Rating agencies: The logic of self-defeating optimism

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Abstract

What is the economic role of credit rating agencies (CRAs), and how should they be paid? We study how credit rating agencies (CRAs) can serve as coordination device, and thereby prevent or cause inefficient liquidation of projects with a long horizon. We show that if CRAs can impact real outcomes, then their reputation concerns hamper truthful revelation of information, and, as a consequence, limit their ratings’ informational content. We then show that equilibrium behavior is shaped by two prominent regimes: (i) an ‘optimistic’ regime in which CRAs tend to give good ratings, and (ii) a ‘pessimistic’ regime in which CRAs tend to give bad ratings. But both regimes are self-defeating: (i) only bad ratings do affect investors’ behavior in an optimistic regime while (ii) only good ratings affect investors’ behavior in a pessimistic regime. Existing evidence on the environment preceding the financial turmoil starting in 2007-2008 fits with our characterization of an optimistic regime. Hence, in order to enable CRAs to play a socially constructive role, we propose that their payoff structure should be altered by giving them stronger incentives to correctly predict project failure rather than project success.

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1 Introduction

The role played by credit rating agencies (CRAs) in financial markets is a matter of considerable debate. Normatively, one would hope that CRAs provide information to investors and thereby facilitate a more efficient allocation of capital. Yet, in practice it is not clear that such an effect actually takes place. A range of empirical studies question the extent to which ratings provide new and relevant information on the fundamentals of investment objects. For instance, Hilscher and Wilson (2012) find that ratings are poor measures of raw default probabilities, and a dominant finding in the earlier literature is that ratings changes are relevant news to markets only when they constitute downgrades, not upgrades. Furthermore, it is conceptually challenging to understand why investors should trust ratings to provide relevant information in the first place, given that CRAs are paid by issuers. Indeed, a widespread concern with CRAs has been that their incentive structure causes a bias toward overly optimistic ratings (‘ratings inflation’). This concern has been particularly prominent in the discussion of the securitization process preceding the financial turmoil starting in 2007-2008, where a widely held view is that CRA incentives are distorted because they are paid by issuers rather than investors (Pagano and Volpin, 2010, Bolton, Freixas and Shapiro 2012). A natural consequence would seem that the payment structure should be altered toward an ‘investor-pay’ model, to make CRAs place higher weight on their reputation for correctly anticipating returns. However, little attention has been directed to how such reputation concerns in fact will influence the role played by CRAs.

In this paper we deal with the above issues, and explore how a rating agency may influence investor choice even though investors are fully aware that its incentive structure is distorted. Crucially, we analyze the effect of the reputation incentives faced by the CRA. In general, the CRA wants to make correct predictions about default, because this is key for their reputation as providers of useful information. However, an agency need not be indifferent about what the outcome actually becomes. For example, a CRA may have a preference of avoiding defaults, so as to be more attractive for issuers. Or it may have a cautious bias, in the sense that it is excessively averse to mistakenly guide investors to finance projects that ultimately fail.

We show that it is difficult for the CRA to transmit precise information, because the reputation concern for making correct predictions will induce the CRA to make polarized predictions, exaggerating the probability of success or failure in either direction. The intuition is as follows. If ratings affect investor choice, and if the CRA observes that default is highly likely, then the CRA will have a tendency to give an overly negative rating in order to prevent investment and increase the default probability further so that the rating proves correct ex post.
We then study the effects of polarized predictions. We show that there will be a tendency for polarized predictions to be biased in one direction, depending on the payoff structure of the CRA. In an 'optimistic regime', which will prevail if the CRA’s payoff structure makes it prefer preventing rather than causing defaults, the CRA will have a tendency toward excessively positive ratings, exaggerating the outlook of the issuer. However, rational investors will realize this, implying that the CRA is unable to influence investors’ beliefs in the optimistic direction. Rather, it is only in the few cases where the CRA observes a sufficiently bad signal to make it report negatively, that investors beliefs are affected. Consequently, in an optimistic regime the CRA cannot reduce the probability of default, but under some circumstances it will in fact increase the probability of default. Thus, when the CRA wants to prevent default, the possible impact is in the opposite direction.

Conversely, in a pessimistic regime, which prevails when the payoff structure motivates the CRA to be cautious, the situation is the opposite. The CRA will tend to report bad news, but such reports will be discounted by investors. Again, it is only when the CRA reports against its bias that the signal has an effect on the beliefs of the investors. Thus, if an issuer’s outlook is sufficiently strong for the CRA to rate is positively, this will change investors’ perceptions, thus reducing the probability of default. Yet again, the outcome is surprising in light of the incentives of the CRA, in the sense that they on average reduce the prevalence of default in the situation where they are biased towards correctly predicting default.

In the environment we study, coordination failure causes inefficient default. Hence, there is a well-defined role for CRAs to potentially affect welfare by coordinating investors to either finance or terminate projects. In a positive regime, CRAs on average raise the incidence of inefficient default. It follows that the payoff structure of CRAs should be designed so that they are in the pessimistic regime, on average giving too negative information. This requires stimulating their incentive to correctly predict project failure rather than success. However, the incentives should not be adjusted too much in this direction, because then the CRA will only give positive ratings when fundamentals are so strong that investors already know it. Hence, to stimulate the functioning of CRAs from a normative perspective, it is not enough to raise their costs of being mistaken ex post, one must also make sure that their incentives are biased in a conservative direction.

Cast against empirical observations made elsewhere, we contend that the 'optimistic' regime illustrates a potentially important background feature of the environment preceding the recent financial crisis, particularly with regard to markets for structured finance products. An illustrative quote is that of Lloyd Blankfein, CEO at Goldman Sachs: 'In January 2008 there were 12 triple A-rated companies in the world. At the same time there were 64,000 struc-
tured finance instruments such as collateralised debt obligations, rated triple A’ (Blankfein, 2009, p.7., Pagano and Volpin, 2010). Arguably, for such instruments the roll-over problem which lies at the heart of our model is particularly severe. Furthermore a dominant view in the earlier empirical literature is that historically, to the extent rating agencies have brought information to the market, it is when they have downgraded issuers, not when they have upgraded them (Liu et al 1999, Goh and Ederington, 1999, 1993, Barron, Ederington and Goh, 1998, Clarke and Thomas, 1997, Matolcsy and Lianto 1995, Impson et al 1992, Wansley et al 1992, Zaima and McCarthy, 1988, Holthausen and Leftwich, 1986, Griffin and Sancivente, 1982).

Our study relates also to several theoretical studies on rating agencies. Particularly relevant is the analysis by Mathis, McAndrews and Rochet (2009), who examine whether reputation concerns are sufficient to discipline rating agencies. They find that there may exist of reputation cycles, where CRAs accumulate a reputation for truth-telling, only to exploit this reputation when it is strong enough. Our study shares with theirs the emphasis on reputational incentives for correct reporting, but we differ in many other respects. A primary difference is that Mathis et al assume that rating agencies may be of a naively honest nature, committed to always telling the truth, and this is why investors might attach weight to their advice. In our model, investors know the opportunistic incentives of the CRAs, but still may be affected by their ratings. Furthermore, we study a setting with an explicit roll-over problem and distinguish between the the reputational incentives to correctly predict success and failure of an issuer.

The role of CRAs as coordination device is at central stage in our paper. Two studies that also consider such a role for rating agencies are Boot, Milbourn and Schmeits (2005) and Carlson and Hale (2006). The former emphasize the monitoring role of CRAs in credit watch procedures and the importance of ratings for institutional investors. Multiple equilibria are possible because issuers choose risk-strategies contingent on their funding costs, while ratings can serve to coordinate markets since institutional investors are required to follow them. This mechanism is distinct from ours, since we study a setting where investors choose whether or not to follow ratings. The approach of Carlson and Hale (2006) is more similar to ours, in that they also use global games tools to study CRAs and a roll-over problem. They show that CRAs may induce multiple equilibria by making public information more precise, and numerically assess how ratings may influence equilibria when they remain unique. What crucially distinguishes our paper from theirs is that rating agencies are non-strategic in their environment, but automatically convey their information truthfully. Hence, they do not address what determines rating agencies’ choice of reporting strategies, which is what we
explore in this paper.

Another issue that has been studied theoretically, is the role of competition among rating agencies and ratings shopping by issuers. Bolton et. al. (forthcoming) show how a duopoly market structure may lead to 'ratings inflation', and worse outcomes than with a monopolist CRA. Skreta and Veldkamp (2009) and Sangiorgi, Sokobin and Spatt (2009) assume that CRAs report truthfully and show that noisier information leads to rating shopping. Grimaud, Peyrache and Quesada (2009) also assume truthful CRAs and find that issuers may suppress ratings that are too noisy.

Finally, our paper constitutes an application of the tools developed in the global games literature. A extensive survey of this literature is given by Morris and Shin (2003). The central ingredient in global games models is that agents are uncertain about each others information sets, and this leads to equilibrium uniqueness rather than equilibrium multiplicity. Other applications of global games that lie particularly close to ours, in addition to Carlson and Hale (2006) mentioned above, are Morris and Shin (2006, 2004) and Corsetti, Guimares and Roubini (2006) who investigate roll-over problems, and Morris and Shin (2001) who analyze currency attacks. However, none of these studies consider a strategic sender of information, such as the rating agency we explore.

Our paper is orgaized as follows. Section 2 presents our model. Section 3 gives our results. Section 4 concludes.

2 The Model

We consider an environment where a continuum of investors of unit mass, are financing a project, similar to Morris and Shin (2004). If the project continues until fruition, it provides a fixed payoff $V$ to each investor. However, in order to last long enough to mature, the project must roll over its debt as creditors have an opportunity to review their investment an interim stage. Investors who liquidate at this stage obtain a payoff $v$, where $v < V$. Hence, investors must choose whether to roll-over the loan until the project’s maturity, or to liquidate and obtain $v$. Refinancing the project is risky since each investor is small and hence faces the possibility that the project is terminated because sufficiently many other investors decline to roll over debt. If the project is unable to refinance, remaining investors get a payoff of zero.

The current cash flow of the project $\theta$, and hence the ability of the firm to meet short-term claims, is uncertain to investors. The cash flow $\theta$ is uniformly distributed in $\mathbb{R}$.\footnote{More generally, one may think of $\theta$ as representing the project’s uncertain ‘fundamental’.} Let $l$ indicate the mass of investors who choose not to refinance the project before its completion.
It follows that the project succeeds if and only if \( \theta > l \), since then the cash flow is sufficient to cover early liquidation. Consequently, if \( \theta > 1 \) the project survives even if all creditors liquidate prematurely. Likewise, the project fails for sure if \( \theta < 0 \). In contrast, if \( \theta \in [0, 1] \), the project may or may not succeed, depending on investors’ behavior. While \( \theta \) is unobserved, each investor receives a private signal \( x \) with uniform distribution on \([\theta - \beta, \theta + \beta]\) conditional on \( \theta \). All signals are assumed independent, conditional on \( \theta \). For expositional purposes we will introduce the variable \( t = 1 - \frac{x}{\beta} \).

A credit rating agency (CRA) receives a private signal \( y \) with uniform distribution on \([\theta - \alpha, \theta + \alpha]\) conditional on \( \theta \). We suppose that a rating from the CRA consists of an interval \( R \subset \mathbb{R} \), with mean \( \mu_R \). We will sometimes use \( r \) when we wish to indicate that a rating consists of a simple scalar. Payoffs of the CRA have two components. One, the CRA receives fee \( \phi \) if the investment project succeeds. Two, the CRA is rewarded to the extent that its rating reflects outcome. This captures the reputation side of CRAs payoff structure. We let \( m_1(R) \) denote payoff in case of success and \( m_0(R) \) the payoff in case of failure, where \( m_1 + m_0 = \rho \), and \( m_1 \geq 0 \). For simplicity, and to fix ideas, we suppose that these payoffs are functions of \( \mu_R \) only. The CRA’s payoff structure is then given by

\[
\Pi = \phi I + m_1(\mu_R) I + m_0(\mu_R)(1 - I),
\]

where \( I = 1 \) if the project is succeeds and \( I = 0 \) if the project fails.

Timing is as follows. At time 0 the CRA receives a private signal \( y \), and it announces its rating \( R \) at time 1. This rating is observed by all investors. Each investor receives his own private signal \( x \) at time 2. Selling orders are sent at time 3, while the project comes to completion at time 4.

The following notation and definitions will be used throughout the paper.

We shall say that an investor is naive if given rating \( R \) he assumes \( y \in R \) and updates his belief on \( \theta \) (i) using Bayes’ rule whenever \( R \cap [x - \beta, x + \beta] \neq \emptyset \), and (ii) by setting probability 1 to \( \theta = \sup R \) if \( x - \beta > \sup R \), and probability 1 to \( \theta = \inf R \) if \( x + \beta < \inf R \).

We refer to the ‘naive investment game’ to indicate the game played among naive investors who observe a rating \( R \) and maximize expected profits taking this \( R \) into account.

We let \( \theta^*(R) \) denote the (Bayes-Nash) equilibrium \( \theta \)-threshold of the naive investment game given rating \( R \). To shorten notation we will sometimes use \( \theta^*_0(y^*) = \theta^*((-\infty, y^*)) \) and \( \theta^*_1(y^*) = \theta^*((y^*, +\infty)) \).

We will say that an equilibrium of the naive investment game is a corner equilibrium if either all investors are sellers, disregarding private information, or no investor sells, again disregarding private information (i.e. \( l = 0 \) always or \( l = 1 \) always). An equilibrium of the
investment game is an interior equilibrium otherwise.

Let $\Pi(R, y)$ denote the RA’s expected payoff with naive investors’ beliefs. A rating strategy $R(.)$ is a PBE strategy if and only if

1. $y \in R(y)$ for all $y \in \mathbb{R}$
2. $\Pi(R(y), y) = \max_{y'} \Pi(R(y'), y)$ for all $y \in \mathbb{R}$

Notice that any PBE strategy is part of a PBE in the game between RA and investors.

We assume throughout that
$$\frac{1}{2} < t < 1 - \frac{1}{2\beta+1}, \phi < \rho.$$

3 Results

Proposition 1 The credit rating affects investor behavior and whether a project is refinanced or not. For $r \in \mathbb{R}$, the cutoff-level of project cash flows below which there is default, is as follows:

$$\theta^*(r) = \begin{cases} 
1 & \text{if } r \leq 1 - \alpha \\
\frac{t(r+\alpha)-(1-t)2\beta}{1-(1-t)(2\beta+1)} & \text{if } 1 - \alpha < r < (2\beta + 1)(1-t) - \alpha \\
1 - t & \text{if } (2\beta + 1)(1-t) - \alpha < r < 1 + \alpha - (2\beta + 1)t \\
\frac{(1-t)(r-\alpha)}{1-t(2\beta+1)} & \text{if } 1 + \alpha - (2\beta + 1)t < r < \alpha \\
0 & \text{if } \alpha \leq r \end{cases}$$

Proof. We first show that a corner equilibrium with $l = 1$ exists if and only if $r < 1 - \alpha$. Suppose $r < 1 - \alpha$ and all investors always sell. Any investor’s belief then entails
$$\theta \leq r + \alpha < 1 = l$$

An equilibrium thus exists in which all investors always sell. Suppose on the other hand $r > 1 - \alpha$. An investor receiving signal $x \geq 1 + \beta$ has posterior belief $\theta \geq 1$, and so chooses not to sell. So an equilibrium in which all investors always sell does not exist.

We next show that a corner equilibrium with $l = 0$ exists if and only if $r > \alpha$. Suppose $r > \alpha$ and no investor ever sells. Any investor’s belief then entails
$$\theta \geq r - \alpha > 0 = l$$
An equilibrium thus exists in which no investor ever sells. Suppose on the other hand 
\( r < \alpha \). An investor receiving signal \( x \leq -\beta \) has posterior belief \( \theta \leq 0 \), and so chooses to sell. So an equilibrium in which no investor ever sells does not exist.

We next explore interior equilibria. An interior equilibrium \((x^*(r), \theta^*(r))\) satisfies

\[
\begin{align*}
\theta^*(r) &= \text{Prob}(x < x^*(r) | \theta = \theta^*(r)) \\
\text{Prob}(\theta < \theta^*(r) | x = x^*(r), y = r) &= t
\end{align*}
\]

which, by (35)-(38), is equivalent to

\[
(x^*(r), \theta^*(r)) \in AB \cap \left( C(r)D(r) \cup D(r)E(r) \cup E(r)F(r) \right)
\]

The proposition now follows from simple computation.

Define \( \mu \) such that \( \phi + m_1(\mu) = m_0(\mu) \), \( \mu \), such that \( m_1(\mu) = \rho \leftrightarrow \mu \geq \mu \), and \( \mu \) such that \( m_0(\mu) = \rho \leftrightarrow \mu \leq \mu \). To save on notation we assume throughout that \( \mu > \alpha \) and \( \mu < 1 - \alpha \). This implies that with a rating of \( \mu \) the project fails to be refinanced with probability one, while with a rating of \( \mu \) the project is able to roll over its debt with certainty.

Let \( p(R, y) \) denote the probability of success given rating \( R \), signal \( y \), and naive investors. Define \( \gamma \) such that \( p(\mu, \gamma) = \frac{1}{2} \). The following proposition shows conditions for when the CRA will pursue a switching strategy

**Proposition 2** \( \max_{r \in \mathbb{R}} \Pi(r, \gamma) = \max\{\Pi(\mu, \gamma), \Pi(\mu, \gamma)\} \). Moreover

1. \( \Pi(\mu, \gamma) = \max_{r \in \mathbb{R}} \Pi(r, \gamma) \Rightarrow \Pi(\mu, y) = \max_{r \in \mathbb{R}} \Pi(r, y) \text{ for all } y < \gamma \)

2. \( \Pi(\mu, \gamma) = \max_{r \in \mathbb{R}} \Pi(r, \gamma) \Rightarrow \Pi(\mu, y) = \max_{r \in \mathbb{R}} \Pi(r, y) \text{ for all } y > \gamma \)

**Proof.** That \( \frac{\partial \Pi}{\partial y}(r, y) \geq 0 \) is obvious. Observe also, using proposition 1, that \( \frac{\partial p}{\partial r}(r, y) \geq 0 \).

Next, we have

\[
\Pi(r, y) = \phi p(r, y) + m_1(r) p(r, y) + m_0(r) \left( 1 - p(r, y) \right) = m_0(r) + p(r, y) \left( \phi + m_1(r) - m_0(r) \right)
\]

So

\[
\frac{\partial \Pi}{\partial r}(r, y) = \frac{\partial p}{\partial r}(r, y) \left( \phi + m_1(r) - m_0(r) \right) + m_1'(r) \left( 2p(r, y) - 1 \right)
\]
By definition of \( \hat{\mu} \) and \( \hat{y} \) we thus have \( \frac{\partial \Pi}{\partial r}(\hat{\mu}, \hat{y}) = 0 \). Observe in addition that \( \frac{\partial \Pi}{\partial r}(r, \hat{y}) \geq 0 \) if \( r > \hat{\mu} \), while \( \frac{\partial \Pi}{\partial r}(r, \hat{y}) \leq 0 \) if \( r < \hat{\mu} \). This finishes to show that \( \max_{r \in \mathbb{R}} \Pi(r, \hat{y}) = \max\{\Pi(\mu, \hat{y}), \Pi(\hat{\mu}, \hat{y})\} \).

Observe next that

\[
\Pi(r, y) - \Pi(r, \hat{y}) = \left( m_1(r) - m_0(r) \right) \left( p(r, y) - p(r, \hat{y}) \right)
\]

Thus, \( y < \hat{y} \) implies

\[
\Pi(r, y) - \Pi(r, \hat{y}) \begin{cases} 
\geq 0 & \text{if } r < \hat{\mu} \\
\leq 0 & \text{if } r > \hat{\mu}
\end{cases}
\]

While \( y > \hat{y} \) implies

\[
\Pi(r, y) - \Pi(r, \hat{y}) \begin{cases} 
\leq 0 & \text{if } r < \hat{\mu} \\
\geq 0 & \text{if } r > \hat{\mu}
\end{cases}
\]

The second part of the proposition now follows once we note using (8) that (i) \( \frac{\partial p}{\partial r}(r, y) \geq 0 \) if \( y > \hat{y} \) and \( r > \hat{\mu} \), while (ii) \( \frac{\partial p}{\partial r}(r, y) \leq 0 \) if \( y < \hat{y} \) and \( r < \hat{\mu} \). □

Consider, to illustrate the intuition, a random variable with two possible outcomes – A and B – and a bettor placing 1 unit of cash across outcomes. Suppose further that the probability \( p \) of A increases with cash \( r \) placed on A. It trivially follows from \( p(\mu, \hat{y}) = 0 > \frac{1}{2} \) that the bettor maximizes his gains by betting all on A.

Proposition 2 is central. It implies in particular that a fully revealing \( (R(y) = y) \) PBE does not exist, while also indicating that polar ratings are particularly attractive from a RA’s viewpoint. RAs receiving low signals place high probability on failure, inducing them to announce (overly) pessimistic ratings with a view to reap high reputation rewards should failure materialize. In a similar way, high signals tend to induce RAs to announce (overly) optimistic ratings with a view to reap high reputation rewards should success materialize. These considerations suggest examining a special class of rating strategies, namely switching strategies.

A strategy \( R(.) \) is a switching rating strategy if and only if there exists \( y^* \) such that \( R(y) = (-\infty, y^*) \) for \( y < y^* \) and \( R(y) = (y^*, +\infty) \) for \( y > y^* \).

We will say that a PBE of the game between RA and investors is a switching-PBE if it entails a PBE strategy on the part of the RA which moreover is a switching rating strategy.

We will say that a switching-PBE is optimistic if the RA tends to report ‘good news’, and pessimistic if it often reports ‘bad news’. Specifically, a switching-PBE is
1. optimistic if $y^* < (2\beta + 1)(1 - t) - \alpha$

2. neutral if $(2\beta + 1)(1 - t) - \alpha \leq y^* \leq 1 + \alpha - (2\beta + 1)t$

3. pessimistic if $1 + \alpha - (2\beta + 1)t < y^*$

Switching rating strategies are particularly attractive analytically, owing to the fact that they induce simple payoff structures. Observe from (1) that, given threshold rating strategy $R(.)$

$$
\Pi = \phi I + \rho_0 (1 - I) L_{(R = (-\infty, y^*)}) + \rho_1 IL_{(R = (y^*, +\infty))}
$$

where $L_{(R = (-\infty, y^*)}) = 1$ if the CRA gives a rating $R = (-\infty, y^*)$, while $L_{(R = (y^*, +\infty))} = 1$ if the rating is $R = (y^*, +\infty)$.

Note that we allow here for the possibility of differentiated reputation rewards associated with success ($\rho_1$) or failure ($\rho_0$). We assume throughout that $\phi < \rho_0$.

Our next proposition shows existence and uniqueness of a threshold-PBE, and provides parametric conditions for different regimes.

**Proposition 3** There exists a unique threshold-PBE. This PBE is

1. Optimistic if $\frac{\rho_0}{\rho_1} < \frac{\beta(1-t)}{1-\alpha(1-t)}$,

2. Neutral if $\frac{\beta(1-t)}{1-\alpha(1-t)} \leq \frac{\rho_0}{\rho_1} \leq \frac{1-\beta t}{\alpha t}$,

3. Pessimistic if $\frac{1-\beta t}{\alpha t} < \frac{\rho_0}{\rho_1}$.

**Proof.** A threshold-PBE with threshold $y^*$ satisfies

$$
\begin{cases}
\Pi((y^*, +\infty), y) > \Pi((-\infty, y^*), y) \implies y > y^*
\\
\Pi((-\infty, y^*), y) > \Pi((y^*, +\infty), y^*, y) \implies y < y^*
\end{cases}
$$

Let $p(R, y)$ denote the probability of success given rating $R$, signal $y$, and naive investors, so that

$$
\Pi((-\infty, y^*), y) = \phi p((-\infty, y^*), y) + \rho_0 [1 - p(0, y^*, y)] = \rho_0 - (\rho_0 - \phi) p((-\infty, y^*), y)
$$

$$
\Pi((y^*, +\infty), y) = (\phi + \rho_1) p((y^*, +\infty), y)
$$
For $R \in \{(-\infty, y^*), (y^*, +\infty)\}$

$$p(R, y) = \begin{cases} 
1 & \text{if } \theta^*(R) \leq y - \alpha \\
\frac{y + \alpha - \theta^*(R)}{2\alpha} & \text{if } y - \alpha < \theta^*(R) < y + \alpha \\
0 & \text{if } y + \alpha \leq \theta^*(R)
\end{cases}$$

(16)

So $p(R, y)$ is continuous in $y$ for $R \in \{(-\infty, y^*), (y^*, +\infty)\}$. A threshold-PBE must therefore satisfy the indifference condition

$$\Pi((-\infty, y^*), y^*) = \Pi((y^*, +\infty), y^*)$$

(17)

We go on to show that there exists a unique $y^*$ satisfying (17).

By (16) and lemmas 1 and 2

$$p((-\infty, y^*), y^*) = \begin{cases} 
0 & \text{if } y^* \leq 1 - \alpha \\
\frac{y^* + \alpha - \theta^*_1(y^*)}{2\alpha} & \text{if } 1 - \alpha < y^* < (1 - t) + \alpha \\
1 & \text{if } (1 - t) + \alpha \leq y^*
\end{cases}$$

(18)

$$p((y^*, +\infty), y^*) = \begin{cases} 
0 & \text{if } y^* \leq (1 - t) - \alpha \\
\frac{y^* + \alpha - \theta^*_1(y^*)}{2\alpha} & \text{if } (1 - t) - \alpha < y^* < \alpha \\
1 & \text{if } \alpha \leq y^*
\end{cases}$$

(19)

Hence,

$$\Pi((-\infty, y^*), y^*) = \begin{cases} 
\rho_0 & \text{if } y^* \leq 1 - \alpha \\
\frac{(\rho_0 - \phi) y^* + \alpha - \theta^*_1(y^*)}{\phi} & \text{if } 1 - \alpha < y^* < (1 - t) + \alpha \\
\phi & \text{if } (1 - t) + \alpha \leq y^*
\end{cases}$$

(20)
\[ \Pi((y^*, +\infty), y^*) = \begin{cases} 
0 & \text{if } y^* \leq (1-t) - \alpha \\
(\phi + \rho_1) \frac{y^* + \alpha - \theta_1(y^*)}{2\alpha} & \text{if } (1-t) - \alpha < y^* < \alpha \\
(\phi + \rho_1) & \text{if } \alpha \leq y^* 
\end{cases} \]  

And, using lemmas 1 and 2 again

\[ \Pi((\infty, y^*), y^*) = \begin{cases} 
\rho_0 & \text{if } y^* \leq 1 - \alpha \\
\rho_0 - (\rho_0 - \phi) \frac{y^* + \alpha - (1-t)}{2\alpha} & \text{if } 1 - \alpha < y^* < (2\beta + 1)(1-t) - \alpha \\
\rho_0 - (\rho_0 - \phi) \frac{y^* + \alpha - (1-t)}{2\alpha} & \text{if } (2\beta + 1)(1-t) - \alpha < y^* < (1-t) + \alpha \\
(\phi + \rho_1) & \text{if } (1-t) + \alpha \leq y^* 
\end{cases} \]  

\[ \Pi((y^*, +\infty), y^*) = \begin{cases} 
0 & \text{if } y^* \leq (1-t) - \alpha \\
(\phi + \rho_1) \frac{y^* + \alpha - (1-t)}{2\alpha} & \text{if } (1-t) - \alpha < y^* < 1 + \alpha - (2\beta + 1)t \\
(\phi + \rho_1) \frac{y^* + \alpha - (1-t)}{2\alpha} & \text{if } 1 + \alpha - (2\beta + 1)t < y^* < \alpha \\
(\phi + \rho_1) & \text{if } \alpha \leq y^* 
\end{cases} \]  

This finishes to prove the claim that there exists a unique \( y^* \) satisfying (17). Moreover, it entails a perfect Bayesian equilibrium since by (14), (15), and (16) we have \( \frac{\partial \Pi((\infty, y^*), y)}{\partial y} \leq 0 \) and \( \frac{\partial \Pi((y^*, +\infty), y)}{\partial y} \geq 0 \).

The PBE is optimistic if and only if

\[ \Pi((y^*, +\infty), y^*)|_{y^*=(2\beta+1)(1-t)-\alpha} > \Pi((\infty, y^*), y^*)|_{y^*=(2\beta+1)(1-t)-\alpha} \]  

It is pessimistic if and only if

\[ \Pi((y^*, +\infty), y^*)|_{y^*=1+\alpha-(2\beta+1)t} < \Pi((\infty, y^*), y^*)|_{y^*=1+\alpha-(2\beta+1)t} \]  

This completes the proof, using (22) and (23) for computation. □

Our next proposition characterizes the different equilibrium regimes, showing that if a regime is optimistic (pessimistic) then investors’ behavior is affected only by ‘bad news’ (‘good
news'). We recall that $\theta^*((-\infty, +\infty)) = 1 - t$.

**Proposition 4**.

1. $\theta_1^*(y^*) = 1 - t < \theta_0^*(y^*)$ in any optimistic regime
2. $\theta_1^*(y^*) = 1 - t = \theta_0^*(y^*)$ in any neutral regime
3. $\theta_1^*(y^*) < 1 - t = \theta_0^*(y^*)$ in any pessimistic regime

**Proof.** This follows from lemmas 1 and 2, and observing that

$$(2\beta + 1)(1 - t) - \alpha \leq 1 + \alpha - (2\beta + 1)t$$

Our next proposition shows that optimistic regimes are never preventive of, but may be conducive to crises.

**Proposition 5** Consider an optimistic regime and let $R^O(.)$ the unique threshold-PBE rating strategy.

1. $\text{Prob}(\theta^*(R^O(y)) < \theta < 1 - t) = 0$
2. $\text{Prob}(1 - t < \theta < \theta^*(R^O(y))) > 0$ if $\rho_0 > (\phi + \rho_1)\frac{t}{2\alpha}$

**Proof.** (1) follows immediately from proposition 4. We next turn to the proof of (2). Suppose we can find parameter values such that the unique threshold-PBE is optimistic with threshold $y^* > 1 - \alpha$. Let $R^O(.)$ denote this hypothetical rating strategy. Notice, using proposition 4, that

$$\text{Prob}(1 - t < \theta < \theta^*(R^O(y))) = \text{Prob}(y < y^*, 1 - t < \theta < \theta_0^*(y^*))$$

But by lemma 1 and given $y^* > 1 - \alpha$

$$1 - t < \theta_0^*(y^*) < 1 < y^* + \alpha$$
So
\[ \text{Prob}(1 - t < \theta < \theta^*(R^O(y))) > 0 \] (29)

The proof is complete once we observe, using (22) and (23), that
\[ \Pi((y^*, +\infty), y^*)_{y^*=1-\alpha} < \Pi((\infty, y^*), y^*)_{y^*=1-\alpha} \Leftrightarrow \rho_0 > (\phi + \rho_1) \frac{t}{2\alpha} \] (30)

Our final proposition shows that pessimistic regimes are never conducive to, but may be preventive of crises.

**Proposition 6** Consider a pessimistic regime and let \( R^P(.) \) the unique threshold-PBE rating strategy.

1. \( \text{Prob}(1 - t < \theta < \theta^*(R^P(y))) = 0 \)

2. \( \text{Prob}(\theta^*(R^P(y)) < \theta < 1 - t) > 0 \) if \( \rho_1 > (\rho_0 - \phi) \frac{1-t}{2\alpha} \)

**Proof.** (1) follows immediately from proposition 4.

We next turn to the proof of (2). Suppose we can find parameter values such that the unique threshold-PBE is pessimistic with threshold \( y^* < \alpha \). Let \( R^P(.) \) denote this hypothetical rating strategy. Notice, using proposition 4, that
\[ \text{Prob}(\theta^*(R^P(y)) < \theta < 1 - t) = \text{Prob}(y > y^*, \theta_1^*(y^*) < \theta < 1 - t) \] (31)

But by lemma 2 and given \( y^* < \alpha \)
\[ y^* - \alpha < 0 < \theta_1^*(y^*) < 1 - t \] (32)
So

\[ \text{Prob}(\theta^*(R^P(y)) < \theta < 1 - t) > 0 \] (33)

The proof is complete once we observe, using (22) and (23), that

\[ \Pi((y^*, +\infty), y^*)_{y^*=\alpha} > \Pi((-\infty, y^*), y^*)_{y^*=\alpha} \Leftrightarrow \rho_1 > (\rho_0 - \phi) \frac{1-t}{2\alpha} \] (34)

An implication of this finding is that CRAs may on average prevent premature liquidation of socially beneficial projects.

4 Conclusion

We have shown how credit rating agencies may affect real outcomes by serving as coordination device in a situation with a roll-over problem. Whether the CRA ultimately causes or prevents early liquidation depends on its payoff structure. We show that if CRA incentives are biased toward excessively positive ratings, they can only affect outcomes by causing inefficient default. Conversely, if CRAs incentives are biased in the opposite direction, toward excessively negative ratings, they may on average improve efficiency by raising the probability that projects with a long horizon receive sufficient financing to mature. Hence, their incentive structure should be biased in a pessimistic direction, placing higher weight on correctly predicting defaults rather than correctly predicting project successes.

5 References


6 Appendix

Let $A$ denote the point in $(x^*, \theta^*)$-space with coordinates $(-\beta, 0)$.

Let $B$ denote the point in $(x^*, \theta^*)$-space with coordinates $(1 + \beta, 1)$.

The equation of the line segment $AB$ is

$$\theta^* = \frac{x^* + \beta}{2\beta + 1} \quad (35)$$

Let $L$ denote the line in $(x^*, \theta^*)$-space with equation

$$\theta^* = x^* - \beta + 2\beta t \quad (36)$$

Let $C(y)$ denote the point in $(x^*, \theta^*)$-space with coordinates $(y - \alpha - \beta, y - \alpha)$.

Let $D(y)$ denote the point in $(x^*, \theta^*)$-space with coordinates $(y - \alpha + \beta, y - \alpha + 2\beta t)$.

Let $E(y)$ denote the point in $(x^*, \theta^*)$-space with coordinates $(y + \alpha - \beta, y + \alpha - 2\beta(1 - t))$.

Let $F(y)$ denote the point in $(x^*, \theta^*)$-space with coordinates $(y + \alpha + \beta, y + \alpha)$.
The equation of the line segment $C(y)D(y)$ is
\[
\theta^* = t(x^* + \beta) + (1 - t)(y - \alpha)
\] (37)

The equation of the line segment $E(y)F(y)$ is
\[
\theta^* = (1 - t)(x^* - \beta) + t(y + \alpha)
\] (38)

Note that:
1. $D(y) \in L$ and $E(y) \in L$ for all $y$
2. $C(y) = A$ for $y = \alpha$, and $F(y) = B$ for $y = 1 - \alpha$.

Let $L(y)$ denote the half-line starting at $D(y)$ and coinciding with $L$ ‘to the right of’ $D(y)$.
Let $L(y)$ denote the half-line starting at $E(y)$ and coinciding with $L$ ‘to the left of’ $E(y)$.

Lemma 1
\[
\theta^*_0(y^*) = \begin{cases} 
1 & \text{if } y^* \leq 1 - \alpha \\
\frac{i(y^*+\alpha)-(1-t)2\beta}{1-(1-t)(2\beta+1)} & \text{if } 1 - \alpha < y^* < (2\beta + 1)(1 - t) - \alpha \\
1 - t & \text{if } (2\beta + 1)(1 - t) - \alpha < y^* < +\infty
\end{cases}
\] (39)

Proof. Clearly, whichever $y^*$, there does not exist a corner equilibrium with $l = 0$.

We next show that a corner equilibrium with $l = 1$ exists if and only if $y^* < 1 - \alpha$. Suppose $y^* < 1 - \alpha$ and all investors always sell. Any investor’s belief then entails
\[
\theta \leq y^* + \alpha < 1 = l
\] (40)

An equilibrium thus exists in which all investors always sell. Suppose on the other hand $y^* > 1 - \alpha$. An investor receiving signal $x \geq 1 + \beta$ has posterior belief $\theta \geq 1$, and so chooses not to sell. So an equilibrium in which all investors always sell does not exist.

We next explore interior equilibria. An interior equilibrium $(x^*_0(y^*), \theta^*_0(y^*))$ satisfies
\[
\begin{align*}
\theta^*_0(y^*) &= \text{Prob}(x < x^*_0(y^*)|\theta = \theta^*_0(y^*)) \\
\text{Prob}(\theta < \theta^*_0(y^*)) &= x^*_0(y^*, y < y^*) = t
\end{align*}
\] (41)

Which, by (35)-(38), is equivalent to
\[
(x^*_0(y^*), \theta^*_0(y^*)) \in AB \bigcap \left( E(y^*)F(y^*) \bigcup L(y^*) \right)
\] (42)
The Lemma now follows from simple computation.

Lemma 2

$$\theta_1^*(y^*) = \begin{cases} 
1 - t & \text{if } y^* < 1 + \alpha - (2\beta + 1)t \\
\frac{(1-t)(y^* - \alpha)}{1-t(2\beta+1)} & \text{if } 1 + \alpha - (2\beta + 1)t < y^* < \alpha \\
0 & \text{if } y^* \geq \alpha 
\end{cases} \quad (43)$$

Proof. Clearly, whichever $y^*$, there does not exist a corner equilibrium with $l = 1$.

We next show that a corner equilibrium with $l = 0$ exists if and only if $y^* > \alpha$. Suppose $y^* > \alpha$ and no investor ever sells. Any investor’s belief then entails

$$\theta \geq y^* - \alpha > 0 = l \quad (44)$$

An equilibrium thus exists in which no investor ever sells. Suppose on the other hand $y^* < \alpha$. An investor receiving signal $x \leq -\beta$ has posterior belief $\theta \leq 0$, and so chooses to sell. So an equilibrium in which no investor ever sells does not exist.

We next explore interior equilibria. An interior equilibrium $(x_1^*(y^*), \theta_1^*(y^*))$ satisfies

$$\begin{cases} 
\theta_1^*(y^*) = Prob(x < x_1^*(y^*)|\theta = \theta_1^*(y^*)) \\
Prob(\theta < \theta_1^*(y^*)|x = x_1^*(y^*), y > y^*) = t 
\end{cases} \quad (45)$$

Which, by (35)-(38), is equivalent to

$$(x_1^*(y^*), \theta_1^*(y^*)) \in AB \bigcap \left( C(y^*)D(y^*) \bigcup \bar{L}(y^*) \right) \quad (46)$$

The Lemma now follows from simple computation.