Financial fragility in emerging markets: firm balance sheets and the sectoral structure

Yannick Kalantzis *
(Banque de France)

Abstract

Capital account liberalization in emerging economies is often followed by a shift of resources from the tradable to the non-tradable sector and sometimes leads to a balance-of-payments crisis. This paper builds a two-sector dynamic model to study the evolution of the sectoral structure and its impact on financial fragility. The model embeds a static mechanism of balance-of-payments crisis which produces multiple equilibria within a single time period when the non-tradable sector is large enough compared to the tradable sector. The paper studies the dynamics induced by an increase in financial openness. It shows that the relative size of the non-tradable sector overshoots, which makes the economy more likely to be financially fragile during the transitory dynamics. An extended version of the model is able to quantitatively reproduce several key features of the Argentinean experience during the nineteen-nineties.

Keywords: two-sector models, capital account liberalization, balance-of-payments crises, foreign currency debt, borrowing constraint.

JEL Classification Numbers: E44, F32, F34, F43, O41

1 Introduction

Capital inflows can have substantial effects on the sectoral allocation of resources. The opening of developing economies to foreign capital flows in the last three decades has been followed in a number of cases by a shift of resources from the tradable to the non-tradable sector. In the first few years after the liberalization of the capital account, the relative size of the non-tradable sector increased on average by about 5% above normal times (see below). During the same period many emerging economies experienced financial and balance-of-payments crises. Among the different factors of fragility that were identified by the empirical literature, sectoral factors also seemed to have played a role. Crises took place in countries and in times where the non-tradable sector was

*Banque de France, 49-1374 DERIE-SEMSI, 31, rue Croix des Petits Champs, 75049 Paris cedex 01, France. E-mail: yannick.kalantzis@banque-france.fr.

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larger than usual compared to the tradable sector. It is tempting to see a link between these two facts. Has capital account liberalization led to financial fragility through the channel of sectoral change?

The paper presents a framework to address this question. It builds a two-sector dynamic model of an emerging economy where balance-of-payments crises can happen within single time periods. The model shows that an increase in financial openness is followed by an increase in the relative size of the non-tradable sector and that this change in the sectoral structure can make crises possible.

The first empirical fact that motivates the paper, the link between capital account liberalization and sectoral dynamics, is illustrated by Figure 1. Panel (a) summarizes an event study covering 18 liberalization episodes in 12 emerging countries between 1973 and 1999. The non-tradable to tradable ratio (measured in constant price value added) is regressed on time dummies indicating the number of years the capital account has been liberalized. The panel regression uses a GLS estimator to allow for country heteroskedasticity, controls for a linear trend interacted with country dummies and for the occurrence of twin crises, both during liberalization episodes and when the capital account is closed (see Appendix A.1 for details). The non-tradable to tradable ratio is found to increase above its trend after the liberalization, peaking at about 5% after five years. Panel (b) focuses on a particularly striking episode, Argentina in the nineteen-nineties. As capital flew in after the opening of the economy, increasing the current account deficit-to-GDP ratio by 8 percentage points over the decade, the N-to-T ratio increased by about 18% (controlling for a time trend as discussed in Appendix A.1).

Some empirical studies have also pointed to the role played by sectoral factors in emerging market crises, the second empirical fact that motivates the paper. Tornell & Westermann (2002) show that the relative size of the non-tradable sector usually increases before twin crises in middle-income countries. Calvo, Izquierdo & Mejía (2004) find that the probability of a sudden stop is higher in economies where the production of tradable goods is small compared to the pre-crisis current-account deficit, a proxy for the size of a possible sudden stop.

To reproduce those two stylized facts, the paper embeds a static model of self-fulfilling balance-sheet crisis into a dynamic two-sector model. The key feature of the dynamic model is the limited mobility of factors between sectors. First, production takes time, which prevents any sectoral reallocation of resources within a single time period. Second, production requires entrepreneurial skill, a sector-specific factor. In the static model, a crisis corresponds to a second depreciated market-clearing real exchange rate. The key features of the static model are borrowing constraints that bind during crises and the absence of a market for bonds denominated in non-tradable goods, which gives rise to balance-sheet effects. Those two frictions are only important in times of crisis: the

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1Tornell & Westermann (2002) find that the production of non-tradable goods falls with respect to that of tradable goods during twin crises.

2All the liberalization episodes last at least three years. Seven of the eighteen last less than seven years. The coefficients of the later dummies in Figure 1 are therefore likely to capture a composition effect in addition to the average country dynamics.
Figure 1: Evolution of the N-to-T ratio after a capital account liberalization. (a) Horizontal axis: time dummies indicating the number of years following a capital account liberalization. The solid line plots point estimates (in log points) in a panel regression of the N-to-T ratio on the time dummies. The dashed lines plot the 95% confidence interval. (b) Solid line: percentage change from 1990 of the N-to-T ratio. Dashed line: absolute change from 1990 of the current account deficit as a share of GDP.

analysis of the no-crisis dynamics is carried out with the assumption that the borrowing constraint does not bind, which makes balance-sheet variables and currency mismatches irrelevant. Financial opening is modeled as a decrease in a transaction cost on international financial flows and results in a lower domestic interest rate. The main findings of the paper are the following. As financial openness increases and foreign capital flows in, the real-exchange rate and the N-to-T ratio overshoot. This overshooting of both variables is persistent. In the medium run, a shift of the sectoral structure towards non-tradable goods, along with the higher leverage induced by cheaper foreign finance, can make the economy fragile to self-fulfilling crises. Such a crisis, should it take place, reproduces several of the stylized facts identified by the empirical literature on emerging market crises: a large real depreciation, a sharp drop in investment, widespread defaults, and sectoral asymmetries (Kaminsky & Reinhart 1999, Tornell & Westermann 2002, Calvo, Izquierdo & Talvi 2006, Calvo et al. 2004).

Those results are first derived analytically in a simple version of the model. The model is then extended to a more realistic set-up to conduct a quantitative analysis. It is calibrated to reproduce the Argentinean economy in the nineteen-nineties. Argentina is chosen since it provides a very striking example of the dynamics predicted by the model, with a large and long-lasting non-tradable boom (as shown in Figure 1b) which ended with a very deep crisis. The results derived in the simple model are first shown to carry over to the extended set-up: the resulting dynamics of the N-to-T ratio actually displays an even stronger overshooting. Then, the model is shown to fit well the experience of Argentina. First, it is able to reproduce the size of the increase in both the current account deficit and the N-to-T ratio over the decade. Second, it correctly predicts the possibility of a crisis around 2000 for reasonable parameter values. Third, it matches the behavior of several key variables during the crisis, in particular the real exchange rate, investment, and employment.
This paper belongs to both the literature studying the sectoral evolution of open economies and the literature on emerging market crises. As regards the former, several works studied how the discovery of natural resources affects the allocation of resources between the tradable and non-tradable sectors, the so-called Dutch disease. The reader may for example refer to Corden & Neary (1982), Bruno & Sachs (1982), and van Wijnbergen (1984). More recently, Hausmann & Rigobon (2002) show how a high concentration of capital in the non-tradable sector increases the volatility of the real exchange rate. This in return induces a shift of resources from the tradable to the non-tradable sector, eventually leading to a complete specialization in non-tradable goods. Caballero & Lorenzoni (2007) study the optimal policy response to episodes of persistent appreciations during which resources move away from the export sector to the non-tradable sector.

The paper is also related to models of balance-of-payments crises based on borrowing constraints, currency mismatches, and balance-sheet effects in the corporate sector. Crises are modeled by the possible existence of multiple equilibria within a single time period, as in Krugman (1999), Schneider & Tornell (2004), and Aghion, Bacchetta & Banerjee (2004). The financial frictions that originate the multiplicity of equilibria have also been incorporated into quantitative real business cycle models where they amplify otherwise small shocks (Mendoza 2002).3

Most of this literature is primarily concerned with modeling the crisis itself and discussing policy options but not with understanding the dynamics that possibly leads to it. By inserting a static crisis mechanism into a dynamic framework, the present paper follows the methodology used by Schneider & Tornell (2004) even though it differs substantially in important ways. These authors study the growth of the non-tradable sector during a transitory lending boom and show that a large enough boom can lead to a self-fulfilling crisis. The boom is driven by a binding borrowing constraint which, together with a linear technology, leads to the cumulative growth of internal funds, investment, and the price of non-tradables: higher internal funds lead to higher investment, which pushes up the price of non-tradables and increases internal funds even more. This dynamics requires a high enough expected future price of non-tradables, induced for example by a reform that is believed to increase the future demand for non-tradable goods. Their model focuses on the non-tradable sector alone and, since it studies a transitory phenomenon, has a finite number of periods. By contrast, this paper studies how an increase in financial openness impacts the allocation of resources between the tradable and non-tradable sectors. The two sectors are therefore modeled explicitly and in a symmetric way. Since borrowing constraints do not bind in normal times, the pre-crisis dynamics reduces to a standard neoclassical two-sector model with a sector-specific factor

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3 Those frictions have been the subject of an extensive literature, both empirical and theoretical. The fact that the domestic agents of a developing country are unable to issue debt denominated in foreign currency on international financial markets has been dubbed the Original Sin (Eichengreen & Hausmann 1999). Alternatively, several authors have proposed arguments to explain why domestic firms choose to take a risky position by issuing debt denominated in foreign currency (Schneider & Tornell 2004, Caballero & Krishnamurthy 2000, Jeanne 2000, Jeanne 2003). Borrowing constraints that introduce a limit on leverage have become common in the macroeconomic literature (Mendoza 2002, Antras & Caballero 2009) and several microfoundations have been developed that could be easily embedded in the current paper (Schneider & Tornell 2004, Aghion, Banerjee & Piketty 1999, Antras & Caballero 2009).
of production: therefore, the mechanism of sectoral reallocation does not rely on financial frictions. Leverage, an important variable to determine financial fragility, is endogenous and not pinned down by the borrowing constraint: this allows to distinguish the effect of a larger non-tradable sector (with respect to the tradable sector) from that of a more indebted non-tradable sector. Finally, the model has an infinite number of periods, which makes it possible to study the effect of financial openness both during the transitory phase of sectoral change and in the new steady state.

The paper is organized as follows. A simple version of the model is presented in section 2. Section 3 studies analytically the equilibrium dynamics of the simple model and checks whether multiple equilibria arise, making self-fulfilling crises possible. Section 4 extends the model to a more realistic set-up, calibrates it on Argentinean data, and simulates it to check whether the previous results still hold. Section 5 uses the extended model to reproduce the experience of Argentina in the nineteen-nineties. Section 6 concludes.

2 A simple framework

2.1 The model

Consider a small open economy. Time is discrete. There are two types of goods: a tradable good T and a non-tradable good N. The tradable good T is chosen as the numeraire. Denote $p_t$ the relative price of the non-tradable good in period $t$. The relative price $p_t$ is a measure of the real exchange rate. A high value of $p_t$ corresponds to an appreciated real exchange rate.

Production

The tradable good T is produced by a tradable sector (sector T). It can also be imported and any excess production of tradable goods can be exported. The non-tradable good N is exclusively produced by a domestic non-tradable sector (sector N) and the whole production has to be used domestically.

Each sector is composed of a continuum of firms of measure one. Production requires entrepreneurial skill and uses an input Z with decreasing returns to scale.$^4$

The model assumes limited mobility of factors. First, entrepreneurial skill is specific to each sector. Second, inputs have to be purchased one period before production can be sold. A firm in sector $s$ buys a quantity $z^{s,t+1}_t$ of inputs in period $t$ and uses entrepreneurial skill to produce a quantity $(z^{s,t+1}_t)^\theta$ sold in period $t + 1$, where $\theta \in (0, 1)$ measures returns to scale with respect to the input Z. The input Z is an intermediate good partly composed of the non-tradable good itself, in a way to be detailed later. Denote $p^Z$ its price.

$^4$This amounts to assuming constant returns to scale with respect to both the input Z—a mobile factor—and entrepreneurial skill—a specific factor.
Agents

In this simple version of the model, there are only two kinds of agents: entrepreneurs and foreign lenders. They are risk neutral with a common discount factor $\beta$.

Entrepreneurs run firms. They are specialized in one sector, tradable or non-tradable. They belong to families with a measure one continuum of members. All members are entitled with equal shares of the family’s wealth. At the beginning of each period, a fraction $1 - \gamma$ of the members exits the economy and is replaced by newly born members.

Entrepreneurs only consume when they exit. The final good they consume is an aggregate of tradable and non-tradable goods to be detailed later, with price $P$. Production decisions are taken by a representative entrepreneur who maximizes utility, defined as the discounted sum of expected future consumptions of the family. To start production, the representative entrepreneur has internal funds equal to beginning-of-period wealth minus the consumption of exiting members plus some possible subsidy from the Government, to be detailed later.

Foreign lenders derive utility from the consumption of tradable goods. They receive a large enough endowment of tradable goods in each period to provide an infinitely elastic supply of funds at the rate of return $1/\beta$.

Financial contracts

Entrepreneurs finance the purchase of inputs out of their internal funds and by issuing one-period bonds denominated in tradable goods. Bonds issued in period $t$ by sector $s$ promise a rate of return $r_s^t$. When the proceeds from the sales of a firm fall short of the promised repayment to bondholders, debt cannot be fully paid back and the entrepreneur is forced to default.

Entrepreneurs are subject to a borrowing constraint. An entrepreneur with internal funds $m_s^t$ can borrow at most

$$p_t^Z z_{t+1}^s - m_s^t \leq (\lambda - 1)m_s^t \quad s = N,T,$$

where $\lambda \geq 1$ is the financial multiplier of internal funds.

To model the degree of financial openness, I assume that there is an iceberg cost $\tau_t^s > 1$ to international financial transactions. When a foreign lender lends $\tau_t^s$ units of tradable good to a domestic agent, the domestic agent only gets 1 unit, and vice versa.

Similarly, domestic agents face an iceberg cost $\tau$ when making loans. I assume that $\tau > \tau_t^s$ because of inefficient domestic financial intermediation.

Crises

A balance-of-payments crisis in this model is defined as the occurrence of widespread defaults in the non-tradable sector. This happens when $p_t(z_t^N)^{\theta} < \tau_t^{N-1}(p_{t-1}^Z z_t^N - m_t^{N-1})$. Let $\zeta_t \in \{0, 1\}$ be a
dichotomic variable indicating a crisis in period $t$:

$$\zeta_t = 1 \text{ if } p_t(z_t^N)^\theta < r_{t-1}^N (p_{t-1} Z_{t-1}^N - m_{t-1}^N), \quad 0 \text{ otherwise.} \quad (2)$$

During crisis times, the Government intervenes to bail out firms producing non-tradable goods, thus preventing the non-tradable sector from completely disappearing (with zero internal funds and a binding borrowing constraint, defaulting entrepreneurs would not be able to start production at all). Entrepreneurs in sector $N$ receive a subsidy $S_N > 0$, financed by a lump-sum tax on entrepreneurs in sector $T$. The subsidy is supposed to be small enough to make the borrowing constraint bind in sector $N$.

### A simple closure

The model can be closed in different ways. I first consider a simple closure with no labor, no capital, and where the non-tradable good is only used as an intermediate good. This makes it possible to solve the model analytically and to study the mechanism of sectoral reallocation in a transparent way. These assumptions will be relaxed in section 4 which studies a more realistic setup where production also requires the use of capital and labor, and the non-tradable good enters in the composition of consumption, capital, and intermediate goods.

In the simple closure, I assume that (i) entrepreneurs only consume tradable goods and (ii) the input $Z$ used in production is a Cobb-Douglas aggregate of tradable and non-tradable goods:

$$Z = \left(\frac{T}{1-\mu}\right)^{1-\mu} \left(\frac{N}{\mu}\right)^\mu.$$

### 2.2 Equilibrium paths

Consider a given exogenous path of the iceberg cost $\{\tau_t^*\}_{t \geq 0}$. Under the simple closure, an equilibrium is a sequence of crisis indicators $\{\zeta_t\}_{t \geq 0}$, prices $\{p_t, r_t^N, r_t^T\}_{t \geq 0}$, allocations $\{z_t^N, z_t^T, m_t^N, m_t^T\}_{t \geq 0}$, and subsidies $\{S_N\}_{t \geq 0}$ such that lenders break even, allocations solve the optimization problems of entrepreneurs given prices and crisis indicators, and the market for non-tradable goods clears.

I now describe in more details the optimization problems of the different agents. Here, I write down these problems in the general case, independently of how the model is closed. Under the simple closure, $P$ is simply equal to 1 since the consumption good is identical to the tradable good and $p^Z$ has the usual expression for the price of a Cobb-Douglas aggregate:

$$P_t = 1,$$

$$p_t^Z = (p_t)^\mu. \quad (3a)$$

$$p_t^Z = (p_t)^\mu. \quad (3b)$$
The entrepreneur’s problem

The representative entrepreneur of sector N makes production decisions to maximize the discounted sum of expected future consumptions of the family, given the borrowing constraint (1). For an entrepreneur producing non-tradable goods, debt denominated in tradable goods is risky since the ability to repay depends on the real exchange rate. In times of crisis, when the price of non-tradable goods is too low to cover the promised debt repayment, the firm defaults, the beginning-of-period wealth is zero, and exiting members of the family do not consume. Hence, the optimization program of the representative entrepreneur in sector N has the following value function:

\[ V_N^t \left( \frac{m_N^t}{p_t^t} \right) = \max \beta E_t \left[ (1 - \gamma) \frac{W_{l+1}^N}{p_{l+1}^t} + V_{l+1}^N \left( \frac{\gamma W_{l+1}^N + \zeta_{l+1} S_{l+1}^N}{p_{l+1}^t} \right) \right] \]

with \( W_{l+1}^N = (1 - \zeta_{l+1}) \left[ p_{l+1}(z_{l+1}^N)^\theta - r_t^N Z_{l+1} \left( p_t^t Z_{l+1} - m_t^N \right) \right] \), and s. t. \( p_t^t Z_{l+1}^N \leq \lambda m_t^N \).

The variable \( W_{l+1}^N \) denotes wealth at the beginning of period \( t + 1 \) and is zero in crisis time (when \( \zeta_{l+1} = 1 \)). The expected value has two terms: the consumption of the \( 1 - \gamma \) exiting family members next period and the continuation value of the measure one family. Denoting \( \Gamma_t^N / p_t^t \) the Lagrange multiplier of the borrowing constraint, the first order condition of this problem is:

\[ \beta E_t \left[ (1 - \zeta_{l+1}) \left[ (z_{l+1}^N)^\theta - r_t^N \right] \frac{\Lambda_t^N}{p_t^t} \right] = \Gamma_t^N p_t^t \]

(4)

where \( \Lambda_t^N \) is the marginal value of beginning-of-period wealth given by

\[ \Lambda_t^N = 1 - \gamma + \gamma \beta E_t \left[ (1 - \zeta_{l+1}) r_t^N \frac{p_t^t}{p_{l+1}^t} \Lambda_t^N \right] + \gamma \lambda \Gamma_t^N \]

(5)

and subject to the usual transversality condition.\(^6\)

In sector T, entrepreneurs produce tradable goods, which makes their debt denominated in tradable goods riskless. The first-order condition is then simply given by

\[ \beta E_t \left[ (z_{l+1}^T)^\theta - r_t^T \frac{\Lambda_t^T}{p_t^t} \right] = \Gamma_t^T p_t^t \]

(6)

with \( \Lambda_t^T = 1 - \gamma + \gamma \beta E_t \left[ r_t^T \frac{p_t^t}{p_{l+1}^t} \Lambda_t^T \right] + \gamma \lambda \Gamma_t^T \).

\(^5\)For expositional convenience, I assume that entrepreneurs do not buy riskless bonds. Not buying riskless bonds is optimal as long as defaults are rare enough events and the iceberg cost on domestic finance, \( \tau \), is large enough.

\(^6\)The marginal value of wealth is formally defined by \( \Lambda_t^N \equiv 1 - \gamma + \gamma (V_t^N)'(1 - \zeta_t^N) \). It has three terms. The first term corresponds to consumption by exiting agents. The second term is the discounted normal return on internal funds. The last term is the excess return on internal funds when the borrowing constraint binds.
**Foreign lenders’ breakeven condition**

Because of their large endowments, foreign lenders set the rate of return in the model. Given the iceberg cost \( \tau_t \), the riskless borrowing rate in the domestic economy is \( r_t \equiv \frac{\tau_t^*}{\beta} \).

The risk-neutrality of foreign lenders implies

\[
r_t = r_t^T, \quad r_t = \mathbb{E}_t \left[ (1 - \zeta_{t+1}) r_t^N + \zeta_{t+1} \frac{p_{t+1} \theta_t}{z_{t+1} - m_t^N} \right].
\]

**(7a)**  

**(7b)**

**The dynamics**

Moving to the simple closure, the dynamics of the model is characterized by the following equations:

\[
z_{t+1}^N = \min \left( \frac{\lambda m_t^N}{p_t^N}, \left[ \frac{\theta}{p_t^N} \mathbb{E}_t ((1 - \zeta_t) \Lambda_t^N [1 + \frac{1}{p_t^N} \frac{p_{t+1} \theta_t}{z_{t+1} - m_t^N}]) \right] \right), 
\]

\[
z_{t+1}^T = \min \left( \frac{\lambda m_t^T}{p_t^T}, \left[ \frac{\theta}{p_t^T} \mathbb{E}_t ((1 - \zeta_t) \Lambda_t^T) \right] \right),
\]

\[
m_t^N = (1 - \zeta_t) \gamma [p_t (z_t^N)^\theta - r_{t-1}^N (p_{t-1}^N z_t^N - m_t^{N-1})] + \zeta_t S_t^N,
\]

\[
m_t^T = \gamma [(z_t^T)^\theta - r_{t-1} (p_{t-1}^T z_t^T - m_t^{T-1})] - \zeta_t S_t^N,
\]

\[
p_t (z_t^N)^\theta = \mu p_t^N (z_{t+1} + z_{t+1}^T),
\]

where \( \Lambda_t^N \) is defined by (4) and (5), and \( r_t^N \) follows from (7b) and (3b). The scales of production in both sectors, \( z_t^N \) and \( z_t^T \), are predetermined, while internal funds \( m_t^N \) and \( m_t^T \) and the real exchange rate \( p_t \) are determined in period \( t \).

Equations (8a) and (8b), which describe the amounts of inputs purchased by entrepreneurs, derive from the first-order conditions (4) and (6) under the simple closure (3). Equations (8c) and (8d) describe the evolution of internal funds, taking into account possible subsidies and taxes during crises. Finally, equation (8e) is the market-clearing condition for non-tradable goods, which implicitly determines the real exchange rate \( p_t \). Under the simple closure, the non-tradable good only enters the composition of intermediate goods used by entrepreneurs, which are themselves a Cobb-Douglas aggregate of tradable and non-tradable goods. Therefore, entrepreneurs spend a constant fraction \( \mu \) of their purchases of intermediate inputs on non-tradable goods.

In general, the model can have multiple market-clearing prices inside a single time period, corresponding to normal times and crisis times, and there could be sunspot-driven shocks on \( \zeta_t \). In the following, I will focus on perfect-foresight dynamics with no uncertainty and zero-probability crises only.

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7 From the breakeven condition (7a), \( r_t^T = r_t \).
2.3 Discussion of the set-up

The model makes two important assumptions regarding the mobility of production factors. First, production takes time. Second, production requires entrepreneurial skill specific to each sector and has decreasing returns to scale with respects to the intermediate input \( Z \). Both assumptions play a key role in the sectoral dynamics of the model.

The assumption that entrepreneurs enter and exit the economy is usual in models with borrowing constraints (as in Bernanke, Gertler & Gilchrist 1999). In my case, this demographic structure makes it possible to consider steady states corresponding to different degrees of financial openness, as internal funds converge to a finite value whatever the level of the real interest rate.\(^8\)

The model assumes two financial frictions: the absence of bonds denominated in non-tradable goods and borrowing constraints. These assumptions are common in models of emerging market crises.

3 Sectoral dynamics and financial fragility

3.1 Solution concept

For the sake of simplicity, my strategy is to focus on perfect-foresight equilibria with non-binding borrowing constraints and no crises. This reduces the model to a simple two-sector small open economy with no financial frictions and makes it easy to study the dynamics. Then, I will ask whether there exists a second market-clearing price, corresponding to a crisis, in a given time period along that perfect-foresight no-crisis dynamics. The proofs of all propositions are in the Appendix.

With no uncertainty, no crises, and assuming that the borrowing constraints (\( \Pi \)) hold as strict inequalities, equations (\( 8 \)) simplify to:\(^9\)

\[
\begin{align*}
z_{t+1}^N &= \left[ \frac{\theta}{\mu_{t+1}} E_t[p_{t+1}] \right]^{\frac{1}{1-\theta}}, \\
z_{t+1}^T &= \left[ \frac{\theta}{\mu_{t+1}} E_t[r_{t+1}] \right]^{\frac{1}{1-\theta}}, \\
m_t^N &= \gamma \left[ \mu_{t}^N (z_{t+1}^N)^\theta - r_{t-1} (p_{t-1}^N z_{t-1}^N - m_{t-1}^N) \right], \\
m_t^T &= \gamma \left[ (z_{t}^T)^\theta - r_{t-1} (p_{t-1}^T z_{t-1}^T - m_{t-1}^T) \right], \\
p_t(z_t^N)^\theta &= \mu_{t}^N (z_{t+1}^N + z_{t+1}^T).
\end{align*}
\]

Definition 1 (No-crisis equilibrium path). Given initial conditions \( z_0^N, z_0^T, m_0^T \), an initial debt

\(^8\)The assumption that entrepreneurs only consume when they exit is a minor restriction. It can be shown that, in any steady state, entrepreneurs would endogenously choose not to consume when they do not exit.

\(^9\)In the absence of crisis, the breakeven condition for foreign lenders implies that \( r_t^N = r_t^T = r_t = \tau^*/\beta \).
repayment $b^N_0$ in sector $N$, and a deterministic exogenous path for the riskless gross interest rate $\{r_t\}_{t \geq 0}$, a no-crisis equilibrium path is a sequence $\{z^N_t, z^T_t, p_t, m^N_t, m^T_t\}_{t \geq 0}$ that satisfies (3a), (3b), (3d), and (3e), such that the borrowing constraints (1) do not bind, and $m^N_0 = \gamma[p_0(z^N_0)^\theta - b^N_0].$

### 3.2 Sectoral dynamics in the absence of crisis

**The steady state**

The following proposition establishes the conditions under which there exists a no-crisis equilibrium steady state with strictly positive debt.

**Proposition 1.** Suppose $\theta > \gamma$.

For $\lambda \geq 1$ and $r > 1$ such that $\max(\frac{\theta}{\gamma+\lambda(1-\theta)}, 1) < r < \frac{\theta}{\eta}$, there is a unique no-crisis equilibrium steady state where entrepreneurs have a strictly positive debt and borrowing constraints do not bind. In this steady state, the debt repayment-to-internal funds ratio, an indicator of leverage, decreases with the gross interest rate $r$. The relative size of both sectors is given by $z^N_t / z^T_t = m^N_t / m^T_t$ and is decreasing in $r$.

The proof consists in first constructing the no-crisis steady state (under the assumption that borrowing constraints do not bind) and then deriving parameter restrictions under which it exists (this includes the fact that borrowing constraints should be non-binding) and debt is strictly positive. One of the conditions of existence is that borrowing constraints (1) are slack, i.e. $p^s z^s < \lambda m^s$ for $s = N, T$. With $m^s = (\frac{1}{\eta^s} - 1)\frac{\gamma}{1-\gamma} p^s z^s$ in the steady state, this requires $r > \frac{\theta}{\eta+\lambda(1-\theta)}$. Debt is strictly positive for $r < \theta / \gamma$. For debt to be positive at positive interest rates (i.e. with $r > 1$) one has to assume that $\theta > \gamma$.

**The transitory dynamics**

I now turn to the transitory dynamics that follows a permanent increase in financial openness, that is, a permanent decrease in the iceberg cost $\tau^*$ and the domestic riskless rate $r$.

**Proposition 2.** Consider an economy in a no-crisis equilibrium steady state corresponding to $r_0 = \bar{r}$ at $t = 0$, hit by an unexpected and permanent negative shock $r_\infty < r_0$ at $t = 1$. Suppose $\lambda$ is large enough so that borrowing constraints always remain slack. Then,

- the real exchange rate $p$ overshoots: it appreciates on impact at $t = 1$ and depreciates thereafter (for $t \geq 2$),
- the scale of production in sector $N$, $z^N$, increases starting at $t = 2$,
- the relative size of sector $N$, measured by $z^N / z^T$, overshoots: it increases at $t = 2$ and decreases thereafter (for $t \geq 3$),
• if $\gamma \bar{r}$ is not too close to either 0 or 1, the ratio of sectoral internal funds, $m_N^N/m_T^T$, overshoots with a hump-shaped response starting at $t = 1$.

To prove the proposition, the dynamics described in equations (8) are first shown to reduce to a simple two-dimensional system $(z_t^N, p_t)$, where $z_t^N$ is a predetermined state variable (similar to a capital stock), while $p_t$ is a non-predicted jump variable that contemporaneously reacts to unexpected shocks. This system is then log-linearized and shown to have saddle-path dynamics, as illustrated by the phase diagram of Figure 2a. The permanent decrease from $r_0$ to $r_\infty$ moves the saddle-path to the north-east in the $(z_t^N, p_t)$ plane, leading to an overshooting real exchange rate as shown in Figure 2b.

![Phase Diagram](image)

Figure 2: Log-linearized dynamics (\(\hat{x}_t\) denotes the log-difference between \(x_t\) and its value in a given steady state). (a) Phase diagram. (b) Dynamics following an unexpected and permanent increase in financial openness.

Figure 4 (dashed lines) illustrates the transitory dynamics after an increase in financial openness. Intuitively, as domestic entrepreneurs have suddenly access to cheaper foreign loans to finance their purchases of inputs at $t = 1$, they are induced to increase their scale of production. With a predetermined supply, the higher induced demand for non-tradable goods bids up the relative price $p$ in the period of the shock. In the long run, however, the scale of production in sector $N$, $z^N$, adjusts to accommodate this larger demand and the real exchange rate subsequently depreciates. Therefore, the real exchange rate overshoots when the shock hits. An important result of the model is that this overshooting lasts for an extended period of time after the shock hits. Indeed, increasing next period’s supply of non-tradable goods pushes down their relative price in the next period: from equation (8a'), this implies a lower optimal scale of production, slowing down the increase. With a slow increase in $z_t^N$, the decrease in $p_t$ from $t = 2$ on has to be gradual.

---

10 The calibration is described in section 4.2. The simulation is performed using Dynare. See Juillard (1996) for details on the algorithm used.

11 In equation (8c), $p_t$ increases with $z_t^N + z_t^T$. 

---

12
As for the evolution of the sectoral structure, the appreciated real exchange rate hurts the tradable sector by raising the price of the input (in terms of tradable goods) whereas in the non-tradable sector this higher cost of inputs is offset by a high expected future price of outputs. So, the relative size of both sectors, measured by their input purchases or by their internal funds, first evolves in a direction favorable to the non-tradable sector until the tradable sector catches up.

The important feature to get these results is the imperfect mobility of factors. The real exchange rate overshoots when the shock hits because production takes time and cannot adjust instantaneously. The overshooting lasts for many periods because entrepreneurs cannot freely move between sectors. If they could, they would switch from the tradable to the non-tradable sector until the expected relative price $E_1[p_2]$ was back to its steady state level. The real exchange rate and the relative size $z^N/z^T$ would overshoot for a single time period. Appendix A.4 formally derives this result.

3.3 Financial fragility and crises

I now look for a second market-clearing price corresponding to a crisis, along the no-crisis equilibrium path.

**Proposition 3.** Consider an economy following a no-crisis equilibrium path. If

$$\frac{r_{t-1}^N(p_{t-1}^N z_t^N - m_{t-1}^N)}{\gamma[(z_{t}^T)^\theta - r_{t-1}(p_{t-1}^T z_{t}^T - m_{t-1}^T)]} > \mu \lambda,$$

(10)

then there exists a second market-clearing price $p_t^{\text{crisis}} < p_t$ associated with a crisis indicator $\zeta_t^{\text{crisis}} = 1$. A necessary condition for the existence of two market-clearing prices associated with $\zeta_t = 0$ and $\zeta_t = 1$ is

$$\mu \gamma \lambda > 1.$$  

(11)

An equilibrium path is said to be **financially fragile** in period $t$ when condition (10) is satisfied. Then, a crisis can be triggered by a non-anticipated expectational shock that makes agents coordinate on the lower market-clearing price. Of course, by definition of the no-crisis equilibrium path, crises are zero-probability events and they remain in the background of the dynamics. Along a no-crisis equilibrium path, condition (10) can be rewritten

$$\frac{r_t^N(p_t^N z_t^N - m_t^N)}{m_t^T} \times \frac{m_t^N}{m_t^T} > \mu \lambda.$$  

(10')

A no-crisis equilibrium path is financially fragile when the product of two factors is large enough. The first factor relates debt service to internal funds and reflects the financial structure of balance sheets in sector $N$. As debt is denominated in tradable goods, it also measures the extent of the
currency mismatch. The second factor describes the relative size of both sectors, measured by their internal funds, and is an indicator of the sectoral structure of the whole economy.\textsuperscript{12}

The steady state value of both factors decrease in \(r\) (see Proposition \textsuperscript{11}). In the long run, more financially opened economies are more leveraged and have a larger non-tradable sector.\textsuperscript{13}

The following proposition shows under what condition this is enough to make the steady state financially fragile.

\textbf{Proposition 4.} Suppose \(\mu \gamma \lambda > 1\). There is a unique \(r_{\text{frag}}\) in \((\max(\frac{\theta}{\gamma + \lambda(1-\theta)} \mu, \frac{\theta}{\gamma} \mu), \frac{\theta}{\gamma})\) such that 
\[
\frac{r(p_t^N z_t^N - m_{t-1}^N)}{m_t} > \mu \lambda \quad \text{if and only if} \quad r < r_{\text{frag}}.
\]
If \(r_{\text{frag}} > 1\), a no-crisis equilibrium steady state is financially fragile for all \(r\) in \((1, r_{\text{max}})\). A sufficient condition for \(r_{\text{frag}} > 1\) is \(\theta > \frac{\lambda}{\lambda(1+1/\gamma)}\).

After a permanent decrease in the transaction cost \(\tau^*\), leverage in sector \(N\), as measured by the ratio of debt repayment to internal funds, increases to its higher new steady state value.\textsuperscript{14}

Section 3.2 showed that the ratio of internal funds \(m_N^T/m_T^T\) displays a hump-shaped dynamics for a wide range of parameters. As a result, the financial fragility ratio \(r_{t-1}(p_{t-1}^N z_t^N - m_{t-1}^N)/m_t^T\) can overshoot in the medium run, as shown in Figure 3 (solid line). This makes it more likely for the economy to become financially fragile during the transition to a more open capital account.

As the financial structure of sector \(N\) and the sectoral structure enter multiplicatively in the financial fragility ratio, the two factors reinforce each other. By how much does each factor contribute to financial fragility? Figure 3 shows the evolution of \(r_{t-1}(p_{t-1}^N z_t^N - m_{t-1}^N)/m_t^T\) with (solid line) and without (dashed line) changes in \(m_N^T/m_T^T\). In the long run the larger value of the ratio mainly comes from a higher leverage but the overshooting of relative internal funds does significantly affect the transitory dynamics.

![Figure 3: Contribution of sectoral change to financial fragility after a permanent increase in financial openness. Solid line: \(r_{t-1}(p_{t-1}^N z_t^N - m_{t-1}^N)/m_t^T\). Dashed line: \(r_{t-1}(p_{t-1}^N z_t^N - m_{t-1}^N)/m_t^T \times (m_N^T/m_T^T)_{\text{constant}}\) where \(m_N^T/m_T^T\) is kept constant at its initial value.](image)

\textsuperscript{12}In condition \textsuperscript{10}, \(m_N^T\) and \(m_T^T\) correspond to normal-time internal funds.

\textsuperscript{13}The monotonicity of \(m_N^T/m_T^T\) in \(r\) depends on the simplifying assumption that the non-tradable good is only used as an input. It does not generalize to the case when it also enters the consumption basket. In such a case, however, \(r(p_t^N z_t^N - m_N^T)/m_T^T\) would still be strictly decreasing in \(r\).

\textsuperscript{14}The leverage ratio in sector \(N\) actually decreases when the shock hits since debt service is predetermined while internal funds increase with the real appreciation. It only starts increasing in the following period. Convergence can be monotonic or slightly hump-shaped, depending on parameter values.
3.4 Discussion of the results

The role of financial frictions  By definition of the no-crisis equilibrium path, financial frictions play no role in the transitory dynamics. However, they are important for financial fragility. As in Krugman (1999), Aghion et al. (2004), and Schneider & Tornell (2004), the crisis mechanism relies on the interplay between currency mismatches and borrowing constraints. Borrowing constraints are in particular necessary to limit the demand for non-tradable goods during crises, not only in sector N but also in sector T: a larger financial multiplier in sector T would weaken the case for financial fragility.\footnote{If entrepreneurs in sector T were not subject to the constraint at all, they would take advantage of the low price of inputs to increase their production during crises. Their higher demand for inputs would partly make up for the lower demand from sector N, dampening the effect of the crisis, or even making it impossible for a crisis to take place at all.}

However, the financial multiplier cannot be too small: borrowing constraints have to remain slack, at least in sector N, so that a normal-time equilibrium exists, a result similar to Aghion et al. (2004) and Schneider & Tornell (2004). More precisely, the financial multiplier λ that enters condition (11) is that of sector N. As regards sector T, the definition of a no-crisis equilibrium path can be easily extended to allow for a binding constraint: this would simply slow down growth in the tradable sector during its catching-up phase.

The role of the sectoral structure  Condition (10') shows that a sectoral structure largely oriented toward the production of non-tradable goods favors the possibility of crises. Intuitively, when borrowing is constrained, the sectoral structure is what determines the level of the real exchange rate needed to adjust the lower demand for non-tradable goods. Then, for a large enough (foreign currency) debt in sector N—the first factor in condition (10')—this depreciated level of the real exchange rate leads to defaults and a crisis.

Anticipated crises  To model anticipated crises, a predictable selection rule should be introduced to coordinate agents across the two possible outcomes when both exist at the same time. Suppose, as Cole & Kehoe (2000), that there is an exogenous sunspot variable independently and uniformly distributed on the interval [0, 1] and denote π the probability of a crisis in a period where a crisis is possible. A possible selection rule could be to coordinate on the crisis-time real exchange rate when (a) it exists, (b) it was predicted to exist with probability π in the previous period, and (c) the sunspot variable is lower than π. The probability of crisis would then be endogenous: equal to 0 or π depending on whether the economy is financially fragile or not. The no-crisis equilibrium path studied in this paper corresponds to the limit \( \pi \to 0 \). By continuity, results concerning financial fragility when \( \pi = 0 \) should also be valid when \( \pi > 0 \) provided that the probability of crisis \( \pi \) is low enough.\footnote{One advantage of studying this limiting case is that the dynamics converges to a steady state, which is not necessarily true when \( \pi > 0 \). If the steady state of the no-crisis equilibrium path is financially fragile, equilibrium}
Crises triggered by shocks on fundamentals  So far, crises were supposed to be triggered by self-fulfilling purely expectational shocks. An alternative is to consider unexpected negative shocks on $\lambda$. If $\lambda < 1/(\mu \gamma)$ when (10) holds, the normal-time within-period equilibrium disappears and the economy jumps to the remaining crisis-time within-period equilibrium.  

4  Quantitative analysis

4.1  The model

A more realistic set-up

To explore the quantitative features of the model, the mechanism has to be embedded into a richer model. In the simple closure studied so far, the focus was exclusively on entrepreneurs, intermediate goods were the only factor of production, and the final good consumed by agents was only made of tradable goods. To close the model presented in section 2 in a more realistic way, three additional elements are introduced: (i) the final good is made of both tradable and non-tradable goods, (ii) production now requires the use of labor and capital, providing new adjustment margins, and (iii) there is a representative household.

The final good $Y$ is a Cobb-Douglas aggregate of tradable and non-tradable goods, of price $P$, with $Y = \left(\frac{T}{1-\mu}\right)^{1-\mu}(\Sigma)^{\mu}$. The tradable and non-tradable goods are produced by entrepreneurs as described in section 2.1, but their input $Z$ is now produced by a competitive sector using capital $K$, labor $L$, financial services $F$, and intermediate goods $X$. The production function of goods $Z$ is Cobb-Douglas:

$$Z_t = AX_t\eta[K_t^\alpha F_t^\epsilon L_t^{1-\alpha-\epsilon}]^{1-\eta}.$$  

Capital is rented from financial intermediaries, who also provide financial services. It depreciates at the rate $\delta$. The production of new capital goods and the repair of depreciated capital goods is carried out by a competitive sector of capital producers. Repairing depreciated capital only requires a corresponding amount of final goods. New capital goods $K_{t+1}$ are produced using the existing stock of capital and an amount $K_t - K_t$ of final goods. In addition, there is an installation cost (in final goods) $\Phi(K_{t+1}, K_t)$, where $\Phi$ is homogeneous of degree one. New capital goods produced in period $t$ can be rented to both sector $Z$ and capital producers from period $t + 1$ on, with respective rental rates $\rho_Z^t$ and $\rho_K^t$. Denote $q^t$ the price of capital goods in period $t$.

In addition to entrepreneurs and foreign lenders, the full closure introduces a representative household who provides labor and owns the capital stock. Within the household, there is a fraction

paths with a small enough $\pi > 0$ never converge to a steady state: instead, convergence is repeatedly interrupted by crises each time the economy is financially fragile and the sunspot is lower than $\pi$.  

17 When (10) does not hold, a shock on $\lambda$ has no effect if the constraints remain slack or triggers a real depreciation if they start binding, but not enough to provoke defaults in sector N.  

18 The simple closure corresponds to the case $\eta = 1$.  

16
of financial intermediaries and a fraction $1 - f$ of workers, with full consumption insurance between members. Workers supply labor to firms in sector $Z$ and return their wage $w_t$ to the household. Financial intermediaries accumulate capital which they rent to sector $Z$ and capital producers, issue debt, and supply a fixed amount of financial services $F$ to sector $Z$. At the beginning of each period, a fraction $1 - \gamma$ of financial intermediaries returns to the household to become new workers, bringing their share of the intermediary’s net wealth back to the household. They are replaced by members previously employed as workers. The household has Greenwood-Hercowitz-Huffman (GHH) preferences given by utility $U_t = E_t \sum_{s\geq 0} \beta^{t+s} u(c^H_{t+s} - h(L_{t+s}))$, where $c^H$ is the consumption of the household and $L$ the labor supplied by workers.

Households are subject to a borrowing constraint: they cannot issue debt. Thus, if $B_t$ denotes the bonds held by households at the end of period $t$ (paying a return $r_t B_t$ in $t + 1$), the constraint is

$$B_t \geq 0.$$  \hfill (12)

Financial intermediaries finance their capital stock with their internal funds $m^F_t$ and by issuing bonds. They are subject to a borrowing constraint. Investment expenditures (both the purchase of new capital goods and the repair of depreciated ones) are assumed to be limited by a multiple of their internal funds, similar to the constraint (1) for entrepreneurs:

$$q_t(K_{t+1} - K_t) + \delta P_t K_t \leq \lambda m^F_t.$$  \hfill (13)

Finally, financial intermediaries have limited liabilities and cannot commit to repay more than the value of their capital stock $(q_t + \rho^Z_t + \rho^K_t - \delta P_t)K_t$. Whenever the value of the capital stock falls short of the promised debt repayment, they are forced to default and debt is renegotiated down to the value of capital. Thus, beginning-of-period wealth is given by

$$W^F_t = (1 - \zeta^F_t) \left[ (q_t + \rho^Z_t + \rho^K_t - \delta P_t)K_t - r^F_{t-1}(q_{t-1}K_{t-1} - m^F_{t-1}) \right]$$

where $\zeta^F_t$ is equal to 1 when financial intermediaries default and 0 otherwise and $r^F$ is the promised rate of return. Internal funds are given by $m^F_t = \gamma W^F_t + p^F_t F$, where $p^F_t$ is the price of financial services. Exiting intermediaries transfer $\Pi_t = (1 - \gamma) W^F_t$ back to the household.

**Equilibrium**

As in section 3, I first focus on perfect foresight no-crisis equilibrium paths where entrepreneurs and financial intermediaries face non-binding constraints and do not default.

Because the final good $Y$ is a Cobb-Douglas aggregate of tradable and non-tradable goods with

19 This structure is analogous to Gertler & Karadi (2010).
parameter $\mu$, the appropriate price index is again given by

$$P_t = (p_t)^\mu$$

(14)

and the market-clearing condition for non-tradable goods becomes

$$p_t(z_t^N)^\theta = \mu p_t^H Y_t.$$  

(15)

The model is closed by the market-clearing conditions for inputs (16) and final goods (17):

$$AX_t^\eta[K_t^\alpha F_t^\epsilon L_t^{1-\alpha-\epsilon}]^{1-\eta} = z_t^N + z_t^T,$$  

(16)

$$Y_t = 1 - \gamma m_t^N + \gamma m_t^T + \frac{c_t^H}{P_t} + X_t + K_{t+1} - (1 - \delta)K_t + \Phi(K_{t+1}, K_t).$$  

(17)

The supply of final goods is equal to the sum of consumption by exiting entrepreneurs—the first term in equation (17)—and by households, intermediate goods, investment, and installation costs.

Given a deterministic exogenous path for the riskless gross interest rate $\{r_t\}_{t \geq 0}$, a no-crisis equilibrium path is then a sequence of allocations $\{z_t^N, z_t^T, Y_t, K_t, L_t, X_t, c_t^H, B_t, m_t^N, m_t^T, m_t^F\}_{t \geq 0}$ and a sequence of prices $\{p_t, p_t^Z, q_t, \rho_t^Z, \rho_t^K, p_t^F, w_t\}_{t \geq 0}$ that solve the optimization problems of all agents and firms under perfect foresight, satisfy market clearing conditions (15), (16), and (17), such that $\zeta_t = \zeta_t^F = 0$ and borrowing constraints for entrepreneurs and financial intermediaries do not bind. Compared to Definition 1, a no-crisis equilibrium path in the full closure now includes the evolution of final good production $Y_t$, capital $K_t$, labor $L_t$, intermediate goods $X_t$, household bond holdings $B_t$, prices of inputs $p_t^Z$ and capital $q_t$, rental rates of capital $\rho_t^Z$ and $\rho_t^K$, and wages $w_t$.

I now briefly discuss the behavior of households and financial intermediaries. The full model is exposed in Appendix A.7. Households choose their labor supply $L_t$ and bond holdings $B_t$ to maximize their intertemporal utility under the borrowing constraint (12) and the budget constraint: $P_t c_t^H + \tau B_t = r_{t-1} B_{t-1} + w_t L_t + \Pi_t$. The two first-order conditions are

$$h(L_t) = \frac{w_t}{P_t},$$  

(18)

$$\tau \Lambda_t^H = \beta E_t r_t \frac{P_t}{P_{t+1}} \Lambda_{t+1}^H + \Gamma_t^H,$$  

(19)

where $\Lambda_t^H = u'(c_t^H - h(L_t))$ is the marginal utility of consumption and $\Gamma_t^H/P_t$ is the Lagrange multiplier associated to the borrowing constraint (12). Since $\beta r_t = \tau_t^H$ and $\tau > \tau_t^*$, this multiplier is strictly positive in the vicinity of any steady state and the borrowing constraint (12) binds. Because domestic finance is less efficient than foreign finance, households do not extend any loans to domestic entrepreneurs and financial intermediaries. They simply consume their entire income,
consisting of both wages and profits repatriated from financial intermediaries.\textsuperscript{20}

The value function of financial intermediaries is

\begin{equation}
V^F_t \left( \frac{m^F}{T^F}, K_t \right) = \max_{K_{t+1}} \mathbb{E}_t \beta \frac{A^F_{t+1}}{A^F_t} \left[ (1 - \gamma) \frac{W^F_{t+1}}{T^F_{t+1}} + V^F_{t+1} \left( \frac{\gamma W^F_{t+1} + p^F_{t+1}}{P^F_{t+1}}, K_{t+1} \right) \right].
\end{equation}

Their first-order condition is very similar to the one of entrepreneurs:

\begin{equation}
E_t \beta \frac{A^H_{t+1}}{A^H_t} \left[ (1 - \zeta_{t+1}) \left[ q_{t+1} + \rho^Z_{t+1} + \rho^K_{t+1} - \delta P_{t+1} - \tau^F_{t+1} q_t \right] \frac{A^F_{t+1}}{T^F_{t+1}} + \Gamma^F_{t+1} (q_{t+1} - \delta P_{t+1}) \right] = \Gamma^F_{t+1} \frac{q_t}{T^F_t}. \tag{20}
\end{equation}

where $\Lambda^F$ is a state-contingent price similar to $\Lambda^N$—see equation (5). Under the assumption of perfect foresight, non-binding borrowing constraints and no crisis, this condition simplifies to the familiar asset-pricing equation

\begin{equation}
E_t [q_{t+1} + \rho^Z_{t+1} + \rho^K_{t+1} - \delta P_{t+1}] = r_t q_t. \tag{21}
\end{equation}

Financial fragility in this extended model is defined as the existence of a second market-clearing price such that both financial intermediaries and entrepreneurs of sector N default and all borrowing constraints are binding. As in the simple model, financial fragility takes place when the ratio $r_{t-1} (p^Z_{t-1} z_t^N - m_t^N) / m_t^T$ exceeds some threshold. See Appendix A.7 for a formal derivation.

To compare the output of the model with macroeconomic data, I define sectoral GDP as the (vertically integrated) value of production net of the consumption of intermediate goods, with inventories valued at current prices:

\begin{equation}
\text{GDP}_t^N = p_t (z_t^N)^\theta + p_t^Z (z_{t+1}^N - z_t^N) - \eta p_t^Z z_{t+1}^N
\end{equation}

and similarly for sector T.\textsuperscript{22} Consistent with national accounts, real GDP growth from one period to the next is computed by keeping prices constant between the two periods.

**Discussion of the extended set-up**

A crisis in this model requires a low demand for non-tradable goods. In the simple model, the only source of demand was the purchase of intermediate inputs by entrepreneurs which was limited by a borrowing constraint. The extended model assumes that consumption and investment, two

\textsuperscript{20} However, the remaining part of those profits is reinvested as equity to finance part of the domestic capital stock.

\textsuperscript{21} As for entrepreneurs, financial intermediaries do not wish to hold riskless bonds if defaults are rare enough events and the iceberg cost $\tau$ is large enough.

\textsuperscript{22} Sectoral GDP is equal to the sales of N or T goods plus the change in inventories of Z minus the intermediate goods X used in the production of Z. I have used the fact that the value spent on intermediate goods is equal to a share $\eta$ of the value of inputs Z produced.
additional sources of demand, are also subject to borrowing constraints. The borrowing constraint on investment is modeled by isolating the investment decision in a financial intermediary owned by the household, a la Gertler & Karadi (2010). The borrowing constraint on households sets an upper bound on their consumption; it also prevents this small open economy model from having a unit root in the evolution of the net foreign position.

As in the simple model, the borrowing constraint on investment has no impact on the no-crisis transition dynamics where, by definition, it does not bind. If anything, relaxing the borrowing constraint on consumption would reinforce the results: as financial openness increases, households would want to consume now part of their higher future income, appreciating the real exchange rate even more.

The iceberg cost on domestic finance \( \tau > \tau^* \) makes sure that the steady state autarky rate \( (\tau/\beta) \) is higher than the rate in the open economy \( (r_t = \tau^*_t/\beta) \). As a consequence, the economy is a net borrower.

With GHH preferences, labor supply does not react to changes in the level of consumption, making employment very procyclical. This specification is often used in models of small open emerging economies. Neumeyer & Perri (2005) show that GHH preferences are essential to get the negative correlation between GDP and the real interest rate that is present in the data.

Financial intermediaries are endowed with a supply \( F \) of financial services so that they have positive internal funds during a crisis.

4.2 Calibration

The model is calibrated to reproduce the Argentinean economy in the nineteen-nineties. This country implemented a reform package, including the opening of the capital account, between 1989 and 1991. Then, the economy experienced a decade of high growth (only interrupted by the “Tequila” crisis of 1995) until the recession of 1999 that culminated in a banking crisis, the abandon of the hard-peg, a default on (mainly external) Government debt, and a collapse of economic activity in 2001-2002. With the exception of the short “Tequila” shock, the period 1991-1998 then roughly corresponds to the model’s no-crisis dynamics that follows an increase in financial openness. Consistent with the prediction of the model, firms increased their leverage during that period: according to firm-level data, the average debt-to-equity ratio rises from about 50% to about 85% between 1992 and 1998.

In addition to the definition of the time period, the parameters of the model fall into three blocks: (i) parameters governing the size of the shock to financial openness, \( r_0 \) and \( r_\infty \), (ii) parameters governing the behavior of entrepreneurs, \( \gamma \) and \( \theta \), and (iii) parameters related to households and to the production of inputs, \( \mu, \eta, \delta, \epsilon, \varphi, \) and \( \psi \). I use macroeconomic, input-output, and firm-level

data to calibrate them (see Appendix A.1 for details on the data sources). Their values are reported in Table 1.

Table 1: Calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>6 months</td>
<td>firm-level working capital/sales</td>
</tr>
<tr>
<td>Final real interest rate</td>
<td>$r_\infty = 1.049$</td>
<td>1991-1998 avg real yield on Gov’t bonds</td>
</tr>
<tr>
<td>Initial real interest rate</td>
<td>$r_0 = 1.071$</td>
<td>firm-level debt/equity in 1992H1</td>
</tr>
<tr>
<td>Survival rate</td>
<td>$\gamma = .85$</td>
<td>firm-level share of retained earnings</td>
</tr>
<tr>
<td>Degree of decreasing returns</td>
<td>$\theta = .94$</td>
<td>firm-level debt/equity in 1998H2</td>
</tr>
<tr>
<td>Share of non-tradable goods</td>
<td>$\mu = .65$</td>
<td>input-output data</td>
</tr>
<tr>
<td>Share of intermediate goods</td>
<td>$\eta = .42$</td>
<td>input-output data</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha = .31$</td>
<td>capital/GDP and</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = .044$</td>
<td>investment/GDP</td>
</tr>
<tr>
<td>Share of financial services</td>
<td>$\epsilon = .04$</td>
<td>input-output data</td>
</tr>
</tbody>
</table>

The time period is calibrated from firm-level data. In the model, the time period determines the amount of working capital (the input Z) required by a given flow of production. In the data, the median firm has an average ratio of working capital to monthly sales equal to 5.9, where working capital is defined as the sum of inventories and short-term commercial credit. Accordingly, the time period is set to 6 months.

The size of the shock to financial openness and the parameters related to entrepreneurs are calibrated using firm-level data. The parameters $\theta$ and $\gamma$, which govern the dynamics of the entrepreneurial sector, are chosen to match the debt-to-equity ratio in 1998 and the empirical share of retained earnings. The size of the shock is then chosen to replicate the increase in the debt-to-equity ratio between 1992 and 1998.

More precisely, $r_\infty$ is set to 10% per year, which is the average value of the real yield of Government bonds from 1991 to 1998. Then, the probability of survival $\gamma$ is chosen to match the share of retained earnings in Argentinean firms. Bebczuk (2005), using data on 65 non financial Argentinean listed companies, reports an average dividend to cash-flow ratio equal to 15% for 1996-2000. Accordingly, I set $\gamma = .85$. The degree of decreasing returns $\theta$ is then set to .94 to match the 1998 debt-to-equity ratio of 85%. Finally, $r_0$ is set to 15% per year to match the 1992 debt-to-equity ratio of 50%. The resulting values for $\theta$ and $r_0$ are reasonable. Atkeson & Kehoe (2005) argue that a value of .95 is appropriate for the degree of decreasing returns at the plant level, based on a survey of the large existing empirical literature. The initial real rate $r_0$ roughly corresponds to the real yield of Government bonds in the mid nineteen-eighties, before the hyperinflation episode.

Bernanke et al. (1999) calibrate three parameters describing the behavior of entrepreneurs, including the survival rate, to match a debt-to-equity ratio, a bankruptcy rate, and an interest rate spread. Similarly, Carlstrom & Fuerst (1997) calibrate the survival rate and a second parameter to match a bankruptcy rate and an interest rate spread. Since borrowing constraints do not bind in the no-crisis dynamics, there is no interest rate spread in my model. I use the share of retained earning instead to directly calibrate $1 - \gamma$.  

24 Bernanke et al. (1999) calibrate three parameters describing the behavior of entrepreneurs, including the survival rate, to match a debt-to-equity ratio, a bankruptcy rate, and an interest rate spread. Similarly, Carlstrom & Fuerst (1997) calibrate the survival rate and a second parameter to match a bankruptcy rate and an interest rate spread. Since borrowing constraints do not bind in the no-crisis dynamics, there is no interest rate spread in my model. I use the share of retained earning instead to directly calibrate $1 - \gamma$.  

21
of 1989.

Another important parameter is the share of non-tradable goods $\mu$ in consumption, investment, and intermediate goods. According to input-output data, it is equal to 62%, but the true share could be even higher. Burstein, Eichenbaum & Rebelo (2005) argue that some goods traditionally classified as tradables are in fact local goods sheltered from foreign competition. They suggest that local goods could represent up to 22% of tradable consumption goods in Argentina. Applying this correction to my estimate would raise the share of non-tradable goods to 70%. However, investment and intermediate goods are likely to have a lower share of local goods than consumption goods. Therefore, I choose a conservative share $\mu$ of 65%.

The rest of the calibration is standard. The share of intermediate goods $\eta$ in the production of the input is chosen to match the empirical share of intermediate goods in total production. This share is equal to 42% in input-output data, but Jones (2010) recommends using an effective share of 38% in Argentina to account for sectoral heterogeneity. I follow this advice. The capital share $\alpha$ and the depreciation rate $\delta$ are chosen to match the average capital-to-GDP and investment-to-GDP ratios for the period 1991-1998, respectively equal to 1.38 and 12.1% $^{25}$ The share of financial services $\epsilon$ is calibrated from input-output data to match a share of inputs from finance and real estate in production equal to 3.6%.

Finally, I choose the following functional forms for the disutility of labor and the installation cost of capital: $h(L) = \frac{\psi}{1+\psi}L^{\frac{1}{1+\psi}}$ and $\Phi(K_{t+1}, K_t) = \frac{1}{2}\varphi K_t (\frac{K_{t+1}}{K_t} - 1)^2$. The value of the parameter $\psi$, the Frish elasticity of labor supply, is uncertain. In several calibrated models of emerging economies, it is assumed equal to 1 (Mendoza 2002, Mendoza & Smith 2006), but a higher value $\psi = 1.67$ is used in several other works (Mendoza 1991, Neumeyer & Perri 2005, Garcia-Cicco et al. 2010) and Chetty (2009) recommends a lower value $\psi = 0.5$ to reconcile micro and macro estimates. I take $\psi = 1$ as a baseline but also report simulations for $\psi = 0.5$ and 2. As regards installation costs, I choose $\varphi = 1$ in the baseline as in Gilchrist, Sim & Zakrajsek (2010) but also consider $\varphi = 0.5$ and 2 given the uncertainty around the true value of this parameter.

### 4.3 Results

**Dynamics under the full closure**

I run the same simulation as in section $3.2$. At $t = 0$, the economy is in the initial steady state corresponding to $\tau^* = \tau_0^*$ and $r = r_0$. At $t = 1$, $\tau^*$ (or $\tau^*$) unexpectedly and permanently jumps to $\tau^*$ (or $\tau^*$). The resulting dynamics is displayed in Figure under both the simple and the full closure.

Two results stand out. First, the mechanism studied under the simple closure carries over to the more realistic closure. As can be seen from Figure the dynamics is qualitatively similar under both closures.

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$^{25}$Indeed, they have a higher share of imported goods.

$^{26}$Capital corresponds to the productive capital of firms and excludes residential real estate.
Second, the initial response of the model is actually stronger under than the full closure: general equilibrium effects tend to amplify the partial equilibrium results of the simple model.

The reason is that the demand for N goods increases more at impact under the full closure, since it also consists of investment and consumption. Investment increases because of the lower riskless rate of return and consumption increases because both wages and profits increase. As a result, the initial appreciation of the real exchange rate is larger and the relative size of sector N, measured by both production and internal funds, displays a stronger overshooting.

In the long run, however, the production of N and T goods is lower under the full closure. This comes from the fact that the input Z is now partly produced by labor, which does not increase as much as capital and intermediate goods.

![Graph](image)

**Figure 4**: No-crisis equilibrium depending on the model closure. Solid line: full closure; dashed line: simple closure. Financial openness permanently increases at \( t = 1 \). Variables are expressed as a percentage deviation from the initial steady state. The current account-to-GDP is expressed as an absolute deviation from the initial steady state.

**Sensitivity analysis**

Figure 5 explores the sensitivity of the dynamics to the key parameters. The dynamics depends on parameters through their effect on the steady state or on the slope of the saddle path. The share of non-tradable goods \( \mu \) seems to mainly affect the latter. A given higher future demand for final goods increases the demand for N goods in all preceding periods through purchases of intermediate goods. The higher \( \mu \), the larger this multiplier effect. For a higher \( \mu \), the resulting dynamics displays a stronger reaction of the relative price \( p \) and the relative size \( z^N / z^T \), and a weaker increase in the scale of production \( z^N \) because of more expensive inputs. On the contrary,

27 The increase in consumption is gradual because of the borrowing constraint but expectation of a future higher consumption also feeds in current prices through larger input purchases by entrepreneurs.

28 The current account-to-GDP ratio is much larger in modulus under the simple closure since GDP is much lower in the absence of capital and labor.
a higher degree of increasing returns $\theta$ mainly affects the dynamics through its effect on the steady state. The optimal scale of production reacts more to the rate of return when $\theta$ is higher—see equation (8a')—which increases the demand for N goods and leads to a larger relative size $z^N/z^T$ in the long run. Because the scale of production is more sensitive to the expected return, this higher relative size requires a lower price increase. A higher Frisch elasticity of labor supply $\psi$ allows to produce more goods in the steady state, resulting in a higher demand for N goods in the short run. While it increases the relative size $z^N/z^T$, the effect on the relative price is dampened because of a lower rise in wages. A higher installation cost of capital $\phi$ mainly affects the scale of production $z^N$ during the transition, by slowing down the increase in the capital stock.

As regards financial variables, the hump-shaped reaction of the relative internal funds $m^N/m^T$ directly depends on how much the relative scale of production $z^N/z^T$ overshoots. It is also very sensitive to the probability of survival $\gamma$, a parameter which hardly affects the dynamics of real variables. Indeed, with a lower value of $\gamma$, internal funds are more sensitive to the dynamics of the relative price $p$ than to their own past values (see the log-linearization in Appendix A.3). The extent of overshooting in $m^N/m^T$ directly translates to the financial fragility ratio $r_{t-1}(p_{t-1} z^N - m_{t-1}^N)/m^T$. While it overshoots in the baseline calibration, this is not always the case. Interestingly, with a low enough share $\mu$ of N goods, which would correspond to a more commercially integrated economy, this ratio does not overshoot any more.

5 Reproducing the experience of Argentina

In this section, I confront the model with the data and check how well I can fit the actual experience of Argentina during the nineteen-nineties. The aim is not to reproduce every aspect of the Argentinean economy but to see if the important ingredients of the model are in line with the data. I compare the model and the data along three dimensions: (i) sectoral dynamics before the crisis, (ii) the possibility of a crisis, (iii) the evolution of key macroeconomic variables during the crisis.

Sectoral dynamics before the crisis

The main prediction of the model is that the relative size of the non-tradable sector moves with capital inflows, increasing when domestic agents borrow more abroad, and decreasing later when they pay their debt back. As a result, the N-to-T ratio follows the same evolution as the current account deficit (see Figure 4). As discussed in the introduction, both the current account deficit and the N-to-T ratio increased in Argentina during the years 1990 until the crisis occurred (Figure 11). I check whether the model can quantitatively reproduce the broad evolution of these two variables from 1991 to the crisis.

To do this, I run a simulation where the time profile of the current account deficit roughly matches the data. Then, I compute the change in both variables between their initial and their
Figure 5: Sensitivity analysis. No-crisis equilibrium under the full closure after a permanent increase in financial openness at $t = 1$. 
peak values and compare it with the observed data. As shown in Figure 4, following a permanent increase in financial openness, the current account deficit peaks after two periods and goes back to zero as entrepreneurs stabilize their debt to its new steady state value. In the data, the current account deficit gradually increased from 1991 to 1998 (with the exception of the Tequila shock of 1995). To broadly reproduce the time profile of the data, I assume that the transition from the initial to the final steady state goes through a sequence of small permanent shocks.

More precisely, assume the iceberg cost follows a random walk \( \tau_t^* = \tau_{t-1}^* + \varepsilon_t \), with \( E_{t-1} \varepsilon_t = 0 \). To obtain a gradual decrease in the current account, I feed the model with shocks \( \{\varepsilon_t\}_{t\in[1,16]} \) that gradually move the iceberg cost from \( \tau_0^* \) to \( \tau_\infty^* \) over 16 periods corresponding to 1991H1–1998H2. As I am only interested in the change over the whole decade, and not the detailed year-over-year changes, I simply consider shocks of equal magnitudes \( |\varepsilon| \), with \( |\varepsilon| = (\tau_0^* - \tau_\infty^*)/16 \). The total size of the shock \(|16\varepsilon|\) comes from the calibration of the initial and final steady states described in section 4.2.

Table 2 reports the observed and the simulated changes in the current account deficit and the N-to-T ratio between their initial value (the end of 1990) and their peak value. The baseline simulation closely reproduces the data for both variables. This is remarkable since, even though the timing of the shock was chosen to get a gradual evolution of the current account as in the data, the size of the shock itself was not calibrated using macroeconomic data but to match firm-level data. Results are only slightly sensitive to the Frisch elasticity \( \psi \) and the installation cost parameter \( \varphi \). This first exercise suggests that the mechanism of sectoral dynamics studied in the model accounts well for the experience of Argentina.

Table 2: Observed and simulated evolution of capital flows and of the sectoral structure during a capital account liberalization (variation between initial and peak value).

<table>
<thead>
<tr>
<th>Observed data</th>
<th>Model simulations</th>
<th>Baseline model</th>
<th>Frisch elasticity</th>
<th>Installation cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA deficit</td>
<td></td>
<td></td>
<td>( \psi = .5 )</td>
<td>( \varphi = .5 )</td>
</tr>
<tr>
<td>(absolute change)</td>
<td>8.1</td>
<td>8.3</td>
<td>7.6</td>
<td>8.6</td>
</tr>
<tr>
<td>N-to-T ratio</td>
<td></td>
<td></td>
<td>( \psi = 2 )</td>
<td>( \varphi = 2 )</td>
</tr>
<tr>
<td>(% change)</td>
<td>17.8</td>
<td>17.5</td>
<td>16.3</td>
<td>19.5</td>
</tr>
</tbody>
</table>

CA deficit: current account deficit over GDP in percentage points. N-to-T ratio: ratio of real GDP in the non-tradable sector to real GDP in the tradable sector. The capital account liberalization begins in 1991. In the data, peak values are reached in 1998 for the current account deficit and in 2001 for the N-to-T ratio. In the simulation, financial openness gradually increases over 16 half-years and peak values are reached in period 16.

\(^{29}\)By doing so, I abstract from short run fluctuations, notably the Tequila shock of 1995, and focus on the trend over the whole decade.
Financial fragility

I now examine whether and when the economy becomes financially fragile in the previous simulation of a gradual increase in financial openness. As in the simple model, financial fragility occurs when the ratio \( r_{t-1}(p_{t-1}z_t^N - m_{t-1}^N)/m_t^T \) exceeds a threshold that depends (approximately) linearly on the financial multiplier \( \lambda \) (see Appendix A.7).

Table 3 reports periods of financial fragility depending on the value of the financial multiplier \( \lambda \) for different simulations. A stronger borrowing constraint (a lower \( \lambda \)) makes financial fragility more likely. In the baseline simulation, financial fragility requires \( \lambda \lesssim 2.20 \). With \( \lambda = 2.10 \), the economy displays financial fragility in the final steady state for all simulations. For \( \lambda = 2.20 \), the baseline simulation displays financial fragility from 2000H2 to 2004H1, a period which includes the actual date of the Argentinean collapse. This is a true result since the shocks fed into the model only use data going up to 1998.

These values of \( \lambda \) are in the range usually found in the literature. They are also consistent with the data: according to firm-level data, the debt-to-equity ratio did not increase much above 100% during the crisis, consistent with \( \lambda \) around 2.

<table>
<thead>
<tr>
<th>Financial multiplier ( \lambda )</th>
<th>2.10</th>
<th>2.15</th>
<th>2.20</th>
<th>2.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi = .5 )</td>
<td>2000H1–∞</td>
<td>never</td>
<td>never</td>
<td>never</td>
</tr>
<tr>
<td>( \varphi = .5 )</td>
<td>1999H1–∞</td>
<td>1999H1–2006H1</td>
<td>2000H2–2001H1</td>
<td>never</td>
</tr>
<tr>
<td>( \varphi = 2 )</td>
<td>1999H1–∞</td>
<td>2000H1–2012H1</td>
<td>2001H1–2006H2</td>
<td>never</td>
</tr>
</tbody>
</table>

How much does sectoral reallocation contribute to financial fragility? When \( \lambda = 2.20 \), the financial fragility ratio has to increase by at least 57 log points from the initial steady state to enter the fragility zone. At the time of entry in the zone, this 57 log points increase can be decomposed into a 40 log points increase in leverage \( r_{t-1}(p_{t-1}z_t^N - m_{t-1}^N)/m_t^T \) and a 17 log points increase in the ratio of internal funds \( m_t^N/m_t^T \). As the increase in leverage goes only up to 51 log points (in the final steady state), the overshooting of \( m_t^N/m_t^T \) is crucial to get financial fragility in this case. For the sake of comparison, the required increase in the financial fragility ratio would be 69 log

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30 For instance, Gertler, Gilchrist & Natalucci (2007) set the constrained debt-to-equity ratio to 110%, corresponding to \( \lambda = 2.10 \). Mendoza (2002) considers constrained debt-to-current income ratios in the range 0.40–1.25, corresponding to \( \lambda \in [1.40, 2.25] \).

31 The average debt-to-equity ratio reaches 122% in 2001, shortly peaks at 238% in 2002H1, and falls back to 118% in the last quarter of 2002. The high ratio at the peak of the crisis comes from equity losses together with the fact that some of the debt is long term and renegotiation takes time, whereas in the model debt entirely consists of one-period bonds.

32 In the baseline simulation, the sectoral ratio \( m_t^N/m_t^T \) peaks at 19 log points above the initial steady state but goes back to 2 log points in the final steady state.
points with $\lambda = 2.50$ and 43 log points with $\lambda = 1.90$.

A stronger overshooting of the sectoral ratio of internal funds makes crises more likely during the transition. Consider for example the case of a high Frisch elasticity where there is a significant overshooting. Then, the economy becomes financially fragile during the transition for a borrowing constraint as weak as $\lambda \approx 2.30$ while financial fragility in the final steady state requires a stronger borrowing constraint ($\lambda \lesssim 2.10$).

### Crises

The aim of the model is to reproduce the pre-crisis dynamics more than the crisis itself. However, it is instructive to look at the evolution of the main variables predicted by the model during a crisis. Using the same simulation as before, I assume that a crisis takes place in 2001H2 as in the data. Table 4 reports the simulated changes in several key variables in the period of the crisis and compares them with the observed peak-to-trough changes in the data. To get comparable results across simulations, I set $\lambda = 2.10$ unless otherwise specified since all simulations display financial fragility in 2001H2 at this value. I also run a simulation where all parameters take their baseline value but $\lambda = 2.20$, the largest value consistent with financial fragility, for which the crisis is slightly less severe. The observed peak-to-trough changes are computed using half-yearly data on a window 1998H1-2002H2.

Table 4: Observed and simulated peak-to-trough evolution of key variables during the Argentinean crisis.

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$P$</th>
<th>$I$</th>
<th>GDP</th>
<th>NX</th>
<th>DD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed data</strong></td>
<td>-53</td>
<td>-47</td>
<td>-63</td>
<td>-11/20</td>
<td>-19</td>
<td>+10</td>
</tr>
<tr>
<td><strong>Model simulations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>baseline, $\lambda = 2.1$</td>
<td>-61</td>
<td>-46</td>
<td>-63</td>
<td>-11</td>
<td>-6</td>
<td>+33</td>
</tr>
<tr>
<td>baseline, $\lambda = 2.2$</td>
<td>-59</td>
<td>-44</td>
<td>-59</td>
<td>-9</td>
<td>-5</td>
<td>+31</td>
</tr>
<tr>
<td>$\psi = .5$</td>
<td>-60</td>
<td>-45</td>
<td>-62</td>
<td>-7</td>
<td>-4</td>
<td>+32</td>
</tr>
<tr>
<td>$\psi = 2$</td>
<td>-62</td>
<td>-47</td>
<td>-64</td>
<td>-14</td>
<td>-8</td>
<td>+34</td>
</tr>
<tr>
<td>$\varphi = .5$</td>
<td>-61</td>
<td>-46</td>
<td>-61</td>
<td>-11</td>
<td>-6</td>
<td>+33</td>
</tr>
<tr>
<td>$\varphi = 2$</td>
<td>-61</td>
<td>-46</td>
<td>-64</td>
<td>-10</td>
<td>-6</td>
<td>+33</td>
</tr>
</tbody>
</table>

I: real productive investment. NX: net export reversal as a share of the initial GDP. DD: domestic demand in tradable goods. All reported figures are % change, except the net export reversal which is an absolute change in percentage points. In the data, $p$ refers to the ratio of the implicit deflator of non-tradable GDP to the implicit deflator of tradable GDP and $P$ to the (trade-weighted) real effective exchange rate. DD is constructed by multiplying domestic demand in current prices by a nominal effective exchange rate index and deflating by the US core CPI. There are two figures for the employment data: the first one corresponds to the rate of employment; the second one corresponds to the rate of employment excluding under-employed workers.

Overall, the simulated magnitudes are close to the data and not very sensitive to changes in parameters. In particular, the model does a good job at producing the right order of magnitude

33When $\psi = 2$, the sectoral ratio $m_t^N/m_t^T$ peaks at 21 log points above the initial steady state.
for the real depreciation. If anything, the drop in the relative price \( p \) is slightly too large compared to the data. However, the simulated decrease in the relative price of final goods \( P \) is close to the observed depreciation of the real effective exchange rate. The reason why \( P \) reacts less to a given change in \( p \) in the model than in the data is the absence of terms-of-trade effects in the model. While there is a single tradable good in the model, the terms of trade in the data decrease by 9% between 1998 and 2002.

With a binding borrowing constraint for financial intermediaries, investment in the model drops sharply during the crisis, by around 60%, consistent with the data. The magnitude of this simulated decrease in investment depends on the strength of the borrowing constraint \( (\lambda) \) and the size of internal funds of intermediaries, driven by the share \( \epsilon \) of financial services. Both parameters were calibrated independently of the observed decrease in investment.

Quite remarkably, the baseline simulation successfully predicts the 11% drop in employment observed in the data. This figure however probably underestimate the decline in hours, as underemployment increased a lot during the crisis. The number of employed workers, excluding the underemployed, decreased by 20%, suggesting that the decrease in the number of hours should be somewhere between 10 and 20%. The model needs a higher value for the Frisch elasticity \( (\psi = 2) \) to produce a 14% drop in employment, more in line with that fact. The mechanism behind the fall in employment is the following. With binding constraints, the demand for inputs drops during the crisis, inducing a lower wage bill in sector \( Z \). For a given supply of labor, the wage \( w \) decreases. The price of the final good, \( P \), also decreases during a crisis, but to a lesser extent since it partly consists of tradable goods. As a result, the real wage \( w/P \) decreases, which induces households to reduce their labor supply. GHH preferences are crucial to get this result.

Given the decrease in employment, the baseline simulation predicts a 6% drop in GDP and the simulation with a high Frisch elasticity of labor, an 8% drop. While this is already significant for an endogenous decrease driven by a self-fulfilling crisis, it still falls short of the 19% collapse observed in the data. The difference comes from the decrease in measured total factor productivity (TFP) which the model was not designed to produce. A plausible explanation for the decline in measured TFP in the data is the banking crisis that took place at the end of 2001 and that very likely disrupted production processes. In the model, on the contrary, the massive defaults of financial intermediaries has no effect on production. Assume on the contrary that financial intermediaries, when they default, can only rent a fraction \( u \) of their capital to sector \( Z \) because of the renegotiation process. This would result in a lower rate of capacity utilization leading to lower measured TFP. In the data, the rate of capacity utilization at the beginning of 2002 was 65% of the average rate over the period 2005–2010. With \( u = .65 \), the model predicts a 15% (17%) GDP decline with \( \psi = 1 \) (\( \psi = 2 \)). Such a mechanism would be enough to reproduce the data.

The model does not do a good job at matching the reversal in net exports. It produces a reversal

\[ \text{34In models with costly state verification, monitoring is assumed to destroy some of the assets.} \]
that amounts to about 30% of the initial GDP, compared to 10% in the data. The difference can be explained by the absence of terms-of-trade effects in the model and by the fact that GDP declines less than in the data as discussed above. As a result, GDP in foreign currency decreases much less in the model than in the data, which for a given change in domestic demand implies a larger net export reversal.\footnote{The net export reversal is equal to \( \Delta \text{NX}/\text{GDP} = \Delta(\text{GDP} - \text{DD})/\text{GDP} = \Delta\text{GDP}/\text{GDP} - (\text{DD}/\text{GDP}) \times \Delta\text{DD}/\text{DD}, \) where GDP and DD are measured in tradable goods.} One way to correct for this is to consider the change in domestic demand measured in tradable goods instead of the net export reversal. The model reproduces the former much better (see the last column of Table 4).

On the whole, these results show how a model with multiple equilibria can successfully reproduce a crisis of a very large magnitude.

6 Final remarks

This paper has built a two-sector model of financial fragility in a small open economy. The model was used to study the effect of an increase in financial openness on the sectoral structure of the economy and the financial structure of its non-tradable sector. In the short to medium run, larger capital inflows lead to a higher relative size of the non-tradable sector. This is in line with the observed behavior of the sectoral structure following a capital account liberalization. At the same time, access to cheaper foreign loans leads firms to increase their leverage. The evolution of these two factors tends to make crises more likely after an increase in financial openness.

The paper sheds some light on the timing of the effects of capital account liberalization. The increase in the relative size of the non-tradable sector is essentially a short to medium run phenomenon, which makes financial fragility more likely to be a concern in the medium run than in the longer run. This result is consistent with the empirical evidence reported by Kaminsky & Schmukler (2003). They argue that the large amplitude of boom-bust cycles in the stock market following financial liberalization is a transitory phenomenon and disappears in the long run. In the model however, when the world interest rate is low enough and the economy opens sufficiently, financial fragility can persist even after the sectoral reallocation has taken place. This could help to explain why in some particular instances, like Argentina in 2001, a crisis can take place a whole decade after the capital account was liberalized.

On the methodological front, the paper shows that a model with multiple equilibria can be a useful tool to quantitatively match the behavior of macroeconomic variables during a large crisis such as Argentina in 2001.

In the model, crises are triggered by self-fulfilling purely expectational shocks but they could also be triggered by exogenous shocks on fundamentals which make the normal-time market clearing price disappear, e.g. a sudden stop that tightens borrowing constraints.\footnote{Calvo et al. (2004) argue that the sudden stop which followed the Russian crisis of 1998 led to episodes of large} This could explain why...
small shocks on fundamentals can have very large effects. It could also explain why two economies can react very differently to the same external shock: a financially fragile economy can jump on the crisis equilibrium, while other economies remain in the normal-time equilibrium, simply experiencing a slight real depreciation and a low decrease of investment. This is fully consistent with the way Argentina and Chile reacted to the 1998 sudden stop, as reported by Calvo & Talvi (2005): the Argentine economy collapsed while Chile went through a mild recession.

As regards policy issues, financial fragility depends on two factors: (a) how large foreign currency liabilities are compared to domestic currency assets in firm balance sheets and (b) how large the non-tradable sector is compared to the tradable sector. While paying attention to mismatches in firm balance sheets is a lesson that is now widely agreed on, this paper suggests that monitoring the evolution of the sectoral structure is also important. If policy makers are trying to prevent balance-of-payments crises, some intervention might be justified to mitigate the sectoral effects of capital inflows. A first way to do it would be to implement macroeconomic policies aimed at decreasing (or not increasing) the size of the financial transfer from abroad (for example by increasing domestic savings or limiting the extent of financial integration). Alternatively, policy makers could resort to sectoral interventions directly aimed at protecting the tradable sector from the effect of the real appreciation. This provides another justification for protecting or promoting the tradable sector to the ones already identified by the literature (increasing returns to scale, sunk costs to enter export markets, financial frictions, etc.) and suggests that there might be some complementarity between financial integration and industrial policy.

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## A Appendix

**A.1 Data**

**Event study** The data covers 12 emerging economies during the period 1973–1999: 7 Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela) and 5 Asian countries (Indonesia, Korea, Malaysia, Philippines, Thailand).

The relative size of the non-tradable sector is constructed with data on sectoral value added in constant local currency units from the World Development Indicators. As the Colombian data from this source displays a strong discontinuity in 1994 (the price index was changed in this year), it was replaced by data from the Colombian Central Bank. The tradable sector is defined as agriculture and manufacturing. The non-tradable sector is defined as services and non-manufacturing industry.

The capital account is defined as liberalized when the *de jure* monthly index of Kaminsky & Schmukler (2003) indicates a “partially liberalized” or “fully liberalized” capital account. The starting date of a liberalization episode is chosen so that the capital account has been liberalized for at least six months. An episode ends if the capital account stops being liberalized for at least a year and a half.

Dates of banking crises are taken from Caprio & Klingebiel (2003). Dates of currency crises are constructed using monthly data from the International Financial Statistics. An exchange market pressure index is defined as the average of the rate of real depreciation (with respect to the US dollar) and the rate of foreign reserve depletion weighted by the inverse of their sample variances. A currency crisis occurs in a given month when this index exceeds the sample mean by two standard deviations. Finally, a twin crisis occurs when a currency crisis takes place in a one-year window around a banking crisis. Twin crises have to be separated by at least three years.
The estimated equation is

$$\log N/T_{it} = \sum_{s=0}^{6} \alpha_s LIB(s)_{it} + \beta_i + \gamma_i t + (\delta + \zeta OPEN_{it}) TWIN_{it} + \varepsilon_{it}$$

where \((i, t)\) denotes a country-year pair, \(OPEN_{it}\) is a dummy indicating a liberalized capital account, \(LIB(s)_{it}\) are equal to 1 when the capital account of country \(i\) has been liberalized for exactly \(s\) years, and 0 otherwise, and \(TWIN_{it}\) is a dummy indicating a twin crisis. The estimated coefficients \(\alpha_s\) are plotted on Figure 1a along with the 95% confidence interval.

**Argentinean data** Most macroeconomic data series come from Ministerio de Economía (MECON): GDP, net exports, employment, EMBI spread, terms-of-trade, rate of capacity utilization. Investment and the capital stock are taken from Maia & Nicholson (2001); they exclude residential real estate. The quarterly real yield of Government bonds is the sum of the rate of the US three-month Treasury bill (Datastream) and the EMBI spread, minus expected US inflation. Expected US inflation next quarter is computed as the change of the core CPI index (Bureau of Labor Statistics) over the four previous quarters. For the period prior to 1996, spreads are taken from Neumeyer & Perri (2005) who reconstruct them from bond data. The (trade-weighted) real effective exchange rate is computed by JP Morgan. To compute the net export reversal and the variation of domestic demand during the crisis, net exports and domestic demand are converted into foreign currency using a (trade-weighted) nominal effective exchange rate also from JP Morgan.

The N-to-T ratio uses annual data from the GGDC 10-Sector Database (Timmer & de Vries 2009). The tradable sector is defined as the sum of “manufacturing”, “agriculture, forestry and fishing”, and “mining and quarrying”; the non-tradable sector is the rest of the economy. The N-to-T ratio is detrended by removing a linear time trend (in log) corresponding to the nineteen-eighties.\(^{39}\) The relative price of non-tradables is computed from implicit sectoral price deflators using similar data, but with a quarterly frequency, from MECON.

Input-output data comes from the OECD Input-Output tables, based on ISIC Revision 3. A sector is defined as tradable if either exports or imports represent at least 10% of production.

The firm-level data was kindly provided by Paula Español. It consists in quarterly balance-sheet data for a panel of 74 firms listed on the Buenos Aires Stock Exchange between 1992 and 2001. See Arza & Español (2008) for details on the data. To compute the average debt-to-equity ratio in a given quarter, I exclude outliers (the top 5% and firms with negative equity).

\(^{39}\)The nineteen-nineties is excluded from the period on which the trend is computed since the theory predicts a long-lasting increase of the ratio over that period, which would create an upward bias in the measured trend.
A.2 Proof of proposition 1

From the first order conditions (8a') and (8b'), we get \( \theta p(z^N)^\theta = rp^\mu z^N \) and \( \theta(z^T)^\theta = rp^\mu z^T \). Plugging this into the market-clearing condition (8e) yields \( z^N = \frac{\theta \mu}{r}(z^N + z^T) \). The relative size of both sectors is then given by

\[
\frac{z^N}{z^T} = \frac{\theta \mu}{r} \left( 1 - \frac{\theta \mu}{1 - r} \right).
\]

For \( r > \theta \mu \), the right-hand size of this expression is strictly positive and the relative size is well defined in the steady state. The relative size of sector N is strictly decreasing in \( r \).

The steady state value of \( p \), \( z^T \), and \( z^N \) can be derived from the relative size \( \frac{z^N}{z^T} \). From the first order conditions (8a') and (8b'), we have \( p = \frac{(z^N + z^T)^{1-\theta}}{(1-n)\theta} \). Knowing \( p \), (8b') gives the value of \( z^T \). The value of \( z^N \) follows.

Then, from equations (8c') and (8d'), internal funds in the steady state are given by \( m^s = \frac{1}{\gamma - 1} \frac{1}{1 - \frac{\gamma}{\gamma - 1}} \frac{1}{1 - \theta} \). They are strictly positive provided that \( r < 1/\gamma \). Note that \( m^N/m^T = z^N/z^T \).

Debt is equal to \( p^\mu z^s - m^s = \frac{1}{\gamma} \frac{1}{\gamma - 1} \frac{1}{1 - \theta} \). Given the restriction \( r < 1/\gamma \), debt is strictly positive for \( r < \theta/\gamma \). The borrowing constraint (1) does not bind when \( p^\mu z^s < \lambda m^s \), i.e. when \( r > \frac{\theta}{\gamma} \frac{1}{\theta + \lambda(1-\theta)} \).

To sum it up, the steady state exists and features strictly positive debts and non-binding borrowing constraints when \( \max(\frac{\theta}{\gamma} \frac{1}{\theta + \lambda(1-\theta)}, \theta \mu) < r < \frac{\theta}{\gamma} \) and is strictly decreasing in \( r \).

A.3 Proof of proposition 2

Under the assumption of perfect foresight and slack borrowing constraints, the dynamics of the model can be reduced to a simple two-dimensional system. First, since the constraints do not bind, balance-sheet variables \( m^N_t \) and \( m^T_t \) do not matter. Second, from (8b), inputs purchased by the tradable sector \( z^T_t \) are completely determined by the current level of the real exchange rate and the real interest rate. Therefore, the no-crisis transitory dynamics can be fully described by the evolution of \( z^N_t \) and \( p_t \). These two variables form a perfect-foresight two-dimensional system \((z^N_t, p_t)\) that satisfies:

\[
p_{t+1} = \frac{1}{\theta} p_t [p_t^\mu (z^N_{t+1})]^{1-\theta}, \tag{22a}
\]

\[
z^N_{t+1} = \frac{1}{\mu} p_t^{1-\mu} (z^N_t)^{1-\theta} - \left[ \frac{\theta}{[p_t^{\mu} r_t]} \right]^{1-\theta}. \tag{22b}
\]
Equation (22a) simply restates (8a'). Equation (22b) follows from (8e) and (8b').

Denote \( \hat{x}_t \equiv \log\left(\frac{x_t}{\bar{x}}\right) \) the log-difference between the variable \( x_t \) and its value \( \bar{x} \) in a particular steady state corresponding to \( r = \bar{r} \). From appendix A.2 we know that \( \frac{z^N}{(z^N + z^T)} = \theta \mu / r \) in the steady state. Using this expression, we can log-linearize equations (22) to get

\[
\begin{align*}
\hat{p}_{t+1} &= \bar{r} + \mu \hat{p}_t + (1 - \theta) \hat{z}_{t+1}^N, \\
\hat{z}_{t+1}^N &= \left[\frac{\bar{r}}{\mu}[1 - \theta(1 - \mu)] - \mu\right] \frac{\hat{p}_t}{1 - \theta} + \frac{\bar{r}}{\mu} \hat{z}_t^N + \left(\frac{\bar{r}}{\mu} - 1\right) \frac{\hat{r}_t}{1 - \theta}.
\end{align*}
\]

Injecting (23b) into (23a) to eliminate \( z_{t+1}^N \) yields the usual Blanchard-Kahn form:

\[
\begin{align*}
\hat{p}_{t+1} &= \frac{\bar{r}}{\mu}[1 - \theta(1 - \mu)] \hat{p}_t + \frac{\bar{r}}{\mu} (1 - \theta) \hat{z}_t^N + \frac{\bar{r}}{\mu} \hat{r}_t, \\
\hat{z}_{t+1}^N &= \left[\frac{\bar{r}}{\mu}[1 - \theta(1 - \mu)] - \mu\right] \frac{\hat{p}_t}{1 - \theta} + \frac{\bar{r}}{\mu} \hat{z}_t^N + \left(\frac{\bar{r}}{\mu} - 1\right) \frac{\hat{r}_t}{1 - \theta},
\end{align*}
\]

where \( \hat{z}^N \) is predetermined while \( \hat{p} \) is a jump variable.

The slope of the loci along which \( \hat{p}_t = \hat{p}_{t+1} \) and \( \hat{z}_t^N = \hat{z}_{t+1}^N \) are given by:

\[
\begin{align*}
\frac{d\hat{p}}{d\hat{z}^N}_{|\hat{p}_{t+1}=\hat{p}_t} &= -\frac{\bar{r}}{\mu}[1 - \theta(1 - \mu)] - 1, \\
\frac{d\hat{p}}{d\hat{z}^N}_{|\hat{z}_{t+1}=\hat{z}_t} &= -\frac{(\bar{r} - 1)(1 - \theta)}{\mu}[1 - \theta(1 - \mu)] - \mu.
\end{align*}
\]

When \( \bar{r} > 1 \), both slopes are strictly negative and the slope of the \( \hat{p}_t = \hat{p}_{t+1} \) locus is steeper. This gives the phase diagram represented in Figure 2a.

To show that the steady state is a saddle-point, we can compute the eigenvalues of the system. The characteristic polynomial is

\[
P(X) = X^2 - \frac{\bar{r}}{\mu}(1 + \theta \mu) X + \bar{r}.
\]

We have \( P(0) = \bar{r} > 0 \) and \( P(1) = 1 - \frac{\bar{r}}{\mu} < 0 \) when \( \bar{r} > 1 \). Thus, the two eigenvalues \( X_1 \) and \( X_2 \) satisfy \( 0 < X_1 < 1 < X_2 \). With one and only one eigenvalue inside the unit circle, the dynamics is saddle-path stable. With one jump variable (\( \hat{p} \)) and one pre-determined variable (\( \hat{z}^N \)), there is a unique path converging to the steady state.

It is easy to show that a decrease in \( \bar{r} \) moves the new steady state to the north-east, as illustrated in Figure 2b. The resulting dynamics proves the first and second points of the proposition.

The dynamics of \( z^N / z^T \) follows from equations (8a) and (8b):

\[
\frac{z^N_{t+1}}{z^T_{t+1}} = \frac{1}{p_{t+1}}.
\]
Starting in period $t = 2$, the ratio $z^N / z^T$ follows the same dynamics as the relative price $p_t$, which proves the third point.

Finally, we can log-linearize the evolution of internal funds (8c') and (8d') to get

\[
\hat{m}_N^t = \gamma \bar{r} \left( \hat{m}_N^{t-1} + \hat{r}_{t-1} \right) + (1 - \gamma \bar{r}) \frac{-\theta}{1 - \theta} \left( \mu \hat{p}_t - (\mu \hat{p}_{t-1} + \hat{r}_{t-1}) \right),
\]

\[
\hat{m}_T^t = \gamma \bar{r} \left( \hat{m}_T^{t-1} + \hat{r}_{t-1} \right) + (1 - \gamma \bar{r}) \frac{-\theta}{1 - \theta} \left( \mu \hat{p}_t - (\mu \hat{p}_{t-1} + \hat{r}_{t-1}) \right).
\]

In the log-linear approximation, the ratio $m_N^t / m_T^t$ evolves according to

\[
\hat{m}_N^t - \hat{m}_T^t = \gamma \bar{r} \left( \hat{m}_N^{t-1} - \hat{m}_T^{t-1} \right) + (1 - \gamma \bar{r}) \frac{\hat{p}_t}{1 - \theta}
\]

and is an exponential smoothing of the relative price $p_t$. Given the dynamics followed by $p_t$, and unless the smoothing factor $\gamma \bar{r}$ is very close to either 0 or 1, the ratio $m_N^t / m_T^t$ has a hump-shaped response to the unexpected and permanent decrease in the domestic interest rate.

A.4 A model with mobile entrepreneurs

Consider the model in its simple version with perfect foresight and non-binding borrowing constraints and assume that entrepreneurs can freely choose their sector in each period. Let $\varphi_{t+1} \in [0, 2]$ the number of entrepreneurs starting production in sector $N$ in period $t$. Assume that the economy is in a steady state corresponding to $r = r_0$ at $t = 0$ and that the gross interest rate unexpectedly and permanently changes to $r_\infty$ at $t = 1$.

Free mobility implies that expected next period profits are the same in both sectors: $p_{t+1} (z^N_{t+1})^\theta - r_t p_t^\mu z^N_{t+1} = (z^T_{t+1})^\theta - r_t p_t^\mu z^T_{t+1}$. Together with the first-order conditions (8a') and (8b'), this implies that entrepreneurs purchase the same amount of inputs in both sectors and that the expected relative price of non-tradable goods is equal to 1:

\[
z^N_{t+1} = z^T_{t+1} = z_{t+1} = \left[ \frac{\theta}{r_t p_t^\mu} \right]^{1/\theta}, \quad \mathbb{E}_t [p_{t+1}] = 1.
\]

The market-clearing condition becomes

\[p_t \varphi_t (z_t)^\theta = 2 \mu p_t^\mu z_{t+1}.
\]

Therefore, for $t \geq 2$, $p_t = 1$, $z_{t+1} = \frac{\theta}{r_\infty}$, and $\varphi_{t+1} = \frac{2 \theta \mu}{r_\infty}$. The real exchange rate, the amount of inputs purchased, and the choice of sector reach their steady state value in period 2, i.e. one period after the shock hits. Note that the relative size of both sectors in the steady state, measured by the amount of inputs used, is the same as in the model with specialized entrepreneurs (see...
of the function $z$ where $r$ threshold under which sector N defaults.

However, like $\varphi$, it reaches its steady state value in the period following the shock.

Before the shock hits, $p_0$, $z_1$, and $\varphi_1$ take the steady state value corresponding to $r_0$. In the period of the shock, $p_1$, $z_2$, and $\varphi_2$ solve the market-clearing conditions in periods 1 and 2 and the first-order condition for $z$. Defining $\eta = \frac{\theta}{\mu + (1- \theta)(1- \rho)}$, we get $p_1 = \left(\frac{\theta}{r_\infty}\right)^\frac{1}{\mu + (1- \theta)(1- \rho)}$, $z_2^{1- \theta} = \left(\frac{\theta}{r_0}\right)\eta \left(\frac{\theta}{r_\infty}\right)^{1- \eta}$, and $\varphi_2 = 2\theta \left(\frac{r_0}{r_\infty}\right)^{\frac{1}{1- \theta}}$. When the shock is a decrease in the rate of return $(r_\infty < r_0)$, the relative price $p_1$ increases above its steady state value in the period of the shock. This slows down the increase in the scale of production, which takes two periods to reach its new steady state value. The relative size of sector N, $\varphi_2$, also overshoots to allow for a sufficient production of non-tradable goods in period 2 despite the small scale of production.

To sum it up, the model with free mobility of entrepreneurs also generates some overshooting of the real exchange rate and the sectoral structure when financial openness increases, but this overshooting only lasts for a single period.

### A.5 Proof of proposition 3

Consider a given time period $t$. The variables $z_t^N$, $W_t^T \equiv [(z_t^T)^\theta - r_{t-1}(p_{t-1}^\mu z_t^T - m_{t-1}^T)]$, $b_t^N \equiv r_{t-1}^N (p_{t-1}^\mu z_t^N - m_{t-1}^N)$, and $p_{t+1}$ are taken as given. A market-clearing real exchange rate $p_t$ is a zero of the function

$$f(p_t) = p_t (z_t^T)^\theta - \mu p_t^\mu [z_{t+1}^T(p_t) + z_{t+1}^N(p_t, p_{t+1})],$$

where $z_{t+1}^N(\cdot)$ and $z_{t+1}^T(\cdot)$ solve the entrepreneurs’ optimization program. Let $p^* = b_t^N/(z_t^N)^\theta$ be the threshold under which sector N defaults.

I first show that condition (11) implies the existence of a crisis-time market-clearing price $p_t^{\text{crisis}} < p^*$. The function $f$ is continuous and strictly increasing on the interval $[0, p^*)$, with $f(0) < 0$. The derivative of $f$ on $[0, p^*)$ is indeed given by

$$f'(p_t) = (z_t^N)^\theta - \mu \frac{\partial}{\partial p_t} p_t^\mu [z_{t+1}^T(p_t) + z_{t+1}^N(p_t, p_{t+1})]$$

$$\geq (z_t^N)^\theta > 0,$$

because $p_t^\mu (z_{t+1}^T + z_{t+1}^N)$ is either strictly decreasing with $p_t$ or constant on $[0, p^*)$ depending on whether the borrowing constraints bind. Besides, for $p_t = 0$, the borrowing constraints are binding and $f(0) = -\mu \lambda (m_t^T + m_t^N) < 0$. Therefore, $f$ has a unique zero on $(0, p^*)$ if $f(\tilde{p}) \geq 0$ for some $\tilde{p} \in (0, p^*)$.

Consider $\tilde{p} = \mu \gamma \lambda W_t^T / (z_t^N)^\theta$. If condition (10) is satisfied, then $\mu \gamma \lambda W_t^T < b_t^N$ so that $\tilde{p} < p^*$. For $0 < p_t < p^*$, the borrowing constraints imply that $f(p_t) \geq p_t (z_t^N)^\theta - \mu \lambda (m_t^N + m_t^T)$. From
equations (8c) and (8d), \( m_t^N + m_t^T = \gamma W_t^T \) since \( \zeta_t = 1 \) on \([0, p^*]\) by definition. Therefore \( f(\bar{p}) \geq 0 \) and \( f \) has a unique zero on \((0, \bar{p}]\). This zero is the crisis-time market-clearing price \( p_{t}^{\text{crisis}} \).

To show that \( \mu \gamma \lambda > 1 \) is a necessary condition for the existence of multiple market-clearing prices corresponding to normal- and crisis-times, I assume that \( \mu \gamma \lambda \leq 1 \) and show that \( f \) cannot have at the same time a zero on the interval \([0, p^*]\) and a zero on the interval \([p^*, +\infty)\). This follows from the fact that (i) \( f \) is strictly increasing on the interval \([0, p^*]\), (ii) \( f \) is increasing on the interval \([p^*, +\infty)\) when \( \mu \gamma \lambda \leq 1 \), and (iii) \( \lim_{p_t \to p^*} f(p_t) \geq \lim_{p_t \leq p^*} f(p_t) \). (i) was established above. I now prove (ii) and (iii).

(ii) \( f'(p_t) = (z_t^N)^\theta - \mu \frac{\partial}{\partial p_t} \mu z_{t+1}^T(p_t) + z_t^N(p_t, p_{t+1}) \)
\[ \geq (1 - \mu \gamma \lambda)(z_t^N)^\theta \geq 0. \]

Here, I have used the fact that \( p_t^\mu z_{t+1}^T \) is either strictly decreasing with \( p_t \) or constant (depending on whether the borrowing constraint binds), that \( \frac{\partial p_t^\mu z_t^N}{\partial p_t} \) is either strictly negative (when the borrowing constraint does not bind) or equal to \( \lambda \gamma (z_t^N)^\theta \) (when it does), and that \( \mu \gamma \lambda \leq 1 \).

(iii) If the borrowing constraint binds in sector \( T \) when \( p_t \leq p^* \), then
\[ \lim_{p_t \leq p^*} f(p_t) = b_t^N - \mu (p^*)^\mu z_{t+1}^T(p^*) \geq b_t^N - \mu \gamma \lambda W_t^T = \lim_{p_t \geq p^*} f(p_t). \]

If it does not,
\[ \lim_{p_t \geq p^*} f(p_t) = b_t^N - \mu (p^*)^\mu z_{t+1}^T(p^*) \geq b_t^N - \mu (p^*)^\mu z_{t+1}^T(p^*) - \mu \lambda S_t^N = \lim_{p_t \leq p^*} f(p_t). \]

Intuitively, the inequality \( \mu \gamma \lambda > 1 \) states that a within-period equilibrium where sector \( N \) faces a binding borrowing constraint but does not default would be unstable (in the sense of any virtual out-of-equilibrium dynamics corresponding to the walrasian auctioneer’s tatonnement), leaving two stable within-period equilibria: one where the borrowing constraint does not bind and one where sector \( N \) defaults.

A.6 Proof of proposition 4

A steady state is financially fragile when \( \frac{\gamma (p^N z_t^N - m_t^N)}{m_t^T} > \mu \lambda \). Denote \( g(r) \) the left-hand side of this inequality. It is continuous on \((\theta \mu, +\infty)\) and strictly decreasing. Indeed, using the steady state values computed in the proof of proposition 4 it reduces to
\[ g(r) = \frac{\theta \mu}{\gamma(1 - r)} \frac{1 - \frac{r}{T}}{r - \theta \mu}. \]
We have $g(\theta) = 0 < \lambda \mu$. To prove the existence and uniqueness of $r_{\text{frag}}$, it is enough to show that $g(r) > \lambda \mu$ when $r > \max(\frac{1}{\gamma \theta + \lambda (1-\theta)}, \theta \mu)$.

When $r > \theta \mu$, $g(r)$ diverges to $+\infty$. This takes care of the case $\theta \mu \geq \frac{1}{\gamma \theta + \lambda (1-\theta)}$. When $\theta \mu < \frac{1}{\gamma \theta + \lambda (1-\theta)}$, we have to show that $g(\frac{1}{\gamma \theta + \lambda (1-\theta)}) > \lambda \mu$. With basic algebra, it is easy to see that

$$g\left(\frac{1}{\gamma \theta + \lambda (1-\theta)}\right) = \frac{\theta \mu (\lambda - 1)}{1 - \mu \gamma \theta + \lambda (1-\theta)} > \lambda \mu \quad \text{if} \quad \mu \gamma \lambda - 1 > 0.$$

The last inequality is the necessary condition for financial fragility which is satisfied by assumption. The threshold $r_{\text{frag}}$ is then the unique root of $g(r) = \lambda \mu$ in $(\max(\frac{1}{\gamma \theta + \lambda (1-\theta)}, \theta \mu), \frac{1}{\gamma \theta + \lambda (1-\theta)})$.

When $\theta > \frac{1}{\lambda \theta (1+1/\gamma)}$, then $\frac{1}{\gamma \theta + \lambda (1-\theta)} > 1 > \theta \mu$ and $r_{\text{frag}} > 1$.

### A.7 The quantitative model

**Optimization problems** The optimization problems for entrepreneurs, households and financial intermediaries are solved in sections 2.2 and 4.1.

Capital producers maximize their profits $q_t(K_{t+1} - K_t) - P_t(K_{t+1} - K_t) - P_t\Phi(K_{t+1}, K_t) - \rho_t^K K_t$. Denote $\phi(K_{t+1}/K_t) = \Phi(K_{t+1}, K_t)/K_t$. The first-order conditions together with the Euler theorem give

$$q_t = P_t[1 + \phi'(K_{t+1}/K_t)], \quad (26)$$
$$\rho_t^K = P_t[(K_{t+1}/K_t - 1)\phi'(K_{t+1}/K_t) - \phi(K_{t+1}/K_t)]. \quad (27)$$

Producers of the input $Z$ maximize their profits $p_t^Z Z_t - \rho_t^Z K_t - w_t L_t - p_t^F F_t - P_t X_t$, yielding the following first-order conditions:

$$\rho_t^Z K_t = \alpha (1-\eta) p_t^Z Z_t, \quad (28)$$
$$w_t L_t = (1 - \alpha - \epsilon) (1-\eta) p_t^Z Z_t, \quad (29)$$
$$p_t^F F = \epsilon (1-\eta) p_t^Z Z_t, \quad (30)$$
$$P_t X_t = \eta p_t^Z Z_t \quad (31)$$

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No-crisis equilibrium path. Under the full closure, a no-crisis equilibrium path is described by (15)–(17), (18), (21), (26)–(31) and

\[ z_{t+1}^N = \left[ \frac{\theta}{p_t} E_t[p_{t+1}] \right]^{\frac{1}{\eta}}, \]  
\[ z_{t+1}^T = \left[ \frac{\theta}{p_t} E_t[r] \right]^{\frac{1}{\eta}}, \]  
\[ m_t^N = \gamma [p_t(z_t^N) - r_{t-1}(p_{t-1}z_{t-1}^N - m_{t-1}^N)], \]  
\[ m_t^T = \gamma [(z_t^T) - r_{t-1}(p_{t-1}z_{t-1}^T - m_{t-1}^T)], \]  
\[ B_t = 0, \]  
\[ p_t^H c_t^H = w_t L_t + (1 - \gamma) [r_{t-1} m_{t-1}^F + (q_t + \rho_t^Z + \rho_t^K - \delta P_t - r_{t-1} q_{t-1}) K_t] \]  
\[ m_t^F = p_t^F F + \gamma [r_{t-1} m_{t-1}^F + (q_t + \rho_t^Z + \rho_t^K - \delta P_t - r_{t-1} q_{t-1}) K_t], \]  

where (Sa)–(Sd) simply restate (Sa)–(Sd) under the full closure, (32) and (33) follow from the discussion of households’ optimization problem in section 4.1, and (34) is the evolution of internal funds of financial intermediaries.

Crises. During a crisis, entrepreneurs from sector N and financial intermediaries default, and borrowing constraints are binding so that \( p_t^Z Z_t = \lambda(m_t^N + m_t^T) = \lambda \gamma W_t^T, \) \( P_t c_t^H = w_t L_t = (1 - \alpha - \epsilon)(1 - \eta) \lambda \gamma W_t^T \) and \( m_t^F = p_t^F F = \epsilon(1 - \eta) \lambda \gamma W_t^T. \) The market-clearing condition for non-tradables is then:

\[ p_t(z_t^N)^\theta = \mu P_t Y_t \]
\[ = \mu W_t^T [(1 - \gamma) + \lambda \gamma [(1 - \alpha - \epsilon)(1 - \eta) + \eta + \lambda(1 - \eta)]] - \mu \rho_t^K K_t \]

where I have used the fact that capital producers make zero profits in equilibrium and that (13) holds with equality. A candidate crisis equilibrium is given by \( (p_t, q_t, \rho_t^K, K_{t+1}) \) satisfying (26), (27), (35), and the binding constraint (13). To be a crisis equilibrium, the two following default conditions must hold:

\[ r_{t-1}(q_{t-1} K_t - m_{t-1}^F) > (q_t + \rho_t^K - \delta p_t^K) K_t + \alpha(1 - \eta) \lambda \gamma W_t^T, \]
\[ r_{t-1}(p_t^H z_t^N - m_{t-1}^N) > p_t(z_t^N)^\theta. \]
By analogy to condition (10), the latter condition can be rewritten

\[
\frac{r_{t-1}(p_{t-1}^\mu z_N^N - m_t^N)}{m_t^T} > \mu \left[ \frac{(1 - \gamma)}{\gamma} + \lambda [(1 - \alpha - \epsilon)(1 - \eta) + \eta + \lambda \epsilon (1 - \eta)] \right] - \mu \frac{\rho_{t}^{K,\text{crisis}}}{m_t^T} K_t
\]

where \( m_t^T \) is the no-crisis level of internal funds (equal to \( \gamma W_t^T \)) and \( \rho_{t}^{K,\text{crisis}} \) is the value of \( \rho^K \) in the crisis equilibrium, which is of second-order so that the right-hand side is almost constant.