Permutation entropies (PEs) of international short-term interest rates and interest rate spreads before the financial crisis of 2007-09 *

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Abstract

Permutation entropy (PE), as a complexity measure for time series proposed by Bandt and Pompe [2002], has been used to detect dynamical changes in time series. In particular, PE can detect nonlinear temporal dependence in contrast to autocorrelation. I compute the PEs of international short-term interest rates and interest rate spreads against the U.S. three-month T-Bill rates by employing a moving window method. One-year and three-year windows are used for the first-differences of the daily data. The threshold is a lower bound under the null hypothesis of temporal independence based on Matilla-García and Marín [2008]. The results identify the period 2005-06 in which the PEs of the three-month U.S. T-Bill rates and interest rate spreads against the U.S. three-month T-Bill rates became lower than a threshold before the financial crisis of 2007-09. The results are robust across different specifications. The method also finds other historical episodes where PEs dropped below the threshold, including Euribor markets in 2009. This paper discusses the potential of PE as a complementary early warning signal in financial markets.

Keywords: Permutation entropy, complexity, market efficiency, interest rate
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1 Introduction

The recent financial crisis of 2007-09 demonstrated the pivotal role of financial stability in maintaining sustainable economic growth. The direct welfare loss from financial crises is enormous. Reinhart and Rogoff [2008] report that the average drop in the real per capita output growth is over 2 percent during 18 post-war banking crises. Moreover, Cerra and Saxena [2008] show there is an indirect and long-lasting effect of crises on economic growth.

The financial crisis of 2007-09 highlighted the necessity of developing tools to work as early warning signals of systemic failures in financial markets (see Gray [2009], for instance). The burgeoning literature on early warning indicators suggests that credit-related variables are the best indicators (Schularick and Taylor [forthcoming]). Whereas the credit-related indicators were successful in predicting financial crises, one of the remaining issues is that an interpretation of the indicators could be somewhat ambiguous. The higher credit-to-GDP ratio, one of the most popular measures for credit booms, can be interpreted as a result of changes in fundamentals like financial development or other institutional changes, for instance (the so-called “This time is different syndrome” as in Reinhart and Rogoff [2010]). In this paper, in order to complement the current literature on early warning indicators, I develop an approach based on the extraordinary dynamics of a variable whereas the focus of current literature is on the extraordinary level of relevant variables.

The Great Moderation suggests that it would be hard to find strong early warning signals in terms of volatility even though measurements of volatility are popular in evaluating the stability of financial markets.\footnote{A popular risk measure in financial markets is VIX (Chicago Board Options Exchange Volatility Index) and it has been known that VIX is correlated with risk premia (see Pan and Singleton [2008] for example). But VIX was low before the financial crisis of 2007-09 and especially in 2005-06.} It could be better to have early warning signals which do not depend on volatility of data. Furthermore, if misalignments in financial markets are growing during good times, then, the distribution of data may be different from it in normal times, particularly in terms of temporal dependence. It would be helpful to have a measurement for detecting the increase in temporal dependence.

There are at least two reasons why abnormally high temporal dependence may be viewed as warning signals. First, abnormally high temporal dependence may be the repercussion of herd behavior\footnote{In this paper, ‘herd’ means ‘converge in behavior’ as in Hirshleifer and Teoh [2003, 26]}. Second, the overly predictable movement of asset prices may induce excessive risk-taking by financial investors. Regarding this issue, Altunbas et al. [2010] indicate the pitfall of highly predictable mone-

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\textsuperscript{2}There exist other sources of temporal dependence. For instance, Billo et al. [2010] interpret positive autocorrelation in returns as a proxy of illiquidity exposure.

\textsuperscript{3}For example, Aitken [1998] finds higher autocorrelation in total returns in emerging stock markets with rising share of institutional investors by employing a variance ratio test.
etary policy in the sense that the resulting lower uncertainty may induce higher risk-taking of banks.\(^5\)

One of simple and popular measures of temporal dependence is autocorrelation. Moreover, according to Scheffer et al. [2009], higher autocorrelation is included in possible early warning signals of bifurcation of dynamical systems. But autocorrelation has the following weak points. First, autocorrelation can detect only linear dependence. In this regard, Mishkin [2011] highlights the nonlinearity of the macro economy as one of lessons from the financial crisis of 2007-09. Carpenter et al. [2011] report the presence of nonlinear dynamics in an experiment about “catastrophic ecological regime shifts.” Second, autocorrelation may be driven by some outliers. Third, autocorrelation is sensitive to structural break like change of mean.

Klonowski et al. [2004] suggest the methods developed in medical diagnosis (particulary regarding epileptic seizures) for diagnosing economic sickness because there is a similarity between economic systems and living organisms in that they both generate state-dependent signals.\(^6\) The complexity measures used in medical diagnosis include dimensions, (permutation) entropies, and Lyapunov exponents.\(^7\) Eckmann and Rulle [1985] provide a classic discussion about the complexity measures. Among them, permutation entropy is the most recent and computationally cheapest measure. In this paper, I investigate the potential of the permutation entropy (PE) as a measure of temporal dependence of financial variables.

In general, higher PE means that the data-generating process is more complex and unpredictable. If PE of a financial variable is significantly low, it implies market inefficiency because the efficient market hypothesis suggests unpredictability of future movements of financial variables (Lo [2008]).\(^8\) In addition, as we see later, PE may complement autocorrelation by overcoming difficulties related with autocorrelation mainly because the concept of PE depends only on ordinal patterns of time series. In this regard, this paper suggests that we need to utilize autocorrelation and PE together for studying temporal dependence of data.

I investigate daily movement of financial variables in order to obtain timely information and relatively large sample size. Instead of credit variables which are not available on a daily basis, I probe into the daily dynamics of nominal interest rates and interest rate spreads against the U.S. interest rates in terms of first-differences. The choice is largely driven by the following considerations:

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\(^5\)The recent discussions on “risk-taking channel” of monetary policy by Borio and Zhu [2008] and Adrian and Shin [forthcoming] highlight the effect of low short-term interest rates on the risk attitudes of investors. This paper, instead, focuses on the effect of low uncertainty of short-term interest rates on them.

\(^6\)Lehnertz [2008] provides an overview on application of nonlinear dynamical system theory in predicting epileptic seizures.

\(^7\)Brock et al. [1996] propose a test (BDS-test) for independence based on correlation dimension. Barnett et al. [1997] contain a useful comparison of several nonlinearity tests including BDS-test and a test based on the maximum Lyapunov exponent.

\(^8\)Among market efficiency measures, serial correlation in returns is one of common ones. See Griffin et al. [2010], for instance
First of all, an interest rate is one of the most important macroeconomic variables. Moreover, as the classic work of Kindleberger and Aliber [2005] shows, financial crisis is often related with boom-burst cycle of credit which, in turn, is related with cyclic movements of interest rates. Finally, daily observations of interest rates are available without a long time-lag, which is important for developing “real time” early warning indicators.

I study, in particular, short-term interest rates rather than long-term ones because of the following reasons: First, short-term interest rates are generally affected by a smaller set of factors and shocks than long-term interest rates, which may make the detection of temporal dependence in the case of short-term interest rates easier. Second, it is also well-known that interest rate spreads reflect the perceived risks in financial markets. Moreover, international short-term interest rate spreads are an important factor in explaining the patterns of capital flows, especially Carry trade. In this regard, the series of interest rate spreads may contain useful information about risk attitudes in financial markets.

A moving window method is employed to detect temporal changes in distribution of data. One-year and three year windows are used. A threshold level as a lower bound is chosen in this paper, based on the asymptotic distribution of PE under the condition of serial independence derived in Matilla-García and Marín [2008]. The threshold is employed as a reference level for detecting abnormal movements of short-term interest rates in terms of PE. To the best of my knowledge, my paper is the first one which employs PE to detect temporal dependence of interest rate dynamics. This paper also compares PE with autocorrelation to see whether PE practically provides additional information not readily available from autocorrelation.

I investigate interest rate dynamics of 10 countries related to Libor markets and, in particular, study domestic interest rates and Libor rates of the 10 countries. It turns out that most of the interest rates pass the independence test based on PE. But I discover some interesting episodes in which PEs suddenly plunge. I find a sudden drop of PEs of the U.S. short-term (especially 3-month and 6-month) T-Bill rates below a threshold before the financial crisis of 2007-09 and notably in 2005-06. Similar movements of PEs are found in international interest rate spreads against the U.S. 3-month T-Bill. This paper also reports similar drops of PEs of Euribors mainly in 2009. The results are robust with several different specifications in computing PE. The results in this paper imply that there were abnormal behaviors in both the U.S. short-term T-Bill markets before the financial crisis of 2007-09 and Euribor markets in 2009. The drops of PEs are even clearer in the case of simple average rates of short-term interest rates with different maturities, which is an interesting property related only with low PE. Violations of temporal independence in terms of PEs also are found in

Martingale-like movements of short-term interest rates in normal times are documented by Mankiw and Miron [1986], Ait-Sahalia [1996], and Bandi [2002] for instance, which suggests that temporal dependence of short-term interest rates may not be severe at least in normal times if there is no change of volatility. As we see later, PEs of short-term interest rates are higher than the threshold in most cases.
Eurodollar deposit rates during the late 1970’s and in Japan’s 3-month Tibor rates after the Asian crisis of 2007.

The composition of the paper is as follows: In Section 2, I overview the literature. Section 3 introduces the concept of PE and relevant computational methods and compares PE with autocorrelation. Section 4 contains the main results of this paper. Section 5 includes concluding remarks.

2 Literature review

This paper is related with the literature on early warning indicators. As in Alessi and Detken [2011], I divide the literature into two strands: signaling approach and discrete choice approach including probit/logit models. Signaling approach issues a warning if a warning indicator crosses a predetermined threshold. Kaminsky and Reinhart [1999] investigate 16 variables as indicators for banking and currency crises. They find that weak economic fundamentals were often followed by crises. Among the indicators, real interest rates and differentials display good performance. Borio and Lowe [2002] insightfully point out that low inflation does not preclude financial imbalances and find that credit gap defined as the deviation of the credit-to-GDP ratio from its trend is the best early warning indicator. Alessi and Detken [2011] discover that the global M1 gap and the global private credit gap are the best indicators among 18 variables (5 real variables and 13 financial variables) with up to 6 transformations. They highlight global financial variables as early warning indicators. Gerdesmeier et al. [2010] are a recent contribution from discrete choice approach. Gerdesmeier et al. [2010] identify credit aggregates, variations in nominal long-term interest rates, and the investment-to-GDP ratio together with asset prices (such as housing price and stock price) dynamics as important indicators of asset price burst. Kannan et al. [2009] show that credit, the investment-to-GDP ratio, current account deficit, and asset prices are good indicators of asset price bust by employing signaling approach and a probit model together. Finally, Schularick and Taylor [forthcoming] look at 12 developed countries over the period of 1870-2008 and document that credit growth is a good indicator for financial crisis. In short, the literature highlights the pivotal role of credit.

This paper is also related with the literature on PE. The concept of PE is introduced in Bandt and Pompe [2002] as a measure of complexity. Cao et al. [2004] apply the Bandt & Pompe method to identify dynamical changes in time series and to detect seizure based on a transient sharp drop of PE. Li et al. [2007] study predictability of absence seizures with permutation entropy. Two findings are particularly interesting: First, PE is higher in the normal state than in the seizure state. Second, the seizure is preceded by decrease of PE. Li et al. [2007, 72] interpret the findings as the decrease of complexity due to the synchronization of neural activity. Matilla-García [2007] and Matilla-García and Marín [2008] develop a non-parametric independence test based on PE. For example, Matilla-García and Marín [2008] show that the powers of the test in
the case of AR processes are higher than 85% with 10% significance level and T=600. Zunino et al. [2009] compute PEs of stock markets regarding market efficiency. They find an evidence for a negative correlation between PE and market efficiency while investigating the equity indices of 32 countries. This paper focuses on the variation of PE over time whereas Zunino et al. [2009] look at the cross-sectional variation of PE. It is interesting to observe the following similarity: Regarding the literature on prediction of seizure one possible driving force of a drop of PE is synchronization. In the literature on asset bubbles, herding behavior as one of the driving forces of asset bubbles may cause a drop of PE due to the inefficient use of information.

Finally, this paper is linked to the literature on early warning signals for critical transitions of complex systems. Mitchell [2009] lists brains and economies as examples of complex systems. According to Scheffer et al. [2009, 53], examples of critical transitions include epileptic seizures and systemic market crashes. In this context, this paper investigates whether an early warning signal in a complex system could be used in another complex system.

3 Symbolic sequences and permutation entropy

For a time series taking finite values \((s_1, s_2, \cdots, s_N)\), a Shannon entropy (Shannon [1948]) is well defined. Let \(p_i\) be the probability of \(s_i\). The Shannon entropy (H, or information entropy) is defined as follows:

\[
H = -\sum_{i=1}^{N} p_i \ln(p_i)
\]

The maximum of H, \(\ln(N)\), is obtained with the uniform distribution, \(p_i = \frac{1}{N}\). Higher H means more uncertainty or complexity (Bandt and Pompe [2002]). If the time series takes infinite values, we can use symbols obtained from a finite partition of a state space. As Bandt and Pompe [2002] explain, permutation entropy uses neighboring values for generating a partition. Let a univariate time series \(\{x_t\}_{1 \leq t \leq T}\) be given. We embed the univariate time series to a m-dimensional space as follows:

\[
X = \{x_{t-i} | \{x_t\}_{1 \leq t \leq T}\}
\]

\[i = 1, \cdots, m\]

10 Zunino et al. [2009, 2862] also point out the potential of PE as a predictor of financial crises.

11 Critical transitions mean sudden qualitative changes of dynamical systems. For detailed introduction, see Scheffer [2009].


13 The method is called the method of delay time coordinate proposed by Takens [1981].
\[ \mathbf{x}_1 = (x_1, x_{L+1}, \ldots, x_{L(m-1)+1}), \]
\[ \mathbf{x}_2 = (x_2, x_{L+2}, \ldots, x_{L(m-1)+2}), \]
\[ \vdots \]
\[ \mathbf{x}_{T-L(m-1)} = (x_{T-L(m-1)}, x_{T-L(m-2)}, \ldots, x_T) \]

where \( L \) is the time-lag and \( m \) is the number of time-lags, also called the embedding dimension. Then, \( \tilde{\mathbf{x}} = \{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{T-L(m-1)} \} \) is a sequence of vectors in \( \mathbb{R}^m \). We write \( \tilde{\mathbf{x}} \in X^m_{T-L(m-1)} \) where \( X^m_{T-L(m-1)} \) is the set of sequences of vectors in \( \mathbb{R}^m \) with the length of \( T-L(m-1) \).

### 3.1 A mapping from a time series to a symbolic sequence

Note that, given the numbers \( (1, 2, \ldots, m) \), there are \( m! \) different permutations:

\[ \mathbf{s}_b_1 = (1, 2, \ldots, m), \]
\[ \mathbf{s}_b_2 = (1, 2, \ldots, m, m-1), \]
\[ \vdots \]
\[ \mathbf{s}_b_{m!} = (m, m-1, \ldots, 1) \]

Let \( S = \{ \mathbf{s}_b_1, \mathbf{s}_b_2, \ldots, \mathbf{s}_b_{m!} \} \) be the set of symbols \( \mathbf{s}_b_1, \mathbf{s}_b_2, \ldots, \mathbf{s}_b_{m!} \). Given \( \mathbf{x} = (x_1, x_2, \ldots, x_m) \in \mathbb{R}^m \), we can arrange elements of \( \mathbf{x} \) in ascending order as follows:

\[ x_{j_1} < x_{j_2} < \cdots < x_{j_m}, \{j_1, j_2, \ldots, j_m\} = \{1, 2, \ldots, m\} \]

Note that \( (j_1, j_2, \ldots, j_m) \in S \). Thus, the operation defines a mapping \( \phi \) from \( \mathbb{R}^m \) to \( S \) as follows:

\[ \phi(\mathbf{x}) = (j_1, j_2, \ldots, j_m) \in S \]

Let \( \tilde{\mathbf{s}} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K) \in X^m_K \) be a sequence of vectors in \( \mathbb{R}^m \) with the length of \( K \). And let \( \Sigma^m_K \) be the space of symbolic sequences where the number of different types is \( m! \) and the length of a symbolic sequence is \( K \). As we see below, \( \text{PE} \) is defined as a mapping from \( \Sigma^m_K \) to \( [0,1] \). Finally, \( \phi \) induces a mapping \( \Phi \) from \( X^m_K \) to \( \Sigma^m_K \) as follows:

\[ \Phi(\tilde{\mathbf{s}}) = (\phi(\mathbf{x}_1), \phi(\mathbf{x}_2), \ldots, \phi(\mathbf{x}_K)) \in \Sigma^m_K \]

### 3.2 PE

Given a symbolic sequence \( \tilde{\mathbf{s}} = (\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K) \in \Sigma^m_K \), let \( 0 \leq p(h) \leq 1 \) be the relative frequency of type \( h \) where \( 1 \leq h \leq m! \). As \( K \to \infty \), the limit of

\[ ^{14} \text{If } x_i = x_j, i < j \text{ then, we may assume that } x_i < x_j \text{ in computation. Alternatively, we may break equalities numerically by adding small random perturbations as Bandt and Pompe \cite{2002} suggest. I follow the latter approach in this paper.} \]
p(h) exists if the following weak stationary condition holds (Bandt and Pompe [2002]):
\[
Pr(x_t < x_{t+j}) \text{ should be independent of } t \text{ for } j \leq m
\] (3.7)

Normalized permutation entropy is defined as a mapping from \( \Sigma^m_K \) to \([0,1]\) as follows:
\[
PE = \left[ -\sum_{h=1}^{m!} p(h) \ln(p(h)) \right] / \ln(m!)
\] (3.8)

To sum up, the algorithm of computing PE from a given time series is as follows:
\[
\text{time series} \rightarrow \text{Embedding (m,L)} \rightarrow X^m_{T-L(m-1)} \Phi (\text{order relation}) \rightarrow \Sigma^m_{T-L(m-1)} \rightarrow PE \rightarrow [0,1]
\] (3.9)

An example Let a time series \{1, 2, 3, 5, 4\} be given. Assume that \((m,L)=(2,1)\). Then,
\[
\tilde{x} = \begin{pmatrix}
(1, 2) \\
(2, 3) \\
(3, 5) \\
(5, 4)
\end{pmatrix}
\] (3.10)

and
\[
\tilde{s} = \begin{pmatrix}
sb_1 \\
sb_1 \\
sb_2 \\
sb_2
\end{pmatrix}
\] (3.11)

Finally we have
\[
PE = \frac{-\left[\frac{1}{4} \ln\left(\frac{1}{4}\right) + \frac{1}{4} \ln\left(\frac{1}{4}\right)\right]}{\ln(2!)} \approx 0.8113
\] (3.12)

The advantages of PE as a measure of complexity are as follows: First, it is conceptually simple in the sense that PE does not presuppose any model. Second, it is easy to compute. PE, in particular, does not require any numerical optimization. Third, PE is robust in the sense that it is invariant with respect to a monotone transformation of data.

Lower bound of PE Let \(PE = \ln(m!)PE\). Then, Matilla-García and Marín [2008] demonstrate that the following statistics asymptotically follows a chi-square distribution with \(m! - 1\) degrees of freedom under the null hypothesis of serial independence,\(^{15}\)
\[
2(T - L(m - 1))(\ln(m!) - \hat{PE}) \sim \chi^2_{m!-1}
\] (3.13)

\(^{15}\)Matilla-García and Marín [2008] assume that \(L=1\). But it is straightforward that the result holds for \(L>1\) as well with \(L(m-1)\) instead of \((m-1)\).
From 3.13, under the null hypothesis of serial independence, the following inequality holds with probability of $1-\alpha$:

$$PE \geq 1 - \frac{\chi^2_{m!-1,\alpha}}{2(T - L(m - 1))ln(m!)}$$ (3.14)

where $\chi^2_{m!-1,\alpha}$ is such that $\Pr(\chi^2_{m!-1} > \chi^2_{m!-1,\alpha}) = \alpha$. In this paper, the quantity in the right hand side of 3.14 is used as a lower bound for PE. I set $\alpha = 0.999$ in this paper for conservative treatment of signals.

**Random data** To illustrate the method employed in this paper, I generate i.i.d. data in MATLAB and compute the PEs of it. Figure 1 shows the results for nine different specifications which are used in this paper. Note that the lower bounds which are indicated by dotted lines work reasonably. Even though there are some cases where PE drops below the lower bounds, we can see that in those cases the PEs in other specifications still stay higher than the lower bounds. The observation suggests that, to avoid sampling error, PEs with different specifications should be checked.

![Figure 1: Permutation entropies](image)
3.3 Comparison between Autocorrelation and PE

The standard measure of temporal dependence of a univariate variable is autocorrelation. If PE always gives us the same result as autocorrelation does, then the usefulness of PE as a measure of temporal dependence would be questionable. In this regard, I provide three mechanisms in which two measures may report different results.

Figure 2 illustrates the effect of outliers on autocorrelation and PE. The top panel shows a randomly generated data except two outliers, -20 and 20 with full sample size of 2,000. The middle panel compares two autocorrelation sequences with and without outliers, respectively. The autocorrelation sequences are generated by using a moving window method with window size of 750. We can see that the outliers have sizable impact on autocorrelation. The bottom panel shows that outliers make little change in computed PEs. Again, the series of PEs in the bottom panel are generated by employing the moving window method. In two cases, two series are almost the same. This is not at all surprising considering the fact that PEs depend only on ordinal patterns. In other words, if we replace -20 and 20 with -100 and 100, the autocorrelation will be changed because the sizes of the two numbers are changed. But PEs will be exactly the same because there is no change in ordinal patterns of data.

Figure 3 illustrates the effect of structural break on autocorrelation and PE. The top panel shows randomly generated data. The first half(1-1000) of the data(= ‘regime1’) is generated from Normal(0,1) and the second half(1001-2000) of the data(= ‘regime2’) is generated from Normal(5,1). So, the mean of
full sample is about 2.5. The middle panel shows the effect of structural break with respect to mean on autocorrelation. As in the above experiment, I employ the moving window method with window size of 750. For sub-samples including both regime1 and regime2, we have a positive correlation since, on average, smaller numbers in regime1 and larger numbers in regime2 realize together, respectively. The bottom panel illustrates the effect of structural break on PE. Note that the effect is negligible since the structural break has only a minor impact on ordinal patterns around the breaking points, t=1,000 and 1001. Hence, Figure 3 reveals different results of autocorrelation and PE regarding structural break like change of mean. By the way, PE would be lower if the structural break happens more gradually and so there is either increasing or decreasing trend.

Figure 3: Structural break

Autocorrelation as a standard measure for linear serial dependence may fail to detect nonlinear dependence because of its dependence on linear relation. On the other hand, PE depends only on ordinal relation of data and hence we may expect that PE works better for discovering nonlinear structure of data. To illustrate the point, I employ a well-known example from chaos theory, a logistic map.

\[ x_{t+1} = Ax_t(1 - x_t) \]  

With A=4, the dynamics of the map is chaotic and has no linear dependence (Sprott [2003, 224]). I generate 10,000 observations with an initial state \( x_0 = 0.1 \). The upper panel of Figure 4 shows the scatter plot of the simulated data. By
employing the moving window method with a sample size of 750, I investigate
the temporal variations of the two measures, autocorrelation and PE. The middle panel of Figure 4 confirms that the autocorrelations of data are negligible. By contrast, in the bottom panel of Figure 4 PEs with time lag=1 are very low, which suggests that the data generating process generates temporal dependence of data. The bottom panel reveals that the choice of time-lag could matter in detecting temporal dependence.

I have provided three mechanisms(outliers, structural break, and nonlinearity) that may result in a discrepancy between two results based on autocorrelation and PE, respectively. Of course, the list is not intended to be exhaustive. Since the two measures approach the problem of temporal dependence by focusing on different aspects of data, it seems desirable to utilize the two measures together to detect and interpret temporal dependence of data.

### 3.4 An experiment: The relation between exogenous random shock and PE in nonlinear dynamical system

To investigate the effect of exogenous random shock on PE when a dynamical system is highly nonlinear, I employ the model used in Hommes and Manzan [2006] which may exhibit chaotic dynamics within some range of parameter values. With the model, Hommes and Manzan [2006] probe into the relationship between exogenous random shock and Lyapunov exponent(LE), another measure of complexity. There are two reasons for choosing the model in an
experiment in this paper regarding the relationship between exogenous random shock and PE in highly nonlinear dynamical systems. First, as Hommes and Manzan [2006, 172] point out, the model allows relatively large exogenous random shock which is indispensable for the experiment in this paper. Otherwise, even very small exogenous shock would make the system quickly diverge as in the logistic map. Second, employing the same model as that used by Hommes and Manzan [2006] makes the comparison of the results in this paper with those in Hommes and Manzan [2006] straightforward.

The model includes four types of heterogeneous beliefs. $\beta$, denoting the intensity of choice, is a bifurcation parameter of the model. The dynamics is chaotic with $\beta = 90$. They find a negative correlation between LE and exogenous random shock in the model. Since adding exogenous random shock into a model results in higher uncertainty, I expect a positive relationship between exogenous random shock and PE in the model. The experiment in this paper confirms this conjecture.

I add different sizes of exogenous random shock into the model as in Hommes and Manzan [2006] and measure PE of the resulting time series. First, I use the moving window method to generate time series of PE. Parameter values for computation are as follows: full sample size = 2,000, window size = 250, $L=1,2,\text{ and }3$, and $m=4$. I move the window by 20 at each step. The results are summarized in Figure 5. The top panel of Figure 5 displays PEs in the case of $L=1$. We can see that PEs increase as exogenous random shock increases. The other panels of Figure 5 exhibit less clear but similar patterns with $L=2$ and 3. Second, I compute the PEs of the whole sample with $m=5$. Table 1 reports the results. Again, the results display a negative relationship between exogenous random shock and PE. Higher exogenous random shock or the inverse of signal-to-noise ratio induces lower PE.

All in all, the experiment suggests the following interpretation of the lower PE: Endogenous dynamics becomes more important in understanding the whole dynamics. Of course, I do not exclude other possible interpretations and mechanisms which may induce changes of PE such as policy interventions. In practice, we need to look at several possible channels before getting to the conclusion.

\footnote{For comparison, the same random sequence was applied for different sizes of exogenous shock. The MATLAB command ‘randn(‘state’,0)’, did the job.}

\footnote{Signal-to-noise ratio means the ratio of $\frac{\sqrt{\text{var}(x_t)}}{\sigma}$ where $\sigma$ is the standard deviation of exogenous random shock and $x_t$ is a state variable.}
Figure 5: The effect of exogenous random shock on PE

Table 1: Exogenous random shock and PE

| σ = 0 | 0.4122 | 0.5507 | 0.6299 | 0 |
| σ = 0.05 | 0.5827 | 0.7736 | 0.8884 | 0.1094 |
| σ = 0.1 | 0.6573 | 0.8640 | 0.9478 | 0.1974 |
| σ = 0.2 | 0.7767 | 0.9437 | 0.9697 | 0.3399 |
| σ = 0.3 | 0.8388 | 0.9635 | 0.9788 | 0.4224 |
| σ = 0.4 | 0.8685 | 0.9733 | 0.9780 | 0.4778 |
4 Results

4.1 Methods

4.1.1 Data

I investigate domestic rates and Libor rates of 10 countries (Australia, Canada, Denmark, Euro area, Japan, New Zealand, Sweden, Swiss, UK, and US). Ten domestic rates include Bank Bill yields (Australia and New Zealand), Interbank rates (Denmark, Euro area, Japan, Swiss, UK), T-Bill rates (Canada, Sweden, US). Data came from central banks of the countries and Bloomberg.\(^{18}\) In this paper, the short-term means the periods no longer than one year. Among several maturities, since three-month interest rates are popular, I will focus on three-month interest rates. But I will also look at simple average rates of short-term interest rates as well. To obtain stationarity of time series, in this paper, I always use first-differences of data. In other words, I compute PEs of first-differences of daily interest rates and interest rate differentials.

4.1.2 Choice of parameter values: $L, m,$ and $T$

For embedding dimension ($m$), Bandt and Pompe [2002] recommend $m=3, \ldots , 7$ and Cao et al. [2004] prefer $m=5, 6, 7$. Given $T$, Matilla-García and Marín [2008] recommend to choose the maximum embedding dimension ($m$) satisfying the following constraint:

$$5m! \leq T \quad (4.1)$$

The rationale for 4.1 is that, for a good approximation of $\chi^2$ distribution, expected frequencies should be no less than 5. One advantage of the strategy proposed by Matilla-García and Marín [2008] is that the choice of the embedding dimension ($m$) becomes trivial given $T$. I follow the strategy in choosing the embedding dimension ($m$).\(^{19}\)

According to Matilla-García and Marín [2008], the independence test based on permutation entropy is reasonable with respect to the size of the test for $T=120$. They also report that the power of the test is sizable for $T=600$ even though the power of the test is moderate for $T=120$ except I(1) process and chaotic process. Taking into account the results, in this paper, I use the two window (or sample) sizes, $T=250$ and 750. $T=250(750)$ roughly corresponds to one (three) year(s). Then, for $T=250$, the possible largest $m$ is 4 from 4.1. Therefore, I specify $m=4$ for $T=250$. Similarly, for $T=750$, the possible largest $m$ is 5 from 4.1. In the case of $T=750$, I specify $m=4$ or 5.

For time-lag ($L$), I investigate up to $L=3$ because I seldom find temporal dependence with the time-lags more than 3 given no evidence for time dependence up to $L=3$. As mentioned before, I set $\alpha = 0.999$ in 3.14 for conservative

\(^{18}\)But, for Denmark and Sweden, I use interbank interest rates instead of Libor rates.

\(^{19}\)It seems that this strategy can be justified because I am interested only in finding temporal dependence of data. In general, the choice of the embedding dimension is important in detecting dynamic structure of data (see Kantz and Schreiber [2004], for instance).
treatment of the signal. Table 2 summarizes the parameter values used in this paper.

<table>
<thead>
<tr>
<th>Table 2: Parameter values</th>
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<tbody>
<tr>
<td>T=250 (≃ 1 Year)</td>
</tr>
<tr>
<td>m</td>
</tr>
<tr>
<td>L</td>
</tr>
<tr>
<td>1-α</td>
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</table>

I move the resulting windows by 20 at each step. Finally, in order to get a unique sequence without common values, I break equalities numerically by adding small random perturbations as Bandt and Pompe [2002] suggest. For the comparison of results, I pick an one-dimensional sequence of random perturbations and apply it to each data set. To quantify the effect of the small random perturbations on computed PEs, I will report the maximum and minimum of the computed PEs obtained by repeating the computation 100 times given a sub-sample.

4.2 PEs of short-term interest rates

I first document the movements of PEs in the U.S. financial markets, the epicenter of the financial crisis of 2007-09. It turns out that the U.S. 3-month T-Bill rates show a sudden drop of PE. Second, I investigate Euribor market for Euro area. Third, I look at domestic short-term interest rates of eight other countries (Australia, Canada, Denmark, Japan, New-Zealand, Sweden, Swiss, and UK). Finally, I examine Libor markets.

4.2.1 The U.S.

Rising interest rates have often played a role in triggering financial crises. For example, Rajan [2006, 519] points out that the rise of the U.S. interest rates increases the possibility of an emerging market crisis. Kaminsky and Pereira [1996, 2] indicate a rapid rise in real interest rate prior to 1982 debt crisis. According to Steigum [2010], the high real interest rates after the German unification in 1990 were an important shock triggering the Nordic crises in 1991-92.

The Fed raised the Federal Funds Target rate by 25bps at seventeen Federal Open Market Committee meetings without an exception from June 30, 2004 to June 29, 2006 after the extraordinary period of loose monetary policy following the collapse of IT bubble. Alan Greenspan, in his congressional testimony on February 16, 2005, indicated the drops of long-term rates and he called the conflicting movements of short-term and long-term rates a conundrum. Furthermore, according to IMF [2009, 112, footnote 2], the peak of the U.S. housing

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20 So, the default state(=‘state0’) in this paper is generated in MATLAB with the commands, ‘rand(‘state’,0)’ and ‘randperm.’


An environment of low interest rates following a period of high rates is particularly problematic, for not only does the incentive of some participants to ‘search for yield’ go up, but also asset prices are given the initial impetus, which can lead to an upward spiral, creating the conditions for a sharp and messy realignment (Rajan [2006, 501-502]).

Figure 6 exhibits the movement of PEs from the year of 2001. The dates in the Figure are end-dates of sub-samples. Roughly speaking, the situation where PEs are lower than the threshold(with respect to L=1, m=4, and window size=250) begins in February 2005 and lasts for more than one year. Figure 20 in the Appendix exhibits PEs of the U.S. 3-month T-Bill rates across different specifications over the whole sample. Note that PEs attain the historically lowest level in 2005 with 1-year window and mainly in 2007 with 3-year window.
Similar patterns are found in the PEs of the U.S. 6-month T-Bill rates as shown in Figure 21 in the Appendix. But for 4-week T-Bill rates the patterns are less clear as we can see in Figure 22 in the Appendix.

Figure 7 displays PEs of the simple mean of interest rates with three different maturities, 4-week, 3-month, and 6-month. We can see that there are drops of PEs during 2005-06. In other words, daily dynamics of short-term risk-free interest rates up to 6 months became more predictable during the period. In particular, in Figure 7, there is no significant drop of PEs below the threshold before 2005 with sample size=750 and L=2 and 3. This feature highlights the observation that the drops of PEs below the threshold are rare events. Therefore,

in addition to interest rate conundrum and housing boom, it seems that attaining the lowest level of PEs of the U.S. 3-month T-Bill rates is another unusual event in 2005. The upper left panel of Figure 23 in the Appendix shows the relative frequencies of monotone sequences from the first-differences of the simple mean of interest rates. In the panel, lower rank means higher occurrence. It turns out that the two types, monotonically increasing or decreasing, happen relatively more often during the period with low PE.

Since I add small random perturbation to break equal numbers, the computation introduces another source of randomness in computed PEs. To gauge the effect of small random shock on computed PEs, I repeatedly compute PEs
of each sub-sample 100 times with different random shocks. Then, I compute maximum and minimum for each sub-sample. Figure 8 displays the results where ‘state0’ indicates one realization of random perturbation. Note that the drops of PEs are more remarkable in the case of average rates. One possible interpretation is the reduction of the effect of exogenous random shock due to averaging. On the contrary, in the cases where PEs are above the thresholds, it seems that averaging does not yield lower PEs.

Figure 8: The effect of random perturbation

![Figure 8: The effect of random perturbation](image)

Figures 24 and 25 in the Appendix show autocorrelation of the first differences of the U.S. 3-month T-Bill rates. We can see that autocorrelation with time lag=2 or 3 is significantly negative. Figure 9 displays the autocorrelation of the first differences of the simple average rates of the U.S. 4-week, 3-month, and 6-month T-Bill rates. Again, with respect to L=2 and 3, there were sizable drops
of autocorrelations around the mid 2005. Considering the concurrent drops of PEs, I may say that the drops of autocorrelations were not entirely driven by outliers. In other words, low PEs and highly negative autocorrelations together imply the following two things: First, the evidence for the temporal dependence is stronger than that obtained by taking into account only the movement of autocorrelation. Second, the driving force of the temporal dependence may not be an outlier.

One possible interpretation of the result is the steady increase in the U.S. policy rates during the period with low PE, which might generate some regularity in data-generating process. I agree with the argument that the change of the U.S. monetary policy stance during the period may be a part of the story. But, I am not completely persuaded by the argument that it is a sufficient condition for the drop of PE. Regarding the issue, I perform a simple experiment by investigating the effect of periodic upward jumps in simulated data. For the experiment, first, I generate random data. Then, I replace the old values in the places of multiples of 10 by the new value, 10. The upper panel of Figure 26 in the Appendix displays the simulated data. And the bottom panel shows the cumulative sum of the data which can be viewed as a realized path of a variable. Note that the path has upward trend because of periodic upward jumps. Figure 27 in the Appendix exhibits the effects of the periodic upward jumps on autocorrelation and PE. Note that there is no significant change in PE whereas autocorrelation decreases a little. This result illustrates the point that PE heavily depends on neighboring values. Periodic upward jumps with the periodicity of 10 do not lead to sizable change of PE because the embedding dimension in this case is 4. This experiment suggests that other factors like responses of financial markets to the U.S. monetary policy would be necessary to fully explain the drop of PE. In this regard, the change of distribution of symbols(or patterns) reported in Figure 23 in the Appendix could be useful guidance for
future research.

Now, I investigate other interest rates of the U.S. with the same maturity. Figure 28 in the Appendix displays PEs of asset-backed commercial paper (ABCP), certificate of deposit (CD), financial commercial paper (FCP), and non-financial commercial paper (NFCP). Note that the minimum PEs of the interest rates are higher than 0.95 for each of three different time-lags. And there is no period in which PEs drop below the threshold for all of the three time-lags. Figure 29 shows that with time-lag=2 or 3, none of PEs of the four interest rates are lower than the threshold.

Figure 10 shows the case of 3-month eurodollar (ED) in London. Note that during the period of 06/1977 - 04/1980, PEs are lower than the thresholds with respect to L=1,2, and 3. According to Bordo [2005], the so-called ‘dollar crisis’ of 1977-79 started in fall 1977 and the famous ‘Volcker shock’ happened on October 8, 1979. Therefore, it seems that the gradual decline of PEs preceded the turbulence of the late 1970’s. The upper right panel of Figure 23 in the Appendix shows the relative frequencies of monotone sequences from the first-differences of the simple mean of interest rates. It turns out that monotonically increasing pattern happens in the least frequency during the period with low PE. Again, to gauge the effect of small random shock on computed PEs, I repeatedly compute PEs of each sub-samples for 100 times with different random shocks. Figure 11 displays the results. Note that the drops of PEs are more remarkable
in the case of average rates as before. Figure 11 also reveals that, during the
financial crisis of 2007-09, PEs of ED in London there were no significant drops
in terms of L=2 or 3. So, we may conclude that T-Bill rates show the most
robust deviation from serial independence before the financial crisis of 2007-09
among six interest rates.

Figure 31 in the Appendix displays autocorrelations of the first differences
of 3-month ED up to three time-lags. For L=1, autocorrelation is significantly
negative in June of 1977. But after that, autocorrelation gradually increases
whereas PE with L=1 decreases for a while. And with L=2 or 3, we can find
only moderate positive autocorrelation even though PEs with the same time-
lags are significantly lower than the threshold. This result indicates that low
PE does not necessarily mean strong autocorrelation.
Figure 30 in the Appendix shows that PEs of interest rate spreads on the three short-term interest rates (ABCP, CD, and ED) in the U.S. financial markets over T-Bill rates drop before the financial crisis of 2007-09. Based on the similar timings of the drops, we may conjecture that the dynamics of T-Bill rates is responsible for the drops. In this sense, these findings support the statement that there was a unusual pattern in dynamics of T-Bill rates, particularly in 2005-06. Note that those spreads reflect the risks of investing in risky assets instead of investing in risk-free T-Bill. The levels of spreads were low from historical perspective in 2005-06. The example supports the conjecture that PE may drop in good times in spite of lower levels of spreads.

4.2.2 Euro area (Euribor market)


Figure 12 displays PEs of 3-month Euribor (Euro Interbank Offered Rate). In contrast to the U.S. 3-month T-Bill rates, PEs of 3-month Euribor do not drop before the financial crisis of 2007-09. Instead, PEs of 3-month Euribor started to plunge during the financial crisis, particularly in 2009.

Figure 12: 3-month Euribor

Figure 13 shows PEs of the average Euribor with maturity up to 12-months.

Figure 13
Note that PEs drop more sharply in this case. The bottom left panel of Figure 23 in the Appendix shows the relative frequencies of monotone sequences from the first-differences of the simple mean of interest rates. It turns out that the two types, monotonically increasing or decreasing, happen relatively very often during the period with low PE.

To gauge the effect of small random shock on computed PEs, I repeatedly compute PEs of each sub-samples 100 times with different random shocks. Then, I compute maximum and minimum for each sub-sample. Figure 13 displays the results where 'state0' indicates one realization of random perturbation. The difference between maximum and minimum is smaller in the case of average rates which reflects a smaller portion of common values in first-differences of average rates.

![Figure 13: The effect of random perturbation](image)

But Figure 32 in the Appendix exhibits similar autocorrelations of the two variables. Again, the observations illustrate the point that PE may provide another information not readily available from autocorrelation.

As Euro area includes France and Germany, it would be interesting to see whether we can observe similar patterns in PEs. Figure 14 displays PEs in three countries: France, Germany and UK. First of all, we can see the absence of significant violation of independence in both 3-month and average interest
rates for three countries. The maxima are always higher than the threshold. For average rates of France, even the minimum are higher than the threshold rates. So, averaging does not lead to a significant drop of PEs in this case. This result reveals that the drop of PEs in Euribor markets was a distinguishing feature in 2009 in contrast to domestic short-term interest markets of the three countries.

Figure 14: The effect of random perturbation (L=2, m=4, sample size=250)

Summing up, I find that PEs of the U.S. 3-month T-Bill rate plunged below the thresholds before the financial crisis of 2007-09 and that PEs of 3-month Euribor similarly dropped before the European sovereign debt crisis of 2010. These results are robust across different specifications.

Moreover, PEs of simple average rates of short-term interest rates dropped even more sharply. We see that the peculiar relationship between a single variable and an average variable with respect to PE does not hold in general, particularly, if there is no evidence of temporal dependence (PE > threshold).

4.2.3 Domestic short-term interest rates of eight countries

Figure 15 exhibits PEs of 3-month interest rates of eight countries. The significant drops of PEs for all three time-lags happen only for Japan. For Australia, Denmark, New-Zealand, Switzerland, and, Sweden, there are the drops of PEs only with L=1. For Canada and UK, there are no persistent drops in any time-lag.

Figure 16 shows Tibor of Japan and PEs of Tibor with sample size=250. We can see the drops of PEs start in 1998 after the Asian crisis of 1997. The bank of Japan adopted the zero interest rate policy (ZIRP) in the spring of 1999. Therefore, the drops of PEs below thresholds preceded the implementation of ZIRP in Japan.
Figure 15: Permutation entropies ($m=5$, sample size=750)

Figure 16: 3-month Tibor
The bottom right panel of Figure 23 in the Appendix shows the relative frequencies of monotone sequences from the first-differences of Tibor. It turns out that the two types, monotonically increasing or decreasing, happen relatively very often during the period with low PE.

Figure 33 displays autocorrelation of the first-difference of Tibor. Note that autocorrelation reaches its peak in 2006 whereas PEs are at the lowest levels until the early 2000’s. In this sense, the two measures do not report the same results. Furthermore, since PEs are not below the thresholds in 2006, the results implies that the rise of autocorrelation around 2006 may be driven mainly by exogenous shock rather than by endogenous mechanisms.

Figure 17 shows PEs of interest rate differentials between the U.S. T-Bill rates and interest rates of other countries with L=2, m=4, and sample size=250. We can find that there are drops of PEs of interest rate differentials below the threshold for all countries in 2005. Since only the U.S. T-Bill rates show significant drops of PEs in 2005, the results seem to be related with the unusual movement of the U.S. T-Bill rates in the period. Figure 34 in the Appendix displays PEs of interest rate differentials with m=5 and sample size=750. Again, we see uniform drops of PEs. Figure 35 in the Appendix exhibits autocorrelations with L=2. Note that all autocorrelations are negative around the periods when PEs are the lowest, which implies a role of a common factor. From Figure 34 and 35, we can see that stronger autocorrelation does not necessarily imply lower PE (see Sweden and UK).

Overall, we can observe that the drops of PEs below thresholds across 1-3 time lags do not happen frequently. Among 10 series of interest rates, the drops of PEs below thresholds across 1-3 time lags have been found in three cases (Euro area, Japan, US).

Figure 17: Permutation entropies (L=2, m=4, sample size=250)
4.2.4 Libor markets

Libor markets are important international financial markets. In particular, TED spread, the interest rate differential between the US Libor and the US T-Bill rate is a popular measure of perceived risk as suggested in Brunnermeier [2009, 85]. Figure 36 exhibits PEs of Libors. There are no significant drops of PEs with time-lag=2 or 3 even though there are several violations of independence with time-lag=1.

Figure 19 displays PEs of interest rate spreads of Libors against the U.S. T-Bill rates. We can observe that PEs drop around 2005 with L=2, m=4, and sample size=250. The results strengthen the finding that there was temporal dependence in the U.S. T-Bill rates because the timings of drops of PEs of interest rate spreads are close to the timing of drops of the U.S. T-Bill rates and because the drops of PEs are uniform across different Libors. In particular, we can observe that PEs of TED spread fall down as well. Figure 37 in the Appendix shows the significant and uniform drops of PEs with L=2, m=5, and sample size=750, which demonstrates that the results are robust with a longer horizon and higher embedding dimension.
Figure 39 in the Appendix exhibits autocorrelations with L=2. Again, all autocorrelations around the periods when PEs are the lowest are negative. Figure 39 also reports an example which shows that autocorrelation and PE may not closely co-move. The autocorrelation of interest rate spreads between Australia dollar Libor and the U.S. T-Bill rate does not exhibit an extraordinary movement whereas PE of interest rate spreads between Australia dollar Libor and the U.S. T-Bill rate drops to the lowest level.

Figure 19: Permutation entropies (L=2, m=4, sample size=250)
5 Conclusion

In this paper, I have proposed a new method for detecting temporal dependence of financial time series by using the permutation entropy developed by Bandt and Pompe [2002] in order to complement the standard measure of temporal dependence, autocorrelation. The method provides temporal variation of PEs by using a moving window method and identifies abnormal behavior of PEs with respect to the threshold based on Matilla-García and Marín [2008].

As an application of the method, I have investigated the temporal dependence of first-differences of daily short-term interest rates to detect abnormal behavior in financial markets. This paper has found irregular movements of PEs of the U.S. 3-month TB particularly during 2005-06 and those of 3-month Euribor particularly in 2009. Those evidences could have been useful information for policy-makers given the growing financial fragility in the periods. The irregular movements of PEs are even stronger for simple average rates of short-term interest rates, which seems to hold only during the periods in which PEs are below thresholds. Significant drops of PEs also have been found in Eurodollar deposit rates during the late 1970’s and in Japan’s 3-month Tibor rates after the Asian crisis of 1997. I leave the issue of whether the drops of PE were related to market inefficiency to future research. Regarding the causes of the drops, this paper has pointed out that the change of distribution of symbols(or patterns) could be useful guidance for future research. But the increase of monotone patterns in some cases reported in this paper should be interpreted with caution since there are also other patterns which occur more often in the same periods. I also have compared PEs with autocorrelation and found that the movements of PEs are not fully explained by autocorrelation, which indicates the usefulness of PEs as a complementary measure to autocorrelation. In particular, if we observe strong autocorrelation and low PE at the same time, those signals could be interpreted as a more robust evidence of temporal dependence.

This paper generates a testable hypothesis regarding the interpretation of low PE. According to Section 3.4, if the role of exogenous shock becomes smaller, we expect decreasing PE and increasing Lyapunov exponent(LE). Based on the observation, it is an interesting future research topic whether we can find rising LE during the periods when PE is decreasing. In this regard, comparing PE and BDS statistic will be also helpful for having a deep understanding of data-generating process as well.

Finally, given the complexity of financial systems, limits of a single indicator should be pointed out. Low PE may be related to market inefficiency but market inefficiency does not necessarily lead to market crashes. On the other hand, it should be clear that high PE does not necessarily mean crisis-free financial systems. When it comes to policy-making, we have to take into account several indicators at the same time, and PE could be one of them particularly because of its robustness in several dimensions. But we need more research on the time-variation of PE in other financial markets for deeper understanding of PE. In future research, the potential of PE as a warning signal could be assessed in several dimensions such as signal-to-noise ratios.
Figure 20: Permutation entropies (the U.S. 3-month T-Bill rates)

Figure 21: Permutation entropies (the U.S. 6-month T-Bill rates)
Figure 22: Permutation entropies (the U.S. 4-week T-Bill rates)

Figure 23: Relative frequencies of monotone sequences
Figure 24: Autocorrelation

$\Delta$ (the U.S. 3-month T-bill), $L=1$, sample size=250

Figure 25: Autocorrelation

$\Delta$ (the U.S. 3-month T-bill), sample size=250
Figure 26: Random data with periodic upward jumps

![Random process with periodic upward jumps](image)

Figure 27: Random data with periodic upward jumps II

![Autocorrelation, L=1](image)  ![PE, L=1, m=4, T=250](image)
Figure 28: Permutation entropies (m=4, sample size=250)

Figure 29: Permutation entropies (m=5, sample size=750)
Figure 30: $PE\ (L=2,\ m=5,\ sample\ size=750)$ and Autocorrelation ($L=2,\ sample\ size=750$)

Figure 31: Autocorrelation
Figure 32: Autocorrelation ($\Delta$(3-month Euribor))

Figure 33: Autocorrelation ($\Delta$(Japan's 3-month Tibor))
Figure 34: PEs (Interest rate spreads against the U.S. T-Bill(3m), L=2, m=5, T=750)

Figure 35: Autocorrelation (Interest rate spreads against the U.S. T-Bill(3m), L=2, T=750)
Figure 36: Permutation entropies(Δ(Libors(3m))), m=4, sample size=250
Figure 37: Permutation entropies, $L=2$, $m=5$, sample size=750
Figure 38: Autocorrelation(Δ(Libor(3m)-the U.S. T-Bill(3m)), L=2, sample size=750
Figure 39: Autocorrelation($\Delta$ (Libor(3m)-the U.S. T-Bill(3m)), L=2, sample size=750
References


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