Quarterly Fiscal Policy in a Multiplier-Accelerator Model: Monte Carlo Results

by

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Abstract

In an earlier paper, i.e. Kendrick and Amman (2011) we raised the question of whether adjusting fiscal policy every quarter rather than once a year could be used to improve stabilization policy. We followed this in a second paper, i.e. Kendrick and Shoukry (2011) where we did counterfactual experiments for the period 2008-2010 with a small multiplier-accelerator model. We performed both annual policy and quarterly policy scenarios in order to see the relative performance of the two policy methods in two experiments. In the first experiment we found that the quarterly policy performed somewhat better than the annual policy in stabilizing output but with a slightly larger increase in debt across the counterfactual period. In a second experiment in which output was forced to track a desired path equally well with the two policies we found that the increase in the debt level was less over the counterfactual period in the quarterly than in the annual scenario.

In this paper we use the same model and repeat the two experiments but do so in a Monte Carlo rather than a counterfactual framework. The results in this more general framework also point the way to a finding that a relatively simple shift from annual to quarterly fiscal policy could provide either better stabilization results with a slightly large increase in the debt level or similar stabilization results but with a smaller increase in the debt level in the period covered by the model.
1. Introduction

In a previous paper, i.e. Kendrick and Amman (2011), we conjectured that quarterly rather than annual fiscal policy changes might help arrest downturns before they become really serious and thereby decrease the magnitude of fluctuations in the economy. This was done in the context of considering a feedback rule for fiscal policy in parallel with the well known feedback rule for monetary policy, i.e. the Taylor Rule, as discussed for example in Taylor (1999).

The Kendrick and Amman paper discusses the mathematics of the fiscal policy feedback rule in the context of a quarterly model of the economy and thus establishes a framework for the comparison of annual and quarterly fiscal policy scenarios. That paper also discusses the institutional and political changes that might be made to facilitate quarterly changes in fiscal policy.

Then in Kendrick and Shoukry (2011) we reported on the development of a small multiplier-accelerator model (viz. Samuelson (1939) and Chow (1967)) that was used to test the relative performance of annual and quarterly fiscal policy scenarios in a counterfactual setting for the U.S. economy in the period 2008-2010. These results indicate that quarterly rather than annual fiscal policy changes result in a smoother path for the economy through the use of smaller but more frequent changes in government appropriations. Thus in the quarterly scenario output tracks a desired path more closely than in the annual scenario but is able to do so with a slightly larger increase in government debt over the counterfactual period. In addition, we did an experiment in which we increased the weights on the output state variable in the annual scenario to force output to track the desired path in the two scenarios in roughly the same way. In this case we found that while output-tracking was about the same in the two scenarios that the increase in government debt over the counterfactual period was somewhat less with the quarterly policy scenario than with the annual policy scenario.

Counterfactual experiments are very interesting but not nearly as general as those which can be obtained with Monte Carlo methods. Thus we report in this paper on stochastic

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1 We are thankful to Hans Amman and Douglas Dacy for comments on this and previous work on quarterly fiscal policy and in particular to Douglas Dacy for suggestions about the institutional and political aspects of the subject.
control experiments with the multiplier-accelerator model in which there is an additive noise in the output equations. These experiments are done with the Duali software developed by Kendrick and Amman and with control theory code developed in MATLAB by Shoukry.

We begin here with a discussion of the mathematics of the model and follow this with a discussion of the results from the Monte Carlo experiments.

2. The Multiplier-Accelerator Model

As discussed in Kendrick and Shoukry (2010) the multiplier-accelerator model used for the counterfactual experiments consist of (1) an income identity, (2) a consumption function and (3) an investment function. Also, there is a lag relationship between government appropriations by the Congress and the expenditure of those funds and there is an equation to compute the increase in government debt over the period covered by the model. In addition, there are a number of equations which are used to total consumption, investment, government expenditures and government revenues over the period covered by the model. The resulting ten equation model is
\[
Y_{t+1} = \alpha_1 + (1-\theta) \beta_1 + \left[ \alpha_2 + (1-\theta) \beta_2 \right] Y_t + \theta I_t - (1-\theta) \beta_3 Y_{t-1} + (1-\theta) \beta_4 R_t \\
+ \gamma_1 A_t + \gamma_2 A_{t-1} + \gamma_3 A_{t-2} + \gamma_4 A_{t-3} \\
G_{t+1} = \gamma_1 A_t + \gamma_2 A_{t-1} + \gamma_3 A_{t-2} + \gamma_4 A_{t-3} \\
C_{t+1} = \alpha_1 + \alpha_3 Y_t \\
I_{t+1} = \theta I_t + (1-\theta) \left[ \beta_1 + \beta_2 R_t + \beta_3 (Y_t - Y_{t-1}) \right] \\
REL_{t+1} = \tau Y_t \\
D_{t+1} = D_t + G_t - \tau Y_t \\
TG_{t+1} = TG_t + G_t \\
TT_{t+1} = TT_t + \tau Y_t \\
TC_{t+1} = TC_t + C_t \\
TI_{t+1} = TI_t + I_t \tag{2.1}
\]

where

\( Y_t \) = income  \\
\( G_t \) = government expenditures  \\
\( A_t \) = appropriations by the Congress  \\
\( C_t \) = consumption  \\
\( I_t \) = investment  \\
\( REL_t \) = lagged tax receipts in period \( t \)  \\
\( D_t \) = government debt  \\
\( TG_t \) = total government expenditures through period \( t \)  \\
\( TT_t \) = total tax receipts through period \( t \)  \\
\( TC_t \) = total consumption through period \( t \)  \\
\( TI_t \) = total investment through period \( t \)

The complete model thus consists of a system of ten fourth order difference equations. These equations are for output, government expenditure, consumption, investment, lagged tax receipts and government debt as well as for the sums of government expenditures, tax receipts, consumption and investment over the time period covered by the model.

The next step then is to convert the model to state space form by changing these fourth order difference equations to first order via augmentation of the state
3. State Space Form of the Model

The first equation in the system of equations (2.1) has four terms that are lagged by multiple periods. Also the second equation and the fourth equation in this system each have terms that are lagged by multiple periods. So in order to convert these equations to first order form we define four new state variables with the equations

\[ YL_t = Y_{t-1} \]
\[ AL_t = A_{t-1} \]
\[ ALL_t = AL_{t-1} = A_{t-2} \]
\[ ALLL_t = ALL_{t-1} = A_{t-3} \]

(2.2)

where

\[ YL = \text{output lagged one period} \]
\[ AL = \text{appropriations lagged one period} \]
\[ ALL = \text{appropriations lagged two periods} \]
\[ ALLL = \text{appropriations lagged three periods} \]

Then substitution of Eq. (2.2) into Eq. (2.1) yields

\[ Y_{t+1} = \mu Y_t + \theta I_t - \eta_3 YL_t + \eta_2 R_t + \gamma_1 A_t + \gamma_2 AL_t + \gamma_3 ALL_t + \gamma_4 ALLL_t + \mu_2 \]
\[ G_{t+1} = \gamma_1 A_t + \gamma_2 AL_t + \gamma_3 ALL_t + \gamma_4 ALLL_t \]
\[ C_{t+1} = \alpha_1 + \alpha_3 Y_t \]
\[ I_{t+1} = \eta_3 Y_t + \theta I_t - \eta_3 YL_t + \eta_2 R_t + \eta_1 \]
\[ REL_{t+1} = \tau Y_t \]
\[ D_{t+1} = D_t + G_t - \tau Y_t \]
\[ TG_{t+1} = TG_t + G_t \]
\[ TT_{t+1} = TT_t + \tau Y_t \]
\[ TC_{t+1} = TC_t + C_t \]
\[ TI_{t+1} = TI_t + I_t \]

(2.3)

where
\[ \eta_1 = (1 - \theta) \beta_1 \]
\[ \eta_2 = (1 - \theta) \beta_2 \]
\[ \eta_3 = (1 - \theta) \beta_3 \]
\[ \mu_1 = \alpha_1 + (1 - \theta) \beta_3 \]
\[ \mu_2 = \alpha_1 + (1 - \theta) \beta_2 \]

Also one can advance the time index by one period in Eq. (2.2) to obtain

\[ YL_{t+1} = Y_t \]
\[ AL_{t+1} = A_t \]
\[ ALLL_{t+1} = ALL_t \]

The model now consists of the fourteen state equations (2.3) and (2.4) which may be written in tableau form as

\[
\begin{bmatrix}
Y \\
G \\
C \\
I \\
REL \\
D \\
TG \\
TT \\
TC \\
TI \\
YL \\
AL \\
ALL \\
ALLL_{t+1}
\end{bmatrix} = \begin{bmatrix}
\mu_i \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\gamma_3 \\
\gamma_3 \\
\gamma_4 \\
\gamma_3 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
-\eta_3 \\
0 \\
0 \\
-\eta_3 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
Y \\
G \\
C \\
I \\
REL \\
D \\
TG \\
TT \\
TC \\
TI \\
YL \\
AL \\
ALL \\
ALLL_{t+1}
\end{bmatrix} + \begin{bmatrix}
A \\
R \\
1
\end{bmatrix} + \begin{bmatrix}
\mu_2 \\
\eta_1 \\
\eta_2 \\
\mu_2
\end{bmatrix}
\]

(2.5)

or in state space form as
\[ x_{t+1} = \hat{A}x_t + Bu_t + Cz_t \]  

(2.6)

(Note that \( \hat{A} \) is used as the symbol for the state variable coefficient matrix here rather than the usual \( A \). This is done to distinguish it from the use of the symbol \( A \) for one of the two control variables, namely appropriations.)

where

\[
\begin{bmatrix}
Y \\
G \\
C \\
I \\
REL \\
D \\
TG \\
TT \\
TC \\
TI \\
YL \\
AL \\
ALL \\
ALLL
\end{bmatrix} = A
\]

\[
\begin{bmatrix}
A \\
R
\end{bmatrix}
\]

\[
\begin{bmatrix}
z
\end{bmatrix} = [1]
\]  

(2.7)
From Kendrick and Shoukry (2011) the estimated coefficients for this model are

\[
\begin{align*}
\alpha_1 &= -0.298 \\
&\quad (0.0144) \\
\alpha_3 &= 0.715 \\
&\quad (0.0025) \\
\beta_1 &= 0.711 \\
&\quad (0.0884) \\
\beta_2 &= -0.004 \\
&\quad (0.0094) \\
\beta_3 &= 4.443 \\
&\quad (0.658)
\end{align*}
\]

Also we have assumed that the tax rate is

\[
\tau = 0.16
\]

The derived coefficients are then

\[
\begin{align*}
\eta_1 &= (1 - \theta) \beta_1 = (1 - \theta) 0.711 \\
\eta_2 &= (1 - \theta) \beta_2 = - (1 - \theta) 0.004 \\
\eta_3 &= (1 - \theta) \beta_3 = (1 - \theta) 4.443 \\
\mu_1 &= \alpha_3 + (1 - \theta) \beta_3 = 0.715 + (1 - \theta) 4.443 \\
\mu_2 &= \alpha_1 + (1 - \theta) \beta_1 = -0.298 + (1 - \theta) 0.711
\end{align*}
\]

Also, the coefficients for the lag structure from appropriations to government expenditure that we are using as a rough guess are

\[
\gamma_1 = 0.20 \quad \gamma_2 = 0.25 \quad \gamma_3 = 0.30 \quad \gamma_4 = 0.25
\]
Thus if we assume that there is a slow adjustment of actual investment to desired investment and use a reaction coefficient of $\theta = 0.95$, then the implied coefficients in the model are

$$\theta = 0.95$$

$$\eta_1 = (0.05) \times 0.711 = 0.035$$

$$\eta_2 = -(0.05) \times 0.004 = -0.002$$

$$\eta_3 = (0.05) \times 4.443 = 0.222$$

$$\mu_1 = 0.715 + (0.05) \times 4.443 = 0.715 + 0.222 = 0.937$$

$$\mu_2 = -0.298 + (0.05) \times 0.711 = -0.298 + 0.035 = -0.263$$

Thus for the $\theta = 0.95$ case

$$\hat{A} = \begin{bmatrix}
0.937 & 0 & 0 & 0.950 & 0 & 0 & 0 & 0 & 0 & 0 & -0.222 & 0.250 & 0.300 & 0.250 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.250 & 0.300 & 0.250 \\
0.715 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.222 & 0 & 0 & 0.950 & 0 & 0 & 0 & 0 & 0 & 0 & -0.222 & 0 & 0 \\
0.160 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.160 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.160 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}$$
Consider next the initial condition for the state vector, i.e.

\[
\begin{bmatrix}
  Y \\
  G \\
  C \\
  I \\
  REL \\
  D \\
  TG \\
  TT \\
  TC \\
  TI \\
  YL \\
  AL \\
  ALL \\
  ALLL
\end{bmatrix}
\begin{bmatrix}
  x_0
\end{bmatrix}
= \begin{bmatrix}
  13.20 \\
  2.41 \\
  9.27 \\
  2.19 \\
  2.64 \\
  0 \\
  0 \\
  0 \\
  0 \\
  13.10 \\
  2.41 \\
  2.41 \\
  2.41
\end{bmatrix}
\]  

(2.9)

The initial condition for the output state variable, \( Y \), is set to the value of the real GDP (in trillions of dollars) in the second quarter of 2007. The level for government
expenditures, $G$, is set to that value in the second quarter of 2007. Also the levels of 
consumption, investment and tax receipts in the base period are all set to their values in 
the second quarter of 2007. In addition, the level of government debt, $D$, is set to zero so 
that the terminal level of this variable will reflect the increase in the debt over the period 
covered by the model. The levels of total government spending, total tax receipts, total 
consumption and total investment are set to zero so that the terminal level of these 
variable will reflect the sum of these variables over the period covered by the model. The level for lagged output, $YL$, is set to the value of real GDP in the first quarter of 2007 and 
the levels for lagged government appropriations, $AL$, $ALL$ and $ALLL$ are set to the value 
of that variable in the first quarter of 2007. The components $C, I$ and $G$ of output do not 
add up to $Y$ primarily because net exports are not included in the model.

This completes the system equations specification. Next we turn to the criterion function 
in the control model.

4. Development of the Criterion Function

The criterion function for the quadratic-linear tracking model may be written as

$$J = \frac{1}{2} (x_N - \tilde{x}_N) W_N (x_N - \tilde{x}_N)$$

$$+ \frac{1}{2} \sum_{t=0}^{N} \left[ (x_t - \tilde{x}_t) W_t (x_t - \tilde{x}_t) + (u_t - \tilde{u}_t) \Lambda_t (u_t - \tilde{u}_t) \right]$$

(3.1)

where

- $x_t$ = state vector - an $n$ vector
- $\tilde{x}_t$ = desired state vector - an $n$ vector
- $u_t$ = control vector - an $m$ vector
- $\tilde{u}_t$ = desired control vector - an $m$ vector
- $W_N$ = symmetric state variable penalty matrix at terminal period, $N$
- $W_t$ = symmetric state variable penalty matrix for period $t$
- $\Lambda_t$ = symmetric control variable penalty matrix for period $t$

The desired path for output in the vector $\tilde{x}$ is set to grow at 1.5 percent per quarter from a 
base of 14.0 trillion in the 3\textsuperscript{rd} quarter of 2007. This process yields the desired time path 
for output which is shown below
The desired path for government expenditures is set to be constant at 2.5 trillion. The desired paths for consumption, investment and tax returns are all set to zero. Also, the desired path for government debt is set to zero in all periods. The desired paths for the totals of government expenditure, tax receipts, consumption and investment are all set to zero. The same is true for the lagged values of output and of government appropriations. Since no weight is assigned to these state variables in the state variable penalty matrix, this specification serves the purpose of the experiments. Finally, in the control vector the desired path for government appropriations is constant at 2.5 trillion and the desired Fed Funds rate is constant at 1 percent.

For this first simplified version of the model the focus is on output as affected by government appropriations. Thus the settings of the penalty weights in the $W_t$ and $\Lambda_t$ matrices reflect this focus. The diagonal elements in the $W_t$ matrices that correspond to output are set to one and all other elements in this matrix in all time periods are set to zero. So the emphasis in this version of the model is on having output track the desired path by growing rapidly at 1.5 percent per quarter. This growth rate is high by usual standards but not by very much in a recovery after a downturn.

Also we have used two different annual policy scenarios. In the first experiment annual policy scenario we used penalty weights of 1 on the output state variable in all time periods as described above. However, in the second experiment in the annual policy scenario we used weights of 25 on the output state variable in all time periods. This was done in order to provide an experiment in which quarterly and annual fiscal policy accomplished roughly the same result in tracking the desired output path.
Different settings of the elements in the $\Lambda_t$ matrices are used for the annual and quarterly scenarios. For both scenarios the elements corresponding to the Fed Funds rate are set to 100. This is done so that there will be little or no use of monetary policy in this simplified version of the model. For the annual scenario the elements of the $\Lambda_t$ matrices corresponding to government appropriations are set to 100 in all except the first quarter of each year (when Congress passed the Bush tax cut in 2008 and the Obama stimulus package in 2009). The elements for the first quarter are set to 1. Thus the intent is that government appropriations in the annual scenario will be at or near 2.5 trillion in all quarters except the first quarter in each year.

In contrast, in the quarterly scenario the weights in the $\Lambda_t$ matrices that correspond to government appropriation are set to one in all time periods. This is sharply different from the weight of 100 that is used for the Fed Funds rate in every period. Thus the intent is that fiscal policy could deviate from the desired path of 2.5 trillion in appropriations in each quarter as needed to offset shocks to the economy but that monetary policy would be inactive.

5. Additive Noise Terms for the Counterfactual Experiments

In the counterfactual experiments in our previous paper (Kendrick and Shoukry (2011)) we used for the additive noise terms in the output equation the first difference of real GDP (GDPC96 from the St. Louis Fed data bank). In contrast in the Monte Carlo experiments reported here we have used additive noise terms that are generated internally in the computer code using the covariance matrix, $Q$, of the additive noise terms. Since the only nonzero noise terms we are using in these experiments is the one in the output equation the only nonzero element in the $Q$ matrix is the $(1,1)$ element and it is set at 0.0155.

This covariance is computed from the shocks for the output equation that were used for the counterfactual example over the periods from the third quarter of 2007 thru the first quarter of 2010 that were in turn calculated by the simple process of computing the first difference in real GDP (GDPC96 from the St. Louis Fed data bank). This process yielded the following results in trillions of dollars.
07-3  0.0743
07-4  0.0950
08-1  -0.0243
08-2  0.0199
08-3  -0.1355
08-4  -0.2298
09-1  -0.1610
09-2  -0.0226
09-3  0.0508
09-4  0.1582
10-1  0.1198

Thus the largest negative shocks were $135.5 billion in the third quarter of 2008, $229.8 billion in the fourth quarter of 2008 and $161.0 billion in the first quarter of 2009. The shocks for all other state variables are set to zero in this experiment. The mean for this series is -0.00502 and the variance is 0.015481 which we have rounded to 0.0155 for use as the (1,1) element in the Q matrix.

6. Experimental Results

Our first experiment was to run the two scenarios as outlined above. The first scenario was for “annual” fiscal policy with substantial changes in the level of government appropriations in only the first quarter of each year. The second scenario was for quarterly fiscal policy with changes in the level of government appropriations permitted in any and all quarters. We ran 1000 Monte Carlo runs in order to compare the results of the annual and quarterly scenarios.

The first result we compared was the average sum of squared difference between the desired and actual output path over the 12 time periods covered by the model. This result was

\[
\begin{align*}
\text{annual} & \quad 1.6905 \\
\text{quarterly} & \quad 0.8439
\end{align*}
\]

Thus the quarterly policy performs about twice as well as the annual policy when measured by the average sum of squared differences. Moreover, it accomplishes this while having a much smaller average sum of squared difference in the separation of the appropriations control variable from its desired path, i.e.

\[
\begin{align*}
\text{annual} & \quad 1.3127 \\
\text{quarterly} & \quad 0.7453
\end{align*}
\]
So a much smoother appropriation path in the quarterly than the annual scenario results in a much smoother output path than in the annual scenario. This is illustrated in the plots of the averages across the 1000 Monte Carlo runs for these paths that are shown below.
Figure 1 contains plots of the government appropriation for the two scenarios. The desired path (Des) of government appropriations is not explicitly shown in the Figure 1. However, recall that the desired path was set at 2.5 trillion dollars and is constant across all periods.

![Graph of Appropriations](image)

Figure 1 Government Appropriations for the Annual and Quarterly Scenarios: Experiment One

The solid red line for the annual scenario shows that in this case government appropriations peak in the first quarter of each year, i.e. in 08-1, 09-1 and 10-1. In contrast the time path for appropriations in the quarterly scenario, the dashed blue line, is smoothed out across all quarters. The magnitude of the changes here are larger than the reality. We think that the time pattern of these results will hold but the magnitudes will become closer to realistic when more complex models are employed.

As is discussed below, the differences in the policy paths in the two scenarios plays an important role in the output results which are shown next in Figure 2.
Figure 2 Real Gross Domestic Product for the Annual and Quarterly Scenarios: Experiment One

Here the straight blue dash-dot line shows the rising desired output level (Des) across the period covered by the model. The dashed blue line for the quarterly scenario is closer to the desired path than the solid red line for the annual scenario in almost all of the time periods covered by the model.

Why does this occur? It seems likely that the accelerator term in the investment function may play an important role. The parameters for the desired investment equation are shown below, i.e.

\[ I_t = 0.711 - 0.004R_{t-1} + 4.443(Y_{t-1} - Y_{t-2}) \]

\[
\begin{pmatrix}
8.04 \\
0.43 \\
6.75
\end{pmatrix}
\]

The t-test for the coefficients are shown in parens under the coefficients.

As discussed in Kendrick and Shoukry (2011), desired investment is a function of the change in GDP. Also recall that in the quarterly scenario there is an immediate – rather
than a delayed – response to the downturns in the economy. Because government expenditures are spread out in a four period lag after the quarter of the appropriations, these increases affect government expenditure levels in 2009 where output in the quarterly scenario does a better job of tracking the desired path in the quarterly scenario than in the annual scenario. Moreover this momentum spreads on into 2010.

Thus it seems possible that “momentum” is one of the keys to these results. Quarterly changes in fiscal policy apparently maintain the economic momentum better than annual changes and thus keep the economy functioning more smoothly.
Figure 3 shows the path of the increase in government debt under the two scenarios.

Figure 3 Accumulated Government Debt for the Annual and Quarterly Scenarios: Experiment One

Recall that the initial government debt variable is set to zero in the first period covered by the model so this figure shows the increase in the government debt during the experiment. The increase in the government debt in the annual scenario is 2.44 trillion dollars and the increase in the quarterly scenario is 2.59 trillion dollars. Thus the debt increase over the counterfactual period is 150 billion more in the quarterly scenario than in the annual scenario. So the quarterly scenario tracks the desired output growth much better than the annual scenario but does so with a somewhat greater increase in the government debt over the twelve periods.

These results raised a new question for us. If we were able to produce a scenario in which the annual and quarterly policies performed equally well in stabilizing output, what would happen to the increase in the debt level? This gave rise to our second experiment.
In order to obtain results in which the annual policy was able to track the rising desired output path roughly as well as the quarter policy it seemed necessary to increase the penalty weight on the output state variable in the criterion function for that scenario. Thus we changed these weights from 1 to 25 for all time periods in the $W_t$ matrices.
For this second experiment the appropriation paths for the two policies are shown in Figure 4.

![Figure 4](image)

**Figure 4** Government Appropriations for the Annual and Quarterly Scenarios: Experiment Two

The annual policy time path is the solid red line and the quarterly policy path is the dashed blue line. These results followed roughly the same pattern as those from the first experiment.

However, the output results are now different as is shown in Figure 5.
Figure 5 Real Gross Domestic Product for the Annual and Quarterly Scenarios: Experiment Two

In these two scenarios the annual policy in the solid red line does about as well in stabilization of output as the quarterly policy in the dashed blue line. The average sum of squared differences for the two scenarios are

- annual: 0.9259
- quarterly: 0.8439

So in rough terms the two policies may be judge to perform about equally well in the stabilization-of-output dimension. But what about in the debt accumulation dimension? This result is shown in Figure 6.
Figure 6 Accumulated Government Debt for the Annual and Quarterly Scenarios: Experiment Two

The increase in the government debt in the annual scenario is 2.73 trillion dollars and the increase in the quarterly scenario is 2.59 trillion dollars. Thus the debt increase over the period covered by the model is about 130 billion dollars more with the annual policy than with the quarterly policy. So in this experiment the quarterly policy tracks the desired output growth about as well as the annual policy but does so with a smaller increase in the size of the government debt.

6. Conclusions

The results from these two experiments indicate two kinds of outcomes. The first is that quarterly rather than annual fiscal policy changes could produce better stabilization results for output but with a somewhat greater increases in debt over the stabilization
period. The second is that roughly similar results for stabilization of output with the two policies could be accomplished but with smaller increases in the debt level with quarterly than with annual changes in fiscal policy.

In our experiments the initial choice of the first quarter as the time when government spending is active in the annual fiscal policy scenario is consistent with the fact that the key fiscal policy interventions in the last year of the Bush Administration and in the first year of the Obama Administration both occurred in February. However, comparison of results across annual and quarterly scenarios in counterfactual experiments like those in Kendrick and Shoukry (2011) will most likely vary substantially depending on which of the four quarters is chosen as the one for active fiscal policy in the annual fiscal policy scenario. This situation is one of the reasons that in the current paper we have shifted from counterfactuals to Monte Carlo experiments. Since the shocks to output are distributed equally across all quarters in the Monte Carlo runs, the choice of the quarter in which fiscal policy is active in the annual scenario will most likely not affect the relative efficacy of the two policy scenarios in this framework.

We have used a somewhat complex model with fourteen state variables in order to capture the necessary lag structures; however, the core of our model is relatively small with only four primary state variables (output, consumption, investment and government expenditures) and one active policy variable (appropriations). Thus results from experiments similar to the two we are reporting on here should become more realistic and may be strengthened or weakened as more complex models are employed.

Though the complexity of the models used to test these results will change in the future, we believe that the basic framework that we proposed in Kendrick and Amman (2011) and employed here will continue to be used to answer the question of the relative efficacy of annual and quarter fiscal policy. This framework includes the use of time varying weights on the control variables so that appropriations are allowed to change only in one quarter each year in the annual scenario but in all four quarters in the quarterly scenario. The framework also includes a lag structure between appropriations and government expenditures in both scenarios.

In future experiments we plan to modify the lag structure between appropriations and government expenditures to see if such changes would have a substantial effect on the comparative advantage of annual and quarterly fiscal policy. Also, we plan to move to
models with active fiscal as well as monetary policy in order to shed light on the question of how the comparative advantage of the two policies changes when one employs quarterly rather than annual fiscal policy and when the lag structure of fiscal policy changes.

At this point we do not know whether or not our results will be robust to different model specifications. However, the results from our first experiment point the way to a finding that the simple process of switching from annual to quarterly fiscal policy could provide, as anticipated, a quicker response to downturns in the economy. This in turn indicates in our results that output remains closer to a desired path but with a somewhat larger increase in debt over the period covered by the model. More interesting perhaps is the result from our second experiment which raises the prospect that quarterly policy changes could be used to stabilize the economy about as well as the annual policy changes but could do so with a smaller increase in debt over the stabilization period. If these results hold over more complex and realistic model specifications this would be good news indeed.
References


