Infrequent Fiscal Stabilization*

Yuting Bai† Tatiana Kirsanova‡
University of Exeter University of Glasgow

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Abstract

We study discretionary non-cooperative monetary and fiscal policy stabilization in the New Keynesian model, where the fiscal authority uses distortionary taxes as policy instrument. We explicitly model different frequencies of fiscal and monetary policy operations. We find that standard models of monetary and fiscal policy interactions may substantially overestimate the social gain from the stabilization. Dynamic complementarities between monetary and fiscal policies are greatly amplified if the fiscal policymaker acts infrequently. We demonstrate the existence of expectations traps, so the economy may experience high volatility of macroeconomic variables. We also find that, in some situations, credible discretionary policy may not exist, and a commitment device is required in order to deliver a socially acceptable outcome.

Key Words: Monetary and Fiscal Policy Interactions, Distortionary Taxes, Discretion, Infrequent Stabilization, LQ RE models

JEL References: E31, E52, E58, E61, C61

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†Address: School of Business and Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU; e-mail yb@exeter.ac.uk.
‡Address: Economics, Adam Smith Building, University of Glasgow, Glasgow G12 8RT; e-mail tatiana.kirsanova@glasgow.ac.uk
1 Introduction

In this paper we study interactions of monetary and fiscal policies in the Blanchard and Kahn (1980) class of infinite horizon non-singular discrete-time linear dynamic models that is typically used to study aggregate fluctuations in macroeconomics. We assess the importance of lower frequency of fiscal policy, which is using distortionary taxes as instrument, for monetary policy decisions and for the dynamics of the economy.

Fiscal and monetary policies operate at different frequencies. Monetary policy sets interest rate every month while fiscal decisions are often taken annually. Changes in fiscal policy, for example in taxation, are normally expected to be relatively long-lasting: the new tax rate is expected to stay for at least one year, sometimes longer, until the next government takes the office.

With longer fiscal cycle bigger fiscal adjustments are optimal; they impact more on monetary policy maker and escalate the conflict between the authorities if the fiscal policy instrument is distortionary tax. Indeed, monetary and fiscal policies are dynamic complements in the sense of Cooper and John (1988): the optimal response of the monetary policy maker reinforces the action of the fiscal policy maker. The task of debt stabilization requires an increase in the tax rate, with a cost-push effect on inflation. The optimal response of monetary policy is to generate a reduction in demand and thus in the tax base. Consequently, the bigger increase in the tax rate may be required in order to achieve the debt stabilization target. In standard quarterly models this reinforcement mechanism is weak; we demonstrate that it is greatly amplified if discretionary fiscal policy operates only infrequently.

We demonstrate that gains from monetary and fiscal policy stabilization of macroeconomic fluctuations can be greatly overestimated, if it is evaluated using models with frequent fiscal policy stabilization. We can fail to account for arising expectations traps (King and Wolman (2004)) with implications of excessive volatility of welfare-relevant economic variables; we can also fail to realize the necessity of policy commitment, as inherently-credible time-consistent policy may not exist.

We use the standard New Keynesian model with monopolistic competition and sticky prices to demonstrate the results. The economy is controlled by monetary and fiscal policy makers which act non-cooperatively at different frequencies. The monetary policy maker optimizes every period while the fiscal policy maker optimizes less frequently, choosing the distortionary tax rate once every several periods. After the tax rate is chosen, it stays at this level until the next fiscal optimization. The fiscal policy maker has an intra-period leadership: the monetary policy maker observes fiscal policy at every period, and the fiscal policy maker knows that the monetary policy maker optimizes every period and takes into account its reaction function when formulating policy.

We demonstrate the existence of expectation traps in case of longer fiscal cycles. However, we also demonstrate that agents might not be able to rationalize some or all of policy equilibria. In the latter case, the time-consistent policy cannot be found, and some form of policy commitment is required. In particular, we demonstrate that quick reduction in the stock of public debt is impossible without policy commitment.

Our model is sufficiently simple to obtain most results for frequent optimization in analytical form. However, with longer periods between the fiscal optimizations we have to rely on numerical
methods. Because the inability to find a solution numerically does not imply its non-existence, we pay particular attention to the role of numerical methods in getting our results.

The paper is organized as follows. In the next Section we present a model of monetary and fiscal policy interactions. Section 3 presents the general framework with infrequent stabilization. Section 4 discusses policy implications in three special cases: quarterly, biannual and annual fiscal stabilization. Section 5 concludes.

2 The Model

We consider the now-mainstream macro policy model, discussed in Woodford (2003), modified to take account of the effects of fiscal policy.\footnote{See e.g. Benigno and Woodford (2003).} It is a closed economy model with two policy makers, the fiscal and monetary authorities. Fiscal policy is allowed to support monetary policy in stabilization of the economy around the non-stochastic steady state.

The economy consists of a representative household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. All agents can observe and affect the accumulation of the real government debt. We assume that the fiscal authority faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes\footnote{We could use distortionary consumption taxes to finance the deficit. The transmission mechanism would be the same.} and by issuing one-period risk-free nominal bonds.

We assume that all public debt consist of riskless one-period bonds. The nominal value \( B_t \) of end-of-period public debt then evolves according to the following law of motion:

\[
B_t = (1 + i_{t-1}) B_{t-1} + PG_t - \tau_t Y_t,
\]

where \( \tau_t \) is the share of national product \( Y_t \) that is collected by the government in period \( t \), and government purchases \( G_t \) are treated as exogenously given and time-invariant. \( P_t \) is aggregate price level and \( i_t \) is interest rate on bonds. The national income identity yields

\[
Y_t = C_t + G_t,
\]

where \( C_t \) is private consumption. For analytical convenience we introduce \( B_t = (1+i_{t-1})B_{t-1}/P_{t-1} \) which is a measure of the real value of debt observed at the beginning of period \( t \), so that (1) becomes

\[
B_{t+1} = (1 + i_t) \left( B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t \right).
\]

Log-linearizing (3) yields

\[
b_{t+1} = \frac{B}{Y} b_t + \frac{1}{\beta} \left( b_t - \frac{B}{Y} \pi_t + \left( 1 - \frac{C}{Y} \right) g_t - \tau \left( \tau_t + y_t \right) \right),
\]
where $b_t = B \ln \left( \frac{B_t}{B_t} \right)$, $c_t = \ln \left( \frac{C_t}{C_t} \right)$, $\iota_t = \ln \left( \frac{1+\iota_t}{1+\iota_{t+1}} \right)$, $\tau_t = \ln \left( \frac{\tau_t}{\tau_{t+1}} \right)$, $g_t = \ln \left( \frac{G_t}{G_t} \right)$, $y_t = \ln \left( \frac{Y_t}{Y_t} \right)$ and letters without time subscript denote steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor $\beta = 1/(1+i)$. To make the presentation of our model particularly simple we assume $B = 0$, which eliminates the first-order effect of the interest rate and inflation on debt, and obtain the final version of linearized debt accumulation equation where we used the linearizes (2) to substitute out output:

$$b_{t+1} = \frac{1}{\beta} \left( b_t + (1-\tau) (1-\theta) g_t - \tau \theta c_t - \tau \tau_t \right), \tag{4}$$

where the parameter $\theta = C/Y$.\(^3\)

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford (2003), Sec. A.5) and marginal cost is a function of output and taxes. A log-linearization of the aggregate supply relationship around the zero-inflation steady state yields the following New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[ \left( \frac{1}{\sigma + \psi} \right) c_t + \frac{(1-\theta)}{\psi} g_t + \frac{\tau}{(1-\tau)} \tau_t \right] + \eta_t, \tag{5}$$

where $\kappa$ is the slope of Phillips curve, and $\sigma$ and $\psi$ are parameters of the private sector utility function. Cost push shock $\eta_t$ follows an autoregressive process.

The social loss is defined by the quadratic loss function\(^4\)

$$L = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \lambda c_t^2 \right), \tag{6}$$

while the monetary and the fiscal policy makers can have different policy objectives, $L^J = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t Q^J (\pi_t, c_t, \tau_t, g_t, b_t)$, $J \in \{M,F\}$. Each policy maker knows the laws of motion (4)-(5) of the aggregate economy and takes them into account when formulating the policy. The following assumption follows Clarida et al. (1999) and substantially simplifies the exposition of the model.

**Assumption 1 (policy instruments)** The monetary policy maker chooses consumption $c_t$ and then, conditional on subsequent optimal evolution of $c_t$ and $\pi_t$, decides on the value of interest rate that achieves the desired $c_t$ and $\pi_t$. The fiscal policy maker uses the tax rate $\tau_t$ as policy instrument and keeps government spending constant $g_t = 0$.

Apart from making the exposition clear, keeping fiscal spending constant allows us to concentrate on the particular transmission mechanism of monetary and fiscal policy.

Despite the simplicity of the model, finding time-consistent optimal policy is not trivial. Of course, the economy can be completely insulated against shocks if the two policy instruments are

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\(^3\)Because we work with one-period debt only, its proportion in the total stock of debt is not very large. We discuss implications of this assumption for policy in Section 3.

\(^4\)The criterion is derived under the assumption of steady state labour subsidy. Here parameter $\lambda$ is a function of model parameters, $\lambda = \theta \kappa / \epsilon$, and $\epsilon$ is the elasticity of substitution between any pair of monopolistically produced goods.
adjusted to offset the effect of shocks on inflation and debt. However, such policy would be (a) time-inconsistent as it would need to offset the effect of expectations $E_t \pi_{t+1}$ on current inflation, and (b) suboptimal as movements in policy instruments are penalized. In what follows we assume that both policy makers act under discretion. Such policy does not require commitment mechanism and is credible by construction: given an opportunity to renege on the chosen plan the policy maker will optimally adhere to the same plan. Formally, we make the following assumption.

**Assumption 2 (policy)** Monetary and fiscal policy mix satisfies the following assumptions.

(i) Monetary and fiscal authorities act non-cooperatively.
(ii) Both authorities are assumed to act under discretion.
(iii) The fiscal authority has intra-period leadership.
(iv) The monetary policy maker optimizes every period, but the fiscal policy maker optimizes once every $N$ periods.

The assumption of fiscal intra-period leadership is motivated by the observation that monetary policy reaction function is much more transparent and predictable, so the fiscal policy maker is able to take it into account when formulating policy.\(^5\) Using the interest rate as an instrument implies that consumption and price-setting decisions are made simultaneously, while in this model they are consecutive decisions taken by relevant agents. This makes no difference for our results.

The assumption of time-consistency and optimality prevents the complete and instantaneous stabilization of the economy. Moreover, the relatively large adjustments of infrequent fiscal policy may create more difficulties for monetary policy to offset the effect of disturbances on the economy. Smooth stabilization may not be possible any more.

**Assumption 3 (policy objectives)** Both policy makers are benevolent.

Different objectives of the two policy makers are likely to result in a conflict between the policy makers as one policy maker tries to ‘undo’ the harm done by the other.\(^6\) We shall demonstrate that a similar conflict exists even if both policy makers are benevolent but operate at different frequencies. The assumption of different frequencies also makes the leadership structure important. If both policy makers are benevolent and face identical constraints, then the intra-period leadership does not play any role. The constraints are different in our case, and we chose to study fiscal leadership.\(^7\)

Finally, we make the assumption which is crucial for clear exposition without the loss of generality.

**Assumption 4** The model is perfect-foresight deterministic.

\(^5\)Simultaneous moves of the two policy makers could be another possibility. Empirical evidence (Fragetta and Kirsanova (2010)) suggests that in countries without fiscal decentralization, like the UK, the regime of fiscal leadership is the most relevant.

\(^6\)See e. g. Dixit and Lambertini (2003), Lambertini (2006).

\(^7\)We leave the question of quantitative significance of the leadership regime under infrequent fiscal stabilization for future research.
If the stochastic model is linear-quadratic then the stochastic component of the solution can be obtained in the unique way once the deterministic component is known.\textsuperscript{5} We are mostly interested in issues of existence and uniqueness of the time-consistent policy and these properties are unchanged with introducing stochastic components in the linear-quadratic framework.

To summarize, the deterministic economy evolves according to system:

\begin{align}
p_t &= \beta p_{t+1} + \kappa c_t + \nu \tau_t, \\
b_{t+1} &= \frac{1}{\beta} (b_t - \tau \theta c_t - \tau \tau_t),
\end{align}

and the initial state \( \bar{b} \) is known to all agents, and coefficients \( \kappa = \kappa \left( \frac{1}{\sigma} + \frac{\theta}{\psi} \right) \), \( \nu = \kappa \frac{\tau}{1 - \tau} \). Debt \( b_t \) is the only endogenous predetermined state variable. Objectives of each policy maker coincide and are given by formula (6).

3 Infrequent Stabilization

The monetary policy maker reoptimizes every period while the fiscal policy maker decides once every \( N \) periods, \( N \geq 1 \). We refer to the period between fiscal reoptimizations as fiscal cycle. We denote the set of numbers \( p \) congruent to a modulo \( N \) as \( \{p\}_N \). There are exactly \( N \) different sets \( \{p\}_N \). We shall identify these sets with the corresponding residue: \( \{p\}_N = p \), so \( p \) denotes the time period after the latest fiscal reoptimization. Both the monetary and fiscal policy makers optimize in period \( 0 = \{0\}_N \) can be described by the following Bellman equation, where the value function depends on the number of periods passed since the last fiscal optimization. Assuming the quadratic form for the appropriate value function we can write the Bellman equation for the monetary policy maker in period \( p \):

\begin{align}
S_p (b_{t+p}^p) &= \min_{c_{t+p}^p} \left( \left( \pi_{b_{t+p}}^{p+1} b_{t+p}^p + \left( \kappa - \pi_{b_{t+p}}^{p+1} \tau \theta \right) c_{t+p}^p + \left( \nu - \pi_{b_{t+p}}^{p+1} \tau \right) \tau_{t+p}^p \right)^2 + \lambda \left( c_{t+p}^p \right)^2 + \beta S_{p+1} \left( \frac{1}{\beta} \left( b_{t+p}^p - \tau \theta c_{t+p}^p - \tau \tau_{t+p}^p \right)^2 \right) \right),
\end{align}

\textsuperscript{8}See Anderson et al. (1996).
where we substituted constraints (8) and (10) written for the appropriate period.

Minimization with respect to $c_{t+p}^p$ yields the following monetary policy reaction function:

\[ c_{t+p}^p = c_{b}^p t_{t+p} + c_{\tau}^p t_{t+p} \]

where

\[ c_b^p = -\frac{\left(\left(\kappa - \tau \theta p_{t+1}^{p+1}\right) + \lambda + \gamma^2 S_{t+1}^{p+1}\right)}{\left(\left(\kappa - \tau \theta p_{t+1}^{p+1}\right)^2 + \lambda + \gamma^2 S_{t+1}^{p+1}\right)}, \]

\[ c_{\tau}^p = -\frac{\left(\left(\kappa - \tau \theta p_{t+1}^{p+1}\right) + \lambda + \gamma^2 S_{t+1}^{p+1}\right)}{\left(\left(\kappa - \tau \theta p_{t+1}^{p+1}\right)^2 + \lambda + \gamma^2 S_{t+1}^{p+1}\right)} \]

The monetary policy maker observes fiscal policy, and takes into account its ‘instantaneous’ influence, measured by $c_{\tau}^p$.

The fiscal policy maker only optimizes in periods $[0]_N$. The Bellman equation which describes the fiscal policy decision can be written as:

\[ V(b_0^t)^2 = \min_{\tau_t^p} \left( \sum_{p=0}^{N-1} \beta^p \left( (\pi_{t+p}^p)^2 + \lambda (c_{t+p}^p)^2 \right) + \beta^N V(b_0^t + N)^2 \right) \]

where constraints (8), (10), and (12) are applied in any period $p = 0, ..., N-1$, because the state in period $[N]_N = [0]_N$ depends on fiscal policy in all intermediate periods, $\tau_{t+p}^p, p = 0, ..., N-1$.

We assume that the fiscal policy maker, when chooses $\tau_{0,t}$ also sets $\tau_{p,t}, p = 1, ..., N-1$ such that

\[ \tau_{t+p}^p = \tau_t^0. \]

This policy has the following representation

\[ \tau_{t+p}^p = \tau_{b}^p t_{t+p}, \quad p = 1, ..., N-1. \]

Indeed, take (17) one period forward and use (16) to obtain

\[ \tau_{t+p+1}^p = \tau_{b}^p t_{t+p+1}^p = \tau_{b}^p \frac{1}{\beta} \left( 1 - \tau \theta c_{b}^p - \tau (1 + \theta c_{\tau}^p) \tau_{t+p}^p \right) b_{t+p}^p \]

from where

\[ \tau_{b}^p = \frac{\beta \tau_{b}^p}{(1 - \tau \theta c_{b}^p - \tau (1 + \theta c_{\tau}^p) \tau_{t+p}^p)} \]

\[ \tau_{t+1}^p \]
The complete set of constraints can be written as

\[
\pi^p_{t+p} = \left( \pi^{p+1}_b + (\pi - \pi^{p+1}_b \pi_p) \right) c^p_b + \left( (\pi - \pi^{p+1}_b \pi_p) \right) c^p_p + \nu - \pi^{p+1}_b \tau_p^{t+p}
\]

\[
= \Pi^p_b \pi^p_{t+p} + \Pi^p \tau_p^{t+p} = ... = \Pi^p_0 b^p_t + \Pi^p_0 \tau^0_t
\]

\[
c^p_{t+p} = c^p_b b^p_{t+p} + c^p_\pi \tau^p_{t+p} = ... = c^p_0 b^0_t + C^0_\pi \tau^0_t
\]

\[
b^p_{t+p+1} = \frac{1}{\beta} \left( (1 - \pi c^p_b) b^p_{t+p} - \tau (\theta c^p_\pi + 1) \tau^p_t \right) = ... = B^0 b^0_t + B^0 \tau^0_t
\]

where the coefficients with superscript 0 are obtained by the recursive substitution \( \tau^k_{t+k}, k = 1..p \).

Substitute these constraints into the Bellman equation (15) and differentiate with respect to \( \tau^0_{t+1} \) to yield:

\[
\tau^0_t = \frac{- \sum_{p=0}^{N-1} \beta^p \left( \Pi^p_b \Pi^p_0 + \lambda C^p_b \right) + \beta^N B^N_0 V B^N_0}{\sum_{p=0}^{N-1} \beta^p \left( \Pi^p_0 \right)^2 + \lambda \left( C^p_\pi \right)^2 + \beta^N V \left( B^N_\pi \right)^2}
\]

From (10), (12) and (17) it follows

\[
\pi^p_b = \pi^{p+1}_b + (\pi - \pi^{p+1}_b) c^p_b + \left( (\pi - \pi^{p+1}_b) \right) c^p_\pi + \nu - \pi^{p+1}_b \tau_p^{t+p}
\]

which determines the time-consistent reaction of the private sector in (10).

The resulting transition of the economy can be written as:

\[
c^p_{t+p} = c^p_b b^p_{t+p}
\]

\[
\pi^p_{t+p} = \pi^p_b b^p_{t+p}
\]

\[
b^p_{t+p+1} = B^p b^p_{t+p}
\]

where

\[
B^p_b = \frac{1}{\beta} \left( (1 - \pi c^p_b) b^p_{t+p} - \tau (\theta c^p_\pi + 1) \tau^p_t \right)
\]

\[
C^p_b = c^p_b + c^p_\pi \tau^p_b
\]

Substitute them into (11) and (15) to yield

\[
S_p = (\pi^p_b)^2 + \lambda \left( C^p_b \right)^2 + \beta S_{p+1} \left( B^p_b \right)^2, \quad p = 0,..,N - 1
\]

and

\[
V = \sum_{p=0}^{N-1} \beta^p \left( (\pi^p_b)^2 + \lambda \left( C^p_b \right)^2 \right) \prod_{j=0}^{p} \left( B^j_b \right)^2 + \beta^N V \prod_{j=0}^{N-1} \left( B^j_b \right)^2
\]

Therefore, for benevolent policy makers \( V = S_0 = S_N \). In periods when both benevolent policy makers reoptimize their value functions are the same.
Proposition 1 Stationary discretionary equilibrium with intra-period fiscal leadership can be described by the set of coefficients \( \{ \pi^p_{b}, c^p_{b},\tau^p_{b}, S_p \}_{p=0}^{N-1} \).

Proof. For a given \( b_0 = \bar{b} \), each trajectory \( \{ b_t, \pi_t, c_t, \tau_t \}_{t=0}^{\infty} \) which solves the system of first order conditions (8), (10), (12), and (17) we can uniquely map into the set of coefficients \( \{ \pi^p_{b}, c^p_{b},\tau^p_{b}, S_p \}_{p=0}^{N-1} \), satisfying (13), (14), (22), (23), (25) and (26). Conversely, if the set of coefficients \( \{ \pi^p_{b}, c^p_{b},\tau^p_{b}, S_p \}_{p=0}^{N-1} \) solves (13), (14), (22), (23), (25) and (26) we can uniquely map it into the trajectory \( \{ b_t, \pi_t, c_t, \tau_t \}_{t=0}^{\infty} \), solving system (8), (10), (12) for given \( b_0 = \bar{b} \). □

4 Policy Implications

In this section we study how the increase in the length of fiscal cycle affects the economy under discretionary policy. We start with the known case of frequent monetary and fiscal policy stabilization.\(^9\) We use this example to discuss the important transmission mechanisms of monetary and fiscal policy interactions in case of distortionary taxes as fiscal instrument. The identified dynamic complementarity between the actions of monetary and fiscal policies will play a crucial role in shaping monetary and fiscal interactions once the fiscal cycle becomes longer. We continue with the case of biannual fiscal optimization, which is instrumental to demonstrate the existence of dynamic complementarity between the optimal actions of consequent monetary policy makers within the fiscal cycle. This results in multiple discretionary equilibria and expectation traps. Finally, we conclude with the case of annual fiscal optimization, which is arguably the most practically relevant setup. We demonstrate how the complementarity between the optimal actions of monetary and fiscal policy makers result in expectation traps, but we also demonstrate that these expectation traps are transitory: for many realistic calibrations of the model the reasonable discretionary policy does not exist. We demonstrate by example that the introduction of a debt target for the fiscal policy maker leads to the non-existence of discretionary equilibrium.

4.1 Quarterly Fiscal Stabilization

In the standard case of frequent stabilization both policy makers operate at the same quarterly frequency. The model is simple enough to prove the following proposition.

Proposition 2 If the stationary discretionary equilibrium exists then it is unique.

Proof. The system of first order conditions (13), (14), (22), (23), (25) and (26) can be written as follows (where we omit index \( p \)):

\[
c_b = -\frac{(\kappa - \pi_b \tau \theta) \pi_b - \frac{\tau \theta V}{\beta}}{(\kappa - \tau \theta \pi_b)^2 + \lambda + \frac{\tau \theta^2 V}{\beta}} \tag{27}
\]

\[
c_\tau = -\frac{(\kappa - \pi_b \tau \theta) (\nu - \pi_b \tau) + \frac{\tau \theta V}{\beta}}{(\kappa - \tau \theta \pi_b)^2 + \lambda + \frac{\tau \theta^2 V}{\beta}} \tag{28}
\]

\(^9\)See Blake and Kirsanova (2011) for a general form solution to this class of problems.
discretionary equilibrium exists and unique for these calibrations. It is labelled Figure 1. The characteristics of the equilibrium are given in Appendix A. If

does not exist. If
condition for the multiplicity of discretionary equilibria, see King and Wolman (2004) and Blake and Kirsanova (2012). However, as we argue next, the interaction of the two mechanisms in this
inflation, which increases the marginal return to a monetary policy decision to reduce demand
rate, set by the fiscal policy maker in response to a higher debt level, generates the cost-push
actions of the monetary and the fiscal policy makers are also dynamic complements. Higher tax
monetary policy decision that increases consumption in response to the higher debt. Optimal
complementarities between the economic agents (Cooper and John (1988)). Optimal actions of
Equation (33) only depends on \( \tau_b + \theta C_b \) and always has two solutions of different signs. If the positive root satisfies \( 0 < \tau_b + \theta C_b \leq (1 - \beta) / \tau \) then stationary discretionary equilibrium does not exist. If \( \tau_b + \theta C_b > (1 - \beta) / \tau \) then equation (32) determines \( C_b \) is the unique way and, therefore, the stationary discretionary equilibrium exists and unique.

Panel I in Figure 1 presents constraints (32)-(33) in \( \{ C_b, \tau_b + \theta C_b \} \) space. Solution to equation (33) is plotted with the dashed line, and solution to equation (32) is plotted with solid line. Condition \( \tau_b + \theta C_b > (1 - \beta) / \tau \) is satisfied if and only if the model parameters satisfy

\[
\frac{\nu (\xi - \theta \nu)}{\lambda^2 (\tau + \theta C_b) + (\xi - \theta \nu)^2} < \frac{1}{\beta} \frac{(1 - \beta) (\xi - \theta \nu)^2}{\lambda^2} = 0
\]

Equation (33) is the unique way and, therefore, the stationary discretionary equilibrium exists and unique.

The result on the uniqueness of the equilibrium is not obvious in the presence of dynamic complementarities between the economic agents (Cooper and John (1988)). Optimal actions of the monetary policy and of the aggregated private sector are dynamic complements. Higher inflation, set by the firms in response to a higher debt level, increases the marginal return to a monetary policy decision that increases consumption in response to the higher debt. Optimal actions of the monetary and the fiscal policy makers are also dynamic complements. Higher tax rate, set by the fiscal policy maker in response to a higher debt level, generates the cost-push inflation, which increases the marginal return to a monetary policy decision to reduce demand and contribute the debt accumulation. The presence of dynamic complementarities is a necessary condition for the multiplicity of discretionary equilibria, see King and Wolman (2004) and Blake and Kirsanova (2012). However, as we argue next, the interaction of the two mechanisms in this model results in the uniqueness of the equilibrium.

First, the complementarity between optimal decisions of the private sector and of the monetary policy maker only results in multiplicity only if fiscal policy responds to debt weakly. Blake and Kirsanova (2012) demonstrate this outcome; the economic behavior in the arising multiple equilibria has a strong resemblance with the one in active-passive monetary regimes identified by Leeper (1991). However, the optimally strong reaction of the fiscal policy maker rules out the equilibrium with passive monetary policy.
Second, the complementarity between optimal decisions of the two policy makers is not strong enough to create the multiplicity. Although the optimal decision of the fiscal policy maker is increasing in the optimal decision of the monetary policy maker, the optimizing fiscal policy maker chooses react to debt strong enough to rule out the equilibrium with passive monetary policy, but weak enough to generate multiple equilibria with active monetary policy.\(^{10}\)

Before we leave this section we comment on the use of numerical methods in finding discretionary equilibria. System of first order conditions (27)-(31) is simple, so we are able to prove the uniqueness of solution in the analytical form. Alternatively, counting points of intersection of solutions to each of the two reduced-form equations gives us all equilibria for a given calibration of the model, but only in the case where such reduction of the system is feasible. More complex models will leave us with the only option to use numerical methods to locate the equilibria.

Many numerical algorithms can be used to find discretionary equilibria, see e.g. Backus and Drifill (1986), Söderlind (1999), Blake and Kirsanova (2011), Blake and Kirsanova (2012). All of them require convergence of at least one iterative routine. Such methods can easily confirm the existence of a solution: the fixed point of an algorithm can be checked to satisfy the first order conditions up to the desired tolerance. However, non-convergence of an iterative algorithm does not imply the non-existence of discretionary equilibrium. In order to address this issue and

\(^{10}\)The strong reaction of taxes to debt may drive the economy close to the instability boundary, see Linne-mann (2006). However, the optimal discretionary policy mix controls the economy at sufficient distance from the boundary.
communicate our findings in cases with more complex models with longer fiscal cycle, we introduce the preferred numerical algorithm and test its capability in the case of quarterly stabilization.

The iterative algorithm which we use is based on the following learning mechanism. Suppose the fiscal policy maker considers implementing policy \( \tau^b \), which is not necessarily optimal. The policy maker attempts to compute the optimal responses of the monetary policy maker and of the private sector, \( \{ c_b, c_r, V, \pi_b \} \) by solving system (27), (28), (30) and (31). Because the system is sufficiently complex the fiscal policy maker may implement different lines of reasoning to obtain and rationalize the optimal response of other agents. If the policy maker can obtain such response, he then updates the initial guess of \( \tau^b \) in the unique way by using (29). For initial guess \( \tau^b \) we denote one-step update \( \mathbb{T}(\tau^b) \). We plot \( \tau^*_b = \mathbb{T}(\tau^b) \) for a range of initial guesses in the right hand panel in Figure 1 with the solid line. All points of intersection of this line with the 45° line are the points of discretionary equilibria. (However, not all equilibria can be obtained up by this method.) Figure 1 demonstrates that the policy maker is able to rationalize and obtain the unique equilibrium described above. It is also clear that this equilibrium is stable in the sense that sequence of fiscal policy updates converges to the optimal coefficient in equilibrium \( A \),

\[
\lim_{n \to \infty} \mathbb{T}^n(\tau_b) = \tau^*_b.^{11}
\]

In many situations, this algorithm has shown greater ability to discover discretionary equilibria than conventional iterative algorithms, as it is also capable to discover some of the unstable equilibria. In what follows we shall use it to illustrate our findings, but we shall also use an array of other numerical techniques. Although we cannot prove the non-existence of equilibria, we will be able to prove the non-existence of equilibria which can be rationalized under any of the available schemes. The next case of biannual fiscal optimization demonstrates an example of discretionary equilibrium which cannot be discovered by the known iterative algorithms.

4.2 Biannual Fiscal Stabilization

Suppose that both policy makers optimize in even periods, and we index all such periods with index 0. Only the monetary policy maker optimizes in odd periods, we index such periods with index 1. To save on notation we use the period index \( p \in \{0, 1\} \) and use \(-p\) to indicate odd periods if \( p = 0 \), and even periods if \( p = 1 \).

Despite we cannot prove analytically the existence and multiplicity of equilibria, we can find all discretionary equilibria numerically.

**Proposition 3** For the base line calibration of the model two discretionary equilibria exist.

**Proof.** The derived in Section 3 first order conditions can be written as

\[
\tau^0_b = -\frac{\left( \Pi^0_1 \Pi^0_2 + \lambda c_r^0 c_b^0 + \beta \Pi^1_1 \Pi^1_0 + \beta \lambda c_r^1 c_b^0 + \beta^2 B^2_2 B^2_0 V \right)}{\left( \Pi^0_1 \right)^2 + \lambda (c^0_r)^2 + \beta \left( \Pi^1_1 \right)^2 + \beta \lambda \left( c_r^1 \right)^2 + \beta^2 V \left( B^2_2 \right)^2}.
\]

\[\tag{34}\]

\[^{11}\]This equilibrium can also be obtained by any of the algorithms mentioned above. All learning mechanisms discussed in Dennis and Kirsanova (2009) deliver IE-stability of the equilibrium.
Fiscal Policy in Even Period, $\tau_b^0$  

**Panel II: Numerical Solution**

Monetary Policy in Even Period, $C_b^0$

Fiscal Policy in Even Period, $\tau_b^0$*

---

Figure 2: Discretionary Equilibria in the Biannual Fiscal Stabilization Model

\[
\tau_b^1 = \frac{\beta \tau_b^0}{(1 - \tau (1 + \theta c_b^0)) (1 - \tau (1 + \theta c_b^0))}
\]

(35)

\[
c_b^p = -\frac{\left(\left(\kappa - \tau \theta \pi_b^{-p}\right) \pi_b^{-p} - \frac{\tau \theta}{\beta} S_p\right)}{\left(\kappa - \tau \theta \pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2 \beta^2}{\beta} S_p}
\]

(36)

\[
c_b^p = -\frac{\left(\left(\kappa - \tau \theta \pi_b^{-p}\right) \left(\nu - \tau \pi_b^{-p}\right) + \frac{\tau^2 \theta}{\beta} S_p\right)}{\left(\kappa - \tau \theta \pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2 \beta^2}{\beta} S_p}
\]

(37)

\[
S_0 = V
\]

(38)

\[
S_1 = \left(\pi_b^1\right)^2 + \lambda \left(C_b^1\right)^2 + \beta S_0 \left(B_b^1\right)^2
\]

(39)

\[
V = \left(\pi_b^0\right)^2 + \lambda \left(C_b^0\right)^2 + \beta \left(\left(\pi_b^1\right)^2 + \lambda \left(C_b^1\right)^2\right) \left(B_b^0\right)^2 + \beta^2 V \left(B_b^0\right)^2 \left(B_b^1\right)^2
\]

(40)

After multiple substitutions the system of first order conditions (34)-(40) can be reduced to the polynomial system of two equations $C_b^0 \left(C_b^0, \tau_b^0\right) = 0$ and $\tau_b^0 \left(C_b^0, \tau_b^0\right) = 0$, although at expense of much complexity. We plot solutions to these equations in Panel I in Figure 2. The curves intersect in several points but only two of them satisfy (i) the system of first order conditions, (ii) the requirement of positive $V, S_1$, and (iii) the requirement of non-explosiveness of the economy, which is $|B_b^0 B_b^1| < 1$. We label these points of intersection as equilibria A and B. 

Multiplicity of discretionary equilibria implies that following a disturbance, for example higher initial debt level, the economy can follow one of multiple paths, each of which satisfies conditions of optimality and time-consistency. Each of these paths is associated with different monetary and
fiscal policies, see Figure 3 which plots two different adjustment paths following the same initial increase in the debt level. For comparison, the Figure also includes responses in case of frequent fiscal stabilization.

Suppose the level of debt is above the steady state and fiscal policy raises the tax rate for two periods. Following the high marginal cost inflation will rise and stay above the steady state for these two periods. Monetary policy maker will find it optimal to intervene. The monetary policy maker at time 0 takes into account monetary policy in period 1. There is a dynamic complementarity between the actions of the two consequent monetary policy makers within the fiscal cycle: the deeper is the future cut in demand, the bigger payoff the current monetary policy maker gets from engineering high demand today. Two point-in-time equilibria arise. In the first such equilibrium, the period-0 monetary policy maker will keep the current demand low and the period-1 monetary policy maker does not generate a big cut in demand. In the second equilibrium, the period-0 monetary policy maker stimulates high demand in anticipation that the period-1 monetary policy maker will implement a cut in demand. The fiscal policy maker when choosing policy in period 0, perceives the both possibilities. The optimal fiscal response in the first point-in-time equilibrium response is to raise the tax rate less than in the second equilibrium. The strong response of the tax rate in the second equilibrium generates a ‘zig-zag’ pattern of adjustment: with low two-period-average demand, the increase in the tax rate generates substantial fall in the stock of debt so that the second half year cycle ‘mirrors’ the first half year one, but with the opposite sign. Figure 3 also demonstrates that in equilibrium A the paths of all variables ‘approximate’ the optimal paths of the corresponding variables under frequent optimization, and we shall call equilibrium A ‘approximating’. We call equilibrium B ‘zig-zag’.

Despite the clearly increased inflation volatility, the loss in the approximating equilibrium is slightly lower than it is in the unique equilibrium under frequent optimization, see Appendix A. The is mainly due to faster stabilization of the economy in this equilibrium. The two-period tax rate increase predominantly determines the two-period speed of debt adjustment $|B^1_b B^1_b|_A = 0.64 < 0.96 = |B^1_b|^2$. This welfare gain of faster stabilization is slightly higher than the welfare loss of higher volatility. The loss in the ‘zig-zag’ equilibrium is much higher than in the ‘approximating’ equilibrium. Not only it generates the relatively slow speed of adjustment, as $|B^1_b B^1_b|_B = 0.79 > |B^1_b B^1_b|_A$, but it also induces very high volatility of economic valuables.

Finally, we argue that equilibrium B is unlikely to be located by any iterative routine from the wide class of numerical routines.\(^\text{12}\) In particular, Panel II in Figure 2 illustrates the application of the described above iterative numerical approach. Suppose the fiscal policy maker considers implementing policy $\tau_0^b$, which is not necessarily optimal. The policy maker attempts to compute the optimal responses of the monetary policy maker and of the private sector, $\{c^p_0, c^p_1, S_p, \pi^p_0\}_{p=0}^1$ by solving system (35) – (40). Because the system is sufficiently complex the fiscal policy maker can implement many different reasoning schemes to obtain and rationalize the optimal response of other agents. If the policy maker can obtain the response, he then updates the initial guess of $\tau_0^b$ in the unique way by using (34). We denote the update $\tau_0^{b*} = T(\tau_0^b)$. For a range of initial guesses we compute the update $T(\tau_0^b)$ and plot it in Panel II in Figure 2 with the solid line and check if the points of intersection with the 45° line constitute a discretionary equilibrium. The line

\(^{12}\)In our work we used routines based on either Backus and Driffill (1986) or Oudiz and Sachs (1985) iterative algorithms.
Figure 3: Impulse responses and counterfactual simulations. Fiscal policy optimizes every other period.
Fiscal policy, $\tau^*_b$ intersects the 45° line in one point, which corresponds approximating equilibrium $A$. This equilibrium is stable, $\lim_{n \to \infty} T^n(\tau^*_b) = \tau^{0A}_b$. With further increase in $\tau^*_b$ the zig-zag equilibrium $B$ cannot be located. There is a neighborhood of $\tau^{0B}_b$ where the fiscal policy maker cannot rationalize the optimal response of the other agents, which is described by system (35) – (40).\footnote{Further research in equilibrium selection is likely to demonstrate that equilibrium $B$ cannot be rationalized by economic agents. See Dennis and Kirsanova (2009) on equilibrium selection in LQ RE models of discretionary policy. Equilibrium $B$ is not rationalizable under the learning schemes discussed in this paper.}

To summarize, the most important conclusion from the example of biannual fiscal stabilization is the demonstration of the existence of the approximating equilibrium. Although the other equilibrium exists, the approximating equilibrium delivers the best possible outcome under the infrequent discretionary fiscal stabilization. In this equilibrium the monetary policy maker can offset most adverse effects of fiscal infrequency on welfare-related macroeconomic variables. In the next example we argue that we should not take this result for granted once the fiscal cycle becomes longer.

4.3 Annual Fiscal Stabilization

4.3.1 Multiplicity of Discretionary Policy Equilibria

Building on results in the previous section we present the third example of infrequent fiscal stabilization. Arguably, this is the most empirically relevant setup in which the monetary policy maker reoptimizes every quarter, but the fiscal policy maker reoptimizes only at the beginning of every four quarters.
In this model we are unable to present the system of first order conditions as a system of two polynomial equations and use the graphical method of finding solutions. We have to resort to numerical technics to find discretionary equilibria.

**Proposition 4** If monetary policy makers take decisions quarterly and the fiscal policy maker optimizes annually then at least two discretionary equilibria exist for the base line calibration of the model.

**Proof.** We use the following algorithm to discover equilibria. Suppose the fiscal policy maker considers implementing policy $\tau^0_b$, which is not necessarily optimal. The policy maker attempts to compute the optimal responses of the monetary policy maker and of the private sector, $\{c^p_b, c^p_p, S_p, \pi^p_p\}_{p=0}^3$ by solving system (13), (14), (18), (23) and (25). Because the system is sufficiently complex the fiscal policy maker may implement different lines of reasoning to obtain and rationalize the optimal response of other agents. If the policy maker can obtain this response, he then updates the initial guess of $\tau^0_b$ in the unique way by using (22). We denote the update $\tau^0_b^* = T(\tau^0_b)$. For a range of initial guesses we compute the update $T(\tau^0_b)$ and plot it in Panel I in Figure 4 with the solid line and check if the points of intersection with the 45° line constitute a discretionary equilibrium. The line $\tau^0_b^* = T(\tau^0_b)$ intersects the 45° line in two points, labelled A and B. Once these equilibria are discovered with the outlined numerical method, we can check that they satisfy the system of the first order condition indeed. With further increase in $\tau^0_b$ the optimal response of other agents cannot be located. There is a range of where the fiscal policy maker cannot rationalize the optimal response of the other agents, described by system (13), (14), (18), (23) and (25).

Figure 5 reports responses of the economy to a one-off increase in the level of debt. We plot the optimal solution (green line with x-markers) but also plot result of counterfactual simulations (red line with v-markers). In counterfactual simulations we assume the monetary policy maker does not try to offset the effects of fiscal policy which are due to infrequent optimization, i.e. it implements policy which would be optimal if fiscal policy maker optimized frequently. The private sector reacts optimally to the state and to policies.

The dynamic complementarity between the optimal actions of the two policy makers is responsible for the multiplicity of equilibria. An optimal response of monetary policy reinforces the action of fiscal policy: higher level of taxation has a cost-push effect and so the optimal monetary response is to reduce demand and so the tax base. Smaller tax base requires higher tax rate to ensure the desired speed of debt stabilization. Both policy makers can coordinate on either slow or fast correction of the level of debt towards the target. Figure 5 illustrates these interactions. Consider equilibrium A. Suppose the initial debt is higher than in the steady state and the tax rate is kept high for four periods. This implies steep reduction in debt. The effect of future high tax rate and high marginal cost creates expectations of high future inflation. If monetary policy does not offset the effect of fiscal “infrequency” then debt and consumption adjust in a linear way between the periods of fiscal optimization. The effect of lower consumption is smaller than the effect of higher tax rate and inflation stays above the frequent optimization solution.

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14In this process the fiscal policy maker may have to rely on the ability of other agents, the consequent monetary policy makers and the private sector to rationalize each other’s decisions and coordinate on the best response. Numerous point-in-time-equilibria may exist, see King and Wolman (2004).
Figure 5: Impulse responses and counterfactual simulations. Fiscal policy optimizes once a year.
benchmark. The tax rate remains high for the four periods and, by the end of the fourth period, it is much higher than it would be if optimization happened every period. The tax correction in the fifth period brings inflation down. Figure 5 demonstrates that the ability of the optimal monetary policy to reduce inflation volatility is limited. Inflation can be reduced if consumption stays below the linear counterfactual reaction. However, only some part of inflation volatility can be eliminated: a further reduction in inflation would require a volatile consumption path. Indeed, it is clear from the picture that consumption should go down first and than up in the first four periods if the inflation hump in first two periods to be eliminated. In what follows we call equilibrium $A$ ‘slow approximating’. This equilibrium is stable, $\lim_{n \to \infty} T^n (\tau_b^0) = \tau_b^0 A$.

In discretionary equilibrium $B$ the tax rate is initially kept above the frequent-optimization benchmark. This generates a much steeper fall in the level of debt than in the slow approximating equilibrium $A$. Higher equilibrium level of taxation results in higher level of inflation and in lower consumption. We call equilibrium $B$ ‘fast approximating’. This equilibrium is unstable, i.e. for any $\tau_b^0 \neq \tau_b^0 B : \lim_{n \to \infty} T^n (\tau_b^0) = \tau_b^0 A \neq \tau_b^0 B$.

To summarize, in case of the annual fiscal cycle there are at least two discretionary policy equilibria. Their existence is a result of strong dynamic complementarity between optimal actions of the two policies, monetary and fiscal, provided that fiscal policy uses distortionary taxes as policy instrument. However, their existence is likely to be non-robust to the model specification; this is suggested by Panel I in Figure 4. Indeed, equilibria $A$ and $B$ are located on the the same curve $T (\tau_b)$, which may not intersect the 45° line. In the next section we discuss why this may occur.

4.3.2 Existence and Uniqueness of the Approximating Policy Equilibrium

We argue that the existence of the approximating equilibrium is not robust to model specifications and policy scenarios. To communicate our argument we present a scenario in which the fiscal policy maker is assigned an additional target to stabilize the debt faster than socially optimal.

Suppose the monetary policy maker is benevolent, but the fiscal authority’s objective function is modified to include the debt target

$$L^F = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda c_t^2 + \mu b_t^2).$$

If the fiscal policy maker is benevolent, then $\mu = 0$.

The strength of the dynamic complementarity depends on the calibration of $\mu$. Both approximating equilibria do not exist if $\mu > 0$ and is sufficiently high. In order to understand this result consider the familiar scenario of high initial debt. Suppose that both policy makers are benevolent and we are in the slow approximating equilibrium $A$, see the left panel in Figure 5. If we impose a debt target for fiscal policy, i.e. start increasing $\mu \geq 0$, the fiscal policy maker will try to speed up the debt stabilization with an increase in the tax rate relative to the benchmark case of $\mu = 0$. The cost-push effect will increase inflation more and so the monetary policy maker will choose to engineer bigger fall in consumption. This, of course, will slow down the debt stabilization and require even higher tax rate. The process converges: each additional reduction in

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15For the benchmark calibration of model parameters this threshold value of $\mu = 0.0003$. 

demand requires smaller increase in the tax rate. Equilibrium exists, in this equilibrium the debt is reduced faster than it is plotted in the left panel in Figure 5.

This contrasts with the effect of introduction the debt target in equilibrium B. Suppose debt is higher than in the steady state by one unit, the policy makers are benevolent and and we are in the fast approximating equilibrium B, see the right panel in Figure 5. Note that in this equilibrium debt is stabilized with an observed overshooting after the first year. If we impose a debt target, i.e. start increasing $\mu \geq 0$, then raising the tax rate in the first several periods becomes counterproductive. If the tax rate is raised higher than in the $\mu = 0$ case, this results in even bigger overshooting of debt, which works towards destabilizing the debt. In order to ensure faster debt convergence the tax rate has to rise less and monetary policy has to engineer smaller fall in consumption. The fiscal policy maker anticipates that demand will not respond much and will lower the tax rate. This process converges: each additional reduction in the size of demand cut requires smaller reduction in the size of the tax rate increase.

To summarize, with the increasing weight on the debt target equilibria A and B move towards each other so that the dynamics of the economy in equilibria A and B becomes similar. The dynamic of the economy in response to the higher debt level in the limiting case $A = B$ is plotted in Figure 6. For comparison we also plot the result of frequent stabilization without the debt target.

If the debt target becomes even stronger, then no approximating equilibrium exist. Any proposed increase of the tax rate $\tau_0$ results in strong optimal rationalizable response of the other agents within the fiscal cycle. To counteract the perceived response requires the bigger initial rise $\tau_0^{\ast} = \tau_0 + T(\tau_0^{\ast}) > \tau_0$. We can summarize this outcome in the form of the following proposition.

**Proposition 5** With sufficiently high weight on the debt target of fiscal authorities the approximating discretionary equilibrium does not exist.

**Proof.** The proof is numerical. Our iterative approach finds two equilibria under the base line calibration with $\mu = 0$. With an increase in parameter $\mu$ the two equilibria eventually coincide and disappear. No other equilibrium can be found numerically with iterative methods.

This result does not imply that there is no discretionary equilibrium if equilibria A and B do not exist. Yet another equilibrium might exist. In particular, Panel II in Figure 4 and its similarity with Panel II in Figure 2 suggests that a ‘zig-zag’ equilibrium might exist. Strategic complementarity between actions of consequent monetary policy maker may lead to a zig-zag adjustment of demand within the fiscal cycle. These adjustments might be ‘fine tuned’ such that the annual average magnitude of them is not too large to provoke the destabilizing increase in the tax rate. However, such equilibrium is unlikely to be rationalized by the economic agents, in particular by the fiscal policy maker. Moreover, it is difficult to argue to call such equilibrium ‘approximating’.

The existing algorithms of finding solutions are not suited to obtain all possible equilibria in a complex case with many states. We could only do this for the quarterly and the biannual models. However, Figure 4 makes it clear that the approximating equilibrium will disappear if the debt target is sufficiently strong, rather than we suddenly became unable to locate it numerically. We can be reasonably sure that if an additional equilibrium exists in the annual optimization model, this equilibrium will generate very low level of social welfare because of the high volatility in macroeconomic variables.
How sensitive our results to calibration of the model? Although we demonstrate the non-existence of approximating equilibrium by introducing the debt target, many other scenarios and calibrations that lead to more active response of policies to debt disturbances result in the non-existence of the equilibrium. In particular, we can mention the following cases.\footnote{We can support these results with the appropriate numerical analysis.}

First, calibrations of the model which result in more active monetary and fiscal policies are likely to lead to non-existence of the approximating equilibrium. For example, higher mark up (or lower elasticity of substitution $\varepsilon$) increases the effect of policies on inflation and increases the strength of complementarity of their actions.

Second, the strength of the dynamic complementarity depends on the level of steady state debt level. The effects of the nominal interest rate and inflation on the process of debt accumulation rises linearly with the steady state level of debt. In response to high inflation the optimal monetary policy will raise interest rate; both the high (real) interest rate and the consequently low tax base increase the rate of debt accumulation, and this effect is stronger with higher steady state level of debt. The higher steady state level of debt the less likely the approximating discretionary
equilibrium exists.

Third, the strength of the dynamic complementarity depends on the frequency of fiscal optimization. The longer the period between the reoptimizations the longer the tax rate remains fixed, and the stronger action of monetary policy is required in order to offset the adverse effect on inflation. The approximating discretionary equilibrium may not exist.

Finally, it is clear from Figure 4 that a move $\tau^0_t = T(\tau^0_b)$ to the right will lead to the non-existence of equilibrium $B$. However, our base line calibration delivers relatively low degree of complementarity between optimal policies. The reduction of the complementarity may be achieved through carefully designed policy delegation schemes. We leave this question for future research.

5 Conclusion

In this paper we study implications of the infrequent discretionary fiscal optimization for the stabilization of the economy. We demonstrate the presence of dynamic complementarity between the optimal monetary and fiscal policies. A higher tax rate, which is required to stabilize higher debt, will have a cost-push effect. The optimal monetary policy response will generate a reduction in demand and in the tax base, and faster debt accumulation. In its turn, the higher tax rate will be required. We argue that infrequent fiscal stabilization amplifies this mechanism.

If both policies operate with the same frequency, this reinforcement mechanism is weak and does not lead to significant adverse effects. However, with longer fiscal cycle this mechanism may lead to expectation traps. Following a disturbance the economy may follow one of the several adjustment paths, each of which satisfies conditions of time-consistency and optimality. Switches between equilibria result in high volatility of macroeconomic variables.

The same transmission mechanism can also lead to the non-existence of the approximating discretionary equilibrium in many practical scenarios. We illustrate this with the example of a too strong debt stabilization target for the fiscal policy maker.

Our results suggest that once the high level of debt is accumulated, its reduction should not be made faster than socially optimal, no additional debt targets should be imposed on the fiscal policy maker. Different delegation schemes should be designed to deal with this issue. We argue that in many practical scenarios the ability of the policy maker to precommit becomes the necessary requirement for stabilization of the economy.
## A Characteristics of Discretionary Equilibria

Table 1.: Characteristics of discretionary equilibria

<table>
<thead>
<tr>
<th></th>
<th>Eq. A</th>
<th>Eq. B</th>
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<tr>
<td><strong>Frequent Fiscal Stabilization</strong></td>
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<td>Fiscal Policy</td>
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<td>Monetary Policy</td>
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<td>Private Sector</td>
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<td>Normalized Loss</td>
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<td><strong>Biannual Fiscal Stabilization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiscal Policy</td>
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<td>0.79;0.96</td>
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<tr>
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<td>-6.41;6.87 $\times 10^{-2}$</td>
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<tr>
<td>Private Sector</td>
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<td>4.7;5.0 $\times 10^{-3}$</td>
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<tr>
<td>Normalized Loss</td>
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<td><strong>Annual Fiscal Stabilization</strong></td>
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<tr>
<td>Fiscal Policy</td>
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<tr>
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## References


