Sales, Inventories, and Real Interest Rates: A Century of Stylized Facts

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Abstract

We use Bayesian time-varying parameters structural VARs with stochastic volatility to investigate changes in both the reduced-form and the structural correlations between business inventories and either sales growth, or the real interest rate, in the United States during both the interwar and the post-WWII periods. We identify four structural shocks by combining a single long-run restriction, to identify a permanent output shock as in ?, with three sign restrictions to identify demand- and supply-side transitory shocks.

We produce several new stylized facts which should inform the development of new models of business inventories. In particular, we show that (i) during both the interwar and the post-WWII periods, the structural correlation between inventories and the real interest rate conditional on identified interest rate shocks has been systematically positive; (ii) the reduced-form correlation between the two series has been positive during the post-WWII period, but—in line with the predictions of theory—it had been robustly negative during the interwar era; and (iii) during the interwar era the correlations between inventories and either of the two other series had exhibited a remarkably strong co-movement with output at the business-cycle frequencies.

Keywords: Bayesian VARs; stochastic volatility; time-varying parameters; structural VARs; long-run restrictions; sign restrictions; inventories; monetary policy; monetary regimes.

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1 Introduction

Inventories have been at the center of modern economics ever since its inception as a quantitative discipline. Starting with accelerator models designed to capture the inventory cycle (Kaldor, 1940), inventory behavior has attracted an enormous amount of attention in macroeconomics. The chief reason is that while inventory investment makes up only a small fraction of GDP, roughly one half of a percent, it can contribute up to ninety percent of its cyclical variation (Blinder and Maccini, 1991). At the same time, the inventory literature is rife with puzzles in the sense that theoretical inventory models cannot reproduce salient empirical facts in the data. Maccini et al. (2010) classify two sets of puzzles. The traditional puzzles tend to describe the unconditional properties of inventories, such as the relative volatilities of inventories, production and sales, their persistence, and their comovement relationships with potential determinants. The second set of puzzles concerns the relationship between interest rates and inventories. While theory derives clear predictions - increases in interest rates lower inventory investment as the cost of holding inventories rises -, the empirical literature is unable to find any relationship.

In this paper we discuss one explanation of why these puzzles exist, namely the surprising difficulty of establishing fixed stylized facts over a century of U.S. data. We focus on the relationship between final sales, inventories and interest rates and utilize data from 1919 on. Using Bayesian time-varying parameter VARs with stochastic volatility we study changes in both the reduced-form and the structural correlations between these variables in the United States during both the interwar and the post-WWII periods. Our estimates are based on the identification of four structural shocks, whereby we combine a single long-run restriction to identify a permanent output shock with three sign restrictions to identify transitory demand- and supply-side shocks.

Our main findings are the following. We first show that the absence of a negative correlation between inventories and interest rates only pertains to the post-WWII period. During the interwar era, the correlation is, in fact, strongly negative. Second, we find that the behavior of this relationship is asymmetric over the business cycle and over subperiods. During the interwar period, the interest rate-inventories correlation exhibits positive co-movement with real output at business-cycle frequencies. Although the correlation is overall systematically negative, it is relatively less negative in business-cycle upswings, and even more negative during downturns. During the post-WWII period, on the other hand, the correlation does not exhibit any clear-cut systematic pattern of co-movement with the cyclical component of real output. Finally, estimates form the structural VAR specification show that the negative reduced-form correlation of the interwar era is induced by demand- and supply-side transitory shocks, and, especially at longer horizons, by the permanent output shock. The contribution of interest rate shocks, on the other hand, is uniformly positive. As for the post-WWII era the positive reduced-form correlation between inventories and the interest rate can be traced back to the positive structural correlations induced
by interest rate shocks, and, to a lesser extent, by demand-side transitory shocks and the permanent output shock.

We also look at the relationship between inventories and sales. During the interwar era the correlation is mostly positive. At the business-cycle frequencies it exhibits strong negative co-movement with the cyclical component of real output. During the post-WWII period, and in line with the evidence reported in Wen (2005), the correlation is mostly negative at very short horizons, and uniformly positive at longer horizons. Evidence from the structural VAR suggests that the positive correlation of the interwar era is due to the demand shock, and in later periods of the sample, to the permanent output shock, whereas the correlation conditional on the other two shocks is negative. As for the post-WWII period, the evolution of the reduced-form correlation closely mirrors the evolution of the structural correlation conditional on the permanent output shock, being strongly positive both in the earlier and in the most recent parts of the sample at all horizons, and, at shorter horizons, being instead comparatively smaller, and sometimes negative, during the Great Inflation years.

Our results suggest that identified interest rate shocks have systematically and robustly induced a positive correlation between business inventories and the real interest rates during both the interwar and the post-WWII periods. The puzzle of the absence of a negative correlation between the two series over the post-WWII period identified in the previous literature is therefore deeper, and more intriguing, than previously thought. Even conditional on a structural shock for which both economic theory, and simple intuition, suggest that the correlation should be negative, our results highlight that, in fact, it has been systematically positive.

We see our paper as making the following contributions to the literature. First, we establish that the reduced form relationships between salient inventory variables vary substantially over the almost 100 years data in our sample. While it is possible to identify periods of stable reduced-form relationships, the changes are often enough that we find it difficult to treat them as invariant stylized facts. We argue that this presents a distinct challenge to theoretical models that is not easy to resolve. Conceptually, changes in reduced-form relationships can be attributed to structural changes in the underlying economic environment. Prime examples are shifts in the behavior of policymakers, technological innovation, or a decline in the incidence and volatility of exogenous shocks. We argue that the pervasive changes in the correlation patterns are unlikely to be explainable by these factors alone.

We conjecture that a solution of this issue is found by recognizing that a positive structural innovation to the interest rate has two effects. First, by raising the real interest rate, it causes, _ceteris paribus_, optimal inventories holdings to be reduced, which turns inventory investment negative. Second, by engineering a decline in output growth it causes an unanticipated fall in sales, which translates into unplanned inventories accumulation. Ultimately, whether an interest rate shock will cause an increase or a decrease in inventories will crucially hinge upon which of these two effects dominates. If the latter effect turns out to be stronger, an interest rate shock
will be associated with an increase in the real interest rate, and with an accumulation of inventories.

The second contribution we make is to the state of empirical modeling of time-varying parameter VARs. We also contribute to the broader empirical inventory literature in that we introduce Bayesian VAR techniques and the study of long time series. To the best of our knowledge, this is the first study of these inventory relationships that allows for time-variation in coefficients and innovations, that is based on Bayesian structural VAR methods, and that utilizes long time series, specifically including the interwar period.

It has been noted before that the behavior of inventories is different across sample periods. Perhaps most notably, changes in inventory management are often associated with changes in the behavior of other aggregate time series, as, for instance, in the Great Moderation. Much of the discussion thus focuses of selected episodes, but does not take into account whether the identified stylized facts are stable over other periods. One approach to the issue of sample selection is to identify time periods of interest, such as the shift from the Great Inflation to the Great Moderation period. this approach rests on the ability of the researcher to identify these episodes extraneously in the data. This may be reasonably plausible in the case of the Great Moderation, but is less convincing otherwise. An alternative is to compute statistics of interest using a rolling window approach. In this paper, we take the latter to its logical conclusion and estimate a Bayesian VAR with time-variation in coefficients and innovation variances for sales, inventories, inflation and nominal interest rates.

Our paper is part of a recent literature that estimates VARs with time-varying parameters. The central papers in this literature are Cogley and Sargent (2005), Primiceri (2005), and Sims and Zha (2006) who study the effects of monetary policy. More recently, Benati (2008) has studied inflation persistence over a long sample using Bayesian VAR methods.

The remainder of the paper is organized as follows. The next section describes the Bayesian methodology we use to estimate the time-varying parameters VARs with stochastic volatility. We detail our identification strategy in the case of the structural VAR, and we discuss the methodology to compute the VAR’s impact matrix. Section 3 discusses estimation results and evidence from the reduced-form specification, while Section 4 examines the structural evidence. We particularly focus on the effects of the identified monetary policy shocks and use the structural evidence to understand time variation in the reduced-form correlations. Section 5 concludes and offers directions for theoretical modeling.
2 Methodology

2.1 A Bayesian time-varying parameter VAR with stochastic volatility

In what follows we will work with the following time-varying parameters VAR($p$) model:

$$ Y_t = B_{0,t} + B_{1,t}Y_{t-1} + \ldots + B_{p,t}Y_{t-p} + \epsilon_t \equiv X_t'\theta_t + \epsilon_t $$

(1)

where the notation is obvious, and $Y_t$ is defined as $Y_t \equiv [\Delta i_t, \Delta s_t, \pi_t, r_t]'$, where $\Delta s_t$ is the log-difference of real sales; $\pi_t$ is inflation, computed as the log-difference of the relevant price index (the GNP deflator for the interwar period, and the GDP deflator for the post-WWII years, respectively); $r_t$ is the short-term rate (the three-month commercial paper rate, and the three-month Treasury bill rate, for the interwar and post-WWII periods, respectively), quoted at a non-annualised rate, in order to make its scale exactly comparable to that of inflation.\footnote{So, to be clear, if $R_t$ is the relevant short-term rate—with its scale such that, e.g., a ten per cent rate is represented as 10.0—$r_t$ is computed as $r_t=(1+R_t/100)^{1/4}-1$.} Finally, $\Delta i_t$ is the change in real inventories normalised by potential output:\footnote{To the very best of our knowledge, no previous study has performed any normalisation of the change in real inventories. Within the present context, however, expressing the change in real inventories in \textit{absolute} terms—rather than in \textit{percentage} terms, as it is the case for sales—would make the interpretation of our results especially difficult. Economic growth causes indeed the variance of the change in real inventories to increase without bounds, thus automatically introducing a systematic element of distortion in any comparison over time and across quarters. Consider for example the response of the economy to an identified interest rate shock. Since the change in sales is expressed in percentage terms, once the impulse-response functions (IRFs) have been appropriately normalised on the interest rate, comparing the response of sales across quarters is indeed appropriate and meaningful. This is however not the case for the change in real inventories: in this case, indeed, what the IRFs is telling us is by how much, in \textit{real dollars}, inventories would have changed in each quarter in response to a normalized interest rate shock. Since the economy in (say) 2009Q1 was significantly larger than it had been in 1959Q1, comparing the response to a normalized interest rate shock in the two quarters of the change in inventories expressed in constant dollars does not provide any meaningful information. This automatically implies that, for our results to be interpretable, the change in real inventories has to be normalised in such a way as to eliminate the impact of economic growth. We have chosen to normalise it by potential, rather than by actual output, in order to avoid distorting the cyclical properties of the resulting series.} for the interwar period we use Balke and Gordon’s (1986) estimate of potential GNP, whereas for the post-WWII years we use the \textit{Congressional Budget Office}’s estimate of potential GDP. For a complete description of the data and of their sources, see Appendix A. The overall sample periods are 1919Q1-1941Q4 for the interwar period,\footnote{This sample period is uniquely dictated by data availability, as the inventories series from Balke and Gordon (1986) starts in 1919Q1, and is not available for the period 1942Q1-1946Q4.} and 1949Q1-2011Q1 for the post-WWII era. For both sample periods, however, we use the first 10 years of data in order to compute the Bayesian priors, so that the effective sample periods are 1929Q1-1941Q4 and 1959Q1-2011Q1, respectively. As it is customary in the literature
on Bayesian time-varying parameters VARs,\(^4\) we set the lag order to \(p=2\). The VAR’s time-varying parameters, collected in the vector \(\theta_t\), are postulated to evolve according to

\[
p(\theta_t \mid \theta_{t-1}, Q) = I(\theta_t) f(\theta_t \mid \theta_{t-1}, Q)
\]

with \(I(\theta_t)\) being an indicator function rejecting unstable draws—thus enforcing a stationarity constraint on the VAR—and with \(f(\theta_t \mid \theta_{t-1}, Q)\) given by

\[
\theta_t = \theta_{t-1} + \eta_t
\]

with \(\eta_t \sim N(0, Q)\). The VAR’s reduced-form innovations in \(1\) are postulated to be zero-mean normally distributed, with time-varying covariance matrix \(\Omega_t\) which, following established practice, we factor as

\[
\text{Var}(\epsilon_t) = A_t^{-1} H_t (A_t^{-1})'
\]

The time-varying matrices \(H_t\) and \(A_t\) are defined as:

\[
H_t = \begin{bmatrix} h_{1,t} & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 \\ 0 & 0 & 0 & h_{4,t} \end{bmatrix}, \quad A_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 \end{bmatrix}
\]

with the \(h_{i,t}\) evolving as geometric random walks,

\[
\ln h_{i,t} = \ln h_{i,t-1} + \nu_{i,t}
\]

For future reference, we define \(h_t = [h_{1,t}, h_{2,t}, h_{3,t}, h_{4,t}]'\). Following \(?\), we postulate the non-zero and non-one elements of the matrix \(A_t\)—which we collect in the vector \(\alpha_t = [\alpha_{21,t}, \alpha_{31,t}, ..., \alpha_{43,t}]'\)—to evolve as driftless random walks,

\[
\alpha_t = \alpha_{t-1} + \tau_t
\]

and we assume the vector \([\epsilon_t', \eta_t', \tau_t', \nu_t']'\) to be distributed as

\[
\begin{bmatrix} u_t \\ \eta_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N(0, V), \quad \text{with } V = \begin{bmatrix} I_4 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & Z \end{bmatrix} \quad \text{and } Z = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}
\]

where \(u_t\) is such that \(\epsilon_t = A_t^{-1} H_t^\frac{1}{2} u_t\). As discussed in \(?\), there are two justifications for assuming a block-diagonal structure for \(V_t\). First, parsimony, as the model is already quite heavily parameterized. Second, ‘allowing for a completely generic correlation

\(^4\)See e.g. \(?\), \(?\), \(?\), \(?\), and \(?\).
structure among different sources of uncertainty would preclude any structural interpretation of the innovations\(^5\). Finally, following, again, \(^7\) we adopt the additional simplifying assumption of postulating a block-diagonal structure for \(S\), too—namely

\[
S = \text{Var}(\tau_t) = \text{Var}(\tau_t) = \begin{bmatrix}
S_1 & 0_{1 \times 2} & 0_{1 \times 3} \\
0_{2 \times 1} & S_2 & 0_{2 \times 3} \\
0_{3 \times 1} & 0_{3 \times 2} & S_3
\end{bmatrix}
\]  \hspace{1cm} (9)

with \(S_1 \equiv \text{Var}(\tau_{21,t})\), \(S_2 \equiv \text{Var}(\tau_{31,t}, \tau_{32,t})\), and \(S_3 \equiv \text{Var}(\tau_{41,t}, \tau_{42,t}, \tau_{43,t})\), thus implying that the non-zero and non-one elements of \(A_t\) belonging to different rows evolve independently. As discussed in Primiceri (2005, Appendix A.2), this assumption drastically simplifies inference, as it allows to do Gibbs sampling on the non-zero and non-one elements of \(A_t\) equation by equation.

### 2.2 Estimation

We estimate (1)-(9) via standard Bayesian methods. Appendix B discusses our choices for the priors, and the Markov-Chain Monte Carlo algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data.

### 2.3 Identification

We identify four shocks. Following \(^7\), the first shock is defined as the only one exerting a permanent impact on log output, and it is identified via a long-run restriction. The other three shocks, which, by construction, only exert a transitory impact on log output, are identified via a standard set of sign restrictions.

#### 2.3.1 The permanent output shock

Table 1 reports, for either the interwar or the post-WWII period, results from augmented Dickey-Fuller (henceforth, ADF) tests with trend for the logarithms of real sales and real output (real GNP for the former period, and real GDP for the latter one), and from ADF tests without trend for their difference. For reasons of robustness \(p\)-values have been computed based on two alternative methodologies, that is, either by (i) simulating 10,000 random-walks (with drift for the ADF tests with a trend, and without drift for the tests without a trend), or (ii) by bootstrapping 10,000 times estimated ARIMA\((p,1,0)\) processes for the relevant series (the lag order of the ARIMA process has been selected based on the Akaike information criterion). In all cases, the simulated or bootstrapped processes are of length equal to the series under investigation.\(^6\) As for the lag order of the ADF tests, since, as it is well known, results

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\(^5\)Primiceri (2005, pp. 6-7).

\(^6\)To be precise, letting \(T\) be the length of the series under investigation, we either simulate or bootstrap an artificial series of length \(T+100\), and we then discard the first 100 observations in order to eliminate dependence on initial conditions.
from unit root tests may be sensitive to the specific lag order which is being used, once again for reasons of robustness we consider three alternative lag orders: one, two, and four quarters.

For either period it is not possible to reject the null of a unit root for the logarithms of either real sales or real output. On the other hand, in line with basic economic logic, which suggests that real sales should perfectly co-move with real output in the very long-run—that is, the two series should be cointegrated with cointegration vector \([1, -1]\)—for either period it is possible to reject at conventional significance levels the null of a unit root in their difference. The rejection is very strong for the post-WWII period, with all \(p\)-values uniformly equal to zero, whereas it is weaker for the interwar period, with \(p\)-values just below 10 per cent for the ADF tests with one lag, and equal instead to 2.5 and 2.9 per cent for the tests with four lags. Overall, the null of a unit root in the difference between the logarithms of real sales and real output for the interwar period can be rejected at the ten per cent level based on any lag order, and it can rejected at the five per cent level based on either two or four lags.\(^7\) In what follows we will therefore work under the assumption that, for either period, real sales and real output share a common stochastic trend, and, in line with ?, we will therefore identify permanent output shocks based on the restriction that they are the only shocks exerting a permanent impact on log sales.

### 2.3.2 The transitory shocks

The other three shocks—which, by construction, only exert a transitory impact on output—are identified based on the standard set of sign restrictions reported in the following table.

<table>
<thead>
<tr>
<th>Shock:</th>
<th>(e_t^R)</th>
<th>(e_t^D)</th>
<th>(e_t^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>(\geq 0)</td>
<td>(\geq 0)</td>
<td>(?)</td>
</tr>
<tr>
<td>Inflation</td>
<td>(\leq 0)</td>
<td>(\geq 0)</td>
<td>(\leq 0)</td>
</tr>
<tr>
<td>Real sales growth</td>
<td>(\leq 0)</td>
<td>(\geq 0)</td>
<td>(\geq 0)</td>
</tr>
<tr>
<td>? = left unconstrained</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^7\)It is important to stress that, since the interwar period is comparatively quite short, results from statistical tests based on interwar data should necessarily be treated with some caution, especially when they deal with properties of the data pertaining to the infinite long-run, such as in the present case. So failure to detect overwhelming evidence of stationarity for the difference between the logarithms of sales and output should be discounted, and should simply be regarded as an illustration of the weak results sometimes produced by unit root tests when applied to comparatively short samples. ? makes this point with reference to results from unit root tests of the logarithm of the consumption-output ratio. As he points out, '[Here look for the exact quotation]' consumption and output should be cointegrated with cointegration vector \([1, -1]\)',' so that failure to overwhelmingly reject stationarity of the ratio should not be regarded as a potential indictment of the permanent income hypothesis, but rather as an illustration of the weak results sometimes produced by unit root tests based on comparatively short samples.
An interest rate shock is identified based on the restriction that it exerts a non-negative impact on the interest rate, and a non-positive impact on both inflation and real sales growth; a demand non-interest rate shock is postulated to have a non-negative impact on either the interest rate, inflation, or real sales growth; finally, a transitory supply shock is sorted out from the other two because it is postulated to be the only one inducing a negative co-movement between inflation and real sales, whereas its impact on the interest rate is left unconstrained. These restrictions are the same used, e.g., by ? and ?—with the only difference that both papers featured output growth instead of real sales growth—and are compatible with a vast class of macroeconomic models.

An important aspect of our identification strategy is that, although we constrain the sign of the response of sales growth to the three transitory shocks, we instead leave the response of the change in inventories unconstrained. As for the permanent output shock, by assumption the responses of all variables to it are left unconstrained. This implies that the structural correlations between inventories and sales, and between inventories and the real interest rate—that is, the correlations conditional on either of the four structural shocks—are left entirely unconstrained. These correlations will be one of the key objects of our investigation.

2.3.3 Computing the VAR’s structural impact matrix

For each quarter, and for each draw from the ergodic distribution, we compute the time-varying structural impact matrix, \( A_{0,t} \), by combining the methodology proposed by ? for imposing sign restrictions \(^8\) and the procedure proposed by ? to impose long-run restrictions within a time-varying parameters VAR context. Specifically, let \( \Omega_t = P_t D_t P_t' \) be the eigenvalue-eigenvector decomposition of the VAR’s time-varying covariance matrix \( \Omega_t \), and let \( \tilde{A}_{0,t} = P_t D_t^{\frac{1}{2}} \). We draw an \( N \times N \) matrix, \( K \), from the \( N(0, 1) \) distribution, we take the QR decomposition of \( K \)—that is, we compute matrices \( Q \) and \( R \) such that \( K = Q \cdot R \)—and we compute the time-varying structural impact matrix as \( \tilde{A}_{0,t} = \tilde{A}_{0,t} \cdot Q' \). Following Gali and Gambetti (2009, Section II), we then compute a local approximation to the matrix of the cumulative impulse-response functions (henceforth, IRFs) to the VAR’s structural shocks as \(^9\)

\[
\tilde{C}_{t,\infty} = \left[ I_N - B_{1,t} - \ldots - B_{p,t} \right]^{-1} \tilde{A}_{0,t}
\]

where \( I_N \) is the \( N \times N \) identity matrix. We then rotate the matrix of the cumulative impulse-response functions \( \text{via} \) an appropriate Householder matrix\(^10\) \( H \) in order to

\(^8\)See at http://home.earthlink.net/~tzha02/ProgramCode/SRestrictRWZalg.m.

\(^9\)The only difference between Gali and Gambetti (2009, Section II) and the present work is that they compute the local approximation to the matrix of the cumulative IRFs based on the companion form of the VAR, whereas we compute it directly based on the VAR itself.

\(^10\)We compute the Householder matrix \text{via} Algorithm 5.5.1 of ?.
introduce zeros in all of the second row of $C_{t,\infty}$—that is, the row corresponding to log real sales—except for the (2,1) entry, so that the second row of the resulting local approximation to the matrix of the cumulative impulse-response functions,

$$C_{t,\infty} = \tilde{C}_{t,\infty} H = C_0 \tilde{A}_{0,t} H = C_0 A_{0,t}$$  \hspace{1cm} (11)$$
is given by $C_{t,\infty}^2 = [x \ 0_{(N-1)\times1}^t]$, with $0_{(N-1)\times1}$ being a vector of $(N-1)$ zeros, and $x$ being a non-zero entry. This implies that the first shock is the only one exerting a long-run impact on the level of log sales, and therefore on the level of log output. If the resulting structural impact matrix $A_{0,t} = \tilde{A}_{0,t} H$ satisfies the sign restrictions we keep it, otherwise we discard it and we repeat the procedure until we obtain an impact matrix which satisfies both the sign restrictions and the long-run restriction at the same time.

We now turn to an analysis of the evidence.

3 Reduced-Form Correlations

3.1 The interwar period

Figures 1 and 2 show, for the interwar period, evidence on changes in the correlation between the reduced-form forecast errors for the change in real inventories and the ex post real interest rate at various horizons,\textsuperscript{11} whereas Figures 3 and 4 show the same objects for the change in real inventories and real sales growth. Specifically, Figures 1 and 3 show, in the top row, the median, and the one- and two-standard deviation percentiles of the posterior distribution of the correlations between the forecast errors for the relevant series generated by the time-varying VAR at each point in time; and in the bottom row the fraction of the draws from the posterior distribution for which the correlation is positive. The top rows of Figures 2 and 4, on the other hand, show the trend components of the series shown in the bottom rows of Figures 1 and 3, respectively. Finally, the bottom rows of Figures 2 and 4 show the business-cycle components of the series shown in the bottom rows of Figures 1 and 3, respectively, together with the business-cycle component of the logarithm of real GNP.\textsuperscript{12} For each

\textsuperscript{11} The long-run correlation has been computed based on the long-run covariance matrix of forecast errors of the time-varying VAR. In turn, this is computed as the limit of the covariance matrix of the forecast errors at horizon $k$, for $k$ which tends to infinity.

\textsuperscript{12} Trends and business-cycle components have been extracted via the band-pass filter proposed by ?. Following established conventions in business-cycle analysis—see e.g. ? and ?—business-cycle frequencies have been defined as those pertaining to fluctuations with frequencies of oscillation between 6 quarters and 8 years, whereas the low (or ‘trend’) frequencies have been defined as those associated with fluctuations with cycles slower than 8 years. Since the fractions of draws for which the correlation has been positive are bounded between 0 and 1, we have performed band-pass filtering of the logit of such fractions, and we have then taken the inverse logit transformation of the resulting components. This has been done in order to eliminate the possibility of ‘non-sensical’ trend estimates, that is, estimated low-frequency components which violate the [0 1] boundaries.
quarter, the correlations between the forecast errors for the relevant series at the various horizons generated by the time-varying VAR have been computed based on the estimated covariance matrix of the forecast errors, which we have computed as in \(^\text{13}\).

Several facts are readily apparent from figures 1-4. Starting from the correlation between the forecast errors for the change in real inventories and the \textit{ex post} real interest rate, as Figures 1 and 2 clearly show, \textit{first}, such correlation had been almost uniformly \textit{negative} during the entire period between the late 1920s and the end of 1941. This stands in marked contrast with the post-WWII years, during which, as it is well known,\(^\text{14}\) the change in real inventories has been systematically \textit{positively} correlated with real interest rates. \textit{Second}, as the bottom panel of Figure 1 shows, the fraction of draws for which the correlation had been positive had exhibited quite significant fluctuations over the sample period. Both on impact, and at all horizons, the fraction of draws had reached a minimum immediately after April 1933, when President Roosevelt took the dollar off gold,\(^\text{15}\) and had instead reached local maxima both in \([???]\) and in \([??]\). An intriguing question is whether such fluctuations in the fractions of draws from the posterior distribution for which the correlation had been positive did bear any systematic relationship with key macroeconomic variables, and in particular with the state of the business cycle. As Figure 2 shows, this actually turns out to be the case. The top row, showing the trend components of the series plotted in the bottom row of Figure 1, highlights in an even starker manner how such correlation had systematically been negative during those years. The most interesting piece of evidence, however, is the one plotted in the bottom row, showing how, at all horizons, the business-cycle component of the fractions of draws for which the correlation had been positive had exhibited a striking positive correlation with the business-cycle component of the logarithm of real GNP.\(^\text{16}\) (On the other hand, casual inspection does not suggest any systematic leading or lagging pattern between the series.) This implies that, although the correlation had systematically been negative during the entire period, it had also tended, equally systematically, to be less negative during economic upswings, and even more negative during downswings. Although we do not have any ready-made explanation for \textit{why}, exactly, this might have been the case, the robustness of this stylized fact naturally suggests—at least, to us, ...—

\(^{13}\)See expressions (9) to (11) of \textsubscript{?}. An important point to stress here is that, given the two-sided nature of the estimates of the time-varying VAR produced by the Gibbs sampler, the VAR-generated forecast errors we are working with (and therefore their estimated covariance matrices) should only be regarded as approximations to the authentic objects we would obtain if we had estimated the VAR based on recursive samples. Such an approximation is routinely used in the literature—beyond \textsubscript{?}, see e.g. \textsubscript{?}—because of the significant computational burden associated with recursive estimation of time-varying parameters VARs.

\(^{14}\)\[Here put a few quotations\]

\(^{15}\)See \textsubscript{?}.

\(^{16}\)\[IMPORTANT: here also look at the correlation with the business-cycle component of the \textit{ex post} real rate\]
that it should find its origin in some deep economic mechanism. Further, as we now discuss with respect to the relationship between the change in real inventories and real sales growth, such a systematic pattern of variation at the business-cycle frequencies in the fraction of draws for which the correlation between the forecast errors is estimated to have been positive does not appear to be isolated, and it rather appears to have been a feature of the interwar period. As Figure 3 shows, with the single exception of the very early quarters of the sample, and of the very last quarter, the correlation between the forecast errors for the change in real inventories and real sales growth at the various horizons had been uniformly positive. Once again, however, the impression from the bottom row is of a quite significant extent of variation over the sample period, with a peak in the fractions of draws for which the correlation had been positive immediately after after April 1933, and, more generally, a pattern of variation which is, roughly, a mirror image of the one we saw for the change in real inventories and the \textit{ex post} real interest rate. As the bottom row of Figure 4 shows, this is indeed, and quite strikingly the case, with the fraction of draws for which the correlation had been positive exhibiting a remarkably strong negative correlation with the logarithm of real GNP at the business-cycle frequencies. Once again, although we do not have any explanation for why this might have been the case, the strength of this correlation, too, naturally suggests that it should find its origin in some deep economic mechanism. Let’s now turn to the post-WWII period, which has been the focus of previous research.

3.2 The post-WWII period

Figures 5 and 6 show evidence on changes in the correlation between the reduced-form forecast errors for the change in real inventories and the \textit{ex post} real interest rate at various horizons, whereas Figures 7 and 8 show the same objects for the change in real inventories and real sales growth. Starting from the former set of variables, evidence suggests that, conceptually in line with previous research\textsuperscript{17}—and in contrast with what we would expect based on basic economic logic—the correlation between reduced-form innovations to the change in real inventories and the \textit{ex post} real rate has been almost uniformly positive at all horizons for the entire post-WWII period. Minor exceptions to this rule are represented by the very first quarters of the sample, for which a majority of the draws from the posterior distribution is associated with a negative correlation for horizons up to one year; and by the first half of the 1980s, for which, on impact, a slight majority of the draws was associated with a negative correlation. Different from previous research, which never investigated time-variation in the correlation between the two variables, our results point however towards non-negligible changes in the correlation over the sample period. Such fluctuations are especially apparent on impact, when, based on the results reported in the bottom row of Figure 5, the correlation turns from negative, during the very first quarters

\textsuperscript{17}[here put references]
of the sample, to strongly positive during the entire period up to the launch of the Volcker disinflation; it then changes sign once again, very briefly, during the first half of the 1980s; and it then exhibits a strong hump-shaped pattern over the following period, with a peak of essentially one around the turn of the century, and a dramatic decrease during subsequent years, with the fraction of draws for which the correlation is positive falling below 50 per cent in the last few quarters of the sample. Evidence for other horizons is weaker, but it replicates the very broad features of the evidence on impact. In line with the previously discussed evidence for the interwar period, Figure 6 splits the series shown in the bottom row of Figure 5 into the components associated with the low and the business-cycle frequencies. In contrast with the interwar period, however, at the business-cycle frequencies the fractions of draws for which the correlation has been positive has not exhibited any stable relationship with the logarithm of real GDP, co-moving with it sometimes positively, and sometimes negatively, but without any systematic discernible pattern. Finally, turning to the relationship between inventories and sales growth (see Figures 7 and 8), first, conceptually in line with the evidence reported in \(^{18}\), the correlation between the two series’ forecast errors has been predominantly negative on impact, and it has instead become more and more positive at longer horizons. Second, evidence points towards a significant extent of time-variation in the correlations between the two series’ forecast errors at the various horizons, which is especially clear from the bottom row of Figure 7. Specifically, both on impact, and at all subsequent horizons, the fractions of draws for which the correlation has been positive has exhibited a broadly V-shaped pattern over the sample period, although with significant short-run fluctuations around such a broad trend. Further, and intriguingly, some of these fluctuations clearly appear to coincide with key events in U.S. post-WWII monetary history, with the fraction of draws for which the correlation has been positive exhibiting significant decreases in correspondence with both the collapse of Bretton Woods, and the Volcker disinflation of the early 1980s (this is especially apparent for horizons up to one year ahead). Such a coincidence provides clear \textit{prima facie} evidence that a non-negligible portion of the time-variation in the correlation between the forecast errors documented in Figure 7 originates from some of the same fundamental macroeconomic forces which have shaped U.S. post-WWII macroeconomic dynamics—in particular, forces connected to the evolution of the U.S. monetary regime. Finally, as shown in Figure 8, the frequency-domain-based decomposition of the evolution of the fractions of draws for which the correlation has been positive points towards a positive co-variation with the logarithm of real GDP at the business-cycle frequencies since the early 1980s, whereas evidence for the former period is not entirely clear-cut, but it seems to suggest some negative co-variation. This points towards an element of continuity between the earliest part of the post-WWII period and the interwar era (during which, as we discussed

\(^{18}\)We say ‘conceptually in line’ because Wen’s (2005) evidence was based on a completely different methodology—band-pass filtering—and he did not explore in any way changes in the correlation over time.
in the previous sub-section, the business-cycle component of the fraction of draws had exhibited a remarkably strong negative correlation with the cyclical component of log real GDP), and suggests that, under this respect, the most recent period represents a discontinuity with the pattern which had been prevailing during previous decades.

Let’s now turn to the structural evidence.

4 Structural Evidence

4.1 Impulse-response functions

4.1.1 The interwar period

Figures 9 to 12 show, for selected quarters of the interwar period, the normalized generalized IRFs of the change in inventories, real sales growth, and the ex post real interest rate to each of the four identified structural shocks. Generalised IRFs have been computed via the Monte Carlo integration procedure described in Appendix C, which allows to effectively tackle the uncertainty originating from future time-variation in the VAR’s structure.

The IRFs to a permanent output shock have been normalized so that the median of the distribution of the cumulative IRFs of sales growth is equal to one (which implies that the median long-run impact of the shock on the logarithm of real GNP is, likewise, equal to one). The IRFs to an interest rate shock have been normalized so that the median impact at zero on the ex post real rate is equal to one. Finally, the IRFs to the transitory supply shock, and to the demand non-interest rate shock, have been normalized so that the median impact at zero on sales growth at zero is equal to one.

Starting from the permanent output shock—whose impact on either series, as we pointed out, is entirely unconstrained—a positive shock is estimated to have caused, with very high probabilities, a temporary fall in the ex post real rate, and a temporary increase in sales growth. As for the change in inventories, the extent of uncertainty surrounding the median estimates is such that, for none of the quarters under examination, it is possible to reject, at conventional significance levels, the null hypothesis of no reaction of inventories to the permanent output shock. The median estimates, however, point towards a difference between the period up to the very early 1930s and subsequent years, with the IRFs for the latter period immediately jumping upwards and then decreasing almost monotonically, and those for the former period exhibiting virtually no reaction on impact, and then a more complex oscillatory pattern at longer horizons. Overall, however, for none of the three series evidence clearly points towards time-variation in their response to the permanent output shock.

\[19\] We normalize the IRFs to an interest rate shock on the ex post real rate—rather than on the nominal rate, as it is customary—because the relationship between inventories and the real rate is one of the key objects of interest in our investigation.
Turning to the interest rate shock, the response of the series upon which the IRFs have been normalized, the *ex post* real rate, appears to have been virtually unchanged over the sample period, with a swift reversion towards zero during the quarters immediately following the impact. As for the other two series, for inventories the extent of uncertainty is so large that it is not possible to make statements with any confidence—in particular, it is not possible to reject the null of no impact at any horizon—but median estimates point towards a *positive* impact.20 As for sales growth, neither the negative impact (which we have imposed in identification), nor subsequent dynamics, exhibit discernible changes over the sample period. A positive demand non-interest rate shock is estimated to have caused, with very high probabilities, a temporary increase in the change in inventories, and a temporary decrease in the *ex post* real rate, which during subsequent quarters rebounds above its equilibrium level, before fading out. None of the IRFs for either of the three series exhibits any discernible time-variation over the sample period. Finally, a transitory supply shock is estimated to have caused temporary increases in both sales growth and the *ex post* real rate, whereas for inventories uncertainty, once again, is so large that it is not possible to reject the null of no impact at any horizon. Median estimates, however, point towards a temporary decrease in the change in inventories. Once again, for none of the series evidence points towards manifest changes in the IRFs over the sample period.

4.1.2 The post-WWII period

Figures 13 to 16 show, for selected quarters of the post-WWII period, the normalized IRFs of the change in inventories, real sales growth, and the *ex post* real interest rate to each of the four identified structural shocks. Normalization has been implemented as for the interwar period. For either series, IRFs to a permanent output shock have exhibited a significant extent of time-variation over the post-WWII period. With very high probability, the response of the change in inventories had been positive and essentially monotonically decreasing at the very beginning of the sample, and positive, and hump-shaped, over the last two decades a half, but it had exhibited a very different pattern during the 1970s, with a significant fraction of the probability mass at short horizon steering into negative territory. The IRFs for sales growth have been uniformly positive over the entire sample period, but the magnitude of the impact at zero has exhibited a significant extent of variation, reaching comparative minima during the Great Inflation years, and maxima both at the beginning of the sample, and since the mid-1980s. Finally, the IRFs of the *ex post* real rate exhibit a significant extent of variation in terms of both the median response, and the uncertainty surrounding it. For both the early part of the sample, and the most recent years, it is not possible to reject the null hypothesis that the permanent output shock did not

20 [Stress the importance of this, because the reduced-form correlation between inventories and the *ex post* real rate had been negative during the interwar period]
have any impact on the real rate. Around the time of the Volcker stabilization, on
the other hand, the IRF had been negative with very high probability.

Turning to the interest rate shock, neither of the three variables’ IRFs exhibits a
significant extent of time-variation. The \textit{ex post} real rate (the variable on which the
IRFs have been normalized) jumps up on impact and then reverts monotonically to
zero. Based on median estimates, the speed of reversion appears to have been greater
during the Great Inflation period than either before or after that, but the extent of
uncertainty is such that, even based on the simple ‘eyeball metric’, it is not possible to
reject the null hypothesis of no variation in the IRFs over the sample period. Mean-
reversion is significantly faster for sales growth, which jumps downwards on impact,
reverts essentially to zero in the following quarter, and then keeps on oscillating
around zero for a few quarters after that. Finally, the IRFs for inventories—which are
of particular interest, as they have been left entirely unconstrained—point towards
a positive reaction to interest rate shocks. This is especially apparent based on
the median estimates, according to which inventories jump upwards on impact, and
then revert to zero almost uniformly monotonically, but with the exception of the
very early part of the sample it clearly emerges even when considering the entire
posterior distributions of the IRFs. The responses of business inventories and the
real interest rate to an identified interest rate shock therefore clearly suggest that the
puzzle of a positive correlation between the two series over the post-WWII period
does not uniquely pertain to their reduced-form relationship, and on the contrary
it also emerges when we consider their joint responses to an interest rate structural
innovation. This finding is even more surprising, and even more puzzling, than the one
of a positive reduced-form correlation between the two series. In principle, a positive
reduced-form correlation is indeed compatible with a negative correlation conditional
on interest rate shocks—which is what we would expect to find—provided that (i)
at least one of the structural correlations conditional on the other shocks is positive,
and (ii) the fractions of variance of inventories and the real interest rate explained
by the shocks generating positive conditional correlations are sufficiently large. To
put it differently, a positive reduced-form correlation may result from the fact that
the negative correlation generated by interest rate shocks gets ‘swamped’ by the
positive correlation induced by at least one of the other shocks. Our results, however,
clearly suggest that this is not the case, and that, on the contrary, even interest
rates shocks generate a positive co-movement between the two series. Why should
that be the case? We conjecture that the mechanism at work may be the following.
An unanticipated structural innovation to the interest rate has two effects. \textit{First}, it
increases both the nominal and the real interest rates. This latter effect, based on
standard economic theory, will cause, ceteris paribus, a decrease in firms’ desired level
of inventories, and therefore a negative change in inventories. \textit{Second}, the interest rate
shock will cause an unexpected fall in sales, and therefore an undesired accumulation
of inventories. So the fact that an interest rate shock causes a positive or a negative
change in inventories crucially hinges upon which of these two effects dominates: if
the extent of the undesired accumulation of inventories due to the recessionary effect of the interest rate shock turns out to be greater than the extent of their planned decrease due to the higher real interest rate, an interest rate shock will turn out to be associated with an increase in inventories, rather than a decrease. An important point to stress, however, is that at the current stage this is just a conjecture, which should be verified within the context of a structural model.

A positive demand non-monetary shock is estimated to have caused, on impact, a small and statistically insignificant decrease in the ex post real rate, whose magnitude has remained essentially unchanged over the sample period. The evolution of the posterior distributions of the IRFs, however, clearly suggests that—conceptually in line with a vast literature on the comparatively weak response of U.S. monetary policy to inflationary shocks during the 1970s\textsuperscript{21}—the response of the real interest rate has been very weak during the Great Inflation period, and comparatively much stronger in the earlier part of the sample, and especially since the mid-1980s. During the 1970s, in particular, the response of the ex post real rate has not been significantly different from zero at any horizon, whereas over the last two decades and a half it has been significantly positive for several quarters, and, based on median estimates, quantitatively large. As for sales growth, based on median estimates the response appears to have been slower, and more drawn out, around the time of the Great Inflation, but the extent of uncertainty is such that it is not possible to reject the hypothesis of no variation in the IRFs over the sample period. Concerning the response of inventories, median estimates point towards a sizeable extent of time-variation over the sample period, with the impact at zero being negative in the early part of the sample, turning clearly positive around the time of the Great Inflation, and then turning again negative, and then positive, over subsequent years. For most quarters, however, the extent of uncertainty is such that, overall, the evolution of the IRFs does not obviously point towards time-variation.

Finally, IRFs to the transitory supply shock point towards no time variation for sales growth, for which the positive impact essentially disappears after one quarter, and, based on median estimates, little time-variation for the ex post real rate. It is notable, however, how for the latter variable the extent of uncertainty has dramatically increased over the last two decades and a half. As for inventories, whose IRFs are entirely unconstrained, first, the magnitude of the impact at zero—which has been consistently negative over the entire sample period—had been comparatively smaller around the time of the Great Inflation, and comparatively larger both before and after that; and second, the peak impact has exhibited an analogous hump-shaped pattern over the sample period, having been comparatively greater during the Great Inflation years, and instead smaller both in previous and in subsequent years.

Let’s now turn to the structural correlations between the series, that is, to the correlations conditional on the identified structural shocks.

\textsuperscript{21}See in particular ?, and the references therein.
4.2 Structural correlations

Our discussion, in Section 3, of the evolution of the reduced-form correlations between business inventories and either the \textit{ex post} real interest rate, or sales growth, naturally raises two questions.

\textit{First}, which structural shocks have historically generated such correlations?

\textit{Second}, what are the causes of time-variation in the two correlations? Specifically, has time-variation been due to changes in the relative importance of the different shocks hitting the economy, or changes in the way their impact has propagated through the system?

In this section we tackle the former question, whereas in the next we address the latter.

4.2.1 The interwar period

Figures 17 and 19 show the structural correlations—that is, the correlations conditional on individual identified shocks—between inventories and either the \textit{ex post} real interest rate or sales growth, whereas Figures 18 and 20 show the fractions of draws for which the structural correlations have been positive. We only consider structural correlations at horizons greater than zero because, on impact, the correlation between two series conditional on a single shock is, by construction, always equal to one. At horizons greater than zero, on the other hand, the dynamics of the system ‘kicks in’, so that even correlations conditional on a single shock are no longer equal to one by construction.

As we saw in Section 3.1, the reduced-form correlation between the change in inventories and the real rate had been systematically negative during the entire interwar period. As Figures 17 and especially 18 show, such a negative correlation had been due to (\textit{i}) demand- and supply-side transitory shocks, uniformly over the entire period, and (\textit{ii}) the permanent output shock, in particular after April 1933, and at longer horizons. In line with our discussion of the IRFs in Section 4.1.1, on the other hand, the correlation conditional on interest rate shocks had been systematically \textit{positive} over the entire sample period, thus highlighting, once again, the puzzling nature of the relationship between interest rates and business inventories.

As for the correlation between inventories and sales, Figures 19 and 20 show how the largely positive correlation of the interwar era had been due to (\textit{i}) demand non-interest rate shocks, over the entire sample, and to (\textit{ii}) the permanent output shock, especially at longer horizons, whereas both interest rate shocks and supply transitory shocks systematically induced a negative correlation over the entire sample period.

4.2.2 The post-WWII period

Turning to the post-WWII era, Figures 21 and 22 show the structural correlations between inventories and either the \textit{ex post} real interest rate or sales growth, whereas
Figures 23 and 24 show the fractions of draws for which the structural correlations have been positive. As we saw in Figure 5, the reduced-form correlation between the real rate and business inventories has been almost uniformly positive during the post-WWII period. Figures 21 and, especially, 22 show a complex picture, according to which such positive reduced-form correlation has been due (i) uniformly, over the entire sample period, to the interest rate shock; (ii) to the permanent supply shock at short horizons, and less so at longer horizons; and (iii) to the demand non-interest rate shock during the period following the beginning of the Volcker stabilization and, to a lesser extent, before the collapse of Bretton Woods, whereas during the Great Inflation period the contribution of this shock was mildly negative. On the other hand, the bottom row of Figure 22 shows how the transitory supply shock has mostly contributed negatively to the correlation.

Figure 7 showed that the reduced-form correlation between inventories and sales has been strongly positive at most horizons, with the exception of the very short ones. Figures 23 and especially 24 show that (i) both the interest rate and the transitory supply shock generated an uniformly negative correlation at all horizons over the sample period; (ii) the demand non-interest rate shock contributed positively to the correlation before the beginning of the Volcker stabilization, and especially during the Great Inflation years; and (iii) the evolution of the reduced-form correlation has closely mirrored the evolution of the correlation conditional on the permanent output shock, being very strongly positive at the very beginning of the sample, oscillating around zero around the time of the Great Inflation, and then becoming, once again, very strongly positive during the most recent part of the sample.

4.3 Understanding time-variation in the reduced-form correlations

Figures 25-31 provide evidence on the underlying causes of the changes in the reduced-form correlations between business inventories and either the ex post real rate or sales growth over the two sample periods. Overall,

(i) as for the interwar period, evidence points towards no role of changes in the VAR’s coefficients in fostering changes in either of the two correlations, which had instead originated from changes in both the volatilities of structural shocks, and the way in which such shocks have impacted upon the economy.

(ii) Concerning the post-WWII period, changes in both the shocks’ volatilities, and the way in which they have impacted upon the variables, appear to have played a dominant role, but it is possible to detect some impact of changes in the VAR’s coefficients for the correlation between inventories and the real rate at longer horizons.

Getting into details, Figure 25 shows, for either period, the medians of the posterior distributions of the VAR’s time-varying coefficients (the $B_{j,t}$’s), of the logarithms of the diagonal elements of the matrix $H_t$ (the $h_{ii,t}$’s), and of the off-diagonal elements of the matrix $A_t$ (the $\alpha_{hk,t}$’s). As for the interwar era, the first column points
towards essentially no time-variation in the VAR coefficients; no time-variation for three volatilities out of four (the one exhibiting time-variation is the one pertaining to the equation for [...]); and a non-negligible extent of time-variation in the off-diagonal elements of the $A_t$. This automatically implies that the VAR coefficients cannot possibly have played any role in fostering changes in the two correlations. On the other hand, it is not possible to further disentangle the separate roles played by changes in the structural shocks’ volatilities, and changes in the way the shocks have impacted upon the economy. Indeed, from equation (4), and from the definition of the structural impact matrix $A_{0,t}$, we have, under the assumption of unit-variance structural shocks, $\Omega_t = A_t^{-1}H_t(A_t^{-1})' = A_{0,t}A_{0,t}'$. However, this is equivalent to

$$\Omega_t = A_t^{-1}H_t(A_t^{-1})' = A_{0,t}A_{0,t}' = A_{0,t}V_t\hat{A}_{0,t}' = \hat{A}_{0,t}$$

where the $v_{j,t}$’s are the non-unitary, and possibly time-varying volatilities of the structural shocks, and $\hat{A}_{0,t}$ is the structural impact matrix associated with such shocks, for any values of the $v_{j,t}$’s. So the bottom line is that time-variation in the elements of the matrices $A_t$ and $H_t$ during the interwar era, which is what the top row of Figure 25 documents, cannot be automatically mapped into time-variation in the $v_{j,t}$’s and the elements of $\hat{A}_{0,t}$, due to an identification problem. The only thing we can say for sure, based on this evidence, is that the VAR coefficients played no role in the variation of the reduced-form correlations. This is also apparent from Figure 26, showing the fractions of draws from the posterior distribution for which the two correlations had been positive. The black lines, which have been computed by keeping, for each draw from the posterior distribution, the VAR coefficients at their estimated values for the first quarter, are identical to the red lines, which are the same shown in Figures 1 and 3.

Turning to the post-WWII era, the bottom row of Figure 25 clearly points towards time-variation in all of the VAR elements, so that, based on this kind of evidence, it is not possible to rule out the impact of any of them. Figure 27, showing the fractions of the draws for which the correlations have been positive computed by keeping the VAR coefficients at their estimated values for the first quarter, show (i) some minor impact for the correlation between inventories and sales, for which the correlation thus computed is uniformly very close to the one we saw in Figure 7, which had been computed by taking into account of time-variation in the VAR coefficients; and (ii) a non-negligible impact for the correlation between inventories and the real rate at longer horizons. It is important to stress, however, that this evidence does not alter a key message of Figure 5: even disregarding time-variation in the VAR coefficients, the correlation is still almost uniformly positive at all horizons.
Finally, Figures 28-31 repeat the same exercise of Figures 26 and 27, but this time for the structural correlations. For the interwar period, in line with Figure 26 even for the structural correlations the impact of changes in the VAR’s coefficients has obviously been *nil*. For the post-WWII period, on the other hand, for the correlation between inventories and the real rate the impact has been essentially *nil* conditional on the interest rate shock; it has been minor, and only at longer horizons, for the one conditional on the transitory supply shock; it has been significant at longer horizons for the demand non-interest rate shock; and it has instead been quite substantial at all horizons for the permanent output shock. For the correlation between inventories and sales, on the other hand, the impact has been negligible for all shocks, and at all horizons.

5 Conclusions

[...]
A The Data

Here follows a detailed description of the dataset.

A.1 Interwar period

Quarterly seasonally adjusted series for real GNP (the acronym is RGNP72), real potential GNP (TRGNP), the GNP deflator (GNPDEF), nominal GNP (GNP), the change in nominal business inventories (DBUSINOM), and the commercial paper rate (CPRATE) are all from the Bureau of Economic Analysis’ National Income and Product Accounts (henceforth, NIPA). The sample period is 1919Q1-1941Q4. Real sales have been computed as the difference between nominal GNP and the change in nominal inventories, deflated by the GNP deflator, whereas the change in real inventories has been computed as the ratio between the change in nominal inventories and the GNP deflator (the series thus obtained is near-identical to Balke and Gordon’s series for the change in real inventories, DBUSI72).

A.2 Post-WWII period

A quarterly seasonally adjusted series for the GDP deflator is from Table 1.1.9 of the Bureau of Economic Analysis’ National Income and Product Accounts (henceforth, NIPA). Quarterly seasonally adjusted series for nominal GDP and the change in nominal inventories are from Table 1.1.5. of the NIPA. The series for potential GDP (‘GDPPOT: Real Potential Gross Domestic Product, U.S. Congress: Congressional Budget Office, Budget and Economic Outlook, Quarterly, Billions of Chained 2005 Dollars’) is from FRED II. The series for real GDP has been computed as the ratio between nominal GDP and the GDP deflator. The series thus obtained is numerically near-identical to the chain-weighted series for real GDP, GDPC96, which can be found, e.g., at the St. Louis FED’s internet data portal, FRED II (‘GDPC96: Real Gross Domestic Product, 3 Decimal, U.S. Department of Commerce: Bureau of Economic Analysis, Gross Domestic Product, Seasonally Adjusted Annual Rate, Quarterly, Billions of Chained 2005 Dollars’). This justifies our use of the Congressional Budget Office’s chain-weighted estimate of potential real GDP in order to normalize the change in real inventories. The series for the change in real inventories has been computed as the ratio between the change in nominal inventories and the GDP deflator. Real sales have been computed as the difference between nominal GDP and the change in nominal inventories, deflated by the GDP deflator. A monthly seasonally unadjusted series for the 3-Month Treasury Bill: Secondary Market Rate (acronym is TB3MS) is from the Board of Governors of the Federal Reserve System, and it has been converted to the quarterly frequency by taking averages within the quarter.
B Details of the Markov-Chain Monte Carlo Procedure

We estimate (1)-(8) via Bayesian methods. The next two subsections describe our choices for the priors, and the Markov-Chain Monte Carlo algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data, while the third section discusses how we check for convergence of the Markov chain to the ergodic distribution.

B.1 Priors

For the sake of simplicity, the prior distributions for the initial values of the states—\(\theta_0\) and \(h_0\)—which we postulate all to be normal, are assumed to be independent both from each other, and from the distribution of the hyperparameters. In order to calibrate the prior distributions for \(\theta_0\) and \(h_0\) we estimate a time-invariant version of (1) based on the first 10 years of data, and we set

\[
\theta_0 \sim N \left( \hat{\theta}_{OLS}^0, 4 \cdot \hat{V}(\hat{\theta}_{OLS}) \right)
\]

(B1)

where \(\hat{V}(\hat{\theta}_{OLS})\) is the estimated asymptotic variance of \(\hat{\theta}_{OLS}\). As for \(h_0\), we proceed as follows. Let \(\hat{\Sigma}_{OLS}\) be the estimated covariance matrix of \(\epsilon_t^t\) from the time-invariant VAR, and let \(C\) be its lower-triangular Cholesky factor—i.e., \(CC' = \hat{\Sigma}_{OLS}\). We set

\[
\ln h_0 \sim N(\ln \mu_0^0, 10 \times I_N)
\]

(B2)

where \(\mu_0\) is a vector collecting the logarithms of the squared elements on the diagonal of \(C\). As stressed by ?, ‘a variance of 10 is huge on a natural-log scale, making this weakly informative’ for \(h_0\).

Turning to the hyperparameters, we postulate independence between the parameters corresponding to the two matrices \(Q\) and \(A\)—an assumption we adopt uniquely for reasons of convenience—and we make the following, standard assumptions. The matrix \(Q\) is postulated to follow an inverted Wishart distribution,

\[
Q \sim IW \left( Q^{-1}, T_0 \right)
\]

(B3)

with prior degrees of freedom \(T_0\) and scale matrix \(T_0Q\). In order to minimize the impact of the prior, thus maximizing the influence of sample information, we set \(T_0\) equal to the minimum value allowed, the length of \(\theta_t\) plus one. As for \(Q\), we calibrate it as \(Q = \gamma \times \hat{\Sigma}_{OLS}\), setting \(\gamma=1.0 \times 10^{-4}\), the same value used in ?, and a slightly more ‘conservative’ prior (in the sense of allowing for less random-walk drift) than the \(3.5 \times 10^{-4}\) used by ?. As for \(\alpha\), we postulate it to be normally distributed with a ‘large’ variance,

\[
f (\alpha) = N(0, 10000 \cdot I_{N(N-1)/2}).
\]

(B4)
Finally, as for the variances of the stochastic volatility innovations, we follow Cogley and Sargent (2002, 2005) and we postulate an inverse-Gamma distribution for \( \sigma_i^2 \equiv \text{Var}(\nu_{i,t}) \):

\[
\sigma_i^2 \sim IG \left( \frac{10^{-4}}{2}, \frac{1}{2} \right)
\]  

\( \text{(B5)} \)

**B.2 Simulating the posterior distribution**

We simulate the posterior distribution of the hyperparameters and the states conditional on the data \( \text{via} \) the following MCMC algorithm, as found in \( ? \). In what follows, \( x^t \) denotes the entire history of the vector \( x \) up to time \( t \) — i.e. \( x^t \equiv [x_1', x_2', \ldots, x_t'] \) —while \( T \) is the sample length.

(a) **Drawing the elements of \( \theta_t \)** Conditional on \( Y^T, \alpha, \) and \( H^T \), the observation equation (1) is linear, with Gaussian innovations and a known covariance matrix. Following \( ? \), the density \( p(\theta^T | Y^T, \alpha, H^T) \) can be factored as

\[
p(\theta^T | Y^T, \alpha, H^T) = p(\theta_T | Y^T, \alpha, H^T) \prod_{t=1}^{T-1} p(\theta_t | \theta_{t+1}, Y^T, \alpha, H^T)  
\]  

\( \text{(B6)} \)

Conditional on \( \alpha \) and \( H^T \), the standard Kalman filter recursions nail down the first element on the right hand side of (A6), \( p(\theta_T | Y^T, \alpha, H^T) = N(\theta_T, P_T) \), with \( P_T \) being the precision matrix of \( \theta_T \) produced by the Kalman filter. The remaining elements in the factorization can then be computed via the backward recursion algorithm found, e.g., in \( ? \), or Cogley and Sargent (2005, appendix B.2.1). Given the conditional normality of \( \theta_t \), we have

\[
\theta_{t|t+1} = \theta_{t|t} + P_{t|t} P_{t+1|t}^{-1} (\theta_{t+1} - \theta_t)  
\]  

\( \text{(B7)} \)

\[
P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}  
\]  

\( \text{(B8)} \)

which provides, for each \( t \) from \( T-1 \) to 1, the remaining elements in (1), \( p(\theta_t | \theta_{t+1}, Y^T, \alpha, H^T) = N(\theta_t, P_{t|t}) \). Specifically, the backward recursion starts with a draw from \( N(\theta_T, P_T) \), call it \( \hat{\theta}_T \) Conditional on \( \hat{\theta}_T \), (A7)-(A8) give us \( \theta_{T-1|T} \) and \( P_{T-1|T} \), thus allowing us to draw \( \hat{\theta}_{T-1} \) from \( N(\theta_{T-1|T}, P_{T-1|T}) \), and so on until \( t=1 \).

(b) **Drawing the elements of \( H_t \)** Conditional on \( Y^T, \theta^T, \) and \( \alpha \), the orthogonalised innovations \( u_t \equiv A(Y_t - X_t' \theta_t) \), with \( \text{Var}(u_t) = H_t \), are observable. Following \( ? \), we then sample the \( h_{i,t} \)’s by applying the univariate algorithm of \( ? \) element by element.\(^{22}\)

(c) **Drawing the hyperparameters** Conditional on \( Y^T, \theta^T, H^T, \) and \( \alpha \), the innovations to \( \theta_t \) and to the \( h_{i,t} \)’s are observable, which allows us to draw the hyperparameters—the elements of \( Q \) and the \( \sigma_i^2 \)—from their respective distributions.

\(^{22}\)For details, see Cogley and Sargent (2005, Appendix B.2.5).
(d) Drawing the elements of $\alpha$ Finally, conditional on $Y^T$ and $\theta^T$ the $\epsilon_t$'s are observable, satisfying

$$A\epsilon_t = u_t$$  \hfill (B9)

with the $u_t$ being a vector of orthogonalized residuals with known time-varying variance $H_t$. Following ?, we interpret (B9) as a system of unrelated regressions. The first equation in the system is given by $\epsilon_{1,t} \equiv u_{1,t}$, while the following equations can be expressed as transformed regressions as

$$
\left( h_{2,t}^{-\frac{1}{2}} \epsilon_{2,t} \right) = -\alpha_{2,1} \left( h_{2,t}^{-\frac{1}{2}} \epsilon_{1,t} \right) + \left( h_{2,t}^{-\frac{1}{2}} u_{2,t} \right) \\
\left( h_{3,t}^{-\frac{1}{2}} \epsilon_{3,t} \right) = -\alpha_{3,1} \left( h_{3,t}^{-\frac{1}{2}} \epsilon_{1,t} \right) - \alpha_{3,2} \left( h_{3,t}^{-\frac{1}{2}} \epsilon_{2,t} \right) + \left( h_{3,t}^{-\frac{1}{2}} u_{3,t} \right) \\
\vdots \\
\left( h_{N(N-1)/2,t}^{-\frac{1}{2}} \epsilon_{N(N-1)/2,t} \right) = -\alpha_{N(N-1)/2,1} \left( h_{N(N-1)/2,t}^{-\frac{1}{2}} \epsilon_{1,t} \right) - \cdots \\
\cdots - \alpha_{N(N-1)/2,N(N-1)/2} \left( h_{N(N-1)/2,t}^{-\frac{1}{2}} \epsilon_{N(N-1)/2,t} \right) + \left( h_{N(N-1)/2,t}^{-\frac{1}{2}} u_{N(N-1)/2,t} \right)
$$

(B10)

where the residuals are independent standard normal. Assuming normal priors for each equation’s regression coefficients the posterior is also normal, and can be computed via equations (77) of (78) in Cogley and Sargent (2005, section B.2.4).

Summing up, the MCMC algorithm simulates the posterior distribution of the states and the hyperparameters, conditional on the data, by iterating on (a)-(d). In what follows, we use a burn-in period of 50,000 iterations to converge to the ergodic distribution, and after that we run 10,000 more iterations sampling every 10th draw in order to reduce the autocorrelation across draws.\footnote{In this we follow ?. As stressed by ?, however, this has the drawback of ‘increasing the variance of ensemble averages from the simulation’}

\section*{C Computing Generalised Impulse-Response Functions}

Here we describe the Monte Carlo integration procedure we use in Section 4.1 in order to compute the generalised IRFs to the structural shocks.

Randomly draw the current state of the economy at time $t$ from the Gibbs sampler’s output. Given the current state of the economy, repeat the following procedure 100 times.

- Draw four independent $N(0, 1)$ variates (the four structural shocks), and based on the relationship $\epsilon_t = A_0.t \epsilon_t$, with $\epsilon_t \equiv [\epsilon_t^{SP}, \epsilon_t^{R}, \epsilon_t^{D}, \epsilon_t^{ST}]'$—where $\epsilon_t^{SP}, \epsilon_t^{R}, \epsilon_t^{D}$, and $\epsilon_t^{ST}$ are the permanent output shock, and the interest rate, demand non-interest rate, and transitory supply structural shocks, respectively—compute the reduced-form shocks $\epsilon_t$ at time $t$.  \footnote{In this we follow ?. As stressed by ?, however, this has the drawback of ‘increasing the variance of ensemble averages from the simulation’}
• Simulate both the VAR’s time-varying parameters and the covariance matrix of its reduced-form innovations, $\Omega_t$, 40 quarters into the future. Based on the simulated $\Omega_t$, randomly draw reduced-form shocks from $t+1$ to $t+40$. Based on the simulated $\theta_t$, and on the sequence of reduced-form shocks from $t$ to $t+40$, compute simulated paths for the four endogenous variables. Call these simulated paths as $\tilde{X}_{t,t+40}$, with $j = 1, \ldots, 100$.

• Repeat the same procedure based on exactly the same simulated paths for the VAR’s time-varying parameters, the $\theta_t$; the same reduced-form shocks at times $t+1$ to $t+40$; and the same structural shocks $e^*_{SP}, e^*_{R}, e^*_{D},$ and $e^*_{ST}$ at time $t$, with the only difference that, in order to compute the GIRF to shock $e^*_x$, with $x = SP, R, D, ST$, you set $e^*_t = 1$. Call these simulated paths as $\bar{X}_{t,t+40}$, with $j = 1, \ldots, 100$.

For each of the 100 iterations define $irf_{t,t+40} = \tilde{X}_{t,t+40} - \bar{X}_{t,t+40}$. Finally, compute each of the 1,000 generalised IRFs as the mean of the distribution of the $irf_{t,t+40}$'s.
## Table 1 Results from augmented Dickey-Fuller tests

<table>
<thead>
<tr>
<th></th>
<th>p-values computed based on:</th>
<th>simulating random walks(^a)</th>
<th>bootstrapping estimated ARIMA processes(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>k=1</td>
<td>k=2</td>
</tr>
<tr>
<td><strong>Interwar period</strong></td>
<td>I: Augmented Dickey-Fuller tests with trend</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log real sales</td>
<td></td>
<td>0.997</td>
<td>0.943</td>
</tr>
<tr>
<td>Log real GNP</td>
<td></td>
<td>0.999</td>
<td>0.975</td>
</tr>
<tr>
<td><strong>Post-WWII period</strong></td>
<td></td>
<td>0.827</td>
<td>0.827</td>
</tr>
<tr>
<td>Log real sales</td>
<td></td>
<td>0.856</td>
<td>0.850</td>
</tr>
<tr>
<td>Log real GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Interwar period</strong></td>
<td>II: Augmented Dickey-Fuller tests without trend</td>
<td>0.095</td>
<td>0.046</td>
</tr>
<tr>
<td>Log real sales minus log real GNP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Post-WWII period</strong></td>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Log real sales minus log real GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Based on 10,000 simulations. Random walks are with drift for ADF tests with trend, without drift for ADF tests without trend. \(^b\) Based on 10,000 bootstrap replications.
Table 2 reports, for either period, results from cointegration tests between the logarithms of real sales and real output, based on Johansen’s trace test statistic of the null of no cointegration against the alternative of one cointegrating vector. For the interwar period the null of no cointegration cannot be rejected at conventional significance levels, with a bootstrapped $p$-value for the trace statistics equal to 0.146. For the post-WWII period the null of no cointegration can be strongly rejected, with a bootstrapped $p$-value equal to zero, and an estimate of the second element of the normalized cointegration vector equal to 0.997, but the 99 per cent confidence interval, computed by bootstrapping the estimated cointegrated VAR, is equal to [-0.998, -0.995], so that, strictly speaking, the null hypothesis that the normalized cointegration vector is equal to $[1, -1]'$ can be rejected at the 1 per cent level.

Overall, statistical tests do not therefore uniformly support the notion that real sales and real output are cointegrated with cointegration vector $[1, -1]'$.

<table>
<thead>
<tr>
<th></th>
<th>interwar period</th>
<th>Post-WWII period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trace statistic</strong></td>
<td>16.142</td>
<td>93.331</td>
</tr>
<tr>
<td><strong>Bootstrapped p-value</strong></td>
<td>0.146</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Estimate of second element of normalized cointegration vector, and bootstrapped</strong> 99% confidence interval</td>
<td>—</td>
<td>-0.997 [-0.998, -0.995]</td>
</tr>
</tbody>
</table>

*a Based on 10,000 bootstrap replications of the estimated cointegrated VAR.