Inflation Bias and Stabilization Bias in Rotemberg Pricing *

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Abstract

In this paper, we study the welfare implication of discretionary and commitment monetary policy in the standard new Keynesian model, where the goods price is sticky due to the price adjustment cost a la Rotemberg (1982). Unlike Calvo price adjustment, Rotemberg price rigidity allows us to employ a projection method to solve the model without linearizing the model. Hence, we can deal with the model even in large monopolistic distortion cases. In this paper, there are two main findings. First, the unconditional expected value of the welfare loss of discretion is almost same as the welfare loss of discretion in the non-stochastic steady state. That is, discretionary monetary policy is welfare inferior, mainly because it results in higher inflation and output gap in level. Certainly, it is inferior, also because the policy maker’s reaction to shocks is different from commitment, generating different fluctuations of output and inflation, but quantitatively the effect of such a difference in the policy reaction is negligible. Second, comparing the results of the projection and the LQ methods, we find that the equilibrium behavior of the model, including the welfare effects, is quite similar to each other. This is somewhat surprising in the sense that this finding holds even in large monopolistic distortion cases. As Gali (2008) and Benigno and Woodford (2003) discuss, when the distortion is large, there is no guarantee that the standard LQ method is a good approximation; we, however, find that, under reasonable parameter sets, it is actually good approximation at least for the standard new Keynesian model with Rotemberg pricing, although it is not clear if we can generalize this conclusion to more large scale models.

KEYWORDS: Inflation bias; Stabilization bias; Projection methods

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1 Introduction

In this paper, we analyze the standard New Keynesian model with Rotemberg (1982) quadratic price adjustment cost, by using a projection method to investigate the inflation and the stabilization biases. By using projection methods, unlike the linear-quadratic (LQ) method, we do not need to linearize equilibrium equations; also, unlike Calvo-pricing, we can easily evaluate the model behavior in stochastic dynamics. This paper has two objectives. First, we decompose the welfare loss of the discretionary policy into its steady state inefficiency, which we call inflation bias, and its fluctuations around the steady state, which we call stabilization bias. Second, while it is well-known that the LQ method is "invalid" when the monopolistic distortion is large, we are also interested in how bad it is.

Since seminal works by Kydland and Prescott (1977) and Barro and Gordon (1983), the optimal monetary policy in dynamic decision making has been investigated. There is a well-known time-inconsistency problem, which is the main motivation of the debate over rules versus discretion in actual conduct of monetary policy. While the commitment has the problem of credibility, the discretionary policy is followed by a certain degree of welfare loss. This welfare loss is usually boiled down to a quadratic loss function in inflation and output gap in the LQ method. The LQ method has been popularized perhaps mainly because of its analytical tractability. However, as pointed by Woodford (2003), Galí (2008), and Anderson, Kim and Yun (2010), the standard LQ method is applicable only if the distortion in the steady state is small enough to approximate the model around the steady state (see also Benigno and Woodford, 2012). Also, it is not a proper framework if the non-linearity raised in the original model is of interest, especially for quantitative analyses.

Intuitively, the inflation bias means that the discretionary policy cannot eliminate the temptation to boost output at the cost of higher inflation. As shown by Anderson, Kim and Yun (2010) for Calvo pricing, the optimal inflation rate under the commitment policy is zero in the steady state for Rotemberg pricing. Hence, often the inflation bias is measured as the steady state inflation under discretion, but in this paper we evaluate its welfare loss. On the other hand, the stabilization bias means that, in response to the markup shock, the discretionary policy cannot stabilize the economy as well as the commitment policy does. For the stabilization bias, we measure its effect by the welfare loss of the discretionary policy compared to the commitment policy, as in Dennis and Söderström (2006). Computationally, we define the inflation bias as $\mathcal{IB} = V_{ss}^{cm} - V_{ss}^{ds}$, where $V_{ss}^{cm}$ and $V_{ss}^{ds}$ are the...
value functions under commitment and discretionary policy regimes at their respective non-stochastic steady state. Since it can be analytically shown that $V^{cm\, ss} = V^{fl\, ss}$, where $V^{fl\, ss}$ is the value function under flexible price, our inflation bias is actually $IB = V^{fl\, ss} - V^{ds\, ss}$. Also, we define the stabilization bias as $SB = (E[V_t^{cm}] - V^{cm\, ss}) - (E[V_t^{ds}] - V^{ds\, ss})$, where $E[V_t^{cm}]$ and $E[V_t^{ds}]$ are unconditional expected value of the value functions, where an unconditional expectation can be regarded as a long-run average. It is obvious that our stabilization bias is the total gap in the unconditional expectation of the two value functions minus inflation bias; $SB = E[V_t^{cm} - V_t^{ds}] - IB$. Note that $E[V_t^{cm}] - V^{cm\, ss}$ and $E[V_t^{ds}] - V^{ds\, ss}$ are determined by the curvature of the value functions and the size of the fluctuations of output, inflation and other variables; hence, if the value function is almost linear and/or if the output and inflation does not fluctuate very much, $E[V_t^{cm}] - V^{cm\, ss}$ and $E[V_t^{ds}] - V^{ds\, ss}$ are both small.

Previewing our main results, our first finding is that quantitatively the stabilization bias is quite small, while the inflation bias is quite significant. The main reasons behind this are as follows. First, the standard new Keynesian model is quite close to linear. As in the standard model formulation, in our model, (i) power utility is linear in log, (ii) production function is linear in log, and (iii) both technology and markup shocks are linear in log (AR(1) processes). Although Rotemberg’s quadratic price adjustment cost is highly non-linear, comparing to the output and inflation shifts, the fluctuation of inflation is quite small, and hence the price adjustment cost is not large. This story is in essence in the same vein as the classic study of the real business cycle model by Lucas (1985). As a consequence, we find that the utility and value functions are almost linear in state variables, and hence $E[V_t^{cm}] - V^{cm\, ss}$ and $E[V_t^{ds}] - V^{ds\, ss}$ are both very small. Our second finding is that, even in the case of large distortion, the LQ and the projection methods generate quite similar results. Qualitatively, comparing to the projection method, the LQ method underestimates the inflation bias, but quantitatively, the difference is quite small. As Gali (2008) and Benigno and Woodford (2003) discuss, when the distortion is large, there is no guarantee that the standard LQ framework is a good approximation; we, however, find that, under reasonable parameter sets, it is actually a good approximation. Although it is not clear if we can generalize this conclusion to more large scale models, at least for the standard new Keynesian model with Rotemberg pricing, given availability of its analytical solution, it is worthwhile to study large distortion cases by using the LQ method.

In this relation, there is one comment on the approximation error of the LQ method. Certainly,
the discussion in Gali (2008) and Benigno and Woodford (2003) is rigorous in the sense that their
discussion is based on the approximation theory of the Taylor expansion, and is perhaps the best
easily available analysis in the sense that their analysis can treat a general class of models with a well
defined concept. However, we must be cautious in interpreting their assertion. Suppose that the order
of the approximation error is cubic: \( O(|\Phi, \xi|^3) \). Then, in the current context, \( O(|\Phi, \xi|^3) \) means that,
as the size of the shocks \( |\xi| \) and distortion \( |\Phi| \) shrink, the approximation error shrinks, and the speed
at which the approximation error shrinks is the cube of the speed at which \( |\xi| \) and \( |\Phi| \) shrink or faster.
However, in many cases, we are not really interested in the limiting case where both \( |\xi| \) and \( |\Phi| \) shrink
toward zero. The actual size of the approximation error with non-shrinking \( |\xi| \) and \( |\Phi| \) depends not
only on the actual sizes of \( |\xi| \) and \( |\Phi| \) but also on the functional forms of a specific model and its
parameter values. A trivial example is that if all model equations are linear or quadratic, whatever
the size of the shocks and distortions, the approximation is perfect. In this sense, what we find does

However, it is dangerous to generalize our results for a general class of models. For example,
Levin et al. (2005) find a much larger welfare loss than ours in the model which has richer structure
and shocks than ours, though, given different motivation and context, it is hard to compare their
results with ours. In this sense, if there is some reasons to stick to the LQ framework, it is safer to
use the method, say, proposed by Benigno and Woodford (2003). Furthermore, it is also dangerous
to generalize our results for Calvo pricing. Also, Anderson, Kim and Yun (2010) find much larger
inflation bias under Calvo pricing than ours. While Calvo and Rotemberg pricing are identical under
the LQ framework (i.e., we can find the parameters that equate them as shown by Nistico, 2007), they
can be very different in the non-linear formulation. Indeed, Calvo pricing requires one additional state
variable that captures the price dispersion, while no such a state variable is present under Rotemberg
pricing. Actually, we suspect that the weak non-linearity in the value function that we find is because
of the quadratic nature of Rotemberg price adjustment cost; that is, it becomes linear in the first order
necessary conditions (FONCs). Note that this weak non-linearity not only makes \( SB \) small but also
the approximation loss of the linearization small. In this sense, it may be worthwhile to investigate the
welfare loss under a generalized Rotemberg price adjustment cost (see Kim and Ruge-Murcia, 2009
and 2011).
There are also some previous works which analyzes the discretionary policy in a non-linear framework. Among them, Anderson, Kim and Yun (2010) studies Calvo pricing, which is the closest to our research, but we investigate the stabilization bias as well. Van Zandweghe and Wolman (2010) studied multiple Markov perfect equilibria. Braun and Waki (2010) did a more comprehensive analysis with the zero lower bound of nominal interest rate in both the non-linear Calvo model and the non-linear Rotemberg model, and saw the difference of the models which appeared in the size of fiscal multiplier. They reported that the use of log-linearization to analyze the effect of the zero-lower bound of nominal interest rate leads to a serious misspecification of the result. However, they assumed the efficient steady state, whereas we are interested in the effects of the distorted steady state, as in Anderson, Kim and Yun (2010).

The plan of this paper is as follows. Section 2 briefly explain the model, which is quite standard, to introduce the notations; it also discusses how to solve the model numerically and reviews the LQ approximation of the model. Section 3 shows the numerical results, and the final section concludes.

2 Model and Its Solution

To introduce our notational convention, this section describes the standard new Keynesian model, which is a workhorse of the study of the monetary policy. Here, we assume the goods price is sticky, because of the quadratic price adjustment cost a la Rotemberg (1982).

Before going on, we want to summarize our notational convention, which are necessary for our decompositions below. Subscripts \( ss \) and \( ef \) indicate the non-stochastic steady state and the efficient steady state, respectively. Lower case letters without decoration are deviations from the efficient non-stochastic steady state; e.g., \( y_t = \ln Y_t - \ln Y_{ef} \), where \( Y_t \) is output under flexible price, discretion or commitment, depending on the context. Lower case letters with bar are deviations from the steady state under flexible price; e.g., \( \bar{y}_t = \ln \bar{Y}_t - \ln \bar{Y}_{ss} \), where \( \bar{Y}_t \) and \( \bar{Y}_{ss} \) are output under flexible price and its non-stochastic steady state value, respectively. Lower case letters with tilde shows the gap between sticky price and flexible price; e.g., output gap is \( \tilde{y}_t = \ln Y_t - \ln \bar{Y}_t \), where \( Y_t \) is output under either discretion or commitment. Every time we need to discriminate discretion and commitment, we do so by adding superscripts such as \( \tilde{y}_t^{cm} \) and \( \tilde{y}_t^{ds} \) for commitment and discretion, respectively.
2.1 The Model

The representative household consumes, saves and works. The firms produce differentiated goods in monopolistic competitive markets subject to Dixit and Stiglitz’ (1997) aggregator of final goods. Firms also set prices subject to Rotemberg’s (1982) quadratic price adjustment cost.

The representative household maximizes

$$\max V_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi H_t^{1+\eta}}{1+\eta} + \beta E_t V_{t+1},$$

s.t. $$C_t + \frac{B_t}{R_t P_t} \leq \frac{B_{t-1}}{P_t} + \frac{W_t}{P_t} H_t + T_t.$$ 

In period t, the household consumes $C_t$ and works $H_t$ hours to earn $W_t H_t / P_t$ real income where $W_t$ is nominal wage and $P_t$ is the general price level. The household also purchases $B_{t+1}$ risk-free bonds with risk-free rate $R_t$. $T_t$ is the sum of dividend from firms and transfer from the government; both of them are lump-sum. The present value of current and future utility is given by $V_t$ with discount factor $\beta \in (0, 1)$. $\sigma > 0$ is a risk aversion parameter, $\eta > 0$ is the inverse of Frisch elasticity, and $\chi > 0$ is a normalization parameter for hours. Then, we have the consumption Euler equation and the labor supply schedule:

$$1 = \beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\},$$

$$\frac{W_t}{P_t} = \chi H_t^\eta C_t^\sigma.$$  

where $\Pi_t = P_t / P_{t-1}$.

There are firms on unit mass $i \in [0, 1]$. Firm $i$ chooses labor input $H_{it}$ so as to minimize production cost subject to production function $Y_{it} = A_t H_{it}$, where $A_t$ is the aggregate technology level which follows AR(1) process $A_{t+1} = A^{1-\rho_A} A^\rho_A \exp \varepsilon_{at+1}$; the firms’ problem is $\min_{H_{it}} (W_t / P_t) H_{it} + MC_{it} (Y_{it} - A_t H_{it})$, where $MC_{it}$ measures real marginal cost. Since the production function is linear homothetic, $MC_{it} = W_t / P_t / A_t$ are the same for all firms. Firm $i$ also chooses price $P_{it}$ so as to maximize the present value of current and future profit subject to the demand function $Y_{it} = (P_{it} / P_t)^{-\theta} Y_t$ with elasticity of demand $-(\partial Y_{it} / \partial P_{it}) P_{it} / Y_{it} = \theta > 1$ and Rotemberg quadratic price adjustment cost with
a parameter $\psi > 0$:

$$E_0 \sum_{t=0}^{\infty} \beta^t Q_{0,t} \left\{ \left( \frac{Z_t P_{it}}{P_t} - MC_t \right) \left( \frac{P_{it}}{P_t} \right)^{-\theta} Y_t - \frac{\psi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right) Y_t \right\},$$

where $Z_t$ is stochastic subsidy to firms which follows AR(1) process $Z_{t+1} = Z_1^{-\rho_s} Z_t^s \exp \varepsilon_{zt+1}$. The FONC of $P_{it}$ is given by

$$(1 - \theta) Z_t \left( \frac{P_{it}}{P_t} \right)^{-\theta} Y_t + \theta M_{C_t} \left( \frac{P_{it}}{P_t} \right)^{-\theta-1} Y_t$$

$$-\psi \left( \frac{P_{it}}{P_{it-1}} - 1 \right) \frac{Y_t}{P_{it-1}} + \beta \psi E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_{it}}{P_{it-1}} - 1 \right) \frac{P_{it+1} Y_{t+1}}{P_{it}^2} = 0.$$ 

Firms choose the same price because they face the same real marginal cost, so index $i$ is dropped:

$$\frac{\psi}{\theta} (\Pi_t - 1) \Pi_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \frac{\psi}{\theta} (\Pi_{t+1} - 1) \Pi_{t+1} \right\} + M_{C_t} - \frac{\theta - 1}{\theta} Z_t. \tag{2}$$

Note that, if $\psi = 0$ (in the flexible-price equilibrium) and $Z_{ef} = \theta/(\theta - 1)$ in the steady state, $M_{C_{ef}} = 1$ holds, implying that the steady state is Pareto optimal. We define $\Phi = 1 - Z_{ss}/Z_{ef} \geq 0$ as a measure of the size of distortion by monopolistic competition. When $\Phi > 0$, there is a positive price markup stemming from monopolistic competition, and the steady state is suboptimal. This entices the policy maker to reduce the output gap and its volatility at the cost of those for inflation. Finally, the resource constraint is given by

$$C_t = \Psi_t Y_t = \Psi_t A_t H_t, \tag{3}$$

where $\Psi_t = 1 - (\psi/2)(\Pi_t - 1)^2$. We call $(1 - \Psi_t) Y_t$ output loss due to the price adjustment. The above FONCs are taken as constraints for the policy maker’s optimization, which we describe next.

### 2.1.1 Optimal Monetary Policy

In the optimal monetary policy, the policymaker chooses the variables in the economy so as to maximize the social welfare, which is measured by the representative household’s utility. There are two types of the policies. In the discretionary policy, the policymaker chooses only current variables in each period though it considers the effects of its current decisions; the discretionary policy is Markov-
perfect and time-consistent. On the contrary, in the commitment policy, the policymaker can exploit the public expectation by specifying the future variables in the initial period; the commitment policy is history-dependent and time-inconsistent. That is, such a commitment ties the policymaker’s hand in the future, but she may have an incentive to deviate from the policy precommitted in the past.

After reducing the number of endogenous variables, the optimal policy is formulated as the following Lagrangian: $L_0 = \min_{\{\lambda_{1,t}, \lambda_{2,t}\}} \max_{\{Y_t, \Pi_t, R_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \Omega_t$, where

$$
\Omega_t = \frac{\Psi_t^{1-\sigma} Y_t^{1-\sigma}}{1-\sigma} - \frac{\lambda_{a,t} Y_t^{1+\eta}}{1+\eta} + \lambda_{1,t} \left\{ \beta E_t \left[ \frac{\Psi_{t+1}^{1-\sigma} Y_{t+1}^{1-\sigma}}{\Pi_{t+1}} - \frac{\Psi_t^{1-\sigma} Y_t^{1-\sigma}}{R_t} \right] \right\} + \lambda_{2,t} \left\{ \beta E_t \left[ \Psi_{t+1}^{1-\sigma} Y_{t+1}^{1-\sigma} \psi (\Pi_{t+1} - 1) \Pi_{t+1} \right] - \Psi_t^{1-\sigma} Y_t^{1-\sigma} \psi \left( \frac{\psi}{\theta} \right) (\Pi_t - 1) \Pi_t \right\},
$$

where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the Lagrange multipliers on consumption Euler equation (1) and new Keynesian Phillips curve (2), respectively. Define $\lambda_{a,t} = \chi/A^{1+\eta}$; thus, $\lambda_{a,t} Y_t^{1+\eta} = MC_t C_t^{-\sigma} Y_t = \chi H_t^{1+\eta}$.

The FONCs are as follows.

$$
0 = \beta E_t \left[ \frac{\Psi_{t+1}^{1-\sigma} Y_{t+1}^{1-\sigma}}{\Pi_{t+1}} \right] - \frac{\Psi_t^{1-\sigma} Y_t^{1-\sigma}}{R_t} \quad (4a)
$$
$$
0 = \beta E_t \left[ \Psi_t^{1-\sigma} Y_t^{1-\sigma} \psi \left( \frac{\psi}{\theta} \right) (\Pi_t - 1) \Pi_t \right] - \Psi_t^{1-\sigma} Y_t^{1-\sigma} \psi \left( \frac{\psi}{\theta} \right) (\Pi_t - 1) \Pi_t + \frac{\theta - 1}{\theta} Z_t - \chi_{a,t} \Psi_t^{1-\sigma} Y_t^{1+\eta} \quad (4b)
$$
$$
0 = \left( \Psi_t + \frac{\sigma}{Y_t} \left( \frac{\lambda_{1,t}}{R_t} - \frac{\lambda_{1,t-1}}{\Pi_t} \right) \right) + (\lambda_{2,t} (1 + \eta) - 1) \chi_{a,t} \Psi_t^{1+\eta} + \lambda_{1,t} - \lambda_{2,t-1} \psi \left( \frac{\psi}{\theta} \right) (\Pi_t - 1) \Pi_t - 1 - 1) \lambda_{a,t} \Psi_t^{1+\eta} + (\lambda_{2,t} - \lambda_{2,t-1}) \Psi_t^{1+\eta} (2 \Pi_t - 1) \quad (4c)
$$

where $\Psi_t = 1 - (\psi/2)(\Pi_t - 1)^2$, and hence $\partial \Psi_t / \partial \Pi_t = -\psi(\Pi_t - 1)$.

There are a couple of remarks in order. First, regardless of the policy regime, without zero lower bound, $\lambda_{1,t} = 0$ for all $t$. Second, for discretion, $\lambda_{2,t-1} = 0$; i.e., the private sector ignores the
"promise" that the policy maker made before. Third, under commitment, in the non-stochastic steady state, $R_{ss} = 1/\beta$, $\chi_{a,ss} = \chi$, $\Pi_{ss} = \Psi_{ss} = 1$ and $\lambda_{ss} = 0$. Also,

$$Y_{ss} = H_{ss} = C_{ss} = \left( \frac{\theta - 1}{\chi \theta} Z_{ss} \right)^{\frac{1}{\pi + \eta}}$$

$$\lambda_{2,ss} = \frac{1}{\sigma + \eta} \left( 1 - \frac{\theta / Z_{ss}}{\theta - 1} \right)$$

These are the same as the steady state for flexible price, and hence $V_{ss}^{cm} = V_{ss}^{fl}$, no matter what the value of $\Phi$ is. We however need to rely on the numerical method to obtain the non-stochastic steady state for discretion.

### 2.2 LQ Approximation

Given its popularity and the availability of the analytical results, we also investigate the gap between the LQ approximation and the projection method. We are especially interested in the approximation error of the standard LQ method for large monopolistic distortion. We are linearizing the model economy around the efficient non-stochastic steady state, where $Z_{ef}$ offsets the distortion by the monopolistic competition exactly. This can lead to a bad approximation when the distortion is large; see Gali (2008) and Benigno and Woodford (2003). However, at our best knowledge, it has not yet been numerically investigated how bad such an approximation is.

#### 2.2.1 Linearized Economy

Here, the monetary policy is absent, which is to be discussed in the next subsection. The model is linearized around the efficient best steady state; $Z_{ef} = \frac{\theta}{\theta - 1}$ and hence $MC_{ef} = 1$.

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (r_t - E_t [\pi_{t+1}] - \rho) \quad \text{(NKIS)}$$

$$\pi_t = \frac{\theta}{\psi} (mc_t - z_t) + \beta E_t [\pi_{t+1}] \quad \text{(NKPC)}$$

$$y_t = c_t \quad \text{(mkt clearing)}$$

$$mc_t = \sigma c_t + \eta h_t - a_t \quad \text{(labour supply)}$$

$$y_t = a_t + h_t \quad \text{(production function)}$$
where \( \rho = (1 - \beta) / \beta \). Reducing the number of endogenous variables,

\[
y_t = E_t [y_{t+1}] - \frac{1}{\sigma} (r_t - E_t [\pi_{t+1}] - \rho) \\
\pi_t = \frac{\theta}{\psi} (mc_t - z_t) + \beta E_t [\pi_{t+1}] \\
mc_t = (\sigma + \eta) y_t - (1 + \eta) a_t
\]

If the price is flexible (\( \psi = 0 \)), \( \tilde{mc}_t = \tilde{z}_t = \tilde{z}_{ss} \), implying \( \tilde{z}_{ss} = \left( \frac{\sigma}{\sigma_c} + \eta \right) \tilde{y}_t - (1 + \eta) a_t \). Note that, following the convention, we assume that the markup shock \( Z_t \) is constant in the flexible price equilibrium so \( \tilde{z}_t = \tilde{z}_{ss} = \ln Z_{ss} - \ln Z_{ef} \); \( z_t \) fluctuates only under sticky price. Since we assume that \( a_t \) and \( z_t \) both follow AR(1) processes, \( (\sigma + \eta) (E_t [\tilde{y}_{t+1}] - \tilde{y}_t) = (1 + \eta) (\rho_a - 1) a_t + (\rho_z - 1) z_{ss} \). This also implies that, \( mc_t = \left( \frac{\sigma}{\sigma_c} + \eta \right) (y_t - \tilde{y}_t) + z_{ss} \). Hence, noting that \( \tilde{y}_t = y_t - \tilde{y}_t = \ln Y_t - \ln \tilde{Y}_t \),

\[
\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \frac{1}{\sigma} (r_t - E_t [\pi_{t+1}] - \rho) + \frac{1 + \eta}{\sigma} (\rho_a - 1) a_t \\
\pi_t = \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t - \frac{\theta}{\psi} z_t
\]

where \( \kappa = \frac{\theta}{\psi} (\sigma + \eta) \).

### 2.2.2 Quadratic Loss

Given NKIS and NKPC above, the value function is \( V_0 = \min_{(\lambda_t, c_t)} \max_{(\tilde{y}_t, \tilde{r}_t, \pi_t)} E_0 \sum_{t=0}^{\infty} \beta^t \Omega_t \)

\[
\Omega_t = -\chi \frac{\tilde{z}_{ss}}{\tilde{\psi}} \left[ (\sigma + \eta) (\tilde{y}_t - y^*)^2 + \psi \pi_t^2 \right] + t.i.p. \quad (5)
\]

\[
+ \lambda_{1t} \left[ -\tilde{y}_t + E_t [\tilde{y}_{t+1}] - \frac{s_c}{\sigma} (r_t - E_t [\pi_{t+1}] - \rho) + \frac{1 + \eta}{\sigma + \eta} (\rho_a - 1) a_t \right] \\
+ \lambda_{2t} \left[ -\pi_t + \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t - \frac{\theta}{\psi} z_t \right]
\]

where \( t.i.p. = \chi \frac{\tilde{z}_{ss}}{\tilde{\psi}} \left( a_t + \frac{1}{2} \frac{(1+\eta)(1-\sigma)}{\sigma+\eta} a_t^2 \right) \) and \( y^* = -\frac{\tilde{z}_{ss}}{\psi + \eta} > 0 \) (see Appendix for derivation). Note that this formulation is the same as the standard one because the optimization is invariant against an increasing linear transformation. We left the quadratic approximation as it is without any simplification, because it makes easier the computation of the consumption equivalent loss.

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1This \( \kappa \) is consistent with the Phillips curve in Calvo pricing. If \( Z_{ss} = \theta / (\theta - 1) \), it is well known that \( \kappa = (1 - \xi t) (1 - \xi) / \xi \) and \( \pi_t = \kappa (mc_t - z_t) + \beta E_t [\pi_{t+1}] \).
Under discretion, because, without binding ZLB, $\lambda_{1,t} = \lambda_{1,t-1} = \lambda_{2,t-1} = 0$,

$$\partial \tilde{y}_t : (\sigma + \eta) (\tilde{y}_t - y^*) = -\kappa \lambda_{2,t}$$

$$\partial \pi_t : \psi \pi_t = \lambda_{2,t}$$

Hence,

$$\tilde{y}_t - y^* = -\theta \tilde{\pi}_t$$

Solving this (see Appendix for details),

$$\tilde{y}_t = \frac{1 - \beta}{1 - \beta + \kappa \theta} y^* + \frac{\theta^2 / \psi}{1 - \beta + \kappa \theta} \tilde{z}_t$$

$$\pi_t = \frac{\kappa}{1 - \beta + \kappa \theta} y^* + \frac{-\theta / \psi}{1 - \beta + \kappa \theta} \tilde{z}_t$$

Under commitment, because, without binding ZLB, $\lambda_{1,t} = 0$,

$$\partial \tilde{y}_t : (\sigma + \eta) (\tilde{y}_t - y^*) = -\kappa \lambda_{2,t}$$

$$\partial \pi_t : \psi \pi_t = \lambda_{2,t} - \lambda_{2,t-1}$$

Hence (see Appendix for details),

$$\tilde{y}_t = \frac{1 - \beta - \beta (M_{\lambda \lambda} - 1)}{\Xi_y} y^* - \theta M_{\lambda \lambda} \frac{\lambda_{2,t-1}}{\psi} + \frac{\theta^2 / \psi}{\Xi_\xi} z_t$$

$$\pi_t = \frac{\kappa}{\Xi_y} y^* + (M_{\lambda \lambda} - 1) \frac{\lambda_{2,t-1}}{\psi} - \frac{\theta / \psi}{\Xi_\xi} z_t$$

$$\frac{\lambda_{2,t}}{\psi} = \frac{\kappa}{\Xi_y} y^* + M_{\lambda \lambda} \frac{\lambda_{2,t-1}}{\psi} - \frac{\theta / \psi}{\Xi_\xi} z_t$$

where

$$M_{\lambda \lambda} = \frac{1}{2\beta} \left( (1 + \beta + \kappa \theta) - \sqrt{(1 + \beta + \kappa \theta)^2 - 4\beta} \right) < 1$$

$$\Xi_y = 1 - \beta + \kappa \theta - \beta (M_{\lambda \lambda} - 1) > 0$$

$$\Xi_\xi = 1 - \beta + \kappa \theta - \beta (M_{\lambda \lambda} - 1) > 0$$
Note that there is one important comment on these analytical solutions. For both discretion and commitment, the coefficient on \( z_t \) is independent from the size of monopolistic distortion \( \Phi \). It only affects constant terms \( y^* = \ln \bar{Y}_{ss} \). This implies that, under our definition, \( SB \) is constant regardless of the size of \( \Phi \). For example, for the inflation term under discretion, we can easily see

\[
E \left[ \pi_t^2 \right] = \left( \frac{\kappa}{1 - \beta + \kappa \theta} y^* \right)^2 + \left( \frac{\theta / \psi}{1 - \beta \rho_z + \kappa \theta} \right)^2 E \left( z_t^2 \right)
\]

Since the first term does not involve any stochastic terms, it is subtracted as \( IB \) under our definition. The second term is a constant whatever \( \Phi \) is. For commitment, though we have \( \lambda_{2,t-1} \), a simple calculation leads us to the same implication.

### 2.3 Additive Decompositions

To sort out concepts, we briefly discuss two simple welfare decompositions. The first one is rather standard to derive the LQ formulation, and the second one is consistent with our definitions of inflation and stabilization biases.

First, consider the case of discretionary monetary policy, though the following is totally in parallel for commitment; \( Y_t = Y_t^{ds} \). It is easy to see that, for both discretion and commitment,

\[
y_t = \ln Y_t - \ln Y_{ef} = \ln Y_t - \ln \bar{Y}_t + \ln \bar{Y}_t - \ln \bar{Y}_{ss} + \ln \bar{Y}_{ss} - \ln Y_{ef}
\]

\[
y_t = \tilde{y}_t + \bar{y}_t + \bar{y}_{ss}
\]

where \( \tilde{y}_t = \ln Y_t - \ln \bar{Y}_t \) is output gap, \( \bar{y}_t = \ln \bar{Y}_t - \ln \bar{Y}_{ss} \) is flexible price output in log deviation from its steady state, and \( \bar{y}_{ss} = \ln \bar{Y}_{ss} - \ln Y_{ef} \) captures the effect of monopolistic distortion which appears even in flexible price steady state. This decomposition leads to the standard LQ representation. Indeed, as shown in Appendix, it is easy to see that \( \tilde{y}_t \) cancels out and \( y^* = \bar{y}_{ss} \) in (5).

Second, the other interesting decomposition is

\[
y_t = \ln Y_t - \ln Y_{ef} = (\ln Y_t - \ln Y_{ss}) - (\ln Y_{ss} - \ln Y_{ef})
\]
Hence, taking the gap of this for discretion and commitment,

\[
E [y_t^{cm}] - E [y_t^{ds}] = \{ E [\ln Y_t^{cm}] - \ln Y_{ss}^{cm} - E [\ln Y_t^{ds}] - \ln Y_{ss}^{ds} \} + \{ \ln Y_{ss}^{cm} - \ln Y_{ss}^{ds} \}
\]

The first curly bracket is related to \( SB \), while the second one is to \( T B \). With our definitions, it is straightforward to implement the computation; we can just compute the equilibrium and its welfare for different regimes, and then compute proper differences. Although these decompositions are rather trivial, keeping this decomposition in mind, it should be easier to grasp the numerical results shown below.

### 2.4 Solution Method

First, we compute the optimal policy function by Euler equation iteration. That is, given (temporary) policy functions for \( t + 1 \), we can compute the expectations in (4). Given expectations, we can find the updated policy functions for \( t \). We iterate this process until policy functions converges (time iteration). The expectation is computed by Gauss-Hermite quadrature in one experiment and by Tauchen’s method in the other experiment. For this paper, two sets of codes are separately written for debugging purpose. The results reported here is based on Gauss-Hermite quadrature. To evaluate the policy functions off the node points, we employ the linear interpolations, which is quite good given near linear nature of the policy functions.

Second, for the LQ method, as shown above, the analytical solution is fully available. So, computation for this is essentially little more than just drawing figures. This simplicity is the one of the strength of the LQ method. To evaluate \( V_t \), \( U_t \) and \( \Psi \), we substitute back the level variables; e.g., \( Y_t = Y_{ef} e^\hat{y}_t = Y_{ef} e^{\hat{y}_t + \hat{y}_t} \).

To evaluate the welfare loss, we compute the (permanent) consumption equivalent loss as well as the value of \( V_t \). As is well known, the consumption equivalent loss is useful, since \( V_t \) has no unit, it is hard to interpret the gap in \( V_t \). In our computation, we first compute \( V_t \) in each policy regime, and then convert it into the consumption equivalent loss. For example,

\[
C_{EquivLss} = (1 - \beta) \exp \{ E [V_t^{cm}] - E [V_t^{ds}] \}
\]
Although there are other possible definitions, this is perhaps the most easy to understand. Now, it is clear why we do not drop off the terms independent from policy; in general, it is necessary to compute the consumption equivalent loss for $E[V_{t}^{cm}] - V_{ss}^{cm}$ for example.

The parameters are chosen as $\sigma = 1$, $\eta = 1$, $\chi = 1$, $\beta = 0.99$, $\theta = 11$ and $\psi = 128.1553$. Our $\psi$ is consistent with Anderson Kin and Yun (2010) for Calvo pricing with the probability of fixing prices $\xi = 0.75$. Both technology shock and markup shock are supposed to follow AR(1) process with $\rho_{A} = 0.95$ and $\rho_{z} = 0.0$, and the standard deviations of the innovations on them are assumed to be 0.01. Note that, under this parameters, it is easy to see that the value of the period utility and the value function with $\Phi = 0$ are $-0.5$ and $-50$, respectively.

3 Numerical Results

The main findings are as follows. First, the unconditional expected value of the consumption equivalent welfare loss is greater than that at the non-stochastic steady state but only slightly, which implies that, in our definition, the discretion policy is inferior mainly because of $TB$. Second, the results of the LQ method is quite similar to those of the projection method (Euler equation iteration), which implies that, at least for this standard new Keynesian model with Rotemberg pricing, the LQ method is a quite good approximation even with a large monopolistic distortion. The following subsections investigate results in details.

3.1 Projection Method

Figure 3 shows that the policy functions of discretion and commitment policies with and without monopolistic distortion. More or less, qualitatively, they are similar to each other. However, there are several points worth discussing. First, for all cases, because of the goods market clearing condition, $C_{t}$ and $Y_{t}$ move in the same direction; so does $H_{t}$, though $H_{t}$ is not shown. For a positive technology shock, all $C_{t}$, $Y_{t}$ and $H_{t}$ increase, of course. For a positive markup shock, again all of them increases. Since an increase in $Z_{t}$ can be also regarded as the upward shift of the labour demand schedule, $H_{t}$ increases along the labour supply curve, and as a result $Y_{t}$ and $C_{t}$ also increase. This per se is not surprising but note importantly that this is one factor to explain small $SB$. Under our definition, $SB$
captures the welfare effect of the fluctuations of output and inflation around the non-stochastic steady state. However, while an increase in $C_t$ improves the utility, that in $H_t$ deteriorates it. Hence, this positive covariation between $C_t$ and $H_t$ offsets their positive and negative effects each other.

Second, for both commitment and discretion, the shape of policy function of $Y_t$ does not change very much. This means that the welfare loss of discretionary policy does not stem from $Y_t$. In contrast, the average level of $\Pi_t$ is higher for larger $\Phi$ for discretion, but it does not change very much against $\Phi$ for commitment. This average increase in inflation under discretion is the main source of the welfare loss of the discretion policy.

Third, the variability of $\Pi_t$ is not very large, especially for commitment; it is very small for any value of $\Phi$. This can be seen the unit of $z$-axis of the output loss $\Psi$. For commitment, in the reasonable state space range, it is less than 0.01% ($1e^{-4}$) of output level. Even in the case that discretion with $\Phi = 0.15$, it is significantly less than 1% of output.

Fourth, importantly, the shape of the period utility and the value function is almost flat. This is mainly because (i) in the standard new Keynesian model, the non-linearity is weak except for the price adjustment cost, but (ii) since the variation of inflation is small, the non-linearity of the price adjustment cost does not make the period utility and the value function concave enough.

Figure 1 implies the similar points as figure 3 does. First, output level, whichever unconditional expectation or steady state values, is similar among flexible price, commitment and discretion. However, the large gap comes from inflation. Under commitment, inflation is near zero, but high inflation causes high output loss $\Psi$ for discretion.

Second, importantly, the shape of the unconditional expectations and the steady state values do not differ very much, again implying $SB$ is small. For both commitment and discretion, the decline in the steady state output deteriorates the welfare equally. However, for discretion, on top of that, high steady state inflation also damages the welfare.

### 3.2 LQ Method

Figures 7 and 4 are for the LQ method, which are corresponding to Figures 3 and 1, though some variables are defined in different way. The main point here is that the results of the LQ approximation are almost identical to those of the projection method even with large distortions. The welfare loss
is slightly smaller under the LQ method, but it is not surprising because the LQ method eliminates
the curvature of equations by linearizing them. Given availability of the analytical solution to the LQ
method, this implies that studying the LQ method may not be a bad choice at least for the standard
new Keynesian model with Rotemberg pricing, which we described in the previous section. Again, the
key intuition seems to be the lack of non-linearity in the new Keynesian model; the original model
is almost linear in log, and in the case of Rotemberg pricing, the price adjustment cost is linear in
the first order conditions. In addition, the output loss due to the price adjustment cost is small given
small fluctuation of inflation.

4 Conclusion

In this paper, we study the standard new Keynesian model with Rotemberg pricing to investigate
the welfare effect of discretionary and commitment monetary policy regimes. There are mainly two
findings. First, under our definition, the welfare loss of discretionary policy almost solely stems from the
inflation bias. Quantitatively, the welfare effect of the stabilization bias is negligibly small. Second,
regardless of the size of the monopolistic distortion, the approximation error of the LQ method is
quite small. Since the analytical solutions are available to the LQ formulation, this may sounds to be
encouraging for the LQ method. Roughly speaking, behind these two findings, the key intuition is the
lack of strong non-linearity. That is, since the original model is close to linear, the linearized version
of the model (i.e., LQ method) is a good approximation. Also, since the model does not have strong
non-linearity, value function $V_t$ is almost linear in state variables; given lack of curvature, $E_t [V_t]$ and
$V_{ss}$ are almost same. In this sense, it is not clear if our two main findings hold for models other than
ours, especially for models with strong non-linearity.
References


A Appendix

A.1 Derivation of Non-Linear FONCs in Detail

Useful expressions:

\[ MC_t = \chi_{a,t} \Psi_t^\sigma Y_t^{\sigma + \eta} = \frac{\chi_{a,t} C_t^{\sigma + \eta}}{\Psi_t^\eta} \]

Lagrangian: \( L_0 = \min_{(\lambda_1,t,\lambda_2,t)} \max_{(Y_t,\Pi_t,R_t)} E_0 \sum_{t=0}^\infty \beta^t \Omega_t \) where

\[ \Omega_t = \frac{\Psi_t^{1-\sigma} Y_t^{1-\sigma}}{1 - \sigma} - \frac{\chi_{a,t} Y_t^{1+\eta}}{1 + \eta} \]

\[ + \lambda_{1,t} \left\{ \beta E_t \left[ \frac{\Psi_t^{\sigma} Y_t^{\sigma}}{\Pi_{t+1}} - \frac{\Psi_t^{-\sigma} Y_t^{-\sigma}}{R_t} \right] \right\} \]

\[ + \lambda_{2,t} \left\{ -\Psi_t^{-\sigma} Y_t^{1-\sigma} \frac{1 - \sigma}{Y_t} \left\{ \frac{\theta - 1}{\theta} Z_t + \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right\} + \lambda_{2,t} (1 + \eta) \chi_{a,t} Y_t^{1+\eta} - \Psi_t^{-\sigma} Y_t^{1-\sigma} \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right\} \]

\( \partial Y_t \) and \( \partial \Pi_t \):

\[ 0 = \Psi_t^{1-\sigma} Y_t^{1-\sigma} - \chi_{a,t} Y_t^{\eta} \]

\[ + \Psi_t^{-\sigma} Y_t^{1-\sigma} \frac{1 - \sigma}{Y_t} \left\{ \frac{\lambda_{1,t}}{R_t} \frac{\lambda_{1,t-1}}{\Pi_t} \right\} \]

\[ - \lambda_{2,t} \Psi_t^{-\sigma} Y_t^{1-\sigma} \frac{1 - \sigma}{Y_t} \left\{ \frac{1 - \sigma}{\theta} Z_t + \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right\} + \lambda_{2,t} (1 + \eta) \chi_{a,t} Y_t^{\eta} \]

\[ 0 = -\Psi_t^{-\sigma} Y_t^{1-\sigma} \frac{1 - \sigma}{Y_t} (\Pi_t - 1) \]

\[ + \Psi_t^{-\sigma} Y_t^{1-\sigma} \left\{ \frac{\lambda_{1,t}}{R_t} \frac{\lambda_{1,t-1}}{\Pi_t} \right\} \psi(\Pi_t - 1) - \frac{\lambda_{1,t-1}}{\Pi_t^2} \]

\[ - \lambda_{2,t} \Psi_t^{-\sigma} Y_t^{1-\sigma} \left( \frac{\sigma}{\Psi_t} \left( \frac{\theta - 1}{\theta} Z_t + \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right) \psi(\Pi_t - 1) + \frac{\psi}{\theta} (2P_t - 1) \right) \]

\[ + \lambda_{2,t-1} \Psi_t^{-\sigma} Y_t^{1-\sigma} \left( \frac{\sigma}{\Psi_t} \psi(\Pi_t - 1) \Pi_t \psi(\Pi_t - 1) + \frac{\psi}{\theta} (2P_t - 1) \right) \]
Simplifying this,

\[ 0 = 1 + \frac{\sigma}{C_t} \left\{ \frac{\lambda_{1,t}}{R_t} - \frac{\lambda_{1,t-1}}{\Pi_t} \right\} - (1 - \lambda_{2,t} (1 + \eta)) \frac{\chi_{a,t} C_t^{\sigma+\eta}}{\Psi_t^{\sigma+\eta}} + \lambda_{2,t} \left( \frac{\theta - 1}{\theta} Z_t + \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right) \frac{\sigma - 1}{\Psi_t} - \lambda_{2,t-1} \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \frac{\sigma - 1}{\Psi_t} \]

\[ 0 = -\psi (\Pi_t - 1) - \frac{\sigma}{C_t} \left\{ \frac{\lambda_{1,t}}{R_t} - \frac{\lambda_{1,t-1}}{\Pi_t} \right\} \psi (\Pi_t - 1) - \frac{\lambda_{1,t-1} \Psi_t}{\Pi_t^2 C_t} - \lambda_{2,t} \left( \frac{\theta - 1}{\theta} Z_t + \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right) \frac{\sigma}{\Psi_t} \psi (\Pi_t - 1) + \lambda_{2,t-1} \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \frac{\sigma}{\Psi_t} \psi (\Pi_t - 1) - (\lambda_{2,t} - \lambda_{2,t-1}) \frac{\psi}{\theta} (2\Pi_t - 1) \]

Hence,

\[ 0 = \left[ 1 + \frac{\sigma}{C_t} \left\{ \frac{\lambda_{1,t}}{R_t} - \frac{\lambda_{1,t-1}}{\Pi_t} \right\} + \left( \lambda_{2,t} \frac{\theta - 1}{\theta} Z_t + (\lambda_{2,t} - \lambda_{2,t-1}) \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right) \frac{\sigma - 1}{\Psi_t} \right] \]

\[ - (1 - \lambda_{2,t} (1 + \eta)) \frac{\chi_{a,t} C_t^{\sigma+\eta}}{\Psi_t^{\sigma+\eta}} \]

\[ 0 = \left[ 1 + \frac{\sigma}{C_t} \left\{ \frac{\lambda_{1,t}}{R_t} - \frac{\lambda_{1,t-1}}{\Pi_t} \right\} + \left( \lambda_{2,t} \frac{\theta - 1}{\theta} Z_t + (\lambda_{2,t} - \lambda_{2,t-1}) \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t \right) \frac{\sigma - 0}{\Psi_t} \right] \psi (\Pi_t - 1) + \frac{\lambda_{1,t-1} \Psi_t}{\Pi_t^2 C_t} + (\lambda_{2,t} - \lambda_{2,t-1}) \frac{\psi}{\theta} (2\Pi_t - 1) \]

Constraints:

\[ \beta E_t \left[ C_t^{\sigma} \frac{C_{t+1}^{\sigma}}{\Pi_{t+1}} \right] = \frac{C_t^{\sigma}}{R_t} \]

\[ \beta E_t \left[ C_t^{\sigma} \frac{\psi}{\theta \Psi_{t+1}} (\Pi_{t+1} - 1) \Pi_{t+1} \right] = C_t^{\sigma} \left( \frac{\psi}{\theta \Psi_t} (\Pi_t - 1) \Pi_t + \frac{\theta - 1}{\theta} \frac{Z_t}{\Psi_t} - \frac{\chi_{a,t} C_t^{\sigma+\eta}}{\Psi_t^{\sigma+\eta}} \right) \]

In SS,

\[ 1 - \frac{\theta / Z_{ss}}{\theta - 1} = \lambda_{2,t} (\sigma + \eta) \]

A.2 Derivation of the Analytical Solution to the LQ Method

For discretion,

\[ \pi_t = \beta E_t [\tilde{\pi}_{t+1}] - \kappa (\theta \tilde{\pi}_t - y^*) - \theta / \psi z_t \]

\[ z_t = \rho_z \tilde{z}_{t-1} + \xi_{z,t} \]
Assuming \( \pi_t = Ay^* + B\tilde{z}_t \) (method of undetermined coefficients), we find \( E_t[\tilde{\pi}_{t+1}] = Ay^* + B\rho_z z_t \), and NKPC implies

\[
Ay^* + B\tilde{z}_t = \beta Ay^* + \beta B\rho_z z_t - \kappa \theta (Ay^* + Bz_t) + \kappa y^* - \theta / \psi \tilde{z}_t
\]

Hence,

\[
0 = (A + \kappa \theta A - \kappa - \beta A) y^* + (B - \beta B\rho_z + \kappa \theta B + \theta / \psi) z_t
\]

Since this must hold for any realization of \( y^* \) and \( z_t \),

\[
A = \frac{\kappa}{1 - \beta + \kappa \theta} \quad \text{and} \quad B = \frac{-\theta / \psi}{1 - \beta \rho_z + \kappa \theta}
\]

Hence,

\[
\pi_t = \frac{\kappa}{1 - \beta + \kappa \theta} y^* + \frac{-1}{1 - \beta \rho_z + \kappa \theta} \psi z_t
\]

\[
\tilde{y}_t - y^* = \frac{-\kappa \theta}{1 - \beta + \kappa \theta} y^* + \frac{\theta}{1 - \beta \rho_z + \kappa \theta} \psi z_t
\]

For commitment, we again use the guess-and-verify method (the method of undetermined coefficients). Letting \( \lambda_{2,t} = \lambda_{2,t} / \psi \),

\[
\pi_t = M_{\pi y} y^* + M_{\pi \lambda} \lambda_{2,t-1} + M_{\pi \xi} z_t
\]

\[
\lambda_{2,t} = M_{\lambda y} y^* + M_{\lambda \lambda} \lambda_{2,t-1} + M_{\lambda \xi} z_t
\]

From NKPC and FONC,

\[
\beta \left( M_{\pi y} y^* + M_{\pi \lambda} \left( M_{\lambda y} y^* + M_{\lambda \lambda} \lambda_{2,t-1} + M_{\lambda \xi} z_t \right) + M_{\pi \xi} \rho_z z_t \right)
\]

\[
= \left( M_{\pi y} y^* + M_{\pi \lambda} \lambda_{2,t-1} + M_{\pi \xi} z_t \right) - \kappa y^* + \kappa \theta \left( M_{\lambda y} y^* + M_{\lambda \lambda} \lambda_{2,t-1} + M_{\lambda \xi} z_t \right) + \theta / \psi \tilde{z}_t
\]

\[
0 = \left( M_{\pi y} y^* + M_{\pi \lambda} \lambda_{2,t-1} + M_{\pi \xi} z_t \right) - \left( M_{\lambda y} y^* + M_{\lambda \lambda} \lambda_{2,t-1} + M_{\lambda \xi} z_t \right) + \lambda_{2,t-1}
\]

Sorting out terms,

\[
0 = [-\kappa + \kappa \theta M_{\lambda y} + M_{\pi y} - \beta (M_{\pi y} + M_{\pi \lambda} M_{\lambda y})] y^* + [\kappa \theta M_{\lambda \lambda} + M_{\pi \lambda} - \beta M_{\pi \lambda} M_{\lambda \lambda}] \lambda_{2,t-1}
\]

\[
+ [\kappa \theta M_{\lambda \xi} + M_{\pi \xi} - \beta M_{\pi \lambda} M_{\lambda \xi} - \beta M_{\lambda \xi} \rho_z + \theta / \psi] z_t
\]

\[
0 = [M_{\pi y} - M_{\lambda y}] y^* + [M_{\pi \lambda} - M_{\lambda \lambda} + 1] \lambda_{2,t-1} + [\psi M_{\pi \xi} - M_{\lambda \xi} \z_t
\]

Obviously, \( \psi M_{\pi y} = M_{\lambda y}, M_{\pi \xi} = M_{\lambda \xi} \) and \( M_{\pi \lambda} + 1 = M_{\lambda \lambda} \). Hence,

\[
y^* = M_{\pi y} + \kappa \theta M_{\pi y} - \kappa = \beta (M_{\pi y} + M_{\pi \lambda} M_{\pi y})
\]

\[
\lambda_{2,t-1} = M_{\pi \lambda} + \kappa \theta (M_{\pi \lambda} + 1) = \beta M_{\pi \lambda} (M_{\pi \lambda} + 1) \quad \text{or} \quad M_{\lambda \lambda} - 1 + \kappa \theta M_{\lambda \lambda} = \beta (M_{\lambda \lambda} - 1) M_{\lambda \lambda}
\]

\[
z_t = M_{\pi \xi} + \kappa \theta M_{\pi \xi} = \beta M_{\pi \lambda} M_{\pi \xi} + \beta M_{\pi \xi} \rho_z - \theta / \psi
\]
Note that the coefficient on $\lambda_{2,t-1}$ implies that $M_{\lambda\lambda}$ is the roots of the following function. Since it is globally convex and $f(0) = 1 > 0$ and $f(1) = -\kappa\theta < 0$, one of the roots is greater than 1 and the other is smaller than 1 (both are positive). Since the state variable $\lambda_{2,t}$ must be stable, we choose the latter: $0 < M_{\lambda\lambda} < 1$.

\[
f(M_{\lambda\lambda}) = \beta M_{\lambda\lambda}^2 - (1 + \beta + \kappa\theta) M_{\lambda\lambda} + 1
\]

Hence,

\[
M_{\lambda\lambda} = \frac{1}{2\beta} \left( (1 + \beta + \kappa\theta) - \sqrt{(1 + \beta + \kappa\theta)^2 - 4\beta} \right), \quad M_{\pi\lambda} = M_{\lambda\lambda} - 1
\]

\[
M_{\pi\gamma} = M_{\lambda\gamma} = \frac{\kappa}{(1 - \beta + \kappa\theta - \beta (M_{\lambda\lambda} - 1))} = \frac{\kappa}{\Xi_y}
\]

\[
M_{\pi\xi} = M_{\lambda\xi} = \frac{\theta}{\psi(1 - \beta \rho_z + \kappa\theta - \beta (M_{\lambda\lambda} - 1))} = -\frac{\theta/\psi}{\Xi_\xi}
\]

Verification:

\[
\beta E_t [\pi_{t+1}] = \beta \frac{\kappa}{\Xi_y} y^* + \beta (M_{\lambda\lambda} - 1) \frac{\kappa}{\Xi_y} y^* + \beta (M_{\lambda\lambda} - 1) M_{\lambda\lambda} \lambda_{2,t-1} - \frac{\beta (M_{\lambda\lambda} - 1)}{\Xi_\xi} \theta \psi z_t - \frac{\beta \rho_z \theta}{\Xi_y} z_t
\]

\[
\pi_t + \kappa \left( \theta \lambda_{2,t} - y^* \right) + \frac{\theta}{\psi} z_t
\]

\[
= \frac{\kappa}{\Xi_y} y^* + (M_{\lambda\lambda} - 1) \tilde{\lambda}_{2,t-1} - \frac{1}{\Xi_\xi} \frac{\theta}{\psi} z_t + \kappa \theta \frac{\kappa}{\Xi_y} y^* + \kappa \theta M_{\lambda\lambda} \tilde{\lambda}_{2,t-1} - \frac{\kappa \theta \theta}{\Xi_\xi} z_t - \kappa y^* + \frac{\theta}{\psi} z_t
\]

Hence,

\[
y^* : \beta \frac{\kappa}{\Xi_y} (M_{\lambda\lambda} + 1 - \beta) = \frac{\kappa}{\Xi_y} (1 + \kappa\theta - \Xi_y)
\]

\[
\tilde{\lambda}_{2,t-1} : \beta (M_{\lambda\lambda} - 1) M_{\lambda\lambda} = (M_{\lambda\lambda} - 1) + \kappa \theta M_{\lambda\lambda}
\]

\[
\frac{\theta}{\psi} z_t : -\frac{\beta (M_{\lambda\lambda} - 1) + \beta \rho_z}{\Xi_\xi} = -\frac{1 - \kappa \theta + \Xi_\xi}{\Xi_\xi}
\]

**A.3 Derivation for Bias Decomposition for LQ framework**

Since we want to evaluate the welfare, we need to track the terms independent from policy, which are not necessary when we are interested only in the equilibrium behavior of the variables.

First note that, in the efficient steady state, $C_{ef} = Y_{ef} = H_{ef} = \chi^{\frac{1}{1+\eta}}$, which implies that $\chi H_{ef}^{1+\eta} = C_{ef}^{1-\sigma} = \chi^{\frac{1-\sigma}{1+\eta}}$. Define $u_t = u[C_t] = C_t^{\frac{1-\sigma}{1+\sigma}}$ and $v_t = v[H_t] = H_t^{\frac{1-\eta}{1+\eta}}$, where $C_t$ and $H_t$ are the realized value of consumption and labour under each model assumption. Applying the second
order Taylor approximation, we obtain:

\[
    u_t \simeq u_{ef} + C_{ef}^{1-\sigma} (\ln C_t - \ln C_{ef}) + \frac{1-\sigma}{2} C_{ef}^{1-\sigma} (\ln C_t - \ln C_{ef})^2 + \mathcal{O} \left( |\Psi, \xi_t|^3 \right)
\]

\[
    v_t \simeq v_{ef} + \chi H_{ef}^{1+\eta} (\ln H_t - \ln H_{ef}) + \frac{1+\eta}{2} \chi H_{ef}^{1+\eta} (\ln H_t - \ln H_{ef})^2 + \mathcal{O} \left( |\Psi, \xi_t|^3 \right)
\]

Hence, the period utility is approximated as

\[
    u_t - v_t \simeq (u_{ef} - v_{ef}) + \frac{c_{t}}{\chi^{1+\eta}} \left( c_t - h_t \right) + \frac{1}{2} \left( (1-\sigma) c_t^2 - (1+\eta) h_t^2 \right) + \mathcal{O} \left( |\Psi, \xi_t|^3 \right)
\]

where \( c_t = \ln C_t \), \( h_t = \ln H_t \) and \( V_{ef} = (u_{ef} - v_{ef}) / (1-\beta) \). Knowing that the solution is linear under the LQ framework, \( c_{ss} = E [c_t] \).

### A.3.1 Quadratic Approximation to Period Utility

We want to express this in terms of output gap \( y_t = \ln Y_t - \ln Y_{ef} \) and inflation \( \pi_t = \ln P_t / P_{t-1} \) by using non-expectational constraints. While the actual values depends on the assumption in each regimes, there are common expressions among them.

\[
    C_t = \Psi_t Y_t \quad \text{where} \quad \Psi_t = 1 - \frac{\psi}{2} (\Pi_t - 1)^2
\]

\[
    H_t = Y_t / A_t
\]

Again, applying the 2nd order Taylor approximation around the efficient steady state:\n
\[
    c_t = y_t - \frac{\psi}{2} \pi_t^2 + \mathcal{O} \left( |\Psi, \xi_t|^3 \right)
\]

\[
    h_t = y_t - a_t
\]

Hence,

\[
    c_t^2 = y_t^2 + \mathcal{O} \left( |\Psi, \xi_t|^3 \right)
\]

\[
    h_t^2 = (y_t - a_t)^2 = y_t^2 - 2a_t y_t + a_t^2
\]

\[2\]

\[
    u_t = \frac{e^{(1-\sigma) \ln C_t}}{1-\sigma}, \quad \frac{\partial u_t}{\partial \ln C_t} = e^{(1-\sigma) \ln C_t}, \quad \frac{\partial^2 u_t}{\partial \ln C_t^2} = (1-\sigma) e^{(1-\sigma) \ln C_t}
\]

\[3\]

\[
    \ln C_t = \ln Y_t + \ln \left( 1 - 0.5 \psi (e^{\pi_t} - 1)^2 \right) \quad \text{where} \quad \pi_t = \ln \Pi_t.
\]

\[
    \frac{\partial \ln C_t}{\partial \ln \Pi_t}_{\Pi_{ef}} = 0 \quad \text{and} \quad \frac{\partial^2 \ln C_t}{\partial \ln \Pi_t^2}_{\Pi_{ef}} = \frac{-\psi^2 (\Pi_{ef} - 1) \Pi_{ef}^2}{(1-0.5 \psi (\Pi_{ef} - 1)^2)^2} + \frac{\psi (\Pi_{ef} - 2) \Pi_{ef}^2}{(1-0.5 \psi (\Pi_{ef} - 1)^2)^2} = -\psi
\]
Hence,

\[ c_t - h_t = a_t - \frac{\psi}{2} \pi_t^2 + \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \]

\[ (1 - \sigma) c_t^2 - (1 + \eta) h_t^2 = - (\sigma + \eta) y_t^2 + 2 (1 + \eta) a_t y_t - (1 + \eta) a_t^2 + \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \]

\[ = - (\sigma + \eta) \left[ y_t^2 - 2 \left( \frac{1 + \eta}{\sigma + \eta} \right) a_t y_t + \left( \frac{1 + \eta}{\sigma + \eta} \right)^2 a_t^2 \right] - (1 + \eta) a_t^2 + \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \]

Hence,

\[ u_t - v_t \simeq (u_{ef} - v_{ef}) - \frac{\chi^{\frac{n-1}{n}}}{2} \left[ (\sigma + \eta) \left( y_t - \frac{1 + \eta}{\sigma + \eta} a_t \right)^2 + \psi \pi_t^2 - \frac{(1 + \eta)(1 - \sigma)}{\sigma + \eta} a_t^2 - 2 a_t \right] + \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \]

(6)

**Remarks:**

1) This expression holds for all cases with discretion, commitment and flexible price. However, of course, the actual values of \( y_t \) and \( \pi_t \) depends on the policy regime and the degree of price stickiness.

2) Term \( \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \) implies that this quadratic approximation is not a good approximation if \( \Psi_t \) is large. This is what Gali (2008, Ch.5) and Benigno and Woodford (2003) discuss. Hence, one of our objectives is to evaluate the approximation error due to this quantitatively.

### A.3.2 Additive Decomposition and LQ approximation

As in the main text, let \( Y_t \) be output under either discretion or commitment, which does not matter because the following discussion is applicable to both. As shown,

\[ y_t = \ln Y_t - \ln Y_{ef} = \ln Y_t - \ln Y_t - \ln \bar{Y}_t + \ln \bar{Y}_t - \ln \bar{Y}_{ss} + \ln \bar{Y}_{ss} - \ln Y_{ef} \]

\[ = \bar{y}_t + \bar{y}_t + \bar{y}_{ss} \]

Substituting this \( y_t \) into (6),

\[ V_0 - V_{ef} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ - \psi \chi^{\frac{n-1}{n}} \frac{\pi_t}{2} \left[ \alpha_y (\bar{y}_t - \bar{y}_{ss})^2 + \pi_t^2 \right] + t.i.p. + \mathcal{O} \left( |\Psi, \xi_t|^{1/3} \right) \right\} \]

\[ t.i.p = \chi^{\frac{n-1}{n}} \left( a_t + \frac{(1 + \eta)(1 - \sigma)}{2(\sigma + \eta)} a_t^2 \right) \]

Note that \( \bar{y}_t = \frac{1 + \eta}{\sigma + \eta} a_t \). Hence, we know \( y^* = \bar{y}_{ss} \).
Figure 1: The unconditional expectations (long-run average) computed by the Euler equation iteration. The upper eight panels show the unconditional expectations, while the lower eight panels show the values in the non-stochastic steady state. The x-axis of all panels is the degree of monopolistic distortion $\Phi = 1 - Z_{ss}/Z_{ef}$. 
Figure 2: The gap between commitment and discretion and between commitment and flexible price computed by the Euler equation iteration. The upper eight panels show the gap of the unconditional expectations between discretion and commitment. The lower eight panels show the same gap between commitment and flexible price. The x-axis of all panels is the degree of monopolistic distortion $\Phi = 1 - Z_{ss}/Z_{ef}$. 
Figure 3: The policy functions at the non-stochastic steady state computed by the Euler equation iteration. The upper half shows the results of the commitment case, of which the first two lines are for $\Phi = 0.0$ and the last two lines are for $\Phi = 0.15$. For commitment, $\lambda_{2,t-1}$ is set at its steady state level. The lower half is for the discretion case. 26
Figure 4: The unconditional expectations (long-run average) computed by the LQ method. The upper eight panels show the unconditional expectations, while the lower eight panels show the values in the non-stochastic steady state. The x-axis of all panels is the degree of monopolistic distortion $\Phi = 1 - Z_{ss}/Z_{ef}$. 


Figure 5: The welfare loss of stochastic variations computed by the LQ method. Each line shows the gap between unconditional expectation and the non-stochastic steady state values. The x-axis of all panels is the degree of monopolistic distortion $\Phi = 1 - \frac{Z_{ss}}{Z_{ef}}$. 
Figure 6: The gap between commitment and discretion and between commitment and flexible price computed by the LQ method. The upper eight panels show the gap of the unconditional expectations between discretion and commitment. The lower eight panels show the same gap between commitment and flexible price. The x-axis of all panels is the degree of monopolistic distortion \( \Phi = 1 - \frac{Z_{ss}}{Z_{ef}} \).
Figure 7: The policy functions at the non-stochastic steady state computed by the LQ method. The upper half shows the results of the commitment case, of which the first two lines are for $\Phi = 0.0$ and the last two lines are for $\Phi = 0.15$. For commitment, $\lambda_{2,t-1}$ is set at its steady state level. The lower half is for the discretion case.