returns to scale	1.87*	1.196	1.363**	1.841**	1.348**	1.937**

Notes: Standard errors in brackets, *** p<0.01, ** p<0.05, * p<0.1. Real value of capital (deflated by the average inflation rate). Returns to scale (log labor + log real capital) and significance level of Wald test of constant returns reported.

(1) Wooldridge (2009); real capital is instrumented using depreciation, employment, and gas consumption.
(2) Wooldridge (2009).
(3) Levinsohn-Petrin (2003).
(4) Levinsohn-Petrin (2003); real capital is instrumented using depreciation, employment, and gas consumption.
(5) Levinsohn-Petrin (2003); linear approximation used in (6).
(6) Levinsohn-Petrin (2003); real capital is instrumented using depreciation, employment, and gas consumption; linear approximation used in (6).

Table 3: Returns to scale in Czech manufacturing industries, 2003–2007

	(1)	(2)	(3)	(4)
Food products, beverages and tobacco products (NACE 15–16)	1.214*	0.968	1.001	1.271**
Textiles, wearing apparel and leather (NACE 17–19)	1.284	0.718***	0.744**	0.891
Wood, pulp and paper, publishing and printing (NACE 20–22)	1.277*	0.859	0.917	0.965
Chemicals (NACE 24)	2.621***	0.948	1.075	1.812***
Rubber and plastic products (NACE 25)	1.281**	0.908	1.106	1.229
Other non-metallic mineral products (NACE 26)	1.148	0.72***	0.695**	0.814
Metals (NACE 27–28)	1.213**	0.906*	0.931	1.052
Machinery and other equipment (NACE 29)	1.344***	0.983	1.041	1.255***
Electrical and optical machinery and equipment (NACE 30–33)	1.475***	0.993	1.049	1.179
Motor vehicles and other transport equipment (NACE 34–35)	1.239	0.777**	0.894	1.261
Furniture, other manufacturing, recycling (NACE 36–37)	1.87*	1.196	1.363**	1.841**

Notes: Returns to scale (log labor + log real capital) and significance level of Wald test of constant returns reported.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$.
(1) Wooldridge (2009); real capital is instrumented using depreciation, employment, and gas consumption.
(2) Wooldridge (2009).
(3) Levinsohn-Petrin (2003).
(4) Levinsohn-Petrin (2003); real capital is instrumented using depreciation, employment, and gas consumption.