A Finite-Time-Horizon Model of Suicide When a Person’s Income is at Risk

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Abstract
The risk of income fluctuations affects suicidal behaviour. First, an increase in this risk makes risk-averse households more likely to commit suicide by reducing their expected utility. Second, the increased risk makes them less likely to commit suicide by creating a value to waiting for the economy to improve. This paper lays out a theoretical model of suicide to assess the net impact of income fluctuations on suicidal behaviour by taking into account a household’s ability to delay the action.

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1. Introduction

Hamermesh and Soss (1974) formalise the theory of suicide within a framework of lifetime-utility-maximisation models. In their model, a household commits suicide when the present value of the expected remaining lifetime utility falls below a certain level. The model is extended in a variety of directions. Crouch (1979) assumes that the household's utility depends upon their income and the income of those they love. Koo and Cox (2007) replace permanent income with relative income, and Marcotte (2003) applies the model to study attempted suicides. Thus, the Hamermesh-Soss model is revolutionary in its economic evaluation of suicide.

This paper adds to the literature by amending the Hamermesh-Soss model, which gives no role to the risk of income fluctuations in suicidal behaviour. Unless households are risk neutral, however, this risk affects their decision. First, an increased risk of income fluctuations makes risk-averse households more likely to commit suicide by reducing their expected utility. Second, the increased risk makes them more likely to stay alive because if something improves, future income is more likely to be high enough to compensate for the current economic hardships. These effects cancel each other out to some extent. To assess the net impact, the model must be improved because of its implicit assumption that those who do not commit suicide never will. As the income fluctuation risk creates a value to waiting, the ability to delay the action of suicide should be taken into account.

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1 In the literature, completed suicides are normally distinguished from attempted ones because those who attempt suicide do not necessarily wish to die.
The next section lays out a theoretical model of suicide under the assumption that a household can choose to die in any period, thereby making it possible to assess the net impact of income fluctuations on suicidal behaviour\(^2\). This is the focus of the paper. As an example, the third section uses a three-period model to simulate how the risk of income fluctuation affects a household's decision of whether to commit suicide.

2. Model

2.1 Assumptions

At the start of each period, a household earns wage income. The household then compares the utility from committing suicide with the expected utility from delaying the action. If the former is greater than the latter, they will commit suicide at the end of the period. Otherwise, they will stay alive, at least until the next period.

The household can, if they wish, live for \( T \) periods beginning at period 1. Their instantaneous utility depends positively upon consumption, \( c \), and negatively upon hours of labour, \( l \). Under the assumption of stationary preferences, the utility flow in period \( t \) is denoted by \( u(c_t, l_t) \). Preferences

\(^2\) Dixit and Pindyck (1994) suggest that the theory of suicide should be formalised within a framework of real options models. Cutler et al. (2000) replace the profit flow from an investment project in a real options model with the utility flow. As the utility flow is assumed to be exogenous, however, their model explains no relationship between economic variables and suicidal behaviour. Suzuki (2008) also applies the real option approach to study suicide but cannot address the question of whether generational differences in suicide exist because the model operates on an infinite-time-horizon.
are also assumed to be separable across time. Let $U$ denote the present value of expected lifetime utility for those who choose to die in period $T^* \leq T$. Then, it is written as

$$U = u(c_1, l_1) + \sum_{t=2}^{T^*} \beta^{t-1} E[u(c_t, l_t)],$$

where $\beta \in [0,1]$ is the rate of time preference, and $E$ is the operator of expectation.

Disutility from working may be interpreted as representing the cost of staying alive. In the context of this paper, however, the household’s choice of labour supply is of secondary importance. Therefore, labour supply is assumed to be inelastic. The household spends the same amount of time on working every period. For the purpose of concreteness, assume the instantaneous utility function exhibiting constant relative risk aversion.

$$u(c_t, l_t) = c_t^{1-\theta}/(1 - \theta) - l^{1+\varphi}/(1 + \varphi),$$

for $t = 1, \cdots, T^*$,

where $\theta > 0$ and $\varphi > 0$ are the elasticity of marginal utility of consumption and that of marginal disutility of labour, respectively. The greater $\theta$ is, the more risk averse the household is. The household chooses consumption so as to maximise the present value of expected lifetime utility subject to some constraints.

Apart from an obvious non-negativity constraint on consumption in each period, there are two constraints that the household faces. One is related to how wage income evolves over time. To introduce a risk of income fluctuations in the following analysis, the evolution of wage income is approximated by a stochastic process. The assumptions are as follows. The initial wage is
exogenously given by \( w_1 \). Given the initial condition, the wage income may increase from \( w_t \) to \( \delta w_{t+1} \) with probability \( p \) or decrease to \( w_{t+1}/\delta \) with probability \( (1-p) \) over each period, where \( \delta \) is greater than one. The probability \( p \) is given by

\[
p = \frac{[(1 + r) - 1/\delta]/(\delta - 1/\delta)},
\]

where \( r \) is the real interest rate\(^3\). Under these assumptions, the expected value of \( w_{t+1} \), conditional upon \( w_t \), is \((1+r)w_t\). Thus, it is possible to investigate how the mean-preserving spread of future wage income influences suicidal behaviour.

The other one is an intertemporal budget constraint. The stochastic process for wage income produces \( 2^{T^* - 1} \) possible paths of wage income for those who live for \( T^* \) periods. When making consumption choice, the household takes into account the evolution of wage income so as to satisfy the intertemporal budget constraint for every possible wage path. Now, index each path of wage income. Then, the set of intertemporal budget constraint is written as

\[
c_1 + \sum_{t=2}^{T^*} c_{t,j}/(1 + r)^t \leq w_1 + \sum_{t=2}^{T^*} w_{t,j}/(1 + r)^t \quad \text{for} \quad j = 1, \ldots, 2^{T^* - 1},
\]

where the subscript \( j \) denotes the \( j \)-th path of wage income. When this set of budget constraints is satisfied, the present value of expected expenditures does not exceed that of expected lifetime income. Subject to the set of constraints, the household chooses consumption so as to maximise the

\(^3\) As \( p \) lies between 0 and 1, the following condition must hold. \( 1/\delta < 1 + r < \delta \).
expected lifetime utility.

In the following analysis, \( \delta \) in the evolution of wage income plays an important role because it may be interpreted as representing a risk of income fluctuations. To see this, let \( \sigma_n^2(h) \) denote the forecast-error variance of \( w_{t+h} \) conditional upon \( w_t \). The variance of the one-step-ahead forecast error, for instance, is calculated as

\[
\sigma_n^2(1) = \{ (1 + r)[ \delta + (1/ \delta) ] - 1 - (1 + r)^2 \} w_t^2.
\]

As \( \delta \) is greater than one by assumption, this variance is increasing in \( \delta \),

\[
d \sigma_n^2(1) / d \delta = (1 + r)[1 - (1/ \delta)^2] w_t^2 > 0.
\]

The subsequent section investigates how an increase in the risk parameter \( \delta \) affects the household’s suicidal decision-making.

2.2. Ability to Delay the Action

Let \( V_k \) denote the maximised present value of expected lifetime utility attained by staying alive for \( k \) periods, where \( k \) is an integer from 1 to \( T \). As the labour supply is assumed to be inelastic, it is written as

\[
V_k = \max_{\{c_1, \ldots, c_k \}} u(c_1, l) + \beta \left[ E \sum_{t=2}^k u(c_t, l) \right],
\]

along with the set of budget constraints. This is a value function for the household’s problem of optimisation. Comparing the value of \( V_k \) for all possible values of \( k \), the household determines
how long they should live. Let $V_T$ denote the greatest one. Then, $T^*$ is the number of periods for which the household prefers staying alive.

The model clearly takes into account the household’s ability to delay the action of committing suicide. If $T^*$ is equal to one, the household will stay alive only for one period, namely, commit suicide. If $T^*$ is equal to $T$, the household will choose a natural death. If $T^*$ is greater than one and less than $T$, however, the household will simply postpone committing suicide. The last case makes the decision making “now or later”. In this sense, the model of this paper is distinguished from the Hamermesh and Soss one and most of its offspring that implicitly make the decision making “now or never”.

3. Simulation Results from a Three-Period Model

This section uses a simple model as an example to derive the threshold wage, $w_1^*$, in the initial period below which a household commits suicide. The number of periods should be more than three to take into account the household’s ability to delay the action of committing suicide. In this example, the number of periods is set as three. The interest rate is set as 5% (i.e., $r=0.05$). The rate of time preference $\beta$ is equal to $1/(1+r)$. In the utility function, $\theta$ and $\phi$ are set as 0.6 and 0.25, respectively. Given these ad hoc values, the model calculates the maximised value of the expected utility for all possible lifespans chosen by the household.
As is mentioned outset, the risk of income fluctuations has two different impacts on the suicidal decision making. Figure 1 illustrates the derived relationship between the risk of income fluctuations and the values of the household’s choices, the latter which measures the expected lifetime utility attained for the chosen lifespan. For a lower range of the wage risk, the household prefers a natural death. On the other hand, it is optimal for them to commit suicide for a higher range of the risk. Thus, the figure clearly shows that the risk averseness dominates the value of waiting. Importantly, the choice to postpone suicide is dominates the other two options for a middle range of the wage risk. This suggests that the ability to delay the action should not be omitted when modelling the suicidal decision making.

[Figure 1 to be inserted here.]

Figure 2 illustrates the derived relationship between the initial wage and the values of the lifespan choices. The household chooses committing suicide and a natural death for a lower range and a higher one of wage income, respectively. This is in line with our perception. The household chooses to postpone committing suicide, however, for a middle range of wage income. This again suggests the importance of the ability to delay the action when modelling the suicidal decision making.

[Figure 2 to be inserted here.]
4. Conclusion

The risk of income fluctuations can affect the decision making of those who contemplate suicide due to economic hardships. First, an increase in this risk makes the risk-averse households more likely to commit suicide by reducing their expected utility. Second, the increased risk makes them less likely to commit suicide by creating a value to waiting for their economic circumstances to improve. These effects cancel out each other to some extent. In the economic literature, the landmark model fails to assess the net impact because of its implicit assumption that those who do not commit suicide never will. In contrast, this paper takes into account the household's ability to delay the action of suicide, thereby making it possible to assess the net impact of income fluctuations on suicidal behaviour.

Using a three-period example, this paper simulates how a risk-averse household will respond to an increase in the risk of income fluctuations. The simulation shows that as the risk of income fluctuations rises, the household is more likely to commit suicide. That is, the risk averseness dominates the value of waiting in the suicidal decision making under the chosen values of parameters. More importantly, the simulation also reveals that it is optimal for the household to postpone suicide for a middle range of the wage risk. This clearly illustrates how wrong it can be to ignore the household’s ability to delay the action.

Some caution should be taken, however. First, the results are certainly sensitive to the choice of
parameter values. Ideally, the parameter values should be consistent with the econometric estimates.

Second, the example model has a very short time-horizon. To make the model more realistic, it must have a longer time-horizon. In addition, a long time-horizon model makes it possible to investigate the generational difference in the suicidal pattern.

References


Figure 1: How Long One should Stay Alive under Risk of Income Fluctuations
Figure 2: How Long One should Stay Alive for a Given Wage