PROFITABILITY OF CONTRARIAN AND MOMENTUM STRATEGIES AND MARKET STABILITY

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Abstract. This paper proposes a continuous-time heterogeneous agent model of investor behaviour consisting of fundamentalist, contrarian, momentum and market maker strategies to study their impact on market stability and profitability. The underlying stochastic delay integro-differential equation model provides a unified approach to model different time horizons of momentum and contrarian strategies, which play an important role in the profitability of these strategies empirically. By including noise traders and imposing a stochastic fundamental value, we demonstrate that momentum and contrarian strategies can be consistent with market efficiency and the model is able to replicate various market phenomena and stylized facts. Finally, we demonstrate the activities of different traders can affect their profitability dramatically, which sheds new light in understanding the profitability mechanism.

Key words: Contrarian, momentum, profitability, market stability and efficiency, stochastic delay integro-differential equations.

JEL Classification: C62, D53, D84, G12

Date: January 12, 2012.

Acknowledgement: Financial support for He from a UTS Business School research grant is gratefully acknowledged. We would like to thank the participants of the 2010 Guangzhou Conference on Nonlinear Economic Dynamics and Financial Market Modelling, the 2011 San Francisco Conference on Computing in Economics and Finance, the 2011 Ancona Annual Workshop on Economic Heterogeneous Interacting Agents and the 2011 Sydney Conference on Quantitative Methods in Finance for helpful comments. The usual caveat applies. Previous versions of this paper were titled “Contrarian, Momentum, and Market Stability”.

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1. Introduction

Despite the efficient market hypothesis of financial markets in the academic finance literature, see Fama (1970), the use of technical trading rules and passive trading strategies based on past returns, in particular momentum and contrarian strategies, still seems to be widespread amongst financial market practitioners, see Allen and Taylor (1990). Earlier empirical literature on stock returns finds evidences that daily, weekly and monthly returns are predictable from past returns (Pesaran and Timmermann 1994, 1995). Recently, the profitability of these strategies and their consistency with the efficient market hypothesis have been investigated extensively in the literature, see for example, Frankel and Froot (1986, 1990), Brock, Lakonishok and LeBaron (1992) and Conrad and Kaul (1998).

In general, contrarian strategies are referred to buying past loser stocks and shorting past winner stocks, while momentum strategies are counter-contrarian strategies, buying past winner stocks and shorting past loser stocks. The profitabilities of these strategies are related to two families of pervasive regularities: overreaction and underreaction as identified by recent empirical research in finance. Under a market overreaction hypothesis, De Bondt and Thaler (1985, 1987) and Lakonishok et al. (1994) find supporting evidence on the profitability of contrarian strategies for a holding period of 3 to 5 years based on the past 3 to 5 years in the U.S. In contrast, Chan et al. (1997), Jegadeesh (1990) and Jegadeesh and Titman (1993, 2001, 2011), among many others, find supporting evidence on the profitability of momentum strategies for a holding period of 3 months to 1 year based on the past 3 months to 1 year in the U.S. and Europe. Instead of studying individual stock momentum, Moskowitz and Grinblatt (1999) demonstrate industry momentum for a holding period of 1 month to 1 year based on the past 1 month to 1 year and long-run reversals in the U.S.\(^1\). George and Hwang (2004) introduce another momentum strategy and also find the momentum in price levels by investigating 52-week high.

\(^1\)Industry momentum appears to be most profitable in the very short term (one month).
These clearly indicate that the time horizons and holding periods play crucial roles in the performance of contrarian and momentum strategies\textsuperscript{2}.

In addition to the time horizons, the state of the market is also a critically important factor affects the profitability of technical trading strategies as shown in Griffin et al. (2003). Hou et al. (2009) find momentum strategies with short time horizon (1 year) are not profitable in “down” market, but return significant profits in “up” market, where the market states are defined in two ways. Similar results of profitability are also reported in Chordia and Shivakumar (2002) that commonly using macroeconomic instruments related to the business cycle can show the positive returns to momentum strategies during expansionary periods and negative returns during recessions. Furthermore, Cooper et al. (2004) find that short-run (6 months) momentum strategies make profits in the up market and lose in the down market, but the up-market momentum profits reverse in the long-run (13-60 months).

The size and apparent persistence of momentum profits have attracted considerable attention, and many behavioral studies have tried to explain the phenomenon. The three-factor model of Fama and French (1996) can explain long-run reversal but not short-run momentum. Ahn et al. (2003) and Yao (2008) employ nonparametric methods and show that systematic factors explain momentum. To the extent that mispricings are systematic, however, the basis assets themselves may also be capturing irrationalities (Hirshleifer 2001). Additionally, Lee and Swaminathan (2000) show that trading volume plays a role in the profits to momentum strategies, which they interpret to mean that prices generally deviate from fundamental values. Grinblatt and Moskowitz (2004) conclude that tax environments affect the profits to both momentum and contrarian strategies. Lesmond et al. (2004) and Korajczyk and Sadka (2004) question if momentum profits are realizable given trading costs. The concern for a data-snooping bias seems small given the foreign-market evidence of Rouwenhorst (1998) and the 1990s United States evidence of Jegadeesh and Titman (2001). Sagi and Seasholes (2007) present a growth options model to identify observable firm-specific attributes that drive momentum. Daniel et al. (1998) and

\textsuperscript{2}Jegadeesh and Titman (2001) also demonstrate a stock may be identified as both a loser and a winner, depending on the time horizons chosen.
Hong and Stein (1999), with single representative agent and different trader types respectively, attribute the under and overreaction to overconfidence and biased self-attribution.

As argued in Jegadeesh and Titman (1993), the “common interpretations of return reversals as evidence of overreaction and return persistence as evidence of underreaction are probably overly simplistic. A more sophisticated model of investor behavior is needed to explain the observed patterns of returns”. They also suggest that such a model should allow “positive feedback traders” on market price (see De Long et al, 1990) “who buy past winners and sell past losers move prices away from their long-run values temporarily and thereby cause prices to overreact...Alternatively, it is possible that the market underreacts to information about the short-term prospects of firms but overreacts to information about their long-term prospects”. Especially, the time horizon should be emphasized in such a model (see Griffin et al, 2003) “the comparison is in some sense unfair since no time horizon is specified in most behavioral models”. Following the recent development in heterogeneous agent models (HAMs) literature, this paper attempts to establish such a model by incorporating momentum and contrarian strategies explicitly to reflect underreaction and overreaction of investor behaviour, trying to provide an explanation on the profitability of these strategies. Over the last two decades, various HAMs have been developed to explain many market behavior. By incorporating bounded rationality and heterogeneity, HAMs have successfully explained the complexity of market price behaviour, market booms and crashes, and long deviations of the market price from the fundamental price. They show some potentials in generating the stylized facts (such as skewness, kurtosis, volatility clustering and fat tails of returns), and various power laws (such as the long memory in return volatility) observed in financial markets. This paper contributes to the literature in three aspects.

First, by using a stochastic delay integro-differential equation, we provide a uniform approach to the current heterogeneous agent-based financial market models.

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3We refer the reader to Hommes (2006), LeBaron (2006), Lux (2009) and Chiarella et al (2009) for surveys of the recent developments in this literature.
in discrete-time framework and develop a simple financial market model consisting of fundamentalists, momentum traders, contrarians and market maker in a continuous-time framework to study the joint impact of all four investment strategies on the market. Most of the HAMs in the literature are in discrete-time rather than continuous-time setup. The discrete-time setup facilitates economic understanding and mathematical analysis, it however faces some limitations when dealing with expectations formed over different time horizons. In discrete-time HAMs, different time horizons used to form the expectation or trading strategy lead to different dimensions of the system which needs to be analysed separately\textsuperscript{4}. Very often, a theoretical analysis on the impact of time horizon is difficult when the dimension of system is higher. Also, due to the complexity in analysis, most of HAMs consider financial markets with only two types of strategies, fundamentalist and momentum (trend chasing) or contrarian strategies\textsuperscript{5}. The continuous-time setup with time delays proposed in this paper provides a uniform treatment on various time horizons used in the discrete-time model. As an extension to the deterministic delay differential equation models in economics\textsuperscript{6}, the model developed in this paper involves two time delays corresponding to two different time horizons used by momentum and contrarian investors, respectively. To our knowledge, this paper is the first one to analyze a financial market model with all three types of strategies in a continuous-time framework. This setup can accommodate easily different time horizons of momentum and contrarian strategies, which play critical roles in the empirical literature.

\textsuperscript{4}For example, to examine the role of moving average (MA) rules used by momentum investors in market stability, Chiarella et al (2006) propose a discrete-time HAM whose dimension depends on the time horizon of the momentum investors used in the MA.

\textsuperscript{5}Brock and Hommes (1998) and Chiarella and He (2002) are a few exceptions.

\textsuperscript{6}Development of deterministic delay differential equation models to characterize fluctuation of commodity prices and cyclic economic behavior has a long history, see, for example, Haldane (1932), Kalecki (1935), Goodwin (1951), Larson (1964), Howroyd and Russell (1984) and Mackey (1989). The development further leads to the studies on the effect of policy lag on macroeconomic stability, see, for example, Phillips (1954, 1957), Yoshida and Asada (2007), and on neoclassical growth model, see Matsumoto and Szidarovszky (2011). We refer to He et al (2009) and He and Zheng (2010) for a recent development along this line when modelling a financial market with fundamentalist and momentum strategies involving one time delay.
Second, roles played by different investment strategies in financial market stability can have significant implications on the profitability of these strategies and, with the model developed in this paper, we provide explanations for the profitability of momentum and contrarian strategies well documented empirically. As we discussed earlier, time horizon plays very important roles in the performance of trading strategies and its impact can be complicated to analyse\(^7\). Intuitively, momentum strategies are based on the hypothesis of underreaction with expectation that the future price will follow the price trend. Consequently the strategies tend to destabilize the market price when they dominate the market, which can generate price overreaction. While contrarian strategies are based on the hypothesis of overreaction with expectation that the future price will go against the price trend. Therefore the strategies can stabilize the market when they dominate the market. However, the joint impact of both strategies on market stability can be complicated. By taking the advantage of continuous-time framework, we examine different role played by different strategy and furthermore, we find momentum traders can make profits for short horizons and lose for long horizons when they dominate the market, and they always lose when the fundamentalists and contrarians dominate the market.

Finally, by including noise traders in the market and imposing a stochastic process on fundamental price, we demonstrate that both contrarian and momentum strategies can be consistent with market efficiency characterized by insignificant return autocorrelations. In addition, the model is able to generate many market phenomena, such as long deviations of the market price from the fundamental price, market bubbles, crashes, and most of the stylized facts, including non-normality, volatility clustering, and long range dependence in volatility with high-frequency returns observed in financial markets.

The paper is organized as follows. We first introduce a stochastic HAM in continuous time with time delays to incorporate fundamentalist, momentum, contrarian

\(^7\)In a discrete-time HAM, Chiarella et al (2006) show that the MA plays a complicated role on the stability of financial markets. In particular, when the activities of the fundamentalist and momentum investors are balanced in certain way, they show that an increase in the time horizon used in the MA can stabilize the market; otherwise, it is a source of market instability.
and market maker strategies in Section 2. In Section 3, we apply stability and bifurcation theory of functional differential equations to examine the impact of these strategies, in particular the different time horizons, on market stability. Section 4 provides some numerical simulation results of the stochastic model in exploring the potential of the model to generate various market behavior and the stylized facts. The profitability is studied in Sections 5 and 6. Section 7 concludes. All the proofs are given in Appendix A. Appendices B and C introduce population evolution and study the impact of the population evolution on the price behaviour and profitability.

2. The Model

Consider a financial market with a risky asset (such as stock market index) and let $P(t)$ denote the (cum dividend) price per share of the risky asset at time $t$. The modelling of the dynamics of the risky asset follows closely to the current HAM framework (see, for example, Brock and Hommes, 1997, 1998; Chiarella and He, 2002, 2003; Chiarella et al, 2006). However, instead of using a discrete-time setup, we consider a continuous-time setup with a fundamentalist component who trades according to fundamental analysis and two chartist components, the momentum traders and contrarians, who trade differently based on price trend calculated from moving averages of history prices over different time horizons. The market price is arrived at via a market maker scenario in line with Beja and Goldman (1980) and Chiarella and He (2003) rather than the Walrasian scenario used in Brock and Hommes (1998) and Chiarella and He (2002). Whilst the market maker is a highly stylized account of how the market price is arrived at, it may be closer to what is going on in real markets.\(^8\) To focus on price dynamics, we motivate the excess demand functions of the four different types of traders by their trading rules directly, rather than deriving the excess demand functions from utility maximization of their portfolio investments with both risky and risk-free assets (as for example in

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\(^8\)As presented in Chiarella et al, (2009), the Walrasian scenario, even though widely used in economic analysis, only plays a part in one real market (the market for silver in London).

\(^9\)In a recent paper by Zhu et al (2009), the roles of the market maker as both liquidity provider and investor are examined. It is found that, in some market conditions, the market maker has an incentive to destabilize the market in order to maximize his profit.
Brock and Hommes, 1998). The market population fractions\textsuperscript{10} of fundamentalists, momentum traders and contrarians are $\alpha_f$, $\alpha_m$ and $\alpha_c$ respectively, where $\alpha_f + \alpha_m + \alpha_c = 1$ and $\alpha_i > 0$, $i = f, m, c$.

2.1. **Fundamentalists.** The fundamentalists believe that the market price $P(t)$ is mean-reverting to the fundamental price $F(t)$ that they estimate based on various types of fundamental information, such as dividends, earnings, exports, general economic forecasts and so forth. They buy the stock when the current price $P(t)$ is below the fundamental price $F(t)$ of the stock and sell the stock when $P(t)$ is above $F(t)$. For simplicity, we assume that the excess demand of the fundamentalists, $D_f(t)$ at time $t$, is proportional to deviation of market price $P(t)$ from the fundamental price $F(t)$, namely,

$$D_f(t) = \beta_f[F(t) - P(t)], \tag{2.1}$$

where $\beta_f > 0$ is a constant parameter, measuring the speed of mean-reversion of $P(t)$ to $F(t)$, which may be weighted by the risk aversion coefficient of the fundamentalists. To focus the analysis on $P(t)$, we assume that $F(t)$ is given by an exogenous random process that will be specified in Section 4.

2.2. **Momentum Traders and Contrarians.** Both momentum traders and contrarians trade based on their estimated market price trends, however they behave differently. Momentum traders believe that future market price follows a price trend $u_m(t)$. When current market price is above the trend, they expect future market price will rise and therefore they take a long position of the risky asset; otherwise, they take a short position. Different from momentum traders, contrarians believe that future market price goes opposite to price trend $u_c(t)$. When current market price is above the trend, they expect future market price will be low and therefore they take a short position of the risky asset; otherwise, they take a long position.

\textsuperscript{10}To simplify the analysis, we first assume that the market fractions are constant parameters as in the market fraction model in He and Li (2008). The population evolution can be introduced based on some performance measure, as in He and Li (2011). The stability of the systems with and without population switching are the same but extensive simulations show the switching can enlarge the profits and losses of the traders (not reported here).
The price trend used for the momentum traders and contrarians can be different in general. We therefore assume that the excess demand of the momentum traders and contrarians are given, respectively, by

\[ D_m(t) = g_m(P(t) - u_m(t)), \quad D_c(t) = g_c(u_c(t) - P(t)), \]  

(2.2)

where the demand function \( g_i(x) \) satisfies

\[ g_i(0) = 0, \quad g_i'(x) > 0, \quad g_i''(0) = \beta_i > 0, \quad x g_i''(x) < 0, \quad \text{for } x \neq 0, \quad i = m, c. \]  

(2.3)

and the parameter \( \beta_i \) represents the extrapolation rate of the price trend when the market price deviation from the trend is small. In the following discussion, we take \( g_i(x) = \tanh(\beta_i x) \), whose slope levels off as the magnitude \( |x| \) increases and which satisfies the condition (2.3)\(^{11}\).

Among various price trends used in practice, standard moving average (MA) rules with different time horizons are the most popular ones, that is,

\[ u_i(t) = \frac{1}{\tau_i} \int_{t-\tau_i}^{t} P(s) ds, \quad i = m, c, \]  

(2.4)

where time delay \( \tau_i \geq 0 \) represents the time horizon of the MA. In particular, when \( \tau_i \to 0, \ u_i(t) \to P(t) \), implying that the price trend is given by the current price.

2.3. Market Price via a Market Maker. Assume a net zero supply\(^{12}\). Then the aggregate market excess demand for the risky asset, weighted by the population weights of the fundamentalist, momentum and contrarian traders, is given by

\[ \alpha_f D_f(t) + \alpha_m D_m(t) + \alpha_c D_c(t). \]

Following Beja and Goldman (1980) and Chiarella\(^{11}\), Chiarella (1992) provides explanations for the increasing and bounded S-shaped excess demand function. For example, each chartist may seek to allocate a fixed amount of wealth between the risky asset and a bond so as to maximize intertemporal utility of consumption. The demand for the risky asset is then proportional to the market price deviation from the trend, but is also bounded above and bounded below due to wealth constraints. For a chartist, the individual demand function would then be piecewise linear, and adding many such individual demand functions together leads approximately to an S-shaped increasing excess demand function. The S-shaped demand function is also consistent with Kahneman and Tversky’s (1979) prospect theory regarding

\[ u_i(t) = P(t), \ i = m, c \]  

as the reference point.

\(^{12}\)When the supply is positive but constant, the fundamental price needs to be adjusted to have the same price dynamics, see for example, Zhu et al (2009).
et al (2006), we assume that the price \( P(t) \) at time \( t \) is set via a market maker mechanism and is adjusted according to the aggregate excess demand, that is

\[
dP(t) = \mu [\alpha_f D_f(t) + \alpha_m D_m(t) + \alpha_c D_c(t)]dt + \sigma_M dW_M(t),
\]

(2.5)

where \( \mu > 0 \) represents the speed of the price adjustment by the market maker, \( W_M(t) \) is a standard Wiener process capturing the random excess demand process either driven by unexpected market news or noise traders, and \( \sigma_M \geq 0 \) is a constant.

Based on Eqs. (2.1)-(2.5), the market price of the risky asset is determined according to the following stochastic delay integro-differential system

\[
dP(t) = \mu \left[ \alpha_f \beta_f (F(t) - P(t)) + \alpha_m \tanh \left( \beta_m \left( P(t) - \frac{1}{\tau_m} \int_{t-\tau_m}^{t} P(s)ds \right) \right) 
+ \alpha_c \tanh \left( -\beta_c \left( P(t) - \frac{1}{\tau_c} \int_{t-\tau_c}^{t} P(s)ds \right) \right) \right] dt + \sigma_M dW_M(t),
\]

(2.6)

where the fundamental price \( F(t) \) is an exogenous random process which will be specified in Section 4. The stochastic model (2.6) will be analyzed in Section 4.

To understand the interaction of the deterministic dynamics and noise processes, in Section 3, we study the dynamics of the corresponding deterministic delay integro-differential equation model.

### 3. Dynamics of the Deterministic Model

By assuming that the fundamental price is a constant \( F(t) \equiv \bar{F} \) and there is no market noise \( \sigma_M = 0 \), the system (2.6) becomes a deterministic delay integro-differential equation

\[
\frac{dP(t)}{dt} = \mu \left[ \alpha_f \beta_f (\bar{F} - P(t)) + \alpha_m \tanh \left( \beta_m \left( P(t) - \frac{1}{\tau_m} \int_{t-\tau_m}^{t} P(s)ds \right) \right) 
+ \alpha_c \tanh \left( -\beta_c \left( P(t) - \frac{1}{\tau_c} \int_{t-\tau_c}^{t} P(s)ds \right) \right) \right].
\]

(3.1)

It is easy to see that \( P(t) = \bar{F} \) is a steady state price of the system (3.1), which is the constant fundamental price. We therefore call \( P = \bar{F} \) the fundamental-steady-state.

In this section, we study the dynamics of the deterministic model (3.1), including the stability and bifurcation of the fundamental steady state\(^{13}\). It is known (see

\(^{13}\)For a general theory of functional differential equations, we refer the reader to Hale (1997).
Gopalsamy, 1992) that the stability is characterized by the eigenvalues of the characteristic equation of the system at the steady state. In the remainder of the paper, we introduce \( \gamma_i = \mu \alpha_i \beta_i \) \((i = f, m, c)\) to characterize the activity of type-i agents\(^{14}\). Then the characteristic equation of the system (3.1) at the fundamental steady state \( P = \bar{F} \) is given by

\[
\lambda + \gamma_f - \gamma_m + \gamma_c + \frac{\gamma_m}{\lambda \tau_m} (1 - e^{-\lambda \tau_m}) - \frac{\gamma_c}{\lambda \tau_c} (1 - e^{-\lambda \tau_c}) = 0.
\] (3.2)

For delay integro-differential equation, the eigenvalue analysis can be complicated. In general, the dynamics depend on the behavior of fundamentalists, momentum traders, contrarians, market maker, and time horizons. To understand the different impact of the fundamentalists, momentum traders and contrarians, we first consider two special cases where only the momentum traders or the contrarians are involved.

3.1. The Stabilizing Role of the Contrarians. Contrarian trading strategies are based on the hypothesis of market overreaction. Intuitively such market overreaction is expected to be corrected by the activity of contrarians. To provide a supporting to the intuition, we consider a market with the fundamentalists and contrarians, that is, \( \alpha_m = 0 \). In this case the system (3.1) reduces to

\[
\frac{dP(t)}{dt} = \mu \left[ \alpha_f \beta_f (\bar{F} - P(t)) + \alpha_c \tanh \left( -\beta_c (P(t) - \frac{1}{\tau_c} \int_{t-\tau_c}^{t} P(s) ds) \right) \right].
\] (3.3)

The following proposition confirms the stabilizing role of the contrarians\(^{15}\).

**Proposition 3.1.** The fundamental steady state price \( P = \bar{F} \) of system (3.3) is asymptotically stable for all \( \tau_c \geq 0 \).

Proposition 3.1 shows that a market consisting of fundamentalist and contrarian investors is always stable, and the result is independent of the time horizon and

\(^{14}\)In fact, market maker set the adjustment speed \( \mu \), which can measure the market activity, according to the number of agents in the market, demand level and other factors. The product of the number of all agents and market fraction \( \alpha_i \) is the number of type-i agents. The demand level multiplied by \( \beta_i \) qualifies the trading behaviour of type-i agents. Therefore, \( \gamma_i \) can well govern the activity of type-i agents.

\(^{15}\)We do not include the proofs and remarks for all propositions in the paper. Please contact the authors for the proofs if needed.
extrapolation of the contrarians. Note that this result is different from the result in discrete-time HAMs, in which market can become unstable when activity of contrarians is strong, see for example, Chiarella and He (2002). This difference is due to the continuous adjustment of the market price. The impact of any strong activity from the contrarians becomes insignificant over a small time increment. Hence the time horizon used to form the MA becomes more irrelative in this case.

3.2. The Double-edged Role of the Momentum Traders. Momentum trading strategies based on the hypothesis of market underreaction are aimed to explore the opportunities of market price continuity. Intuitively, when the market is dominated by fundamentalist traders, the market is expected to reflect the fundamental price, consequently, the impact of the momentum traders on market stability can be very limited. However, when the market is dominated by momentum traders, the extrapolation of the market price continuity can have significant impact on market stability. To explore the impact of the momentum traders, we now consider a market consisting of the fundamentalists and the momentum traders only, that is \( \alpha_c = 0 \).

In this case, the system (3.1) reduces to

\[
\frac{dP(t)}{dt} = \mu \left[ \alpha_f \beta_f (\bar{F} - P(t)) + \alpha_m \tanh \left( \beta_m \left( P(t) - \frac{1}{\tau_m} \int_{t-\tau_m}^t P(s) \, ds \right) \right) \right],
\]

and the price dynamics can be described by the following proposition.\(^\text{16}\)

**Proposition 3.2.** The fundamental steady state price \( P = \bar{F} \) of system (3.4) is

(i) asymptotically stable for all \( \tau_m \geq 0 \) when \( \gamma_m < \frac{\gamma_f}{1 + a} \);

(ii) asymptotically stable for either \( 0 \leq \tau_m < \tau_{m,1}^* \) or \( \tau_m > \tau_{m,h}^* \), and unstable for \( \tau_{m,l}^* < \tau_m < \tau_{m,h}^* \) when \( \frac{2\gamma_m}{1 + a} \leq \gamma_m \leq \gamma_f \); and

(iii) asymptotically stable for \( \tau_m < \tau_{m,1}^* \) and unstable for \( \tau_m > \tau_{m,l}^* \) when \( \gamma_m > \gamma_f \).

\(^{16}\)Let \( a = \max\{-\sin x/x; x > 0\} \approx 0.2172 \), \( \tau_{m,1}^* := 2\gamma_m/(\gamma_f - \gamma_m)^2 \), and \( \tau_{m,l}^*, \tau_{m,h}^*(\in (\tau_{m,l}^*, \tau_{m,1}^*)) \) be the minimum positive root and the largest of the roots which are less than \( \tau_{m,1}^* \), respectively, of the following equation

\[
f(\tau_m) := \frac{\tau_m}{\gamma_m} (\gamma_f - \gamma_m)^2 - \cos \left[ \sqrt{2\gamma_m \tau_m - (\gamma_f - \gamma_m)^2 \tau_m^2} \right] = 0.
\]
Proposition 3.2 shows that the impact of the time horizon used in forming the MA by the momentum traders depends on their extrapolation activity. On the one hand, when fundamentalist traders dominate over momentum traders (so that $\gamma_m < \gamma_f/(1 + a)$), the market is always stable and time horizon plays no role in market stability. On the other hand, when momentum traders dominate over fundamentalists (so that $\gamma_m > \gamma_f$), the market is stable when time horizon is small (so that $\tau_m < \tau_{m,l}^*$), but becomes unstable when the time horizon is large (so that $\tau_m > \tau_{m,l}^*$). The difference between price and the price trend based on the MA is insignificant when the time horizon is small and a strong activity from the momentum traders has a very limited impact on market stability, yielding the stability for small horizon. But the difference can become significant when the time horizon is large which, together with strong activity from the momentum traders, makes the market become unstable. However, when the activity of the momentum traders is balanced by that of the fundamentalist traders (so that $\gamma_f/(1 + a) \leq \gamma_m \leq \gamma_f$), the market is stable when the time horizon is either small (so that $\tau_m < \tau_{m,l}^*$) or large (so that $\tau_m > \tau_{m,h}^*$), and unstable with medium time horizon (so that $\tau_{m,l}^* < \tau_m < \tau_{m,h}^*$).

Intuitively, when time horizon is large, the price trend becomes significant, resulting in strong trading signals. However, the activity of the trend followers, measured by $\gamma_m$, is limited by the activity of the fundamentalists, measured by $\gamma_f$. Therefore, the market is dominated by the fundamentalists, leading to a stable market.

3.3. The General Case. The analysis in the previous subsections illustrates different role of the time horizon used in the MA by either the contrarians or momentum traders. When both strategies are employed in the market, the market stability of the system (3.1) can be characterized by the following proposition and corollary\textsuperscript{17}.

**Proposition 3.3.** The fundamental steady state price $P = \bar{F}$ of system (3.1) is

(i) asymptotically stable for all $\tau_m, \tau_c \geq 0$ when $\gamma_m \leq \gamma_c + \frac{\gamma_f}{1 + a}$;

\textsuperscript{17}Let

$$h(\tau) := \frac{\tau}{\gamma_m - \gamma_c}(\gamma_f - \gamma_m + \gamma_c)^2 - 2\left(\frac{2(\gamma_m - \gamma_c)}{\gamma_m - \gamma_c} - \gamma_f + \gamma_c\right)\tau \cos\left(\sqrt{2(\gamma_m - \gamma_c)\tau - (\gamma_f - \gamma_m + \gamma_c)^2}\right) - 1 = 0. \quad (3.6)$$
(ii) asymptotically stable for either $0 \leq \tau_m, \tau_c < \tau_1^*$ or $\tau_m, \tau_c > \tau_1^*$ when $\gamma_c + \frac{\gamma_f}{1+\alpha} \leq \gamma_m \leq \gamma_c + \gamma_f$; and

(iii) asymptotically stable for $\tau_m, \tau_c < \tau_1^*$, and unstable for $\tau_m, \tau_c > \tau_1^*$ when $\gamma_m > \gamma_c + \gamma_f$.

**Corollary 3.4.** If $\tau_m \equiv \tau_c := \tau$, then the fundamental steady state price $P = \bar{F}$ of system (3.1) is

(i) asymptotically stable for all $\tau \geq 0$ when $\gamma_m < \gamma_c + \frac{\gamma_f}{1+\alpha}$;

(ii) asymptotically stable for either $0 \leq \tau < \tau_1^*$ or $\tau > \tau_1^*$, and unstable for $\tau_1^* < \tau < \tau_2^*$ when $\gamma_c + \frac{\gamma_f}{1+\alpha} \leq \gamma_m \leq \gamma_c + \gamma_f$; and

(iii) asymptotically stable for $\tau < \tau_1^*$ and unstable for $\tau > \tau_1^*$ when $\gamma_m > \gamma_c + \gamma_f$.

Despite the activity of both momentum and contrarian traders, Proposition 3.3 and especially Corollary 3.4 share the same message as Proposition 3.2 with respect to the joint impact of the time horizons and activity of the momentum traders on market stability, except that the activity of the fundamentalists in Proposition 3.2 is measured by the joint activities of the fundamentalist and contrarian traders in Proposition 3.3 and Corollary 3.4. Given the stabilizing nature of the contrarian strategy indicated in Proposition 3.1, this is not unexpected.

**Figure 3.1.** (a) The function $h(\tau)$; (b) the bifurcation diagram of the market price. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 1.67$, $\beta_m = 25$, $\beta_c = 6.67$, $\mu = 10$, and $\bar{F} = 1$.

Fig. 3.1 illustrates the case when $\gamma_m > \gamma_c + \gamma_f$ and $\tau_m = \tau_c = \tau$. It is clearly observed that the first Hopf bifurcation value $\tau_1^* \approx 0.026$ from Fig. 3.1 (a) and the
Hopf bifurcation leads to stable limit cycles for $\tau > \tau^*_l$, as shown in Fig. 3.1 (b). The stability switches only once at $\tau^*_l$.

**Figure 3.2.** (a) The function $h(\tau)$; (b) the corresponding bifurcation diagram; and the market price for (c) $\tau = 0.05$; (d) $\tau = 0.5$; (e) $\tau = 1.25$. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 21.1$, $\beta_m = 20$, $\beta_c = 8.5$, $\mu = 10$, and $\bar{F} = 1$.

Fig. 3.2 illustrates the case when $\gamma_c + \gamma_f/(1 + a) \leq \gamma_m \leq \gamma_c + \gamma_f$ and $\tau_m = \tau_c = \tau$. There are three Hopf bifurcation values $\tau^*_l \approx 0.07$, $\tau^*_3 \approx 0.15$, and $\tau^*_h \approx 1.08$ (Fig. 3.2 (a)) and the fundamental steady state price $P = \bar{F}$ is stable when $\tau \in [0, \tau^*_l) \cup (\tau^*_h, \infty)$ and unstable when $\tau \in (\tau^*_l, \tau^*_h)$ (Fig. 3.2 (b)). Figs. 3.2 (c)-(e) illustrate that the fundamental steady state is asymptotically stable\(^{18}\) for $\tau = 0.05$ ($< \tau^*_l$) and

\(^{18}\)The speed of the convergence when the fundamental steady state price becomes stable after switching from instability as $\tau$ increases can be very slow.
\( \tau = 1.25 (> \tau^*_h) \), and unstable for \( \tau = 0.5 (\in (\tau^*_l, \tau^*_h)) \) as stated in Proposition 3.3. The stability switches twice\(^{19}\).

The implications of the results in Proposition 3.3 and Corollary 3.4 on profitability and the intuitions behind are more interesting. When the market is dominated jointly by the fundamentalist and contrarian traders (so that \( \gamma_m < \gamma_c + \gamma_f/(1 + a) \)), the stability of the market in this case is independent of the time horizon. This implies the profits of contrarians from any market overreaction and the losses of momentum traders no matter how long the time horizon they choose. However, the opportunity of making profit for the contrarians is rather limited due to the limited activity of momentum trading that limits market price overreaction.

Although the time horizon can affect the stability when \( \gamma_c + \gamma_f/(1 + a) \leq \gamma_m \leq \gamma_c + \gamma_f \), the limited activity of momentum traders by fundamentalists and contrarians only arises the instability existing for time horizon in a short interval \((\tau^*_l, \tau^*_h)\) and the quite small oscillation amplitudes. So this market is still flat and consequently the profitabilities are still the same as the case \( \gamma_m < \gamma_c + \gamma_f/(1 + a) \).

When \( \gamma_m > \gamma_c + \gamma_f \), momentum traders dominate the market. A strong extrapolation from the momentum traders can generate price overreaction. In this case, the market, in which the market price fluctuates widely, is different from previous two cases in nature. Then momentum traders make profits and contrarians lose money over short time horizons, which become self-fulfilling for the momentum traders.

\(^{19}\)There are some interesting properties on the nature of bifurcations related to Corollary 3.4, including the number of bifurcations, stability switching and the dependence of the bifurcation values on the parameters. In summary, (i) all roots of \( h(\tau) = 0 \), except \( \tau = \tau^*_1 \), are Hopf bifurcation values; (ii) the stability switching happens only once when \( \gamma_m > \gamma_c + \gamma_f \) and only twice when \( \gamma_c + \gamma_f/(1 + a) \leq \gamma_m \leq \gamma_c + \gamma_f \), though the number of the Hopf bifurcations that the fundamental steady state can undergo increases when \( \gamma_f + \gamma_c \to \gamma_m \). Moreover, the number of the Hopf bifurcations that the fundamental steady state undergoes must be odd; (iii) the first bifurcation value \( \tau^*_1 \) depends on the population fractions, the extrapolation rates and the speed of the price adjustment. It increases as \( \gamma_f \) or \( \gamma_c \) increases, or \( \gamma_m \) decreases. However it is always bounded away from zero and infinity. These properties are interesting both theoretically and empirically, which may deserve some further study.
But for the long time horizon, the underreaction has been well exploited and market becomes overreacted, momentum traders cannot make profits any more in this case and contrarian strategies start to win.

In addition, it follows from Figs. 3.1 (b) and 3.2 (b) that the oscillation amplitudes when momentum traders dominate the market \((\gamma_m > \gamma_c + \gamma_f)\) are much bigger than those when they do not dominate the market \((\gamma_m \leq \gamma_c + \gamma_f)\), implying that the price trend is stronger for \(\gamma_m > \gamma_c + \gamma_f\) than \(\gamma_m \leq \gamma_c + \gamma_f\), and yielding higher trading volume for \(\gamma_m > \gamma_c + \gamma_f\). This is similar to the “ostrich effect” in Karlsson et al. (2009) that investors trade more actively and frequently in rising markets (momentum traders dominating the market), but “put their heads in the sand” in flat markets (momentum traders not dominating the market).

We finish the discussion of this subsection by considering a very special case when \(\alpha_m = \alpha_c\), \(\beta_m = \beta_c\) and \(\tau_m = \tau_c\), that is the momentum traders and the contrarians have the same population, extrapolation rate and time horizon. In this case, the system (3.1) reduces to

\[
\frac{dP(t)}{dt} = \gamma_f(\bar{F} - P(t)).
\]

The destabilizing effect of momentum traders is completely offset by contrarians, which leads to the global stability of the fundamental price.

4. Price Behavior of the Stochastic Model

In this section, we conduct some numerical simulations for the stochastic model (2.6). The focus is on the interaction between the market dynamics of the deterministic model and noise processes in order to explore the potential capability of the model to generate various market behaviors, such as the long deviation of the market price from the fundamental price, and the stylized facts, including the skewness, fat tails, volatility clustering, and long range dependence in volatility observed in financial markets.

To complete the stochastic model (2.6), we introduce the stochastic fundamental price process

\[
dF(t) = \frac{1}{2} \sigma_F^2 F(t) dt + \sigma_F F(t) dW_F(t), \quad F(0) = \bar{F},
\]

(4.1)
where $\sigma_F > 0$ represents the volatility of the fundamental return and $W_F(t)$ is a standard Wiener process, which is independent of the standard Wiener process for the market noise $W_M(t)$ introduced in (2.6)\(^a\). It follows from Eq. (4.1) that the fundamental return defined by $d(\ln(F(t)))$ is a pure white noise process following the normal distribution with mean of 0 and standard deviation of $\sigma_F \sqrt{dt}$. This ensures that any non-normality and volatility clustering of market returns that the model could generate are not carried from the fundamental returns.

Firstly, we explore the joint impact of the time horizon and the two noise processes on the market price dynamics. In the following numerical simulations, we choose $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 1.67$, $\beta_m = 25$, $\beta_c = 6.67$, $\mu = 10$, $\bar{F} = 1$, $\sigma_F = 0.15$ and $\sigma_M = 0.05$\(^b\). For the corresponding deterministic model (3.1), we see that the fundamental steady state price is stable for $\tau = 0.02$ and unstable for $\tau = 0.5$ as illustrated in Figs. 3.2 (c) and (d) respectively. The market price converges to $\bar{F}$ when $\tau = 0.02$ and there are significant fluctuations of the market price when $\tau = 0.5$. For the stochastic model, with the same random draws of the fundamental and market noise processes, we plot the market price (the red solid line) and the market price (the red solid line) for two values (a) $\tau = 0.02$; (b) $\tau = 0.5$.

\(^a\)The two Wiener processes can be correlated. We refer the reader to He and Li (2011) for the study of correlated Wiener processes.

\(^b\)In the simulations in this section, time unit is a year, an annual volatility is given by $\sigma_F = 0.15$, and the time step of numerical simulations is 0.004, corresponding to one day.
line) and the fundamental price (the blue dotted line) in Fig. 4.1 for two different values of $\tau$. For $\tau = 0.02$, Fig. 4.1 (a) demonstrates that the market price follows the fundamental price closely. For $\tau = 0.5$, Fig. 4.1 (b) indicates that the market price fluctuates around the fundamental price in cyclic way. The stochastic price behaviour is underlined by the dynamics of the corresponding deterministic model.

Secondly, we explore the potential of the stochastic model in generating the stylized facts for daily data observed in financial markets. For the stochastic model with both noise processes, Fig. 4.2 represents the results of a typical simulation where the time step is one day\(^{22}\). Fig. 4.2 (a) shows that the market price (the red solid line) follows the fundamental price (the blue dotted line) in general, but companied with large deviations from time to time. The returns of the market prices in Fig. 4.2 (b) show significant volatility clustering. Comparing to the corresponding normal distribution, the return density distribution in Fig. 4.2 (c) displays high kurtosis. The returns show almost insignificant autocorrelations (ACs) in Fig. 4.2 (d), but the ACs for the absolute returns and the squared returns in Figs. 4.2 (e)-(f) are significantly with strong decaying patterns as time lag increases. The result demonstrates that the stochastic model has a great potential to generate most of the stylized facts observed in financial markets\(^{23}\).

Thirdly, the above analysis is based on the daily market return. When it comes to weekly and monthly market returns, Fig. 4.3 provides the autocorrelations of the weekly and monthly market returns by engaging the same price data as Fig. 4.2 and shows that the weekly and monthly returns are predictable from past returns.

\(^{22}\)We choose $\tau = 0.02$ so that the fundamental steady state is locally asymptotically stable as illustrated in Fig. 3.2 (c). We can also obtain the stylized facts for $\tau$ in its unstable interval and the same random seeds.

\(^{23}\)We may argue that the above features of the stochastic model is a joint outcome of the interaction of the nonlinear HAM and the two stochastic process as mentioned in He and Zheng (2010). The numerical simulations (not reported here) when there is only one stochastic process involved show that the potential of the model in generating the stylized facts is not due to either one of the two stochastic processes, rather than to both processes. The underlying mechanism in generating the stylized facts and long range dependence and the interplay between the nonlinear deterministic dynamics and noises are very similar to the one for a discrete-time HAM in He and Li (2007).
(a) The market price and the fundamental price

(b) The market return ($r$)

(c) The density of the market return

(d) The ACs of the market return

(e) The ACs of the absolute return

(f) The ACs of the squared return

**Figure 4.2.** The time series of (a) the market price (red solid line) and the fundamental price (blue dotted line) and (b) the market return; (c) the density distribution of the market returns; the ACs of (d) the market returns; (e) the absolute returns, and (f) the squared returns.
as stated in Pesaran and Timmermann (1994, 1995). Furthermore, the positive ACs for initial small lags reveal the underreaction over short time horizons and negative ACs for the following bigger lags demonstrate the overreaction of the market over longer time horizon (Hong and Stein 1999).

5. Profitabilities

To simplify the problem and capture the mechanism of profitability, we consider at first a special case in which momentum and contrarian traders use the same time horizon and holding period. It then follows from Eq. (2.2) that the excess demands of an individual momentum and contrarian trader using time horizon $\tau$ are given, respectively, by

$$D_m(t) = \tanh \left( \beta_m (P(t) - \frac{1}{\tau} \int_{t-\tau}^{t} P(s) ds) \right),$$

$$D_c(t) = \tanh \left( -\beta_c (P(t) - \frac{1}{\tau} \int_{t-\tau}^{t} P(s) ds) \right).$$

(5.1)

Based on buy and hold strategy, which is what empirical study used, the spot profits yielded by the investments following fundamentalist, momentum, contrarian and market maker strategies at time $t$ can be characterized by

$$U_i(t) = D_i(t)(P(t + \tau) - P(t)), \quad i = f, m, c, M,$$

(5.2)
where the excess demand of fundamentalists $D_f(t)$ has been given by Eq. (2.1) and the excess demand of market maker $D_M(t)$ at $t$ is given by

$$D_M(t) = -(\alpha_f D_f(t) + \alpha_m D_m(t) + \alpha_c D_c(t)).$$

In another direction, the average accumulated profits yielded over a time interval $[t_0, t]$ of the four types of traders can be given by

$$\bar{U}_i(t_0, t) = \frac{1}{t - t_0} \int_{t_0}^{t} D_i(s)(P(s + \tau) - P(s)) ds, \quad i = f, m, c, M. \quad (5.3)$$

![Graphs showing average profits for different traders and time periods.](image)

**Figure 5.1.** (a) the average spot profits of different traders for different entering market time based on 1000 simulations and (b) average accumulated profits along one group price series. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 1.67$, $\beta_m = 25$, $\beta_c = 6.67$, $\mu = 10$, $\bar{F} = 1$, $\sigma_F = 0.15$ and $\sigma_M = 0.05$.

Under the parameters in Fig. 3.1, that is the momentum traders dominate the market ($\gamma_m > \gamma_c + \gamma_f$), Fig. 5.1 reports not only the average spot profits of the
four types of traders for different entering market time based on 1000 simulations (Figs. 5.1 (a) and (c)), where the profits are characterized from Eq. (5.2), but also the average accumulated profits along one group price series described by Eq. (5.3) (Figs. 5.1 (b) and (d)). Simulations clearly show fundamentalists and momentum traders can gain money comparing with the losses of contrarians and market maker for $0 < \tau < 0.7$ as illustrated in Figs. 5.1 (a) and (b). But for the long horizons $1.15 < \tau < 5$, fundamentalists and contrarians make profits however, momentum traders and market maker lose money, see Figs. 5.1 (c) and (d). We do not do the simulation for $\tau > 5$.

![Graphs showing average spot profits and accumulated profits for different traders](image)

**Figure 5.2.** (a) the average spot profits of different traders for different entering market time based on 1000 simulations and (b) average accumulated profits along one group price series. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 21.1$, $\beta_m = 20$, $\beta_c = 8.5$, $\mu = 10$, $\bar{F} = 1$, $\tau = 0.5$, $\sigma_F = 0.15$ and $\sigma_M = 0.05$.

Under the parameters used in Fig. 3.2, so that $\gamma_c + \gamma_f/(1 + a) \leq \gamma_m \leq \gamma_c + \gamma_f$, both Figs. 5.2 (a) and (b) illustrate that fundamentalists and contrarians can make profits, however momentum traders and market maker lose money when all technical traders use short time horizon and holding period ($\tau = 0.5$). Notice the profit level of market maker is much smaller than those of fundamentalists, momentum traders and contrarians. Furthermore, numerical simulations show that using different time
horizons does not change the profitabilities but the losses/profits increase as time horizon increases (not reported here)\textsuperscript{24}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig53}
\caption{(a) the average spot profits of different traders for different entering market time based on 1000 simulations and (b) average accumulated profits along one group price series. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 13.33$, $\beta_m = 12.5$, $\beta_c = 13.33$, $\mu = 10$, $\bar{F} = 1$, $\tau = 0.5$, $\sigma_F = 0.15$ and $\sigma_M = 0.05$.

In addition, when the market is dominated jointly by the fundamentalist and contrarian traders, so that $\gamma_m < \gamma_c + \gamma_f/(1 + a)$, the contrarian and fundamentalist strategy can make profit, however momentum traders and market maker loses for time horizons $0 < \tau < 5$ as illustrated by Fig. 5.3. Although the profitabilities are the same as those shown by Fig. 5.2, the profit/loss of contrarian/momentum traders is smaller than that in Fig. 5.2.

We also find that the profitability of momentum (contrarian) strategy is positive (negative) correlated with $\beta_m$ and negative (positive) correlated with $\beta_c$. Secondarily, the time delay $\tau$ can also affect the profitability greatly. But the impacts of the seeds and other parameters on the profitability are much smaller than $\beta_m$, $\beta_c$ and $\tau$. Recall that the stability of the system depends on $\gamma_i = \mu \alpha_i \beta_i$ ($i = f, m, c$) and $\tau$ completely. So the profitability can be correlated with the stability but can be not determined by the stability totally.

\textsuperscript{24}The oscillation amplitudes in this case are much smaller than those in the case of Fig. 5.1, but this is not the reason for the difference of the profitabilities between the two cases. In fact, an increase in $\mu$ can lead to an increase in the amplitudes, but this can not affect the profitabilities.
In all, (i) extensive numerical simulations show that the fundamentalist strategy can win and market maker loses in general. (ii) When momentum traders dominate the market, they can make profits for small time horizon but lose money for big time horizon; contrarians with long time horizon can win but lose with short time horizon. (iii) When momentum traders do not dominate the market, contrarian strategy can always win but momentum strategy always loses.

6. Momentum in Index

We apply the momentum strategy in our paper to stock indices and study its profitability. The mean profits and t statistic of momentum strategy with \(i\)-months horizons and \(j\)-months holding periods (\(i, j = 1, 2, \ldots, 60\)) are calculated in this way: we firstly calculate the demand \(D_m(t)\) of momentum traders at time \(t\) based on past \(i\)-month prices using the first equation of Eq. (5.1), and hold the asset for \(j\)-months. So at each time \(t\), we have \(j\) demands (except for the previous \(j - 1\) time periods). The average profit at time \(t\) is given by

\[
\left[ \frac{1}{j} \sum_{k=1}^{j} \text{sign}(D_m(t - k)) \right] \times \log \frac{P(t)}{P(t - 1)}
\]

\(= \left[ \frac{1}{j} \sum_{k=1}^{j} \text{sign} \left( \tanh \left( \beta_m \left( P(t) - \frac{1}{\tau} \int_{t-\tau}^{t} P(s) ds \right) \right) \right) \right] \times \log \frac{P(t)}{P(t - 1)}.\)

Then the mean profits of momentum strategy and the corresponding t statistic of \(i\)-months horizons and \(j\)-months holding periods are calculated based on the average profits at all time periods.

\(\text{The results above still hold when the market only consists of two types of traders, fundamentalists and momentum traders.}\)

\(\text{The integral is calculated by using the price at the start of each time period and time step is one month.}\)

\(\text{The profitability of contrarians is opposite to momentum traders’ due to the opposite strategy.}\)

\(\text{The advantages of the above method to measure profitability are that (1) the profitability is irrelevant to parameters. In fact, the behavioral parameter } \beta_m, \text{ even the nonlinear function } \tanh(x) \text{ cannot affect the average profit in Eq. (6.1); (2) a large number of numerical simulations show that the profitability is insensitive to the random seeds while applying the method to the experimental data generated from our model. So we only report the results based on one trial instead of Monte Carlo simulations.}\)
(a) Mean profits with respect to 1-60 month holding periods for different horizons

(b) T statistic with respect to 1-60 month holding periods for different horizons

**Figure 6.1.** The profitability of momentum strategy in Dax 30 (Dec. 1965—Aug. 2011).

Fig. 6.1 illustrates the mean profits and corresponding t statistic of the momentum strategy in our model investing in Dax 30. The strong decaying patterns in mean profits and t statistics can be observed. Fig. 6.1 also shows the statistically significant profits of momentum traders with small horizons and holding periods.

Figure 6.2. The profitability of momentum strategy when momentum traders dominate the market. Here $\alpha_f = 0.3$, $\alpha_m = 0.4$, $\alpha_c = 0.3$, $\beta_f = 1.67$, $\beta_m = 25$, $\beta_c = 6.67$, $\mu = 10$, $\bar{F} = 1$, $\sigma_F = 0.15$ and $\sigma_M = 0.05$.

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29The profitability of the momentum strategy investing in S&P 500, FTSE 100 and Nikkei 225 also exhibits similar patterns.
Now we analyze the experimental data generated from our model. For a fixed \( \tau \in \{1/12, 0.25, 0.5, 1, 1.5, 3, 5\} \) (1, 3, 6, 12, 18, 36, 60 months), the model can generate a group of price series. Then we calculate the profitability of momentum strategy with horizon equal to \( \tau \) and holding period from 1 to 60 months applying the above method. First, when momentum traders dominate the market, we can see the long reversal in the momentum profits that they make profits with short horizon and holding period, and lose for large horizon and holding period statistically significantly as illustrated in Fig. 6.2. However, when momentum traders do not dominate the market, they always lose, see Fig. 6.3. It can be observed that Fig. 6.2, that is, momentum traders dominating the market, can match the profitability results on the Dax 30 better than the case where momentum traders do not dominate the market. But different from the Dax 30, the decaying patterns are faster in Fig. 6.2, leading to losses for large horizon and holding period.

7. Conclusion

Based on market underreaction and overreaction hypotheses, momentum and contrarian strategies are widely used by financial market practitioners and their profitability has been extensively investigated by academics. However, common
interpretations of return reversals as evidence of overreaction and return persistence as evidence of underreaction are probably overly simplistic. Following the recent development in heterogeneous agent models literature, this paper proposes a continuous-time heterogeneous agent model of investor behaviour consisting of fundamentalist, contrarian, momentum and market maker strategies. The stochastic delay integro-differential equation used in the model provides a unified approach to deal with different time horizons of momentum and contrarian strategies, which play an important role in the profitability empirically. By examining their impact on market stability, we show that momentum traders win for short horizons and lose for large horizons while they dominating the market, however always lose while they not dominating the market, and the opposite profitability of contrarians. By including noise traders in the market and imposing a stochastic process on fundamental prices, we demonstrate that both contrarian and momentum strategies can be consistent with market efficiency. In addition, the model is able to generate long deviations of the market price from the fundamental price, market bubbles, crashes, and most of the stylized facts, including non-normality, volatility clustering, and power-law behavior of high-frequency returns, observed in financial markets.

The modelling approach used in this paper demonstrates the advantage of continuous-time models over the discrete-time models and provide an alternative approach to the current discrete-time financial market modelling with bounded rational and heterogeneous agents. The model proposed in this paper is very simple and some extensions would be of interest. One of them is to extend the market of one risky asset to many risky assets so that we can examine the profitability of portfolios constructed from momentum and contrarian strategies. We leave these to the future research.

**References**


PROFITABILITY OF CONTRARIAN AND MOMENTUM STRATEGIES AND MARKET STABILITY


