Decomposing the effects of the updates in the framework for forecasting

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Abstract

In this paper, we develop a forecasting decomposition analysis framework that can be applied to decompose the differences between two forecasts generated by any linearized state-space model with multivariate linear filter (such as Kalman filter). Differences in the forecasts are expressed as the contributions of various determinants, such as forecasting tool, observed data, expert judgments on history and future. We also address the problem of mutual interactions among individual factors affecting the forecast. The core contribution of this paper lies in the design of a set of partial decompositions that simplify the decomposition of changes in specific groups of factors. These partial decompositions are particularly useful when the filtration and projection ranges in the forecasts differ. Main advantage of the presented approach is thus its flexibility, as it can be applied to a variety of decomposition problems.

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1. Introduction

In recent years, structural models, and the DSGE models in particular, has become a standard tool applied within a forecasting framework of most of the central banks. Although every model is a simplified representation of real economy, ongoing attempts to incorporate more and more stylized facts often lead to creation of complex models. In such models interactions among variables, data, and expert judgments are not straightforward.[1] The forecast decomposition analysis shed light on these interactions as it separates the effects of changes in various factors entering the forecasting process on all variables and shocks in the model. Therefore, it is one of the focal points since the early age of forecasting as presented in Mincer and Zarnowitz (1969).

Forecast decomposition analysis is used in wide variety of applications. It also helps to evaluate or adjust expert judgements. Also, as the forecasting framework usually consists of periodically repeating forecasting exercises, explaining differences between consecutive forecasts is a natural part of every forecasting exercise. The improvement in understanding of forecasting procedure properties motivates forecast decomposition analysis since the early days of structural model forecasting as it is discussed by Todd (1990).

Based on the work of Andrle (forthcoming), we developed a complex framework for forecast decomposition analysis which allows us to evaluate the effects of different factors - changes in expert judgments, data revisions, new observations and updated outlooks - between two forecasts. This framework can be used even in applications when the filtration and projection ranges in the forecasts differ. Moreover, we take into consideration possible mutual interactions among factors and adjust our decomposition procedure to account for the possible endogenous character of factors. Illustration with the simple examples is provided. To our knowledge, there is no systematic description of the forecast decomposition analysis that incorporates both past estimates of unobserved variables and shocks and their future projections. We believe that the presented procedure thus improves our understanding of the forecast and expert judgments implementation. [2]

In order to proceed with the analysis of forecast decomposition, we divide the factors into the following groups: model changes, data revisions, new data releases and changes in expert judgments. In our analysis the considered model change is related to a change in the parameters but not the structure of the model. Data revisions group covers revisions in observed data that are considered to be past realizations in both forecasts. On the other hand, new data releases group includes observed data which are available only in one of the forecasts. The expert judgments group covers the information delivered by expert judgments on both past and future realizations of variables or shocks.

The developed procedure divides the forecast decomposition analysis into several partial decompositions using auxiliary artificial forecasts. These auxiliary forecasts are described in section 4 and the forecast decomposition analysis using partial decompositions is described in section 5.

[1] Expert judgment as seen by Svensson (2005) is information, knowledge, and views outside the scope of a particular model. Svensson (2005) argues that judgmental adjustments are a necessary and essential ingredient in modern monetary policy.
[2] As the focus of this paper is not the forecast evaluation, we do not present any analysis of the forecast accuracy which is also a very popular approach to evaluate inclusion of judgment.

The decomposition revision procedure described in this paper was implemented in the IRIS Toolbox.[3] Its applicability is documented by the examples from the forecasting framework of a sticky-price DSGE model of the Czech Republic.

2. Notation, Assumptions and Expert Judgment

The forecast decomposition analysis can be performed on any structural model in a state-space form. To demonstrate our approach, we use the specific example of the dynamic stochastic general equilibrium (DSGE) model as used by the Czech National Bank in its forecasting process. Andrle et al. (2009) present the relevant decisions problems of agents, the solution of the model and its properties. The solution of the model provides the transformation of the decision problems to laws of motion and representation of the model in a state-space form. In this section we introduce notation for the rest of the paper.

By forecasting we understand the use of the model as a tool to map observed data on unobserved variables and shocks using multivariate linear filter and project their future development using the model structure and expert judgments. We consider data generating process (DGP) for observable variables $D_{\mathbf{x}_t}$ and its realizations $\mathbf{x}_t$, such that $\mathbf{x}_t \sim D_{\mathbf{x}_t}(\mathbf{x}_t \mid \mathbf{X}_{t-1})$, where, $\mathbf{x}_t$ is a $n \times 1$ vector and $\mathbf{X}_{t-1}$ is a history of realizations of the data, such that $\mathbf{X}_{t-1} = (\mathbf{x}_1^{t-1}, \ldots, \mathbf{x}_{t-1}^{t-1})$. In this notation, the revisions of data from period 1 to period t that are realizations of the DGP can be denoted as follows: $(\mathbf{x}_1^1, \ldots, \mathbf{x}_1^t)$ for the first period data point, $(\mathbf{x}_2^2, \ldots, \mathbf{x}_2^t)$ for the second period and the data point at t is the recent observation $(\mathbf{x}_t^t)$.

We denote end of the history T as the last period when any realization of the DGP exists. Using a model structure and observed data, filtration is a vector $\widetilde{\mathbf{x}}_{T-h}^T$ at time T that consist of all observed and unobserved variables and shocks, vector $\widetilde{\mathbf{x}}_{T+h}^T$ is then the projection at time period T. The subscript refers to the period of the projected data point, while the superscript is the end of the history. The h is the number of periods between the projected data point and end of the history, it can be called projection or forecast span as in Mincer and Zarnowitz (1969). For the computational purposes, we limit the h so that $h \leq H$, where H is a finite number known as the end of the projection and defines the forecast horizon.

In this notation, the forecasting tool f_T is a function of the model and its parameters, so that $f_T = F(m_T, \phi_T)$, where m_T is the model parameterized by parameters ϕ_T in the period T.

Additional off-model information may be incorporated in the forecast in the form of expert judgements. The expert judgments on the history takes the form of unobserved variables and shocks realizations, so $\mathbf{Tunes}_T$ is the set of expert judgments for each period $\mathbf{Tunes}_T = (Tune_1^T, \ldots, Tune_T^T)$. Similarly, to data revisions the the revisions in these expert judgments can be collected in the in the following structure: $(Tune_1^1, \ldots, Tune_1^1)$ for the first period expert judgment, $(Tune_2^1, \ldots, Tune_2^T)$ for the second period and the data point at T is the recent observation $(Tune_T^T)$.

[3] IRIS is a free, open-source toolbox for macroeconomic modeling and forecasting in Matlab, developed by Jaromír Beneš. IRIS integrates core modeling functions (such as flexible model file language, simulation, estimation, or forecasting) with a wide range of supporting features (such as time series analysis, data management, or reporting) in a user-friendly command-oriented environment. Toolbox and its documentation is available at `http:\\www.iris-toolbox.com`

Expert judgments on the forecast horizon $EJ_T()$ are described by the extra information that forecasters apply to components of the forecast. $EJ_T()$ is a mapping of forecast plan $\mathbf{FP}_T$, expert judgments on variables $\mathbf{Z}_T$ and expert judgments on the structural shocks $\mathbf{E}_T$. The forecast plan $\mathbf{FP}_T$ is a series of indicators, so $\mathbf{FP}_T = (I^T_{T+1}, \dots, I^T_{T+H})$. These indicators identify how the expert judgments on the variables $\mathbf{Z}_T$ is treated, for each corresponding expert judgment on variable it defines the shock and its type that used to implement it. The forecast plan $\mathbf{FP}_T$ specifies how the expert information on the variables is spread within the model. The forecast plan describes how the constraints imposed on the future paths of the variables $\mathbf{Z}_T$ are modifying the structural shocks. The expert judgments on variables $\mathbf{Z}_T$, often referred as the implementation of outlooks is again a set of per period expert judgments in the following form $\mathbf{Z}_T = (z^T_T, \dots, z^T_{T+H})$, where z^i_t is the expert judgment as seen in time t applied in period i.

In here, the expert judgements $\mathbf{Z}_T$ is in the form of hard tunes as defined by Beneš et al. (2010). This is the form of the judgment when the imposed restrictions on the future paths of some of the variables hold exactly under any circumstances. In the newer works this may be referred as the judgment without uncertainty. In the case discussed by Beneš (forthcoming), there may some uncertainty associated with the restrictions, and it is allowed for some flexibility in deviating from them. Further, the problem $EJ_T(\mathbf{FP}_T, \mathbf{Z}_T, \mathbf{E}_T)$ is exactly determined as defined by Beneš (forthcoming) i.e. the degrees of freedom exceeds the number of constraints.

The expert judgment is complemented by the information about the future shocks that are driving the system $\mathbf{E}_T$. The set of future shocks has a richer structure than the other components of the forecast. The set of shocks $\mathbf{E}_T = (\epsilon^T_T, \dots, \epsilon^T_{T+H})$ collects the shocks that will enter the system. As in our modeling practice, we distinguish between the two levels of information anticipation. The deepest level (man mode) of anticipation is the expectation of the agents in the model m_T. At this level the forecaster has anticipation what shocks will be expected by model agents. At the higher level (demigod mode) shocks are anticipated by forecasters but model agents do not know about these shocks until they realize in the model framework. At the top level (god mode) shocks are not expected even by the forecaster. Therefore, in the forecasting framework for each shock $\epsilon^i_t = (\epsilon^{UE,i}_t, \epsilon^{E,i}_t)$, where $\epsilon^{UE,i}_t$ is the shock that is unexpected to model agents but anticipated by forecaster. Shock $\epsilon^{E,i}_t$ is shock expected by model m_t agents and by the forecaster. The $\epsilon^{E,i}_t$ shocks over the forecasting horizon allow the future data to affect the current behavior of the agents as they anticipate these shocks.

By forecast we mean the whole realization of all data points $-$ unobserved variables and shocks in the past and their projection on future. Therefore, we define the forecast at time T as the $\mathbb{X}^T_{T+H} = (\widetilde{\mathbf{x}}^T_1, \dots, \widetilde{\mathbf{x}}^T_{T+H})$. A forecast is then a solution to this problem $\mathbb{X}^T_{T+H} = f_T(\mathbf{X}_T, \mathbf{Tunes}_T, EJ_T(\mathbf{FP}_T, \mathbf{Z}_T, \mathbf{E}_T))$, where $\mathbf{X}_T$ is the actual history of the data $\mathbf{Tunes}_T$ is expert judgments filter modifications of unobserved variables and shocks in the past and $EJ_T(\mathbf{FP}_T, \mathbf{Z}_T, \mathbf{E}_T)$ denotes expert judgments on the future.

The forecast problem can be easily solved and the solution to this problem $\mathbb{X}^T_{T+H}$ consist of the and variables and set of structural shocks. The solution of forecast problem transforms the data realizations, expert judgements on history in the form of tunes, expert information from forecast plan $\mathbf{FP}_T$, and expert judgment on variables $\mathbf{Z}_T$, into the paths of variables and shocks $\mathbb{X}^T_{T+H}$. If the number of constraints originating from the forecast plan $\mathbf{FP}_T$ and the expert judgment $\mathbf{EJ}_T$, is less then the number of degrees of freedom number originating from model structure then the uncertainty in solution exists. In this case, likelihood maximization method, as described by Beneš (forthcoming), is used to solve the forecasting problem. This solution depends on the stochastic properties of the shocks and initial state. Beneš (forthcoming) also shows that in case when number of degrees of freedom equal the number of constraints then the system is exactly determined and the unique solution exists independently of the stochastic properties of the shocks and initial state.

3. Forecast decomposition analysis

As presented above, every forecast is driven by number of determinants – forecasting tool f_T, observed data $\mathbf{X}_T$, expert judgments on history $\mathbf{Tunes}_T$ and expert judgments on the future $EJ_T(\mathbf{FP}_T, \mathbf{Z}_T, \mathbf{E}_T)$. The forecasting process based on the forecasting tool f_T takes the following steps. At first, observed data $\mathbf{X}_T$ are filtered using the forecasting tool f_T over the history range and expert judgements on history $\mathbf{Tunes}_T$; the initial state at time T is identified. At second, projection is simulated using the forecasting tool f_T conditionally on expert judgments on future $EJ_T(\mathbf{FP}_T, \mathbf{Z}_T, \mathbf{E}_T))$. We assume that the same model and its parametrization is used for projection computation and the initial state identification.

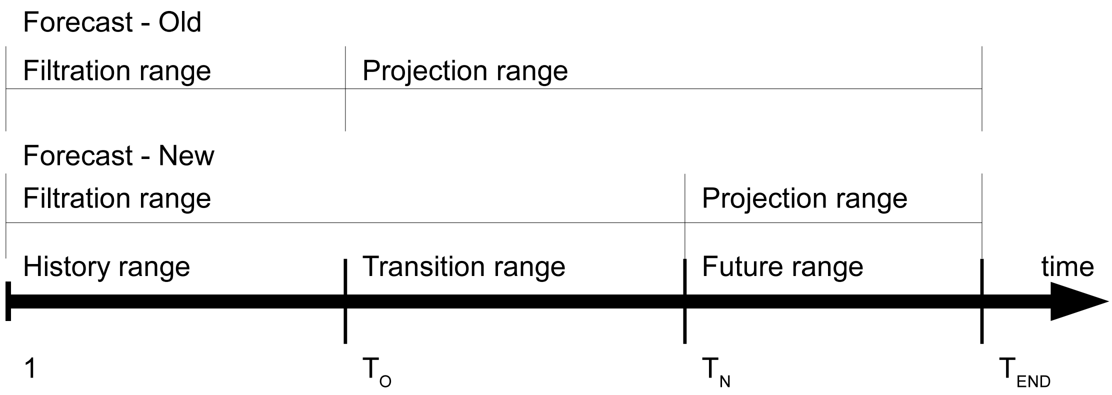
The decomposition is generally not a linear problem. Nevertheless, as shown by Andrle (forthcoming), decomposition using multivariate linear filter (such as Kalman filter/smoother) is a linear problem and can be solved numerically one group of factors after each other. Therefore, under linearized state-space model, linear multivariate filter and same model parametrization of both forecasts, the forecast decomposition analysis is also a linear problem.

The forecast decomposition analysis identifies the effects of the changes in the factors or groups of factors (observed data, tunes, expert judgments on history range, expert judgements on projection range) on the difference between the new forecast $\mathbb{X}^N_{T_N,T_{END}}$ and old forecast $\mathbb{X}^O_{T_O,T_{END}}$, where $T_N \geq T_O$ and T_{END} denote end of the projection range. The two forecasts are not necessary produced in the same period. However, in case of $T_N = T_O$ the decomposition problem boils down to a simplified task already discussed in Andrle et al. (2009) and used during the standard CNB forecasting process. Our paper focuses primarily on the forecast decomposition problem when $T_N > T_O$. Under these circumstances, a problem arises as due to time shift filtration has to be compared to projection.

Figure 3.1 shows the timing of the forecasts that we analyze. Interval $\langle T_O, T_N\rangle$ denotes the transition range which corresponds to intersection of the projection range of the old forecast $\mathbb{X}^O_{T_O,T_{END}}$ and the filter range of the new forecast $\mathbb{X}^N_{T_N,T_{END}}$. History range $\langle 1, T_O\rangle$ denotes the intersection of filtration ranges of both forecasts. Finally, the future range $\langle T_N, T_{END}\rangle$ is the intersection of projection ranges of both of the analyzed forecasts.

In previous applications, factors driving the forecast were assumed to be orthogonal. That is, factors were treated as exogenous inputs entering the forecast process. There may exist, however, some factors which realizations are influenced by other factors. In our methodology we take these interactions into account and divide factors into two broad groups - exogenous and endogenous. Exogenous factors group includes factors that are exogenous either with respect to the structural model (objectively exogenous) or from the point of view of the forecaster (subjectively exogenous). This group, therefore, consists of observed data realizations and subjective expert judgments on the filter range and projection range. Based on this assumption, realization of the exogenous factor is not affected by any other factor. The rest of the factors is considered to be endogenous.

Figure 3.1: Time conventions



Using the grouping of the factors two types of forecast decomposition analysis are considered:

1. **Treating all factors as exogenous**
   Effects of all factors are orthogonal. The effect of the chosen factor in the decomposition is computed given that the rest of the factors remain unchanged (the ceteris paribus assumption). The linearity of the decomposition problem allows us to compute the effect of chosen factor as the difference between its new and old realization holding all other factors unchanged.
2. **Treating exogenous and endogenous factors separately**
   Effects of all exogenous factors are orthogonal. The effect of the chosen exogenous factor is computed given that the rest of the exogenous factors remain unchanged (endogenous factors may change). The linearity of the decomposition problem allows us to compute the effect of chosen exogenous factor as the difference between its new and old realization holding all other exogenous factors unchanged.

   The effect of the chosen endogenous factor is then computed as the difference between its new realization and its old realization corrected for the changes induced by exogenous variables, holding other endogenous factors unchanged.

4. Auxiliary artificial forecasts

In this section we define auxiliary artificial forecasts used in forecast decomposition analysis of differences between new forecast $\mathbb{X}^{N}_{T_N,T_{END}}$ and old forecast $\mathbb{X}^{O}_{T_O,T_{END}}$, where $T_N \geq T_O$.

New forecast without expert judgments on projection is the solution to the following problem: $\mathbb{X}^{N,freeP}_{T_N,T_{END}} = f_{T_N}(\mathbf{X}_{T_N}, \mathbf{Tunes}_{T_N}, EJ_{T_N}(\varnothing, \varnothing, \overline{\mathbf{E}}_{T_N}))$, where there are no expert judgments on the variables on the projection range and the forecast plan is also omitted from the simulation. In here, the error structure $\overline{\mathbf{E}}_{T_N}$ is defined. In this simulation, there are no expected shocks, so $\overline{\mathbf{E}}_{T_N} = (\epsilon^{T_N}_{T_N}, \ldots, \epsilon^{T_N}_{T_{END}})$, where $\epsilon^{T_N}_i = (\epsilon^{UE,T_N}_i, \varnothing)$ for i in the projection range. The unexpected shocks realize according to scheme $E_{i|i-1}(\epsilon^{T_N}_i) = E_{i|i-1}(\epsilon^{UE,T_N}_i) = 0$ for all i in the projection range. The purpose of this auxiliary forecast is to remove all expert information from the projection. Similarly we define old forecast without expert judgments on projection:

$\mathbb{X}^{O,freeP}_{T_O,T_{END}} = f_{T_O}(\mathbf{X}_{T_O}, \mathbf{Tunes}_{T_O}, EJ_{T_O}(\varnothing, \varnothing, \overline{\mathbf{E}}_{T_O}))$. As in the previous simulation, in the structural error structure $\overline{\mathbf{E}}_{T_O}$ there are no expected shocks and the unexpected shocks realize at their mean values.

New forecast without expert judgments on projection and filtration is a solution to the following problem: $\mathbb{X}^{N,freeFP}_{T_N,T_{END}} = f_{T_N}(\mathbf{X}_{T_N}, \varnothing, EJ_{T_N}(\varnothing, \varnothing, \overline{\mathbf{E}}_{T_N}))$. This forecast originates from the $\mathbb{X}^{N,freeP}_{T_N,T_{END}}$, where the expert judgments on the history and transition range are removed. Therefore, this forecast uses the historical data of the new forecast to identify the initial state and runs on the projection horizon without fixing of variables or expected shocks. Unexpected shocks realize at their mean values. As there are no expert judgments, this forecast just follows the observed data using the forecasting tool. Similarly we define old forecast without expert judgments on projection range: $\mathbb{X}^{O,freeFP}_{T_O,T_{END}} = f_{T_O}(\mathbf{X}_{T_O}, \varnothing, EJ_{T_O}(\varnothing, \varnothing, \overline{\mathbf{E}}_{T_O}))$.

Old forecast without expert judgment on projection and filtration with revised history is a solution to the following problem: $\mathbb{X}^{rev,freeFP}_{T_O,T_{END}} = f_{T_O}(\mathbf{X}^{rev}_{T_N}, \varnothing, EJ_{T_O}(\varnothing, \varnothing, \overline{\mathbf{E}}_{T_O}))$. In this forecast the expert judgments on the history and transition range are removed and the historical data of the **new** forecast on the history (and not on the transition) range are used, i.e. $\mathbf{X}^{rev}_{T_N} = (\mathbf{x}^{T_N}_1, \ldots, \mathbf{x}^{T_N}_{T_O})$. Just to emphasize, the filtration is applied only on the history range. It differs from the new forecast without expert judgments on projection and filtration $\mathbb{X}^{N,freeFP}_{T_N,T_{END}}$ just in the filtration range, where it spans over history and transition range, so it uses also new releases of observed data.

We assume that all auxiliary forecasts are made using the same forecasting tool f_T. Since this assumption is quite restrictive and unrealistic in practical policy applications, we suggest the approach that allows for different parametrization. The forecaster may choose her preferred parametrization of the model for forecast decomposition analysis (either parametrization used in one of the forecasts, or any other). Obviously, the change of the parametrization of the model is a non-linear problem. We avoid it by re-simulating both forecasts with the preferred parametrization and we assign the effects of these changes as a special factor to the final decomposition. Then, we can decompose re-simulated forecasts, keeping the assumption of linearity valid.

5. Forecast decomposition analysis using partial decompositions

With auxiliary forecasts in hand, we define partial decompositions, which enable us to compute the effects of changes in specific factors groups between the two forecasts.

The partial decomposition of expert judgments on projection $\mathbb{D}^{freeP}_{T,T_{END}}$ is the decomposition of the differences between the forecast $\mathbb{X}_{T,T_{END}}$ and forecast without expert judgments in projection $\mathbb{X}^{freeP}_{T,T_{END}}$.

The partial decomposition of expert judgments on filtration $\mathbb{D}^{freeF}_{T,T_{END}}$ is the decomposition of the differences between forecast without expert judgments in projection $\mathbb{X}^{freeP}_{T,T_{END}}$ and forecast without expert judgments in projection and filtration $\mathbb{X}^{freeFP}_{T,T_{END}}$.

The partial decomposition of revisions $\mathbb{D}^{(N-O),rng,freeFP}_{T,T_{END}}$ is the decomposition of the differences between two forecasts without expert judgments in projection and filtration that have the same filtration range and differ only in observed data in the range $rng \in \{transition, history\}$: $\mathbb{X}^{N,freeFP}_{T,T_{END}}$ and $\mathbb{X}^{O,freeFP}_{T,T_{END}}$.

The linearity of the forecast decomposition problem ensures that the overall forecast decomposition $\mathbb{D}_{T_{END}}$ can be computed as the sum of partial decompositions as follows:

$$\begin{aligned} \mathbb{D}_{T_{END}} &= \mathbb{D}^{N,freeP}_{T_N,T_{END}} + \mathbb{D}^{N,freeF}_{T_N,T_{END}} + \mathbb{D}^{(N-O),transition,freeFP}_{T_N,T_{END}} + \\ &+ \mathbb{D}^{(N-O),history,freeFP}_{T_O,T_{END}} - \mathbb{D}^{O,freeF}_{T_O,T_{END}} - \mathbb{D}^{O,freeP}_{T_O,T_{END}} \end{aligned} \tag{5.1}$$

$$\begin{aligned} \mathbb{D}_{T_{END}} &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{O}_{T_O,T_{END}}) = \\ &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{N,freeP}_{T_N,T_{END}}) + (\mathbb{X}^{N,freeP}_{T_N,T_{END}} - \mathbb{X}^{N,freeFP}_{T_N,T_{END}}) + \\ &+ (\mathbb{X}^{N,freeFP}_{T_N,T_{END}} - \mathbb{X}^{rev,freeFP}_{T_N,T_{END}}) + (\mathbb{X}^{rev,freeFP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - \\ &- (\mathbb{X}^{O,freeP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - (\mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}) \end{aligned} \tag{5.2}$$

Note that every partial decomposition is straightforward to compute, since both forecasts considered have the same filtration range. Moreover, relevant factors are decomposed either on filtration range (i.e. filtration is applied) or on projection range (simulation is applied).

Next subsections discuss in a greater detail the use of the partial decompositions in some practical applications.

5.1 Expert judgements on future range

We start the description with the simplest example: decomposition of differences between two forecasts with the same filtration and projection ranges that differ only in the expert judgments on projection. One possibility is to compute the decomposition by finding the differences in these factors and then simulating them (factor by factor and period by period) with the forecasting tool, tracking the changes in all variables. Nevertheless, the same results can be acquired by using the partial decompositions: we decompose both forecasts to their respective forecast without expert judgements on projection. Since $T_N = T_O$ and all observations and expert judgments on filtration are same, $\mathbb{X}^{N,freeP}_{T_N,T_{END}} = \mathbb{X}^{O,freeP}_{T_O,T_{END}}$.

$$\begin{aligned} \mathbb{D}_{T_{END}} &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{O}_{T_O,T_{END}}) = \\ &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{N,freeP}_{T_N,T_{END}}) - (\mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}) \\ &= \mathbb{D}^{N,freeP}_{T_N,T_{END}} - \mathbb{D}^{O,freeP}_{T_O,T_{END}} \end{aligned} \tag{5.3}$$

The forecaster has to be careful while grouping the effects of factors from partial decompositions to get comparable results.

5.2 Expert judgements on filtration range

Now we decompose two forecasts which differ not only in expert judgments on projection, but also on filtration range. All other factors driving the forecasts remain the same, including the end of the history for both forecasts. This is still a simplified type of decomposition and can be computed directly when all the factors are treated as exogenous. However, the factors on filtration range usually affect the factors on projection range. If the forecaster would like to treat some of the factors as endogenous, the decomposition has to be computed using partial decompositions.

In order to decompose expert judgments on filtration robustly, we first need to remove all expert judgements on projection for both forecasts. Then we decompose the forecasts without expert judgments on projection against the forecasts without expert judgments on projection and filtration. These decompositions are computed using differences in $Tunes$ (factor by factor and period by period), using artificial values of $Tunes$ for the forecasts without any expert judgments. These artificial values of $Tunes$ impose the same forecasts as those without all expert judgments, and serve the sole purpose of keeping same structure of the model for filtration purposes. Such a procedure again assures that the linearity of the problem is held. Since $T_N = T_O$ and all observations and expert judgments on filtration are same, $\mathbb{X}^{N,freeFP}_{T_N,T_{END}} = \mathbb{X}^{O,freeFP}_{T_O,T_{END}}$.

$$
\begin{aligned}
\mathbb{D}_{T_{END}} &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{O}_{T_O,T_{END}}) = \\
&= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{N,freeP}_{T_N,T_{END}}) + (\mathbb{X}^{N,freeP}_{T_N,T_{END}} - \mathbb{X}^{N,freeFP}_{T_N,T_{END}}) - \\
&- (\mathbb{X}^{O,freeP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - (\mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}) \\
&= \mathbb{D}^{N,freeP}_{T_N,T_{END}} + \mathbb{D}^{N,freeF}_{T_N,T_{END}} - \\
&- \mathbb{D}^{O,freeF}_{T_O,T_{END}} - \mathbb{D}^{O,freeP}_{T_O,T_{END}}
\end{aligned} \tag{5.4}
$$

5.3 Data revisions with the same filtration ranges

Real world data is imperfectly measured and therefore subject to revisions. Let us now decompose two forecasts that differ in all expert judgments and in observed data, still holding the assumption that both forecasts have the same filtration range.

We can use the same partial decompositions as in the previous step to decompose effects of expert judgments both on filtration and projection range, ending up with the two artificial forecasts without expert judgments $\mathbb{X}^{N,freeFP}_{T_N,T_{END}}$ and $\mathbb{X}^{O,freeFP}_{T_O,T_{END}}$. These forecasts differ only in historical data realizations on the same filtration ranges. Again, they can be decomposed simply by finding the differences in these particular factors and then, using the linearity of the system, filtering them (factor by factor and period by period) with the forecasting tool, tracking the changes in all variables.

$$\begin{aligned}
\mathbb{D}_{T_{END}} &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{O}_{T_O,T_{END}}) = \\
&= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{N,freeP}_{T_N,T_{END}}) + (\mathbb{X}^{N,freeP}_{T_N,T_{END}} - \mathbb{X}^{N,freeFP}_{T_N,T_{END}}) + \\
&+ (\mathbb{X}^{N,freeFP}_{T_N,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - \\
&- (\mathbb{X}^{O,freeP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - (\mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}) \\
&= \mathbb{D}^{N,freeP}_{T_N,T_{END}} + \mathbb{D}^{N,freeF}_{T_N,T_{END}} + \\
&+ \mathbb{D}^{(N-O),history,freeFP}_{T_O,T_{END}} - \mathbb{D}^{O,freeF}_{T_O,T_{END}} - \mathbb{D}^{O,freeP}_{T_O,T_{END}}
\end{aligned} \tag{5.5}$$

5.4 New data release on transition period

The most complicated case of forecast decomposition analysis is when the filter ranges of both forecasts differ. A new problem arises, because the old forecast is projected and new forecast is filtered on the same - transition - range, and these are inconsistent due to handling of expected shocks. However, if the same forecasting tool f_T is used for filtration as well as for projection and neither observations nor expert judgments are applied in transition and future ranges, then the filtration and projection on the transition range are interchangeable.

Therefore, we may freely change the end of the history of the forecast without expert judgments on projection and filtration with revised history, i.e., $\mathbb{X}^{rev,freeFP}_{T_O,T_{END}} = \mathbb{X}^{rev,freeFP}_{T_N,T_{END}}$. The decomposition is then computed as follows:

$$\begin{aligned}
\mathbb{D}_{T_{END}} &= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{O}_{T_O,T_{END}}) = \\
&= (\mathbb{X}^{N}_{T_N,T_{END}} - \mathbb{X}^{N,freeP}_{T_N,T_{END}}) + (\mathbb{X}^{N,freeP}_{T_N,T_{END}} - \mathbb{X}^{N,freeFP}_{T_N,T_{END}}) + \\
&+ (\mathbb{X}^{N,freeFP}_{T_N,T_{END}} - \mathbb{X}^{rev,freeFP}_{T_N,T_{END}}) + (\mathbb{X}^{rev,freeFP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - \\
&- (\mathbb{X}^{O,freeP}_{T_O,T_{END}} - \mathbb{X}^{O,freeFP}_{T_O,T_{END}}) - (\mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}) \\
&= \mathbb{D}^{N,freeP}_{T_N,T_{END}} + \mathbb{D}^{N,freeF}_{T_N,T_{END}} + \mathbb{D}^{(N-O),transition,freeFP}_{T_N,T_{END}} + \\
&+ \mathbb{D}^{(N-O),history,freeFP}_{T_O,T_{END}} - \mathbb{D}^{O,freeF}_{T_O,T_{END}} - \mathbb{D}^{O,freeP}_{T_O,T_{END}}
\end{aligned} \tag{5.6}$$

Apart from the problem with decomposition on the transition range, the interpretation of effects of factors becomes an issue. Forecaster has to distinguish which part of the effects of the expert judgments on the projection range in the old forecast should be attributed to the transition range, i.e., corresponds with the effects of the new released data on the transition range in the new forecast. In other words, the question is what part of the effects of the expert judgments on the projection range in the old forecast can be attributed to the changes in initial states in the new forecast, and what part of these effects can be attributed to the changes in expert judgments on the projection range from old forecast to the new forecast.

We define another partial decomposition, similar to the partial decomposition of expert judgments on projection for the old forecast $\mathbb{D}^{O,freeP}_{T_O,T_{END}} = \mathbb{X}^{O}_{T_O,T_{END}} - \mathbb{X}^{O,freeP}_{T_O,T_{END}}$, where we change the end of the history to the T_N. Then the part of the effects of the expert judgments on the projection range in the old forecast that is attributed to the *future range* consist of the effects of all changes in the expert judgments on projection when the end of the history is set to T_N

$$\mathbb{D}^{O,future,freeP}_{T_N,T_{END}} = \mathbb{X}^{O}_{T_N,T_{END}} - \mathbb{X}^{O,freeP}_{T_N,T_{END}}. \tag{5.7}$$

Consequently, the part of the effects of the expert judgments on the projection range in the old forecast that is attributed to the *transition range* is the rest:

$$\mathbb{D}^{O,transition,freeP}_{T_N,T_{END}} = \mathbb{D}^{O,freeP}_{T_O,T_{END}} - \mathbb{D}^{O,future,freeP}_{T_N,T_{END}}. \tag{5.8}$$

DISCUSSION OF THE EXOGENOUS AND ENDOGENOUS GROUPS AND EXAMPLES WILL BE PROVIDED IN THE FINAL VERSION OF THE PAPER

6. Conclusions

Following the work of Andrle et al. (2009) and Andrle (forthcoming) we develop a forecasting decomposition analysis framework that can be applied to decompose the differences between two forecasts generated by any linearized state-space model with multivariate linear filter. We express the differences in the forecasts as the contribution of various factors. We also acknowledge the problem of mutual interactions among factors and propose adjustment of the forecast decomposition analysis by assigning them into endogenous and exogenous groups.

We design a set of partial decompositions that simplify the decomposition of changes in specific groups of factors even when the filtration and projection ranges in the forecasts differ. Main advantage of the presented approach is its flexibility as it can be applied to a variety of decomposition problems.

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