Financial frictions and shocks

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Abstract

This paper aims to quantify the extent to which sources of economic fluctuations generate in the financial markets. First, a novel identification method is introduced into a Bayesian VAR model in order to identify a financial type shock which we refer to as a ‘risk news’ shock. We identify the risk news shock in macroeconomic time series for the US, while simultaneously identifying other standard macroeconomic shocks with the use of sign restrictions. Motivated by Barsky and Sims (2011), the identified risk news shock is constructed to be uncorrelated with today’s volatility, but to contribute maximally to future volatility. Our VAR evidence suggests that the risk news shock is small, but yet contributes significantly to movements in the business cycle. Second, we estimate a standard New Keynesian DSGE model with financial frictions, modified as in Christiano, Motto, and Rostagno (2010) to incorporate a risk news shock, and compare the impulse responses to those implied by the VAR. We use simulation methods to check for the suitability of our identification strategy. The main message of the paper is that it provides evidence that a small shock in the financial system can cause substantial effects on real variables.

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1 Introduction

Could a small shock generated in the financial system have large effects on the macroeconomy? Classic papers such as Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999) have analyzed frictions in the financial system through which traditional shocks such as productivity shocks are amplified, leading to large effects on the macroeconomy. However, subsequent literature struggled to find empirical evidence in support of the importance of financial frictions in the presence of traditional shocks, which even lead to the questioning of the relevance of such frictions. We argue that it is the combination of financial frictions and the shocks generated within the financial system that leads to empirically significant amplification mechanisms.

Our starting point is the definition of a risk shock proposed by Christiano, Motto, and Rostagno (2010). They introduce an anticipated, news-type of shock that affects the future dispersion of returns to the projects that the credit constrained entrepreneurs have access to. We aim to identify this shock in the data in a way that is less dependent on the structure of a complicated DSGE model.

As a first step, we take a 10-variable VAR for the US, comprising the 7-variable data-set of Smets and Wouters (2007), augmented with 3 financial variables. We then identify a shock to anticipated future risk motivated by Christiano, Motto, and Rostagno (2010). Following the terminology used by the authors in the ‘news’ literature, we shall refer to this identified shock as a ‘risk news’ shock. Our identification strategy is based on the method of Uhlig (2004) and more directly on (Barsky and Sims, 2011) who aimed to identify news shocks in case of productivity. We make a methodological contribution by identifying a risk news shock while simultaneously identifying other structural shocks by incorporating sign restrictions.

We find that the risk news shock appears to be relatively small - in that it contributes in a small way to fluctuations in the proxy for volatility we introduce into the VAR - but yet accounts for quite large amounts of the fluctuations in observables such as inflation, consumption and investment. In addition, if our proposed identification strategy is to be believed, it appears to perform well at generating impulse responses that are in line with the observed dynamics during the recent financial crisis.

As a second step, we compare these impulses responses to those implied by a structural model. We take a standard New Keynesian model based on Smets and Wouters (2007), and extend it with the financial frictions outlined by Bernanke, Gertler, and Gilchrist (1999). Based on Christiano, Motto, and Rostagno (2010), we introduce the risk news shock to the model and we bring the specification of this shock process as close as possible to resembling that used in the VAR. Subsequently, we use a limited
information econometric method to estimate the model. We compare the estimated impulse responses to those implied by the VAR, and find that the dynamics implied by the DSGE model match well with the dynamics implied by the VAR.

As a final step, we follow the recommendation of Chari, Kehoe, and McGrattan (2008), and we apply our proposed identification strategy to a set of simulated data-sets generated by the DSGE model. We find that the simulated impulse responses are very close to the impulse responses of the DSGE model. We interpret this results as a confirmation of our identification strategy.

What is our ‘risk news shock’ and why is it an interesting thing to study? One way of answering this is to give an account of the econometric mechanics of the shock and its identification. Barsky and Sims (2011) identified a productivity news shock as a shock that was constructed to be (i) uncorrelated with today’s productivity, yet contributed maximally to the forecast error variance of productivity at some finite horizon in the future. Our ‘risk news’ shock is identified as (i) uncorrelated with contemporaneous values for our proxy for uncertainty, (ii) contributing maximally to forecast error variances of that volatility proxy at some future horizon and (iii) satisfies the sign restriction that increases in future uncertainty raise the spread, reduces GDP, investment, net worth and inflation.

A second part of the explanation of what our ‘risk news’ shock is follows from work by Christiano, Motto, and Rostagno (2010) that introduces an anticipated risk shock into a medium scale DSGE model - with various real and nominal frictions - modified to include the financial accelerator proposed by Bernanke, Gertler, and Gilchrist (1999). In the financial accelerator mechanism, a key object is the distribution of returns from which entrepreneurs’ projects will be drawn. Christiano, Motto, and Rostagno (2010) allow for the variance of this distribution itself to be a stochastic process. Moreover, they allow that entrepreneurs receive advance signals about the variance of returns in the future which will prevail. These signals constitute one possible theoretical counterpart to the empirical object which we uncover from our VAR: hence our desire ultimately to fit a DSGE model with such shocks is implied by our identified risk news shock.

The motivation for studying risk news shocks is twofold. First, our work flows from the literature on other types of news shocks. The news literature takes as its motivation the simple observation that if agents can be forward-looking and are to be considered intelligent gatherers of information about now and the future, we may think of them as potentially receiving signals about future states of the world. Barsky and Sims (2011) look for a productivity news shock on the grounds that just as the possibility frontier may be shifted around by contemporaneous changes in technology, agents may receive news about future such
changes. Our hunt for ‘risk news’ shocks is motivated by applying this logic to economic uncertainty. If we can think of this as itself random, and agents are forward looking, we might think that they get signals about future uncertainty.

A second motivation is that we know - partly from Christiano, Motto, and Rostagno (2010) - that risk news shocks improve the ability of DSGE models with financial frictions to match the macro data. A one period contemporaneous shock to the variance of the returns to entrepreneurs’ returns reduces the return on capital for one period only. If it is simultaneously assumed that there are investment adjustment costs - as Smets and Wouters (2007) assumed, on the grounds that investment dynamics are a puzzle without such costs - investment will not respond to this one period fall in the return on capital. It will not be worth paying the adjustment costs to reduce and, soon after, increase investment again. However, if a shock brings about unexpected news about future risks, then this will induce a more persistent fall in the return on capital, thus giving firms an incentive to reduce investment. This risk news shock will cause spreads to rise and investment to fall, something that is taken by many to be a stylized fact of the business cycle.

Our paper is related to two main strands of literature. Firstly, our paper builds on the macroeconomic literature with financial frictions. Our model is a variant of Smets and Wouters (2007) and Justiniano and Preston (2010), and builds on models with the financial accelerator mechanism as in Bernanke, Gertler, and Gilchrist (1999), Meier and Muller (2006), De Graeve (2008), Gilchrist, Yankov, and Zakrajsek (2009), von Heideken (2009) and Gelain (2010). In addition, we build on Nolan and Thoenissen (2009) and Christiano, Motto, and Rostagno (2010) that focus on shocks generated within the financial sector.

Secondly, our method of identifying financial shocks contributes to the literature on news shocks as in Beaudry and Portier (2004), Schmitt-Grohe and Uribe (2008), Jaimovich and Rebelo (2008), Blanchard, L’Huillier, and Lorenzoni (2009), Leeper and Walker (2011) and Barsky and Sims (2011). More specifically, we follow Christiano, Motto, and Rostagno (2010) in defining financial shock as that of an anticipated, news-type, which may be thought of as series of signals about the cross-section dispersion of entrepreneurial productivity. This shock causes surprise changes in the current perception about the future conditions of financial markets, and to this extent, it complements the volatility shocks analyzed by Bloom (2009) and Villaverde (2010).

The rest of the paper is organized as follows. Section 2 explains the empirical strategy to identify a financial risk shock as that of a news type, and presents the evidence of the VAR. Section 3 presents the log-linearized version of a standard DSGE model with financial frictions. It explains the estimation
strategy that is used to match the impulse responses of the VAR to those implied by the DSGE, and
does not result in order to test the suitability of our identification strategy. Section 5 concludes.

2 Empirical evidence

2.1 Data

In this section, we provide time series evidence on the effects of a financial and other structural shocks.
We use quarterly US data on seven macroeconomic variables and three financial variables, and the sample
period is 1980Q1 - 2010Q2. The seven macroeconomic variables are the same as used by Smets and
Wouters (2007): log difference of real GDP, real consumption, real gross investment and real wage, log
hours worked, the log difference of the GDP deflator and the federal funds rate. The financial series are the
difference between BBA and AAA corporate bond yields – a measure for the external finance premium,
the per capita Dow Jones Wilshire index deflated by the GDP deflator\textsuperscript{1} – a proxy for entrepreneurial net
worth – and VIX\textsuperscript{2} – it illustrates the degree of implied financial uncertainty. A more detailed description
of the data can be found in the appendix.

2.2 VAR model

The starting point of our empirical analysis is a vector autoregressive model of order \( K = \text{VAR}(K) \)

\[
y_t = \sum_{i=1}^{K} \Theta_i y_{t-i} + u_t
\]

where \( u_t \) is the \( N \times 1 \) vector of reduced-form errors that is normally distributed with zero and \( \Sigma \) variance-
covariance matrix. The regression-equation representation of the latter system is

\[
Y = X\Psi + V
\]

where \( Y = [y_{h+1},..,y_T] \) is a \( N \times T \) matrix containing all the data points in \( y_t \), \( X = Y_{-h} \) is a \( (NK) \times T \) matrix containing the \( h \)-th lag of \( Y \), \( \Theta = \begin{bmatrix} \Theta_1 & \ldots & \Theta_K \end{bmatrix} \) is a \( N \times (NK) \) matrix, and \( U = [u_{h+1},..,u_T] \) is a \( N \times T \) matrix of disturbances.

\textsuperscript{1}As in Christiano, Motto, and Rostagno (2010).
\textsuperscript{2}VIX is a popular measure of the implied volatility of S&P 500 index options.
The model has seven variables and three lags and this immediately implies a large number of parameters, whose estimation poses serious difficulties even with 50 years of macroeconomic data. Classical inference techniques will deliver estimates that are subject to enormous uncertainty, meaning that Bayesian procedures are required. Priors are used to shrink the number of the estimated parameters by focusing on some of them and ignoring others. An obvious choice can be Minnesota type priors as in \textcite{DoanLittermanSims1983} and \textcite{Litterman1986}, since they shrink the VAR($K$) model towards independent autoregressive of order one - AR(1) - models. Furthermore, evidence provided by \textcite{BanburaGiannoneReichlin2010} suggests that large VAR models achieve very good forecasting properties when they combined with Minnesota type prior information.

The posterior inference is obtained as follows. It is assumed that the prior distribution of the VAR parameter vector has a Normal-Wishart conjugate form

\[
\theta | \Sigma \sim N(\theta_0, \Sigma \otimes \Omega_0), \quad \Sigma \sim IW(v_0, S_0). \tag{2}
\]

where $\theta$ is obtained by stacking the columns of $\Theta$. The prior moments of $\theta$ are given by

\[
E[(\Theta_k)_{i,j}] = \begin{cases} 
\delta_i & i = j, k = 1 \\
0 & \text{otherwise} 
\end{cases}, \quad \text{Var}[(\Theta_k)_{i,j}] = \lambda \sigma_i^2 / \sigma_j^2,
\]

and as it is explained by \textcite{BanburaGiannoneReichlin2010} they can be constructed using the following dummy observations

\[
Y_D = \begin{pmatrix} 
\text{diag}(\delta_1 \sigma_1, \ldots, \delta_N \sigma_N) \\
0_{N \times (K-1)N} \\
\ldots \\
\text{diag}(\sigma_1, \ldots, \sigma_N) \\
\ldots \\
0_{1 \times N} 
\end{pmatrix} \quad \text{and} \quad X_D = \begin{pmatrix} 
J_K \otimes \text{diag}(\sigma_1, \ldots, \sigma_N) \\
0_N \times NK \\
\ldots \\
0_{1 \times NK} 
\end{pmatrix} \tag{3}
\]

where $J_K = \text{diag}(1, 2, \ldots, K)$ and $\text{diag}$ denotes the diagonal matrix. The prior moments of (2) are just functions of $Y_D$ and $X_D$, $\Theta_0 = Y_D X_D' (X_D X_D')^{-1}$, $\Omega_0 = (X_D X_D')^{-1}$, $S_0 = (Y_D - \Theta_0 X_D) (Y_D - \Theta_0 X_D)'$ and $v_0 = T_D - N K$. Finally, the hyper-parameter $\lambda$ controls the tightness of the prior.

Since the normal-inverted Wishart prior is conjugate, the conditional posterior distribution of this
model is also normal-inverted Wishart [Kadiyala and Karlsson (1997)]

\[ \theta | \Sigma, Y \sim N(\tilde{\theta}, \Sigma \otimes \tilde{\Omega}), \Sigma | Y \sim IW(\tilde{v}, \tilde{S}), \] (4)

where the bar denotes that the parameters are those of the posterior distribution. Defining \( \hat{\Theta} \) and \( \hat{U} \) as the OLS estimates, we have that \( \tilde{\Theta} = (\Omega_0^{-1}Y_0 + Y'X)(\Omega_0^{-1} + X'X)^{-1}, \tilde{\Omega} = (\Omega_0^{-1} + X'X)^{-1}, \tilde{v} = v_0 + T, \)

and \( \bar{S} = \hat{\Theta}XX'\hat{\Theta'} + \Theta_0\Omega_0^{-1}\Theta_0 + S_0 + \hat{U}U' - \tilde{\Theta}\tilde{\Omega}^{-1}\Theta'. \)

The values of the persistence \( - \delta_i \) and the error standard deviation \( - \sigma_i \) parameters of the AR(1) model are obtained from its OLS estimation. Sensitivity analysis reveals that the results are robust to different selections of VAR lags. Finally, \( \lambda \) has been set equal to 4, implying relatively loose priors.

2.3 Identification

Consider moving average representation of the VAR(\( K \))

\[ y_t = B(L)u_t \] (5)

Under the assumption that a mapping between the reduced-form errors and the structural shocks exists:

\[ u_t = A\varepsilon_t \] (6)

such as \( AA' = \Sigma \), the \( h \) step ahead forecast error can be expressed as

\[ y_{t+h} - E_{t-1}y_{t+h} = \sum_{\tau=0}^{h} B_\tau A\tilde{Q}(\omega) \tilde{\varepsilon}_{t+h-\tau} \]

where \( \tilde{A} \) is the lower triangular matrix obtained from the Cholesky decomposition of \( \Sigma \) and \( Q \) is an orthonormal matrix such as \( Q(\omega)Q(\omega)' = I_{dy} \), where \( I_{dy} \) is the \( dy \times dy \) identity matrix and \( \omega \in \left[ 0 \ 2\pi \right] \) denotes the rotation angles.

The share of the forecast error variance of variable \( i \) attributable to the structural shock \( j \) at horizon \( h \) is written as:

\[ \Omega_{i,j}(h) = \frac{e_i' \left( \sum_{\tau=0}^{h} B_\tau A\tilde{Q}(\omega) e_j e_j' Q(\omega)' \tilde{A}' B_\tau \right) e_i}{e_i' \left( \sum_{\tau=0}^{h} B_\tau \Sigma B_\tau \right) e_i} \] (7)

where \( e_i \) denotes the selection vector with one in the \( i \)-th place and zeros elsewhere.
Similarly to Barsky and Sims (2011) and consistently with model discussed below we assume that the uncertainty process is exogenous and driven by two random disturbances the unanticipated shock $- \varepsilon_{\sigma,t}$ and the anticipated one $- \eta_{\text{news},t-1}$.

\[
\ln \sigma_{\omega,t} = (1 - \rho_\sigma) \sigma_\omega + \rho_\sigma \ln \sigma_{\omega,t-1} + \varepsilon_{\sigma,t} + \varepsilon_{\text{news},t-1} \quad (8)
\]

By allowing $\varepsilon_{\sigma,t}$ to be the first element of $\varepsilon$ and $\varepsilon_{\text{news},t-1}$ and the second then by assumption we get that

\[
\Omega_{1,1}(h) + \Omega_{1,2}(h) = 1 \quad (9)
\]

However, it is unlikely that equation (11) holds at all horizons in a multivariate VAR model. Hence, as suggested by Barsky and Sims (2011), we select the second column of the impact matrix $- \tilde{A}Q(\omega)$ -- that comes as close as possible to making equation (9) hold over a finite set of horizons.

Since we intend to identify other structural shocks as well, further conditions need to be imposed on the rotation matrix $Q(\omega)$. We identify three more “standard” macroeconomic -- productivity, demand, policy-rate -- shocks and we chose for this “the scheme known as sign-restrictions” introduced by Uhlig (2005) and Canova and Nicoletti (2002). Table 1 reports the “qualitative” restrictions that shocks need to satisfy.

<table>
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<tr>
<th>Table 1: Sign-Restrictions</th>
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<tr>
<td>VAR</td>
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<td>Uncertainty</td>
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<td>Spread</td>
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<td>GDP-Growth</td>
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<td>Consumption-Growth</td>
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<td>Investment-Growth</td>
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<td>Hours</td>
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<td>Wages-Growth</td>
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<td>Inflation</td>
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<td>Policy-Rate</td>
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<td>Net-Worth</td>
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In words, a positive supply shock increases output and net worth and decreases inflation and interest rates, while a negative demand shocks rises spreads and lowers gdp, investment, inflation and interest rate. Finally, a positive policy-rate shock increases interest rates, higher policy rates lower the demand – gdp and investment – and cause lower inflation. Table 1 presents the sign restrictions we impose in order to
identify the structural shocks. The anticipated, news-type shock increases the spread, and has negative contemporaneous effects on GDP, investment and net worth. The productivity shock has the standard characteristics: it increases GDP and net worth and lowers inflation and the interest rate. The demand shock is assumed to increase the spread and it lowers GDP, investment and the net worth. Finally, the monetary policy shock that increases the interest rate has a negative contemporaneous effect on GDP, investment, inflation and net worth. As explained earlier, all the structural shocks are assumed to have no contemporaneous effect on volatility - a key assumption that guarantees sufficient degree of exogeneity in the volatility variable so that equation (??) holds.

A by-product of our identification strategy is the surprise shock to volatility that we shall refer to as the contemporaneous risk shock. This shock by construction is orthogonal to the anticipated shock, and it captures contemporaneous innovations in the volatility. To this extent, this shock resembles the uncertainty shock analyzed by Bloom (2009).

2.4 Estimation

The VAR is estimated using Bayesian methods which helps overcome the curse of dimensionality resulting from the large number (10) of observables in the model. Based on Litterman (1986), Kadiyala and Karlsson (1997) and following the approach and the notation of Banbura, Giannone, and Reichlin (2010), we apply Minnesota priors to impose prior beliefs on the parameters. More specifically, we use a diagonal matrix as a prior, whereby the diagonal elements of the prior matrix are estimated AR(1) parameters corresponding to each individual variable in the VAR. In addition, we assume that the more recent lags should provide more information about the current values of the variables compared to distant lags. These beliefs are incorporated in the formation of the prior of the variance-covariance matrix $\Sigma$. Formally, it can be written as:

$$
B_{1,ij} = \begin{cases} 
\hat{\delta}_i, & j = i \\
0, & \text{otherwise}
\end{cases}, \quad \Sigma_{ij} = \begin{cases} 
\frac{\lambda^2}{k^2}, & j = i \\
\vartheta \frac{\lambda^2 \sigma_i^2}{\sigma_j^2}, & \text{otherwise}
\end{cases}
$$

(10)

where $B_{1,ij}$ is the prior of the parameter matrix $B_1$, whose diagonal elements are the individual AR(1) estimates, $\hat{\delta}_i$. The prior of the variance-covariance matrix is $\Sigma_{ij}$, and the hyper-parameter $\lambda$ controls the overall tightness of the prior distribution.\footnote{For $\lambda = 0$, the posterior equals the prior and the data is not informative about the estimates. For $\lambda = \infty$, the posterior coincides with the OLS estimates.} The factor $k^2$ captures the effect of the lag length, and $\frac{\sigma_i^2}{\sigma_j^2}$ controls for the different scale and variability of the data. To incorporate these beliefs to the VAR, we
first rewrite the model in (??) as a system of multivariate regressions:

\[ Y_{T \times n} = X_{T \times k} \cdot B_{k \times n} + U_{T \times n} \]  \hspace{1cm} (11)

where \( Y = (Y_1, \ldots, Y_T)' \) is the contemporaneous data matrix, \( X = (X_1, \ldots, X_T)' \) is the lagged data matrix with a constant, \( X = (Y_{t-1}', \ldots, Y_{t-p}', 1)' \) and \( U = (u_1, \ldots, u_T)' \) is the vector of residuals. The parameter matrix \( B \) contains \( k = np + 1 \) of parameters for each of the \( n \) equations. Prior information is incorporated through a multivariate Normal inverted Wishart of the following form:

\[
vec(B) \mid \Psi \sim N(vec(B_0), \Psi \otimes \Omega_0) \quad \text{and} \quad \Psi \sim iW(S_0, \alpha_0) \]  \hspace{1cm} (12)

where \( B_0, \Omega_0, S_0 \) and \( \alpha_0 \) are selected in a way so that prior moments of \( B \) coincide with those implied by the conditions described in (10), and the expectation of \( \Psi \) is equal to the fixed residual covariance matrix \( \Sigma \). As in Banbura, Giannone, and Reichlin (2010), we impose the priors as dummy observations, \( T_d \), to the data matrices, \( X \) and \( Y \). The dummy matrices, \( X_d \) and \( Y_d \), take the following form:

\[
Y_d = \begin{pmatrix}
    \text{diag}(\delta_1 \sigma_1, \ldots, \delta_n \sigma_n) / \lambda \\
    0_{n(p-1) \times n} \\
    \vdots \\
    \text{diag}(\sigma_1, \ldots, \sigma_n) \\
    \vdots \\
    0_{1 \times n}
\end{pmatrix} \\
X_d = \begin{pmatrix}
    J_p \otimes \text{diag}(\sigma_1, \ldots, \sigma_n) / \lambda & 0_{np \times 1} \\
    \vdots & \vdots \\
    0_{n \times np} & 0_{n \times 1} \\
    \vdots & \vdots \\
    0_{1 \times np} & \epsilon
\end{pmatrix} \]  \hspace{1cm} (13)

where \( J = \text{diag}(1, 2, \ldots, p) \). Using the definition of the augmented data matrices, \( Y_* = (Y', Y_d') \) and \( X_* = (X', X_d') \), the regression model (11) is rewritten as:

\[
Y_{* \times n} = X_{* \times k} \cdot B_{k \times n} + U_{* \times n} \]  \hspace{1cm} (14)

where the dimension of the data matrices and the residual vector, \( U_* \), has increased, \( T_* = T + T_d \). The posterior means of the parameter matrix \( B \) is then obtained by running OLS on (14).\(^4\)

\(^4\)See Banbura, Giannone, and Reichlin (2010) for an explanation of why these OLS estimates coincide with the posterior means of the Bayesian VAR with the Minnesota setup.
2.5 Empirical results

This section presents the empirical results obtained by estimating the Bayesian VAR and using the identification strategy explained in section 2.3. The magnitudes of all the shocks are normalized such that GDP growth increases by 1% on impact. Figure 1 displays the impulse response functions of all variables following an anticipated risk shock, which can be thought of as surprise news about the future conditions of financial markets. The shock causes a 1% increase in GDP growth, decreases spreads by 5 basis points, and raises consumption, investment and hours, while the impact on wages and inflation is uncertain. The interest rate increases on impact, whereas net worth displays a larger immediate increase.

The impulse responses to a contemporaneous shock are displayed on figure 2. A 1% increase in GDP growth is associated with a 40% drop in volatility and 25 basis points fall in the spread. Consumption, investment and hours respond positively, while wages fall on impact suggesting a shift in the labor supply curve.

It is interesting to compare the impulse responses induced by the anticipated and surprise risk shocks. To yield a relatively small (1%) increase in GDP growth, the anticipated risk shock has a substantially smaller impact on the financial variables than the contemporaneous risk shock does. Should these two financial shocks have the same magnitude in terms of their impact on the financial variables, the anticipated risk shock would have an enormous effect on other macroeconomic variables compared to the anticipated risk shock. We argue that these empirical results are in line with the implication of Christiano, Motto, and Rostagno, 2010, who pointed out that the anticipated nature of the risk shock is fundamental in generating large effects on the macroeconomy as a whole.

The impulse responses to a productivity shock are depicted in figure 3. A positive productivity shock causes a sizable and protracted fall in volatility and spreads which lasts throughout the forecast horizon. Consumption, wages and to a lesser extent investment increase on impact, while hours fall which corroborates the findings of the New Keynesian literature (Smets and Wouters, 2007). The accumulated negative impact on interest rates is larger than on inflation suggesting a fall in the real interest rate. Net worth increases by 10%, and falls back after six quarters.

The impulse responses to a monetary policy shock are shown in figure 4. An around 75 basis points unexpected decrease in the policy rate is associated with 1% and 0.5% increases in GDP and consumption, whereas the impact on investment and wages is surrounded by much uncertainty. Following the monetary policy loosening, hours fall persistently, while inflation and net worth increase sharply. Due to the expansionary policy, volatility and, to a smaller extent, spreads fall.
Figure 5 shows the impulse responses to a demand shock. Similar to GDP, consumption increases by 1%, while investment and net worth rise by more than 2% on impact. Hours worked increase significantly, while wages remain unchanged suggesting a shift in the labor demand curve. Inflation does not seem to respond, while the nominal interest rate displays a positive and protracted response implying an increase in the real interest rate.

Figure 6 shows the historical contribution of each of the five identified structural shocks (anticipated risk, contemporaneous risk, net worth, productivity and monetary shocks) to the variables under consideration over the past five years. This exercise can be thought of as a tentative attempt to identify the possible causes of the great recession. One of the key findings is that the anticipated risk shock explains relatively little of the variation in the financial variables (volatility and net worth), but it has a sizable contribution to variation in the macroeconomy as a whole, especially during the deepening of the crises.

The financial variables have been driven mainly by the surprise uncertainty shock. Productivity shocks seem to have played no role in the current recession exerting virtually no impact on output and investment, and affect hours, interest rates and inflation only marginally. Similarly, monetary policy shocks have only had a small impact on macroeconomic dynamics, and the effects on interest rates have vanished.

2.6 The anticipated risk shock in 2008

The magnitude of the risk shock is normalized such that the response of the volatility variable increases to the level observed in the 2008Q4.\(^{5}\) Spreads increase by almost 150 basis points. Output and consumption growth rates fall by about 5%, whereas investment drops by almost 30%. Hours fall significantly and persistently, while the response of real wages is uncertain. Inflation and the interest rates show highly persistent falls over the forecast horizon. Finally, real net worth experiences an immediate and large (20%) drop, but it seems to recover after 3 quarters.

To match these empirical results and the newly identified financial shocks to a more structural framework, the next section presents a DSGE model with financial frictions and shocks.

3 Fitting a DSGE to the risk news shock

This section proposes to fit our identified shocks to a DSGE model with financial frictions. We apply minimum distance estimation, a limited information method, to estimate the model for two related reasons.

\(^{5}\)Thereby, the response to of the volatility is set to 38 units - the peak of the detrended volatility variable during the crisis.
stochastic structure of the model. With 10 variables and potentially 10 independent sources of randomness, we have to find the same number of shocks as measurement errors. Limited information methods like the one we pursue are justified, if we do not have sufficient confidence to make the model choices necessary, and to make this structure explicit in its entirety. Christiano, Motto, and Rostagno (2010) and others have done just that in similar situations, but it seems to us that a complementary approach is justified.

3.1 Model Economy

The section presents the log-linearized DSGE model with financial frictions. The main structure of the model is based on Smets and Wouters (2007) (SW henceforth) and Justiniano and Preston (2010) (JPT henceforth), which is extended by the financial accelerator mechanism described by Bernanke, Gertler, and Gilchrist (1999) (BGG henceforth). The introduction of the BGG follows the treatment of De Graeve (2008) and Christiano, Motto, and Rostagno (2010), but a major difference is that we allow the presence of non-stationarity in our model. A detailed description of the equilibrium conditions is provided by section 6.3 in the appendix.

On the demand-side of the economy, the household maximizes intertemporal utility by trading-off current and future consumption and working hours in the presence of some degree of habit formation. The dynamics of aggregate consumption can be written as:

$$\hat{\lambda}_t = \frac{h\beta e^\gamma}{(e^\gamma - h\beta)(e^\gamma - h)}\hat{c}_{t-1} - \frac{e^{2\gamma} + h^2\beta}{(e^\gamma - h\beta)(e^\gamma - h)}\hat{c}_t + \frac{he^\gamma}{(e^\gamma - h\beta)(e^\gamma - h)}E_t\hat{c}_{t+1}$$

$$+ \frac{h\beta e^\gamma \rho_z - he^\gamma}{(e^\gamma - h\beta)(e^\gamma - h)}\hat{z}_t + \frac{e^\gamma - h\beta \rho_b}{(e^\gamma - h\beta)}\hat{b}_t$$

(15)

where $\beta$ is the subjective discount factor, $h$ is the degree of habit formation, $\gamma$ is the growth rate of aggregate productivity, $\hat{\lambda}_t$ is the shadow price of the budget constraint, $\hat{c}_t$ is aggregate consumption, $\hat{z}_t$ is the productivity shock with the corresponding AR(1) parameter, $\rho_z$, and $\hat{b}_t$ is the intertemporal preference shock with the corresponding AR(1) parameter $\rho_b$. The Euler-equation is written as:

$$\hat{\lambda}_t = \hat{R}_t + E_t (\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{\pi}_{t+1})$$

(16)

where $\hat{R}_t$ is the nominal interest rate and $\hat{\pi}_t$ denotes price inflation. Equation 15 and 16 show that aggregate consumption is a function of past consumption, expected future consumption, and unexpected changes in productivity and preferences. The household is a monopolistic supplier of differentiated labor,
which leads to certain levels of markup on real wages. However, real wages adjust only gradually to desired markups because of wage rigidity and indexation of wages to inflation:

\[
\dot{w}_t = \frac{1}{1 + \beta} \dot{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \ddot{w}_{t+1} + \frac{\tau_w}{1 + \beta} \pi_t - \frac{1 + \beta \tau_w}{1 + \beta} \pi_{t-1} + \frac{1}{1 + \beta} E_t \ddot{\pi}_{t+1}
\]

\[
+ \frac{\tau_w}{1 + \beta} \dot{z}_{t-1} - \frac{1 + \beta \tau_w}{1 + \beta} \dot{z}_t - \kappa_w \dot{g}_{w,t} + \kappa_w \dot{\lambda}_{w,t}
\]

where the real wage, \( \dot{w}_t \), is a function of past and expected future real wages and past, present and expected future inflation. In addition, the real wage depends on the wage mark-up, \( \dot{g}_{w,t} \), and the wage mark-up shock, \( \dot{\lambda}_{w,t} \). The parameter \( \tau_w \) denotes the degree of wage indexation, and \( \kappa_w \) represents the degree of wage-rigidity determining the speed of adjustment to the desired wage mark-up.\(^6\) The wage mark-up can be written as:

\[
\dot{g}_{w,t} = \dot{w}_t - (\dot{v}_t + \dot{b}_t - \dot{\lambda}_t)
\]

The household transforms purchased final consumption goods into investment goods. The capital transformation is subject to quadratic investment adjustment costs. The dynamics of aggregate investment is given by Tobin’s \( q \):

\[
\dot{q}_t = \dot{\mu}_t - e^{2\gamma} \varphi \left( \dot{z}_{t-1} - \dot{z}_t - \dot{z}_t \right) + \beta e^{2\gamma} E_t \left( \dot{i}_{t+1} - \dot{i}_t + \dot{z}_{t+1} \right)
\]

where \( \dot{q}_t \) is the Tobin’s \( q \) representing the price of newly installed capital relative to that of the consumption good. Moreover, \( \varphi \) is the steady-state elasticity of the investment adjustment cost function, \( \phi_t \) is the associated shadow price, and \( \dot{\mu}_t \) is the investment-specific technology shock that affects the efficiency with which final goods can be transformed into productive capital. The evolution of the stock of physical capital can be written as:

\[
\dot{k}_t = (1 - \delta) e^{-\gamma} \left( \dot{k}_{t-1} - z_t \right) + \left( 1 - (1 - \delta) e^{-\delta} \right) \left( \dot{\mu}_t + \dot{\lambda}_t \right)
\]

where \( \dot{k}_t \) is physical capital and \( \delta \) is the depreciation rate.

On the supply-side of the economy, the entrepreneur buys the physical capital stock, \( \dot{k}_t \), from the household at price \( \dot{q}_t \) by using both internal and external funds. Following the capital purchase, as in BGG, the entrepreneur is hit by an idiosyncratic shock affecting its physical capital holdings. He then

\(^6\)The degree of wage rigidity is written as: \( \kappa = \frac{(1 - \xi_w)(1 - \nu)}{\xi_w(1 + \beta)(1 + \nu(1 + \lambda_w))} \), where \( \xi_w \) is the Calvó-elasticity for the degree of wage stickiness.
transforms the physical capital into productive capital by choosing the capital utilization rate, \( \hat{u}_t \):

\[
\dot{k}_t = u_t \hat{k}_t
\]

(21)

Productive capital is then rented out to the intermediate goods producing firm at rate, \( \hat{r}_t^k \). The aggregate expected real return on productive capital is defined as:

\[
E_t \hat{R}_{t+1}^k = (1 - (1 - \delta) \beta e^{-\gamma}) \hat{r}_t^k + (1 - \delta) \beta e^{-\gamma} \hat{q}_{t+1} - \hat{q}_t
\]

(22)

which can be viewed as the traditional demand curve for new capital, suggesting that the expected return on capital decreases in the level of investment reflecting diminishing returns. The idiosyncratic shock hitting the entrepreneur, \( \omega \), is log-normally distributed with a CDF \( F(\omega) \) and parameters \( \mu_{\omega,t} \) and \( \sigma_{\omega,t} \).

Following the literature, we refer to the standard deviation of the stochastic process, \( \sigma_{\omega,t} \), as the 'risk shock', which captures the idea that the riskiness of the entrepreneurs vary over time. The law of motion for \( \sigma_{\omega,t} \) is specified as follows:

\[
\sigma_{\omega,t} = \rho_\sigma \sigma_{\omega,t-1} + \varepsilon_{\omega,t} + \eta_{\text{news},t-1}
\]

(23)

where \( \varepsilon_{\omega,t} \sim N(0, \sigma_{\omega}^2) \), and the news term, \( \eta_{\text{news},t} \), evolves according to the following AR(1) process:

\[
\eta_{\text{news},t} = \rho_{\text{news}} \eta_{\text{news},t-1} + \varepsilon_{\text{news},t}
\]

(24)

where \( \varepsilon_{\text{news},t} \sim N(0, \sigma_{\text{news}}^2) \). This specification is different from Christiano, Motto, and Rostagno (2010) which characterized the news shock as an innovation in the 8-period ahead anticipation about a future increase in \( \sigma_{\omega,t} \). Instead, we define the news shock as an innovation in the one-period ahead change in \( \sigma_{\omega,t} \). We do this in order to keep consistency with the identification and timing assumptions of our VAR outlined in section 2.3. Moreover, to maintain the sufficiently high impact that the risk news shock exerts on the return on capital and investment dynamics as shown by Christiano, Motto, and Rostagno (2010), we introduce persistence (\( \rho_{\text{news}} \)) in our new shock specification.

The financing of capital purchase is subject to a moral hazard friction between the banks and the entrepreneurs which is modeled by the costly state verification framework of BGG. The major implication of this friction is that if the idiosyncratic shock, \( \omega \), will be severe enough, the entrepreneur will default on its debt obligations. The optimal debt contract between the bank and the entrepreneur accounts for
this default probability, therefore the entrepreneur cannot borrow at the riskless rate. This gives rise to
the presence of the external finance premium, \( \hat{s}_t \), as an equilibrium wedge between the expected return on
capital and the riskfree rate:

\[
\hat{s}_t = E_t \hat{R}^k_{t+1} - E_t (\hat{r}_t - \hat{\pi}_{t+1})
\] (25)

The presence of the premium is caused by the monitoring (state verification) cost that the bank would
have to pay in case of entrepreneurial default. The optimal debt contract and the entrepreneur’s first-
order condition result in the following relationship between the premium and the entrepreneur’s financing
position:

\[
\hat{s}_t = \hat{n}_t - \hat{q}_t - \hat{k}_t + \Xi_1 E_t \hat{\omega}_{t+1} + \Xi_2 E_t \hat{\sigma}_{\omega,t+1}
\] (26)

where \( \Xi_1 \) and \( \Xi_2 \) are the elasticities of the risk premium to changes in the expected mean \( E_t \hat{\omega}_{t+1} \) and the
variance \( E_t \hat{\sigma}_{\omega,t+1} \) of the distribution of the idiosyncratic entrepreneurial productivity shock. \( \Xi_1 \) and \( \Xi_2 \) are key parameters that measure the elasticity of the premium to changes in the entrepreneur’s financial
position\(^7\).

The accumulation of the aggregate entrepreneurial net worth is written as follows:

\[
\hat{n}_t = + \gamma^e (1 - \mu \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \hat{\omega}}) \left[ \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \sigma_{\omega}} \frac{\hat{k}_t}{\hat{t}_t} \right] - R_B \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \sigma_{\omega}} \hat{\sigma}_{\omega,t} \right] + \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \hat{\omega}} \hat{\omega}_t \right] \right]
\] (27)

where \( \gamma^e \) is the entrepreneurial survival rate following a stationary stochastic process around its steady-
state value, \( \gamma^e \). \( \frac{K}{N} \) and \( \frac{B}{N} \) are the steady state ratios of capital and credit to net worth respectively. The
other Greek letters are functions of the log-normally distributed idiosyncratic productivity shock, \( \hat{\omega}_t \). A
detailed description is found in appendix 6.3.4.

The aggregate production function of the intermediate goods producers is written as:

\[
\hat{y}_t = \frac{y + \left[ \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t \right]}{y}
\] (28)

where output, \( \hat{y}_t \), is produced by using capital, \( \hat{k}_t \), and labor, \( \hat{l}_t \), and \( \alpha \) is the share of capital in production.

The steady-state level of production is denoted by \( y \), and \( F \) is the fixed cost of production. The firm’s

\(^7\)The exact forms of these elasticities are: \( \Xi_1 = \frac{K}{N} \left( \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \hat{\omega}} + \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \sigma_{\omega}} \right) \hat{\omega} \) and \( \Xi_2 = \frac{K}{N} \left( \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \hat{\omega}} + \frac{\partial G(\hat{\omega}, \sigma_{\omega})}{\partial \sigma_{\omega}} \right) \sigma_{\omega}. \)

The Greek letters in parentheses refer to functions of the log-normally distributed idiosyncratic productivity shock, \( \hat{\omega}_t \). The
detailed derivation of \( \Xi_1 \) and \( \Xi_2 \) is provided in section 6.3.4.
profit-maximization problem yields the following condition for the rental rate of physical capital, \( \hat{r}^k_t \):

\[
\hat{r}^k_t = \hat{w}_t + \hat{l}_t - \hat{k}_t \tag{29}
\]

In addition, the marginal cost of production is defined as:

\[
m\hat{c}_t = \alpha \hat{r}^k_t + (1 - \alpha) \hat{w}_t \tag{30}
\]

whereby the marginal cost, \( m\hat{c}_t \), is a linear combination of the rental rate of capital and the real wage, weighted by the shares of inputs in production. The aggregate resource constraint is defined as follows:

\[
\hat{y}_t = \hat{g}_t + CG + IG + \rho K + \phi \pi \pi_t + \phi X (x_t - x^*_t) + \epsilon^m_t \tag{31}
\]

where aggregate output is divided into consumption, \( \hat{c}_t \), investment, \( \hat{i}_t \), government consumption, \( \hat{g}_t \). The term \( \frac{\rho K}{Y} \) captures capital utilization costs, and the coefficients \( \Upsilon_1 \), \( \Upsilon_2 \) and \( \Upsilon_3 \) are associated with the aggregate resources used up in monitoring bankrupt entrepreneurs.\footnote{The exact forms of these elasticities are: \( \Upsilon_1 = \frac{\mu G G K R^k K}{Y^2} \), \( \Upsilon_2 = \frac{\mu G R^k K G (\varphi \sigma \omega)}{\sigma \omega} \) and \( \Upsilon_3 = \frac{\mu G R^k K \sigma G (\varphi \sigma \omega) \sigma \omega}{\sigma \omega} \). The Greek letters in parentheses refer to functions of the log-normally distributed idiosyncratic productivity shock, \( \bar{\omega}_t \). See section 6.3.4 of the appendix.}

The model is closed by defining the behavior of the central bank with the following monetary policy reaction function:

\[
\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) [\phi \pi \pi_t + \phi X (x_t - x^*_t) + \epsilon^m_t] \tag{32}
\]

where \( \rho \) denotes the degree of interest rate smoothing.

### 3.2 Estimation

In this section we explain our methodology for estimating and evaluating our model. We apply minimum distance estimation which is a limited information method often applied in the literature (Christiano, Eichenbaum, and Evans, 2005). In contrast to full information methods such as Maximum Likelihood and Bayesian methods, partial information methods make smaller demands on the model.\footnote{See Theodoridis (2011) for a detailed review.}

We estimate the following group of parameters:

\[
\varrho \equiv (\rho_{sp}, \sigma_{sp}, \sigma_{\omega}, \bar{\gamma}^F, \rho_{sp}, \mu, \bar{\omega}, \rho, \phi, \phi X, \phi d X, \phi sp)
\]
The estimation is based on minimizing a measure of the distance between the DSGE model and the impulse response functions of the VAR. Let $\Theta(\varrho)$ denote the mapping from $\varrho$ to the DSGE model impulse response function, and let $\Theta$ denote the corresponding empirical estimates. The estimator for $\varrho$ is the solution to the following minimization problem:

$$J = \min (\Theta - \Theta(\varrho))^\prime V^{-1} (\Theta - \Theta(\varrho))$$

where $V$ is a diagonal matrix with the sample variances of the $\Theta$'s along the diagonal. These variances are the basis for the empirical confidence bands. The estimation results are summarized in table 2.

### 3.3 Estimated impulse responses

This section presents the impulse responses of the DSGE model using the estimated parameters in table 2. Figure 8 shows the effects of a one standard deviation shock to anticipated risk on the key macroeconomic variables. The three colored lines represent the impulse responses under three different specifications of the monetary policy rule. To compare the performance of the DSGE model to that of the VAR, impulse responses implied by the DSGE model are plotted against those implied by VAR as well.

### 4 Simulation

The DSGE model provides a natural starting point for checking the suitability of our VAR identification strategy. Following Chari, Kehoe, and McGrattan (2008) and Barsky and Sims (2011), we simulate data from the DSGE model to which we apply our identification method explained in section 6.5. We simulate 1000 different data sets with 120 observations each, which corresponds to the sample size in our benchmark estimation. Moreover, we use the same variables and model specification as used in the benchmark 10-variable VAR. Figure 9 shows the impulse responses of the key macroeconomic variables following an anticipated risk news shock. Impulse responses from both the empirical VAR and the simulated VAR are shown. All the estimated impulses responses are within the simulation bands of the theoretical impulse responses. We interpret these results as a confirmation that our empirical approach is successful in identifying a risk news shock.
5 Conclusion

This paper has proposed a method to identify a ‘risk news’ shock in macro time series for the US. This shock is meant to capture revelation about future uncertainty surrounding the returns to entrepreneurial activity. The risk news shock is estimated to be very small, in the sense that it contributes very little to fluctuations in the uncertainty proxy, but yet we estimate that it has sizable impacts on standard macro observables like inflation and output, and, if the VAR is to be believed, this shock seems to have been a significant factor during the recent crisis.

References


6 Appendix

6.1 Data description

6.2 Impulse response of the VAR

Figure 1: Impulse Responses to an Anticipated Risk Shock

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
Figure 2: Impulse Responses to a Contemporaneous Risk Shock

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
Figure 3: Impulse Responses to a Productivity Shock

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
Figure 4: Impulse Responses to a Monetary Policy Shock

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
Figure 5: Impulse Responses to a Demand Shock

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
Figure 6: Historical Decomposition

Volatility

Spread

Output

Consumption

Investment

Hours

Net worth

Inflation

Interest rate
Figure 7: Impulse Responses to an Anticipated Risk Shock

Volatility, spread, ygrowth, cgrowth, invgrowth, hours, Wage, inflation, interest, networth

Note: The solid line is the mean impulse response; the shaded area is the 90 percent posterior interval. The vertical axes are in percentages; the horizontal axes denote quarters.
6.3 Equilibrium conditions of the DSGE model

Most of the equilibrium conditions are identical to Justiniano and Preston (2010) with the exception of the financial frictions block.

### 6.3.1 Production function and cost minimization

Final good producers are perfectly competitive and produce final consumption good $Y_t$ by combining intermediate goods $Y_t(i)$ using the following CES production function:

$$
Y_t = \left[ \int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} \right]^{1+\lambda_{p,t}}
$$

where the $\lambda_{p,t}$ is often referred to as the price-markup shock. Profit maximization of the final goods producers implies that the aggregate price level, $P_t$, is a CES aggregate of the prices of the intermediate goods, $P_t(i)$:

$$
P_t = \left[ \int_0^1 P_t(i)^{\frac{1}{1+\lambda_{p,t}}} \right]^{\lambda_{p,t}}
$$

and the corresponding demand function for the intermediate good $i$ is:

$$
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t
$$

A monopolist produces the intermediate good $i$ according to a production function is of constant-returns-to-scale form with capital, $K_t$, and labor, $L_t$, as inputs. It is subject to an aggregate technology shock, $A_t$, and a fixed cost, $F$, so that profits are zero in steady state.

$$
Y_t(i) = A_t^{1-\alpha} K_t^\alpha (i) L_t^{1-\alpha} (i) - A_t F
$$

The growth rate of the exogenous technological progress is defined as $z_t = \Delta \log A_t$, which follows a stationary AR(1) process, implying that the level of technology is non stationary. The cost minimization of the monopolist yields the following condition for the capital-labor ratio:

$$
\frac{K_t(i)}{L_t(i)} = \frac{W_t}{r_t^K} \frac{\alpha}{1-\alpha}
$$

29
where $W_t$ is the aggregate wage that the monopolist pays for the labor input, and $r^k_t$ is the per-period rental rate of capital. The common marginal cost is written as:

$$MC_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} (r^k_t)^\alpha \left( \frac{W_t}{A_t} \right)^{1-\alpha}$$  \hspace{1cm} (39)

### 6.3.2 Price-setting

The price-setting behavior of the firms follows Calvo (1983), so that each period a fraction $\xi_p$ of the intermediate firms cannot choose its prices optimally, but has to reset them according to the following indexation rule:

$$P_t (i) = P_{t-1} (i) \pi_t^{1-\xi_p}$$  \hspace{1cm} (40)

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate with the corresponding steady-state level, $\pi$. The rest of the firms choose their prices $P_t (i)$ optimally, by maximizing the present discounted value of future profits:

$$\max_{P_t(i)} E_t \left\{ \sum_{s=0}^{\infty} \xi_p^s \beta^s \Lambda_{t+s} \left[ P_t (i) \Pi_{t,t+s} - MC_{t+s} \right] Y_{t+s} (i) \right\}$$  \hspace{1cm} (41)

s.t. $Y_{t+s} (i) = \left( \frac{P_t (i) \Pi_{t,t+s}}{P_{t+s}} \right)^{1+\lambda_{p,t+s}} Y_{t+s}$

and the first-order condition is written as:

$$0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_p^s \beta^s \Lambda_{t+s} \tilde{Y}_{t+s} \left[ \tilde{P}_t \Pi_{t,t+s} - (1 + \lambda_{p,t+s}) MC_{t+s} \right] \right\}$$  \hspace{1cm} (42)

where $\tilde{P}_t$ is the optimally chosen price, which is the same for all producers, and $\tilde{Y}_{t+s}$ is the demand they face in $t + s$. The aggregate price index is written as:

$$P_t = \left[ (1 - \xi_p) \left( \frac{\tilde{P}_t}{\pi_p^{1-\xi_p}} \right)^{\frac{1}{\lambda_{p,t}}} + \xi_p \left( \frac{\pi_{t-1}^{1-\xi_p} P_{t-1}}{\pi_{t-1}} \right)^{\frac{1}{\lambda_{p,t}}} \right]^{\lambda_{p,t}}$$  \hspace{1cm} (43)

### 6.3.3 Households

Each household maximizes the following utility function:

$$\max_{P_t(i)} E_t \left\{ \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[ \log (C_{t+s} - hC_{t+s-1}) - \varphi \frac{L_{t+s} (j)^{1+\nu}}{1+\nu} \right] \right\}$$  \hspace{1cm} (44)
where $C_t$ is consumption, $h$ is the degree of habit formation and $b_t$ is a shock to the discount fact which is referred to as the intertemporal preference shock. The household’s budget constraint is:

$$P_tC_t + P_tI_t + T_t + B_t \leq Q_t(j) + \Pi_t + W_t(j) L_t(j) + r_k^t u_t \bar{K}_t - P_t a(u_t) \bar{K}_{t-1} \quad (45)$$

where $I_t$ is investment, $T_t$ is lump-sum taxes, $B_t$ is holdings of government bonds, $R_t$ is the gross nominal interest rate, $Q_t(j)$ is the net cash flow from household’s $j$ portfolio of state contingent securities, and $\Pi_t$ is the per capital profit accruing to the household from ownership of the firms. The household’s first-order conditions yield the well-known Euler equation:

$$\Lambda_t = \beta R_t E_t \Lambda_{t+1} \quad (46)$$

where $\Lambda_t$ is the shadow price of the borrowing constraint (45). The marginal utility of consumption is written as:

$$P_t \Lambda_t = \frac{b_t}{C_t - hC_{t-1}} - h\beta E_t \frac{b_{t+1}}{C_{t+1} - hC_t} \quad (47)$$

In addition, households choose the capital utilization rate, $u_t$ leading to the following optimality condition:

$$r_k^t = P_t a'(u_t) \quad (48)$$

which transforms physical capital into effective capital according to the following condition:

$$K_t = u_t \bar{K}_{t-1} \quad (49)$$

Effective capital is then rented out to the intermediate goods producers at the rate $r_k^t$. The cost of capital utilization is $a(u_t)$ per unit of physical capital. In steady state, $u = 1$, $a(1) = 0$ and $\chi \equiv \frac{a''(1)}{a'(1)}$. In the log-linear approximation of the model solution this curvature is the only parameter that matters for the dynamics.

The physical capital accumulation is subject to quadratic investment adjustment costs:

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \mu_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \quad (50)$$

where $\delta$ is the depreciation rate. The function $S$ captures the quadratic adjustment costs with the following
steady-state properties: \( S = S' = 0 \) and \( S'' > 0 \). The term \( \mu_t \) is often referred to as the investment efficiency shock which determines how efficiently final good can be transformed into physical capital, and this into tomorrow’s capital input in production. The optimal choice of the physical capital stock is given by:

\[
\Phi_t = \beta E_t \left[ \Lambda_{t+1} \left( \tau_{t+1}^k u_{t+1} - P_{t+1} a (u_{t+1}) \right) \right] + (1 - \delta) \beta E_{t} \Phi_{t+1}
\]

(51)

where \( \Phi_t \) is the shadow value of installed physical capital associated with the capital accumulation constraint (49). The optimal choice of investment is written as:

\[
P_t \Lambda_t = \Phi_t \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S' \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta \left[ \Phi_{t+1} \mu_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{I_t} \right) \right]
\]

(52)

Tobin’s \( q \) is then defined as the relative marginal value of installed capital with respect to consumption:

\[
Q_t = \frac{\Phi_t}{P_t \Lambda_t}
\]

(53)

Each household is a monopolistic supplier of specialized labor, \( L_t (j) \). As in Erceg, Henderson, and Levin (2000), a large number of competitive employment agencies combine the specialized labor into a homogenous labor input and sold to the intermediate goods producer:

\[
L_t = \left[ \int_0^1 L_t (i) \frac{1}{1 + \lambda_{w,t}} \right]^{1 + \lambda_{w,t}}
\]

(54)

where the \( \lambda_{w,t} \) is often referred to as the wage-markup shock. Profit maximization of the perfectly competitive employment agencies implies the following labor demand function:

\[
L_t (i) = \left( \frac{W_t (i)}{W_t} \right)^{-\frac{1 + \lambda_{w,t}}{\lambda_{w,t}}} L_t
\]

(55)

where \( W_t (j) \) is the wage received from employment agencies by the supplier of household \( j \). The wage paid by the intermediate producer is written as:

\[
W_t = \left[ \int_0^1 W_t (i) \frac{1}{1 + \lambda_{w,t}} \right]^{1 + \lambda_{w,t}}
\]

(56)

Each period, \( \xi_w \) fraction of the households cannot freely set its wage, but follows the indexation rule:
\[ W_t(j) = W_{t-1}(j) (\pi_{t-1}e^{z_{t-1}})^{\iota^w} (\pi e^\gamma)^{1-\iota^w} \]  

The remaining fraction of households sets its wage optimally by maximizing the following:

\[
Et \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ -b_{t+s} \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} + \Lambda_{t+s} W_t(j) L_{t+s}(j) \right] \right\} 
\]  

subject to the labor demand function (55). The optimality condition for household wages is then written as:

\[
W_t = \left\{ (1 - \xi_w) \left( \bar{W}_t \right)^{\frac{1}{\lambda^w}} + \xi_w \left[ (\pi e^\gamma)^{1-\iota^w} (\pi_{t-1}e^{z_{t-1}})^{\iota^w} W_{t-1} \right]^{\frac{1}{\lambda^w}} \right\}^{\lambda^w} 
\]

### 6.3.4 Entrepreneurs and financial frictions

Entrepreneurs use their end-of-period wealth, \( N_t \), and nominal loan, \( B_t \), to purchase installed capital \( K_t \):

\[ Q_t K_t = N_t + \frac{B_t}{P_t} \]  

The newly purchased capital is hit by an idiosyncratic shock, \( \omega \), that is log-normally distributed with a cumulative density function of \( F(\omega, \sigma_{\omega,t}) \). The entrepreneur rents out the purchased capital at a rate \( r_k^t \), and at the end of the period he sells the depreciated capital at price \( Q_{t+1} \). The average return on purchasing a unit of capital over period \( t \) is written as:

\[ R_{t+1}^k = \frac{r_k^t + Q_{t+1} (1 - \delta)}{Q_t} \]  

The average return on capital, \( R_{t+1}^k \), is a major component for the equilibrium debt contract, which determines the financing conditions for entrepreneurial investment. The financial intermediary lends to the entrepreneur at a rate \( R_{t+1}^l \), knowing the distribution of the idiosyncratic shock, \( \omega \). Knowledge of \( F(\omega, \sigma_{\omega,t}) \) allows the financial intermediary to weight \( R_{t+1}^k \) with the probability of the entrepreneur defaulting on his debt obligation. In case of entrepreneurial default, the lender seizes all the remaining revenues of the entrepreneur, following \( \mu \) fraction of these revenues being lost in bankruptcy procedures. The literature often refers to \( \mu \) as the cost of state verification [Townsend (1979)]. There is a cut-off value of \( \bar{\omega}_{t+1} \), at which the entrepreneur can just pay back the debt:
\[ \bar{\omega}_{t+1} = \frac{R^l_t B_t}{R^k_{t+1} Q_t K_t} \]  

(62)

This condition allows us to characterize the debt contract in terms of the threshold productivity level, \( \bar{\omega}_{t+1} \), instead of characterizing it in terms of the cost of borrowing, \( R^l_t \). The debt contract determines \( R^l_t \) such that the zero profit condition for financial intermediaries is satisfied:

\[
R_t B_t = [1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})] R^l_t B_t + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1}) R^k_{t+1} Q_t K_t
\]

(63)

which states that the cost of funds, \( R_t B_t \), must equal the revenues the loan pays to the financial intermediary in the case of non-default and the value of the acquired revenues in case default default. Non-default is captured by the term \( 1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \), which is the log-normal cumulative density function of \( \bar{\omega}_{t+1} \) above the threshold probability \( \bar{\omega}_{10} \). Default is captured by the partial expectation term of the log-normal density \( \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1}) \). Combining equations (60), (62) and (63) results in the participation constraint of the optimal contract:

\[
\left[ 1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \right] + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1}) R^k_{t+1} Q_t K_t = R_t (Q_t K_t - N_t)
\]

(64)

To simplify the notation, we introduce the following two auxiliary variables:

\[
G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t+1})
\]

\[
\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) = \bar{\omega}_{t+1} (1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})) + G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})
\]

where \( \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \) represents the share of entrepreneurial earnings accrued to the financial intermediary.

The optimal contract (64) can therefore be rewritten as:

\[
\left[ \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \right] R^k_{t+1} Q_t K_t = R_t B_t
\]

(65)

\[ ^{10} \text{Given that } \bar{\omega}_{t+1} \text{ is log-normally distributed, } F(\bar{\omega}) = \int_0^\infty \frac{1}{\bar{\omega} \sqrt{2\pi} \sigma_{\omega}} e^{-\frac{(\ln \bar{\omega} - \frac{1}{2} \sigma_{\omega}^2)^2}{2\sigma_{\omega}^2}} d\omega. \]

\[ ^{11} \text{Given that the expected value of a log-normally distributed random variable is } E_t \omega = e^{\mu + \frac{1}{2} \sigma_{\omega}^2}, \text{ the partial expectation is written as: } \int_0^{\bar{\omega}} \omega dF(\bar{\omega}) = e^{\mu + \frac{1}{2} \sigma_{\omega}^2} \Phi \left( \frac{\ln \omega - \ln \bar{\omega}}{\sigma_{\omega}} \right), \text{ where } \Phi \text{ is the CDF of } \bar{\omega} \text{ so that } \int_0^{\bar{\omega}} \omega dF(\bar{\omega}) = e^{\mu + \frac{1}{2} \sigma_{\omega}^2} \left[ 1 - \Phi \left( \frac{\ln \omega - \ln \bar{\omega}}{\sigma_{\omega}} \right) \right]. \]
The optimization problem of the entrepreneur is then to choose $K_t$ and a schedule for $\bar{\omega}_{t+1}$ to maximize its expected net worth given the zero-profit condition of the financial intermediary:

$$\max_{\bar{\omega},K_t} \left[ 1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) R_{t+1}^k Q_t K_{t+1} \right]$$

subject to equation (65). The first-order conditions are given as follows:

$$\bar{\omega}_{t+1} : \Gamma'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) = \eta_t \Gamma'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu G'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})$$  \hspace{1cm} (66)

$$K_t : \eta_t = \frac{R_{t+1}^k}{R_t} \left\{ 1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) + \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) \right\}$$  \hspace{1cm} (67)

where $\Gamma'_\omega$ and $G'_\omega$ are the partial derivatives of the densities $\Gamma$ and $G$ with respect to $\bar{\omega}$, and $\eta_t$ is the Lagrange multiplier of the constraint (65). Combining the above conditions yields the fundamental relationship which links the external finance premium, $S_t$, to the ratio of purchased capital to net worth:

$$S_t = E_t \frac{\eta(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}{(1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})) Q_t K_t}$$  \hspace{1cm} (68)

where the external finance premium is the expected excess return on capital above the risk free rate, $S_t = \frac{E_t R_{t+1}^k}{R_t}$. Condition (68) shows that the premium is a function of the ratio of net worth and the value of the purchased capital.

After entrepreneurs have settled their debt, and capital has been resold, the aggregate net worth is determined. At this point, $\gamma_t^e$ fraction of the entrepreneurs survives to the next period, while the rest die and their capital is taxed at a 100 percent and gets transferred as a lump-sum payment to the households. The evolution of net worth is written as:

$$N_t = \gamma_t^e \left[ (1 - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})) R_{t-1}^k Q_{t-1} K_{t-1} - R_{t-1} B_{t-1} \right]$$  \hspace{1cm} (69)

where $\gamma_t^e$ is referred to as the net worth shock, which follows an AR(1) process with a mean $\gamma^e$.

---

12 See appendix A of Bernanke, Gertler, and Gilchrist (1999).

13 Making the dependence on $\omega_{t+1}$ and $\sigma_{\omega,t+1}$, the Lagrange multiplier can be written as: $\eta(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) =$$\frac{\Gamma'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1}) - \mu G'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}{\Gamma'_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}$
6.3.5 Monetary policy and other conditions

Monetary policy sets the nominal interest rate following a feedback rule of the form:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{X_t}{X^*} \right)^{\phi_X} \left( \frac{R^k_t/R_{t-1}}{R^k/R} \right)^{\phi_{sp}} \right]^{1-\rho_R} \left[ \frac{X_t}{X_{t-1}}/X_t^*/X_{t-1}^* \right]^{\phi_{dX}} \eta_{mp,t} \tag{70}
\]

where \( R \) is the steady state of the gross nominal interest rate, and \( \rho_R \) is the interest rate smoothing parameter. The interest rate responds to the level and the growth rate of the GDP gap \( (X_t/X_t^*) \) as well as inflation deviation from its steady state \( (\pi_t/\pi) \). A key assumption of our model is that monetary authority responds to deviations of the external premium \( (R^k_t/R_{t-1}) \) from its steady state level. The monetary policy shock is denoted by \( \eta_{mp,t} \).

The model is closed by defining the GDP as:

\[
X_t = C_t + I_t + G_t \tag{71}
\]

and defining the aggregate resource constraint as follows:

\[
\frac{1}{g_t} Y_t = C_t + I_t + a(u_t) K_{t-1} + \mu G(\bar{\omega}_t, \sigma_{\omega,t}) R^k_t Q_{t-1} K_{t-1} \tag{72}
\]

where \( a(u_t) \) denotes the cost of utilization and the term \( \mu (\bar{\omega}_t, \sigma_{\omega,t}) \) is associated with the aggregate resources used up in bank monitoring costs.

6.3.6 Exogenous processes

The price-markup shock follows a stationary ARMA(1) process:

\[
\log (1 + \lambda_{p,t}) = (1 - \rho_p) \log (1 + \lambda_p) + \rho_p \log (1 + \lambda_{p,t-1}) + \varepsilon_{p,t} - \theta_p \varepsilon_{p,t-1} \tag{73}
\]

where \( \varepsilon_{p,t} \sim N(0, \sigma_p^2) \). The change in technology, \( z_t = \Delta \log A_t \), follows an AR(1) process:

\[
z_t = (1 - \rho_z) \gamma + \rho_z z_{t-1} + \varepsilon_{z,t} \tag{74}
\]

where \( \varepsilon_{z,t} \sim N(0, \sigma_z^2) \). The intertemporal preference shock follows an AR(1) process:

\[
\log b_t = \rho_b b_{t-1} + \varepsilon_{b,t} \tag{75}
\]
where $\varepsilon_{b,t} \sim N\left(0, \sigma_b^2\right)$. The investment shock follows an AR(1) process:

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}$$  \hspace{1cm} (76)

where $\varepsilon_{\mu,t} \sim N\left(0, \sigma_\mu^2\right)$. The wage mark-up shock follows an ARMA(1) process:

$$\log (1 + \lambda_{w,t}) = (1 - \rho_w) \log (1 + \lambda_w) + \rho_w \log (1 + \lambda_{w,t-1}) + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}$$  \hspace{1cm} (77)

where $\varepsilon_{w,t} \sim N\left(0, \sigma_w^2\right)$. The monetary policy shock follows an AR(1) process:

$$\log \eta_{mp,t} = \rho_{mp} \log \eta_{mp,t-1} + \varepsilon_{mp,t}$$  \hspace{1cm} (78)

where $\varepsilon_{mp,t} \sim N\left(0, \sigma_{mp}^2\right)$. The government spending shock follows an AR(1) process:

$$\log g_t = \rho_g \log g_{t-1} + \varepsilon_{g,t}$$  \hspace{1cm} (79)

where $\varepsilon_{g,t} \sim N\left(0, \sigma_g^2\right)$. The net worth shock follows an AR(1) process:

$$\log \gamma^e_t = \rho_\gamma \log \gamma^e_{t-1} + \varepsilon_{\gamma,t}$$  \hspace{1cm} (80)

where $\varepsilon_{\gamma,t} \sim N\left(0, \sigma_\gamma^2\right)$. The risk news shock is written as follows:

$$\log \sigma_{\omega,t} = \rho_\sigma \log \sigma_{\omega,t-1} + \varepsilon_{\omega,t} + \log \eta_{\text{news},t-1}$$  \hspace{1cm} (81)

where $\varepsilon_{\omega,t} \sim N\left(0, \sigma_\omega^2\right)$, and the news term, $\eta_{\text{news},t}$, evolves according to the following AR(1) process:

$$\log \eta_{\text{news},t} = \rho_{\text{news}} \log \eta_{\text{news},t-1} + \varepsilon_{\text{news},t}$$  \hspace{1cm} (82)
6.4 Estimation results

Table 2: Estimated Parameter values

<table>
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<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Median</th>
<th>Upper Bound</th>
</tr>
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<tbody>
<tr>
<td>$\rho_{sp}$</td>
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<td>0.925</td>
<td>0.933</td>
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<td>$\sigma_{sp}$</td>
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<td>0.153</td>
<td>0.162</td>
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<td>$\sigma_{\omega}$</td>
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<td>1.506</td>
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<td>0.032</td>
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<td>0.027</td>
</tr>
<tr>
<td>$\phi_{sp}$</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimated are obtained using identity weighting matrix; lower and upper bounds are 10% and 90% respectively.

Figure 8: Estimated impulses responses

spread

ygrowth

cgrowth

invgrowth

hours

Wage

inflation

interest

networth

38
6.5 Simulation results

Figure 9: Model and Monte Carlo Estimated Impulse Responses to a Risk news shock