Should Financially Constrained Consumers Ask for High Public Spending? *

Preliminary and Incomplete

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Abstract

We study optimal government spending policy in a New Keynesian setting. We augment the standard model with limited asset market participation as in Bilbiie (2008). Furthermore we allow fiscal authority to levy a proportional labour income tax to finance its public consumption. We show that limited asset market participation makes it optimal to reduce public spending compared to the first best allocation. This is so as to dampen the strongly distortionary effects that fiscal policy has on the welfare of the financially constrained agents. Moreover we show that replacing labour income tax with consumption tax allows the Ramsey Planner to set the public spending-to-GDP ratio at its first best regardless of limited asset market participation or long run public debt.

Keywords: Limited asset market participation, Ramsey policy, Optimal public spending, distortionary taxation.


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1 Introduction

We identify two important consequences of the 2008 ‘Great Recession’. First, there has been a strong increase in credit constraints. The trigger of the crisis was the housing bubble burst in the US, which affected deeply the financial market and the international banking system. The direct consequences of these facts were liquidity shortage and stock markets downturns. Many financial institutions collapsed around the world, contributing to the failure of key businesses, declines in consumer wealth and a significant decline in economic activity. Questions regarding bank solvency have caused not only an interbank credit crunch but also a decline in credit availability for both firms and households. The main factors contributing to the decline in credit availability were the bad expectations regarding general economic activity and housing market prospects as well as cost of funds and balance sheet constraints for banks.

In this section we show some empirical evidence on the decline of banking lending to households, for housing and other consumer credit, in the Euro area and in the US. Figures 1 and 2 show the behavior of credit standards for the period 2003-2010. As shown in Figure 1 credit standards tightened in the Euro area since the first months of 2008. The tightening reached its maximum value in April 2009 and then started decreasing. Nevertheless, in December 2009, the tightening was still higher than in the pre-crisis period. The US credit standards feature a very similar behavior. However, as shown in Figure 2 the tightening of credit standards started in the mid of 2007, before the EU. Moreover, the tightening was even stronger than in the Euro area. These features of the US credit standards are not surprising since the financial crisis was triggered by a liquidity shortfall in the United States banking system at the beginning of the summer 2007, which afterwards spread all over the Euro area and most of the industrialised countries. Overall, the evidence on credit standards shows a sharp decline of credit to households since the beginning of the crisis.

Second, governments in many OECD economies have implemented expansionary fiscal policy measures. These decisions have lead to a considerable increase the level of government indebtedness, triggering for some countries even fears about the sustainability of public finances. Table 1 illustrates this fact by depicting the evolution of the central governments’ liabilities in relation to GDP for a selected group of OECD economies. Debt levels strongly increased over the period 2007-2009 and the OECD forecasts for the years 2010 and 2011 show that debt levels are expected to increase even further.

<table>
<thead>
<tr>
<th>Year \ Country</th>
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<th>Ireland</th>
<th>Italy</th>
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<tr>
<td>2011</td>
<td>85%</td>
<td>72%</td>
<td>93%</td>
<td>130%</td>
<td>74%</td>
<td>94%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1: Public Debt as fraction of the GDP, 2007-2011.
In this paper we develop a New Keynesian model which takes into account these two facts. First, we introduce public debt steady state and labour income taxation as in Schmitt-Grohë and Uribe (2004, 2007) and Adam (2011). Second we add, alongside traditional optimising agents a set of rule-of-thumb consumers as in Galì et al. (2007), Bilbiie (2008) and Motta and Tirelli (2011). With a fraction of consumers which are liquidity constrained, the Ricardian equivalence in the sense of Barro (1974) does not hold anymore. This feature brings different results on the dynamics and the long run of the economic system with respect to the standard framework.

To keep the exposition clear, the paper considers three government instruments that are generally considered relevant for the conduct of stabilisation policy, namely (1) monetary policy defined as control of the short-term nominal interest rate, (2) fiscal policy in the form of spending decisions on public goods, and (3) a fiscal financing decision determining whether to use labor income taxes, consumption tax or government debt as means to finance current expenditure, where government debt is assumed to be nominal and non-state contingent. The paper determines how these tools should be used as instruments and how they should depend on the level of government debt and on the share of limited asset market participation. In particular we intend to address two set of policy questions. First, how should public spending be used in the short and in the long run if the policy maker has social preferences which reflects the actual welfare of the society? Second, does the optimal provision of public goods increase in the share of limited asset market participation? Third, should stabilization attempts in the future depend on the fact that government debt is higher now? Is it optimal to keep government debt at these elevated levels or should it be reduced over time?

In the Social Planner equilibrium, government spending is kept at the social optimum. This is identified via the social preferences. In the Ramsey equilibrium with labour income tax this allocation is not achievable. This is due to four major frictions present in the economy. First, market power by firms generally implies that wages fall short of their marginal product, so that labor supply and therefore output is too low relative to the optimal allocation. Second, the requirement to finance government expenditure and interest payments on outstanding government debt with distortionary income taxes additionally depresses labor supply and output. Third, the presence of limited asset market participation induces an extra element in the distortionary effects of fiscal policy. Fourth, the presence of nominal rigidities may prevent the price system from providing the appropriate scarcity signals. Monetary and fiscal policy will seek to minimise the effects of all these distortions.

We find that the Ramsey planner implements a zero long run inflation policy. We further show that it is optimal in the Ramsey steady state to reduce government spending below its social optimum level. In particular the Ramsey public spending is (slightly) decreasing in the level of long run public debt and (greatly) decreasing in the share of limited asset market participation. This is due to the fact that public spending (and the service of public debt)
must be financed by a proportional distortive taxation. This type of taxation is particularly
distortive in terms of welfare for the financially constrained households. Reducing public
spending allows to reduce long run taxation thus increasing the limited asset market partici-
pation welfare. Finally, we show that the Ramsey Planner can set the government spending
to GDP ratio at its first best level when she can access to consumption tax only. The reason
is that the consumption tax scheme is less distortive. In particular, the optimisers are less
sensitive to consumption taxation compared to the labour income one. This allow the Ramsey
planner to keep public spending to GDP ratio at the Social planner level without incurring
in welfare losses. For this reason, using the consumption tax is also welfare improving with
respect to labour income tax.

2 The model

We consider a cashless DSGE model where nominal rigidities are introduced with a quadratic
cost of price adjustment à la Rotemberg (1982). Following Gali et al. (2004 and 2007), house-
holds are characterised by the same utility function, but a distinction can be drawn between
the fraction $\theta$ of rule-of-thumb consumers (ROTC) and the $(1 - \theta)$ Ricardian agents who have
unrestricted access to financial markets. The key difference between the two groups concerns
the intertemporal consumption optimisation, which is precluded to the $\theta$ households who have
no access to financial markets. Variables with a suffix ‘$o$’ or ‘$r$’ identify optimising agents and
ROTC respectively. A variable without time index identifies its steady state level while the
suffix ‘*’ identifies the Social Planner equilibrium. Besides presenting the model ingredients,
this section derives the implementability constraints characterising optimal private sector be-
behavior, i.e., derives the optimality conditions determining households’ consumption and labor
supply decisions and firms’ price setting decisions. One should note that we also abstract
from money holdings. This should be interpreted as the ‘cashless limit’ of an economy with
money, see Woodford (1998).

2.1 Private Sector

2.1.1 Household preferences

Preferences are defined as follows:

$$U^j_t = E_0 \sum_{t=0}^{\infty} \beta^t u \left( c^j_t, h^j_t, g_t \right)$$

(1)

Where $j: o, r$ stands for the household type (optimisers and ROTC consumers respectively),
$\beta$ is the discount factor, $c^j_t$ represents total individual consumption, $h^j_t$ denotes hours. $E_t$
identifies the rational expectation operator. We impose that the utility is separable in its

4
three arguments and \( U_c > 0, U_{cc} < 0, U_g > 0, U_{gg} < 0, U_h < 0 \) and \( U_{hh} \leq 0 \).

The consumption bundle can be written as

\[
c^i = \left[ \int_0^1 \left( c^i_{i,t} \right)^{\frac{s-1}{s}} \frac{d\theta}{\theta^s} \right]^{\frac{s}{s-1}}; \quad i \in [0,1], \tag{2}
\]

while the aggregate consumption price index is

\[
P_t = \left[ \int_0^1 P^{1-\eta}_t \frac{d\theta}{\theta^s} \right]^{\frac{1}{1-\eta}} \tag{3}
\]

while demand for good \( i \) follows

\[
c^j_{i,t} = c^j_i \left( \frac{P^+_{i,t}}{P_t} \right)^{-\eta} \tag{4}
\]

where \( \eta \in (1, \infty) \) is the price elasticity for differentiated goods. Importantly, the previously stated assumptions about the demand function are consistent with optimising individual behavior when private (and public later on) consumption goods are Dixit-Stiglitz aggregates.

2.1.2 Ricardian households

Ricardian households maximise (1) subject to the following period budget constraint:

\[
P_t c^o_t (1 + \tau^c_t) + \frac{B_t}{1-\theta} + E_t \Lambda_{t,t+1} Q_{t+1} = R_{t-1} \frac{B_{t-1}}{1-\theta} + Q_t + P_t \frac{d_t}{1-\theta} + P_t w_t h^o_t \left( 1 - \tau^h_t \right). \tag{5}
\]

In each time period \( t \), Ricardian agents can purchase any desired state-contingent nominal payment \( Q_{t+1} \) in period \( t+1 \) at the dollar cost \( E_t \Lambda_{t,t+1} Q_{t+1} \). The variable \( \Lambda_{t,t+1} \) denotes the stochastic discount factor between period \( t \) and \( t+1 \). Real dividends are denoted by \( d_t \), while \( B_t \) is the quantity of nominally riskless bonds purchased in period \( t \) at price \( R^{-1}_t \) and paying one unit of the consumption numeraire at period \( t+1 \). Taxes on consumption and labour income are respectively \( \tau^c_t \) and \( \tau^h_t \), and \( w_t \) is the real wage.

The solution for the optimising household problem is standard (\( U^o_{x,t} \) defines the derivative of the utility for agent of type \( i \) with respect to the generic variable \( x \)):

\[
U^o_{c,t} = \lambda^o_t (1 + \tau^c_t) \tag{6}
\]

where \( \lambda^o_t \) stands for the Lagrangian multiplier associated with this programme. Labor supply is determined by

\[
\frac{-U^o_{h,t}}{U^o_{c,t}} = \frac{w_t (1 - \tau^h_t)}{(1 + \tau^c_t)} \tag{7}
\]

While the Euler equation is
\[
\frac{U^o_{c,t}}{(1 + \tau^o_t)} = \beta E_t \left( \frac{U^o_{c,t+1}}{(1 + \tau^o_{t+1})} \frac{R_t}{\pi_{t+1}} \right) .
\]

(8)

The stochastic discount factor is defined as \( E_t \Lambda_{t,t+1} = \beta E_t \frac{U^o_{c,t+1}}{P_{t+1}} \frac{P_t}{U^o_{c,t}} \frac{(1+\tau^o_t)}{(1+\tau^o_{t+1})} \) and absence of arbitrage profits in the asset markets implies that \( E_t \Lambda_{t,t+1} = R_t^{-1} \).

2.1.3 Rule-of-thumb households

As pointed out above, rule-of-thumb consumers neither save nor borrow. In each period they consume entirely their labor income net of taxes:

\[
c^*_t = w_t h^*_t \left( \frac{1 - \tau^h_t}{1 + \tau^h_t} \right) .
\]

(9)

and supply labor according to

\[
-\frac{U^r_{h,t}}{U^r_{c,t}} = w_t \left( \frac{1 - \tau^h_t}{1 + \tau^h_t} \right)
\]

(10)

Note that since firms are indifferent with respect to the type of household they hire, they offer the same wage \( w_t \) to each worker irrespective to its type. Therefore, it must be the case that

\[
\frac{U^r_{h,t}}{U^r_{c,t}} = \frac{U^o_{h,t}}{U^o_{c,t}}
\]

(11)

2.2 Firms

A generic good \( i \) is produced in a monopolistically competitive market with the following technology:

\[
y_{i,t} = a_t h_{i,t},
\]

(12)

where \( a_t \) is a common exogenous technology process. Firm \( i \)'s real marginal costs are:

\[
mc_t = \frac{w_t}{a_t}
\]

(13)

We assume firms, when reset their prices, incur in a quadratic adjustment cost as,

\[
\frac{\varphi}{2} P_t \left( \frac{P_{i,t+s}}{P_{h,t+s-1}} - 1 \right)^2,
\]

(14)

where \( \varphi \) represents the degree of price stickiness. The profit maximising generic firm \( i \)'s problem can be expressed as,
\[
\max_{\{P_{i,t}\}} \sum_{s=0}^{+\infty} \beta^s \left( \frac{U_{c,t+s}^o}{U_{c,t}^o} \frac{(1 + \tau_{t+s}^e)}{1 + \tau_{t+s}^e} \right) \left[ \frac{P_{t+s}}{P_{t+s}} - mc_{t+s} \right] - \frac{\varphi}{2} \left( \frac{P_{t+s}}{P_{t,s-1}} - 1 \right)^2
\]

s.t. \( y_{t+s} = \left( \frac{P_{t,s}}{P_{t+s}} \right)^{-\eta} y_{t+s} \)

hence

\[
\max_{\{P_{i,t}\}} \sum_{s=0}^{+\infty} \beta^s \left( \frac{U_{c,t+s}^o}{U_{c,t}^o} \frac{(1 + \tau_{t+s}^e)}{1 + \tau_{t+s}^e} \right) \left[ y_{t+s} \left( \frac{P_{t,t+s}}{P_{t+s}} \right)^{1-\eta} - mc_{t+s} \left( \frac{P_{t,t+s}}{P_{t+s}} \right)^{-\eta} y_{t+s} - \frac{\varphi}{2} \left( \frac{P_{t,t+s}}{P_{t,t+s-1}} - 1 \right)^2 \right]
\]

In the symmetric equilibrium holds that \( P_{it} = P_t \).

Substituting for \( mc_t \), the optimal pricing decision can be written as

\[
(1 - \eta) a_t + \eta w_t \left( \theta h_t^r + (1 - \theta) h_t^o \right) - \varphi (\pi_t - 1) \pi_t + \varphi \beta E_t \left( \frac{U_{c,t+1}^o}{U_{c,t}^o} \frac{(1 + \tau_{t+1}^e)}{1 + \tau_{t+1}^e} \right) (\pi_{t+1} - 1) (\pi_{t+1}) = 0
\]

(16)

The aggregate production function can be expressed as

\[
y_t = a_t h_t
\]

(17)

Furthermore, using the optimisers’ budget constraint, we can obtain aggregate profits as

\[
d_t = \left( 1 - \frac{w_t}{a_t} - \frac{\varphi}{2} (\pi_t - 1)^2 \right) y_t
\]

(18)

Finally, we close the private sector equilibrium by imposing the no-Ponzi game condition

\[
\lim_{s \to +\infty} E_t \left[ \left( \prod_{i=0}^{t+s-1} \frac{1}{R_i} \right) B_{t+s} \right] = 0
\]

(19)

and the transversality condition

\[
\lim_{s \to +\infty} \left[ \beta^{t+s} \left( \frac{U_{c,t+s}^o}{1 + \tau_{t+s}^e} \right) B_{t+s} \right] = 0
\]

(20)

2.3 Government sector

The government consists of two authorities. First, there is a monetary authority which controls the nominal interest rates on short-term nominal bonds through open market operations. Second, there is a fiscal authority deciding on the level of government expenditures, labor income or consumption taxes and on debt policy. Government expenditures consist of spending
for the provision of public goods and for interest payments on outstanding debt. The level of
public goods provision is a choice variable, i.e. a policy instrument of the government. The
government finances current expenditures by raising labor income or consumption taxes and
by issuing new debt so that its budget constraint is given by

\[
\frac{b_t}{\pi_t} + g_t = \frac{b_{t+1}}{R_t} + w_t h_t \tau_t^h + c_t \tau_t^c
\]  \hspace{1cm} (21)

The government can credibly commit to repay its debt and public debt is assumed to be
nominal and not state-contingent, consistent with the type of debt typically issued by gov-
ernments around the globe. These features imply, however, that monetary policy decisions
affects the government budget through two channels: first, the nominal interest rate policy
of monetary authority influences directly the nominal return the government has to offer on
its instruments; second, nominal interest rate decisions also affect the price level and thereby
the real value of outstanding government debt. Thus, to the extent that the monetary policy
can affect the real interest rate or the price level, it will affect the government budget. In
what follows we assume that government debt and tax policies are such that the no-Ponzi
constraint and the transversality constraint are both satisfied.

2.4 Aggregate resource constraint

Combining the government budget constraint, the definition of profits and the optimisers’
budget constraint, one can obtain the aggregate resource constraint as

\[
y_t = a_t (\theta h_t^o + (1 - \theta) h_t^p) = \theta c_t^o + (1 - \theta) c_t^p + g_t + \frac{\varphi}{2} (\pi_t - 1)^2
\]  \hspace{1cm} (22)

2.5 Rational Expectation Equilibrium

The private sectors’ optimality conditions can then be condensed into a (non-linear) Phillips
curve

\[
((1 - \eta) a_t + \eta w_t) h_t - \varphi (\pi_t - 1) \pi_t + \varphi \beta E_t \left[ \frac{U^o_{c,t+1} (1 + \tau_{t+1}^c)}{U^o_{c,t} (1 + \tau_{t+1}^c)} (\pi_{t+1} - 1) (\pi_{t+1}) \right]
\]  \hspace{1cm} (23)

The Euler Equation

\[
\frac{U^o_{c,t}}{(1 + \tau_t^c)} - \beta E_t \left( \frac{U^o_{c,t+1}}{(1 + \tau_{t+1}^c)} \frac{R_t}{\pi_{t+1}} \right)
\]  \hspace{1cm} (24)

the ROTC budget constraint

\[
c_t^c = w_t h_t^c \frac{(1 - \tau_t^h)}{(1 + \tau_t^c)}. \hspace{1cm} (25)
\]
the labour supply of the optimisers

\[ \frac{U^o_{h,t}}{U^o_{c,t}} = w_t \frac{1 - \tau^h_t}{1 + \tau^c_t} \]  \hspace{1cm} (26)

and the one of ROTC

\[ \frac{U^r_{h,t}}{U^r_{c,t}} = w_t \frac{1 - \tau^h_t}{1 + \tau^c_t} \]  \hspace{1cm} (27)

the government budget constraint

\[ \frac{b_t}{\pi_t} + g_t = \frac{b_{t+1}}{R_t} + w_t h_t \tau^h_t + c_t \tau^c_t \]  \hspace{1cm} (28)

and the aggregate resource constraint

\[ y_t = a_t (\theta h_t^r + (1 - \theta) h_t^o) = \theta c_t^r + (1 - \theta) c_t^o + g_t + \frac{\varphi}{2} (\pi_t - 1)^2 \]  \hspace{1cm} (29)

**Definition 1 (Rational Expectation Equilibrium)** Given the initial outstanding debt level \((b_{-1})\), a Rational Expectations Equilibrium (REE) consists of a sequence of government policies \(\{\tau^h_t, \tau^c_t, R_t, g_t, b_{t+1}\}_{t=0}^{\infty}\) and private sector choices \(\{c_t^r, c_t^o, h_t^r, h_t^o, w_t, \pi_t\}_{t=0}^{\infty}\) satisfying equations (23)-(29), the no-Ponzi game (19) and the transversality condition (20).

### 3 Social Planner Problem

**Definition 2 (Social Planner Problem)** The Social Planner program consists in choosing \(\{c_t^r, c_t^o, h_t^r, h_t^o, c_t^*, h_t^*, g_t^\}_{t=0}^{\infty}\) taking as given the technology process \(\{a_t\}_{t=0}^{\infty}\), in order to maximise the weighted average of the utility function of the two agents as in (1) subject to the aggregate resource constraint (22) and the aggregate production function (17).

**Proposition 1** The Social Planner allocation can be expressed as

\[ (U^r_{c,t})^* = (U^o_{c,t})^* = -\left(\frac{U^r_{h,t}}{a_t}\right)^* = -\left(\frac{U^o_{h,t}}{a_t}\right)^* = (U^*_{g,t}) \]  \hspace{1cm} (30)

**Proof.** See Appendix. ■

**Corollary 1** From Proposition 1 it follows that

\[ c_t^* = c_t^o = c_t^* \]  \hspace{1cm} (31)

\[ h_t^* = h_t^o = h_t^* \]  \hspace{1cm} (32)

Hence it is optimal to equate the marginal utilities across agents. Furthermore it is optimal to equate the marginal utilities of private and public consumption to the marginal disutility of
labour, where the latter is scaled by total productivity. This simple allocation rule is optimal because it is equally costly to produce public and private consumption goods and the two type of households have the same utility function.

4 Ramsey Policy

This section describes the monetary and fiscal policy problem. It is important to note that - due to the existence of a number of important economic distortions - policy can generally not achieve the first best allocation determined in the previous section. First, market power by firms generally implies that wages fall short of their marginal product, so that labor supply and therefore output is too low relative to the optimal allocation. Second, the requirement to finance government expenditure and interest payments on outstanding government debt with either distortionary income taxes or consumption taxes additionally depresses labor supply and output. Third, the presence of limited asset market participation induces an extra element in the distortionary effects of fiscal policy. Fourth, the presence of nominal rigidities may prevent the price system from providing the appropriate scarcity signals. Monetary and fiscal policy will seek to minimise the effects of all these distortions. As we will see below, the implementation of the Ramsey equilibrium will crucially depends on the type of fiscal instrument adopted, i.e. labour or consumption taxes, as well as the stock of outstanding debt and the intensity of limited asset market participation.

**Definition 3 (Ramsey Problem):** The optimal policy problem (Ramsey problem) is a unique RE competitive equilibrium which maximises the total welfare defined as

\[
E_0 \left\{ \max_{c_t^r, c_t^o, h_t^r, h_t^o, \pi_t, g_t} \sum_{t=0}^{\infty} \beta^t \left[ \theta U_t^r (c_t^r, h_t^r, g_t^r) + (1 - \theta) U_t^o (c_t^o, h_t^o, g_t^o) \right] \right\}.
\]

In the Ramsey program the Lagrangian multipliers associated with forward looking variables, i.e. $E_t \pi_{t+1}$ and $E_t y_{t+1}$, are two additional state variables. As it is known, assuming these states initial values equal to zero gives rise to transitory non-stationary components in the solution to the optimal policy problem, even in the absence of shocks. Specifically, in the initial period the policy-maker may find it optimal to generate ‘surprise’ inflation so as to erode the real value of any outstanding government debt. Likewise, the policy-maker may find it optimal to transitory increase taxes. In what follows, we abstract from these non-stationary deterministic components of optimal policy and focus instead on the time-invariant deterministic long-run outcome. This outcome will be called the Ramsey steady state. Technically, time-invariance can be achieved by setting the time zero values of the two additional initial states equal to their steady state value rather than to zero. Economically, this amounts to
imposing an initial commitment on the policy-maker not to generate ‘surprise’ movements in
taxes, government spending, or nominal interest rates in period zero. This is standard prac-
Implicitly, we also impose the constraint that the policy-maker at time $t = 0$ is required to
repay the outstanding debt $b_{-1}$. As will be shown below, this constraint is binding; without
this constraint it would be optimal to default on the inherited debt level at time $t = 0$.

### 4.1 Ramsey Steady State

There is a continuum of deterministic Ramsey steady states, each of which is associated with
a different level of government debt. Note that the first order condition with respect to bonds
from the optimal policy problem (33) is given\footnote{In the Appendix the reader can find a detailed description of the Ramsey plan.} by

$$
\frac{\gamma_{3,t}}{R_t} - \beta E_t \frac{\gamma_{3,t+1}}{\pi_{t+1}} = 0
$$

while the Euler equation at steady state reads as

$$
R = \frac{\pi}{\beta}
$$

It follows that the foc for public debt does not restrict the steady state equilibrium. This
implies an indeterminacy problem. In other words, in order to calculate the unique Ramsey
steady state we have to impose an exogenous steady state outstanding value of public debt.

### 5 Results

#### 5.1 Calibration and Numerical Solution

Where not differently stated, we solve the non-stochastic steady state numerically. We adopt
our version of the OLS projection approach proposed by Schmitt-Grohè and Uribe (2004,
2007, 2012). This consists in exploiting the insight that the Ramsey equilibrium conditions
are linear in the vector of Lagrange multipliers, $\gamma_i$.

We specify preferences in such a way that satisfy the conditions explained above and are
consistent with balanced growth,

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \theta \left[ \log c_t^r - \omega_h \frac{(h^r)^{1+\phi}}{1+\phi} + \omega_g \log (g_t) \right] + (1-\theta) \left[ \log c_t^o - \omega_h \frac{(h^o)^{1+\phi}}{1+\phi} + \omega_g \log (g_t) \right] \right\}
$$

We calibrate the model to a quarterly frequency. We fix the discount factor in order
to have an ex-post real interest rate of 3.2% which is consistent with the three months US
interest rate ($\beta = 0.9913$). The elasticity of demand is chosen in order have a steady state gross markup of $1.2$ ($\eta = 6$), which is in line with the macro literature. We solve the model with different values of $\phi$, i.e., $0.2, 1, 3$. The utility parameter $\omega_h$ is chosen so that the two types of households supply in the decentralised equilibrium with full participation and no steady state public debt between one fifth and one third of their time to work. We further fix $\omega_g$ in order to have in the first best allocation a ratio of government spending over total output of $30\%$ ($\omega_h = 19.792$, $\omega_g = 0.2641$). The price stickiness parameter is selected such that the log-linearized version of the Phillips curve (16) is consistent with the estimates of Sbordone (2002), as in Schmitt-Grohé and Uribe (2004) ($\varphi = 17.4$). The quarterly standard deviation of the technology shocks is $0.6\%$ and the shocks have a quarterly persistence equal to $0.9$. The estimates of the share of limited asset market participation ranges between $0$ and $60\%$ of the total population. We show how our results change by varying the parameter ruling the share of limited asset market participation, $\theta$. we use as a benchmark a value of $0.3$, which is generally considered as a conservative parametrisation of LAMP behaviour. Furthermore we show the implications of varying the long run level of public debt. The benchmark employed of debt to total output of $60\%$. Table 2 summarises the calibration adopted

<table>
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<th>Parameter Definition</th>
<th>Assigned Value(s)</th>
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<td>Discount factor</td>
<td>$\beta = 0.9913$</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\phi = 1, 3, 4$</td>
</tr>
<tr>
<td>Demand Elasticity</td>
<td>$\eta = 6$</td>
</tr>
<tr>
<td>Utility weight on labor effort</td>
<td>$\omega_h = 19.792$</td>
</tr>
<tr>
<td>Utility weight on government spending</td>
<td>$\omega_g = 0.2641$</td>
</tr>
<tr>
<td>Sticky price parameter</td>
<td>$\varphi = 17.4$</td>
</tr>
<tr>
<td>Limited asset market participation</td>
<td>$\theta = 0 - 0.6$</td>
</tr>
<tr>
<td>Steady state public debt (%of GDP)</td>
<td>$0 - 200%$</td>
</tr>
</tbody>
</table>

### 5.2 Steady State Results

This section explores the quantitative implications of different levels of limited asset market participation for the Ramsey steady state allocations (with particular focus on the Ramsey public spending) and steady state welfare. In particular, here we show that limited asset market participation gives rise to quantitatively important steady state effects. Using the calibration from the previous section, Figures (2) and (3) report the Ramsey steady state outcomes for government to GDP ratio under labour income tax and consumption tax, the associated tax rates and the loss with respect to the first best welfare.

A few things are worth of notice.

First, when only labour income tax are available, for a given level of outstanding public debt, the Ramsey steady state ratio of public spending to GDP decreases in the share of
ROTC. As a consequence, the labour income tax rate decreases in \( \theta \). This is the result of two combined effects. On one hand, labour income taxes depresses labour supply. This gives to the policy-maker an incentive to reduce public spending below first best. In this way tax rates are lower than it would have implied with \( \frac{g}{y} = \frac{\varphi^*}{y} \). This holds even when \( \theta = 0 \) (Adam, 2011).

On the other hand, increasing the share of ROTC, labour income tax become more distortive, as part of the tax revenues has to finance the service of public debt. This in turn means a pure wealth transfer from ROTC to optimising agents. Therefore an higher incentive for the Ramsey planner to further decrease public spending left panel in figure (1).

Interestingly, this stops being true when the government has access to consumption tax only. With this instrument, the policy maker has the possibility to keep public spending at its first best. The reason is that the consumption tax scheme is less distortive. In particular, the optimisers are less sensitive to consumption taxation compared to the labour income one. This allow the Ramsey planner to keep public spending at the Social planner level without incurring in welfare losses. For this reason, using the consumption tax is always welfare improving with respect to labour income.

Third, both under labour income and consumption taxes, the increase in the share of ROTC generates a aggregate welfare loss. The reason is that the Ramsey planner does not have enough instruments to compensate for the wealth effects of fiscal policy which imply an increase in the inequality between consumer types. This is clearly suboptimal since the first best allocation would imply perfect equality.

Similarly, for a given share of ROTC, increasing the steady state level of public debt, implies a lower public spending to GDP ratio when labour income taxes are used and an increase in the aggregate loss. As before, the loss is higher under the labour income tax scheme figure (4) and figure (5).

5.3 Ramsey Dynamics

[In progress]

6 Conclusions

We study optimal government spending policy in a New Keynesian setting. We augment the standard model with limited asset market participation as in Bilbiie (2008). Furthermore we allow fiscal authority to levy a proportional labour income tax to finance its public consumption and public debt service as in Schmitt-Grohë and Uribe (2004, 2007). We show that limited asset market participation makes it optimal to reduce public spending compared to the first best allocation. This is so as to dampen the strongly distortionary effects that fiscal policy has on the welfare of the financially constrained agents. Moreover we show that
replacing labour income tax with consumption tax allows the Ramsey Planner to set public spending at its first best regardless of limited asset market participation or long run public debt.
References


Appendix

Structural equations:

Rule-of-thumb consumption

\[
 c^r_t = w_t h_t^r \frac{(1 - \tau^h_t)}{(1 + \tau^c_t)} \tag{36}
\]

Aggregate consumption:

\[
 c_t = (\theta c^r_t + (1 - \theta) c^o_t) \tag{37}
\]
Euler Equation

\[
\frac{(c_t^0)^{-1}}{(1 + \tau_t^0) R_t} = \beta E_t \left[ \frac{(c_{t+1}^0)^{-1}}{(1 + \tau_{t+1}^0)} \right]
\]  \quad (38)

Aggregate hours

\[
h_t = \theta h_t^R + (1 - \theta) h_t^o
\]  \quad (39)

Optimising agents labour supply

\[
\omega (h_t^o) c_t^o = w_t \frac{(1 - \tau_t^h)}{(1 + \tau_t^h)}
\]  \quad (40)

Rule-of-thumb consumers labour supply

\[
\omega (h_t^R) c_t^R = w_t \frac{(1 - \tau_t^h)}{(1 + \tau_t^h)}
\]  \quad (41)

New Keynesian Phillips Curve

\[
((1 - \eta) a_t + \eta w_t) h_t - \varphi (\pi_t - 1) \pi_t + \varphi \beta E_t \left[ \frac{c_t^o (1 + \tau_t^c)}{c_{t+1}^o (1 + \tau_{t+1}^c)} (\pi_{t+1} - 1) (\pi_{t+1}) \right] = 0
\]  \quad (42)

Government budget constraint

\[
\frac{1}{\pi_t} b_t + g_t = R_t^{-1} b_{t+1} + w_t h_t \tau_t^h + c_t \tau_t^c
\]  \quad (43)

Aggregate output

\[
a_t h_t = c_t + g_t + \frac{\varphi}{2} (\pi_t - 1)^2
\]  \quad (44)

Profits

\[
d_t = \left( 1 - \frac{w_t}{a_t} - \frac{\varphi}{2} (\pi_t - 1)^2 \right) a_t h_t
\]  \quad (45)

Technology

\[
\log (a_t) = \rho_a \log (a_{t-1}) + \varepsilon_t
\]  \quad (46)
Social Planner Problem

The social planner problem can be characterised as:

\[
\max_{c_t^o,c_t^r,h_t^o,h_t^r,g_t} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \theta \left( \log c_t^r - \omega_h \frac{(h_t^r)^{1+\phi}}{1+\phi} + \omega_g \log g_t \right) + (1 - \theta) \left( \log c_t^o - \omega_h \frac{(h_t^o)^{1+\phi}}{1+\phi} + \omega_g \log g_t \right) + \right. \\
\left. - \lambda_t \left[ \theta c_t^r + (1 - \theta) c_t^o + g_t - a_t \left( \theta h_t^r + (1 - \theta) h_t^o \right) \right] \right\}
\]

The first order conditions to the social planner optimization problem are the following:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial c_t^o} &= 0 : (c_t^o)^{-1} = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial c_t^r} &= 0 : (c_t^r)^{-1} = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial h_t^o} &= 0 : \omega_h (h_t^o)^{1+\phi} = a_t \lambda_t \\
\frac{\partial \mathcal{L}}{\partial h_t^r} &= 0 : \omega_h (h_t^r)^{1+\phi} = a_t \lambda_t \\
\frac{\partial \mathcal{L}}{\partial g_t} &= 0 : \omega_g (g_t)^{-1} = \lambda_t 
\end{align*}
\]

By assumption, the two household groups have symmetrical preferences, but ROTC consumers have no access to financial markets. As a result, from the social planner perspective both consumption and worked hours should be identical for the two groups, i.e. \((c_t^o)^* = (c_t^r)^* = (c_t)^*\) and \((h_t^o)^* = (h_t^r)^* = (h_t)^*\)

Moreover at steady state the following result holds:

\[
\frac{c^*}{h^*} = \psi^* \\
\psi^* = \frac{1}{(\omega_g + 1)}
\]

This completes the proof of Proposition 1.
Ramsey problem

\[ \mathcal{L} = \max_{c_t^r, c_t^o, h_t^r, h_t^o, \pi_t, g_t, b_t} \left\{ \begin{array}{l}
R_t \geq 1, \tau_t^h, c_t, h_t, w_t, d_t,
\end{array} \right\} + \sum_{t=0}^{\infty} \beta^t \left[ \begin{array}{l}
\theta \left( \log c_t^r - \omega h_t^r \frac{(h_t^o)^{1+\phi}}{1+\phi} + \omega_g \log g_t \right) \\
+ (1 - \theta) \left( \log c_t^o - \omega h_t^o \frac{(h_t^o)^{1+\phi}}{1+\phi} + \omega_g \log g_t \right)
\end{array} \right] + \gamma_1^t \beta^t \left[ \begin{array}{l}
((1 - \eta) a_t + \eta w_t) \left( \theta h_t^r + (1 - \theta) h_t^o \right) - \varphi (\pi_t - 1) \pi_t^+ \\
+ \varphi \beta E_t \left[ \frac{c_{t+1}^r (1 + \tau_t^r)}{c_{t+1}^o (1 + \tau_t^o)} (\pi_{t+1} - 1) (\pi_{t+1}) \right]
\end{array} \right] + \gamma_2^t \beta^t \left[ \begin{array}{l}
\left[ \frac{1}{(1 + \tau_t^r)^2} \right] - \beta E_t \left[ \frac{1}{(1 + \tau_t^r)^2} \right] \frac{R_t}{\pi_t}
\end{array} \right] + \gamma_3^t \beta^t \left[ \begin{array}{l}
-\pi_t^{-1} b_t - g_t + R_t^{-1} b_{t+1} + w_t (\theta h_t^r + (1 - \theta) h_t^o) \tau_t^h + (\theta c_t^r + (1 - \theta) c_t^o) \tau_t^o
\end{array} \right] + \gamma_4^t \beta^t \left[ \begin{array}{l}
a_t \left( \theta h_t^r + (1 - \theta) h_t^o \right) - (\theta c_t^r + (1 - \theta) c_t^o) - g_t - \frac{\varphi^2}{2} (\pi_t - 1)^2
\end{array} \right] + \gamma_5^t \beta^t \left[ \begin{array}{l}
\omega h (h_t^o)^{\phi} c_t^r \left( 1 - \tau_t^r \right) - w_t \left( 1 - \frac{b_t}{h_t^o} \right)
\end{array} \right] + \gamma_6^t \beta^t \left[ \begin{array}{l}
c_t^o \left( 1 + \tau_t^o \right) - h_t^r w_t \left( 1 - \tau_t^h \right)
\end{array} \right] + \gamma_7^t \beta^t \left[ \begin{array}{l}
\omega h (h_t^o)^{\phi} c_t^o \left( 1 + \tau_t^o \right) - w_t \left( 1 - \tau_t^h \right)
\end{array} \right] \]

focs:

\[ \tau_t^o = \gamma_1, t \varphi \beta E_t \left[ \frac{c_{t+1}^o}{c_t^o (1 + \tau_t^o)} (\pi_{t+1} - 1) (\pi_{t+1}) \right] - \gamma_{1,t-1} \varphi \left[ \frac{c_{t-1}^o (1 + \tau_t^o)}{c_t^o (1 + \tau_t^o)^2} (\pi_t - 1) (\pi_t) \right] - \gamma_{2,t} \left( \frac{1}{c_t^o (1 + \tau_t^o)^2} \right) + \gamma_{2,t-1} \left( \frac{1}{c_t^o (1 + \tau_t^o)^2} \frac{R_t}{\pi_t} \right) + \gamma_{3,t} (\theta c_t^r + (1 - \theta) c_t^o) + \gamma_{5,t} \omega h (h_t^o)^{\phi} c_t^r + \gamma_{6,t} c_t^r + \gamma_{7,t} \omega h (h_t^o)^{\phi} c_t^o
\]

\[ R_t = -\gamma_{2,t} \beta \left( \frac{1}{c_{t+1}^o (1 + \tau_t^o)} \frac{1}{\pi_{t+1}} \right) - \gamma_{3,t} \frac{b_{t+1}}{R_t^2}
\]

\[ \pi_t = -\gamma_{1,t} \varphi \left( 2\pi_t - 1 \right) + \varphi \gamma_{1,t-1} \left( \frac{c_{t-1}^o (1 + \tau_t^o)}{c_t^o (1 + \tau_t^o)} \left( 2\pi_t - 1 \right) \right) + \gamma_{2,t-1} \left( \frac{1}{c_t^o (1 + \tau_t^o)} \frac{R_{t-1}}{\pi_t} \right) + \gamma_{3,t} \frac{b_t}{\pi_t^2} - \gamma_{4,t} \varphi (\pi_t - 1)
\]
\[c_t^c: \theta \frac{1}{c_t^c} + \gamma_{3,t} \theta \tau_{t}^c - \gamma_{4,t} \theta + \gamma_{5,t} \omega h (h_t^c)^\phi (1 + \tau_t^c) + \gamma_{6,t} (1 + \tau_t^c)\]  
\hspace{15em} (50)

\[h_t^c: -(1 - \theta) \omega h (h_t^c)^\phi + \gamma_{1,t} (1 - \theta) ((1 - \eta) a_t + \eta w_t) + \gamma_{3,t} (1 - \theta) w_t \tau_t^h + \gamma_{4,t} a_t (1 - \theta) + \gamma_{7,t} \phi \omega h (h_t^c)^{\phi-1} c_t^c (1 + \tau_t^c)\]  
\hspace{15em} (51)

\[c_t^c: (1 - \theta) \frac{1}{c_t^c} + \gamma_{1,t} \left[ \varphi \beta E_t \left[ \frac{(1 + \tau_t^c)}{c_t^c (1 + \tau_{t+1}^c)} (\pi_{t+1} - 1) (\pi_{t+1}) \right] \right] - \gamma_{1,t-1} \left[ \varphi \left( \frac{c_t^{c-1}(1 + \tau_{t-1}^c)}{(c_t^c)^2 (1 + \tau_t^c)} (\pi_t - 1) \pi_t \right) \right] - \gamma_{2,t} \left( \frac{1}{1 + \tau_t^c} (c_t^c)^2 \right) + \gamma_{2,t-1} \left( \frac{1}{1 + \tau_t^c} (c_t^c)^2 \frac{R_{t-1}}{\pi_t} \right) + \gamma_{3,t} (1 - \theta) \tau_t^c - \gamma_{4,t} (1 - \theta) + \gamma_{7,t} \omega h (h_t^c)^\phi (1 + \tau_t^c)\]  
\hspace{15em} (52)

\[w_t: \gamma_{1,t} \eta (\theta h_t^c + (1 - \theta) h_t^c) + \gamma_{3,t} (\theta h_t^c + (1 - \theta) h_t^c) \tau_t^h + \gamma_{5,t} \omega h (h_t^c)^\phi (1 + \tau_t^c) \hspace{15em} (53)

\[-\gamma_{5,t} \left( 1 - \tau_t^h \right) - \gamma_{6,t} \left( 1 - \tau_t^h \right) - \gamma_{7,t} \left( 1 - \tau_t^h \right) \hspace{15em} (54)

\[h_t^c: -\theta \omega h (h_t^c)^\phi + \gamma_{1,t} \theta ((1 - \eta) a_t + \eta w_t) + \gamma_{3,t} w_t \tau_t^h \theta + \gamma_{4,t} a_t \theta + \gamma_{5,t} \omega h \phi (h_t^c)^{\phi-1} c_t^c (1 + \tau_t^c) - \gamma_{6,t} w_t (1 - \tau_t^h)\]  
\hspace{15em} (55)

\[g_t: \omega_g \frac{1}{g_t} - \gamma_{3,t} - \gamma_{4,t}\]  
\hspace{15em} (56)

\[b_{t+1} = \frac{1}{R_t} - \frac{\beta E_t}{\pi_t} \gamma_{3,t+1}\]  
\hspace{15em} (57)

\[\tau_t^h: \gamma_{3,t} w_t (\theta h_t^c + (1 - \theta) h_t^c) + \gamma_{5,t} w_t + \gamma_{6,t} h_t^c w_t + \gamma_{7,t} w_t\]  
\hspace{15em} (58)
Steady State

We guess and verify that in the non-stochastic Ramsey steady state the optimal inflation rate is $\pi = 1$, hence the steady state version of the NKPC yields

$$w = \frac{\varepsilon - 1}{\varepsilon} \tag{62}$$

Combining the ROTC budget constraint and their labour supply it yields

$$h^r = \left(\frac{1}{\omega_h}\right)^{\frac{1}{\tau + \phi}} \tag{63}$$

due to

$$e^r = h^r w \frac{1 - \tau^h}{(1 + \tau_c)} \tag{64}$$

using the initial values and the optimiser’s labour supply we get

$$c^o = \frac{w (1 - \tau^h)}{\omega_h (h^o)\phi (1 + \tau_c)} \tag{65}$$

Given $b_0$, from the $R$ FOC we get

$$\gamma_2 = -\gamma_3 b_0 \beta c^o (1 + \tau_c) \tag{66}$$

From the FOC for public spending we get

$$\gamma_4 = \omega_g \frac{1}{g} - \gamma_3 \tag{67}$$

From the ROTC consumption

$$\gamma_6 = \left[-\theta \frac{1}{c} - \gamma_3 \theta \tau^c + \gamma_4 \theta - \gamma_5 \omega_h (h^r)\phi (1 + \tau_c)\right] \frac{1}{1 + \tau_c} \tag{68}$$
and the FOC w.r.t. $\tau_t$

$$\gamma_7 = -\gamma_5 - h^r \gamma_6 - \gamma_3 (\theta h^r + (1 - \theta) h^o) \tag{69}$$

via the foc w.r.t. $w_t$

$$\gamma_1 = \left[ \gamma_5 (1 - \tau) + \gamma_7 (1 - \tau^h) + \gamma_6 h^r (1 - \tau^h) - \gamma_3 (\theta h^r + (1 - \theta) h^o) \tau^h \right] \frac{1}{\eta (\theta h^r + (1 - \theta) h^o)} \tag{70}$$

so we pinned down $w, h^r, c^r, c^o, \gamma_1, \gamma_2, \gamma_4, \gamma_6, \gamma_7$.

We use the other six equations to solve for the initial guesses for $\tau^h, \tau^c, g, \gamma_3, \gamma_5, h^o$ as

$$(1 - \theta) \frac{1}{c^o} + \gamma_2 \left( \frac{1}{(1 + \tau^c) (c^o)^2} \left( \frac{1}{\beta} - 1 \right) \right) + \gamma_3 (1 - \theta) \tau^c - \gamma_4 (1 - \theta) + \gamma_7 \omega_h (h^o)^\phi (1 + \tau^c) = 0 \tag{71}$$

$$\gamma_2 \left( \frac{1}{c^o (1 + \tau^c)^2} \left( \frac{1}{\beta} - 1 \right) \right) + \gamma_3 (\theta c^r + (1 - \theta) c^o) + \gamma_5 \omega_h (h^r)^\phi c^r + \gamma_6 c^r + \gamma_7 \omega_h (h^o)^\phi c^o = 0$$

$$(1 - \theta) \omega_h (h^o)^\phi + \gamma_1 (1 - \theta) ((1 - \eta) + \eta w) + \gamma_3 (1 - \theta) w \tau^h + \gamma_4 (1 - \theta) + \gamma_7 \phi \omega_h (h^o)^{\phi - 1} c^o (1 + \tau^c) = 0 \tag{72}$$

$$(\theta h^r + (1 - \theta) h^o) - (\theta c^r + (1 - \theta) c^o) - g = 0 \tag{73}$$

$$-b - g + \beta b + w (\theta h^r + (1 - \theta) h^o) \tau^h + (\theta c^r + (1 - \theta) c^o) \tau^c = 0$$

$$-\theta \omega_h (h^r)^\phi + \gamma_1 \theta ((1 - \eta) + \eta w) + \gamma_3 w \tau^h + \gamma_4 \theta + \gamma_5 \omega_h (h^r)^{\phi - 1} c^r (1 + \tau^c) - \gamma_6 w (1 - \tau^h) = 0 \tag{74}$$
Figures

Figure 1: Bank Tightening Credit Standard
Figure 2: Steady state public debt 100% of total output
Figure 3: Steady state public debt 100% of total output.
Figure 4: $\theta = 0.3$
Figure 5: $\theta = 0.3$