Price Differentiation and Menu Costs in Credit Card Payments

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Abstract: We build a model of credit card payments where the retailers are allowed to charge differential prices depending on the instrument of payment chosen by the consumer. We follow the approach in Rochet and Wright (2010) but assuming a credit card system without any type of no-surcharge rule. In a Hotelling competition framework, the competitive equilibrium prices are computed assuming that the store credit provided by the retailer is less cost efficient than the one provided by the credit card. In accordance with the literature, we obtain that the interchange fee became neutral if we eliminate the no-surcharge rule, when the interchange fee loses its ability to distort the individual consumers’ decisions and displace the aggregated consumers’ welfare from its maximum level. We prove that the average price obtained under price differentiation is smaller than the unique retail price under no-surcharge rule, despite the retailer’s margins are the same in both scenarios. In addition, we introduce menu costs to prove that there is a value for the interchange fee such that there is equilibrium with price differentiation if and only if that fee is above this value. It must be interpreted as the existence of an endogenous cap for the interchange fee fixed by the credit card industry. Finally, we also obtain that under price differentiation with menu costs there is a non cooperative (Nash) equilibrium as in the well known “prisoner dilemma” game.

Keywords: Credit cards, Payments, Two-sided markets.

JEL Classification: L11; E42; G18.

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1. Introduction

There is an intensive international debate involving industry members, regulators and representatives of the society about the structure of the credit card market, as well as, the behavior of its players, and the consequences on competitiveness and, most importantly, on social welfare. Actually, the social welfare maximization should be the ultimate goal of any regulator, a difficult challenge that encompasses the assessment of distributive aspects, like defining weights on welfare for each segment of the society.

Among the most instigating issues in this debate is the one about the effects on consumers’ welfare of the presence, or absence, of no-surcharge rules in the credit card payments. A distinctive feature of card payment system is that, despite the cardholders make the choice of payment instrument, the significant part of the transaction cost is incurred by the merchant. In practice, the fee structure has triggered the use of merchant fees to reward the issuance (interchange fee) and the usage of cards (card rewards), which is a typical behavior in two-side markets structures. The central question is if the credit card industry can exert market power imposing scheme rules prohibiting surcharging of credit card purchases by merchants. In other words, if no-surcharge rules could prevent price signaling to cardholders about the relative costs of different payment methods, reinforcing the pattern where the higher the merchant fee, the greater is the capacity to reward cardholders, leading to a less efficient allocation of resources in the payment system (“excess” of card usage).

Other important issue is that, under no-surcharge rule, merchants recover the average cost of different instruments of payment charging all consumers equally. Consequently, consumers who do not use credit cards pay more that they would otherwise, by the other side, consumers who use credit cards are subsidized in their purchases. There are empirical studies that measure those subsidies in some jurisdictions, in general indicating that cross subsidies are not negligible.

4 Schuh, S., Stavins, J., (2010) and Central Bank of Brazil (2011a) and (2011b) are examples of empirical studies that estimate cross subsidies in United States and Brazil, respectively.
Because of its anti-competitive nature, the no-surcharge rule has been prohibited in some jurisdictions, for instance, in United Kingdom since 1991, in Netherlands since 1994, in Sweden since 1995 and in Australia since 2003. The authorities judged that merchant pricing freedom is essential for effective price competition and competition between payment systems.

In Australia, for instance, the prohibition on no-surcharge rules is stated in the Standards as “Neither the rules of the Scheme nor any participant in the Scheme shall prohibit a merchant from charging a credit cardholder any fee or surcharge for a credit card transaction”. However, despite they still recognize that the surcharging reforms in Australia have been successful and have provided significant public benefit, they become concerned about cases where surcharges seem to be higher than the acceptance costs. Since the evidences obtained are that in some instances surcharging has developed in a way that potentially compromises price signals and reduces the effectiveness of the reforms, they are currently reviewing the no-surcharge standards in order to provide card schemes with the ability to constraint the level of surcharges to something close to the merchant acceptance costs, whose main component is the merchant fees\(^5\).

In those discussions, it is not uncommon to find arguments in favor or against the price differentiation which are not always connected with the economic welfare. For instance, when analyzing the convenience of eliminating the no-surcharge rule for credit card payments, it is quite reasonable to affirm (as we prove in this paper) that some merchants will have incentives to surcharge unilaterally credit cards transaction above the unique price level, without any reduction on prices of other types of transactions. Notwithstanding, it is far from being a valid argument against differentiation, because this assertion, does not take into account the existence of new equilibrium prices in the absence of the no-surcharge rule and its effects on consumers’ welfare with the scenario of no-surcharge rule.

\(^5\) Consultation documents of Reserve Bank of Australia (2011a) and (2011b).
As we will illustrate in the next section, through our theoretical analysis of a simple model, the fact that each merchant has the possibility to obtain a profit increase, when he individually deviates from the unique price, is not a guarantee that this extra profit is sustainable. Actually, a complete and coherent analysis needs, first of all, to find the new equilibrium prices, which will depend on the specific competitive environment and its effects on profit possibilities, in order to measure the welfare gains, or losses, comparing both equilibria.

The main reference of our work is Rochet and Wright (2010). They model the credit card explicitly, allowing a separated role for the credit functionality of credit cards, which is modeled apart from other payment cards (i.e., debit cards). They assume impossibility (or lack of incentives) of retailers to differentiate prices accordingly to the instrument of payment chosen by consumers. Under those assumptions, they showed how a monopoly card network could select an interchange fee higher enough to promote the utilization of credit cards in a level that exceeds the level that maximizes the aggregated consumer surplus. They show how a regulatory cap for the interchange fee could be used to increase consumer surplus.

Our work aims at the extent of the Rochet and Wright model, to give a distinctive subsidy to the debate, helping to clarifying, through a simple theoretical model, the implications on consumers’ welfare of price differentiation and of menu costs faced by merchants in credit card payments. With that extension we are able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable to improve consumers’ welfare, reducing the market power of banks through the interchange fee\(^6\), even under the assumption that retailers face menu costs associated to price differentiation. In this case, we prove that retailers will differentiate prices as long as the menu costs are not high enough.

The paper is divided into four sections. Section 2 describes the model, which is similar to that defined in Rochet and Wright (2010). In Section 3, we present the main

\(^{6}\) See Gans and King (2003) about the neutrality of the interchange fee under price differentiation.
results of the paper, first considering the absence of frictions given by the menu costs of price differentiation and secondly including such costs to analyze the effects of that market imperfection. In section 5, we summarize the main conclusions. The Appendix contains the detailed proofs of all the propositions presented in the paper.

2. The model

It is introduced two distinctive changes in the model proposed by Rochet and Wright (2010). In a first version of our model we only allow retailers to differentiate price of credit card payments from the price of the other payment instruments (store credit, cash, debit cards and others). The second specification introduces menu costs in the former version of the model.

As in Rochet and Wright (2010), we assume here that there is a continuum of consumers, all distributed uniformly in an unitary length interval. All consumers are identical with quasi-linear preferences, spending their income on retail goods costing $\gamma$ to produce. There are two payment technologies. The first one corresponds to a group of “cash” payment technologies, which could include money, checks, debit cards or other instruments not involving any credit functionality. The second one corresponds, exclusively, to the credit cards’ payment technology. As an alternative to both technologies, each retailer can directly provide credit to the consumer, which is called “store credit”. Credit cards are held by a constant fraction $x$ of consumers and assumed to be more costly than cash, and both costs are normalized assuming cash has zero cost. Credit cards allow consumers to purchase on credit and entail a cost (or benefit, if negative) $f$ for the consumer (buyer), which is received (or paid) by the issuer, and a cost $m$ (merchant fee) for the retailer (seller). The store credit is an alternative for the credit function of the credit card and entails a random and transaction specific cost (or benefit, if negative) $c_B$ for the consumer and cost $c_S$ to the retailer.
Each consumer purchases one unit of the retail good, called “ordinary purchases”, providing him utility $u_0 > \gamma$, but, in addition, and with probability $\theta$, he also receives utility $u_1 > \gamma$ from consuming another unit of the retail good called “credit purchases”. It is assumed that merchants cannot bundle the two transactions and also cannot distinguish between “ordinary” and “credit” purchases.

When making ordinary purchases, all consumers can choose between cash or store credit, but only a fraction $x$ of them have possibility to choose credit cards. By the other side, when making a credit purchase, cash is not an option for any consumer. Additionally, it is assumed that each consumer always has sufficient cash to pay for his ordinary purchases, but must rely on credit for credit purchases.

The transaction specific cost $c_B$ of a store credit is observed by the consumer only when he is in the store, which is drawn from a continuous distribution with the cumulative distribution function $H$. We assume the distribution has full support over some range $(c_B, \overline{c_B})$, where $c_B$ is sufficiently negative, such that cardholders will sometimes choose to use store credit even if cash can be used instead, and $\overline{c_B}$ is positive but not too high (in comparison with $u_1 - \gamma$), such that consumers will always prefer to make the credit purchase even if they have to pay with store credit rather than not buy at all. The draw $c_B$ is the net cost of using store credit rather than credit cards or cash. A negative draw of $c_B$ could represent a situation whereby a cardholder needs to preserve his cash or credit card balance for some other contingencies and so values the use of store credit.

If the merchant fee $m$ of credit cards purchase is smaller than the cost of store credit $c_S$ ($m < c_S$), accepting credit cards is a potential mean for merchants to reduce their transaction costs of accepting credit purchases. But if $m > c_S$, to accept credit cards increase merchant’s transaction costs.
In general, consumers will prefer credit cards rather than store credit when \( c_B > f \), for both ordinary and credit purchases. In particular, when issuers give benefits (\( f < 0 \), i.e., cash back bonuses) to consumers in each credit card purchase, consumers will prefer to use their credit cards rather than cash for ordinary purchases. It was proved in Rochet and Wright (2010) that, from the point of view of aggregated consumers, excessive incentives for credit card use could be socially wasteful.

The bank of the merchant, or acquirer of the transaction, incurs in an acquiring cost \( c_A \) and an interchange fee \( a \) (which is paid to the bank of the consumer) for each credit card transaction. It is assumed that acquirers are perfectly competitive, which implies that the merchant fee \( m \) is equal to the sum of the acquiring cost \( c_A \) and the interchange fee \( a \),

\[
m = c_A + a
\]  

\hspace{1cm} (1)

The bank of the cardholder, or issuer of the card, incurs in an issuing cost \( c_I \) and receive the interchange fee \( a \) from the acquirer. It is assume that issuers are imperfectly competitive, which implies that the cardholder fee \( f \) is equal to the net issuer cost \( c_I \) plus a constant profit margin \( \pi \),

\[
f = c_I - a + \pi
\]  

\hspace{1cm} (2)

Thus, the total cost of a credit card transaction is

\[
c = c_A + c_I
\]  

\hspace{1cm} (3)

Denote by \( \delta \) the excess cost of the store credit with respect to the total cost credit card transaction, which is define by
We will restrict our analysis to the situation where, from the point of view of the suppliers of the credits (merchants or credit card industry), a credit card transaction is more costly efficient than the store credit, or, equivalently, \( \delta > 0 \).

Competition between retailers occurs as in the standard Hotelling model: consumers are uniformly distributed on an interval of unitary length, with one retailer \((i = 1, 2)\) located at each extremity of the interval. There is a transport cost \( t \) for consumers per unit of distance. Unlike Rochet and Wright (2010) we are interested here in the situation where retailers have the option to charge different retail prices according to the instruments of payment. For simplify, we restrict to the particular situation where it is allowed the retailer to charge a price \( p^c \) for a credit card transactions which may be different from the price \( p' \) charged for a cash or store credit transaction. We denote by \( \Delta^c \) the spread between these two prices, namely:

\[
\Delta^c := p^c - p'
\]

Figure 1 – Prices, costs and fees of instruments of payment

\( \gamma \) Merchant \( c_s \) \( p^r \) Consumer \( c_B \) random with c.d.f. \( H \)

\( m = c_A + a \) Acquirer \( c_A \) \( a \)

\( f = c_I + \pi - a \) Issuer \( c_I \)
The Figure 1 illustrates the interconnection between participants of the credit card market, as well as, the respective prices, costs and fees charged by each one.

The timing of the decisions is as in the model of Rochet and Wright (2010), which can be divided in 9 steps, grouped in two periods: 5 steps before the arriving of consumer to the store and 4 steps once the consumer is in the store.

Before arriving to the store:
1. The card network sets the interchange fee \( a \);
2. Banks set their fees: \( f \) for cardholders and \( m \) for retailers;
3. Retailers independently choose their card acceptance policies: \( L_i = 1 \) if retailer \( i \) accepts credit cards, 0 otherwise;
4. Retailers independently set retail prices \( p_i' \) and \( p_i = p_i' + \Delta_i \);
5. Consumers select one retailer to patronize, after observing the observed retail prices, retail’s acceptance policies, issuer’s fee, the distribution of store credit cost and transport cost.

Once the consumer is in the store:
6. Consumer buys a first unit of the retail good (‘‘ordinary purchase’’), and pays it using cash or credit card (if he has one);
7. Nature decides whether consumer has an opportunity for an additional credit purchase, which will occur with probability \( \theta \);
8. The cost \( c_B \) of using store credit for the buyer is drawn according to the c.d.f. \( H \), with full support on \((c_B,\bar{c}_B)\);
9. Cardholders then select their mode of payment. We set \( L_i^c = 1 \) if the consumer prefers credit card rather than cash when buying at the retailer \( i \), or 0 otherwise, in other words:

\[
L_i^c := \begin{cases} 1 & \text{if } f + \Delta_i^c \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)
\]
3. Analysis and results

Assuming the merchant charge a price \( p_i' \) for cash and store credit, and charge an additional spread \( \Delta_i \) specifically on credit card transactions, we obtain (see Appendix) that the expected margin of the retailer \( i \) is given by

\[
M_i = (1 + \theta)(p_i' - \gamma) - (H(0) + \theta)c_s - xL_i' \overline{\Gamma}(a, \Delta_i) \tag{7}
\]

where

\[
\overline{\Gamma}(a, \Delta_i) := [1 - H(0)]L_i', c_s + [1 - H(f + \Delta_i)](L_i' + \theta)(m - c_s - \Delta_i) \tag{8}
\]

The first two terms at the right hand side of (7) corresponds to the expected revenue of the retailer \( i \), net of the cost of the products and net of the cost of the instruments of payment, if there is no credit card users (\( x = 0 \)) or the retailer decide not to accept credit cards (\( L_i' = 0 \)). The third term corresponds to the expected margin reduction associated to the use of credit cards.

We obtain (see Appendix) that the utility of the consumer that choose to purchase the good from the retailer \( i \) is given by

\[
U_i = u_o + \theta.u_i - (1 + \theta).p_i' - \int_{c_B}^0 c_B.dH(c_B) - \theta.E(c_B) + xL_i'.\overline{S}(a, \Delta_i) \tag{9}
\]

where

\[
\overline{S}(a, \Delta_i) := (L_i' + \theta)\left(\int_{f + \Delta_i}^a (c_B - f - \Delta_i).dH(c_B)\right) - L_i' \int_0^a c_B.dH(c_B) \tag{10}
\]

The first five terms at the right side of (9) Corresponds to the expected utility of the consumption, net of the cost of the products and net the cost of the store credit, when there are no credit card users or the retailer decides not to accept credit cards. The sixth
term corresponds to the additional welfare associated exclusively to the use of credit cards.

The market share of both retailers is determined computing the position of the indifferent consumer in the region (interval of size one) where all consumers are uniformly distributed. Since there is a cost $t$ for unit of displacement, the welfare, net of the displacement cost, of a consumer who decides to purchase the good retailer $i$ is $U_i - s_it$. Therefore, the distance $s_i$ between the indifferent consumer and the retailer $i$ is equal to the proportion of consumers choosing retailer $i$. Figure 2 shows net utilities and the market shares of both retailers.

**Figure 2 – Indifferent consumer in the Hotelling model**

\[
U_1 - s_1t = U_2 - s_2t
\]

The market share $s_i$ of the retailer $i$ depends on the interchange fee, prices and spreads, and is given by the following expression

\[
s_i = \frac{1}{2} + (1 + \Theta) \left( \frac{p_j - p_i}{2t} \right) + x \left( \frac{L_j^i S(a, \Delta_j) - L_j^i S(a, \Delta_j)}{2t} \right)
\]  \hspace{1cm} (11)

Note that, for a fixed interchange fee $a$, the (Nash) equilibrium price $\bar{p}$ when differentiation is not allowed, as defined by equation (5) in Rochet and Wright (2010), satisfies the following equation
\[(1 + \theta) \bar{p} = t + (1 + \theta) \gamma + (H(0) + \theta) c_s + \frac{x}{3} \left( L_j - L_i \right) \bar{\phi}(a, 0) + x L_j \bar{\Gamma}(a, 0) \tag{12}\]

where

\[\bar{\phi}(a, \Delta_i) := \bar{S}(a, \Delta_i) - \bar{\Gamma}(a, \Delta_i) \tag{13}\]

is the difference between the additional welfare of the consumer and additional cost of the retailer associated to each credit card transaction.

For each \( \delta > 0 \), as defined in (4), consider the following parameter definition

\[\bar{\phi}_\delta := (1 + \theta) \left( \int_{-\delta}^{\delta} (c_B + \delta) \, dH(c_B) \right) - \int_{0}^{\infty} (c_B + c_S) \, dH(c_B) \tag{14}\]

and note that \( \bar{\phi}_\delta = \bar{\phi}(a, a + c_A - c_S) \).

Assuming \( \bar{\phi}_\delta > 0 \) means that, if the spread is equal to \( m - c_S \ (= a + c_A - c_S) \), the benefit of a credit card transaction for consumers is bigger than the cost of the same transaction for the retailers. Note that if \( \delta > 0 \) is sufficiently small then the assumption \( \bar{\phi}_\delta > 0 \) is equivalent to

\[\theta \left( \int_{0}^{\infty} (c_B + c_S) \, dH(c_B) \right) > (1 + \theta) \int_{0}^{\infty} (c + \pi) \, dH(c_B) \tag{15}\]

where the left hand-side term above corresponds to the economy of costs in using credit cards for extraordinary purchases and right hand-side term corresponds to the cost of using credit cards in both types of purchases.
The results in this paper are obtained under the three basic assumptions below. The first one relaxes the non-surcharge rule allowing retailers to charge different prices for credit card transactions, and represents the main assumption.

**Assumption 1:** Price differentiation of credit cards transactions is allowed.

The second assumption is related with the cost efficiency of the credit card industry compared with the store credit instrument.

**Assumption 2:** The parameter $\delta$, defined by (4), is strictly positive.

The Assumption 2 above means that from the point of view of the lenders, the total bank costs and profits of credit card transactions $c_A + c_I + \pi$ is smaller than the retailer’s cost of providing store credit $c_S$. In this specific sense, the credit cards’ industry is more cost efficient than retailers in providing credit.

As demonstrated in Rochet and Wright (2010), under no-surcharge rule, if the consumer’s benefit of a credit card transaction is equal to the economy of costs of generating credit through credit card transaction instead of store credit ($f = \delta$), consumers obtain the maximum aggregated welfare. They proved that any other level of credit cards’ costs/benefits $f$ (which depends on the interchange fee, since $f = c_I + \pi - a$) will generate a loss in the consumers’ welfare. In other words, in spite of consumers are deciding individually their instruments of payments in an optimal way, from the aggregated point of view, those decisions generate an inefficient level of credit card usage, if compared with the optimal situation when $f = \delta$.

The third assumption has a more sophisticated interpretation, and it essentially imposes restrictions on the retailers’ average cost and the consumers’ average benefits of a credit card transaction.
**Assumption 3:** The parameter $\bar{\phi}_d$, defined in (14), is strictly positive.

The Assumption 3 above is equivalent to impose that, when spread charged by both merchants is equal to $m - c_5$, the consumers’ average benefit from credit card transactions is bigger than the retailers’ average cost from the same credit card transactions. Note that, if retailers recover those costs thought the average price paid by consumers, Assumption 3 implies that consumers have positive average benefit in using credit cards.

### 3.1. Equilibrium prices under price differentiation

In this subsection we provide some propositions that allow us to analyze the impacts of ruling out the no-surcharge rules in the credit card systems. All the propositions are obtained under Assumptions 1, 2 and 3.

**Proposition 1:** If both retailers charge the unique price $\bar{p}$, as defined in equation (12), and the merchant fee is bigger than the cost of the store credit ($m > c_5$), no matter the payment instrument, and differentiation is allowed, retailers have incentives impose a surcharge over the unique price.

**Proof:** See Appendix 1.

An immediate consequence of Proposition 1 above is that the unique price $\bar{p}$ charged for every instrument of payment is not (Nash) equilibrium price under price differentiation.

The proof of Proposition 1 uses the fact that the profit function is strictly increasing in the price spread, meaning that it is desirable for the merchant to surcharge credit cards transactions above $\bar{p}$. However, it is worth noting that this result is only a
static comparative assessment, whose utility is exclusively to prove that the unique price strategy is no longer an equilibrium under price differentiation assumption.

Any policy assessment needs to address more relevant questions like: is there a competitive equilibrium with that price differentiation characteristic? and if the response is positive, how is the consumer welfare in that equilibrium if compared with that of a unique price? The results in the following proposition help us to clarify those questions.

Proposition 2: For each interchange fee $a$ defined by the banks, there exist a pair of prices $(\bar{p}^r, \bar{p}^c)$, the price charged for cash/store credit transactions and the one charged exclusively for credit card transactions, and a neighborhood of it, where this pair is a Nash equilibrium in that neighborhood. Namely, if both retailers are charging those prices, none of them, has incentives to deviate from those prices. The prices are given by

$$\bar{p}^r = \gamma + \frac{t + [H(0) + x(1 - H(0)) + \theta]c_s}{(1 + \theta)} \quad (16)$$

and

$$\bar{p}^c = \bar{p}^r + m - c_s \quad (17)$$

Proof: See Appendix.

Note that the price $\bar{p}^r$ does not depend on interchange fee $a$, actually, only the price of credit cards transactions $\bar{p}^c$ depends on it.

From equations (17) and (1), we obtain that the equilibrium spread is given by

$$\Delta^c = c_A - a - c_S \quad (18)$$
and, consequently, using (18) and (2), we conclude that $f + \overline{X} = \delta$. In other words, the interchange fee losses its capability of affecting the consumers’ net benefit of a credit card transaction ($f + \overline{X}$), under differentiation, which becomes constant and equal to the optimal benefit value ($\delta$) under no-surcharge rule. This is a remarkable difference with respect to the Rochet and Wright (2010) findings.

Note that the average equilibrium price is

$$\overline{p}^m = (1 - \alpha_\Delta) \cdot \overline{p}' + \alpha_\Delta \cdot \overline{p}'^c \quad (19)$$

where $\alpha_\Delta := x \cdot [1 - H(f + \overline{X})]$ is the proportion of card owners that, under price differentiation, prefer to use credit card than store credit or cash.

**Proposition 3:** The price $\overline{p}$ is a convex combination of the prices $\overline{p}'$ and $\overline{p}'^c$. More specifically,

$$\overline{p} = (1 - \alpha_0) \cdot \overline{p}' + \alpha_0 \cdot \overline{p}'^c \quad (20)$$

where $\alpha_0 := x \cdot [1 - H(f)]$ corresponds to the proportion of credit card owners that prefer credit cards than any other instruments under no-surcharge rule.

**Proof:** See appendix.

An important and immediate consequence of Proposition 3 is that, when $\overline{X} > 0$, we have $\alpha_\Delta = x \cdot [1 - H(f + \overline{X})] < x \cdot [1 - H(f)] = \alpha_0$, and using (19) and (20) we obtain that the average price $\overline{p}^m$ under price differentiation is smaller than the unique price $\overline{p}$ under no-surcharge rule.
Figure 3 below illustrates how the price $\bar{p}$ can be decomposed in order to turn explicit the subsidy components that are eliminated in the new scenario with differentiation.

**Figure 3 – Equilibrium prices decomposition**

1) Cash:

$$\bar{p} = \gamma + \frac{t}{1 + \theta} + \left(1 - \frac{(1 - x).[1 - H(0)]}{1 + \theta}\right)c_s + x[1 - H(f)](m - c_s)$$

2) Store credit:

$$\bar{p} = \gamma + \frac{t}{1 + \theta} + c_s - \frac{(1 - x).[1 - H(0)]}{1 + \theta}c_s + x[1 - H(f)](m - c_s)$$

3) Credit card:

$$\bar{p} = \gamma + \frac{t}{1 + \theta} + m - \frac{(1 - x).[1 - H(0)]}{1 + \theta}c_s - x[1 - H(f)](m - c_s)$$

Note that part of the total subsidy is eliminated with the price differentiation of credit cards transactions, but one component of subsidy prevails. The remaining component corresponds to the one associated with the group of consumers that do not have credit cards (fixed proportion $1 - x$) and, at the same time do not see any benefit in using store credit, thus they use cash. This particular group of consumers pays a subsidy to the other consumers. The subsidy occurs because they have fewer options of payment instruments and, as a consequence, less market power than the others.

Notice that the remaining subsidy from cash users to the others is eliminated if we suppose that every consumer has a credit card ($x = 1$). However, since we are not allowing for price differentiation between cash and store credit transactions, a subsidy
between them persists. This is because the price $\bar{p}$, as an average price, does not reflect the different costs of each instrument (cash has zero cost and store credit has cost $c_s$).

The following proposition shows that there is the positive impact on consumers’ welfare as a consequence of the elimination of barriers to price differentiation of credit card transactions.

**Proposition 4:** The consumers’ welfare (aggregate utility) in the equilibrium under price differentiation is equal to that under the no-surcharge rule only if the interchange rate $a$ is equal to $c_s - c_A$, and bigger for all other values.

**Proof:** See appendix.

With respect to the merchants’ profits the following proposition shows that retailers are indifferent with respect to the no-surcharge rule.

**Proposition 5:** The equilibrium merchant profit is the same under price differentiation and under the no-surcharge rule.

**Proof:** See appendix.

In order to analyze the effect of the interchange fee $a$, fixed by banks, on the incentives of retailers to deviate for a unique price, it can be useful to compute the profit of a retailer resulting from this deviation despite the fact that the other retailer remains charging the differentiated prices $\bar{p}'$ and $\bar{p}^r$. The Figure 4 illustrates the situation when Retailer 1 moves unilaterally to the unique price strategy, whilst the Retailer 2 stays charging two prices.
The following proposition shows that the merchant has no incentive to decide, individually, to charge the unique price $\bar{p}$. In the next subsection (Proposition 7), the same type of analysis is revisited, but in a context of existence of menu costs when considering the possibility of differentiate prices.

**Proposition 6:** Suppose that both retailers are initially charging differentiated prices $\bar{p}'$ and $\bar{p}^c$, with a positive spread $\bar{\Delta}^c$, if one of them decides to charge the unique price $\bar{p}$, then the profit of this retailer will decrease and the size of the reduction $\varepsilon(a)$ is given by

$$
\varepsilon(a) = \frac{x.(1+\theta)}{2}\int_{\delta-c_{B}-c_{A}-a}^{\delta}(-\delta-c_{B})dH(c_{B})
$$

(21)

which depends on the value of the interchange fee $a$.

**Proof:** See appendix.

A consequence of Propositions 5 and 6 is that, despite both retailers having the same profit under price differentiation and under a unique price, no one have, without cooperation, incentives to move individually the unique price.
The Figure 5 below illustrates the variations on profits under price differentiation assumption when retailers change from differential prices equilibrium to unique price equilibrium. The gray cell indicates, respectively, the profits of Retailer 1 and Retailer 2, which are equal to $t/2$.

**Figure 5 – Profits under price differentiation assumption**

<table>
<thead>
<tr>
<th>Retailers’ profits under price differentiation</th>
<th>Retailer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>two prices $p'$ and $p^c$</td>
</tr>
<tr>
<td>Retailer 1</td>
<td>$t/2$ ; $t/2$</td>
</tr>
<tr>
<td>unique price $p$</td>
<td>$t/2 - \varepsilon(a)$ ; $t/2 + \varepsilon(a)$</td>
</tr>
</tbody>
</table>

In Figure 5, the cell below the gray one indicates that, if Retailer 1 decides, individually, to charge the unique price, his profit reduce from $t/2$ to $t/2 - \varepsilon(a)$. The same occurs with Retailer 2, as indicated in the cell at the right side of the gray one. The cell at the opposed side of the diagonal indicates the fact, if both decides to charge the unique price $p$, both profits are the same and equal to $t/2$. We can observe here that the unique price decision, actually, is not an equilibrium.

### 3.2. Menu costs

The last result of this work analyzes the effects on the model resulting from the introduction of menu costs associated to the price differentiation. It is not uncommon the situation where a retailer faces costs associated with the strategy of differentiate prices of credit cards’ transactions. As usual menu cost may be related to the cost of market research in order to define correctly the price for each instrument of payment. In addition, retailer can realize that significant part of their clients prefer to pay with credit cards and do not feel comfortable with the idea of paying more than other clients, situation that could cause frictions in the communication with those clients, which can be interpreted as
a cost. Other example, the retailer can be subject to a penalty due to price differentiation in an environment of no-surcharge contract or regulation.

Suppose that the margin of the Retailer $i$ is given by

$$M_i^m := (1 + \theta)(p_i^r - \gamma) - (H(0) + \theta)c_s - xL_i^sI(\Delta_i^r) - \mu_iI(\Delta_i^r)$$

where $I(\Delta_i^r) := \begin{cases} 0 & \text{if } \Delta_i^r = 0 \\ 1 & \text{if } \Delta_i^r \neq 0 \end{cases}$

**Proposition 7:** Suppose that retailers face menu costs $\mu_1$ and $\mu_2$ respectively per transaction, where both margins are given by (22). If both retailers are charging differential prices $\bar{p}^r$ and $\bar{p}^c$, none of them, independently, has incentives to deviate (locally) from those prices. Additionally, if both menu costs are smaller than $\varepsilon(a)$, none of them, independently, has incentives to charge the unique price $\bar{p}$.

**Proof:** See appendix.

The Figure 6 below shows the profits of both retailers depending on their individual decisions with respect to their pricing strategies, in the case we assume the existence of menu costs.

**Figure 6 – Profits under price differentiation and menu costs assumptions**

<table>
<thead>
<tr>
<th>Retailers' profits under price differentiation and menu costs</th>
<th>Retailer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>two prices $p^r$ and $p^c$</td>
</tr>
<tr>
<td>Retailer 1</td>
<td>$t/2 - \mu_1$ ; $t/2 - \mu_2$</td>
</tr>
<tr>
<td></td>
<td>$t/2 - \varepsilon(a)$ ; $t/2 - \mu_2 + \varepsilon(a)$</td>
</tr>
</tbody>
</table>
Since the function \( \epsilon(a) \) is a increasing function of \( a \), we can define

\[
\tilde{a} := \min\{\epsilon^{-1}(\mu_1); \epsilon^{-1}(\mu_2)\}
\]

(23)

**Proposition 8:** If the interchange fee \( a \) is bigger than \( \tilde{a} \), the unique price \( \bar{p} \) is not an equilibrium price, since at least one of the retailers has incentives to charge, independently, the differential prices \( p' \) and \( p'' \).

**Proof:** See appendix.

In other words, the Proposition 7 asserts that in the absence of a no-surcharge rule and in the presence of menu costs, if banks decide to set a sufficiently high interchange fee \( (a > \tilde{a}) \), some retailers will have incentives to deviate from the unique price strategy, deciding to charge differential prices in order to increase profits.

In particular, suppose that the threshold \( \tilde{a} \), as defined in Proposition 7, is smaller than the interchange fee \( \bar{a} \), which is the maximizing profit decision of banks under no-surcharge rule, as obtained in Rochet and Wright (2010). In this situation, the possibility and desire to differentiate prices from part of retailers can destabilize the equilibrium price \( \bar{p} \), forcing banks to set a interchange fee at \( \tilde{a} \), reducing their market power and, consequently, improving consumers’ average welfare.

**4. Conclusions**

In this paper, we adapt the framework of Rochet and Wright (2010), to the absence of no-surcharge rules for prices of credit cards’ transactions. In this setting we prove that the equilibrium prices for the purchases using credit cards and using cash or store credit are not the same. In particular, we obtain that the equilibrium surcharge
spread is the difference between the merchant fee and the retailers’ cost of providing the store credit.

The result about the equilibrium price spread is remarkable, especially in jurisdictions where the debate agenda is the necessity of defining a merchant’s surcharge cap (as in Australia). Our results assert that the surcharge cap should not exceed the competitive equilibrium price spread that we found, namely, that surcharge cap must be lower or equal to the difference between the merchant fee and the store credit cost faced by merchants. In particular, this result implies that only if the cost of store credit is equal, or greater, than the merchant fee, the no-surcharge rule should be acceptable from the point of view of who seeks to preserve consumers’ welfare.

Initially, we prove that the unique price is not an equilibrium, which is a consequence of each retailer be willing to surcharging unilaterally the credit card payments and deviate from the unique price equilibrium stated by the no-surcharge rules.

The result given above leads us to the following question: if price differentiation is allowed, might it provide some degree of market power to the merchants such that they will keep a surcharge on credit card transaction with an average price of all transactions greater than the price found under the no-surcharge rule? The answer is definitely no. In order to show that, we computed the new equilibrium prices when price differentiation of credit card payments is allowed, and proved that the average price is smaller than the unique price under no-surcharge rule. Moreover, the new consumers’ average welfare in the price differentiation equilibrium is, in general, greater than the corresponding in the unique price equilibrium, the equality takes place only if the interchange fee is at the level that maximizes the consumers surplus under the no-surcharge rule framework.

We also obtain that the merchants’ profits under price differentiation are equal to those under unique price equilibrium. This result brought the question if one retailer would have incentives to deviate independently towards the unique price. We found that
it does not happen, in fact, we proved that none of the retailers has incentives to deviate individually from the equilibrium with price differentiation to the unique price.

In the last exercise we introduce menu costs to analyze if this sort of friction may inhibit retailers’ incentives to differentiate prices. In this new context, it is simple to conclude that the strategy of a single price turns out to be equilibrium, which is independent on the existence of a no-surcharge rule. In fact, if both retailers charge differential prices, their, their profits will fall below the attained levels under a single price. Furthermore, we obtained that, for interchange fee values large enough, differential prices remains as an equilibrium of the type of the “prisoner dilemma” game. Actually, we proved that if both retailers are charging differential prices none of them has incentives to change unilaterally to the unique price strategy. The intuition behind is the following, if only one merchant charges the unique price, he loses market share and, consequently, reduce his profit. If this reduction of profit is bigger than the economy of menu costs, there will be a net cost associated to the decision of moving toward the unique price strategy.

In summary, by using a simple model of credit card market we were able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable to improve consumers’ welfare. This may reduce the market power that banks have by using the interchange fee, even under the presence of menu costs associated to price differentiation that the retailers may face.

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REFERENCES


APPENDIX

To prove that, under price differentiation, the unique price $\bar{p}$ is not an equilibrium and, in addition, to find the new equilibrium prices, we first derive the merchant margin, the consumer utility and merchant market share, and use them to find the profit function. Subsequently, we compute the derivatives of the profit function with respect to the base price $p'_i$ and the spread $\Delta'_i$.

The Figure 7 below shows a decomposition of the merchant’s expected margin into several components. The first group of components corresponds to the margin obtained by merchants from consumers that cannot use credit cards, either by not having, or because the retailer did not adhere to the credit card system. The second group of components corresponds to the retailer margins from the consumers who have a credit card conditioned to his adherence to the credit card system.

**Figure A1 – Merchant’s expected margin**

\[
M_i = (1 - x_i \Delta'_i) \left[ H(0) \left( p'_i - \gamma - c_s \right) + (1 - H(0)) \left( p'_i - \gamma \right) \right] + \theta (p'_i - \gamma - c_s)
\]

\[
M_i = (1 - x_i \Delta'_i) \left[ H \left( \Delta'_i (f + \Delta'_i) \right) \left( p'_i - \gamma - c_s \right) + (1 - H \left( \Delta'_i (f + \Delta'_i) \right)) \left( p'_i - \gamma + \Delta'_i - m \right) \right] + \theta \left[ H \left( f + \Delta'_i \right) \left( p'_i - \gamma - c_s \right) + (1 - H \left( f + \Delta'_i \right)) \left( p'_i - \gamma + \Delta'_i - m \right) \right]
\]

The formula in Figure A1 can be simplified to derive equation (7). Then the partial derivatives with respect to the base price and the spread are given by
\[
\frac{\partial M_i}{\partial p_i'} = (1 + \theta)
\]

and
\[
\frac{\partial M_i}{\partial \Delta_i'} = x_i \lambda_i' (L_i' + \theta) \left[ h(f + \Delta_i')(m - c_i - \Delta_i') + \left[ 1 - H(f + \Delta_i') \right] \right]
\]

Consumers decide which retailer to patronize computing its utility and subtracting the transportation costs of each choice. To calculate the market share we need to identify the consumer that is indifferent between both retailers to purchase the good.

Analogously, the Figure A2 below shows a decomposition of the consumers expected utility into several components. The first group of components corresponds to the utility obtained by consumers that cannot use credit cards, either by not having, or because the retailer did not to adhere to the credit card system. The second group of components corresponds to the utility of consumers that have a credit card and the chosen retailer adhered to the system.

**Figure A2 – Consumer’s utility**

\[
U_i = (1 - x_i \lambda_i')(u_0 + \theta u_i - (1 + \theta) p_i' - \int_{\Sigma_2} c_{\theta} dH(c_{\theta}) - \theta E(c_{\theta}))
\]

cannot use credit card

\[
+ x_i \lambda_i'(u_0 + \theta u_i - (1 + \theta) p_i' - \int_{\Sigma_2} c_{\theta} dH(c_{\theta}) - \theta E(c_{\theta}) - \int_{\Sigma_{f + \Delta_i'}} c_{\theta} dH(c_{\theta}) + \int_{\Sigma_{f + \Delta_i'}} \lambda_i'(f + \Delta_i') dH(c_{\theta}))
\]

can use credit card

utility of consumption product cost of store credit (ordinary and credit purchases)

cost of store credit (ordinary purchases) cost of credit card (ordinary purchase)

cost of store credit (credit purchase) cost of credit card (credit purchase)
The expression in Figure A2 can be simplified to derive equation (9), which is used to obtain the market share equation (11). The derivatives of the merchant’s market share (11), with respect to the base price and the spread are given by

\[
\frac{\partial s_i}{\partial p'_i} = -(1 + \theta) \frac{2t}{2t} \quad (A3)
\]

and

\[
\frac{\partial s_i}{\partial \Delta'_i} = -(L'_i + \theta) \frac{xL'_i}{2t} \left[1 - H \left(f + \Delta'_i \right) \right] \quad (A4)
\]

The merchant’s expected profit is the product of the margin and the market share. As we can see below, the expressions (A5) to (A8) are useful to compute the derivatives of the profit function.

We can use the equations (7), (11), (A1) and (A3) to derive the expressions below:

\[
\frac{2t}{(1 + \theta)} \frac{\partial M_i}{\partial p'_i}.s_i = t + (1 + \theta)(p'_j - p'_i) + x(L'_i,\bar{S}(\Delta'_i) - L'_i,\bar{S}(\Delta'_i)) \quad (A5)
\]

and

\[
\frac{2t}{(1 + \theta)} M_i, \frac{\partial s_i}{\partial p'_i} = -(1 + \theta)(p'_j - \gamma) + (H(0) + \theta)c_s + xL'_i,\bar{T}(\Delta'_i) \quad (A6)
\]

We can use the equations (7), (11), (A2) and (A4) to derive the expressions below:

\[
\frac{2t}{(L'_i + \theta)} \frac{\partial s_i}{\partial \Delta'_i} M_i = -xL'_i \left[1 - H \left(f + \Delta'_i \right) \right] \\
\left[1 + \theta)(p'_j - \gamma) - (H(0) + \theta)c_s - xL'_i,\bar{T}(\Delta'_i) \right] \quad (A7)
\]

and
Summing up (A5) and (A6), we obtain the derivative of the profit function with respect to the base price

$$\frac{2t}{(L^*_i + \theta)} \cdot \frac{\partial M_i}{\partial \Delta^*_i} \cdot s_i = x_i L_i \left\{ h(f + \Delta^*_i)(m - c_s - \Delta^*_i) + \left[ 1 - H(f + \Delta^*_i) \right] \right\}$$

$$\cdot \left\{ t + (1 + \theta)(p^*_j - p_i) + x_i \left( L_i \cdot \Delta^*_i - L_i \cdot \Delta^*_i \right) \right\}$$

(A8)

Analogously, we find the derivatives of the profit function with respect to the spread from (A7) and (A8)

$$\frac{2t}{(1 + \theta)} \cdot \frac{\partial \pi_i}{\partial \Delta^*_j} = \left\{ -x_i L \left[ 1 - H(f + \Delta^*_i) \right] \left[ 1 + (1 + \theta)(p^*_j - \gamma) - (H(0) + \theta)c_s - x_i \cdot \Delta^*_j \cdot \Gamma(a, \Delta^*_i) \right] \right\}$$

$$\cdot \left\{ 1 + (1 + \theta)(p^*_j - p_i) + x_i \left( \Delta^*_i - \Delta^*_j \right) \right\}$$

(A9)

(A10)

Proof of Proposition 1:

Substituting (12) in (A10), with $p^*_j = p^*_j = \bar{p}, \Delta^*_j = 0, L^*_i = 1$ and $L^*_j = 1$, we obtain that the derivative of the merchant’s expected profit with respect to the spread satisfy the following equation

$$\frac{2t}{(1 + \theta)} \cdot \frac{\partial \pi_i}{\partial \Delta^*_j} = t \cdot x_i \cdot h(f) \cdot (m - c_s)$$

(A11)

where $h$ represents the density function of the cumulative distribution function $H$.

Note that the right hand side of (A11) is positive if the all following conditions are satisfied:
• There are transportation costs \( t > 0 \);
• There are card users \( x > 0 \);
• The density of consumers that are indifferent between the cost of a store credit or a credit card \( c_B = f \) is positive \( h(f) > 0 \);
• The merchant fee is bigger than the cost of the store credit \( m > c_s \).

We conclude that, if price differentiation is allowed, merchants have incentives to surcharge and, consequently, the unique price \( \bar{p} \) is not equilibrium.

**Proof of Proposition 2:**

The first order conditions with respect to the base price and the spread, using equations (A9) and (A10), are given, respectively, by

\[
t + (1 + \theta)\left( p_j^i - p_i^c \right) + x \left( L_j^i \bar{S}(a, \Delta_j^i) - L_j^i \bar{S}(a, \Delta_j^c) \right) \\
= (1 + \theta)(p_i^c - \gamma) - (H(0) + \theta)c_s - x L_j^i \bar{S}(a, \Delta_j^c) \tag{A12}
\]

and

\[
x L_j^i \left[ 1 - H(f + \Delta_j^i) \right] \left[ (1 + \theta)(p_j^i - p_i^c) - (H(0) + \theta)c_s - x L_j^i \bar{S}(a, \Delta_j^c) \right] \\
= x L_j^i \left[ H(f + \Delta_j^i)(m - c_s - \Delta_j^c) + \left[ 1 - H(f + \Delta_j^i) \right] \left[ t + (1 + \theta)(p_j^c - p_i^c) + x \left( L_j^i \bar{S}(\Delta_j^c) - L_j^i \bar{S}(a, \Delta_j^c) \right) \right] \right] \tag{A13}
\]

Substituting (A12) in (A13) we obtain that \( h(f + \Delta_j^i)(\Delta_j^i - m + c_s) = 0 \) and that the spread in equilibrium is \( \Delta_j^c = m - c_s \).

Using (A12) we obtain the following expressions containing the base prices of both retailers.
\begin{equation}
(1+\theta)(2.p'_r - p'_j) = t + (1+\theta)\gamma + (H(0) + \theta)c_s + x.S(a,\Delta^i)(L'_i - L'_j) + x.L'_i \Gamma(a,\Delta^j)
\end{equation}

and

\begin{equation}
(1+\theta)(4.p'_j - 2.p'_i) = 2.t + 2.(1+\theta)\gamma + 2.(H(0) + \theta)c_s \\
+ 2.x.S(a,\Delta^j)(L'_j - L'_i) + 2.x.L'_j \Gamma(a,\Delta^i)
\end{equation}

Summing up the equations above and rearranging we obtain that the equilibrium price satisfy the following equation

\begin{equation}
(1+\theta).p'_r = t + (1+\theta)\gamma + (H(0) + \theta)c_s + x/3 \left( L'_j - L'_i \right) \bar{\phi}(a,\Delta^j) + x.L'_i \bar{\Gamma}(a,\Delta^j)
\end{equation} \tag{A14}

From equation the equation (A14) above, we can obtain

\begin{equation}
(1+\theta).\left( p'_j - p'_i \right) = -2x/3 \left( L'_j - L'_i \right) \bar{\phi}(a,\Delta^j) - x \left( L'_i - L'_j \right) \bar{\Gamma}(a,\Delta^j)
\end{equation}

which can be included in the market share equation (11) to obtain

\begin{equation}
 s_i = \frac{1}{2} + \frac{x.\bar{\phi}(a,\Delta^i) \left( L'_i - L'_j \right)}{6.t}
\end{equation} \tag{A15}

If $\bar{\phi}(a,\Delta^i) = \bar{\phi} > 0$, we conclude from formula (A15) that in equilibrium both retailers adhere to the credit card system, $L'_i = 1$. This occurs because if one of them decides the contrary, he will lose market share. Consequently, the market share of both retailers are the same, $s_i = s_j = \bar{s} = \frac{1}{2}$.
Therefore, using \( L'_i = L'_j = 1 \) and \( \Delta'_j = \bar{\Delta} = m - c_s \), we conclude from (A14) and (8) that the equilibrium value for the base price is given by equation (16).

**Proof of Proposition 3:**

From (12) and (A13), since \( L'_i = L'_j = 1 \) and \( \Delta'_j = \bar{\Delta} = m - c_s \), we conclude that

\[
\bar{p} - \bar{p'} = x[1 - H(f)](m - c_s) \tag{A16}
\]

which can be rearranged, using equation (17), to obtain equation (20).

**Proof of Proposition 4:**

We can use equation (9) to show that the difference between the consumers’ aggregated utilities (surplus) in both equilibria is given by

\[
\Delta U_i = (1 + \theta)(\bar{p} - \bar{p'}) - x(1 + \theta)\left[ \int_f^{f + \bar{\Delta}} (c_B - f) dH(c_B) + \bar{\Delta} \cdot [1 - H(f + \bar{\Delta})] \right]
\]

Substituting equation (A16) into the equation above we obtain

\[
\Delta U_i = x[H(f + \bar{\Delta}) - H(f)].(1 + \theta)\bar{\Delta} - x(1 + \theta)\left[ \int_f^{f + \bar{\Delta}} (c_B - f) dH(c_B) \right]
\]

Rewriting the equation above we obtain that

\[
\Delta U_i = x(1 + \theta)\left[ \int_f^{f + \bar{\Delta}} (f + \bar{\Delta} - c_B) dH(c_B) \right] \rightarrow \begin{cases} 0 & \text{se } \bar{\Delta} = 0 \\ > 0 & \text{se } \bar{\Delta} > 0 \end{cases}
\]

or
\[ \Delta U_i = x.(1 + \theta) \left[ \int_{-\delta}^{\delta + c_s} (c_B + \delta) dH(c_B) \right] \begin{cases} = 0 & \text{se } a = c_s - c_A \\ > 0 & \text{se } a < c_s - c_A \end{cases} \]

**Proof of Proposition 5:**

We use equation (7) to show that the difference between the retailers’s margins in both equilibria is given by

\[ \Delta M_i = (1 + \theta) \left[ x \left[ 1 - H(f) \right] \bar{\Delta} - (\bar{p} - \bar{p}') \right] \]

which is equal to zero by equation (A16). Since the markets shares are equal in both equilibria (\( \bar{s} = 1/2 \)), we conclude that merchants’ profits are also the same. In other words merchants are indifferent between both equilibria.

**Proof of Proposition 6:**

Supposing that the retailer \( i \) decides to charge the unique price \( \bar{P} \) and the retailer \( j \) charge differential prices \( \bar{p}' \) and \( \bar{p}'' \), and that \( \bar{\Delta}_j = m - c_s > 0 \), we will compute both market shares and margins to compare their profits.

Using equations (10), (11) and (A16), we obtain the market share of the retailer \( i \)

\[ s_i = \frac{1}{2} + (1 + \theta) \left( \frac{\bar{p}_j - \bar{p}}{2.\bar{x}} \right) + x \left( \frac{\bar{S}(a,0) - \bar{S}(a, \bar{\Delta}_j^c)}{2.\bar{x}} \right) \]

where

\[ \bar{p}_j - \bar{p} = -x.\left[ 1 - H(f) \right] \bar{\Delta}_j \]

and

\[ \bar{S}(a,0) - \bar{S}(a, \bar{\Delta}_j^c) = (1 + \theta) \int_{f}^{f + \bar{\Delta}_j} (c_B - f) dH(c_B) + \left[ 1 - H(f + \bar{\Delta}_j) \right] \bar{\Delta}_j \]

Consequently, we obtain that

\[ s_i = \frac{1}{2} - x.(1 + \theta) \left[ \int_{f}^{f + \bar{\Delta}_j} (f + \bar{\Delta}_j - c_B) dH(c_B) \right] < \frac{1}{2} \]
since the term inside the integral is positive. In other words, the retailer who decides to charge unique price will lose market share for the one charging differential prices.

Using equations (7), (8) and (A16) we compare both margins

\[ M_i - M_j = (1 + \theta)(\bar{p} - \bar{p}_j) - x\left\{ \bar{\Gamma}(a,0) - \bar{\Gamma}(a,\bar{\Delta}_j) \right\} \]

where

\[ \bar{p} - \bar{p}_j = x[1 - H(f)]\bar{\Delta}_j \]

and

\[ \bar{\Gamma}(a,0) - \bar{\Gamma}(a,\bar{\Delta}_j) = (1 + \theta)x[H(f + \bar{\Delta}_j) - H(f)](m - c_s) + [1 - H(f + \bar{\Delta}_j)]\bar{\Delta}_j \]

Consequently, we obtain that both margins are equal, since

\[ M_i - M_j = -x(1 + \theta)x[H(f + \bar{\Delta}_j) - H(f)](m - c_s - \bar{\Delta}_j) = 0 \]

Indeed, it is easy to compute both the margins using (7), which are equal to the transportation cost \( t \). Then comparing the profits we obtain that

\[ \pi_i(\bar{\Delta}_j) - \pi_i(0) = \frac{t}{2} - \left( \frac{t}{2} = \varepsilon(a) \right) = \varepsilon(a) \]

where the decrease on market share is

\[ \varepsilon(a) := \frac{x(1 + \theta)}{2} \int_{f}^{f + \bar{\Delta}_j} (f + \bar{\Delta}_j - c_s) dH(c_b) \]

which is equivalent to equation (21).

Notice that the decrease on market share can be rewritten as equation (21) and that

\[ \varepsilon(a) = \begin{cases} > 0 & \text{if } \bar{\Delta}_j = c_A + a - c_s > 0 \\ = 0 & \text{if } \bar{\Delta}_j = c_A + a - c_s = 0 \\ < 0 & \text{if } \bar{\Delta}_j = c_A + a - c_s < 0 \end{cases} \]
Proof of Proposition 7:

From equations (22) and (23), we obtain that, if both retailers charge differential prices, the menu costs ($\mu_i$) will decrease the margins. However, since the reduction in margin is constant with respect to small perturbations, they do not affect the first order conditions used to find equilibrium prices.

Consequently, the prices $\bar{p}'$ and $\bar{p}''$ remain as equilibrium prices, only if the impact on profit of the unilateral deviation to the unique price $\varepsilon(a)$ is bigger than the correspondent economy obtained by the retailer with the elimination of the menu cost.

Proof of Proposition 8:

If both retailers charge the unique price and banks set an interchange fee $a$ bigger than $\bar{a}$, at least one retailer $i$ will have a menu cost $\mu_i$ smaller than the extra profit $\varepsilon(a)$ as consequence of the increase of market share obtained from charging unilaterally differential prices.