Estimating dynamic macroeconomic models:
How informative are the data?

Daniel O. Beltran

Federal Reserve Board of Governors, USA

and David Draper

University of California, Santa Cruz, USA

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Summary. Central banks have used complicated time series models — in particular, dynamic stochastic general equilibrium (DSGE) models — for some time to inform key policy decisions such as the setting of interest rates, but insufficient attention has sometimes been given to important estimation issues. For researchers estimating such models using Bayesian techniques, this paper offers an empirical strategy that strives to unveil the information content of the data relative to that of the prior distribution. We illustrate our approach using the widely cited Smets and Wouters [2003] model for the Euro Area. Before taking the model to the real data, we use artificial data generated by the model to uncover parameters for which the model’s structure obscures identification. Then, after verifying that the model’s structure is not creating an identification problem, we begin the empirical analysis by integrating over the likelihood function of the data. Our plots of the marginal likelihood densities derived from MCMC methods reveal that some parameters are weakly informed by the data. After having identified the dimensions along which the likelihood function is relatively flat, we conduct a Bayesian sensitivity analysis using three sets of priors, which differ in how much confidence is placed around their means. The Bayesian analysis complements the analysis of the likelihood function in helping to diagnose which parameters are weakly identified by the data, and also provides a “reality check” on whether the posterior estimates are being driven mainly by the prior or the data. Although parameter identification is model- and data-specific, the weak identification of some key structural parameters in this “off-the-shelf” macroeconomic model should raise a red flag to researchers trying to draw valid inferences from, and to base policy upon, similarly large-scale models featuring many parameters.

Keywords: Bayesian estimation, econometric modeling, identification, Euro Area.

1 Address for correspondence: Department of Applied Mathematics and Statistics, Baskin School of Engineering, University of California, 1156 High Street, Santa Cruz CA 95064 USA.
E-mail: draper@ams.ucsc.edu
1. Introduction

Large-scale time series models have played a major role for the last several decades in the setting of macro-economic policy in the advanced and emerging economies. In particular, one class of such models — dynamic stochastic general equilibrium (DSGE) models (e.g., Kydland and Prescott [1982], Rotemberg and Woodford [1997]) — has gained increasing prominence in macroeconomics and econometrics over the past 25 years. DSGE models, which describe macro-level economic phenomena using micro-economic principles, are used as forecasting tools by central banks worldwide. These models typically examine inter-relationships over time among key economic indicators such as output, inflation, and interest rates.

In the last decade, the monetary and fiscal-policy literature has moved forward from calibrating DSGE models — placing point-mass distributions on parameter values based on a-priori beliefs or long-term features of the data — to estimating them, often using Bayesian techniques. DSGE models were calibrated mainly because researchers were interested (a) in examining their dynamics, to see how closely they resembled those of actual data, and (b) in evaluating policy implications under reasonable assumptions about the parameter values. However, as DSGE models have grown in complexity to incorporate more realistic features of the data, it has become less obvious how to calibrate many of the new parameters that have emerged. Furthermore, analyses of calibrated DSGE models are not always robust to alternative calibrations. Bayesian techniques are well suited to address this calibration problem, because they provide a formal way to estimate the parameters by combining prior information about them with the data, as viewed through the lens of the model being analyzed. This offers hope that calibration may no longer be needed, as long as the data do indeed have something to say about plausible parameter values.

When maximum-likelihood techniques are used to estimate DSGE models (e.g., Ireland [2003]), the parameter estimates come purely from the data (although, as we discuss below, if the likelihood function is essentially flat for a parameter, the precise “maximum” found by numerical maximization may be largely arbitrary), and without (explicit) controversy over the role of priors. But if the data alone are not sufficient to identify all parameters of a model, using priors and Bayesian techniques is sensible. The main message of this paper is that if one is to achieve identification through the Bayesian approach, this should be done transparently. That is, the Bayesian approach should not only be used to derive parameter estimates, but should also reveal which parameters are most sensitive to the prior choice. Having confidence of how strongly the empirical results depend on the prior assumptions is crucial if the model is to be used for policymaking. For researchers estimating DSGE models using Bayesian techniques, this paper offers an empirical strategy that strives to unveil the information content of the data relative to that of the prior distribution.

Our empirical strategy begins by integrating over the likelihood function of the data. Our plots of the marginal likelihood densities derived from MCMC methods reveal that some parameters are weakly informed by the data. After having identified the dimensions along which the likelihood function is relatively flat, we conduct a Bayesian sensitivity analysis using three sets of priors, which differ in how much confidence is placed around their means. The Bayesian analysis complements the analysis of the likelihood function in helping to diagnose which parameters are weakly identified by the data, and also provides a “reality check” on whether the posterior estimates are being driven mainly by the prior or the data.
Researchers estimating DSGE models are generally aware that identification problems can prevent certain parameters from being consistently estimated. In fact, most of the existing literature on parameter identification in DSGE models has focused on diagnosing lack of identification that is inherent in the model’s structure—that is, when the model’s structure induces the same impulse response to a given shock under different parametrizations. This problem can and should be flagged prior to taking the model to the data. Canova and Sala [2009] provide an informal approach to diagnosing this problem which involves computing impulse responses using random draws of the model’s parameters and measuring their distance from the benchmark impulse responses obtained using the “true” parameter vector (that is, the parameters assumed as the data generating process). If the model is weakly identified, parameter values far from the true ones will generate impulse responses with small distances from the benchmark responses. One problem with this approach is that identification depends on the objective function chosen (e.g., the impulse responses the researcher is trying to match, and the weights associated with them in the objective function). Also, identification problems detected when using limited-information techniques do not necessarily carry over to full-information methods.

Other studies have derived more formal tests for local identification that can be performed prior to taking the model to the data. These tests are designed to produce a “yes or no” answer as to whether a given parameter is identified. Iskrev [2010] uses the rank of the information matrix, and demonstrates how to derive it analytically. However, as later recognized in Iskrev and Ratto [2010], computing the analytical derivatives of the Jacobian with respect to the deep parameters is computationally inefficient and requires a large amount of memory allocation because sparse Kronecker-product matrices are used extensively. Similarly, Komunjer and Ng [2011] obtain necessary and sufficient rank conditions for identification using the spectral density of the endogenous variables in the model. This approach also requires extensive use of numerical derivatives and Kronecker-product matrices. Komunjer and Ng [2011] find that computation of the rank is sensitive to the tolerance level that is used to determine whether the eigenvalues are sufficiently small. They address this issue by performing the analysis using different tolerance levels.

Our approach to diagnosing identification problems that are inherent to the model’s structure before taking the model to the data is similar to that of Adolfson and Lindé [2011]. This approach is much easier to implement than the formal approaches of Iskrev [2010] and Komunjer and Ng [2011] because it only requires simulating the model to generate artificial data and maximizing the likelihood function. In the case of the Smets and Wouters [2003] (hereafter SW) model that we use, this approach correctly reveals the parameters that are not identified due to the structure of the model.

The plan for the remainder of the paper is as follows. In Section 2 we describe our data resources and sketch the model we fit (with more details on the model and the fitting process given in Appendices A.1–A.2). In Section 3 we estimate the model using artificial data to check that the model’s structure is not creating an identification problem. In Section 4 we provide results from likelihood analysis. Section 5 provides results from the Bayesian sensitivity analysis (with prior elicitation details given in Section 2 of the supplementary material in

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2Although it is not difficult to find published Bayesian studies in which the marginal prior and posterior essentially coincide and yet it is claimed that the relevant parameter has been estimated from the data.
2. Data resources and the Smets-Wouters model

Building on work by Christiano et al. [2005], SW developed and estimated a large-scale dynamic stochastic general equilibrium model for the Euro Area. Using summary statistics such as the marginal likelihood, Bayes factor, and root mean squared errors they show that their DSGE model fits the data as well as standard nontheoretical VAR models and Bayesian VAR models estimated using the same data. This implies that their DSGE model does at least as good a job as the VAR models in predicting the observable data series over the sample period. The model’s good fit largely derives from a number of frictions that generate persistence in the propagation of shocks, such as sticky prices and wages, external habit persistence, investment adjustment costs, and other features such as variable capacity utilization. SW describe the full non-linear model, and derive a linear version which we summarize in section 1 of the supplement to this paper (Beltran and Draper [2012]). Its basic structure involves 28 log-linearized time series which link output, inflation, real wages, investment, capital stock, hours worked, firms’ marginal cost, the real interest rate, and employment to ten structural shocks, six of which are allowed to be serially correlated.

The model has 32 parameters. The key structural parameters include include the degree of relative risk aversion \( \eta_C \), external habit-formation \( h \), elasticity of work effort with respect to the real wage \( \eta_L \), rigidity of goods prices \( \xi_p \) and wages \( \xi_w \), productivity persistence \( \rho \), and three policy parameters relating to the lagged interest rate \( \rho_i \), inflation \( r_\pi \), and the output gap \( r_y \).

The log-likelihood is computed using the Kalman filter (Hamilton [1994]) and the Euro Area dataset described in Fagan et al. [2001]. The data comprise quarterly time series for seven key macroeconomic variables: real GDP, real consumption, real investment, real wages, employment, the GDP deflator, and the nominal interest rate. Following SW, we detrend real variables by their linear trend, and we detrend inflation and the nominal interest rate by the same linear trend in inflation. We extended the sample period used in SW to cover the most recent decade, so our data span the period 1970:3 to 2009:4, with the first 12 observations (3 years) being used to initialize the Kalman filter.

The prior-posterior plots shown in SW suggest that data are not informative about 10 out of the 32 parameters, which makes the likelihood function nearly flat along these dimensions. That is, the estimated marginal posterior closely resembles the prior distribution. Onatski and Williams [2010] also find evidence of weak parameter identification when they re-estimate the same model; when using uniform priors they obtain “substantially different” parameter estimates.

In re-estimating the SW model, Onatski and Williams [2010] paid special attention to how strongly the empirical results depend on the prior assumptions. However, in using “less informative” (uniform) priors they encountered two problems. First, they found “numerous local minima which confounded many optimization methods.” We also find that standard methods for maximizing the log-likelihood function are sharply inadequate with this model, and develop a maximization/adaptive-MCMC/maximization algorithm to address this problem. The second problem Onatski and Williams [2010] encountered is that for 9 of the 32 parameters, the maximum likelihood estimates are on the boundaries of the prior range. Onatski
and Williams [2010] acknowledge this boundary problem but dismiss it by stating that “To the extent that we set our prior to reflect reasonable ranges of estimates, this is not troubling in itself. But it does suggest that the data may favor some parameter values which may be implausible from an economic viewpoint, and hence are not in the support of our prior.” (p.154) More worrisomely, because the data seem to favor parameter values that are far from the arbitrary prior boundaries they chose, their point estimates depend critically on their boundary assumptions. That is, while Onatski and Williams [2010] sought to make their priors “less informative” by using a uniform distribution, the end result is that they inadvertantly calibrated these parameters at the prior boundary. By integrating over the likelihood function (instead of maximizing it as Onatski and Williams [2010] did), our estimation strategy will gauge the degree of parameter uncertainty, even for those that seem implausible from an economic viewpoint.

3. Checking identification problems due to the model’s structure

Before estimating the model, we check for the presence of identification problems that are coming from the structure of the model. The model’s structure could imply, for example, that a parameter is indistinguishable from another in terms of how they propagate the exogenous shocks to the state variables. Iskrev [2010] examines local identification by checking that the Jacobian matrix of the mapping of the deep parameters to the parameters that determine the first two moments of the data is not rank deficient. Because numerical derivatives tend to be inaccurate for highly non-linear functions, Iskrev [2010] develops a method for computing the Jacobian analytically. Iskrev [2010] apply their method to the Smets and Wouters [2007] model (an extended version of model examined in this paper), and find that the Jacobian is rank deficient. They attribute the lack of identification to the similar roles played by the two curvature parameters for the goods and labor markets, and the Calvo wage and price parameters. These parameters play similar roles in the nonlinear version of the model, but become equivalent in the linearized version that is estimated. To fix the identification problem, Iskrev [2010] (as well as Smets and Wouters [2007]) calibrate these curvature parameters.

As recognized in Iskrev and Ratto [2010], computing the analytical derivatives of the Jacobian with respect to the deep parameters is computationally inefficient and requires a large amount of memory allocation because sparse Kronecker-product matrices are used extensively. To check if local identification is possible, we use artificial data to estimate the model’s parameters. We created two simulated data sets, one with $T = 118$ (same number of observations in the original SW dataset) and another with $T = 1,000$ (an obviously infeasible length of time series); in both cases the artificial data were generated by our basic model, which allowed us to check if the model could be identified even in the absence of model mis-specification. For the data-generating process (DGP) we chose a particular calibration of the model parameters, denoted by $\theta_{DGP}$. The artificial data were then generated by simulating random draws for the IID shocks and feeding them into the state-space representation of the model equations. Using this artificial data set, we then found the parameter vector $\hat{\theta}$ that maximized the log-likelihood. If $\theta_{DGP} \approx \hat{\theta}$, then we can be confident that the parameters are locally identified by the model structure. Note that the goal here to is determine which parameters do not pass a local identification test; testing for global identification is not feasible given the number of parameters and the wide range of possible values they can take.
For both simulations, Table 1 shows the maximum likelihood estimates of \( \hat{\theta} \), and the associated 95% confidence intervals based on MCMC draws of the likelihood function. With \( T = 118 \) quarters of data, the MLEs for \( \varphi \), \( \lambda \), \( \psi \), \( \rho \), \( \sigma \), and \( \pi \) are moderately far from their true values and have wide likelihood intervals. But for most of these parameters (the exception being \( \rho \) and \( \sigma \)), with \( T = 1,000 \) observations the MLEs are similar to the true values and the likelihood intervals are sufficiently narrow. However, for \( \rho \) and \( \sigma \), the likelihood intervals are wide even with \( T = 1,000 \) observations, suggesting that the structure of the model is likely preventing these two parameters from being identified. These two parameters govern the persistence and the standard deviation of the inflation objective shock that enters the interest rate equation. The inflation objective shock and the policy shock enter additively in the same equation. In principle, the two shocks should be able to be identified because the inflation objective shock is assumed to be autocorrelated in the DGP while the policy rule shock is not. However, when \( \rho \) is high the model can generate an autocorrelated interest rate even when the inflation objective shock is not autocorrelated, making it difficult to distinguish the two shocks, as evidenced by our simulation with \( T = 1,000 \). SW address this identification problem by placing a tight prior on the autocorrelation parameters, whereas Onatski and Williams [2010] fix the problem by eliminating the policy rule shock. We take the latter approach, because we will later want to compare the results of our estimation exercise with those of Onatski and Williams [2010].

Although removing the policy rule shock fixes the lack of identification inherent in the model’s structure, when taking the model to the actual data we may still find that some parameters are weakly identified. Weak identification could arise because we are using data on only seven observable variables to estimate a model that has 9 structural shocks (even after eliminating the policy rule shock). SW attempt to identify the 10 shocks in their model by assuming that 1) they are uncorrelated with each other, 2) six of them are autocorrelated, and 3) four of them are white noise processes. For the six autocorrelated shocks, SW impose fairly strong persistence by setting the prior mean on the autocorrelation coefficient to 0.85, with standard errors of 0.10. However, without these priors, if the actual data favor low values for these autocorrelation parameters, some of the shocks will be difficult to identify. In the next section we show how a close examination of the log-likelihood function can reveal which parameters are weakly informed by the data.

4. Likelihood analysis

The “less informative” uniform prior used by Onatski and Williams [2010], shown in column (1) of Table 2 restricts the range of values the parameters can take to the “plausible” region. To derive their point estimates, which are shown in column (2), Onatski and Williams [2010] maximize the log-likelihood function in the region defined by their prior bounds. As highlighted in bold, many of these estimates are at the prior boundary, suggesting that the data favor implausible values for these parameters. When using the same data and our own maximization algorithm (described in section 3 of the supplement), we obtain similar results (column 3). In column 4 we extend the sample period by a decade and re-estimate the parameters using the same prior; many of the estimates remain at the prior boundary. It is interesting to check just how implausible the estimates become when the prior bounds are widened considerably (while still imposing some theoretical constraints). As shown in the last column, the estimates
obtained when using the wider prior range are strikingly distant from the ones obtained using the OW prior. In particular, the estimates for $\phi$, $\lambda_c$, $\lambda_l$, $\sigma_L$, and $\sigma_Q$ are clearly implausible.

Integrating over the unbounded likelihood function using MCMC methods reveals that these parameters whose maximum likelihood estimates are implausible are also weakly informed by the data. Figure 1 shows the marginal likelihood plots for the 31 parameters in our model. For each parameter, the shaded region denotes the histogram of the MCMC draws of the likelihood function. The thick line just above the horizontal axis denotes the Onatski and Williams [2010] prior range, and the dashed line shows their point estimate. The wide range spanned by the histograms of $\phi$, $\lambda_c$, $\lambda_l$, $\psi$, $r$, $\rho_l$, $\sigma_B$, $\sigma_I$, $\sigma_L$, and $\sigma_Q$ suggests that these parameters are weakly informed by the data. These likelihood plots also reveal some contradictions with the prior. For example, the likelihood function strongly favors values for $h$ between 0 and 0.4, which is in contrast with the Onatski and Williams [2010] prior range of 0.4 to 1. It is therefore not surprising that Onatski and Williams [2010] arrived at a point estimate of 0.4 for this parameter, effectively calibrating it at the prior boundary. The data also strongly favor values for $\xi_p$, $\xi_e$, and $\psi$ that are at odds with the prior used by Onatski and Williams [2010].

With insufficient data, some parameters that play similar roles in DSGE models can be difficult to identify. In the case of $\lambda_c$, the curvature parameter in the household’s utility function, the histogram of the MCMC-based likelihood draws shown in figure 1 ranges from 200 to 1000, well above the plausibility range of 1-4 typically used in the literature. In the linearized model $\lambda_c$ governs how sensitive the household’s optimal consumption choice is to the real interest rate and the preference shock. As shown in equation 1 of the supplement, a high value of $\lambda_c$ will dampen the response of consumption to the preference shock ($\epsilon^p_t$). In the same equation, a high value for $h$, which governs the persistence of the habit formation in consumption, would also dampen the response of consumption to a preference shock. It is therefore not surprising that the data favor implausibly high values for $\lambda_c$ that are compensated (in terms of the log-likelihood value) with values of $h$ that are much lower than those typically used in the literature.

If we had strong prior knowledge that $h$ should be closer to one (which we do), would such a prior help identify $\lambda_c$? One way to address this question is to examine the log-likelihood surface as a function of $h$ and $\lambda_c$, while keeping the other parameters fixed at their maximum likelihood values. As shown in the left panel of figure 2, when $h$ is high (closer to 1), the curvature of the log-likelihood surface increases with respect to $\lambda_c$, meaning that $\lambda_c$ is indeed better identified when $h$ is high. Because the SW and Onatski and Williams [2010] priors for $h$ rule out low values for this parameter, this restriction likely helped them identify the curvature parameter in the household’s utility function. The lesson to take from this example is that if one has good prior knowledge for a parameter, one should use it. Such a prior would not only help to identify the parameter in question, but may also help identify other parameters in the model as well.

If there is little prior knowledge for a parameter, one must be careful in choosing the prior when performing Bayesian estimation, specially if the parameter plays a key role in the model dynamics. One such parameter in the SW model is $\psi$, the elasticity of the capital utilization cost function. A low value of $\psi$ implies that the cost of utilizing capital does not change much as the utilization rate of capital increases. A high value of $\psi$ means that the cost of
utilizing capital increases rapidly with its utilization rate. King and Rebelo [2000] show that variable capacity utilization makes the labor demand curve more elastic with respect to the real wage (or the marginal product of labor). Similarly, Francis and Ramey [2005] and Smets and Wouters [2007] show that capital adjustment costs can help explain the empirical finding of (Gali [1999]) that productivity shocks have a negative impact on hours worked.

The impact of productivity shocks on hours worked also depends on the elasticity of work effort with respect to the real wage, or $\lambda^{-1}$. Figure 2 plots the likelihood surface as a function of $\psi$ and $\lambda$ while holding the others fixed at their maximum likelihood values. The log-likelihood surface shows an inverse relationship between these two parameters. Because they are poorly informed by the data (figure 1), placing a strong prior on one of them will likely influence the estimate of the other.

Summing up, the histograms of the MCMC-based likelihood draws reveal that some parameters in the Onatski and Williams [2010] model cannot be identified by the data alone. Furthermore, the surface plots suggest that identification of some parameters could be achieved by placing a strong prior on other parameters. Both SW and Onatski and Williams [2010] achieve identification by incorporating priors into their analysis. However, as evidenced by the disparity in their estimates, when identification is achieved through the use of priors, the results can be fragile. This is important because having confidence of how strongly the empirical results depend on the prior assumptions is crucial if the model is to be used for policymaking.

5. Bayesian sensitivity analysis

When the data alone are not sufficient to identify all of the model’s parameters, using priors and Bayesian techniques is sensible. However, if one is to achieve identification through the Bayesian approach, this should be done transparently. That is, the Bayesian estimation exercise should reveal which parameters are most sensitive to the prior choice. To perform this sensitivity analysis, we estimate the model using three sets of priors: the SW prior which we refer to as the “informative” prior, a looser version of their prior which we term “somewhat informative,” and a uniform prior. The bounds of the uniform prior are wider than those used by Onatski and Williams [2010]. In section 2 of the supplement, we examine the background literature used in specifying the prior distributions for each parameter; the exact prior specifications are reported in Table 3.

For some parameters, there is practically no previous research to base a prior on. Often, the priors for these parameters are chosen for convenience. In some cases, even though the data have little to say about a given parameter, its posterior estimate is used to inform the prior in subsequent studies. An example is the parameter that governs the elasticity of the capital utilization cost function ($\psi$). Smets and Wouters [2007] normalize this parameter so that it lies in the unit interval and center it at 0.5 because they did not have any previous research to base it on. As recognized by Smets and Wouters [2007], their posterior estimate for $\psi$ largely coincides with their prior, casting doubt on the sensitivity of this estimate to the prior. Even so, Onatski and Williams [2010] use the Smets and Wouters [2007] posterior estimate for $\psi$ to inform the boundaries of their uniform prior, only to arrive at a point estimate which is at the prior boundary. By using three sets of priors, we will determine how strongly our empirical results depend on the prior assumption.
For the three prior specifications, table 4 summarizes the marginal posterior distributions, and figure 3 plots the marginal posterior densities. The marginal posteriors for the weakly identified parameters ($\phi$, $\lambda_c$, $\lambda_l$, $\psi$, $\rho_\pi$, $\sigma_B$, $\sigma_I$, $\sigma_L$, and $\sigma_Q$) vary tremendously when we change the prior, confirming the diagnosis from the previous analysis of the likelihood function. In particular, the posterior estimates of $\psi$ and $\phi$ crucially depend on the prior choice, which is unfortunate, because there is practically no information to base the prior on. For parameters that are well-identified by the data, such as $\rho_g$, $\sigma_G$, and $\sigma_P$, the marginal posterior is practically invariant to the prior choice.

When trying to infer how informative the data are about the parameters of the model, it has become common practice in the literature to compare the moments of the posterior and prior distributions using just one set of priors. Our findings demonstrate that this approach would mislead one to believe that a parameter is well identified just because its posterior and prior distributions are different. For example, the SW prior for $\phi$ is quite different than its posterior estimate. However, it is incorrect to assume that this difference is because the likelihood (data) swamped the prior. To the contrary, the prior and likelihood are at odds with each other and their interaction resembles a tug-of-war. The more informative the prior, the more the posterior mean is shifted in toward the prior. The huge interval spanned by the MCMC draws of the likelihood function for $\phi$ (figure 1) and the high sensitivity of its posterior distribution to the prior choice (3) confirm that this parameter is actually weakly informed by the data.

6. Conclusions and discussion

Recent contributions to the literature on parameter identification in DSGE models has provided several useful tools for flagging lack of identification due to the structure of the DSGE model (e.g. Iskrev [2010], and Komunjer and Ng [2011]). But existing research has paid little attention to the issue of weak identification in DSGE models that arises from insufficient or inadequate data. The main contribution of this paper is to provide an empirical strategy for estimating DSGE models that is aimed at obtaining reasonable parameter estimates through the use of priors while at the same time revealing the information content of the data relative to that of the prior distribution. Previous studies have focussed on obtaining reasonable estimates, which is fairly straightforward to achieve in a Bayesian setting if one uses reasonable priors to begin with. But the latter goal of transparency, which has largely been ignored in the DSGE literature, is crucial if the estimates are to be taken seriously for analysis of policy.

We demonstrate our approach by estimating an “off-the-shelf” DSGE model, applied to the Euro Area over the period 1970–2009. The empirical strategy has three parts: simulation to uncover lack of identification from the model’s structure, MCMC-based likelihood estimation determine the dimensions along which the likelihood function is relatively flat, and Bayesian sensitivity analysis to gauge the information content of the data relative to that of the prior. Using this approach, we first find that a few parameters are not identified due to the model’s structure. After eliminating these parameters from the model, we then take the model to the data and perform an MCMC-based likelihood analysis and a Bayesian sensitivity analysis. We find that roughly one third of model’s parameters are weakly identified by the data. One such parameter is the elasticity of the capital utilization cost function ($\psi$), which plays a key role in the dynamics of the model because it determines (among other things) the impact
of productivity shocks on hours worked. Weak identification of this parameter is troubling because there is not much information to base a prior on. Despite the lack of prior information regarding this parameter, previous studies have estimated it using informative priors, obtaining narrow posterior intervals which, in turn, have been used as the prior for subsequent studies.

When estimating DSGE models, it is sensible to incorporate priors by using Bayesian techniques when the data alone are not sufficient to identify all of the model’s parameters. But unless this is done in a transparent manner, it is virtually impossible to judge how well the posterior estimates are informed by the data. This is crucial if the estimates are to be used for policymaking, and also if the estimates will be used to inform the priors in subsequent studies. We have shown how integrating over the likelihood function and performing a Bayesian sensitivity analysis can help diagnose weak identification coming from the data. This approach is natural from a statistical perspective but, surprisingly, is not in routine use by investigators fitting complicated econometric models.

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Table 1: Results of a simulation study comparing the parameters $\theta_{DGP}$ of the data-generating process with their maximum likelihood estimates $\hat{\theta}$, using $T = 118$ and $T = 1,000$ quarters of simulated data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_{DGP}$</th>
<th>$T = 118$</th>
<th>$T = 1,000$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>95% Interval</td>
<td>95% Interval</td>
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<tr>
<td></td>
<td></td>
<td>2.5%</td>
<td>97.5%</td>
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<td>$\varphi$</td>
<td>Inv. adj. cost</td>
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<td>$\lambda_y$</td>
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<td>$h$</td>
<td>Cons. habit</td>
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<td>Calvo prices</td>
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<td>Calvo empl.</td>
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<td>$r_{\Delta \pi}$</td>
<td>Inflation gr.</td>
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<td>$\rho$</td>
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<td>$r_{\Delta y}$</td>
<td>Output gap gr.</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Shocks, autocorr.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Productivity</td>
<td>0.90</td>
<td>0.93</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>Preference</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Gov. spending</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Labor supply</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>Investment</td>
<td>0.82</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>Infl. objective</td>
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<td>0.34</td>
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<td><strong>Shocks, std. dev.</strong></td>
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<td></td>
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<td>Productivity</td>
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<td>Preference</td>
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<td>Gov. spending</td>
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<td>$\sigma_I$</td>
<td>Investment</td>
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<td>$\sigma_L$</td>
<td>Labor supply</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Price markup</td>
<td>0.20</td>
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<tr>
<td>$\sigma_w$</td>
<td>Wage markup</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>Interest rate</td>
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<tr>
<td>$\sigma_Q$</td>
<td>Equity premium</td>
<td>0.95</td>
<td>0.60</td>
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<tr>
<td>$\sigma_\pi$</td>
<td>Infl. objective</td>
<td>0.06</td>
<td>15.79</td>
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</table>
Table 2: Maximum-likelihood estimates of the parameters in the DSGE model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>OW prior range</th>
<th>OW estimate</th>
<th>Our estimates</th>
<th>Wider prior</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Inv. adj. cost</td>
<td>3.57-8.33</td>
<td>6.579</td>
<td>6.332</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Cons. utility</td>
<td>1.00-4.00</td>
<td>2.178</td>
<td>2.953</td>
</tr>
<tr>
<td>$h$</td>
<td>Cons. habit</td>
<td>0.40-0.90</td>
<td>0.400</td>
<td>0.400</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Calvo wages</td>
<td>0.65-0.85</td>
<td>0.704</td>
<td>0.708</td>
</tr>
<tr>
<td>$\lambda_l$</td>
<td>Labor utility</td>
<td>1.00-3.00</td>
<td>3.000</td>
<td>3.000</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Calvo prices</td>
<td>0.40-0.93</td>
<td>0.930</td>
<td>0.930</td>
</tr>
<tr>
<td>$\xi_e$</td>
<td>Calvo empl.</td>
<td>0.40-0.80</td>
<td>0.400</td>
<td>0.800</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Wage indexation</td>
<td>0.00-1.00</td>
<td>0.000</td>
<td>0.242</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Price indexation</td>
<td>0.00-1.00</td>
<td>0.323</td>
<td>0.307</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Capital util. cost</td>
<td>2.80-10.00</td>
<td>2.800</td>
<td>2.800</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fixed cost</td>
<td>1.00-1.80</td>
<td>1.800</td>
<td>1.800</td>
</tr>
<tr>
<td><strong>Policy rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_\pi$</td>
<td>Inflation</td>
<td>1.00-4.00</td>
<td>4.000</td>
<td>4.000</td>
</tr>
<tr>
<td>$r_{\Delta \pi}$</td>
<td>Inflation gr.</td>
<td>0.00-0.20</td>
<td>0.181</td>
<td>0.169</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Lag interest rate</td>
<td>0.60-0.99</td>
<td>0.962</td>
<td>0.957</td>
</tr>
<tr>
<td>$r_y$</td>
<td>Output gap</td>
<td>0.00-1.00</td>
<td>0.062</td>
<td>0.000</td>
</tr>
<tr>
<td>$r_{\Delta y}$</td>
<td>Output gap gr.</td>
<td>0.00-1.00</td>
<td>0.319</td>
<td>0.434</td>
</tr>
<tr>
<td><strong>Shocks, autocorr.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Productivity</td>
<td>0.00-1.00</td>
<td>0.957</td>
<td>0.961</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>Preference</td>
<td>0.00-1.00</td>
<td>0.876</td>
<td>0.913</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>Gov. spending</td>
<td>0.00-1.00</td>
<td>0.972</td>
<td>0.901</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Labor supply</td>
<td>0.00-1.00</td>
<td>0.974</td>
<td>0.986</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>Investment</td>
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<td>0.943</td>
<td>0.967</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>Inflation obj.</td>
<td>0.00-1.00</td>
<td>0.582</td>
<td>0.746</td>
</tr>
<tr>
<td><strong>Shocks, std.dev.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Productivity</td>
<td>0.00-6.00</td>
<td>0.343</td>
<td>0.542</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Preference</td>
<td>0.00-4.00</td>
<td>0.240</td>
<td>0.220</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>Gov. spending</td>
<td>0.00-4.00</td>
<td>0.354</td>
<td>0.352</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>Investment</td>
<td>0.00-1.00</td>
<td>0.059</td>
<td>0.075</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>Labor supply</td>
<td>0.00-36.00</td>
<td>2.351</td>
<td>2.724</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Price markup</td>
<td>0.00-2.00</td>
<td>0.172</td>
<td>0.197</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>Wage markup</td>
<td>0.00-3.00</td>
<td>0.246</td>
<td>0.267</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>Infl. objective</td>
<td>0.00-1.00</td>
<td>1.000</td>
<td>0.716</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>Equity premium</td>
<td>0.00-7.00</td>
<td>7.000</td>
<td>7.000</td>
</tr>
</tbody>
</table>
Table 3: Prior distributions used in the Bayesian sensitivity analysis; see Section 2 of the supplement for specification details.

<table>
<thead>
<tr>
<th>Parameter($\theta$)</th>
<th>Uniform</th>
<th>Somewhat Informative</th>
<th>Informative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lb</td>
<td>ub</td>
<td>$p(\theta)$</td>
</tr>
</tbody>
</table>

- **$\varphi$** Inv. adj. cost
  - $lb$: 1
  - $ub$: 100
  - $p(\theta)$: IG
  - $E[\theta]$: 4
  - 5%: 1.49
  - 95%: 8.96

- **$\lambda_c$** Cons. utility
  - $lb$: 0
  - $ub$: 50
  - $p(\theta)$: IG
  - $E[\theta]$: 1.5
  - 5%: 0.36
  - 95%: 4.07

- **$h$** Cons. habit
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.7
  - 5%: 0.32
  - 95%: 0.96

- **$\xi_w$** Calvo wages
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.75
  - 5%: 0.47
  - 95%: 0.95

- **$\lambda_l$** Labor utility
  - $lb$: 0
  - $ub$: 50
  - $p(\theta)$: IG
  - $E[\theta]$: 1.5
  - 5%: 0.36
  - 95%: 4.07

- **$\xi_p$** Calvo prices
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.75
  - 5%: 0.47
  - 95%: 0.95

- **$\xi_e$** Calvo empl.
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.5
  - 5%: 0.04
  - 95%: 0.96

- **$\gamma_w$** Wage indexation
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.5
  - 5%: 0.04
  - 95%: 0.96

- **$\gamma_p$** Price indexation
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.5
  - 5%: 0.04
  - 95%: 0.96

- **$\psi$** Capital util. cost
  - $lb$: 1
  - $ub$: 100
  - $p(\theta)$: N
  - $E[\theta]$: 10
  - 5%: 1.78
  - 95%: 18

- **$\phi$** Fixed cost
  - $lb$: 1
  - $ub$: 2
  - $p(\theta)$: N
  - $E[\theta]$: 1.5
  - 5%: 1
  - 95%: 2

**Policy rule**

- **$r_\pi$** Inflation
  - $lb$: 1
  - $ub$: 10
  - $p(\theta)$: N
  - $E[\theta]$: 2
  - 5%: 1
  - 95%: 3

- **$r_{\Delta\pi}$** Inflation gr.
  - $lb$: -1
  - $ub$: 1
  - $p(\theta)$: N
  - $E[\theta]$: 0.3
  - 5%: -0.03
  - 95%: 0.63

- **$\rho$** Lag interest rate
  - $lb$: 0.5
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.8
  - 5%: 0.38
  - 95%: 1

- **$r_y$** Output gap
  - $lb$: -1
  - $ub$: 1
  - $p(\theta)$: N
  - $E[\theta]$: 0.13
  - 5%: -0.04
  - 95%: 0.29

- **$r_{\Delta y}$** Output gap gr.
  - $lb$: -1
  - $ub$: 1
  - $p(\theta)$: N
  - $E[\theta]$: 0.3
  - 5%: -0.03
  - 95%: 0.63

**Shocks, autocorr.**

- **$\rho_a$** Productivity
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

- **$\rho_b$** Preference
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

- **$\rho_G$** Gov. spending
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

- **$\rho_L$** Labor supply
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

- **$\rho_I$** Investment
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

- **$\rho_\pi$** Inflation obj.
  - $lb$: 0
  - $ub$: 1
  - $p(\theta)$: B
  - $E[\theta]$: 0.85
  - 5%: 0.41
  - 95%: 1

**Shocks, std.dev.**

- **$\sigma_a$** Productivity
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_b$** Preference
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_G$** Gov. spending
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_I$** Investment
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_L$** Labor supply
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_p$** Price markup
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_w$** Wage markup
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_\pi$** Infl. Objective
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

- **$\sigma_Q$** Equity premium
  - $lb$: 0
  - $ub$: 20
  - $p(\theta)$: E
  - $E[\theta]$: 2
  - 5%: 0.10
  - 95%: 6

*Note: B = Beta, IG = Inverse-gamma, N = Normal, and E = Exponential.*
Table 4: Summaries of marginal posterior distributions of the parameters under the three prior distributions described in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uniform Percentiles</th>
<th>Somewhat Informative Percentiles</th>
<th>Informative Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5% 97.5% Median</td>
<td>2.5% 97.5% Median</td>
<td>2.5% 97.5% Median</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>9 22 16</td>
<td>3 11 5</td>
<td>5 10 7</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>12 26 19</td>
<td>8 27 15</td>
<td>1 2 2</td>
</tr>
<tr>
<td>$h$</td>
<td>0.04 0.35 0.19</td>
<td>0.1 0.38 0.23</td>
<td>0.5 0.7 0.6</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.72 0.94 0.79</td>
<td>0.76 0.93 0.86</td>
<td>0.7 0.82 0.76</td>
</tr>
<tr>
<td>$\lambda_l$</td>
<td>11 37 23</td>
<td>5 23 10</td>
<td>2 5 3</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.92 0.98 0.94</td>
<td>0.94 0.98 0.96</td>
<td>0.92 0.95 0.94</td>
</tr>
<tr>
<td>$\xi_e$</td>
<td>0.79 0.9 0.86</td>
<td>0.82 0.9 0.87</td>
<td>0.76 0.85 0.81</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>0.01 0.44 0.15</td>
<td>0.01 0.31 0.1</td>
<td>0.17 0.59 0.36</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.04 0.37 0.19</td>
<td>0.03 0.36 0.18</td>
<td>0.18 0.45 0.31</td>
</tr>
<tr>
<td>$\psi$</td>
<td>5.91 95.04 47.57</td>
<td>4.03 20.65 11.88</td>
<td>2.74 9.13 5.71</td>
</tr>
<tr>
<td>$\phi$</td>
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<td>1.34 1.97 1.74</td>
<td>1.41 1.86 1.63</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.67 9.47 4.42</td>
<td>1.33 3.17 2.05</td>
<td>1.52 1.9 1.71</td>
</tr>
<tr>
<td>$\Delta\pi$</td>
<td>-0.05 0.09 0.02</td>
<td>-0.03 0.13 0.04</td>
<td>0.05 0.19 0.11</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.97 1 1</td>
<td>0.91 1 0.98</td>
<td>0.84 0.96 0.91</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>-0.55 0.88 0.19</td>
<td>-0.05 0.32 0.13</td>
<td>0.06 0.2 0.13</td>
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<tr>
<td>$\gamma_p$</td>
<td>0.23 0.39 0.29</td>
<td>0.27 0.48 0.36</td>
<td>0.23 0.32 0.27</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.96 0.98 0.97</td>
<td>0.97 0.99 0.98</td>
<td>0.96 0.99 0.98</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9 0.99 0.96</td>
<td>0.92 1 0.97</td>
<td>0.84 0.94 0.89</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.91 0.96 0.94</td>
<td>0.92 0.96 0.94</td>
<td>0.91 0.96 0.94</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>0.97 1 0.99</td>
<td>0.98 1 0.99</td>
<td>0.93 0.99 0.97</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>0.75 0.97 0.89</td>
<td>0.87 0.99 0.96</td>
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</tr>
<tr>
<td>$\rho_{\pi}$</td>
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<td>0.7 1 0.94</td>
<td>0.71 0.96 0.88</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.48 0.94 0.68</td>
<td>0.43 0.74 0.54</td>
<td>0.54 0.86 0.68</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.2 1.06 0.49</td>
<td>0.11 0.57 0.23</td>
<td>0.19 0.47 0.3</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.35 0.44 0.39</td>
<td>0.35 0.44 0.39</td>
<td>0.35 0.44 0.39</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>0.18 2.06 0.57</td>
<td>0.1 0.5 0.18</td>
<td>0.06 0.31 0.13</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>15 20 19</td>
<td>8 21 13</td>
<td>2 4 3</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.21 0.29 0.25</td>
<td>0.21 0.29 0.24</td>
<td>0.2 0.27 0.23</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>0.34 0.96 0.55</td>
<td>0.18 2.06 0.57</td>
<td>0.21 0.94 0.61</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>0.18 0.21 0.09</td>
<td>0.08 0.44 0.39</td>
<td>0.06 0.31 0.13</td>
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<tr>
<td>$\sigma_{\pi}$</td>
<td>0.34 0.96 0.55</td>
<td>0.18 0.24 0.08</td>
<td>0.21 0.94 0.61</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>0.34 0.96 0.55</td>
<td>0.18 0.24 0.08</td>
<td>0.21 0.94 0.61</td>
</tr>
</tbody>
</table>
Figure 1: MCMC-based Marginal Likelihood Plots

*Note:* Grey region shows histogram of the MCMC draws from the likelihood function. Thick horizontal line denotes prior range from Onatski and Williams [2010], and the dashed vertical line shows their point estimate.
Figure 1: (continued). MCMC-based Marginal Likelihood Plots

Note: Grey region shows histogram of the MCMC draws from the likelihood function. Thick horizontal line denotes prior range from Onatski and Williams [2010], and the dashed vertical line shows their point estimate.
Figure 3: MCMC-based Marginal Posterior Plots

Note: Dashed red line denotes marginal posterior under uniform prior, thin blue line under somewhat informative prior, and thick green line under the informative prior of Smets and Wouters [2003].
Figure 3: (continued). MCMC-based Marginal Posterior Plots

Note: Dashed red line denotes marginal posterior under uniform prior, thin blue line under somewhat informative prior, and thick green line under the informative prior of Smets and Wouters [2003].

References


D. Beltran and D. Draper, 2012. Supplement to “Estimating dynamic macroeconomic models: How informative are the data?”.


