Matching efficiency and business cycle fluctuations*

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Abstract

A large decline in the efficiency of the U.S. labor market in matching unemployed workers and vacant jobs has been documented during the Great Recession. We use a simple New Keynesian model with search and matching frictions in the labor market to study the propagation of matching efficiency shocks. We show that the transmission of these disturbances and their importance for business cycle fluctuations depend crucially on the form of hiring costs and on the presence of nominal rigidities.

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1 Introduction

Between 2009-Q3 and 2010-Q4 the US labor market has been characterized by an increase in the vacancy rate of 20 per cent whereas the unemployment rate has not decreased at all. This fact can simply reflect insufficient aggregate demand and be part of the painful adjustment to a large negative shock like the recent Great Recession or it can be due to an outward shift in the Beveridge curve caused by structural factors. In particular, some policy-makers have related the absence of a decrease in unemployment to a less efficient matching process in the labor market (cf. Bernanke, 2010, Kocherlakota, 2010, Evans, 2010 among others for an overview on the debate). This view has received some support from recent empirical work by Barnichon and Figura (2011b) who find that a large decline in matching efficiency added 1.5 percentage points to the unemployment rate during the Great Recession.

Fluctuations in matching efficiency can be interpreted as variations in the degree of search and matching frictions in the labor market and reflect all the hiring behavior that cannot be explained by the stocks of unemployment and vacancies. Unemployment, vacancies, matching efficiency and hiring behavior are usually related through the aggregate matching function, one of the building blocks of models with search and matching frictions in the labor market (cf. Blanchard and Diamond, 1989 and Petrongolo and Pissarides, 2001). When matching efficiency is low, for given stocks of unemployment and vacancies, few new matches will be created. The opposite is true when matching efficiency is high. Barnichon and Figura (2011a) have estimated the aggregate matching function for the US over the period 1976-2010 by using data on the job finding rate and labor market tightness. The regression residual, that represents fluctuations in matching efficiency, is relatively stable over time except during the recent Great Recession, when the matching efficiency is at historically low levels.¹

¹A substantial decline in matching efficiency during the Great Recession is documented also by Barlevy (2011), Borowczyk-Martins, Jolivet and Postel-Vinay (2011), Elsby, Hobijn and Sahin (2010) and Sedláček (2011). Notice that the large decline in matching efficiency is a feature specific to the Great Recession.
Several factors could explain a lower degree of matching efficiency: skill mismatch (cf. Sahin, Song, Topa and Violante, 2011 and Herz and van Rens, 2011), geographical mismatch, possibly exacerbated by house-locking effects (cf. Nenov, 2011), reduction in search intensity by workers because of extended unemployment benefits (cf. Kuang and Valletta, 2010), reduction in firm recruiting intensity (cf. Davis, Faberman and Haltiwanger, 2010), shifts in the composition of the unemployment pool due, for example, to a larger share of long-term unemployment or to a larger share of permanent layoffs (cf. Barnichon and Figura, 2011a).

Importantly, in the framework of the aggregate matching function, matching efficiency has the same interpretation of the Solow residual in the context of the production function. Therefore, shocks to the matching efficiency play the same role as technology shocks in the production function and can be interpreted as structural shocks in modern business cycle models.\(^2\) However, while the literature has devoted a substantial effort to studying the properties of technology shocks, little is known of the effects of shocks to the matching efficiency. This paper aims at filling this gap by providing a careful analysis of the transmission mechanism for shocks to the matching efficiency in the context of a very simple New Keynesian model with search and matching frictions in the labor market.\(^3\)

Two contributions emerge from our analysis. First, the propagation of shocks to the matching efficiency depends crucially on the form of hiring costs. When we consider post-match hiring costs, in the form of training costs as in Gertler and Trigari (2008), we show analytically that the shock does not even propagate and unemployment is invariant to fluctuations in matching efficiency. Given that in the data post-match hiring costs

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\(^2\) The residual of the matching function can have an endogenous component, as it is the case for the Solow residual in the production function (cf. Basu, Fernald and Kimball, 2006, among others). How to purify the Solow residual of the matching function is an interesting area for future research that is outside the scope of the current paper. Here we concentrate on the transmission mechanism for the exogenous component.

happen to be the main component of total hiring costs (cf. Pissarides, 2009, Silva and Toledo, 2009, and Yashiv, 2000), our analysis seems to indicate a rather limited role for shocks to the matching efficiency in explaining business cycle fluctuations. When we consider pre-match hiring costs, in the form of linear costs of posting a vacancy as in Pissarides (2000), the shock propagates and unemployment declines in response to a positive impulse. However, the importance of these shocks is limited by the fact that they imply a large positive correlation between unemployment and vacancies whereas it is well known that this correlation is strongly negative in the data. Therefore, shocks to the matching efficiency cannot be a main driver of unemployment fluctuations although they can be seen as shifters of the Beveridge curve.

The second contribution of this paper is to show that when matching efficiency shocks propagate, i.e. under pre-match hiring costs, the presence of nominal rigidities is crucial for the transmission mechanism. In fact, the response of vacancies can be positive or negative depending on the presence of nominal rigidities in the model. The sign of the vacancy response is important because it determines the slope of the Beveridge curve conditional on matching efficiency shocks. We show that when nominal rigidities are present, as in our baseline model, vacancies decrease and the conditional Beveridge curve has a positive slope. When prices are flexible, instead, vacancies increase and the conditional correlation between unemployment and vacancies declines substantially and can even become negative when the shock has limited persistence. Interestingly, nominal rigidities are also a feature that determine the sign of the hours worked response to a technology shock (cf. Basu, Fernald and Kimball, 2006, Chang, Hornstein and Sarte, 2009, Christiano, Eichenbaum and Vigfusson, 2003, Galf, 1999, and McGrattan, 2005, among many others).4 We show analytically that the features that induce a negative response of hours worked to a positive technology shock also imply a negative response of vacancies to a positive matching efficiency shock.

Shocks to the matching efficiency were already present in the seminal paper by Andolfatto (1996) that introduced search and matching frictions in the standard RBC model.

4See also Francis and Ramey (2005) for an alternative mechanism based on real rigidities (habit persistence and capital adjustment costs) that can deliver a negative response of hours even in a RBC model.
Since then, these shocks have also been considered in Arsenau and Clugh (2007), Beauchemin and Tasci (2008), Krause, Lubik and Lopez-Salido (2008), Lubik (2009), Chermukhin and Restrepo-Echevarria (2011), Justiniano and Michelacci (2011) and Mileva (2011). However, none of these papers relates matching efficiency shocks to the form of hiring costs or to the degree of nominal rigidities or to the slope of the Beveridge curve. Importantly, our theoretical analysis of the transmission mechanism can in part reconcile very different results on the importance of matching efficiency shocks that explain 92% of unemployment fluctuations in Lubik (2009), 37% in Krause, Lubik and Lopéz-Salido (2008) and only 11% in Michelacci and López-Salido (2011).

Our paper is also related to the literature initiated by Lilien (1982) on the importance of reallocation shocks for business cycle fluctuations. Shocks to the matching efficiency, in fact, can be considered as reallocation shocks, at least as long as they capture some form of mismatch (in skills, in geography or in other dimensions), as argued in Andolfatto (1996) and Pissarides (2011). Abraham and Katz (1986) suggested that reallocation shocks play a limited role in explaining aggregate fluctuations because they imply a positive correlation between unemployment and vacancies (unlike aggregate demand shocks). However, that argument was not based on a general equilibrium analysis. Here, we confirm the statement by Abraham and Katz (1986) in the context of our New Keynesian model, but we show that the slope of the conditional Beveridge curve can become negative when prices are flexible and the shock has low persistence.

The paper proceeds as follows: Section 2 briefly describes the model, section 3 presents our results, section 4 relates our results to the literature and section 5 concludes and offers an outline of our ongoing research.

2 The model

The model economy consists of a representative household, a continuum of wholesale goods-producing firms, a continuum of monopolistically competitive retail firms, and monetary and fiscal authorities, which set monetary and fiscal policy respectively. The model is deliberately simple. We ignore features such as capital accumulation, real rigidities
(such as habit persistence and investment adjustment costs) and wage rigidities. We include all these features in a companion paper (Furlanetto and Groshenny, 2012), where we estimate a medium-scale version of our model to study the evolution of unemployment during the Great Recession and to quantify the importance of structural factors for unemployment dynamics. Based on the results from our companion paper, we can safely concentrate only on the features that are critical for the transmission of matching efficiency shocks and ignore the unnecessary complications. Our model is very similar to Kurozumi and Van Zandwaghe (2010) in the version with pre-match hiring costs and is a simplified version of Gertler, Sala and Trigari (2008) in the version with post-match hiring costs.

The representative household  There is a continuum of identical households of mass one. Each household is a large family, made up of a continuum of individuals of measure one. Family members are either working or searching for a job. Following Merz (1995), we assume that family members pool their income before allowing the head of the family to choose optimal per capita consumption.

The representative family enters each period \( t = 0, 1, 2, \ldots \), with \( B_{t-1} \) bonds. At the beginning of each period, bonds mature, providing \( B_{t-1} \) units of money. The representative family uses some of this money to purchase \( B_t \) new bonds at nominal cost \( \frac{B_t}{R_t} \), where \( R_t \) denotes the gross nominal interest rate between period \( t \) and \( t+1 \).

Each period, \( N_t \) family members are employed. Each employee works a fixed amount of hours and earns the nominal wage \( W_t \). The remaining \((1 - N_t)\) family members are unemployed and each receives nominal unemployment benefits \( b \), financed through lump-sum nominal taxes \( T_t \). Unemployment benefits \( b \) are proportional to the steady-state nominal wage: \( b = \tau W \). During period \( t \), the representative household receives total nominal factor payments \( W_t N_t + (1 - N_t) b \) as well as profits \( D_t \). The family purchases retail goods for consumption purposes.

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\(^5\)The model abstracts from the labor force participation decision.
The family’s period $t$ budget constraint is given by

$$P_t C_t + \frac{B_t}{R_t} \leq B_{t-1} + W_t N_t + (1 - N_t) b - T_t + D_t.$$  

(1)

where $C_t$ represents a Dixit-Stiglitz aggregator of retail goods and $P_t$ is the corresponding price index.

The family’s lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s \ln C_{t+s}$$  

(2)

where $0 < \beta < 1$.

**The representative intermediate goods-producing firm** Each intermediate goods-producing firm $i \in [0, 1]$ enters in period $t$ with a stock of $N_{t-1} (i)$ employees. Before production starts, $\rho N_{t-1} (i)$ old jobs are destroyed. The job destruction rate $\rho$ is constant. The workers who have lost their jobs start searching immediately and can possibly still be hired in period $t$ (cf. Ravenna and Walsh, 2008). Employment at firm $i$ evolves according to $N_t (i) = (1 - \rho) N_{t-1} (i) + M_t (i)$ where the flow of new hires $M_t (i)$ is given by $M_t (i) = Q_t V_t (i)$. $V_t (i)$ denotes vacancies posted by firm $i$ in period $t$ and $Q_t$ is the aggregate probability of filling a vacancy defined as $Q_t = \frac{M_t}{V_t}$.

$$M_t = \int_0^1 M_t (i) \, di$$ and $$V_t = \int_0^1 V_t (i) \, di$$ denote aggregate matches and vacancies respectively. Aggregate employment $N_t = \int_0^1 N_t (i) \, di$ evolves according to

$$N_t = (1 - \rho) N_{t-1} + M_t.$$  

(3)

The matching process is described by an aggregate constant-returns-to-scale Cobb Douglas matching function

$$M_t = L_t S_t^\sigma V_t^{1-\sigma}.$$  

(4)
where $S_t$ denotes the pool of job seekers in period $t$

$$S_t = 1 - (1 - \rho) N_{t-1}. \quad (5)$$

and $L_t$ is a time-varying scale parameter that captures the efficiency of the matching technology. It evolves exogenously following the autoregressive process

$$\ln L_t = (1 - \rho_L) \ln L + \rho_L \ln L_{t-1} + \varepsilon_{Lt}. \quad (6)$$

where $L$ denotes the steady-state value of the matching efficiency, while $\rho_L$ measures the persistence of the shock and $\varepsilon_{Lt}$ is $i.i.d.$ $N(0, \sigma_L^2)$.

The job finding rate ($F_t$) is defined as $F_t = \frac{M_t}{S_t}$ and aggregate unemployment is $U_t \equiv 1 - N_t$. Newly hired workers become immediately productive. Hence, the firm can adjust its output instantaneously through variations in the workforce. However, firms face hiring costs, measured in terms of the finished good ($H^k_t(i)$) where $k$ is an index to distinguish the two kinds of hiring costs that we consider.

The first specification is a post-match hiring cost ($H^\text{post}_t(i)$) in which total hiring costs are given by

$$H^\text{post}_t(i) = \phi_N \left[ \frac{Q_t V_t(i)}{N_t(i)} \right]^2 N_t(i). \quad (7)$$

The parameter $\phi_N$ governs the magnitude of the post-match hiring cost. This kind of adjustment cost was used by Gertler and Trigari (2008) because it makes possible the derivation of the wage equation with staggered contracts and helps the model fit the persistence and the volatility of unemployment and vacancies that we observe in the data (Pissarides, 2009). Since then, this feature has become standard in the empirical literature (cf. Christiano, Trabandt and Walentin, 2011, Gertler, Sala and Trigari, 2007, Groshenny, 2009 and 2011, Sala, Söderström and Trigari, 2008). The post-match hiring cost can be interpreted as a training cost: it reflects the cost of integrating new employees into the employment pool.

The second specification that we consider is the hiring cost that is commonly used in
the literature on search and matching frictions (Pissarides, 2000). Following the classification in Pissarides (2009), it is a pre-match hiring cost \( H_{t}^{pre} (i) \) and it represents the cost of posting a vacancy. We use a standard linear specification that reads as follows

\[
H_{t}^{pre} (i) = \phi_N V_t (i)
\]

The parameter \( \phi_N \) governs the magnitude of the pre-match hiring cost.

Each period, firm \( i \) uses \( N_t (i) \) homogeneous employees to produce \( Y_t (i) \) units of intermediate good \( i \) according to the constant-returns-to-scale technology described by

\[
Y_t (i) = A_t N_t (i).
\] (8)

\( A_t \) is an aggregate labor-augmenting technology shock that follows the exogenous stationary stochastic process

\[
\ln (A_t) = (1 - \rho_A) \ln (A) + \rho_A \ln (A_{t-1}) + \varepsilon_A,
\] (9)

where \( \varepsilon_A \) is i.i.d. \( N (0, \sigma_A^2) \).

Each wholesale goods-producing firm \( i \in [0, 1] \) chooses employment and vacancies to maximize profits and sells its output \( Y_t (i) \) in a perfectly competitive market at a relative price \( Z_t (i) \). The firm maximizes

\[
E_t \sum_{s=0}^{\infty} \beta^s \frac{A_{t+s+1}}{A_{t+s}} \left( Z_{t+s} (i) Y_{t+s} (i) - \frac{W_{t+s} (i)}{P_{t+s}} N_{t+s} (i) - H_{t+s}^{k} (i) \right).
\]

**Wage setting** \( W_t (i) \) is determined through bilateral Nash bargaining,

\[
W_t (i) = \arg \max \left[ \Delta_t (i)^{\eta} J_t (i)^{1-\eta} \right],
\] (10)
where $0 < \eta < 1$ represents the worker’s bargaining power. The worker’s surplus, expressed in terms of final consumption goods, is given by

$$\Delta_t(i) = \frac{W_t(i)}{P_t} - b + \beta E_t [\left(1 - \rho\right) \left(1 - F_{t+1}\right)] \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \Delta_{t+1}(i).$$  \hspace{1cm} (11)

The firm’s surplus in real terms is given by

$$J_t(i) = Z_t(i) A_t - \frac{W_t(i)}{P_t} + \frac{\partial H^k_t(i)}{\partial N_t(i)} + \beta \left(1 - \rho\right) E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}(i)\right].$$  \hspace{1cm} (12)

**Retail firms** There is a continuum of retail goods-producing firms indexed by $j \in [0, 1]$ that transform the wholesale good (bought at price $Z_t$, which is common across wholesale goods-producing firms) into a final good $Y^f_t(j)$ that is sold in a monopolistically competitive market at price $P_t(j)$. Demand for good $j$ is given by $Y^f_t(j) = C_t(j) = \left(P_t(j)/P_t\right)^{-\theta}C_t$ where $\theta$ represents the elasticity of substitution across final goods. Firms choose their price subject to a Calvo (1983) scheme in which every period a fraction $\alpha$ is not allowed to re-optimize whereas the remaining fraction $1 - \alpha$ chooses its price by maximizing the following discounted sum

$$E_t \sum_{s=0}^{\infty} (\alpha \beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_t(j)}{P_{t+s}} - Z_{t+s}\right) Y^{f}_{t+s}(j)$$

**Monetary and fiscal authorities** The central bank adjusts the short-term nominal gross interest rate $R_t$ by following a Taylor-type rule

$$\ln \left(\frac{R_t}{R}\right) = \rho_r \ln \left(\frac{R_{t-1}}{R}\right) + (1 - \rho_r) \left[\rho_\pi \ln (\Pi_t) + \rho_y \ln (Y_t/Y_{t-1})\right],$$ \hspace{1cm} (13)

where $\Pi_t = P_t/P_{t-1}$. The degree of interest-rate smoothing $\rho_r$ and the reaction coefficients to inflation and output growth ($\rho_\pi$ and $\rho_y$) are all positive.

The government budget constraint is of the form

$$(1 - N_t) b = \left(\frac{B_t}{R_t} - B_{t-1}\right) + T_t.$$ \hspace{1cm} (14)
3 Results

Our calibration is based on the US economy. A first set of parameters is taken from the literature on monetary business cycle models. The discount factor is set at $\beta = 0.99$, the elasticity of substitution final goods at $\theta = 11$ implying a steady-state markup of 10 percent. The parameters in the monetary policy rule are $\rho_r = 0.8$, $\rho_\pi = 1.5$, $\rho_y = 0.5$. The average degree of price duration is 4 quarters, corresponding to $\alpha = 0.75$.

<table>
<thead>
<tr>
<th>Table 1: equilibrium equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler equation $c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1})$ (T 1)</td>
</tr>
<tr>
<td>production function $y_t = a_t + n_t$ (T 2)</td>
</tr>
<tr>
<td>law of motion for employment $n_t = (1 - \rho) n_{t-1} + \rho(q_t + v_t)$ (T 3)</td>
</tr>
<tr>
<td>Definition of unemployment $u_t = -\left(\frac{N}{S}\right) n_t$ (T 4)</td>
</tr>
<tr>
<td>Probability of filling a vacancy $q_t = l_t - \sigma \left( v_t + \left(\frac{(1-\rho)N}{S}\right) n_{t-1} \right)$ (T 5)</td>
</tr>
<tr>
<td>Job finding rate $f_t = l_t + (1 - \sigma) \left( v_t + \left(\frac{(1-\rho)N}{S}\right) n_{t-1} \right)$ (T 6)</td>
</tr>
<tr>
<td>Definition of the hiring rate $x_t = q_t + v_t - n_t$ (T 7)</td>
</tr>
<tr>
<td>New Keynesian Phillips curve $\pi_t = \beta E_t \pi_{t+1} + \kappa z_t$ (T 8)</td>
</tr>
<tr>
<td>Monetary policy rule $r_t = \rho_r r_{t-1} + (1 - \rho_r) \left( \rho_\pi \pi_t + \rho_y (y_t - y_{t-1}) \right)$ (T 9)</td>
</tr>
<tr>
<td>Matching efficiency shock $l_t = \rho_L l_{t-1} + \epsilon_{L,t}$ (T 10)</td>
</tr>
<tr>
<td>Technology shock $a_t = \rho_A a_{t-1} + \epsilon_{A,t}$ (T 11)</td>
</tr>
</tbody>
</table>

A second set of parameter values is taken from the literature on search and matching in the labor market. The degree of exogenous separation is set at $\rho = 0.08$, the steady-state value of the unemployment rate is $U = 0.06$. The parameter $\tau$ that governs the value of unemployment benefits is set equal to 0.6 whereas the elasticity in the matching function is $\sigma = 0.65$, in keeping with recent estimates by Barnichon and Figura (2011a). We target a vacancy filling rate $Q$ equal to 0.70 and we set the steady state degree of matching efficiency $L$ accordingly. The two remaining parameters, the one that governs
the size of hiring costs ($\phi_V$ or $\phi_N$) and the degree of bargaining power of workers $\eta$, are linked by steady state conditions. Given the lack of convincing empirical evidence on the value of $\eta$, we follow Blanchard and Gali (2010) and we set $\phi_V$ and $\phi_N$ such that total hiring costs in steady state are equal to one percent of steady state output in both models (or, equivalently, the consumption to output ratio is set at 0.99). This implies that $\eta$ has to be equal to 0.83 in the model with pre-match hiring costs and to 0.71 in the model with post-match hiring costs. These choices avoid indeterminacy issues that are widespread in this kind of model, as shown by Krause and Lubik (2010) and Kurozumi and Van Zandweghe (2010).

Finally, the degree of persistence for the shock processes is set at 0.7.

The log-linear first order conditions that do not depend on the form of the hiring cost function are listed in table 1 where we define $x_t$ as the hiring rate, the ratio between new matches and employment.

### 3.1 Matching efficiency shocks and post-match hiring costs

In this section we look at the transmission mechanism for the shock to the matching efficiency when the hiring cost is in the form of a training cost, as in Gertler and Trigari (2008).

In table 2 we report the three loglinearized first order conditions that depend on the form of the hiring cost function (the job creation condition, the wage equation and the market clearing condition):

<table>
<thead>
<tr>
<th>Table 2: additional equations for the model with post-match hiring cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t = -\left(\frac{W}{\phi_N\rho(1-2\rho)^{\rho}}\right)rw_t + \left(\frac{Z}{\phi_N\rho(1-2\rho)^{\rho}}\right) (z_t + a_t) - \frac{\beta(1-\rho)(1-2\rho)}{(1-2\rho)} (i_t - E_t \pi_{t+1} + x_{t+1}) \quad (T \ 12)$</td>
</tr>
<tr>
<td>$rw_t = \left(\frac{\eta^2}{W} \right) (z_t + a_t) + \left(\frac{\eta^2\phi_N\rho^2}{W} \right) x_t \quad - \left(\frac{\eta^2(1-\rho)\phi_N\rho^2}{W} \right) (E_t - E_t \pi_{t+1} + E_t x_{t+1} - E_t f_{t+1}) \quad (T \ 13)$</td>
</tr>
<tr>
<td>$y_t = \left(1 - \frac{\phi_N\rho^2}{2}\right) c_t + \phi_N \rho^2 x_t + \frac{\phi_N\rho^2}{2} n_t \quad (T \ 14)$</td>
</tr>
</tbody>
</table>
Impulse responses in figure 1 show that only vacancies and the probability of filling a vacancy react to the shock. A positive shock to the matching efficiency makes it easier to fill a vacancy because the job market is more efficient (\( q_t \) increases) but firms react by posting fewer vacancies (\( v_t \) decreases). Importantly, with post-match hiring costs the response of the two variables is of the same magnitude. This implies that employment does not react (see T.3) and, in turn, unemployment and output are also invariant to the shock (see T.4 and T.2). All variables unrelated to the matching process are invariant to the matching efficiency shock or, in other words, the shock does not propagate.

This neutrality result hinges on the form of the hiring costs function. In a model with post-match hiring decision, the choice variable for firms is the hiring rate (\( x_t \)). Vacancy posting, which is now costless, is determined residually from the matching function equation, once the decision on hiring has been made. This point can be seen analytically by using the list of equilibrium conditions in tables 1 and 2. By substituting T7 into T3, we obtain

\[
  n_t = n_{t-1} + \frac{\rho}{1 - \rho} x_t
\]  

(15)
and by substituting T.5, T.6 and T.7 into T.13, we have

\[ rw_t = \left( \frac{\eta ZP}{W} \right) (z_t + a_t) + \left( \frac{\eta 2 \phi_N \rho^2 P}{W} \right) x_t \]

\[ - \left( \frac{\eta \beta (1 - \rho) \phi_N F \rho P}{W} \right) \left( r_t - E_t \pi_{t+1} - E_t n_{t+1} - \frac{(1 - \rho) N}{1 - (1 - \rho) N} n_t \right) \]

(16)

In the system of 9 equilibrium conditions (T1, T2, T4, T8, T9, T12, T14, 15 and 16) with 9 endogenous variables, \( q_t, f_t \) and \( v_t \) never appear. Therefore, that block of equations is not affected by how the matching function is specified. More specifically, unemployment dynamics are invariant to shocks to the matching efficiency and to different values of the elasticity in the matching function (\( \sigma \)). \( q_t, f_t \) and \( v_t \) are determined residually by T5, T6 and T7.\(^6\)

To sum up, our model predicts that the greater the importance of post-match hiring costs is in total hiring costs, the lower the propagation of shocks to the matching efficiency will be. Importantly, Silva and Toledo (2009) and Yashiv (2000) have looked at the importance of post-match hiring costs in the data. Both studies find that post-match hiring costs are substantial, accounting for at least 70 percent of total hiring costs. The same result is confirmed in an estimated New Keynesian model for Sweden by Christiano, Trabandt and Walentin (2011). Therefore, according to our analysis, given that the post-match component is dominant in the data, we should expect a very limited role for shocks to the matching efficiency in explaining business cycle fluctuations.

### 3.2 Matching efficiency shocks and pre-match hiring costs

In this section we look at the transmission mechanism for the shock to the matching efficiency when the hiring cost is in the form of a linear cost of posting a vacancy, as it is standard in the literature on search and matching frictions in the labor market (Pissarides, 2000).

In table 3 we report the three loglinearized first order conditions that depend on the

\(^6\)This point was brought to our attention by Larry Christiano in a private conversation few years ago. The same concept is expressed in a note written by Thjis Van Rens (2008) who also refers to a conversation with Larry Christiano. At that time the point was relevant to understand why unemployment volatility was higher in the model by Gertler and Trigari (2008) rather than in standard search and matching models and there was no discussion on shocks to the matching efficiency.
form of the hiring cost function:

$$q_t = \left( \frac{WQ}{FQ} \right) rw_t - \left( \frac{2Q}{\phi v} \right) (z_t + a_t) + \beta (1 - \rho) (r_t - E_t \pi_{t+1} + E_t q_{t+1}) \quad \text{(T 15)}$$

$$rw_t = \left( \frac{2ZF}{W} \right) (z_t + a_t) - \left( \frac{w^\beta(1-\rho)\phi_v FP}{WQ} \right) (r_t - E_t \pi_{t+1} + E_t q_{t+1} - E_t f_{t+1}) \quad \text{(T 16)}$$

$$y_t = \left( 1 - \frac{\phi_v V}{N} \right) c_t + \frac{\phi_v V}{N} v_t \quad \text{(T 17)}$$

In figure 2 we plot impulse responses to a matching efficiency shock and we see that it propagates, in contrast to the model with post-match hiring costs. A positive shock implies that the labor market is more efficient at matching workers and firms and, in fact, the probability of filling a vacancy and the probability of finding a job both increase. This expands the production possibilities in the economy, unemployment decreases and output increases.

We can understand why the shock propagates under pre-match hiring costs by comparing the non-linear version of job creation conditions (17 and 18) that we report in the appendix. In a model with pre-match hiring costs, the average cost of hiring a worker...
includes a component that depends on the expected duration of a vacancy, that itself depends on labor market tightness, which is taken as given by the firm. In a model with post-match hiring costs, instead, the average cost of hiring a worker does not depend on labor market tightness but only on the hiring rate which is a firm-specific variable. In a model with pre-match hiring costs, search frictions imply a congestion externality in the job creation condition, whereas in a model with post-match hiring costs, search frictions are not active and the model is equivalent to a model with quadratic employment adjustment costs.

Importantly, even though it is easier to fill a vacancy, firms react by posting fewer vacancies, as in the model with post-match hiring costs. This fact reminds us of the debate on the response of employment/hours worked to a positive technology shock in the standard New Keynesian model. The analogy is justified by the fact that a matching efficiency shock can also be seen as a technology shock in the production of new hires. Galí (1999) and Galí and Rabanal (2005) have linked the sign of the employment/hours worked response to the presence of nominal rigidities and inertia in monetary policy. Interestingly, the same is true for the response of vacancies to a matching efficiency shock. The dotted line in figure 2 represents impulse responses in our model when prices are flexible: the response of vacancies is positive, as is the response of employment when we simulate our model in response to a positive technology shock (see figure 3).

The relationship between the sign of the vacancy response and the degree of nominal rigidity can also be shown analytically in an extreme (but still interesting) case, closely following Galí (1999). For the sake of the argument, we consider the case of exogenous monetary policy (instead of an interest rate rule) and fixed prices (instead of sticky prices) and we postulate the following equation for money demand in log-linear terms

\[ m_t - p_t = y_t \]

The assumptions of exogenous money and fixed prices imply that output is fixed in the period. Given fixed output and exogenous technology, employment is also fixed (see T.2). Then, from (T.3) there will be no job creation in response to the shock. Finally, the
response of vacancies to matching efficiency shocks can be derived by using the matching function. Being new hires fixed in the period and searchers a predetermined variable, the following is true:

\[ v_t = -\frac{1}{(1 - \sigma)} l_t \]

According to our calibration \((\sigma = 0.65)\), a one percent increase in the matching efficiency will be accompanied by a 2.85 percent decline in vacancies. Therefore, under the extreme case of exogenous money and fixed prices, the vacancy response will be always negative.\(^7\) This is also true in our model although the decline in vacancies is of course lower, given that monetary policy is endogenous and prices are not fixed. Nevertheless, the larger the degree of price rigidity is (and the more inertial monetary policy is), the more negative the vacancy response will be (as the more negative the effect of a positive technology shock on the labor input will be).\(^8\)

Although a quantitative evaluation of the importance of matching efficiency shocks

\(^7\)Notice that in this special case the distinction between pre-match and post-match hiring costs vanishes: in both cases unemployment is invariant to shocks to the matching efficiency.

\(^8\)Notice that the negative response of vacancies can be even larger in models with additional nominal (sticky wages) and real rigidities (habit persistence) and with capital accumulation (cf. Furlanetto and Groshenny, 2012). Here, we prefer to use the simplest set-up to make our point more transparent.
is not the objective of this paper, impulse responses in figure 2, and in particular the sign of the vacancy response, can give some insights on the relevance of this shock. In fact, unemployment and vacancies move in the same direction and they are almost perfectly positively correlated. Instead, it is well known that in the data unemployment and vacancies are strongly negatively correlated. This simple observation brings us to the conclusion that shocks to the matching efficiency cannot be an important source of aggregate fluctuations in a New Keynesian model with pre-match hiring costs, although they can be seen as shifters of the Beveridge curve. Interestingly, Galí (1999) used the same argument to limit the importance of technology shocks in New Keynesian models.

Therefore, the argument based on the sign of the Beveridge curve reinforces even further the argument based on the importance of post-match hiring costs that we used in the previous section to downplay the importance of shocks to the matching efficiency in a New Keynesian model of the business cycle.

4 Our results in perspective

Our results from the previous section can be related to the literature on the importance of reallocation shocks initiated by Lilien (1982). Sectoral shifts in demand can have consequences for aggregate macroeconomic variables if resources are not instantaneously mobile across sectors. The shock to the matching efficiency can be seen as a reallocation shock: if job creation is easier within sectors than across sectors, as seems plausible, reallocation shocks will affect aggregate matching efficiency.

Lilien (1982) emphasizes the importance of reallocation shocks that could explain up to 50 percent of unemployment fluctuations in the postwar period. The empirical regularity underlying that result is a positive correlation between the dispersion of employment growth rates across sectors and the unemployment rate. However, Abraham and Katz (1986) show that this positive correlation is consistent not only with reallocation shocks but also with aggregate demand shocks under general conditions. Moreover, according to Abraham and Katz (1986), data on unemployment and vacancies are more useful to disentangle the importance of reallocation shocks. In fact, they argue that reallocation
shocks deliver a positive correlation between unemployment and vacancies as reallocation shocks can be seen as shifters of the Beveridge curve along a positively sloped job creation line. Instead, aggregate demand shocks produce an inverse relationship between unemployment and vacancies, as observed in the unconditional data (summarized by a negatively sloped Beveridge curve). Therefore, according to Abraham and Katz (1986), data on unemployment and vacancies suggest the primacy of aggregate shocks, rather than reallocation shocks. That argument has been used as an identifying assumption in VARs (vector autoregressions) to reevaluate the importance of reallocation shocks. Blanchard and Diamond (1989) conclude that reallocation shocks play a minor role in unemployment fluctuations, at least at business cycle frequencies.

Our paper contributes to the literature on the relationship between reallocation shocks and the slope of the Beveridge curve by highlighting the different role of pre-match and post-match hiring costs and by using a fully specified general equilibrium model rather than a partial equilibrium model as in the previous literature. On the one hand, the distinction between pre-match and post-match hiring costs is crucial: post-match hiring costs generate a vertical conditional Beveridge curve (given that unemployment is invariant to the shock) whereas pre-match hiring costs imply that unemployment and vacancies move in the same direction delivering a positively sloped conditional Beveridge curve. On the other hand, the general equilibrium aspect becomes important when we investigate further the model with pre-match hiring costs. Our baseline model with sticky prices is fully consistent with the argument in Abraham and Katz (1986): conditional on matching shocks, unemployment and vacancies are almost perfectly correlated and, importantly, the correlation does not depend on the autocorrelation in the shock process (see figure 4 and table 4).

9 The statement makes reference to a partial equilibrium model of the labor market with search and matching frictions (cf. Jackman, Layard and Pissarides, 1989).

10 A useful review of empirical results in this literature is proposed in Gallipoli and Pelloni (2008).
Figure 4: Impulse-responses in the model with pre-match hiring cost for different degrees of shock persistence

<table>
<thead>
<tr>
<th>Persistence</th>
<th>corr(U_t, V_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.96</td>
</tr>
<tr>
<td>0.9</td>
<td>0.94</td>
</tr>
<tr>
<td>0.5</td>
<td>0.98</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

However, this result is not as general as the previous literature has taken for granted. In fact, it relies on the presence of nominal rigidities. From figure 5 and table 5, we see that in an RBC version of our model (α = 0) the correlation between unemployment and vacancies depends on degree of autocorrelation in the shock process. When the shock process is very persistent, we confirm the finding by Abraham and Katz (1986) also in an RBC set-up and the matching shock can be seen as a shifter of the Beveridge curve. But for lower degrees of persistence, the correlation between unemployment and vacancies declines and becomes negative for values of ρ_m lower than 0.6. When the shock is iid, the
conditional correlation between unemployment and vacancies is -0.64, meaning that the conditional Beveridge curve has a negative slope, as in aggregate data. This point was also raised by Hosios (1994) but in a partial equilibrium model where the reallocation shock was modeled as a shock to the relative price dispersion across firms.\textsuperscript{11} In his model, as in the flexible price version of our model with pre-match hiring costs, data on unemployment and vacancies are not conclusive to disentangle aggregate shocks and reallocation shocks. As far as we know, this is the first paper that shows this point when the reallocation shock is given by a shock to the matching efficiency.

\textbf{Table 5:} \textit{corr}(U_t, V_t) with pre-match hiring costs and flexible prices

<table>
<thead>
<tr>
<th>$\rho_\zeta$</th>
<th>corr(U_t, V_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.85</td>
</tr>
<tr>
<td>0.7</td>
<td>0.21</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.23</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.56</td>
</tr>
<tr>
<td>0</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

\textsuperscript{11}Hosios (1994) also considers a second kind of reallocation shock, a shock to the job separation rate. That shock always generates a positively sloped Beveridge curve in his model. This is also the case in our model (results are available upon request).
We believe that our result has two implications. First, most of the literature on reallocation shocks is based on real business cycle models. We show that the assumption of flexible prices is not innocuous and that the interpretation of reallocation shocks as shifters of the Beveridge curve is robust only in a model with sticky prices. Second, this paper provides additional evidence that the presence of nominal rigidities crucially changes the transmission mechanism of shocks. This has already been shown for technology shocks (Galí, 1999), financial and different kind of investment shocks (Christiano, Motto and Rostagno, 2011, Del Negro, Eggertsson, Ferrero and Kiyotaki, 2011, Furlanetto and Seneca, 2010 and 2011), fiscal shocks (Christiano, Eichenbaum and Rebelo, 2011). Here we show that this is also relevant for shocks to the matching efficiency.

Finally, our paper contributes to the literature on DSGE models with unemployment. Lubik (2009), Krause, Lubik and Lopez-Salido (2008), and Justiniano and Michelacci (2011) include shocks to the matching efficiency in estimated business cycle models for the US, although none of these papers focuses on the transmission mechanism. Importantly, the three studies reach very different conclusions on the role of matching efficiency shocks. Lubik (2009) finds that they explain 92 percent of unemployment and 38 percent of vacancy fluctuations in a RBC model very similar to our baseline model. Justiniano and Michelacci (2011) also estimate an RBC model for the US and for several other countries. However, in contrast to Lubik (2009), they find that matching efficiency shocks explain only 11 percent of unemployment and 3 percent of vacancy fluctuations in the US.\textsuperscript{12} Our model can, at least in part,\textsuperscript{13} reconcile these results: in Lubik (2009) hiring costs are only pre-match whereas in Justiniano and Michelacci (2011) there is also a post-match component. According to our analysis the larger the weight of the post-match component is, the lower the importance of matching efficiency shocks should be, in keeping with results in Lubik (2009) and Justiniano and Michelacci (2011). Finally, Krause, Lubik and López-Salido (2008) estimate a sticky price version of the model in Lubik (2009) where prices are flexible. They find that matching efficiency shocks explain 37 percent of

\textsuperscript{12}Similar numbers are found for Germany, Norway and Sweden, whereas there is evidence of a somewhat more important role for the shock in France and in the UK.

\textsuperscript{13}The two models are similar but not identical. These differences can also influence the propagation of matching efficiency shocks.
unemployment and only 1 percent of vacancy fluctuations. According to our analysis, the model with sticky prices implies a positively sloped conditional Beveridge curve whereas this is not always the case in a model with flexible prices (it depends on the persistence of the shock, that is not reported in Lubik, 2009). Therefore, our results can rationalize a more important role for matching shocks in RBC models.

5 Conclusion

Our analysis of the transmission mechanism for shocks to the matching efficiency emphasize the importance of the form of the hiring cost function and of the presence of nominal rigidities. When hiring costs are only post-match, the shock does not propagate and matching efficiency shocks are irrelevant for business cycle fluctuations. When hiring costs are pre-match, the shock propagates but generates a positively sloped Beveridge curve, in contrast to the unconditional empirical evidence but in keeping with Abraham and Katz (1986), at least insofar as prices are sticky and the shock is persistent.

More generally, our analysis shows that empirical models of the business cycle with unemployment should consider pre-match and post-match hiring costs in an integrated framework. This is the way we follow in a companion paper (cf. Furlanetto and Groshenny, 2012) where we use a generalized hiring cost function that combines the pre-match and the post-match components (cf. Yashiv, 2000). The generalized hiring function is important to obtain meaningful estimates in a medium-scale model that we use to study the evolution of unemployment during the Great Recession and to quantify the importance of structural factors for unemployment dynamics.

A further avenue for future research is to consider some of the determinants of matching efficiency in isolation. For example, the length of the unemployment benefit duration and the search effort of workers and firms can be modeled explicitly in simple extensions of the standard model. This exercise can be seen as a way to purify the Solow residual of the matching function, as has been done for the production function. In that sense, the role of endogenous search effort can play the same role as endogenous capital utilization in the production function. We leave these extensions for future research.
References


Appendix: equilibrium conditions and steady-state

List of common equilibrium conditions:

\[ \Lambda_t = (C_t)^{-1} \]

\[ \frac{\Lambda_t}{R_t} = \beta E_t \left( \frac{\Lambda_{t+1}}{\Pi_{t+1}} \right) \]

\[ Y_t = A_t N_t \]

\[ N_t = (1 - \rho) N_{t-1} + Q_t V_t \]

\[ U_t = 1 - N_t \]

\[ S_t = 1 - (1 - \rho) N_{t-1} \]

\[ Q_t = L_t \left( \frac{V_t}{S_t} \right)^{-\sigma} \]
\[ F_t = L_t \left( \frac{V_t}{S_t} \right)^{1-\sigma} \]

\[ P_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s \Lambda_{t+s} P_{t+s}^\theta C_{t+s} Z_{t+s}}{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s \Lambda_{t+s} P_{t+s}^{\theta-1} C_{t+s}} \]

Conditions specific to the model with post-match hiring costs

\[ Y_t = C_t + \frac{\phi_N}{2} \left( \frac{Q_t V_t}{N_t} \right)^2 N_t \]

\[ \frac{W_t}{P_t} = \eta \left[ Z_t A_t + \phi_N X_t^2 + \beta (1 - \rho) \phi_N E_t \frac{\Lambda_{t+1}}{\Lambda_t} F_{t+1} X_{t+1} \right] + (1 - \eta) b \]

\[ \phi_N X_t = Z_t A_t - \frac{W_t}{P_t} + \phi_N X_t^2 + \beta (1 - \rho) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \phi_N X_{t+1} \quad (17) \]

Conditions specific to the model with pre-match hiring costs

\[ Y_t = C_t + \phi_V V_t \]

\[ \frac{W_t}{P_t} = \eta \left[ Z_t A_t + \beta (1 - \rho) E_t \frac{\Lambda_{t+1}}{\Lambda_t} F_{t+1} \frac{\phi_V}{Q_{t+1}} \right] + (1 - \eta) b \]
\begin{equation}
\frac{\phi_V}{Q_t} = Z_t A_t - \frac{W_t}{P_t} + \beta (1 - \rho) \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\phi_V}{Q_{t+1}}
\end{equation}

Steady state equations: common conditions

\begin{align*}
N &= 1 - U \\
Y &= N \\
S &= 1 - (1 - \rho) N \\
V &= \frac{\rho N}{Q} \\
Z &= \frac{\theta - 1}{\theta} \\
R &= \frac{1}{\beta}
\end{align*}
\[ L = Q \left( \frac{V}{S} \right)^\sigma \]

\[ F = L \left( \frac{V}{S} \right)^{1-\sigma} \]

Steady state equations: conditions specific to the model with post-match hiring costs

\[ \frac{W}{P} = Z - \phi_N \rho (1 - \rho) (1 - \beta) \]

\[ \phi_N = \frac{Z \left( 1 - \frac{\eta}{1-\tau(1-\eta)} \right)}{\rho (1 - \rho)(1 - \beta) + \left( \frac{\eta}{1-\tau(1-\eta)} \right)(\rho^2 + \beta (1-\rho) F \rho)} \]

\[ C = Y - \frac{\phi_N}{2} \rho^2 N \]

Steady state equations: conditions specific to the model with pre-match hiring costs

\[ \frac{W}{P} = Z - \frac{\phi_V}{Q} (1 \beta (1 - \rho)) \]

\[ \phi_V = \frac{QZ (1 - \tau (1 - \eta) - \eta)}{(1 - \beta (1 - \rho))(1 - \tau (1 - \eta)) + \beta (1-\rho) F \eta} \]
\[ C = Y - \phi_V V \]