Price-Setting with Unobservable Elasticities of Demand: The Business-Cycle Effects of Heterogeneous Expectations

Christian Jensen*
University of South Carolina

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Abstract
In a dynamic stochastic general equilibrium model with monopolistic competition and flexible prices, we show how more realistic price-setting, discarding the conventional assumption of known exogenous demand elasticities, can distort production through relative prices, thereby generating business cycle fluctuations indistinguishable from those produced by traditional productivity shocks. We do this by assuming firms must estimate their exogenous demand elasticities, which due to idiosyncratic shocks leads to heterogeneous estimates and expectations, and distorts relative prices and production. We then argue that more generally, price-setter’s heterogeneous expectations about the contemporary values of aggregate variables will distort production. The reason is that these expectations shape their perceptions of relative prices and market shares, which affect their perceived demand elasticities, and thereby, their price-setting and production.

*Department of Economics; University of South Carolina; 1705 College Street; Columbia, SC 29208; cjensen@alumni.cmu.edu.
1 Introduction

The mechanisms by which prices change over time to clear markets are not well-understood. Even with monopolistic competition, where prices are set endogenously by sellers so as to maximize profits, our models tend to oversimplify, assuming that price-setters merely apply a known exogenous mark-up to their nominal production costs. In reality, price-setting is not this simple. The optimal mark-up typically depends on the size of the market, the number of competitors and their mark-ups, which might be unknown at the time prices are set. The convenience of assuming monopolistic competition with exogenously given mark-ups, which is what has spurred its popularity and widespread use in the literature, especially in sticky-price models, is that in the absence of nominal rigidities, relative prices are set optimally through the unsynchronized actions of individual producers, as a result of the coordination of their nominal prices through production costs.\footnote{Sticky-price and sticky-information models introduce menu costs and informational delays into the price-setting problem, but otherwise assume that price-setters simply apply an exogenously given mark-up. In contrast to our model, distortions to relative prices arise due to the staggered adjustment of prices that arises due to the costs of updating information and adjusting prices. See Calvo (1983), Mankiw and Reis (2002) and Rotemberg (1982) for examples.}

However, moving away from this simplistic framework, even as slightly as assuming that the exogenously optimal mark-ups are not known by price-setters, but must be estimated, the effect on relative prices can be so large as to have a significant impact on aggregate output. Moreover, we show that these distortions can explain a considerable part of the fluctuations observed in real GDP, and be indistinguishable from the productivity shocks used to explain business cycle fluctuations in the tradition of Kydland and Prescott (1982).

Idiosyncratic shocks, to consumer tastes or the production technology for final goods, generate dispersion in a priori identical monopolistically competing intermediate-good producers’ perceived elasticities of demand, distorting their relative prices, thus affecting the cost-minimizing composition of final goods. By substituting intermediate goods that are priced too high with those that are priced too low, final-good producers keep costs down, sacrificing productivity. In a more general setup that captures the strategic interaction between competitors and the interdependence across markets, a monopolistic seller’s optimal price depends on the simultaneous pricing decisions of other sellers and the contemporary
size of the market, which, in turn, depend on the concurrent actions of all sellers. This interdependence, and the simultaneity of the price-setting, implies that pricing decisions must be based on expectations of market-wide, or even economy-wide, aggregate variables.\(^2\)

When such expectations differ across price-setters, relative prices of intermediate goods become distorted, affecting the productivity with which they are combined into final goods. As the dispersion in expectations of aggregate variables fluctuates over time with changing levels of uncertainty and shock-heterogeneity, the productivity with which final goods are produced also varies, making aggregate output fluctuate.

While surveys show that expectations of aggregate economic variables differ across both households and professional forecasters, it is not immediate what could generate such heterogeneity in a model with rational expectations.\(^3\) In practice, dispersion arises from forecasts being based on different models, ideas and idiosyncratic observations. There could be many reasons why such diverse practices, including non-rational ones, persist, but without doubt an important factor is that there is still too much we do not understand about the workings of the economy, in part due to poor, or non-existent, measures of key variables. For example, using the neoclassical model to forecast aggregate output requires projecting total factor productivity, but without a good understanding of the underlying causes of productivity shocks, these have to be estimated from Solow residuals. This, in turn, requires having a reliable measure of the capital stock, which we lack. As a result, our GDP forecasts tend to be based on ad-hoc approaches that to a great extent ignore the theory. In the model presented here, the capital stock is assumed to be perfectly observable (it is a choice variable of the households), and together with the labor input, it can be used to deduce total factor productivity. However, this aggregate productivity depends on the efficiency with which each of the intermediate goods are produced and combined into final goods, and the mark-ups applied by each of their producers, which we assume is never measured.

\(^2\)As in the sticky price and information models, price-setting depends on expectations of aggregate variables, but with some key distinctions. The source of this dependency, and the distortions to relative prices, is that no one can know the values of contemporary aggregates that rely on the sum of all individuals’ actions before making their own decisions, instead of staggered price-setting and updating of information over time. As a result, the dynamic implications and the distributions of relative prices and expectations are quite different.

\(^3\)Mankiw, Reis and Wolfers (2003) not only document the disparities that exist in inflation forecasts but also show that these vary greatly over time.
across all the heterogenous firms. Hence, while individual price-setters can easily obtain a history of aggregate productivity, we assume they cannot observe all of the underlying variables affecting it, which would require obtaining a cross-section of all their competitors. Because of this, they have to form their expectations of aggregate productivity based on past observations of its value, which may or may not be rational, and on estimates of the underlying processes produced with their own histories of shocks. Since each price-setter will have a unique history of idiosyncratic shocks, they will produce different estimates of the processes generating these, even if these are a priori identical for all, and thus generate heterogeneous expectations of aggregate productivity and all other aggregate variables that depend on it.\footnote{Expectations would become more homogenous as price-setters obtained longer histories of their own observations, or if information were shared across firms.}

As in the misperception model of Lucas (1972), it is incomplete information that distorts relative prices and output in our setup. While this information problem is not caused by (random monetary) policy in our model, there is still a role for policy to improve the situation by contributing towards homogenizing expectations. Though factor prices serve as a coordination device for simultaneous price-setters, helping these achieve their optimal relative prices, this coordination is insufficient when the optimal mark-up depends on the simultaneous actions of one’s competitors, and thus, contemporary aggregate variables that have not yet been realized. As a result, there is scope for improving welfare by coordinating expectations about such variables, thereby minimizing the distortions to relative prices, and increasing productivity. In particular, we argue this is relevant for the aggregate price and output levels, since sellers cannot determine their elasticity of demand, and thus their optimal price, without knowing their own relative price and market share.

Our dynamic general equilibrium model builds on that of Blanchard and Kiyotaki (1987) and consists of an infinite number of a priori identical monopolistically competing producers that rent capital and labor from households in competitive factor markets to produce differentiated intermediate goods that households purchase to compose final goods. The production side of the economy is presented in the next section, including intermediate-
good producers’ price-setting decisions, which are initially functions of the expected values of their exogenous elasticities of demand, which they need to estimate. The following two sections describe the households, which face a standard dynamic decision problem, and the equilibrium conditions for the model economy, respectively. The subsequent section motivates how idiosyncratic shocks can make a priori identical intermediate-good producers obtain different estimates for their exogenous demand elasticities, and the distribution of these. The following sections study the impact this dispersion has on aggregate output for different values of the elasticity of substitution between intermediate goods. This is first done for an inelastic labor supply (steady state), as this only requires specifying a distribution for the expected demand elasticities across intermediate-good producers and the weight of capital versus labor in the production of intermediate goods ($\alpha$). The analysis is then repeated for the case where households adjust their labor supply optimally over time, which requires a more detailed specification and calibration of the model. The subsequent section studies intermediate-good producers’ elasticities more generally, arguing that dispersion in their perceived values can arise from aggregate uncertainty, that is, from expectations of contemporary aggregates differing across price-setters. We conclude that variations in the dispersion of heterogeneous expectations produced by changes in the degree of uncertainty can contribute significantly to business cycle fluctuations, and be almost indistinguishable from the productivity shocks traditionally used to explain these.

2 Producers

Each of the continuum of measure one of identical households produces $y$ units of final good by combining a continuum of differentiated intermediate goods $x_i$ indexed by $i \in [0, 1]$, so that

$$y_t = \left( \int_0^1 (\gamma_i x_i)^{\theta - 1} \, di \right)^{\frac{1}{\theta - 1}}$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between any two intermediate goods. This is the Dixit-Stiglitz aggregator modified to incorporate productivity shocks $\gamma_i$ that can change the relative weight of each intermediate good in the production of the final good, as well as the general productivity of intermediate goods in the production of the final good,
see Dixit and Stiglitz (1977). Assuming intermediate goods are the only inputs required to produce final goods, each household chooses the optimal mix of these so as to minimize the cost of providing final goods by solving

$$\min_{\{x_i\}_{i=0}^1} \int_0^1 P_i x_i di$$ \hspace{1cm} (2)$$

subject to the production function (1), where \(P_i\) is the price of intermediate good \(i\). The resulting demand for intermediate good \(i\) from each of the households is

$$x_i = \left(\frac{P_i}{P}\right)^{-\theta} \gamma_i^{\theta-1} y$$ \hspace{1cm} (3)$$

for all \(i \in [0, 1]\), where all variables are known by the household, since it is assumed to know its demand \(y\) of the final good, the prices \(P_i\) of the intermediate goods it buys, the shocks \(\gamma_i\), and the marginal cost of producing the final good

$$P = \left(\int_0^1 \left(\frac{P_i}{\gamma_i}\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$$ \hspace{1cm} (4)$$

which is derived by inserting the demand for intermediate goods (3) into the production function for final goods (1). Because all households are identical, they compose identical final goods at identical cost, and since they can all produce this good, its market price equals its cost of production (4). Aggregating intermediate-good demands (3) across all households, we find the aggregate demand for intermediate good \(i\) to be

$$X_i = \left(\frac{P_i}{P}\right)^{-\theta} \gamma_i^{\theta-1} Y$$ \hspace{1cm} (5)$$

where \(Y\) is the aggregate demand for final goods.

Intermediate-good producer \(i\) finds the optimal mix of inputs, capital \(k_i\), labor \(n_i\) and

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5The \(\gamma_i\) shocks could also be interpreted as taste shocks, but then the composition of the final good would change over time, making it difficult to compare final goods produced at different times.
land \( l_i \), so as to minimize its production costs by solving

\[
\min_{k_i, n_i, l_i} Rk_i + Wn_i + Fl_i
\]  

subject to its production technology

\[
X_i = z_i k_i^{\alpha} n_i^{1-\alpha-\nu} l_i^\nu
\]

where \( W \) is the nominal wage, \( R \) is the nominal rental rate of capital, while \( F \) is the nominal rental rate of land, \( \alpha \in (0, 1) \), \( \nu \in (0, 1) \), and \( z_i \) is an exogenous productivity shock that is known by producer \( i \), but no one else. The resulting first-order conditions yield its factor demands

\[
k_i = \alpha \frac{\lambda_i X_i}{R}
\]

\[
n_i = (1 - \alpha - \nu) \frac{\lambda_i X_i}{W}
\]

\[
l_i = \nu \frac{\lambda_i X_i}{F}
\]

where

\[
\lambda_i = \frac{1}{z_i} \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F}{\nu} \right)^\nu
\]

is the marginal cost of producing intermediate good \( i \). In addition to choosing the cost-minimizing input mix, intermediate-good producer \( i \) needs to price its good. It does so by choosing the price \( P_i \) that maximizes its expected profits given the demand it faces (5), and thus solves

\[
\max_{P_i} E_i \left[ (P_i - \lambda_i) \left( \frac{P_i}{P} \right)^{-\theta} \gamma_i^{\theta-1} Y \right]
\]

where \( E_i \) is the expectations operator, which is needed because intermediate-good producer \( i \) cannot observe the shock \( \gamma_i \), the elasticity \( \theta \), aggregate demand \( Y \), or the aggregate price level \( P \). Exploiting that it can observe the demand \( X_i \) for its good and the marginal cost \( \lambda_i \) of producing it, the profit-maximizing condition can be written as

\[
X_i \left( 1 - \frac{P_i - \lambda_i}{P_i} E_0 \theta \right) = 0
\]
Intermediate-good producer \( i \) continuously adjusts its price \( P_i \) until this condition is satisfied, which makes the optimal price

\[
P_i = \lambda_i \frac{E_i \theta}{E_i \theta - 1}
\]  

(14)

a gross mark-up \( \frac{E_i \theta}{E_i \theta - 1} \in (1, \infty) \) of the marginal cost of production \( \lambda_i \).

Inserting the optimal pricing condition (14) into the price aggregator (4), after substituting for the marginal cost of production (11), yields the price level

\[
P = \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F}{\nu} \right)^{\nu} \left( \frac{1}{\gamma z_i E_i \theta - 1} \right)^{1-\theta} \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1} di \right)^{\frac{1}{1-\theta}}
\]  

(15)

and thus the relative price

\[
\frac{P_i}{P} = \frac{\frac{1}{z_i E_i \theta - 1}}{\left( \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} di \right)^{\frac{1}{1-\theta}}}
\]  

(16)

Substituting this relative price into the demand function for intermediate good \( i \) (5), and substituting the resulting equation and the marginal production cost (11), into the factor demands (8), (9) and (10), and aggregating over all intermediate-good producers, we find the aggregate demands for capital, labor and land,

\[
K = \left( \frac{R}{\alpha} \right)^{\alpha-1} \left( \frac{W}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F}{\nu} \right)^{\nu} Y \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} \left( \frac{E_i \theta}{E_i \theta - 1} \right)^{-\theta} \left( \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} di \right)^{\frac{1}{1-\theta}}
\]  

(17)

\[
N = \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - \nu} \right)^{-\alpha-\nu} \left( \frac{F}{\nu} \right)^{\nu} Y \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} \left( \frac{E_i \theta}{E_i \theta - 1} \right)^{-\theta} \left( \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} di \right)^{\frac{1}{1-\theta}}
\]  

(18)

\[
L = \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F}{\nu} \right)^{\nu-1} Y \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} \left( \frac{E_i \theta}{E_i \theta - 1} \right)^{-\theta} \left( \int_0^1 \frac{1}{\gamma z_i E_i \theta - 1}^{1-\theta} di \right)^{\frac{1}{1-\theta}}
\]  

(19)

respectively. Since producers’ decisions are not dynamic, the time-subscripts have been
suppressed throughout this section.

3 Consumers

In addition to effortlessly composing final goods, households rent their labor \( N \), capital \( K \) and land \( L \) to the collectively owned intermediate-good producers in order to provide for consumption \( C \) and the accumulation of assets: physical capital \( K \), money \( M \) and bonds \( B \). Since households are assumed to be identical, aggregation is trivial, so we focus on aggregates directly. Putting money in the utility function \( u \) as a short-cut, each of the continuum of measure one of identical households solves the dynamic problem

\[
\max_{C_t, N_t, K_t, B_t, M_t} E_0 \sum_{t=0}^{\infty} \beta^t u \left( C_t, 1 - N_t, \frac{M_t}{P_t} \right) \tag{20}
\]

subject to

\[
K_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + C_t =
\]

\[
\frac{W_t}{P_t} N_t + \frac{R_t}{P_t} K_{t-1} + \frac{F_t}{P_t} + (1 - \delta) K_{t-1} + \frac{(1 + \Re_t) B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \Pi_t + S_t
\]

given a discount rate \( \beta \in (0, 1) \), depreciation rate \( \delta \in (0, 1) \), and initial conditions \( K_{t-1}, B_{t-1}, M_{t-1} \) and \( \Re_{t-1} \), where \( \frac{W_t}{P_t} \) is the real wage, \( \frac{R_t}{P_t} \) is the real rental rate of capital, \( \frac{F_t}{P_t} \) is the real rental cost of land, \( \Re_t \) is the nominal interest rate on bonds, \( \Pi_t \) are profits from the production of intermediate goods, \( S_t \) are transfers from the government, and \( P_t \) is the aggregate price level, equal to the price of the final good. To simplify, the supply of land is normalized to one.\(^6\) The first-order conditions are given by the budget constraint (21) and

\[
u'_2 \left( C_t, 1 - N_t, m_t \right) = u'_1 \left( C_t, 1 - N_t, m_t \right) \frac{W_t}{P_t} \tag{22}\]

\[
u'_1 \left( C_t, 1 - N_t, m_t \right) = \beta E_t u'_1 \left( C_{t+1}, 1 - N_{t+1}, m_{t+1} \right) \left( \frac{R_{t+1}}{P_{t+1}} + 1 - \delta \right) \tag{23}\]

\(^6\)The assumption of an inelastic supply of land and the inclusion of this input allows us to obtain an explicit solution for aggregate output. The reason is that the production side only pins down the optimal factor mix, not the levels. However, by fixing the level of one of the inputs, all other levels are pinned down. We let the importance of land converge to zero below.
\[ u'_1(C_t, 1 - N_t, m_t) = (1 + \Re_t) \beta E_t u'_1(C_{t+1}, 1 - N_{t+1}, m_{t+1}) \left( \frac{P_{t+1}}{P_t} \right)^{-1} \] (24)

\[ u'_1(C_t, 1 - N_t, m_t) = u'_3(C_t, 1 - N_t, m_t) + \beta E_t u'_1(C_{t+1}, 1 - N_{t+1}, m_{t+1}) \left( \frac{P_{t+1}}{P_t} \right)^{-1} \] (25)

where \( m_t = \frac{M_t}{P_t} \) is the demand for real money balances.

4 Equilibrium

Exploiting the government budget constraint

\[ P_t S_t + P_t G_t = M_t - M_{t-1} + B_t - (1 + \Re_{t-1}) B_{t-1} \] (26)

and that total profits for intermediate-good producers are

\[ \Pi_t = \int_0^1 P_{i,t} z_{i,t} k_{i,t-1}^{\alpha} n_{i,t}^{1-\alpha-\nu} \nu_{i,t} \theta_{i,t} d\theta_{i,t} - R_t K_{t-1} - W_t N_t - F_t \] (27)

households’ budget constraints (21) simplify to

\[ K_t + C_t + G_t = Y_t + (1 - \delta) K_{t-1} \] (28)

where

\[ Y_t = \int_0^1 P_{i,t} z_{i,t} k_{i,t-1}^{\alpha} n_{i,t}^{1-\alpha-\nu} \nu_{i,t} \theta_{i,t} d\theta_{i,t} \] (29)

is the real value of production.\(^7\) Setting aggregate demand for land (19) equal to the inelastic unitary supply yields aggregate output

\[ Y_t = K_{t-1}^{\alpha} N_t^{1-\alpha} \left( \int_0^1 \left( \frac{\gamma_{i,t} z_{i,t} E_{i,t}^{\theta-1}}{E_{i,t}^{\theta}} \right)^{\theta-1} d\theta_{i,t} \right)^{\frac{\theta}{\theta-1}} \int_0^1 \left( \frac{\gamma_{i,t} z_{i,t}}{E_{i,t}^{\theta}} \right)^{\theta-1} \left( \frac{E_{i,t}^{\theta-1}}{E_{i,t}^{\theta}} \right)^{\theta} d\theta_{i,t} \] (30)

\(^7\)We use the convention that it is capital \( K_{t-1} \) that is available to produce in period \( t \), so that intermediate-good producer \( i \) is renting \( k_{i,t-1} \) units of capital in period \( t \).
after exploiting that the aggregate demands for factors of production (17)-(19) imply that

\[
\frac{R_t}{F_t} = \frac{\alpha}{\nu K_{t-1}} \tag{31}
\]

\[
\frac{W_t}{F_t} = \frac{1 - \alpha - \nu}{\nu N_t} \tag{32}
\]

which guarantees an optimal input mix in the production of intermediate goods. Combining these two conditions with the one for the price level (15), yields

\[
\frac{R_t}{P_t} = \alpha K_{t-1}^{\alpha-1} N_t^{1-\alpha-\nu} \left( \int_0^1 \left( \frac{\gamma_{i,t} z_{i,t}}{E_{i,t}} \right) \, di \right)^{\frac{1}{1-\theta}} \tag{33}
\]

\[
\frac{W_t}{P_t} = (1 - \alpha - \nu) K_{t-1}^{\alpha} N_t^{-\alpha-\nu} \left( \int_0^1 \left( \frac{\gamma_{i,t} z_{i,t}}{E_{i,t}} \right) \, di \right)^{\frac{1}{1-\theta}} \tag{34}
\]

\[
\frac{F_t}{P_t} = \nu K_{t-1}^\alpha N_t^{1-\alpha-\nu} \left( \int_0^1 \left( \frac{E_{i,t}}{E_{i,t}} \right) \, di \right)^{\frac{1}{1-\theta}} \tag{35}
\]

which are the real rental rates and the real wage. Finally, the equilibrium price level is determined by the demand and supply for money;

\[
P_t = \frac{M_t}{m_t} \tag{36}
\]

so that the rate of inflation satisfies

\[
\frac{P_t}{P_{t-1}} = \mu_t \frac{m_{t-1}}{m_t} \tag{37}
\]

where \(\mu_t = \frac{M_t}{m_{t-1}}\) is the rate of money-printing.

Assuming that government spending \(G_t\) and the rate of money-printing \(\mu_t\) are exogenous, the first-order conditions for the households (22)-(25), the resource constraint (28), the production function (30), the factor-pricing equations (33)-(35) and the inflation equation (37), jointly determine the equilibrium values of \(N_t, C_t, R_t, m_t, K_t, Y_t, R_t, W_t, F_t, E_t, \) and \(\frac{P_t}{P_{t-1}}\), given \(K_{t-1}, m_{t-1}, G_t, \mu_t, \gamma_{i,t}, z_{i,t}\) and \(E_{i,t}\), for all \(i\). In addition, the equilibrium
price level $P_t$ can be determined given an exogenous money supply $M_t$ (or $M_{-1}$). Due to the lack of significance of land as a source of fluctuations, we let $\nu$ converge towards zero, so that land is eliminated from the model henceforth. Heterogenous expectations among intermediate-good producers about their demand elasticities $E_{i,t}\theta$ directly affects aggregate output (30) and the real payments to the factors of production, capital (33) and labor (34), and all other aggregate variables indirectly through these.

5 Dispersion

As the equilibrium conditions above show, there is no way to deduce the value of the elasticity $\theta$ from aggregate data without knowing the distributions of $\gamma_{i,t}$, $z_{i,t}$ and $E_{i,t}\theta$ across all producers $i$. Since these distributions can change over time, in particular the one for expectations $E_{i,t}\theta$, they are not likely to be known. However, when $\theta$ is constant over time, intermediate-good producer $i$ could estimate it from the demand equation (5), using historical data of aggregate output $Y$ and the price level $P$, as these become available, combined with past realizations of its own price $P_i$ and sales $X_i$. Applying the logarithm to both sides of its demand function (5) yields the linear equation

$$\ln \frac{X_i}{Y} = -\theta \ln \left( \frac{P_i}{P} \right) + (\theta - 1) \ln \gamma_i$$

(38)

where $(\theta - 1) \ln \gamma_i$ is an unobservable error term, since the $\gamma_i$-shocks are never observed by intermediate-good producers. Whatever the standard deviation of $\ln \gamma_i$ is, the shock to the demand equation would be magnified by a factor of $\theta - 1$. Hence, the larger the elasticity $\theta$, the more imprecise its estimate would be, by a factor of $(\theta - 1)$. Since the $\gamma_i$-shocks are heterogenous, the estimates of $\theta$ would differ across firms. Borrowing from this, we assume in our simulations below that intermediate-good producers’ expectational errors $E_{i,t}\theta - \theta$ are Normally distributed with mean zero and standard deviation $(\theta - 1) \sigma_\gamma$, where $\sigma_\gamma$ is the standard deviation of $\ln \gamma_i$ across firms. Having the standard error of the forecasts increase with $\theta$ is necessary in order to generate dispersion in intermediate-good producers’ mark-ups also for larger values of $\theta$. 

12
When the forecast errors $E_i\theta - \theta$ are independently and identically distributed across producers according to a Normal distribution with mean zero and standard deviation $(\theta - 1) \sigma_\gamma$, the distribution of the actual mark-ups $\frac{1}{E_i\theta - \theta}$ relative to the median $\frac{1}{\theta - 1}$ are independent of $\theta$. This is illustrated in table 1, which shows how the 5th and 95th percentile mark-ups are distributed around the median for different values of $\sigma_\gamma$, reporting these as a fraction of the median mark-up. The implication is that the difference between the 5th and 95th percentile mark-ups is smaller the higher the value of $\theta$. For example, when $\theta = 20$, the median mark-up is 5.26%, so with $\sigma_\gamma = .5$ the 5th to 95th percentile mark-up interval is (2.89%, 29.61%), but when $\theta = 2.51$ and the median mark-up is 66.23% the same interval is (36.43%, 372.87%). Hence, $\sigma_\gamma = .5$ generates less dispersion in the mark-ups when $\theta = 20$ than when $\theta = 2.51$.

<table>
<thead>
<tr>
<th>$\sigma_\gamma$</th>
<th>.05</th>
<th>.10</th>
<th>.15</th>
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<th>.25</th>
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Table 1: Effect of $\sigma_\gamma$ on 5th and 95th percentile mark-ups relative to median.

The productivity shocks $\gamma_i$ and $z_i$ are assumed to be independently distributed across firms according to a Normal distribution with a unitary mean. To simplify, these shocks are also assumed to be independent of the expectational errors $E_i\theta - \theta$.

6 Inelastic labor supply (steady state)

While discerning all the effects of dispersion requires solving the dynamic model, the implications for equilibrium real output and factor payments can be obtained without doing so when the labor supply is inelastic ($N_t = .3$). The steady state with constant dispersion is a special case of this ($N = .3$), so the results in this section also represent the long-run effects when the labor supply is elastic.

A mean-preserving spread of productivity, $z_{i,t}\gamma_{i,t}$, across firms would have a positive

---

8 According to the linearized demand function (38), forecast errors $E_i\theta - \theta$ could be correlated with autocorrelated $\gamma_i$-shocks.
effect on both the equilibrium output and real return on capital, as a result of inputs flowing from low-productivity to high-productivity firms. This effect is greater the larger the elasticity $\theta$, that is, the easier it is to substitute between intermediate goods. Since our primary interest is to determine the effects of heterogeneous expectations (of perceived elasticities $E_{i,t}\theta$), we control for the effects of dispersion in productivity, while at the same time allowing for these two types of heterogeneities to interact. To do so, our benchmark assumes that the perceived elasticities are constant across firms, $E_{i,t}\theta = \theta$ for all $i$, while maintaining the dispersion in terms of the productivity shocks $z_{i,t}$ and $\gamma_{i,t}$.

When $E_{i,t}\theta = \theta$ and $N_t = .3$, aggregate output (30) is

$$Y_t = K_{t-1}^\alpha (.3)^{1-\alpha} \left( \int_0^1 (\gamma_{i,t} z_{i,t})^{\theta-1} di \right)^{\frac{1}{\theta-1}} \tag{39}$$

Dividing output with dispersion in perceived elasticities (30) by output without this dispersion (39), yields the fraction

$$\bar{r}_t = \frac{\left( \int_0^1 (\gamma_{i,t} z_{i,t})^{\theta-1} di \right)^{\frac{\theta}{\theta-1}}}{\left( \int_0^1 (\gamma_{i,t} z_{i,t})^{\theta-1} di \right)^{\frac{1}{\theta-1}}} \leq 1 \tag{40}$$

by which output is reduced due to dispersion in perceived elasticities, for any given capital level. The effect of dispersion in perceived elasticities is to immediately reduce aggregate output, independently of whether or not the perceived elasticities are centered around the true value, only the dispersion matters. The intuition is that since producers face the same elasticity $\theta$, disparities in their beliefs $E_{i,t}\theta$ make them choose suboptimal relative prices, which in turn makes the composition of the final good inefficient, resulting in less of it being produced. Errors in beliefs that are homogenous across producers have no effect (given the inelastic supply of the factors of production), because such errors do not affect relative prices. The larger the disparities in the estimated elasticities, $E_{i,t}\theta$, the greater the distortions to relative prices, and the less final good is produced. This negative impact on output is larger the smaller the value of $\theta \in (1, \infty)$, since this reduces the substitutability
between intermediate goods, making it more difficult to adjust the mix of these in reaction to distortions to relative prices.

In addition to the immediate effect on output through the mix of intermediate goods, dispersion also affects output over time through capital accumulation. It does so not only through the wealth effect from the immediate drop in output, a rise in heterogeneity across intermediate-good producers’ perceived elasticities that is expected to endure also affects capital accumulation through its rental rate. Comparing, as above, the rental rate with dispersion in expectations (33) with the one where $E_{i,t}\theta = \theta$,

$$\frac{R_t}{P_t} = \frac{\theta - 1}{\theta} \alpha K_{t-1} \left( \int_0^1 (\gamma_{i,t} z_{i,t})^{\theta - 1} \, dt \right) \theta^{-1}$$

(41)

yields the multiple

$$\bar{q}_t = \left( \frac{\int_0^1 \left( \frac{E_{i,t} \theta - 1}{E_{i,t} \theta} \right)^{\theta - 1} \, dt}{\theta - 1} \right) \left( \int_0^1 (\gamma_{i,t} z_{i,t})^{\theta - 1} \, dt \right) \theta^{-1}$$

(42)

which measures the impact dispersion in perceived elasticities $E_{i,t}\theta$ has on the equilibrium real rental rate of capital (and the real wage) for a given capital level. It can be larger or smaller than one, since the effect of dispersion can be positive or negative. From the previous paragraph, we know there is a negative effect from the inefficient mix of intermediate goods that dispersion in relative prices gives rise to, which makes capital be used less efficiently in producing the final good, thus reducing its rental rate. In addition, dispersion makes some firms apply a mark-up that is higher, and others apply a mark-up that is lower, than they otherwise would. This affects the rental rate, because the higher a mark-up an intermediate-good producer applies, the less capital it employs, as the equation for factor-demand (8) shows, thus reducing the demand for capital and its rental rate. Whether this effect though the mark-ups is positive or negative depends on the skewness of the distribution of the mark-ups, and since firms that apply low mark-ups become larger than those that apply high ones, a negative effect requires a positive skewness. In our case, this positive skewness arises due to the nonlinearity of the mark-ups in terms of the perceived elasticities, and because the perceived elasticities $E_{i,t}\theta$ are required to be larger than one for the corresponding mark-ups.
to be positive. Even errors in beliefs that are homogenous across firms have an impact on the rental rate, as they affect the average mark-up, and thereby, the demand for capital.

While the immediate effect dispersion in beliefs $E_{i,t}\theta$ has on aggregate output with an inelastic labor supply can be measured through the ratio $\bar{r}_t$, the long-run one, assuming the change in dispersion is permanent, needs to take into account the impact on the capital stock induced by the change in its rental rate. In a steady state with constant productivity and beliefs, so that $E_{i,t}\theta = E_i\theta$ and $\gamma_{i,t}z_{i,t} = \gamma_i z_i$, implying that the dispersion across intermediate-good producers is constant over time, where, in addition, $\mu_t = \mu$, $G_t = G$ and $N_t = .3$, aggregate output is

$$Y = .3 \left( \frac{1}{\beta} - 1 + \delta \right) \frac{\alpha}{\alpha - 1} \left( \int_0^1 \left( \gamma_i z_i E_i \theta - 1 \right) \frac{d\theta}{\beta} \right)^{\frac{\theta + (1 - \theta)\alpha}{(1 - \alpha)(\theta - 1)}}$$

In an identical steady state without dispersion in perceived elasticities, so that $E_i\theta = \theta$, we have

$$Y = .3 \left( \frac{1}{\beta} - 1 + \delta \right) \frac{\alpha}{\alpha - 1} \left( \theta - 1 \right) \left( \int_0^1 \left( \gamma_i z_i \right) \frac{d\theta}{\beta} \right)^{\frac{\theta + (1 - \theta)\alpha}{(1 - \alpha)(\theta - 1)}}$$

Thus, steady-state output with dispersion in beliefs $E_{i,t}\theta$ is a fraction

$$\bar{r} = \frac{\left( \int_0^1 \left( \gamma_i z_i E_i \theta - 1 \right) \frac{d\theta}{\beta} \right)^{\frac{\theta + (1 - \theta)\alpha}{(1 - \alpha)(\theta - 1)}}}{\left( \frac{\theta - 1}{\beta} \right) \left( \int_0^1 \left( \gamma_i z_i \right) \frac{d\theta}{\beta} \right)^{\frac{\theta + (1 - \theta)\alpha}{(1 - \alpha)(\theta - 1)}}} \leq 1$$

of what it would be in a steady state without such dispersion.

With a constant labor supply, we can quantify the impact dispersion in perceived elasticities $E_i\theta$ has on output, through $\bar{r}_t$ immediately, and through $\bar{r}$ in the long run, by computing these. Doing so only requires specifying the distributions for productivity $\gamma_i z_i$ and perceived elasticities $E_i\theta$, and the true value of the elasticity $\theta$, and, for the long-run effects, the value of $\alpha$. Quantifying the impact of dispersion when the labor supply is elastic

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9 Modeling dispersion in terms of the mark-ups directly without any skewness makes dispersion have a positive impact on the rental rate.
requires specifying the model in greater detail and solving out for the dynamics (see the next section).

A steady-state income share of .6 for labor implies that

\[
\alpha = 1 - .6 \int_0^1 \left( \gamma_i z_i \frac{E_i^{\theta-1}}{E_i^{\theta-1}} \right)^{\theta-1} \frac{di}{\int_0^1 \left( \gamma_i z_i \right)^{\theta-1} \left( \frac{E_i^{\theta-1}}{E_i^{\theta-1}} \right)^{\theta} di}
\]

which in a steady state without dispersion means

\[
\alpha = 1 - .6 \frac{\theta}{\theta - 1}
\]

Ignoring the effects of dispersion, a strictly positive value of \( \alpha \) requires \( \theta > 2.5 \).\(^{10}\) The same is, more or less, required with dispersion. Lowering the elasticity \( \theta \) raises intermediate-good producers’ mark-ups, and thus the share of income that goes to profits. Since the income share of labor is fixed at .6, this means that the share of capital gets squeezed as more and more of the income goes to profits as the elasticity \( \theta \) is reduced, making the importance of capital in production (\( \alpha \)) fall.

Figure 1 plots \( \bar{r}_t \), our measure of the effect dispersion in the perceived elasticities \( E_{i,t} \theta \) has on aggregate output, for given levels of capital and labor, as a function of the standard deviation \( \sigma_{\gamma} \), defined above, for different values of \( \theta \). As expected, it shows that this effect is larger the lower the elasticity \( \theta \) and the larger the dispersion \( \sigma_{\gamma} \). At most, dispersion in perceived elasticities reduces output by about 8.5%, which is only attainable with minimal competition, \( \theta = 2.51 \). With a moderate level of competition, \( \theta = 5 \), the negative impact on output is at most 4.1%, while it is at most 1% with high competition, \( \theta = 20 \). Figure 2 plots \( \bar{q}_t \), the measure of the impact dispersion in perceived elasticities has on the rental rate of capital, for given levels of capital and labor. With the distributions assumed above, dispersion in perceived elasticities reduces the rental rate, however, it does so by at most

\[^{10}\text{The income share of labor is usually taken to be between .6 and .7, with } \frac{2}{3} \text{ being the most popular value. We have chosen a value at the lower end because it allows the elasticity } \theta \text{ to be smaller without } \alpha \text{ going negative. With a labor share of .7, having a strictly positive value of } \alpha \text{ requires } \theta > 3.33, \text{ with a share of } \frac{2}{3} \text{ it requires } \theta > 3. \text{ The value of } \alpha \text{ affects the steady-state effects of dispersion } \bar{r}, \text{ but has no impact on the immediate effects } \bar{r}_t \text{ and } \bar{q}_t.\]
5.5% for $\theta = 2.51$ and .5% for $\theta = 20$. Figure 3 plots $\bar{r}$, the measure of the impact dispersion in perceived elasticities has on steady-state output. It shows that the impact on output is amplified by allowing capital to adjust, making the maximum reduction in output be 8.7% for $\theta = 2.51$, 4.8% for $\theta = 5$ and 1.3% for $\theta = 20$. The effect on output from letting capital adjust is greater the larger is $\theta$, despite that this makes dispersion in the perceived elasticities have a smaller impact on the rental rate, because $\alpha$ is larger the higher $\theta$ is, making capital carry a larger weight in production.

Intuitively, the effects of dispersion should be greater when the labor supply is elastic. The reason is that dispersion in perceived elasticities has the same impact on the real wage as on the rental rate of capital, and therefore discourages the labor supply, which adjusts immediately, instead of gradually, as capital does. This is quantified in the following section.

7 Elastic labor supply

To simplify the dynamic analysis, and because our main focus is on the impact of dispersion in perceived elasticities $E_{it}\theta$, we henceforth eliminate the dispersion in productivity by letting $\gamma_{it}z_{it} = \gamma_t z_t = Z_t$ for all $i$, noting that these two variables have identical effects on the aggregate variables of interest. This shuts down any interaction there might be between the two types of dispersion, but this interaction appears to be insignificant with a large enough number of firms.$^{11}$ As a result, aggregate output (30) simplifies to

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha} \frac{\int_0^1 \left( \frac{E_{it}\theta}{E_{it}z_{it}} \right)^{\theta-1} \frac{\theta}{\theta-1} \, di}{\int_0^1 \left( \frac{E_{it}\theta-1}{E_{it}z_{it}} \right)^{\theta} \, di}$$

while the real rental rate of capital and real wage are

$$\frac{R_t}{P_t} = \alpha Z_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} \left( \int_0^1 \left( \frac{E_{it}\theta - 1}{E_{it}z_{it}} \right)^{\theta-1} \, di \right)^{1-\alpha}$$

$^{11}$We use 25 million firms in our simulations, but the results do not depend on this number as long as we have enough firms, or replications, for the simulated samples to be representative for the complete distribution.
\[
\frac{W_t}{P_t} = (1 - \alpha) Z_t K_{t-1}^{\alpha} N_t^{-\alpha} \left( \int_0^1 \left( \frac{E_{i,t} \theta - 1}{E_{i,t} \theta} \right)^{\theta - 1} di \right)^{\frac{1}{\theta - 1}} \tag{50}
\]
respectively. These are the only equilibrium conditions where dispersion enters directly, so we focus on these. In particular, we are interested in the fraction
\[
r_t = \frac{\left( \int_0^1 \left( \frac{E_{i,t} \theta - 1}{E_{i,t} \theta} \right)^{\theta - 1} di \right)^{\frac{\theta - 1}{\theta - 1} \theta}}{\int_0^1 \left( \frac{E_{i,t} \theta - 1}{E_{i,t} \theta} \right)^{\theta - 1} di} \leq 1 \tag{51}
\]
and the factor
\[
q_t = \left( \int_0^1 \left( \frac{E_{i,t} \theta - 1}{E_{i,t} \theta} \right)^{\theta - 1} di \right)^{\frac{1}{\theta - 1}} \tag{52}
\]
where, \(r_t\) measures the impact of heterogenous expectations on output \(Y_t\) for given levels of capital \(K_{t-1}\), work effort \(N_t\), and productivity \(Z_t\), while \(q_t\) measures the impact of dispersion on the factor prices under the same conditions. Due to the negligible effects of the interaction between dispersion in productivity and in the perceived elasticities, figure 1 also represents \(r_t\), while figure 2 also applies for \(\frac{\theta}{\theta - 1} q_t\).

The approximation
\[
q_t \simeq \left( 1 - \frac{1}{\theta} \right) r_t \tag{53}
\]
is accurate for \(\sigma_{\gamma} \leq .55.\)\(^{12}\) With it, we can write the equations for aggregate output (48), real rental rate (41) and real wage (50) as
\[
Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha} r_t \tag{54}
\]
\[
\frac{R_t}{P_t} = \alpha Z_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} \left( 1 - \frac{1}{\theta} \right) r_t \tag{55}
\]
\[
\frac{W_t}{P_t} = (1 - \alpha) Z_t K_{t-1}^{\alpha} N_t^{-\alpha} \left( 1 - \frac{1}{\theta} \right) r_t \tag{56}
\]
respectively. Since the productivity shock \(Z_t\) and the dispersion shock \(r_t\) enter the model in exactly the same way, their effects on the aggregate variables will be identical if they posses

\(^{12}\)When \(\sigma_{\gamma} \leq .55\), this approximation is at most 1.31% off for \(\theta \geq 7.5\) and less than 3.2% off for \(\theta \geq 2.51\). It is more accurate the lower the value of \(\sigma_{\gamma}\) and the higher the value of \(\theta\). The approximation is imperfect because \(r_t\) drops faster than \(q_t\) when \(\sigma_{\gamma}\) increases, as is reflected in figures 1 and 2.
the same statistical properties.\textsuperscript{13} The standard deviation of the productivity shock $Z_t$ is typically assumed to be about .0225, implying it is within ±4.5% of its steady-state value 95% of the time. Looking at figure 1, it is evident that swings in $r_t$ of this magnitude cannot be generated unless $\sigma_\gamma$ is increased beyond .55. Hence, it would take an implausible amount of dispersion for such $r_t$-shocks to be the sole source of productivity shocks. Exactly what fraction of these could be due to dispersion shocks depends on $\theta$, and how $\sigma_\gamma$ changes over time, which determines the variance and autocorrelation of $r_t$.

With the utility function

$$u(C_t, 1 - N_t) = b \left( C_t^a (1 - N_t)^{1-a} \right)^{\frac{1-\phi}{1-\phi}} + (1 - b) m_t^{1-\phi}$$ \hspace{1cm} (57)$$

where $a$ measures the weight of consumption relative to leisure, $1 - b$ measures the weight of real money balances relative to consumption and leisure, and $\phi$ is the inverse of the intertemporal elasticity of substitution, we calibrate the model for $\beta = .989$, $\delta = .028$, $\phi = .5$, $\frac{G}{Y} = .2$ and $N = .3$, see Cooley and Hansen (1995) and Hansen (1997).\textsuperscript{14} In addition, we choose $\alpha$ so as to yield an income share of .6 in a steady state (46) with constant dispersion $\sigma_\gamma$. Log-linearizing around such a steady state, we can measure the impact dispersion shocks $r_t$ have on aggregate output, taking into account the reaction of the supplies of labor and capital to changes in factor prices spurred by variations in the dispersion of perceived elasticities.

When dispersion shocks $r_t$ are highly correlated over time, with an autocorrelation coefficient $\rho_r = .95$, we find that a 1% deviation in $r_t$ from steady state makes contemporaneous aggregate output deviate about 1.44% from steady state for $\theta \in [5, \infty)$.\textsuperscript{15} For lower values of the elasticity $\theta$, the amplification of dispersion shocks $r_t$ are somewhat lower, 1.41 for

\textsuperscript{13}Comparing figures 1 and 2 shows that dispersion actually has a slightly larger immediate impact on output than on the rental rates, which makes it differ from the impact of a traditional productivity shock, $Z_t$. The approximation (53) between $q_t$ and $r_t$ eliminates this difference, which is quite small.

\textsuperscript{14}The values of $b$ and $\mu$ have no effect on the impact dispersion $r_t$ has on output $Y_t$, just as they have no effect on the impact the productivity shock $Z_t$ has on $Y_t$. For $\sigma_\gamma \leq .55$, these effects are also independent of the calibrated value of $\sigma_\gamma$, which affects the steady-state value of $r_t$.

\textsuperscript{15}This number is consistent with what is commonly found for traditional productivity shocks in models with perfect competition, see for example Cooley (1995) and Walsh (2003). The impact is largest immediately, and gradually fades away.
\( \theta = 4, 1.31 \) for \( \theta = 3 \) and \(.9 \) for \( \theta = 2.51 \). The reason is that while the effect of an \( r_t \) shock on aggregate output through productivity is the same for all values of \( \theta \), the impact through the inputs is lower the smaller is \( \theta \), since this makes intermediate-good producers limit the quantity produced more tightly. These results are quite robust in that for \( \theta \in [3, \infty) \), achieving an output response below 1% or above 2% would require an extreme calibration for \( \beta, \delta, \phi, \frac{G}{F}, N \) and the income share of labor. The results are, however, fairly sensitive to the autocorrelation \( \rho_r \) of the shocks (see below). Hence, we find that when \( \rho_r = .95 \), taking into account the effect of an elastic labor supply at most allows for doubling the effects seen in figure 1, even though magnifying these by a factor of .9-1.44, depending on the value of \( \theta \), is more realistic. This implies that when \( \sigma_\gamma \leq .55 \), dispersion in the perceived elasticities \( E_{it,\theta} \) can reduce aggregate output by about 7.7% when \( \theta = 2.51 \), 9.2% when \( \theta = 3 \), 7.2% when \( \theta = 4 \), 5.8% when \( \theta = 5 \), 3.8% when \( \theta = 7.5 \), 2.9% when \( \theta = 10 \) and 1.4% when \( \theta = 20 \). Assuming that the steady-state value of dispersion \( \sigma_\gamma \), in each case is such that the impact on output is half of the maximum impact, we find that fluctuations in dispersion, that is, changes in \( \sigma_\gamma \) between 0 and .55, can generate output deviations from steady state of \( \pm 3.8\% \) when \( \theta = 2.51 \), \( \pm 4.6\% \) when \( \theta = 3 \), \( \pm 3.6\% \) when \( \theta = 4 \), \( \pm 2.9\% \) when \( \theta = 5 \), \( \pm 1.9\% \) when \( \theta = 7.5 \), \( \pm 1.4\% \) when \( \theta = 10 \) and \( \pm 1.7\% \) when \( \theta = 20 \). For comparison, table 2 tabulates the deviations from trend of U.S. real GDP. It shows that from the first quarter of 1960 to the fourth quarter of 2007, real GDP was within \( \pm 1.5\% \) of its trend 74.5% of the quarters, and within \( \pm 3\% \) of trend 93.8% of the time.

<table>
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<tr>
<th>% dev. trend</th>
<th>( \pm .5 )</th>
<th>( \pm 1.0 )</th>
<th>( \pm 1.5 )</th>
<th>( \pm 2.0 )</th>
<th>( \pm 2.5 )</th>
<th>( \pm 3.0 )</th>
<th>( \pm 3.5 )</th>
<th>( \pm 4.0 )</th>
<th>( \pm 4.5 )</th>
<th>( \pm 4.7 )</th>
</tr>
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<tr>
<td>% of quarters</td>
<td>28.7</td>
<td>54.7</td>
<td>74.5</td>
<td>85.4</td>
<td>88.5</td>
<td>93.8</td>
<td>95.8</td>
<td>97.9</td>
<td>99.5</td>
<td>100</td>
</tr>
</tbody>
</table>


The autocorrelation coefficient \( \rho_r \) has a large impact on the effects of \( r_t \)-shocks on output, since it determines their persistence, which shapes the labor response. The more persistent the \( r_t \)-shocks are, the less intertemporal substitution of labor they generate, reducing their impact on output. The calibration \( \rho_r = .95 \) is the standard one for productivity shocks, but it is unclear how persistent the dispersion shocks we have in mind should be. When
\( \rho = .5 \), a 1% deviation in \( r_t \) from steady state makes aggregate output deviate about 1.12% from steady state with \( \theta = 2.51 \), 2.26% with \( \theta = 3 \), 2.15% with \( \theta = 4 \), 2.06% with \( \theta = 5 \), 1.94% with \( \theta = 7.5 \), 1.89% with \( \theta = 10 \) and 1.82% with \( \theta = 20 \). In this case, dispersion in the perceived elasticities \( E_{i,t} \theta \) can reduce aggregate output by at most 9.5% when \( \theta = 2.51 \), 15.8% when \( \theta = 3 \), 11.0% when \( \theta = 4 \), 8.3% when \( \theta = 5 \), 5.2% when \( \theta = 7.5 \), 3.7% when \( \theta = 10 \) and 1.8% when \( \theta = 20 \). Assuming, as above, that the steady state is at the midpoint, fluctuations in dispersion, \( \sigma_\gamma \) varying between 0 and .55, can generate output deviations from steady state of \( \pm 4.8\% \) when \( \theta = 2.51 \), \( \pm 7.9\% \) when \( \theta = 3 \), \( \pm 5.5\% \) when \( \theta = 4 \), \( \pm 4.2\% \) when \( \theta = 5 \), \( \pm 2.6\% \) when \( \theta = 7.5 \), \( \pm 1.9\% \) when \( \theta = 10 \) and \( \pm .9\% \) when \( \theta = 20 \).

Despite the robustness mentioned above, the calibration affects the results in the following ways. Increasing the labor share amplifies the effects dispersion shocks have on output, since it reduces the importance of capital, which cannot immediately react to shocks, and raises that of labor, which does adjust instantly. Increasing the discount rate \( \beta \), or decreasing the depreciation rate \( \delta \), raises the steady-state capital stock, thus raising the steady-state marginal product of labor, which increases the impact that changes in the labor input have on output, thereby raising the amplification of dispersion shocks. Similarly, lowering the steady-state value of the labor input \( N \), raises the steady-state marginal product of labor, and therefore also raises the amplification. Lowering \( \phi \) raises the intertemporal substitution, making households vary their labor supply more in response to changes in the real wage, which is determined by the labor productivity and the dispersion shock. Since government spending \( G \) is exogenous and constant in the model, dispersion shocks have smaller effects the larger the government sector is.\(^{16}\)

While dispersion has a greater impact on aggregate output for lower values of the elasticity \( \theta \), we should keep in mind that in terms of the mark-ups, the dispersion is also larger for these, as discussed above. It follows that obtaining comparable numbers for different values of \( \theta \) might arguably require letting \( \sigma_\gamma \) reach higher values the larger \( \theta \) is. This would make our results more similar across \( \theta \) values. There would be no effect on the amplification of

\(^{16}\)With \( \rho_Z = .95 \), \( \beta = .995 \), \( \delta = .019 \), \( \phi = .2 \), \( G = .15 \), \( N = .2 \) and a labor share of \( \frac{2}{3} \), a 1% deviation in \( r_t \) from steady state makes output deviate 1.92-1.97% from steady state for \( \theta \geq 4 \). With \( \rho_Z = .95 \), \( \beta = .96 \), \( \delta = .04 \), \( \phi = .9 \), \( G = .4 \), \( N = 4 \) and a labor share of \( \frac{4}{5} \), we get an output response of 1.00-1.06% for \( \theta \geq 4 \).
the dispersion shocks \( r_t \), just how extreme these can be. The magnitude of the fluctuations in aggregate output that dispersion shocks can generate would still depend crucially on the width of the range we allow for the dispersion \( \sigma \), the autocorrelation \( \rho_r \), and the value of the elasticity \( \theta \).\(^\text{17}\)

8 Demand elasticities in general

While the elasticity \( \theta \) is exogenous, constant and identical for all producers of intermediate goods above, it is not possible to deduce, or estimate, its value from aggregate data without knowing the distribution of its perceived value \( E_{i,t}\theta \) across all \( i \). As a result, intermediate-good producers are forced to form their expectations about its value from estimates of the demand (38) for their good \( i \). Due to unobservable heterogeneous shocks, the estimates they produce differ, which leads to dispersion in perceived elasticities. This is crucial, since the results above rely on this dispersion to distort relative prices of intermediate goods, which in turn affects aggregate productivity. With identical \( \theta \) values for all producers, these could get data from each other to produce more accurate estimates. However, in a world where the elasticities vary across producers, such information sharing would not be useful, and each would be stuck with their own estimation problems and estimates. In addition, the fact that intermediate-good producers compete against each other is likely to limit information sharing.

In a more general setup, where the production of the final good is given by

\[
y = f (x_0, ... , x_1, \gamma_0, ... , \gamma_1)
\]

minimizing the cost of producing the final good (2) yields an aggregate demand function

\(^{17}\)Allowing for variable capital utilization would raise the impact that a dispersion shock \( r_t \) has on output. However, making capital utilization endogenous would not change our results much for lower values of \( \theta \), since capital carries a smaller weight in production then. Allowing for variable effort in labor would have a greater impact, a situation which can be approximated by raising the intertemporal substitution, that is, lowering \( \phi \). With \( \phi = .2 \), the numbers for the impact of dispersion shocks presented above increase only by 7-8% for \( \theta \geq 3 \) when \( \rho_r = .95 \). When \( \rho_r = .5 \) the numbers increase by 11-18% for \( \theta \geq 3 \), with the larger increases occurring for lower values of \( \theta \). When \( \theta = 2.51 \), changing \( \phi \) has practically no impact.
for intermediate good $i$

$$X_i = D_i \left( \frac{P_0}{P}, \ldots, \frac{P_i}{P}, \ldots, \frac{P_1}{P}, \gamma_0, \ldots, \gamma_i, \ldots, \gamma_1, Y \right)$$ \hspace{1cm} (59)$$

where $P$ is the price of the final good, equal to its marginal cost of production. Maintaining the Cobb-Douglas production technology (7) for intermediate goods, the marginal cost of producing good $i$ remains as above (11), and the price-setting problem of producer $i$ is

$$\max_{P_i} E_i \left[ (P_i - \lambda_i) D_i \left( \frac{P_0}{P}, \ldots, \frac{P_i}{P}, \ldots, \frac{P_1}{P}, \gamma_0, \ldots, \gamma_i, \ldots, \gamma_1, Y \right) \right]$$ \hspace{1cm} (60)$$

The optimal price of intermediate good $i$ is in this case

$$P_i = \frac{1}{z_i} \left( \frac{R^i}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha} \right)^{1-\alpha} \frac{E_i \Theta_i}{E_i \Theta_i - 1}$$ \hspace{1cm} (61)$$

where

$$\Theta_i = -\frac{\partial D_i \left( \frac{P_0}{P}, \ldots, \frac{P_i}{P}, \ldots, \frac{P_1}{P}, \gamma_0, \ldots, \gamma_i, \ldots, \gamma_1, Y \right)}{D_i \left( \frac{P_0}{P}, \ldots, \frac{P_i}{P}, \ldots, \frac{P_1}{P}, \gamma_0, \ldots, \gamma_i, \ldots, \gamma_1, Y \right)}$$ \hspace{1cm} (62)$$

is the elasticity of demand for intermediate good $i$. This elasticity is a function of all the relative prices $\frac{P_0}{P}, \ldots, \frac{P_i}{P}$, all shocks $\gamma_0, \ldots, \gamma_1$ and the aggregate demand for goods $Y$, which are all unknown to producer $i$ at the time it needs to set its price $P_i$.

Intermediate-good producer $i$ could learn about its elasticity $\Theta_i$ by studying how the demand for its product changes as its relative price changes, computing

$$\frac{\Delta X_i}{\Delta \frac{P_i}{P}}$$ \hspace{1cm} (63)$$

where $\Delta X_i$ denotes the change in its demand, while $\Delta \frac{P_i}{P}$ is the change in its relative price. This would require knowing its relative price $\frac{P_i}{P}$, which requires knowing the aggregate price level $P$.\textsuperscript{18} In addition, it would require that all other variables in the expression for

\textsuperscript{18}Even if the aggregate price level $P$ were constant, producers would need to know its level to know how much their relative price is changing. The assumption that intermediate-good producers do not know the aggregate price level, while final-good producers do, is a simplification. In reality, with heterogenous households, each would know how the cost of their particular consumption bundle evolves, but not how that
\[ \Theta_i \] (62) remain constant over the period studied, as well as the demand function \( D_i \) itself, or that the producer control for the impact of such changes on the measured elasticity (63). In particular, this applies to aggregate output \( Y \) when the elasticity \( \Theta_i \) varies over the business cycle, as it is likely to do. Hence, even if price-setters simply mark-up their nominal production costs, which we assume they can observe perfectly, their pricing decisions depend on aggregate variables they cannot immediately observe, such as the aggregate price and output levels, and would therefore have to be based on their expectations of these. When such expectations differ across individuals, as surveys show they do, relative prices are distorted, with some producers applying mark-ups that are too high and others applying mark-ups that are too low, affecting output through productivity.

As an example of how the elasticity of substitution can vary over the business cycle and how this could make price-setting depend on the expected aggregate output, imagine that instead of being constant, the elasticity \( \theta \) varies over time with entry and exit spurred by profits. Hence, \( \theta_t \) continually adjusts so as to make gross aggregate profits (27) match real fixed costs \( V \) of being in business (lump-sum transfers to households or the government that have no impact on the equations above), so that

\[
\frac{\Pi_t}{P_t} = \left( 1 - \frac{\int_0^1 (\gamma_{i,t} z_{i,t}) \theta_t - 1 \left( \frac{E_{i,t} \theta_t - 1}{E_{i,t} \theta_t} \right) \theta_t - 1 \frac{d\theta_t}{d\theta_t}}{\int_0^1 (\gamma_{i,t} z_{i,t} \frac{E_{i,t} \theta_t - 1}{E_{i,t} \theta_t} \theta_t - 1 \frac{d\theta_t}{d\theta_t}} \right) Y_t = V \tag{64}
\]

where the first equality is obtained by substituting equations (29), (30), (33), (34) and (35) into (27). Given (perceived) distributions for \( \gamma_{i,t} z_{i,t} \) and \( E_{i,t} \theta_t \) across producers, determining the ever-changing \( \theta_t \) amounts to predicting the contemporary aggregate output \( Y_t \), since \( \theta_t \) can be computed from the entry condition (64) with this information. Hence, heterogenous expectations about aggregate output, or the distributions of productivity and perceived elasticities, generate dispersion in the perceived elasticities, which distorts relative prices and reduces output.\(^{19}\)

\(^{19}\)While the equations in previous sections assume the elasticity \( \theta \) is constant over time, they would look

of households with dissimilar tastes behaves. A household’s demand for a particular intermediate good then depends on its price relative to the cost of composing that household’s preferred bundle, which will vary across households. Hence, intermediate good producers would need to know the costs of all households’ bundles, not just the aggregate price level. The same applies to other aggregate variables.
By reducing the effectiveness with which labor and capital can be used to produce final goods, heterogenous expectations have a negative impact on consumers’ lifetime utility (20), just as a standard negative productivity shock would. As a result, there is scope for improving welfare by reducing such dispersion. Since heterogenous expectations about aggregates can be one of its sources, there would be gains to homogenizing these. In particular, the aggregate price level lends itself to this, since according to the money-market clearing condition (36), it can be controlled through the money supply. The more credible a target the monetary authority can provide for the aggregate price level, or the rate at which it changes, the rate of inflation, the less dispersion is generated by the uncertainty inherent in its contemporary value, and the more efficient the production of final goods.\textsuperscript{20,21} While the dispersion is independent of the degree to which such a target is met, as long as it synchronizes expectations, the credibility of such a regime going forward, and therefore its ability to reduce future dispersion, require matching the target closely.

9 Conclusions

In a general equilibrium model with perfectly flexible prices, we show how heterogenous expectations can lead to reduced output by distorting relative prices and reducing productivity. Moreover, we find that shocks to the dispersion in expectations can be indistinguishable from standard productivity shocks, and be a significant source of business-cycle fluctuations.

\textsuperscript{20}See Bernanke and Mishkin (1997), Bernanke and Woodford (2005), McCallum (1996) and Svensson (1997 and 1999) for discussions of inflation-targeting.

\textsuperscript{21}With lump-sum taxation, the welfare maximizing inflation target in the model is the Friedman (1969) rule, that is, making the rate of inflation equal the negative of the real rate of return on capital. Without lump-sum taxation, the optimal inflation rate would be somewhat higher. It would also be higher with menu costs and price-stickiness.
10 References


Mankiw, N. G., Reis, R. and Wolfers, J. (2003), “Disagreement about Inflation Expec-


Figure 1: Effect of dispersion in perceived elasticities on output, $\tilde{r}_t$ & $r_t$

$\sigma_\gamma = 2.51$
$\theta = 3$
$\theta = 4$
$\theta = 5$
$\theta = 7.5$
$\theta = 10$
$\theta = 20$
Figure 2: Effect of dispersion in perceived elasticities on factor prices, $\tilde{q}_t \& \frac{\theta}{\bar{y}_t} q_t$.
Figure 3: Effect of dispersion in perceived elasticities on steady-state output, $\tilde{r}$