Beyond Computable General Equilibrium:
Simulating non-equilibrium dynamics and money,
an agent-based Computational Complete Economy

Davoud Taghawi-Nejad

02/06/2012

Abstract

This agent-based model uses the building blocks of a Computable General Equilibrium model, but instead of assuming equilibrium, it explicitly models the price formation and investment processes using money as the medium of exchange.

This model gives us several insights that CGE models do not: first, we can check whether or not the results of the CGE models still hold without the equilibrium assumption. Second, as the price formation and investment processes are explicitly modeled, this model reveals non-equilibrium dynamics - the evolution of the economy over time. This allows us to conduct experiments that cannot be conducted using CGE models. We can show how the economy reacts to demand or supply shocks and to inflation.

In Computational Complete Economy simulations firms act in a physical world. The physical world is derived following the Computable General Equilibrium approach. Based on the social accounting matrix and input-output data, we calibrate the production functions of firms in specific sectors, and use industry data to establish their capacity. Household utility functions are modeled following the CGE approach. On the basis of these physical characteristics, the 30,000,000 firms of the three north American economies are represented as software agents. These agents buy intermediary goods or resources directly from other firms and hire labour from the households. Intermediary goods are sold to companies, and final goods to households. Prices are set by the strategic interaction of firms and by the interaction of firms with households. Thus the modeled economy is a network. Firms can also expand their capacity or abandon establishments.

The simulation is based on the ABCE simulation system (Agent-Based Complete Economy) a multi-core extension of the SWARM like agent-based modeling protocol for Python. ABCE automatically enforces macroeconomic closedness. The simulation will be open-sourced and can be modified with minimal programming skills.
1 introduction

CGE models are a standard tool of policy analysis. Such models are attractive because they can be used to model a large amount of detail based on empirical data. This data can be found in a standardised form for many countries. The GTAP database\footnote{Global Trade Analysis Project from Purdue University \url{www.gtap.org}} for example, offers social accounting data for over one hundred countries. The data we are using for the current model allows us to calibrate production functions for 128 sectors.

The disadvantage of using CGE models is the strong assumption of equilibrium. In a world that is not in equilibrium, every result we derive from this model could be a result of the equilibrium assumption rather than simulated policy. Naturally, equilibrium models predict the long-run better than they do the short-run.

We therefore propose a pragmatic hybrid between computational general equilibrium and agent-based models: computationally complete economy. This model class calibrates household and firm behaviour on the basis of the same data: social accounting matrices. However, the policy simulation is then done with an agent-based model in which the equilibrium assumption is replaced by adaptive behaviour.

Traditional CGE models assume utility maximizing households and one representative firm per sector. They also assume market clearance. Given this, the modeling procedure has two steps. First, CGE modelers calibrate the coefficients of households and firms based on the social accounting matrix (SAM). The SAM is a table that quantifies the circular flow of goods and money in the economy. After calibration, we have a model of the economy in which we know all production and utility functions, together with their coefficients. In the second step, this calibrated model of the economy is used to simulate policy changes.

Our Computational Complete Economy models follow the same procedure until the calibration. The only modification is that sectors and households are disaggregated into individual firms and households. Using calibrated production, utility, and functions, we create an agent-based model. In this agent-based model, agents produce, invest, trade, consume and save, given the current level of prices. They update their prices by learning from past actions. Prices and the economy move towards equilibrium, but it is never reached. After a policy change and subsequent sector shifts, prices change, and goods and labour have to be reallocated. Here, non-equilibrium unemployment emerges, without the imposition of the modeler.

The model is constructed from North American (Canada, US, Mexico) data. In this first attempt at Computational Complete Economy modeling, we make the simplified assumption of a closed economy. Treating North America as one is a gross simplification, which will be alleviated in future models.

The model can be found at \url{www.taghawi-nejad.de} and the source code at \url{github.com/DavoudTaghawiNejad/cce}
1.1 Methodology

Agent-based computational economics is the study of economic or social processes with the use of computer-generated autonomous agents. These agents execute their functions autonomously. In the model examined in this paper, for example, the agents trade with their trading partners, produce and consume.

Agent-based computational economics models combine the flexibility of a verbal model with the enforced consistency of a mathematical model. The sacrifice is the possibility of being able to solve it analytically. Holland and Miller argue in favour of this type of modeling:

Artificial adaptive agent models, that a pure [verbal] representation, would give the model the possibility to be flexible enough to accommodate a number of rules, uncertainty and complexity of the model. However, pure verbal models, are in danger of being logically unsound. Mathematical models on the other hand lose flexibility. The AAA models, specified in a computer language, retain much of the flexibility of the original [verbal] model, while having the precision and consistency enforced by the [computer] language.

Holland and Miller further argue that the models are dynamic and executable in the sense that they can be observed step by step, and checked for plausibility and logical consistency. Furthermore, agent-based computational economic models give us more insight than mathematical models in that we can not only find equilibria, but we can obtain a complete dynamic history of all the processes in the model.

There are several methodological approaches used in agent-based modeling. The most prominent one is generative social science. He shows that it is as third form of scientific inference in addition to deduction and induction. In generativist reasoning, explaining macroscopic regularities, such as norms and price equilibria, requires us to answer the following question:

How could the decentralized local interactions of heterogeneous agents generate the given regularity?

A generative model situates an initial population of autonomous heterogeneous agents in an environment, and allows them to act and interact. This model then generates the macroscopic regularity, “from the bottom up”.

From the generativist point of view the classical analytical approach is not sufficient - it is not sufficient to establish that, if deposited in some macro configuration, the system remains there. Generativists demand that it is shown that the configuration is attained through a decentralized system of heterogeneous agents.

Although in his paper Epstein demonstrates with formal logic that agent-based modeling is a proof, we follow the methodology of Kirman and Friend. The purpose of agent-based computational economic models is to explain social phenomena. The computer program itself is the model that explains the social phenomenon under consideration. In their words:
An [Agent-based Computational Economics] model stands to executing its program as a mathematical model stands to solving its equations.

Executing an agent-based model is a consistency check, for the expected behavior of the program and its consistency with reality. Doing research is building a model of reality and checking its consistency.

We may not be able directly to confirm, that the causal mechanism determining the phenomenon in question is the same as that of our model. But we know that, if the mechanism is the same, the observed structures must be capable of showing some kinds of action and unable to show others; and if, and so long as, the observed phenomena keep within the range of possibilities indicated as possible, that is so long as our expectations derived from the model are not contradicted, there is good reason to regard the model as exhibiting the principle at in the more complex phenomenon. (Hayek (1967)[3], reported in Kirman and Vriend (2001) [6])

This resembles Friedman’s (1953)[2] 'as if' argument. Reality’s social process gives results as if it used the model.

2 Model

2.1 Introduction

We create an agent-based economy calibrated on empirical data by creating a computational general equilibrium (CGE) model that represents the agent-based model’s long-run equilibrium. The first step is to create the theoretical CGE and the agent-based model. The second step is to calibrate the CGE model using conventional calibration techniques. This gives us the calibrated exponents of the production and utility functions. In the third step, these are plugged into the agent-based model. With this calibrated agent-based model, we can now undertake policy experiments.

2.2 Calibration

2.2.1 SAM

The SAM is a table that quantifies the circular flow of goods and money in the economy. Rows and columns contain activities (aggregated sectors), products, factors, households, the government and other agents, as well as the rest of the world. The row and column titles are identical. Every cell in the table represents the monetary value of the flow, from one entry to another. Take, for example, the cell on the intersection of the row household and column labour. This is the value of labour provided by the household and also the wages paid for labor to the households. The cell labour - agricultural sector contains the value of the
labour provided to the agricultural sector, and the wages paid for this service. The cell where products intersect with sectors, show us the intermediate goods used in that particular sector. The intersection with household show the value of the final goods delivered to the households.

2.2.2 Disaggregation of the SAM

The SAM is usually compiled by the country’s statistical office. All its entries are aggregations. In computable general equilibrium modeling, it is a common practice to disaggregate row/columns that are of interest, and aggregate entries that are not. Sectors are aggregations of individual firms. For our purpose, we need to disaggregate the SAM. If we do not have firm level data, we disaggregate sectors into firms. For this, we have to make assumptions on the relative output share. Usually we can gain information on the market share from statistical data or approximate it by the distribution of the relative firm sizes. Similarly we disaggregate households. This disaggregated SAM is conceptually the same as the conventional SAM. We still can use the same CGE model.

2.2.3 CGE Model

In order to calibrate the disaggregated CGE model, we follow the large country CGE model proposed by Hosoe, Gasawa and Hashimoto (2010) [5]. In figure 1 we see the flows of goods and factors in the economy. All nodes are agents and we will describe their behaviour herein.

Households

Households are specified by a Cobb-Douglas utility function. The household’s budget constraint is the income from the factor endowment,

\[^2\text{For the exact algebraic derivation see [5] chapter 2 and 6.}\]
minus constant taxes and savings. Households maximize utility by choosing the optimal consumption bundle from the available final goods, given prices for goods and factors.

**Investment and savings** The calibration process does essentially calibrate a static model of the economy. This means that there is no growth or decline in the capital stock. However, the social accounting matrix contains an investment entry. We assume that the investment agent collects funds from private and public savings, plus the current account balance. He spends the saving proportionally on investment goods. Total savings always equal total investment. Investment and saving in the Agent-Based model will be incorporated into the decision of the households and the government.

**Firms** A firm’s production process is modeled in two stages:

- In the first stage, capital and labour are used for the production of a composite factor (or value added). The production process of the composite factor can be regarded as the behaviour of a virtual factory which maximizes its profits by choosing its output (composite factor) level and inputs (capital and labour) use, depending on their relative prices, subject to its technology. In the second stage, the composite factor is combined with intermediates in order to produce the gross domestic output, as indicated by the gross domestic output production function. As for technology in this two-stage production process, we assume a Cobb-Douglas-type production function for the first stage, and a Leontief-type production function for the second stage. They are both homogeneous of degree one and thus characterized as constant returns to scale. The Cobb-Douglas-type production function allows us to describe substitution between inputs, while the Leontief-type production function does not.

Firms maximize their utility by first choosing the composition and quantity of the composite factor, and subsequently choose the output of the firm.

**Government**

We assume that the government levies a direct tax on household income at the tax rate $\tau^d$, an ad valorem production tax (an indirect tax) on gross domestic output at the tax rate $\tau^j$ tariffs on imports at the rate $\tau^m$. Further, we assume that the government spends all tax revenues in a fixed proportion on consumption goods.

---

3 For a detailed treatment of the apparent inconsistency, we refer the reader to Chapter 6.4.1.

4 There is also a government sector in the SAM. We treat the government sector like an industrial sector.
**Closed economy assumption** In this model we make the strong assumption of a closed economy. In the computational general equilibrium model there are standard procedures with regard to modeling import and export goods as imperfect substitutes: following Armington, imported and domestic goods are combined by virtual companies. However, this would mean that we would have to create these virtual ‘Armington’ agents in the agent-based policy model. As of now, this has not been done and therefore we leave this for future research. The assumption of a closed economy restricts our choice of regions to regions which do not have too much foreign trade. Therefore we chose North America (Canada, the US and Mexico) as a relatively closed region.

**Market clearing** The quantity supplied of each of good must be equal to the consumption by the Households plus the government plus savings and the use as of this good as an intermediate goods. The use of factors by the sectors must be equal to the factor endowment.

### 2.3 Agent-Based Model

#### 2.3.1 Agents

The model consist of the same agents we have seen in the calibration: households, firms and the government.

**Households** Households have three functions in the agent based model: they save and invest, consume, and provide land, capital and labour. The agents consumption is modeled on the basis of the utility function that is calibrated for the CGE’s household agent. The agents investment and saving on the other hand makes use of the parameters calibrated for the CGE’s investment agent. (1) Agents determine how much to save. We use the average propensity for private saving, that we have estimated in the calibration part of our model. (2) The agents have a Cobb-Douglas utility function that determines their consumption behavior. Households face the prices announced by the firms. Agents buy the utility maximizing bundle. (3) Agents offer their labour services. They use reinforcement learning to set the price of such labour. The do not face a labor leisure choice. (4) Agents invest their saved money proportionately in the different sectors. For this we use the proportions calibrated with regard to the investment agent. The price setting is done by reinforcement learning.

**Sectors / Firms** Depending on the data available in the social accounting matrix, we have a number of different sectors. Each firm is part of one sector. The number of firms in each sector is a fixed proportion, corresponding to the industry data we have obtained. Firms in a particular sector have different capacity constraints. The distribution of firms capacities is also taken from industry data.

Initially firms produce at full capacity. (1) They set their price as a function of their inventory. (2) When the price is smaller to, or equal to, the variable cost,
and the firm’s inventory grows nevertheless, firms decrease their production. When firms make profits and the inventory is zero they increase their production. The magnitude of the change in terms of prices and production is subject to reinforcement learning. Agents learn what magnitude of change is best.

(1) Given their production quantity, they buy their intermediate goods on the basis of the Leontief production function and their capital and labour on the basis of the Cobb-Douglas production function. (2) They offer and sell the products on the market for a set price.

Every 30 rounds, firms can engage in a capacity expansion. They do this when they are producing profitable at full capacity. The magnitude of the capacity expansion is determined on the basis of the history of the profitability of previous production quantity expansions and, where available, capacity expansions.

**Government** The government buys goods given the proportions estimated in the calibration process.

**Market interaction** Firms, consumers, and the government do not interact within centralised markets. Instead they have bilateral relationships. As searching for new trading partners is costly and time consuming, agents use reinforcement learning to determine what amount of price change justifies the need to start searching for new trading partners.

### 2.4 Timeline of the agent-based model

At the beginning of the simulation agents, prices and quantities are at the equilibrium values. Prices are set to the normalized price of one from the CGE model. Firms are endowed with the output of the corresponding SAM entry.

1. households offer labour and capital
2. firms decide on production quantity
3. firms offer output
4. • households buy consumption goods simultaneously
   • firms buy labour, and capital and intermediary goods
   • every 30 rounds firms decide whether or not to expand their capacity and buy the necessary intermediate goods
5. firms produce

### 2.5 Learning-Calibration

Given the calibration with the social accounting matrix, the agent-based model corresponds to the physical reality of year for which the data for the SAM has been collected. However, in reality, firms and households in that year have
already learned how to set their prices and to make their production decisions. Therefore we need to calibrate the learning parameters that govern the price and production decisions. For this we run the agent-based model, given the calibrated firms and households. During this run the agents will learn how to set prices and make production decisions.

These learned parameters for the prices and production decisions, can be used for a policy simulation. The policy starts again from the year of the SAM data. This is a slight inaccuracy in that we should have calibrated the learning on the time before the start of the policy simulation, but that is not feasible.

3 Policy Simulation

3.1 Theoretical considerations

We have calibrated the firm, household and government behaviour. When we run the simulation, the model will remain in its equilibrium position, and every round will be the same. There will be no investment because no profits are being made. When we introduce a change, the agents will respond by changing their prices, buying behaviour and production quantities. The agent-based economy is out of equilibrium. In the following paragraphs we will outline some policy simulations.

3.2 Sectoral Schock

Modeling a demand or supply shock is straightforward. If, for example, we want to test a policy that decreases the agents propensity to invest in real estate, we constrain the agents’ behaviour accordingly. We now run the simulation and compare it with the baseline run.

We could also test the impact of environmental policies that affect the supply of resources. In this case, we change the production functions of the oil firms accordingly.

Policy simulations can be found at [www.taghawi-nejad.de](http://www.taghawi-nejad.de)

4 Shocks

In order to study the effects of technology shocks, we need to make the agents production functions heterogeneous. With the available data of the SAMs, all firms in a sector are homogeneous. A firm that innovates would out-compete its complete sector. Once we have introduced this heterogeneity, we can follow [8] in order to simulate a shock triggered by an innovation on the part of a single firm. Technological shocks, can lead to a temporary decrease in GDP and hence to the creation of business cycles. We neither assume nor explicitly model business-cycles. Business cycles are generated by the actions and interactions of autonomous agents producing and trading in an endogenously changing network. We model a change in production technology as a change in the production
When a firm innovates, it can sell more cheaply than its competitors. It might buy fewer expensive inputs and might buy more cheaper inputs. This triggers the other agents to react and change their trading partners or change their consumption production bundles. The other agents change in behaviour might even affect other agents. What we have is a snowball of change. During this time of change, some firms will be unable to sell their inventory or may not be able to find the goods needed to produce their final output at prices, that allow the firm to produce profitable. Those agents negatively affected by the changes will produce less. Therefore the GDP decreases during changes in the economy. Once the economy has stabilized, the GDP increases once more, and we observe a business cycle.

5 Future Extensions

Investment is rudimentary modeled and estimated. In agent-based models, institutions can be modeled as agents. It is therefore interesting to introduce financial intermediaries which mediate between the households savings and the firms investment. That would include a central bank that can change monetary supply, as part of policy experiments.

References