Trend Inflation and the Unemployment Volatility Puzzle

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Abstract

I show that the combination of small positive trend inflation with staggered prices may account for the large relative volatilities found in US labor market data. The model does not have any wage rigidity and is hit only by an aggregate technology shock. The calibration procedure uses standard parameter values. Controlling for the sample average of the CPI inflation rate and the degree of price stickiness, the model solves the Shimer (2005) puzzle and explains the volatilities observed during two important sample periods: full sample (1951-2005) and Great Moderation (1985-2005).

Keywords: Trend inflation, Staggered prices, Unemployment volatility puzzle. JEL codes: E2, E3, E5, J6.

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1 Introduction

The DMP model (after Diamond (1982), Mortensen (1982) and Pissarides (1985)) incorporates search and matching frictions into general equilibrium macroeconomic models to account for equilibrium unemployment. It is a simple but elegant description of the labor market. While searching for jobs, unemployed workers earn monetary transfers and leisure benefits. Firms search for workers and post job vacancies at a cost. Search frictions prevents all unemployed workers from getting a job and firm from filling all available vacancies. Instead, the probability that an unemployed worker is matched into a new job depends on the total unemployed labor force and on the total number of vacancies. After a match occurs, individual wages are set by a Nash bargaining between the newly hired worker and the firm.

The DMP model predicts that the standard deviations of key labor market variables (unemployment rate, vacancy postings, tightness ratio and job finding rate) are of the same order of magnitude as the standard deviation of the productivity shock. In contrast with this prediction, recent empirical evidence from US data strongly suggests that the standard deviations of the labor market variables are about ten times as large as what the model implies (see Shimer (2005), Hall (2005) and Costain and Reiter (2008)); hence, a puzzle.

In this paper, I contribute to the discussion on the unemployment volatility puzzle by showing that the combination of small positive trend inflation with staggered prices is able to generate the relative standard deviations of labor market quantities (with respect to the standard deviation of labor productivity) compatible with US evidence.

Most of the existing literature agrees that the unemployment volatility puzzle is best resolved by incorporating wage rigidities. As Pissarides (2009, pg. 1340) states, the “canonical model can deliver nontrivial volatility in unemployment only if there is at least some wage stickiness.” I depart from the literature by considering a model without any wage rigidity. By doing that, I am not implying that those rigidities are not important. Indeed, empirical results in the US strongly suggest that wage rigidities are not negligible. However, the evidence also points to relevant price staggering and to a non negligible positive inflation rate after the second world war. My main point is to show that trend inflation is just as relevant to explain the puzzle as wage rigidities are.

To formalize my analysis, I use a simple model with one sector and an aggregate technology shock, as described in section 2. For simplification purposes, staggered pricing is obtained by Calvo (1983) nominal rigidity with no indexation. Hours and wages are decided every period by Nash bargaining. Because I use quarterly frequency calibration, I follow Ravenna and Walsh (2010) to describe the labor flows and account for time-aggregation issues. Using both the extensive and intensive labor margins, the productivity metric is output per total hours worked, which I use for computing the endogenous relative volatilities to compare with the empirical assessments. Each differentiated firm decides simultaneously on vacancy posting and price setting as in Thomas (2008b). I depart from him mainly by adding a trend inflation analysis, which allows me to achieve the large relative volatilities. Thomas, on the other hand, assesses his model only under zero trend inflation. Even though his setup amplifies the relative volatilities in the labor market, the

1For papers with wage rigidities in all jobs, see e.g. Hall (2005), Gertler et al. (2008), Gertler and Trigari (2009), Thomas (2008a), Blanchard and Gali (2010). For wage rigidities in ongoing jobs only, see e.g. Haefke et al. (2008) and Pissarides (2009). Alternative approaches avoiding wage rigidities include: adding heterogeneous worker productivity (e.g. Pries (2008)); adding time-varying bargaining power (e.g. Ravenna and Walsh (2011)).
amplification is still very small.

In a slightly modified version of that model, Thomas (2011) shows that his setup is able to match the empirical values of an alternative measure of relative volatilities (normalized by the unconditional standard deviation of output) when the only exogenous shock hitting the economy is the technology one. However, under Calvo (1983) price stickiness and flexible wages, the output volatility will be much smaller than the productivity one when the economy is hit only by technology shocks. Thus normalizing the volatilities by the standard deviation of output pushes the relative volatilities upward. That is the reason his setup is able to match the empirical values of the alternative relative volatilities. I show that Thomas’ setup lacks sufficient amplifiers to match empirical assessments of the standard relative volatilities (normalized by the unconditional standard deviation of labor productivity) – section 3 presents the empirical evidence on the relative volatilities. For that, positive trend inflation is an important contribution.

Trend inflation is not an extra variable added to the model. It is only the level of the inflation rate in the steady state equilibrium. I show in section 4 that the reason why small levels of trend inflation are able to generate higher relative volatilities in market quantities is indeed simple. For that, I identify two groups of multipliers through which trend inflation increases the relative volatilities: (i) labor multipliers and (ii) pricing multipliers.

Labor multipliers are well understood in the labor market literature but, to the best of my knowledge, were never linked to the trend inflation literature. They were first identified by Hagedorn and Manovskii (2008) and Costain and Reiter (2008). They show that employment becomes more volatile as the total surplus decreases. If it is sufficiently small, then small movements in labor productivity have relevant impact on the total surplus and are able to cause proportionally larger fluctuations in the labor market.

I show that trend inflation is an important cause of small surpluses. As known in the literature on Dynamic Stochastic General Equilibrium (DSGE) models with staggered prices, inflation reduces welfare and profits. Therefore, the workers’ and firms’ surpluses from job matches fall.

Pricing multipliers, on the other hand, are well explored in the recent literature on trend inflation, but were never linked to the literature on search frictions in the labor market. Responses of the inflation rate and the output gap to technology shocks tend to be more volatile and take longer to fade as trend inflation rises in DSGE models with staggered prices. If the monetary authority follows a Taylor-type interest rate rule, as Ascari and Ropele (2009) find, larger levels of trend inflation strongly shrink the determinacy region of the policy parameter space under rational expectations equilibria (REE). This effect is relevant even for moderate levels of annual trend inflation such as 2 percent. As a consequence, the determinacy frontier progressively closes in around the point characterizing any determinacy-consistent policy rule as trend inflation rises. As that happens, fluctuations under determinate REE are amplified as the proximity to the indeterminacy region increases.

Adding a labor sector with search frictions to standard DSGE models with trend inflation, I show that the pricing channel also affects the volatilities of the labor variables.

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2Broadly speaking, the non-stochastic steady state equilibrium is defined as the one achieved when all disturbances are fixed at their means.

3See e.g. Woodford (2003) and Walsh (2010).

Firms simultaneously decide on vacancy postings and price setting, and hence the decisions are affected by the same environment conditions. Of course, the latter includes the trend inflation level.

As it turns out, the pricing multipliers have stronger effects than the labor ones.

I test my model by comparing its theoretical relative volatilities with empirical evidence in section 5. I consider two important periods: (a) the full sample period 1951:Q1-2005:Q4, which is often used in the literature for computing the empirical relative volatilities; and (b) the Great Moderation period 1985:Q1-2005:Q4. In terms of the standard relative volatilities (relative to labor productivity), the labor market has become slightly less volatile during the Great Moderation. However, the reduction is very small and the puzzle remains an important feature of the period. As for the alternative relative volatilities (relative to output), the empirical assessments parallel Thomas (2011) findings: they have actually increased during the Great Moderation.

The periods had slightly different average annual inflation rates (CPI): about 4 percent (full sample period) and 3 percent (Great Moderation period). Associated with the inflation reduction, Smets and Wouters (2007) find an increase in the degree of price stickiness. Thus, I consider specific calibration sets to reflect each period. The sets have the same parameter values, in line with the literature, except for the trend inflation levels and the degree of price stickiness, which are typical of each period.

I evaluate the theoretical relative volatilities implied by two policy frameworks that are often used in DSGE models: (i) a Taylor rule with contemporaneous inflation; and (ii) a Taylor rule with expected future inflation. The monetary policy implemented from the late 1960s to the 1970s, if described by a general Taylor rule, was probably not consistent with determinacy under rational expectations (see e.g. Coibion and Gorodnichenko (2011)). That fact may explain part of the observed large volatilities. However, the fact that I use Taylor rules consistent with determinacy is a parsimonious choice. The analysis under the Great Moderation period, on the other hand, is fully consistent with modeling monetary policy with the generalized Taylor rules.

I find that the theoretical volatilities are very close to their empirical counterparts in the labor market after controlling for the level of trend inflation and the degree of price stickiness. This control is sufficient for the model to achieve the slightly smaller relative volatilities of the Great Moderation period. Independently of the period studied, a general feature of the results is that the relative standard deviations of the labor market variables and the inflation rate monotonically increase with trend inflation. The relative volatility of the output gap, on the other hand, does not always increase as trend inflation rises. Its behavior depends mostly on the way monetary policy is implemented.

2 The model

The economy consists of a central bank that implements monetary policy, a representative household with a continuum of workers, and a unit mass of differentiated firms $z \in (0, 1)$. Each firm produces using labor in both the extensive and the intensive margins, posts job vacancies at a cost and makes price decisions, subject to Calvo (1983) price stickiness.\footnote{Following Thomas (2008b), I depart from the alternative approach that considers two production sectors for considering that individual firms indeed face the simultaneous problem of setting prices and posting vacancies. For the literature on two production sectors, see e.g. Blanchard and Gali (2010), Christoffel and Linzert (2006), Faia (2008), Gali (2010), Ravenna and Walsh (2010, 2011), Thomas} Workers can be hired or lose their jobs. The labor market is subject to search
frictions captured by a matching function. Wages and hours are decided in a flexible Nash bargaining framework.

2.1 Labor flows

At the end of period $t$, a fraction $n_t$ of household members is employed in existing jobs, where $n_t = \int_0^1 n_t(z) \, dz$ aggregates all end-of-period specific labor force $n_t(z) \in (0, 1)$ at firm $z$. At the beginning of each period, employed members separate from their jobs at the exogenous rate $\rho \in (0, 1)$. Therefore, the beginning-of-period unemployment rate $u_t$ accounts for the unemployed members at the end of last period $u^t_{t-1}$ and the recently separated workers. During the period, $m_t = \int_0^1 m_t(z) \, dz$ workers are matched into new jobs, where $m_t(z)$ is the number of matches into firm $z$. Firm $z$ posts $\nu_t(z)$ job vacancies at the end of each period. Therefore, $\nu_t = \int_0^1 \nu_t(z) \, dz$ is the total number of vacancy postings. Let $v_t(z) \equiv v^t_{t-1}(z)$ denote the number of job openings at firm $z$ available at the beginning of period $t$, and $v_t \equiv v^t_{t-1}$ the total number of job openings available at the same time. The laws of motion are described by

$$n_t(z) = (1 - \rho) n_{t-1}(z) + m_t(z) \quad ; \quad u_t = 1 - (1 - \rho) n_{t-1}$$

$$n_t = (1 - \rho) n_{t-1} + m_t \quad ; \quad u^t_t = 1 - n_t$$

(1)

In this context, $p_t \equiv m_t / u_t$ is the job-finding rate within the period, $q_t \equiv m_t / v_t$ is the matching rate for vacancies and $\theta_t \equiv v_t / u_t$ is the labor market tightness. As in Pissarides (2000), $m_t$ is given by the Cobb-Douglas matching function $m_t = \eta v_t^{1-a} u_t^a$. Assuming that the aggregate matching rate is given, the number of matches into firm $z$ is $m_t(z) = q_t v_t(z)$.

2.2 Households

The representative household has unions specialized in negotiating wage and hours with each firm. Union $z$ represents all $n_t(z)$ workers when bargaining with firm $z$ on hours per worker $h_t(z)$ and the total nominal wage $W_t(z) = P_t w_t(z)$ to be paid over the period, where $P_t$ is the aggregate price and $w_t(z)$ is the real wage.\(^6\)

The union’s disutility to its members’ hours worked $H_t(z) \equiv n_t(z) h_t(z)$ is $v_t(z) \equiv \chi H_t(z)^{1+\nu} / (1 + \nu)$. Since the unions belong to the representative household, its aggregate disutility function is $v_t \equiv \int_0^1 v_t(z) \, dz$.\(^7\) As in Merz (1995), I assume full risk sharing of consumption among household members, employed and unemployed.\(^8\) All members pool their income and evenly consume $c_t(z)$ units of good $z$. Unemployed workers earn monetary transfers from the government until they are matched into a firm. That generates $P_t w^n (1 - n_t)$ in nominal income for the household, where $w^n$ is the fixed real unemployment compensation. Consumption over all differentiated goods is aggregated

\(^6\)Note that $w_t(z)$ is not an hourly real wage.

\(^7\)Using a unions-based aggregate disutility function instead of a workers-based one allows me to derive closed form equations describing the dynamics of the aggregate disutility to work in Section 2.4, which is an important variable for understanding the amplified volatilities under trend inflation. The dynamics implied by the labor flows and by the Calvo price setting convolute in such a way that the derivation is not possible otherwise. The unions-based disutility also allows me to obtain the firms’ supply equations with no need to guess the loglinearized function forms to deal with the issue on firms’ specific labor, as done in Thomas (2008b).

\(^8\)Some authors have been making efforts to model imperfect consumption insurance and fully capture the distortions caused by unemployment. See e.g. Christiano et al. (2010).
into a bundle $C_t$, as in Dixit and Stiglitz (1977), and provides utility $u_t \equiv C_t^{1-\sigma} / (1 - \sigma)$. The household instantaneous utility is $u_t = v_t$. Aggregation and expenditure minimization relations are described by:

\[
(C_t)^{\frac{\omega-1}{\omega}} = \int_0^1 c_t(z)^{\frac{\omega-1}{\omega}} dz \quad ; \quad P^1_t - \phi = \int_0^1 p_t(z)^{1-\phi} dz
\]

\[
c_t(z) = C_t \left( \frac{p_t(z)}{P_t} \right)^{-\phi} \quad ; \quad P_1 C_t = \int_0^1 p_t(z) c_t(z) dz
\] (2)

The budget constraint is $P_t C_t + E_t Q_{t+1} A_{t+1} \leq A_t + \int_0^1 n_t(z) W_t(z) d_z + P_t w^u (1 - n_t) + \Xi_t$, where $\Xi_t$ denotes nominal profits net of lump-sum taxes that finance the unemployment transfers, $E_t$ is the time-$t$ expectations operator, $A_t$ is the state-contingent value of the portfolio of financial securities held at the beginning of period $t$, $Q_{t+1}$ is the stochastic discount factor from $t + 1$ to $t$ and satisfies $E_t Q_{t+1} = 1 / I_t$, where $I_t \equiv 1 + i_t$ is the gross riskless one-period nominal interest rate and $i_t$ is the nominal interest rate.

The household chooses $C_t$ and $A_{t+1}$ to maximize its welfare measure $W_t \equiv u_t - v_t + \beta E_t W_{t+1}$, subject to the budget constraint. The first-order conditions are the Euler equations $1 = \beta E_t \left[ (u'_{t+1} / u_t') (I_t / \Pi_{t+1}) \right]$ and $Q_{t+1} = \beta \lambda_{t+1} / \lambda_t$, where $u_t' \equiv \partial u_t / \partial C_t$ is the marginal utility to consumption, $\Pi_t \equiv 1 + \pi_t$ is the gross inflation rate and $\lambda_t = u_t' / P_t$ is the Lagrange multiplier on the budget constraint.

### 2.3 Firms

Firm $z$ uses $H_t(z)$ hours to produce its differentiated good with technology $y_t(z) = A_t H_t(z)^{\varepsilon}$, where $A_t$ is the aggregate technology shock and $\varepsilon \in (0, 1)$. As in Ravenna and Walsh (2010), posting $v_t(z)$ end-of-period vacancies requires $k v_t^e(z)$ units of the final aggregate good.

The firm optimally posts $v_t(z)$ vacancies every period and, with probability $(1 - \alpha)$, optimally readjusts its price to $p_t(z) = p_t'$. With probability $\alpha$ its price is fixed at $p_t(z) = p_{t-1}(z)$. For that, the firm maximizes its expected discounted flow of profits subject to its demand curve and to the law of motion of $n_t(z)$.

Wages $w_t(z)$ and hours per worker $h_t(z)$ are decided by Nash bargaining and maximize $b \log (u_t(z)) + (1 - b) \log (j_t(z))$, where $b$ is the workers’ bargaining power, and $u_t(z)$ and $j_t(z)$ are the worker’s and firm’s real match surpluses when the marginal worker is matched into firm $z$. See appendix C for details on computing the surpluses and deriving the aggregate wage curve (3) and the aggregate job creation curve (10), shown below. For that, I closely follow the approach of Thomas (2008b).

Note that $u_t, v_t, p_t$ and $q_t$ are predetermined variables. In this context, $\theta_t^f \equiv \theta_{t+1}$ and $q_t' \equiv q_{t+1}$ are key in deriving the quarterly wage and the aggregate job market curves.

As usual, optimal Nash bargaining requires $u_t(z) = b s_t(z)$, where $s_t(z) \equiv u_t(z) + j_t(z)$ is the total surplus of each match. Let $w_t \equiv (1/n_t) \int_0^1 w_t(z) n_t(z) d_z$ denote the aggregate wage. The bargaining solution implies the aggregate wage curve:

\[
w_t = b \frac{v_t/n_t}{u_t'} + (1 - b) w^u + b (1 - \rho) k \theta_t^f
\] (3)

where $v_t/n_t$ is the average disutility per worker and $b \equiv (1 - b) (1 + \nu) / [1 - b (1 + \nu)]$. As in Thomas (2008b), $w_t$ increases with hours only if the workers’ bargaining power
satisfies \( b (1 + \nu) < 1 \).

The first order condition for vacancies implies the aggregate job creation curve:

\[
\frac{k}{q^*_t} = \frac{E_t Q_{t+1} \Pi_{t+1}}{(1 + \nu) b^{u_{t+1}/n_{t+1}} - u_{t+1} + (1 - \rho) \frac{k}{q^*_t}}
\]

The Calvo (1983) pricing structure implies that the gross inflation rate evolves as

\[
1 = (1 - \alpha) \left( \frac{N_t}{D_t} \right)^{\frac{-(\psi-1)}{1+\phi}} + \alpha \Pi_t^{(\psi-1)}
\]

where \( \omega \equiv (1 + \nu) / \varepsilon - 1 \). The first order condition on the readjusting price \( p^*_t \) can be conveniently written as \((p^*_t / P_t)^{1+\phi \omega} = N_t / D_t\), in which \( N_t \) and \( D_t \) are written in recursive forms to avoid infinite sums:

\[
\begin{align*}
N_t &= (X_t)^\omega (X_t^c)^\sigma + \alpha E_t n_{t+1} \quad ; \quad n_t = Q_t G_t (\Pi_t)^{1+\phi(1+\omega)} N_t \\
D_t &= 1 + \alpha E_t d_{t+1} \quad ; \quad d_t = Q_t G_t (\Pi_t)^{\phi} D_t
\end{align*}
\]

where \( G_t \equiv Y_t / Y_{t-1} \) is the gross output growth rate, \( X_t^c \equiv C_t / C_t^n \) is the gross consumption gap and \( X_t \equiv Y_t / Y_t^n \) is the gross output gap. The natural (flexible prices) consumption \( C_t^n \) and output \( Y_t^n \) satisfy

\[
(Y_t^n)^\omega (C_t^n)^\sigma = \frac{\varepsilon}{\lambda \mu b} A_t^{(1+\omega)}
\]

The aggregate market clearing condition is \( Y_t = C_t + \kappa v_t \), where \( Y_t^{\phi-1} = \int_0^1 y_t(z)^{\phi-1} dz \).

### 2.4 Aggregates and productivity

Consider the aggregate disutility function \( v_t \equiv \int_0^1 v_t(z) \, dz \) and the aggregate hours worked \( H_t \equiv \int_0^1 H_t(z) \, dz \), and let \( P_t \) and \( P_{Ht} \) denote aggregate relative prices:

\[
\begin{align*}
P_t^{-(1+\omega)} &= \int_0^1 \left( \frac{p_t(z)}{P_t} \right)^{-\phi(1+\omega)} dz, \\
P_{Ht}^{-(1+\omega)} &= \int_0^1 \left( \frac{p_t(z)}{P_t} \right)^{-\phi(1+\omega)} dz
\end{align*}
\]

where \( \tilde{\omega} \equiv (1/\varepsilon) - 1 \).

Using the structure of the Calvo (1983) price setting, I am able to derive the laws of motion of the aggregate relative prices in a way that is very similar to how Schmitt-Grohe and Uribe (2006) derive the aggregate relative price relevant for the resource constraint in their model. The following propositions describe the evolution of the aggregate disutility function \( v_t \), the aggregate total hours \( H_t \) and the aggregate relative prices \( P_t \) and \( P_{Ht} \). The results are general and independent of the level of trend inflation.

**Proposition 1** The aggregate disutility \( v_t \) and its corresponding aggregate relative price \( P_t \) evolve according to

\[
\begin{align*}
v_t &= \frac{\chi}{1 + \nu} \left( \frac{Y_t}{A_t} \right)^{(1+\omega)} P_t^{-(1+\omega)} \\
P_t^{-(1+\omega)} &= (1 - \alpha) \left( \frac{N_t}{D_t} \right)^{-\phi(1+\omega)} + \alpha (\Pi_t)^{\phi(1+\omega)} P_{t-1}^{-(1+\omega)}
\end{align*}
\]
Proposition 2 The aggregate worked hours $H_t$ and its corresponding aggregate relative price $P_{Ht}$ evolve according to

\[ H_t = \left( \frac{Y_t}{A_t} \right)^{(1+\hat{\omega})} - \phi^{(1+\hat{\omega})} \]  
\[ P_{Ht}^{-(1+\hat{\omega})} = (1 - \alpha) \left( \frac{N_t}{D_t} \right)^{-\phi^{(1+\hat{\omega})}} + \alpha (\Pi_t)^{\phi^{(1+\hat{\omega})}} P_{Ht-1}^{-(1+\hat{\omega})} \]  

The proofs are shown in appendix D.

Note that $P_t$ and $P_{Ht}$ will decrease if the price dispersion rises. Therefore, $\nu_t$ and $H_t$ increases with price dispersion. I highlight that the inflation rate has first order effects on the two measures of price dispersion under positive trend inflation (see the log-linearized system (14 - 15)). As my analysis in section 4 shows, this effect is very important to explain most of the fluctuations that are transmitted into the labor market. It is important to note that this effect is completely absent under zero trend inflation.

Since the model has extensive and intensive labor margins, the appropriate measure of labor productivity is the output per total hours ratio $\varphi_t \equiv Y_t / H_t$. Aggregate hours per worker are computed as $h_t \equiv H_t / n_t$.

2.5 Monetary policy

For any variable $X_t$, the hatted representation $\hat{X}_t$ is its log-deviation from its steady state level $\bar{X}$. See appendices A and B for all steady state levels, a complete description of the composite parameters, and the list of the log-linearized equations.

Since monetary policy has an important role in the dynamics of the model, I present its log-linearized structure here. The monetary authority is assigned an inflation target $\pi_t \geq 0$ to pursue and implements monetary policy according to the generalized Taylor rule:

\[ \hat{i}_t = \phi_i \hat{i}_{t-1} + (1 - \phi_i) \left[ \phi_{\pi} \hat{\pi}_t + \phi_{\pi_1} E_t \hat{\pi}_{t+1} + \phi_{gy} (\hat{y}_t - \hat{y}_{t-1}) \right] \]  

in which the parameters $\phi_i$, $\phi_{\pi}$, $\phi_{\pi_1}$ and $\phi_{gy}$ are consistent with stability and determinacy in trend inflation equilibria with rational expectations.

This specification is in line with the one presented by Coibion and Gorodnichenko (2011). They find that reacting to the observed output growth rather than to the level of the output gap has two major advantages: (i) it has more stabilizing properties when the trend inflation is not zero; and (ii) it is empirically more relevant.\footnote{To test the empirical dominance, they estimate the generalized Taylor rule using Greenbook forecasts prepared for each meeting of the Federal Open Market Committee (FOMC) as real-time measures of expected inflation, output growth, and the output gap. This approach is advantageous because it avoids any extra assumption on how the FED’s expectations are formed.}

I consider two variations of the generalized Taylor rule: (i) responding to contemporaneous inflation ($\phi_{\pi_1} = 0$); and (ii) responding to expected future inflation ($\phi_{\pi} = 0$).

3 Data, stylized facts and calibration

Empirical evidence on the labor market is commonly inferred in terms of end-of-period variables. In this context, I define the end-of-period job-finding rate $p^*_t \equiv m_t / u^*_t$ and the end-of-period market tightness $\theta^*_t \equiv v^*_t / u^*_t$.\footnote{Note that he end-of-period variable $\theta^*_t$ is not the same as the lead variable $\theta^*_t$.} Those variables complement the set...
formed by the previously defined end-of-period unemployment rate \( u_t \) and the end-of-period vacancy postings \( v_t \).

The measure of labor productivity is the (quarterly seasonally adjusted) output per hours series in the nonfarm business sector as constructed by the Bureau of Labor Statistics (BLS) Major Sector Productivity program. End-of-period unemployment \( u_t \) is the quarterly average of the seasonally adjusted monthly series constructed by the BLS from the Current Population Survey (CPS). Following Shimer (2005), I compute the quarterly average of the seasonally adjusted monthly end-of-period job-finding rate \( p_t \). As in Gervais et al. (2011), end-of-period vacancy postings \( v_t \) are the quarterly average of the monthly composite Help-Wanted Index, provided by Barnichon (2010), adjusted to the Job Openings and Labor Turnover Survey (JOLTS) units of measurement.\(^{12}\) The composite Help-Wanted Index combines the information on the old newspaper and current online (since 2005) job advertisements provided by the Conference Board. Prior to 1995, the series is the newspaper Help-Wanted Index. The end-of-period tightness ratio \( \theta_t \) is simply computed as vacancies over unemployment. Finally, paralleling Shimer (2005), I Hodrick Prescott filter the quarterly artificial series with smoothing parameter \( 10^{5} \).\(^{13}\)

Table 1: Sample averages

<table>
<thead>
<tr>
<th>Period</th>
<th>( \pi_t )</th>
<th>( u_t^e )</th>
<th>( \theta_t^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>full sample (1951 : Q1 – 2005 : Q4)</td>
<td>3.84</td>
<td>0.057</td>
<td>0.71</td>
</tr>
<tr>
<td>great moderation (1985 : Q1 – 2005 : Q4)</td>
<td>3.05</td>
<td>0.057</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Note: Variables: end-of-period unemployment rate (\( u_t^e \)), end-of-period labor market tightness (\( \theta_t^e \)), inflation rate (\( \pi_t \)).

Table 2: Relative volatilities

<table>
<thead>
<tr>
<th>Period</th>
<th>( u_t^e )</th>
<th>( v_t^e )</th>
<th>( \theta_t^e )</th>
<th>( p_t^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>full sample (1951 : Q1 – 2005 : Q4)</td>
<td>st</td>
<td>10.6</td>
<td>10.8</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>alt</td>
<td>5.9</td>
<td>6.0</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td>alt</td>
<td>5.3</td>
<td>6.9</td>
<td>11.8</td>
</tr>
<tr>
<td>benchmark (flexible prices)</td>
<td>1.0</td>
<td>1.2</td>
<td>2.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Note: Standard relative volatility to productivity (st), alternative relative volatility to output (alt). Variables: end-of-period unemployment rate (\( u_t^e \)), end-of-period vacancy postings (\( v_t^e \)), end-of-period labor market tightness (\( \theta_t^e \)), end-of-period job-finding rate (\( p_t^e \)).

Table 1 shows the sample averages, including that of the average CPI inflation rate. The unemployment rate remained at the same level over both periods. The inflation rate and the tightness ratio, on the other hand, fell to slightly lower levels. As table 2 shows, the standard relative volatilities (relative to labor productivity) have slightly decreased over the Great Moderation period. However, their values have remained much larger than the benchmark ones, defined as what the theoretical model predicts for the case of flexible

\(^{12}\)Pissarides (2009) use a similar approach to rescale the number of vacancy postings into JOLTS units of measurement.

\(^{13}\)I use this value to follow the literature. After Shimer (2005) published his results, this specific detrending approach has become very common in the labor literature.
prices. Most of the alternative measures (relative to output), on the other hand, have actually increased.

The calibration is described as follows. As in Cooley and Prescott (1995), the technology shock follows an AR(1) process \( \hat{A}_t = \rho_a \hat{A}_{t-1} + \epsilon_t^a \), where \( \epsilon_t^a \) is a white-noise disturbance. I set the subject discount factor at \( \beta = 0.99 \), the risk aversion parameter at \( \sigma = 1 \), the elasticity to hours at the production function at \( \varepsilon = 1 - 0.36 \) and the autoregressive coefficient of the technology shock at \( \rho_a = 0.95 \). The variance of \( \epsilon_t^a \) is irrelevant for computing the relative volatilities.

As for the elasticity \( \nu \) of the disutility from hours worked \( H_t(z) \), note that it is the reciprocal of the Frisch elasticity of aggregate hours. Thus, I follow the micro evidence collected by Chetty et al. (2011) and set \( \nu = 1.2 \). As Thomas (2008b) highlights, the fact that firms make both pricing and vacancy posting decisions implies that the workers’ bargaining power \( b \) and the hours elasticity \( \nu \) cannot be simultaneously large: \( b(1 + \nu) < 1 \). Following Hosios (1990) condition, I impose \( b = a = 0.45 \), a value that is close to the midpoint estimate and satisfies \( 0.45(1 + 1.2) < 1 \). Since jobs last about 10 quarters in the US (e.g. Shimer (2005)), I set the average quarterly separation rate at \( \rho = 1/10 = 0.10 \).

Following Cogley and Sbordone (2008), I set the elasticity of substitution at \( \phi = 10 \). On the generalized Taylor rule, I use central estimates from Coibion and Gorodnichenko (2011) for the 1983–2002 period. The Contemporaneous Taylor rule (current inflation) is defined by \( \phi_i^c = 0.90, \phi_n^c = 1.58, \phi_{\pi 1}^c = 0 \) and \( \phi_{gg}^c = 2.21 \). The Mixed Taylor rule (future inflation) is characterized by \( \phi_i^m = 0.86, \phi_n^m = 0, \phi_{\pi 1}^m = 2.20 \) and \( \phi_{gg}^m = 1.56 \). There is a small drawback from using their estimates in my simulations. Even though they are consistent with the Great Moderation period, I use the same parameters for assessing the theoretical volatilities in the full sample. I am aware of this problem, but I choose this way due to the lack of better suited policy rules for the full sample period, i.e. rules that guarantee determinate equilibrium under rational expectations in trend inflation models.

As for the nuisance parameters \( [\eta, w^w, k, \chi] \), I follow the literature and calibrate them based on steady state values. I set the steady state aggregate hours per worker is at \( \bar{h} = 0.3 \). As in Ravenna and Walsh (2011), I fix the replacement ratio \( w^w/\bar{w} \) at 0.54. Since the average unemployment rate in both periods is the same, the steady state level of the end-of-period unemployment rate is fixed at \( \bar{u} = 0.057 \). I set the steady state level of the end-of-period tightness rate according to the averages observed in each period: \( \bar{\theta}_c^e = 0.71 \) (full sample), and \( \bar{\theta}_c^e = 0.64 \) (Great Moderation sample). The trend inflation level is also set according to the observed averages: \( \bar{\pi}^t = 3.84 \) (full sample), and \( \bar{\pi}^t = 3.05 \) (Great Moderation sample). As for the degree of price stickiness, I use the central estimates obtained by Smets and Wouters (2007), whose sample periods used for estimation are very similar to mine. The values are set at: \( \alpha = 0.65 \) (full sample), and \( \alpha = 0.72 \) (Great Moderation sample).

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14 More specifically, I refer to their model with no government consumption.

15 The authors conduct meta analyses of existing micro evidence. Their point estimate of the Frisch elasticity of aggregate hours is 0.82.


17 Flihn (2006) estimates the workers’ bargaining power at 0.4.

18 More specifically the modes of the posterior distribution.
4 Volatility multipliers

I start this section by summarizing the distortions caused by inflation. As it rises, the dispersion of relative prices $p_t(z) / P_t$ increases. Due to monopolistic competition, this dispersion translates into dispersion in production $y_t(z)$. By means of the production function, this dispersion is passed through hours worked $H_t(z)$, which also become more dispersed. Because disutility to work $v_t(z)$ is a convex function of $H_t(z)$, the aggregate disutility $v_t$ increases as inflation rises. The increase in the aggregate disutility deteriorates welfare while profits fall.

An important feature of staggered prices under trend inflation is that the steady state is still dynamic at the firm level. The positive trend inflation induces a stationary dispersion of relative prices and the deterioration effects of positive trend inflation holds even in the steady state.\footnote{Even if the shocks remains at their means, positive trend inflation implies that there is always a fraction of firms whose relative prices lag behind their optimal levels. Consequently, firms adjust above the aggregate price trend when resetting their prices. Interestingly, the aggregate variables converge to time invariant steady states – the individual level dispersion cancels out.} Figure 1 depicts how the steady state levels of important quantities behave as trend inflation rises, using the Great Moderation calibration described in section 3. In the labor market, note that inflation initially produces a small benefit to employment at the vicinities of zero trend inflation. However, the disutility effect dominates at very small levels of inflation. If the annual level is greater than 0.6 percent, the labor market variables deteriorate as trend inflation rises. Hours per worker, vacancy postings and wages are reduced. Matches and surpluses fall and unemployment increases. Empirically, my theoretical predictions are backed by Berentsen et al. (2011), who find a strong and positive long-run relationship between inflation and unemployment in quarterly US data from 1955–2005.

I define multipliers as the elasticities in the log-linearized equations. They change the way the fluctuations are transmitted into the variables. Short-run multipliers are defined as the coefficients in the log-linearized equations. Long-run multipliers are defined as the elasticities in the stationary distribution implied by the dynamic equations, i.e. they are the ratio of the unconditional standard deviations of hatted variables $\tilde{z}_t$. Even though the all log-linearized equations are listed in appendix B, I focus here on the ones containing the relevant multipliers.

Positive trend inflation is a fluctuation amplifier, which is particularly important in the labor market. This effect is obtained because some multipliers increase as trend inflation rises. Of course, the total fluctuation created in general equilibrium also depends on the variances and covariances of all variables, and on the way the multipliers interact. The general equilibrium amplification is larger than what the multipliers suggest. For instance, the law of motion of employment is highly inertial and this fact alone amplifies the short-run volatilities when computing the unconditional standard deviations. Since the general equilibrium solution with rational expectations cannot be derived algebraically, I focus my analysis on the multipliers.

I classify the multipliers affected by trend inflation as: (i) labor multipliers; and (ii) pricing multipliers. In particular, I show that pricing multipliers are much stronger than labor multipliers and explain most of the large relative volatilities found in the labor market.

Labor multipliers are the ones that depend on the steady state levels of the labor variables, which can always be written as functions of the total surplus level $\bar{s}$ in the...
steady state. Those multipliers are found in the aggregate wage curve, the job creation curve and the market clearing identity.

The most important one amplifies most of the fluctuations brought into $q_t^f$, which is then transmitted into the remaining labor variables without further amplification. To obtain the short-run labor multiplier $m_q$, I use equation (3) to eliminate the aggregate wage from the job creation curve (4) and obtain:

$$q_t^f = f_q \beta E_t \tilde{q}_{t+1}^f + E_t (\tilde{i}_t - \tilde{\pi}_{t+1}) - m_q \beta E_t (\tilde{v}_{t+1} + \sigma \tilde{c}_{t+1} - \tilde{n}_{t+1})$$

(10)

where

$$f_q = (1 - \rho) \left(1 - \frac{b}{a} \bar{p}\right) ; \quad m_q = \frac{w^u}{\beta} + \frac{w^u}{\beta} + \left(\frac{\eta b (1-b)}{k} \right)^{1-a} \frac{\eta h (1-\rho) \beta^{1/a}}{\sigma}$$

The behavior of $m_q$ depends on the calibration. In particular, $m_q$ increases as trend inflation rises as long as the real unemployment compensation $w^u$ is sufficiently large. Under reasonable calibration, that seems to be the case as the first panel in the last row of figure 1 depicts. However, the level of the labor multiplier $m_q$ and its amplification
caused by higher trend inflation is very small.\footnote{Note that the coefficient $f_{\theta}$ on $\beta E_t \hat{x}_{t+1}$ may amplify the long-run fluctuations brought into $\hat{q}'_t$. This coefficient increases as trend inflation rises because the job-finding rate $\bar{\sigma}$ falls. However, its amplification contribution is not large. As depicted in the second panel in the last row of figure 1, $f_{\theta}$ increases by a small amount as trend inflation rises.}

Pricing multipliers are the ones which can always be written as direct functions of the trend inflation level, by means of the composite parameters $\vartheta$ and $\bar{\alpha}$, and do not depend on the steady state levels of the labor variables. Those three composite parameters are key in describing the dynamics under trend inflation: $\vartheta \equiv \bar{\Pi}^{(1+\omega)} \geq 1$ and $\bar{\alpha} \equiv \bar{\Pi}^{(1+\phi \omega)} \geq 1$ are positive transformations of the trend inflation $\tilde{\pi}$, and $\tilde{\alpha} \equiv \alpha \bar{\Pi}^{\theta-1} \geq \alpha$ is the effective degree of price stickiness.\footnote{The existence of an equilibrium with trend inflation requires max $(\bar{\alpha}, \vartheta) < 1$.} All increase as trend inflation rises.

Those multipliers are found in system (11–13), describing the Phillips curve under trend inflation, and in system (14–15), describing the laws of motion of the aggregate relative prices $\bar{P}_t$ and $\bar{P}_{H_1}$:

\begin{align}
\bar{\pi}_t &= \beta E_t \bar{\pi}_{t+1} + \bar{k} \left[ \varrho \hat{x}_t + (1-\varrho) \hat{x}_t^c \right] + \frac{(\vartheta - 1)}{(1 - \bar{\alpha} \vartheta L^{-1})} \beta E_t \hat{x}_{t+1} \\
\bar{\omega}_t &= \bar{k}_1 \bar{\pi}_t + \bar{\alpha} \bar{k} \left[ \varrho \hat{x}_t + (1-\varrho) \hat{x}_t^c \right] + \bar{k}_2 \left[ (\hat{x}_t - \hat{x}_{t-1}) - \sigma \left( \hat{x}_t^c - \hat{x}_{t-1}^c \right) \right] + \bar{k}_2 \hat{x}_{t}^{cps} \\
\xi_{t}^{cps} &= (\hat{y}_t^m - \hat{y}_{t-1}^m) - \sigma (\hat{c}_t^m - \hat{c}_{t-1}^m)
\end{align}

\begin{align}
\bar{P}_t &= \bar{\alpha} \theta \bar{P}_{t-1} - (\vartheta - 1) \frac{\bar{\alpha}}{(1 - \bar{\alpha})} \bar{\pi}_t \\
\bar{P}_{H_1} &= \bar{\alpha} \theta \bar{P}_{H_{t-1}} - (\bar{\vartheta} - 1) \frac{\bar{\alpha}}{(1 - \bar{\alpha})} \bar{\pi}_t
\end{align}

where $L^{-1}$ is the lead operator, and the composite parameters are

\begin{align*}
\bar{k} &\equiv \frac{(1-\bar{\alpha} \vartheta \omega)}{\varrho} \bar{k}_2 \quad ; \quad \bar{k}_1 \equiv \varphi (1+\omega) \bar{k}_2 \quad ; \quad \bar{k}_2 \equiv \frac{(1-\bar{\alpha})}{1+\phi \omega} \quad ; \quad \varrho \equiv \frac{\omega}{\bar{\omega} + \sigma}
\end{align*}

In system (11–13), the fluctuation induced by the endogenous cost push shock $\xi_{t}^{cps}$ is amplified by the short-run multiplier $\bar{m}_t \equiv \beta \bar{k}_2 (\vartheta - 1)$. In system (14–15), the equations are simple enough to allow me to compute the unconditional standard deviations as $sd \left( \bar{P}_t \right) = m_p sd \left( \bar{\pi}_t \right)$ and $sd \left( \bar{P}_{H_1} \right) = m_{p_H} sd \left( \bar{\pi}_t \right)$, where $m_p$ and $m_{p_H}$ are long-run pricing multipliers from inflation fluctuations into the aggregate relative prices:

\begin{align*}
m_p &\equiv \frac{\bar{\alpha} \vartheta - 1}{(1 - \bar{\alpha}) \sqrt{1 - (\bar{\alpha} \vartheta)^2}} \quad ; \quad m_{p_H} \equiv \frac{\bar{\alpha} \bar{\vartheta} - 1}{(1 - \bar{\alpha}) \sqrt{1 - (\bar{\alpha} \bar{\vartheta})^2}}
\end{align*}

Figure 2 shows how the pricing multipliers increase as trend inflation rises. In panel A, I compare $m_t$ with the coefficient $\bar{k}$ of the average gap $(\varrho \hat{x}_t + (1-\varrho) \hat{x}_t^c)$ in the Phillips curve. Note that as the inflation trend rises to the average inflation rate during the Great Moderation period, $m_t$ increases to the same order of magnitude as $\bar{k}$. The main message is that the short-run effect of this pricing multiplier can be as large as the effect of ordinary variables in the Phillips curve.

Panel B shows that the multiplier $m_p$ quickly increases to a higher level than the the one achieved by the multiplier $m_{p_H}$ as trend inflation rises. Most importantly, $m_p$ quickly increases to a much higher level when compared to the labor multipliers. This result is
the key to explain why the volatilities of the labor market quantities grow faster than the productivity volatility as trend inflation rises. Indeed, consider the following log-linearized equations describing the aggregate disutility, aggregate worked hours, aggregate hours per worker and labor productivity:

\[
\begin{align*}
\dot{\bar{v}}_t &= (1 + \omega) \left( \bar{y}_t - \phi \bar{P}_t - \bar{A}_t \right); \quad \ddot{h}_t = \bar{H}_t - \bar{n}_t \\
\hat{H}_t &= (1 + \hat{\omega}) \left( \bar{y}_t - \phi \hat{P}_{Ht} - \hat{A}_t \right); \quad \hat{\phi}_t = \hat{y}_t - \hat{H}_t
\end{align*}
\] (16)

The amplified fluctuations of \( \hat{P}_t \) are transferred to the aggregate disutility \( \dot{\bar{v}}_t \), which in turn are weakly amplified by labor multipliers and transmitted to the labor market by means of the aggregate wage and job creation curves, as summarized by equation (10). The weaker volatility of \( \bar{P}_{Ht} \), on the other hand, is transmitted to the labor productivity variable \( \dot{\bar{P}}_t \) (output per hours) by means of the aggregate hours worked \( \hat{H}_t \).

Therefore, I identify the multipliers of the aggregate relative prices as contributing the most for the large increase of the relative volatilities in the labor market as trend inflation rises. The strong effect of the pricing multipliers is in line with the findings of Ascarì and Ropele (2009) for the case in which the monetary authority follows Taylor-type interest rate rules. Larger levels of trend inflation strongly shrink the determinacy region of the policy parameter space under rational expectations equilibria (REE). As a consequence, the determinacy frontier progressively closes in around the point characterizing any determinacy-consistent policy rule as trend inflation rises. As that happens, fluctuations under determinate REE are strongly amplified.

Finally, monetary policy also has a role in defining how the fluctuations are transmitted into the inflation rate and the demand variables (output and consumption gaps). There are two channels by which the volatility of the real interest rate \( \dot{i}_t = \bar{i}_t - E_t \hat{i}_{t+1} \) increases: (i) a direct one caused by the effect of the expected inflation rate; and (ii) an indirect one caused by the responses to the inflation rate and output growth in the Taylor rule. The real interest rate affects the expected growth of the consumption gap \( E_t \left( \hat{c}_{t+1} - \bar{c}_t \right) \) via
the log-linear Euler equation, in which there is no multiplier.

In equilibrium, production relates to the household demand for goods and to the vacancy postings by means of the market clearing condition. The interactions in general equilibrium are such that it is cumbersome to explain how the volatility of the output gap evolves as trend inflation rises. As it turns out, the numerical simulations of section 5 indicate that there are regions of trend inflation in which the volatility of the output gap decreases and others in which it increases. This is caused by the fact that the monetary policy rule is invariant with respect to the level of trend inflation, and does not internalize the fact that the coefficient $\kappa$ of the average gap in the Phillips curve decreases as inflation trend rises.

## 5 Simulations

I simulate the log-linearized model with rational expectations, considering different levels of trend inflation. In each simulation, I proceed as follows: (i) I HP-filter the quarterly artificial series with smoothing parameter $10^5$, paralleling the detrending approach in Shimer (2005); and (ii) compute the unconditional standard deviations of the filtered endogenous variables.

The simulations of the general equilibrium allows me to compute the total amplification effects of trend inflation on the volatilities, defined as the unconditional standard deviations of the HP-filtered simulated variables normalized by the unconditional standard deviation of the HP-filtered technology shock $\hat{A}_t$.

For illustration purposes, figure 3 shows the total amplification effect of trend inflation on end-of-period unemployment $\hat{u}_t$, labor productivity $\hat{v}_t$ (output per hours) and output $\hat{y}_t$. As the analysis on the pricing multipliers of the aggregate relative prices suggests, the total amplification of the unemployment volatility quickly increases as trend inflation rises and reaches a point in which it is about ten times as large as the total amplification of the labor productivity volatility when the trend inflation matches the sample average of the inflation rate. At this point, the output volatility is still about one half of the technology volatility and about one fifth of the productivity volatility. Due to price stickiness, output does not rise (fall) as much after a positive (negative) technology shock. Dividing by a small output volatility is the reason why it is easier to match alternative relative volatilities in the labor market (relative to output) in models with price stickiness.

![Figure 3: Amplification effect](image)

**Note:** Current inflation Taylor rule (full line), future inflation Taylor rule (dashed line), points at the Great Moderation average inflation rate (stars).
The relative volatilities of HP-filtered labor market variables are computed as their unconditional standard deviations divided by the unconditional standard deviation of the HP-filtered productivity measure $\hat{\phi}_t$. As for the HP-filtered goods market variables (inflation and output gap), I divide their unconditional standard deviations by the unconditional standard deviation of the HP-filtered technology shock $\hat{A}_t$.

Figure 4 shows standard relative volatility (relative to labor productivity) schedules of variables from the labor and goods markets. I also plot the theoretical values when inflation trend set at the sample averages (black stars), the theoretical values in the equilibrium with flexible prices (red circles), and the empirical values (blue dots). As expected, the equilibrium with flexible prices implies very small relative volatilities when compared to the empirical assessments.

If $\bar{\pi} = 0$, my model is similar to Thomas (2008b) and I obtain identical results: the fact that firms make both the pricing and vacancy postings decision does help increase the relative volatilities when compared to the flexible price equilibrium, but the volatility amplification is still too small to explain the puzzle. As trend inflation rises, on the other hand, the relative volatilities significantly increase and reach the region where the empirical values are located.

The model is better fitted to match the empirical volatilities of the Great Moderation period – in particular, using the current inflation Taylor rule (column C) provides the best fit. This is expected because the calibration of the monetary rule is based on the estimates obtained by Coibion and Gorodnichenko (2011) for the Great Moderation period. Note, however, that the theoretical values in the full sample approach the empirical ones by slightly increasing the trend inflation from the full sample average $3.8$ to about $4.2$. Another feature of the model is that the theoretical relative volatilities of the end-of-period job-finding rate is always larger than the empirical ones.

Figure 5 shows alternative relative volatility (relative to output) schedules of variables from the labor markets. As in Thomas (2011), I find that the empirical values of the alternative measure are closely matched, and sometimes even surpassed, by the theoretical assessments at the zero trend inflation. As the pictures show, the effects of positive trend inflation on the relative volatilities are much stronger using the alternative measures. In general, the alternative values are matched reasonably well at very small levels of trend inflation.

Interestingly, the labor market volatilities monotonically increase as trend inflation rises. This outcome is not the case in the goods market. Their volatilities are remarkably effected by different policy rules and calibration sets, and the shapes have points with large curvature. This result suggests that the effects of trend inflation on the labor market schedules are not particular cases, and are robust to different monetary policy frameworks.

The correlations between the technology shock and labor market variables can be at odds with the prediction of RBC models with search frictions: the unemployment rate may rise on impact after a positive technology shock. This happens whenever the price stickiness is sufficiently large. Since prices do not adjust instantaneously, it is costly for firms to increase hours and post more vacancies to accommodate the increase in production. As a consequence, employment falls in the short-run. Empirical evidence for that has been found in many postwar countries, including the US, as the findings of Gali (1999, 2010) strongly suggest.
Figure 4: Standard relative volatilities (to productivity)

Note: Theoretical values at the average inflation rate (stars), theoretical values with flexible prices (circles), empirical values (dots). Variables: end-of-period unemployment rate ($u^e$), end-of-period vacancy postings ($v^e$), end-of-period job-finding rate ($p^e$), end-of-period labor market tightness ($θ^e$), inflation rate ($π$), output gap ($x$).
Figure 5: Standard relative volatilities (to output)

Note: Theoretical values with flexible prices (circles), empirical values (dots).
Variables: end-of-period unemployment rate ($u_e$), end-of-period vacancy postings ($v_e$), end-of-period job-finding rate ($p_e'$), end-of-period labor market tightness ($\theta_e'$), inflation rate ($\pi$), output gap ($x$).
6 Conclusion

Positive trend inflation has an important role in amplifying the effects of the technology shocks in the fluctuation in the labor market. Labor and pricing (fluctuation) multipliers increase as trend inflation rises. In particular, the level of the aggregate disutility to work increase, which in turn deteriorates the welfare and the surpluses. This result is reflected in some labor multipliers, which increase as the total surplus fall.

However, labor multipliers do not increase as much as trend inflation rises. The major part of the large fluctuations in the labor market are generated by pricing multipliers, which magnify the fluctuations coming from the goods market. Under positive trend inflation, the inflation rate generates first order effects on the dispersion of relative prices. The first order effects quickly increase as trend inflation rises, and are captured by important pricing multipliers that magnify the fluctuations transmitted from the inflation rate to the aggregate dispersion of relative prices. The amplified fluctuations are in turn transmitted into the aggregate disutility to work and the aggregate hours worked. The highly amplified volatility of the aggregate disutility is finally transmitted into labor market quantities by means of the aggregate wage and job creation curves, where labor multipliers have a minor role.

Controlling for the observed average of the CPI inflation rate and degree of price stickiness, the model is successful in explaining the large relative volatilities and resolve the unemployment volatility puzzle. Without any wage rigidity, the model generates the large volatilities observed in US labor quantities during two sample periods: full sample (1951-2005) and Great Moderation (1985-2005).

Different modeling assumptions and calibration sets may lead to different quantitative results. Nevertheless, the general property is that trend inflation increases the fluctuations when prices are staggered. This leads to increased dispersion of relative prices and amplified fluctuations.

References


A Steady state levels and composite parameters

Let $\bar{\Pi} = 1 + \bar{\pi}$ denote the gross trend inflation. The composite parameters are

<table>
<thead>
<tr>
<th>Table 3: Composite parameters</th>
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<tbody>
<tr>
<td>$\psi \equiv 1 - \beta (1 - \rho)$</td>
</tr>
<tr>
<td>$\varrho \equiv \frac{\omega}{\omega + \sigma}$</td>
</tr>
<tr>
<td>$\mathcal{C}_1 \equiv \frac{1 - \alpha \beta \theta}{1 - \alpha \beta}$</td>
</tr>
<tr>
<td>$\bar{\alpha} \equiv \alpha (\bar{\Pi})^{(\theta - 1)}$</td>
</tr>
<tr>
<td>$\omega \equiv \frac{(1+\nu)}{\nu} - 1$</td>
</tr>
<tr>
<td>$\mu_\nu \equiv \frac{1+\nu}{\nu}$</td>
</tr>
<tr>
<td>$\mathcal{C}_2 \equiv \frac{1 - \alpha}{1 - \alpha \theta}$</td>
</tr>
<tr>
<td>$\bar{\vartheta} \equiv \frac{1}{\bar{\Pi}}$</td>
</tr>
<tr>
<td>$\bar{\mathcal{C}} \equiv \frac{1 - \alpha}{1 - \alpha \theta}$</td>
</tr>
<tr>
<td>$\bar{\theta} \equiv \frac{1}{\bar{\Pi}}$</td>
</tr>
<tr>
<td>$b \equiv \frac{(1-b)(1+\nu)}{1-b(1+\nu)}$</td>
</tr>
<tr>
<td>$\bar{\mathcal{K}} \equiv \frac{(1-\alpha \beta \theta) \omega}{\alpha \theta} \bar{\mathcal{K}}_2$</td>
</tr>
<tr>
<td>$\mathcal{C}_3 \equiv \frac{1 - \alpha}{1 - \alpha \theta}$</td>
</tr>
<tr>
<td>$\bar{\mathcal{C}} \equiv \frac{1 - \alpha}{1 - \alpha \theta}$</td>
</tr>
<tr>
<td>$\mu \equiv \frac{\phi}{\phi - 1}$</td>
</tr>
<tr>
<td>$\bar{\mathcal{K}}_2 \equiv \frac{(1-\alpha)}{f w}$</td>
</tr>
<tr>
<td>$\mathcal{C}_5 \equiv \frac{1 - \alpha}{1 - \alpha}$</td>
</tr>
</tbody>
</table>

max $(\bar{\alpha}, \bar{\alpha} \bar{\theta}) < 1$

The steady state levels $\bar{Y}$, $\bar{C}$, $\bar{\theta}$, $\bar{\varrho}$, $\bar{\nu}$, $\bar{\pi}$ and $\bar{\mathcal{C}}$ are found by solving the non-linear system:

$$\frac{\psi}{\bar{\varrho}} = \frac{\beta \varrho \mathcal{C}_2 \bar{Y}}{\mu \Pi} - \beta (1 - b) w^a - \beta b (1 - \rho) k \bar{\theta} \quad ; \quad \bar{Y} \bar{C}^\sigma = \frac{\beta \varrho \mathcal{C}_2 \bar{Y}}{\mu \Pi}$$

The remaining levels are\(^{22}\)

$$\bar{Y} = \bar{C} + k \bar{V} \quad ; \quad \bar{V} = \bar{\pi} \bar{\varrho} \quad ; \quad \bar{\mathcal{C}} = \frac{2 \beta^a}{1 + (1 - \rho)^{2 \beta^a - 1}} \quad ; \quad \bar{\mu} = \frac{\mu \Pi}{1 - (1 - \rho)^{\beta^a - 1}} \quad ; \quad \bar{\theta} = \frac{\theta}{\bar{\Pi}}$$

B The log-linearized model

For any variable $X_t$, the hatted representation $\hat{X}_t \equiv \log (X_t / \bar{X})$ is its log-deviation from its steady state level $\bar{X}$.

The Beveridge equations:

$$\hat{n}_t = (1 - \rho) \hat{n}_{t-1} + \rho \hat{m}_t \quad ; \quad \hat{u}_t = -\frac{(1 - \rho) \hat{P}}{\rho} \hat{n}_{t-1} \quad ; \quad \hat{p}_t = (1 - a) \hat{\theta}_t$$

$$\hat{m}_t = (1 - a) \hat{\theta}_t + \hat{u}_t \quad ; \quad \hat{v}_t = \hat{\theta}_t + \hat{u}_t \quad ; \quad \hat{q}_t = (1 - \alpha) \hat{\theta}_t$$

$$\hat{\nu}_t = (1 - \rho) \hat{\nu}_t - \frac{1}{\alpha} \hat{\nu}_t \quad ; \quad \hat{P}_t = \hat{m}_t - \hat{v}_t \quad ; \quad \hat{q}_t = \hat{v}_t - \hat{\nu}_t$$

$$\hat{\mu}_t = (1 - \rho) \hat{\mu}_t \quad ; \quad \hat{\mu}_t = \hat{m}_t - \hat{v}_t \quad ; \quad \hat{q}_t = \hat{v}_t - \hat{\nu}_t$$

The aggregate wage and job creation curves:

$$\hat{\mathcal{F}} \hat{\omega} \hat{\omega}_t = \frac{1}{\nu} \mathbf{m}_t (\hat{u}_t + \sigma \hat{c}_t - \bar{n}_t) - (1 - \rho) \frac{\beta}{\nu} \mathbf{p}_t$$

$$\hat{\mathcal{F}} \hat{\mathcal{Q}}_t = (1 - \rho) \beta E_t \hat{q}_{t+1} + E_t (\hat{u}_t - \bar{n}_{t+1}) + \frac{\beta}{\nu} \hat{\omega} \beta E_t \hat{\omega}_{t+1}$$

\(^{22}\)The equilibrium with flexible prices is determined by solving the equations replacing $\alpha = 0$ for the Calvo rigidity parameter.
Aggregates and productivity:
\[
\begin{align*}
\hat{v}_t &= (1 + \omega) \left( \hat{y}_t - \phi \hat{P}_t - \hat{A}_t \right) \quad ; \quad \hat{P}_t = \hat{\alpha} \hat{\rho} \hat{P}_{t-1} - \frac{(\theta - 1)\hat{\alpha}}{1 - \hat{\alpha}} \hat{\pi}_t \\
\hat{H}_t &= (1 + \omega) \left( \hat{y}_t - \phi \hat{P}_{Ht} - \hat{A}_t \right) \quad ; \quad \hat{P}_{Ht} = \hat{\alpha} \hat{\rho} \hat{P}_{Ht-1} - \frac{(\theta - 1)\hat{\alpha}}{1 - \hat{\alpha}} \hat{\pi}_t \\
\hat{h}_t &= \hat{H}_t - \hat{n}_t \quad ; \quad \hat{\gamma}_t = \hat{y}_t - \hat{H}_t
\end{align*}
\]

The trend inflation new-Keynesian Phillips curve (NKPC):
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left( q \hat{x}_t + (1 - q) \hat{x}^c_t \right) \\
+ \frac{(\theta - 1)\hat{\sigma}}{(1 - \hat{\alpha}L^{-1})} \beta E_t \hat{x}_{t+1} + \frac{(\theta - 1)\hat{\sigma}}{(1 - \hat{\alpha}L^{-1})} \beta E_t \left( q \hat{x}_{t+1} + (1 - q) \hat{x}^c_{t+1} \right) \\
+ \frac{(\theta - 1)\hat{\sigma}}{(1 - \hat{\alpha}L^{-1})} \beta \left[ E_t \left( \hat{x}_{t+1} - \hat{x}_t - \sigma \left( \hat{x}^c_{t+1} - \hat{x}^c_t \right) \right) + \xi^c_{t+1} \right]
\]
where \( L^{-1} \) is the lead operator\(^{23}\), \( \xi^c_{t+1} \equiv E_t \left[ \left( \hat{y}_{t+1}^c - \hat{y}_t^c \right) - \sigma \left( \hat{c}_{t+1}^c - \hat{c}_t^c \right) \right] \) is an endogenous cost push shock, and \( \hat{x}_t = \hat{y}_t - \hat{y}^c_t \) and \( \hat{x}^c_t = \hat{c}_t - \hat{c}^c_t \) are the output and consumption gaps.

The IS curve and market clearing identity:
\[
\hat{x}^c_t = E_t \hat{x}^c_{t+1} - \frac{1}{\sigma} \left( i_t - E_t \hat{\pi}_{t+1} - \hat{r}^*_t \right) \quad ; \quad \hat{y}_t \equiv \hat{s}_t \hat{c}_t + (1 - \hat{s}_t) \hat{\nu}_t
\]
where \( \hat{r}^*_t \equiv \sigma \left( E_t \hat{c}^*_t - \hat{c}^*_t \right) \) is the real interest rate under flexible prices.

In the equilibrium with flexible prices, the NKPC is replaced by
\[
\theta \hat{y}_t + (1 - \theta) \hat{c}^*_t = \frac{1 - \omega}{\omega} \hat{A}_t
\]

C The aggregate wage and job creation curves

Individuals take the predefined variables \( u_t, v_t, p_t \) and \( q_t \) as given. In this context, the number of workers matched into firm \( z \) is \( m_t(z) = q_t v_t(z) = p_t u_t v_t(z) / v_t \). Likewise, \( p_t(z) \equiv m_t(z) / u_t \) is the job-finding rate for being matched at firm \( z \) and satisfies \( p_t = \int_0^1 p_t(z) \, dz \). Let \( s_t(z) \equiv v_t(z) / v_t \) denote the firm’s vacancy share. Conditioned on obtaining a new job, the probability that the worker is matched into firm \( z \) is \( p_t(z) / p_t \). Note that \( p_t(z) / p_t = v_t(z) / v_t = s_t(z) \). For notation purposes, let \( w_t(z) \equiv w(h_t(z)) \) denote the wage schedule, \( u_t^t \equiv \partial u_t / \partial C_t \), \( u_t^t(z) \equiv \partial u_t(z) / \partial H_t \) and \( w_t^t(z) \equiv \partial w(h_t(z)) / \partial H_t \) denote derivatives, and \( Q^t_t = Q^t_H \) denote the real stochastic discount factor.

Let \( W_t \equiv u_t - \int_0^1 v_t(z) \, dz + \beta E_t W_{t+1} \) describe the welfare dynamics. As in Thomas Thomas (2008a,b), \( U_t(z) \equiv \partial W_t / \partial m_t(z) \) is the value enjoyed by the marginal worker matched into firm \( z \), and \( U_t \equiv \int_0^1 s_t(z) U_t(z) \, dz \) is the average value enjoyed by marginal workers being matched into all firms, conditional on the workers leaving unemployment at the beginning of the period. Note that
\[
U_t(z) = u_t^t \frac{\partial C_t}{\partial m_t(z)} - \frac{\partial u_t^t}{\partial m_t(z)} \int_0^1 v_t(z) \, dz + \beta E_t \frac{\partial W_{t+1}(z) / \partial m_t(z)}{\partial m_t(z)} + \beta E_t \int_0^1 \frac{\partial W_{t+1}(z)}{\partial m_t(z)} \frac{\partial m_{t+1}(z) / \partial m_t(z)}{\partial m_t(z)} \frac{\partial m_{t+1}(z) / \partial m_t(z)}{\partial m_t(z)} dz
\]
\[
= [w(h_t(z)) - w^u u_t^t - u_t^t(z) h_t(z) + (1 - \rho) \beta E_t (U_{t+1}(z) - p_{t+1} U_{t+1})]
\]

Let \( u_t(z) \equiv U_t(z) / \lambda_t P_t \) denote the real match surplus to the marginal worker and

\(^{23}\)I chose a representation that makes it easier to see the effect of \( \hat{\pi} \) (captured by means of \( \theta - 1 \)).
\[ u_t = \frac{U_t}{\lambda_t P_t} \text{ its aggregate. Note that} \]
\[ u_t(z) = w_t(z) - w^n - \frac{v_t(z)h_t(z)}{q_t^z} + (1-\rho)E_tQ_{t+1}^u(u_{t+1}(z) - p_{t+1}u_{t+1}) \]

In order to post \( v_t^z(z) \) end-of-period vacancies, firm \( z \) consumes \( kv_t^z(z, \tilde{z}) \) units from each good \( \tilde{z} \), i.e. a total \( kv_t^z(z) \) units of the final aggregate good. The posting expenditure is \( kP_t v_t^z(z) = k \int_0^1 p_t(\tilde{z}) v_t^z(z, \tilde{z}) \, d\tilde{z} \), where \( v_t^z(z) = \int_0^1 v_t^z(z, \tilde{z}) \, d\tilde{z} \). Minimizing the total posting expenditure implies \( v_t^z(z) = v_t^z(z) (p_t(\tilde{z}) / P_t)^{-\phi} \).

Let \( y_t(z)^d \) denote the total demand for good \( z \). As in Thomas Thomas (2008b), once the firm sets the price \( p_t(z) \) it must meet the demand \( y_t(z)^d \) by adjusting hours per worker, taking as given its labor force \( n_t(z) \). In particular, the real revenue \( (p_t(z) / P_t) y_t(z)^d \) is independent of \( n_t(z) \) and \( h_t(z) \). Independently of the price setting structure, its expected present discounted sum of real profits \( \mathcal{J}_t(z) \) evolves as

\[ \mathcal{J}_t(z) = \frac{p_t(z)}{q_t^z} y_t(z)^d - w_t(z)n_t(z) - kv_t^z(z) + E_tQ_{t+1}^n \mathcal{J}_{t+1}(z) \]

In this setup, \( w_t(z) \) is treated as a wage schedule \( w_t(z) = w(h_t(z)) \), i.e. unions and firms regard the wage as a function of the hours per worker.

Let \( j_t(z) \equiv \partial \mathcal{J}_t(z) / \partial n_t(z) \) denote the value of the marginal worker to the firm. From the production function, we know that \( \partial h_t(z) / \partial n_t(z) = -h_t(z) / n_t(z) \). Using the law of motion of \( n_t(z) \), the evolution dynamics of \( j_t(z) \) is then described by

\[ j_t(z) = w_t(z)h_t(z) - w_t(z) + (1-\rho)E_tQ_{t+1}^n j_{t+1}(z) \]

The firm chooses \( v_t^z(z) \) to maximize \( \mathcal{J}_t(z) \) subjected to the law of motion of \( n_t(z) \). The first order condition is \( k/q_t^z = E_tQ_{t+1}^n j_{t+1}(z) \). The last two results yield

\[ \frac{k}{q_t^z} = E_tQ_{t+1}^n w_{t+1}(z)h_{t+1}(z) - E_tQ_{t+1}^n w_{t+1}(z) + (1-\rho)E_tQ_{t+1}^n \frac{k}{q_{t+1}^z} \]

The solution to the Nash bargaining is \( u_t^z(z) = b s_t(z) \), where \( s_t(z) \equiv u_t(z) + j_t(z) \) is the total surplus of each match. The last results imply

\[ E_tQ_{t+1}^n u_{t+1}(z) = \frac{bk}{(1-b)q_t^z} \quad ; \quad E_tQ_{t+1}^n b_{t+1}(z) = \frac{k}{(1-b)q_t^z} \]

Since \( p_t \) is a predetermined variable, I obtain \( E_tQ_{t+1}^p P_{t+1}u_{t+1}(z) = bk\theta_t^f / (1-b) \). Plugging this result into the equation for \( u_t(z) \), I obtain

\[ w_t(z) = b u_t^z(z)h_t(z) + \frac{(1-k)}{q_t^z} j_t(z) + (1-b)w^n + b(1-\rho)k\theta_t^f \]

whose solution is the firm’s wage curve

\[ w_t(z) = b \frac{u_t^z(z)h_t(z)}{q_t^z} + (1-b)w^n + b(1-\rho)k\theta_t^f \]

Plugging it into the result for \( k/q_t^z \), I obtain the firm’s job creation curve

\[ \frac{k}{q_t^z} = E_tQ_{t+1}^n \left[ (1+\nu)b \frac{u_{t+1}(z)/q_{t+1}(z)}{w_{t+1}(z)} - w_{t+1}(z) + \frac{(1-\rho)b}{q_{t+1}^z} \right] \]

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Let $I$ derive the law of motion of aggregate disutility $z$. Thus, the aggregate market clearing condition is the expenditure matches the total revenue:

$$\text{PROPOSITION 1:}$$

The aggregate disutility $D$.

Proofs of Propositions 1 and 2

Since $n_{t+1}(z)$ is predetermined, I multiply the curves by $n_{t+1}(z)$ and integrate across $z$ to obtain the aggregate wage and job creation curves:

$$w_t = b\frac{\nu_{t+1}}{\nu_t} + (1-b)w^* + b(1-\rho)k\theta I,
\frac{k}{\rho} = E_tQ_{t+1}^i \left[(1+\nu)b\frac{\nu_{t+1}/n_{t+1}}{\nu_t} - w_{t+1} + (1-\rho)\frac{k}{\rho}\right]$$

In equilibrium, total demand $y_t(z)^d$ for good $z$ matches the production $y_t(z)$. Total expenditure on good $z$ is $E_t(z) \equiv p_t(z) c_t(z) + kp_t(z) \nu_t(z)$, where $\nu_t(z) \equiv \int_0^1 \nu_t(\tilde{z},z) d\tilde{z}$. The expenditure matches the total revenue $R_t(z) \equiv p_t(z) y_t(z)$ of firm $z$. Therefore, $y_t(z) = c_t(z) + k\nu_t(z)$.

Aggregating $\int_0^1 R_t(z) dz = \int_0^1 E_t(z) dz$ over all firms gives

$$P_tY_t = P_tC_t + k \int_0^1 p_t(z) \int_0^1 \nu_t(\tilde{z},z) d\tilde{z} = P_tC_t + \int_0^1 \left(k \int_0^1 p_t(z) \nu_t(\tilde{z},z) dz\right) d\tilde{z} = P_tC_t + \int_0^1 kP_t\nu_t(\tilde{z}) d\tilde{z}$$

Therefore, $P_tY_t = P_tC_t + kP_t\nu_t$, where $Y_t \equiv \frac{1}{P_t} \int_0^1 p_t(z) y_t(z) dz$ is the aggregate output. Thus, the aggregate market clearing condition is $Y_t = C_t + k\nu_t$.

D Proofs of Propositions 1 and 2

PROPOSITION 1: The aggregate disutility $v_t$ and its corresponding aggregate relative price $P_t$ evolve according to

$$v_t = \frac{\chi}{1 + \nu} \left(\frac{Y_t}{A_t}\right)^{(1+\omega)} P_t^{-\phi(1+\omega)}$$

$$P_t^{-\phi(1+\omega)} = (1 - \alpha) \left(\frac{N_t}{D_t}\right)^{-\phi(1+\omega)(1+\omega)} + \alpha (\Pi_t)^{\phi(1+\omega)} P_t^{-\phi(1+\omega)}$$

Proof. Let $\omega \equiv \frac{(1+\nu)}{\nu} - 1$. Using the production and demand functions, I rewrite the aggregate disutility $v_t \equiv \int_0^1 v_t(z) dz$ as follows:

$$v_t = \frac{\chi}{1 + \nu} \int_0^1 h_t(z) dz = \frac{\chi}{1 + \nu} \int_0^1 \left(\frac{A_t^2}{P_t} v_t(z) dz\right) dz = \frac{\chi}{1 + \nu} A_t^{-1} (1+\omega) v_t \int_0^1 \left(\frac{Y_t}{A_t}\right)^{-\phi(1+\omega)} dz$$

I derive the law of motion of $P_t$ using the the structure of the Calvo price setting and the first order condition of price setting firms:

$$P_t^{-\phi(1+\omega)} = (1-\alpha) \int_0^1 \left(\frac{P_t}{P_{t-1}}\right)^{-\phi(1+\omega)} dz + \alpha \int_0^1 \left(\frac{P_t}{P_{t-1}}\right)^{-\phi(1+\omega)} dz = (1-\alpha) \int_0^1 \left(\frac{N_t}{D_t}\right)^{-\phi(1+\omega)} dz + (1-\alpha) \int_0^1 \left(\frac{N_t}{D_t}\right)^{-\phi(1+\omega)} dz + \alpha (\Pi_t)^{\phi(1+\omega)} P_t^{-\phi(1+\omega)}$$

PROPOSITION 2: The aggregate worked hours $H_t$ and its corresponding aggregate
relative price $P_{Ht}$ evolve according to

$$H_t = \left( \frac{Y_t}{A_t} \right)^{(1+\bar{\omega})} P_{Ht}^{-(\phi(1+\bar{\omega}))}$$

$$P_{Ht}^{-(\phi(1+\bar{\omega}))} = (1 - \alpha) \left( \frac{N_t}{D_t} \right)^{-\frac{\phi(1+\bar{\omega})}{1+\bar{\omega}}} + \alpha \left( \Pi_t \right)^{\phi(1+\bar{\omega})} P_{Ht-1}^{-(\phi(1+\bar{\omega}))}$$

**Proof.** Let $\bar{\omega} \equiv \frac{1}{\hat{e}} - 1$. Closely following my steps in the last proof, I rewrite the aggregate hours worked $H_t \equiv \int_0^1 H_t(z) \, dz$ as follows:

$$H_t = \int_0^1 A_t^{-\frac{1}{2}} y_t(z)^{\frac{1}{2}} \, dz = A_t^{-(1+\bar{\omega})} \int_0^1 Y_t \left( \frac{p_t(z)}{P_t} \right)^{-(1+\bar{\omega})} \, dz = A_t^{-(1+\bar{\omega})} Y_t^{(1+\bar{\omega})} \int_0^1 \left( \frac{p_t(z)}{P_t} \right)^{-(1+\bar{\omega})} \, dz = \left( \frac{Y_t P_{Ht}}{A_t} \right)^{(1+\bar{\omega})}$$

The derivation of the law of motion of $P_{Ht}$ are almost identical to the ones leading to the law of motion of $P_t$. ■