A Unified Theory of Changes in Labor Productivity and Inventory Dynamics*

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Abstract

*The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.
1 Introduction

Post-war U.S. economic data is characterized by a noticeable fall in the volatility of key macroeconomic aggregates in the mid-1980s known as the Great Moderation. Recent work by, amongst others, McConnell and Kahn (2001), Francis and Ramey (2009), and Gordon (2010), has pointed out that the period after 1984 is also characterized by notable changes in the comovement properties of some of these aggregates. In particular, labor productivity becomes considerably less procyclical and, more surprisingly, starts to comove negatively with employment. At the same time, the volatility of sales increases relative to that of output while inventory investment becomes less positively correlated with sales.

As pointed out in Gordon (2010), changes in the comovement properties of labor productivity cannot be easily reconciled with conventional real business cycle models driven by technology shocks. Francis and Ramey (2009) argue that the negative comovement of labor productivity with employment can be rationalized by a model in which capital and labor are complements and technology shocks are labor-saving. Alternatively, Gali (1999) has pointed out that models with nominal rigidities driven by supply shocks can also account for labor productivity and employment moving in opposite direction over the business cycle. By and large, this literature abstracts from inventories and, therefore, cannot address corresponding facts in the data. One exception is Chang, Hornstein, Sarte (2009), who show that once inventories are introduced in conventional sticky-price models, the implications for comovement between labor productivity and employment are overturned.

McConnell and Kahn (2001) interpret changes in the dynamics of inventories as a reflection of improvements in management inventory techniques. They suggest that these same improvements could also account for the reduced volatility of output and the Great Moderation. However, subsequent work by Khan and Thomas (2007), Wen (2009), as well as Iacoviello, Schiantarelli, and Schuh (2010), has shown that changes in inventory management technology tend to have moderate effects on output volatility.

This paper investigates the extent to which a generalized version of a canonical business cycle model, such as first presented in Long and Plosser (1982), can explain salient changes in the comovement properties and dynamics of both labor productivity and inventories. The key aspects of this generalization are the introduction of time to produce and its interpretation as the reason underlying the presence of inventory holdings. The model relies on only two shocks to address the stylized facts, a supply shock capturing fluctuations in the productivity of inputs, and a demand shock that affects the discount rate of a representative household. Because Long and Plosser (1982) features an environment with multiple sectors,
our framework allows us to treat durable and non-durable production separately. This distinction is important because inventories play a disproportionately greater role in the production of durable goods relative to non-durable goods and services.

One appeal of the set-up we study is that it nests, in addition to Long and Plosser (1982), a large class of frictionless business cycle models such as frameworks with time to build as in Kydland and Prescott (1982), and Rouwenhorst (1991), as well as models where input-output linkages are explicitly taken into account such as Horvath (1998, 2000), Dupor (1999), Foerster, Sarte, and Watson (2010). A key insight is that this class of models has implications for inventory dynamics that, to this point, have been overlooked. Specifically, we show that implications for how inventory dynamics have changed after 1984 are consistent with changes in the comovement properties of labor productivity over the same period.

Our preliminary findings indicate that the relative importance of demand shocks relative to supply shocks has grown over time. In particular, within the context of the model we present, changes in the dynamics of inventories and comovement properties of labor productivity can be accounted for by a shift in the composition of these two shocks. In addition, aside from a fall in the volatility of key macroeconomic aggregates consistent with the Great Moderation, the model delivers predictions for changes in the relative volatility of consumption and investment consistent with U.S. data. Moreover, the model further implies an increase in the relative importance of low frequency movements in output, consumption, and investment, that are also found in the data. Most notably, the rich set of stylized facts we address derives from a model that is estimated using only two time series, namely aggregate output and investment.

The rise over time in the importance of demand shocks for business cycle fluctuations finds support in the work Iacoviello, Schiantarelli, and Schuh (2010), but in a model that focuses more exclusively on inventory behavior. The increase in the importance of low frequency movements of main macroeconomic aggregates is echoed in Pancrazi (2010), who concentrates on the behavior of consumption in a partial equilibrium model and its implications for sources of welfare gain of the Great Moderation. More generally, both the data and the model indicate that business cycle fluctuations place greater emphasis on slower moving components relative to the pre-1984 period. Thus, expansions and downturns in economic activity now take longer to turn around. Finally, as pointed out in Hornstein (1993), taking account of input-output linkages is key in ensuring that our model, while bridging a large literature on the effects of technology shocks and its implications for the comovement properties of labor productivity with one that has addressed changes in inventory behavior,
remains consistent sectoral comovement properties of employment.

2 A Sketch of the Model

The economy is populated by a representative agent with utility function:

\[ E_t \sum_{t=0}^{\infty} \zeta_t \beta^t \left( C^\theta_t (1 - L_t)^{1-\theta} \right)^{1-\sigma} \]

where \( C_t \) is aggregate consumption and \( L_t \) is the amount of labor supplied to the firms. \( \zeta_t \) is a discount factor shock which we interpret as a demand disturbance. Aggregate consumption is a geometric composite of sector-specific consumption:

\[ C_t = \prod_{i=1}^{N} C_{it}^{\eta_{i,t}} \]

where the weights sum to one, \( \sum \eta_{i,t} = 1 \).

The household faces a sequence of budget constraints:

\[ C_{jt} + \sum_{i=1}^{N} I_{jit} + \sum_{i=1}^{N} \sum_{v=0}^{V} M_{jit} (v) = Y_{jt}, \forall j, \]

where \( Y_{jt} \) is the output of sector \( j \). Similarly, \( I_{jt} = \prod_{i=1}^{N} I_{ijt}^{\theta_{ij}}, \) with \( \sum \theta_{ij} = 1 \), is sector-specific investment. \( M_{jt} (v) = \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}} (v), \sum \gamma_{ij} = 1 \), is the amount of materials sourced from all sectors \( i \) for usage in sector \( j \). Capital in each sector then evolves according to:

\[ K_{jt+1} = I_{jt} + (1 - \delta) K_{jt}. \]

The production function in sector \( j \) is given by:

\[ Y_{jt} = A_{jt} \sum \left[ \omega_j (v)^{\frac{1}{\rho}} Z_{j,t-v} (v) \frac{\rho-1}{\rho} \right]^{\frac{\rho}{\rho-1}}, \]

where \( A_{jt} \) is sector-specific productivity. In the benchmark version we restrict this to be identical for all sectors, \( A_{jt} = A_t \). In each sector \( j \), the production of final goods takes place using as materials the amount \( M_{ijt} \) of output produced in sector \( i \). In addition, investment goods in sector \( j \), \( Z_{jt} \), are produced using the amount \( X_{ijt} \) of output produced in sector \( i \). We assume that it takes time to produce, with the production function at stage \( \nu \) being:

\[ Z_{jt} (v) = B_{jt} K_{jt} (v)^{\alpha_j} L_{jt} (v)^{\xi_j} M_{jt} (v)^{1-\alpha_j-\xi_j} \]
An input-output matrix for this economy is an $N \times N$ matrix $\Gamma$ with typical element $\gamma_{ij}$, the share of sector $i$ in output of sector $j$. A sectoral investment matrix is an $N \times N$ matrix $\Theta$ with typical element $\theta_{ij}$, the share of sector $i$ in total investment made by sector $j$.

Finally, we aggregate sector- and production-stage-specific quantities as:

$$L_t = \sum_{j=1}^{N} \sum_{v=0}^{V} L_{jt}(v),$$

and:

$$K_{jt} = \sum_{v=0}^{V} K_{jt}(v)$$

### 3 Methodology

Our approach is parsimonious in that we only use two observables in the estimation, although our model has predictions for a far wider range of variables. We pursue this strategy for two reasons. First, data availability, data construction, and the mapping of data into the corresponding model concepts is fraught with pitfalls in the context of inventory models (see Iacoviello, Schiantarelli, and Schuh, 2010). While we do not necessarily want to preclude the introduction of more data sources, we will only pursue this for robustness checks. Second, using more data series in the estimation necessitates the introduction of additional exogenous shocks, which raises issues of interpretation, identification, and plausibility. Our use of output and investment as observables allows us to fairly clearly identify productivity (“supply”) and preference shocks (“demand”) from the respective optimality conditions. We will then use the estimated model to extract information for the unobserved variables and confront them with a broad set of facts. In other words, we use structural estimation with a parsimonious data set to put discipline on the exercise, but not by so much as to force the model to match potentially problematic data series.

We estimate the structural model using Bayesian methods. This involves setting a prior distribution for the structural parameters, computing and maximizing the likelihood function, and then evaluating the posterior distribution by means of Markov-chain-Monte-Carlo methods. The posterior distribution can then be used to perform a posterior predictive density analysis regarding in various directions. We choose a Bayesian approach for two reasons. First, it allows us to bring in a host of extraneous information. While we do not directly incorporate many available series into the estimation, we use this information to guide our choice of priors. The second reason is that a Bayesian approach enables model comparison for models that are not nested.
4 Preliminary Results

As a preliminary step to a full empirical analysis of the model and the question at hand, we calibrate the model and simulate it for various shocks. We then contrast the simulated data with the stylized facts we identify in the data. The calibration of the model is standard, the only non-standard parameter is the one governing time to produce. An essential insight is that this parameter can be pinned down using the average inventory to sales ratio. Sales in any given sector is simply $Y^i_t$. Inventories can be calculated from the following accounting identity:

$$\text{Inventories}_t = \text{Inventories}_{t-1} + \text{Current Costs} - \text{Cost of Goods Sold}.$$ 

In the table below we compare moments derived from different versions of the model with equivalent moments in the data. In the model we simulate shocks as AR(1)-processes with quarterly mean reversion coefficient of 0.95. Our main interest is in cross-moments and relative volatilities. With one exception, all moments are calculated based on band-pass filtered data on a 6-32 quarter window.

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<th>Table 1: Selected Moments</th>
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<tr>
<td>Standard RBC TFP Discount</td>
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<tr>
<td>corr(GDP / L, GDP)</td>
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<tr>
<td>var(Sales)/var(GDP)</td>
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<tr>
<td>var(I)/var(GDP)</td>
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<tr>
<td>var(GDP^{HF})/var(GDP^{LF})</td>
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The first two statistics reflect the main facts from the previous literature that a shift in the composition of shocks away from technology towards preferences is able to capture: the reduction in the correlation between labor productivity and output and the increase in the volatility of sales relative to that of output. The intuition for the correlation between output and labor productivity is straightforward and can be gleaned from a standard RBC model.
Productivity shocks naturally increase both labor productivity and labor supply, since wages go up. On the other hand, a shock that makes households more impatient will lead them to reduce their labor supply since they will choose to enjoy leisure in the present rather than work to accumulate assets for future consumption. Since the capital stock does not change over the short run, a reduction in labor supply increases the marginal product of labor. The intuition goes through once we allow for intersectoral linkages and time to produce.

In order to discuss inventory dynamics we must by necessity look at the results with time to produce, since with instantaneous production there are no inventories. It is immediate that with shocks to discount rates, the volatility of sales is much higher relative to that of output, whereas with productivity shocks they are roughly the same. The reason is that shocks to the discount rate generate instability in sales. At first, sales overshoot as households choose to consume more. However, households also decide to produce less, which lowers output. Thus, over a longer period, sales end up declining. Output is much smoother since the early desire for greater sales can be satisfied by reallocating resources towards finishing older production lines rather than starting new ones.

The following two lines show results which have not been emphasized in the literature so far. The first is that in the post-1984 period, the volatility of investment has increased relative to that of output. As it turns out, compared to productivity shocks, discount rate shocks imply a higher volatility of investment relative to that of output. The intuition is straightforward and can be derived immediately from a standard RBC model. When it comes to productivity shocks, investment is more volatile than output because the capital stock is durable. Small changes in the stock require large changes in the flow of investment. When it comes to discount rate shocks, an additional mechanism is present. Discount rates affect exactly the desire for saving. If agents discount the future more heavily, they will want to invest less.

The last line compares the volatility of output at high frequencies (2-3 quarters) with the volatility at business cycle frequencies (6-32 quarters). It shows that after 1984, volatilities at high frequencies dropped by a greater amount than volatilities at business cycle frequencies. This, again, is consistent with a shift towards discount rate shocks so long as the model has time to produce. The reason is that over the short run much of current production is complementary to production decisions made before the economy was hit by the shock and thus will react less. While this will tend to make reactions to shocks more hump-shaped regardless of the shock, the effect is less prominent with the productivity shock, since productivity has a direct impact on output over and above any changes in factor allocation.