Liquidity, Term Spreads and Monetary Policy

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Abstract

We propose a model that delivers endogenous variations in term spreads driven primarily by banks’ portfolio decision and their appetite to bear the risk of maturity transformation. We show that fluctuations of the future profitability of banks’ portfolios affect their ability to cover for any liquidity shortage and hence influence the premium they require to carry maturity risk. During a boom, profitability is increasing and thus spreads are low, while during a recession profitability is decreasing and spreads are high, in accordance with the cyclical properties of term spreads in the data. We also present empirical evidence on the linkages between yield spreads and financial businesses’ expected profitability. Finally, we use the model to look at monetary policy and show that allowing banks to sell long-term assets to the central bank after a liquidity shock leads to a sharp decrease in long-term rates and term spreads. Furthermore, such interventions have significant impact on long-term investment, decreasing the amplitude of output responses after a liquidity shock. The short-term rate does not need to be decreased as much and inflation turns out to be much higher than if no QE interventions were done.

JEL Codes: E43, E44, E52, G20

Keyword: Yield Curve, Quantitative Easing, Financial Business Profits, Bank Portfolio

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1. Introduction

The presence of term spreads has implications on financial transactions, macroeconomic outcomes and policy design. The existing literature finds that the slope of the yield curve has significant predictive power in explaining US business cycle fluctuations (see, for instance, Estrella and Hardouvelis (1991) or more recently Rudebusch and Williams (2009)). This is not only linked to the fact the slope is related to the future path of short-term interest rates but also due to the changes in the term spread. Moreover, recent large scale purchase programmes adopted by the Federal Reserve Bank, the Bank of England and the European Central Bank have had significant impacts on the shape of the yield curve, increasing the importance of understanding and modeling the fluctuations in term spreads. While there is a growing literature on term spreads and macroeconomic outcomes, there is no consensus as to which are the determinants of time varying risk premia, one of the key components behind the variations in the shape of the yield curve. In this paper we propose a model that provides an explanation for endogenous variations in term spread driven primarily by changes in banks’ balance sheets, their expected profitability and their appetite to bear the risk of maturity transformation.

The first structural models that focus on the term structure of interest rates rely on the expectations hypothesis limiting the analysis to cases of either no or constant risk premium. There is evidence, however, that the term premium is time varying; therefore these models are far from satisfactory. In a natural development from these earlier contributions, structural models were built focusing on the variability of the stochastic discount factor and its links with macroeconomic variables (macro-finance or affine term structure models). The literature is extensive. Recent work by Piazzesi and Schneider (2007) and by Rudebusch and Swanson (2008a) model risk premium as an outcome of the negative covariance between inflation and consumption growth. In this framework financial investors demand a higher risk premium as a hedge against (long-term) inflation risk.

Although supporting the view that long run inflation risk is an important determinant of term spread fluctuations, three main empirical facts motivate the search for additional factors external to monetary policy. First, dynamics of short-run rates and inflation expectations do not explain all the variability of long-term rates, particularly in the last decade (De Graeve, Emiris, and Wouters (2009)). Second, Benati and Goodhart (2008) observe that during the 2000’s the marginal predictive content of term spreads to future output increased, although monetary policy uncertainty remained low. Finally, as stressed by Gürkaynak and Wright (2010), the US treasury inflation protected securities (TIPS) forward rate dynamics have not been that different than their nominal counterparts, indicating the term premia are also influenced by real factors.

As a result, we focus on the role of financial intermediation on the determination of term spreads. We develop a DSGE model with endogenous term spreads derived from bank’s portfolio choice and risk assessment of potential liquidity shortages impacting their profitability and balance sheets. The funding or banking structure of our model, which focuses on potential liquidity risks,
relies on the contributions\(^4\) of Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005). We assume that firms’ long-term projects may suffer from potential liquidity shortages during their first periods, thus these are only completed if a liquidity injection, ultimately made by the bank, is done. Banks fund their portfolio of equities, short and long-term lending with short-term borrowing, thus they bear the risk of maturity transformation. Term spreads are then determined by the volatility of future short-term rates, as usual in the macro finance literature, and the additional element introduced here, the premium for bearing the maturity risk. Another relevant feature of our framework, which is not present in macro finance models, is that the term spread has an explicit impact on the fundamental variables, since long-term rates influence investment on capital formation.

Confirming the stylized facts we obtain counter-cyclical term spread movements. Furthermore, we observe that term premia are good predictors of future activity. On the one hand, as the economy approaches the peak of the business cycle spreads tend to be at their lowest and will tend to increase thereafter. On the other hand, as the economy approaches the bottom of the business cycle, spreads tend to be at their highest. The main driver of these endogenous movements in long-term rates and term spreads is the fluctuations of the future profitability of banks’ portfolios. Banks rely on the overall profitability of their portfolio to assess their ability to cover for any liquidity shortages. Hence, future profitability relates directly to their appetite to bear maturity risk and thus the risk premia they require to commit to provide long-term funding to firms. As output is increasing, profitability is expected to be high and hence spreads are low. During recessions, profitability is low and thus spreads tend to be high.

We also show that our model delivers considerably more volatile term spreads compared to the affine term structure models although we do not assume higher degree of risk aversion nor introduce implausibly high variance for exogenous disturbances. The main reasons for this result are (i) spread movements in our model are driven by bank profits which are more volatile than consumption (the determinant of spread movements in affine models such as Rudebusch and Swanson (2008b)) and (ii) short-term and long-term markets are segmented in our model.

The importance of bank balance sheets and bank risk taking has been recently stressed by Adrian and Shin (2009, 2010), and Adrian, Shin, and Moench (2010). They show that financial intermediary balance sheets contain strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios; higher banking asset growth is related to decreasing risk premia. This link between bank’s balance sheets and risk premia is indeed present in our model. As the bank expects higher profits, it increases asset holdings and consequently spreads decrease. Thus, the bank’s portfolio choice in the model provides a rationale for the link between balance sheet asset growth and risk premia. While those authors focus more on leveraging and asset growth rates, we look particularly at the role of the variability of profits and the maturity transformation risk. Complementing their macro empirical evidence we find a statistically significant link between yield spreads and changes in expected financial sector profitability using macro data from the U.S. economy, giving support to our main theoretical channel. It is important to remark that our bank’s portfolio choice mechanism does not depend on the nominal frictions introduced in the model for the purpose of analyzing monetary policy.

\(^4\)Although we simplify the model structure excluding moral hazard problems and the potential for bank failures to focus particularly on term spreads.
Finally, we use the model to consider the impact of both conventional and unconventional monetary policies. Although we find that the base rate dynamics are implicitly influenced by movements in term spreads due to their impact on output and inflation, we do not find that responding explicitly to their movements (altering the standard (Taylor) monetary rule to include term spreads) leads to greater stabilization. We then look at the impact of unconventional monetary policy similar to the recent quantitative easing (QE) adopted in the US and the UK. Note that the channel through which QE affects the economy in our framework is distinct to the one stressed in some of the recent theoretical papers (see, among others, Gertler and Kiyotaki (2009)). In the latter the mechanism is generally through a direct replenishing of banking capital, covering for current shortages. In contrast, unconventional monetary policy in our framework aims at protecting banks from potential liquidity shortages in the future, increasing their willingness to carry maturity transformation risk, or in other words, reducing term spreads. We find that allowing banks to sell long-term assets to the central bank after a liquidity shock leads to a sharp decrease in long-term rates and term spreads matching the results found by several empirical studies on the recent QE policies in the US, UK and the Eurozone. Furthermore, such interventions have significant impact on long-term investment, decreasing the amplitude of output responses after a significant liquidity shock. The base rate does not need to decrease as much and inflation turns out to be higher than if no QE interventions were implemented.

The paper is organized as follows. Section 2 presents a simple partial equilibrium model to highlight the relationship between banks’ profitability, balance sheets and term spreads. The empirical evidence on the link between financial business profitability, output and spreads in support of our channel is presented in Section 3. Section 4 describes the general equilibrium model of endogenous term spreads. We start presenting our results in Section 5, focusing on the main drivers of the endogenous movements in term spreads, their potential to predict future output growth and their amplification effects. Section 6 presents the volatility in term spreads at different points of the yield curve, comparing to the data and the figures obtained by Rudebusch and Swanson (2008b). Section 7 discusses conventional and unconventional monetary policies. Finally, Section 8 concludes.

2. A Simple Model of Bank’s Portfolio Choice

We start by presenting a simple partial equilibrium model of bank’s portfolio decision aiming at explaining the basic link between bank’s balance sheet (portfolio), their profits and term spreads. In the main part of the paper we embed a similar decision problem into a general equilibrium model to explore the effects of endogenous fluctuations of spreads on economic activity and monetary policy.

The simple model has three periods. At period zero the bank selects a portfolio of assets and holds them until maturity. While assets are in the balance sheet the bank must fund them with deposits. We assume bank’s portfolio may contain three assets, namely, a long-term asset ($X_L$), a short-term asset ($X_S$) and equity (or a portfolio of the rest of risky short-term assets available in the economy), denoted $Z$. The short-term asset pays out a certain return of $R_S$ one period after the portfolio has been set. Equities also pay out in the period 1 but their return $R_Z$ is uncertain; we assume $R_Z \sim N(\bar{R}_Z, \sigma_Z^2)$. Long-term assets mature and pay out a certain return of $R_L$ two periods
after the portfolio decision, but the bank might be forced to make an injection of liquidity ($\rho$) to keep the asset in the portfolio during period 1; we assume $\rho \sim N(\bar{\rho}, \sigma_\rho^2)$. This liquidity injection, also used in Holmstrom and Tirole (1998), effectively implies that the bank may be exposed to cash flow shocks at period 1 and leads to an ex-post revaluation of the overall return on long-term asset holdings, replicating problems of balance sheet funding. Finally, we denote $\text{cov}_{Z, \rho}$ the correlation index between the two disturbances. The bank fully funds its portfolio with deposits that provide the holder a gross return of $R_D$.

In order to depict the basic channel as clearly as possible we simplify a number of features that are part of the bank’s portfolio choice in our main model (Section 4). Firstly, we focus on the portfolio decision at time zero only, with short-term assets and equity remaining in the balance sheet for one period and long-term assets for two periods. Secondly, we assume deposits are in infinite supply at the equilibrium short-term rate and $R_D$ is exogenously set and constant for the two periods the long-term assets are held. These simplifications allow us to concentrate on the bank’s decision of how much long-term asset to hold at time zero, the point when long-term rates, and thus term spreads, are set. The main factors affecting this decision will then be the bank’s expected profitability and the liquidity risk the long-term asset holder bears.

If banks have no regard for risk, being risk neutral and/or not having to abide by any constraint on risk taking, term spreads would be constant (this can be easily seen by setting $\sigma_B = 0$ in (6)). Hence, in order to study term spread fluctuations we assume banks care about risk in two possible ways. In the first case, also used in our general equilibrium model, we assume banks are risk averse, maximizing a constant relative risk aversion function of profits. In the second case, we assume banks are risk neutral, thus maximizing expected profits, but are subject to a value-at-risk constraint. Both cases deliver a similar association between expected portfolio returns and term spread fluctuations. The bank problem, formally, is

\[
\max_{\{X_S, X_L, Z\}} \quad E[\Pi^B] \\
\text{s.t.} \quad \Pi^B = \frac{(\Pi_1^B)^{1-\sigma_B}}{1-\sigma_B} + \beta \frac{(\Pi_2^S)^{1-\sigma_B}}{1-\sigma_B} \\
\Pi_1^B = (R_S - 1)Z + (R_S - 1)X_S - \rho X_L - (R_D - 1)D_0 \\
\Pi_2^B = (R_L - 1)X_L - (R_D - 1)D_1 \\
D_0 = Z + X_L + X_S \\
D_1 = X_L \\
\text{VaR}(\Pi^B) \geq \Xi \text{ iff } \sigma_B = 0
\]  

where $\text{VaR}(\Pi^B)$ is the value-at-risk of the bank’s portfolio defined as the expected minimum portfolio return over the two periods within a 1% confidence interval and $\Xi$ is the limit on that minimum return imposed on the bank when risk neutrality is assumed.  

\footnote{Note that ex-post revaluations might occur if a portion of long-term assets must be sold due to lack of funding. Hence, although the liquidity injection in the model occurs on the asset side of the bank’s balance sheet, it can also be understood as a reduced form liability shortage shock.}

\footnote{Note that volatility of short-term funding costs could also generate increased maturity transformation risk. In the general equilibrium model in Section 4 short-term rates will be endogenous and thus we incorporate this risk there.}
It is straightforward to see that short-term rates will be equal to the return on deposits. The key equations to determine the bank’s portfolio and the return on risky assets are, therefore, given by

\[
- \mathbb{E} \left[ \left( \Pi^B_1 \right)^{-\sigma_B} (R_D - 1 + \rho) \right] + \beta \mathbb{E} \left[ \left( \Pi^B_2 \right)^{-\sigma_B} (R_L - R_D) \right] + 2 \frac{\partial \text{VaR}}{\partial X_L} \zeta = 0 \quad (2)
\]

\[
- \mathbb{E} \left[ \left( \Pi^B_1 \right)^{-\sigma_B} (R_Z - R_D) \right] + 2 \frac{\partial \text{VaR}}{\partial Z} \zeta = 0 \quad (3)
\]

where \( \mathbb{J} \) is an indicator function that takes the value of 1 when \( \sigma_B = 0 \) and zero otherwise\(^7\) and \( \zeta \) is the lagrange multiplier of the value-at-risk constraint. In effect, these two equations above determine the bank’s demand for equity and long-term assets. In order to obtain an equilibrium for these two markets we assume that\(^8\)

\[
X_L = \frac{\gamma_S S}{R_L + \mathbb{E} |\rho|} \quad (4)
\]

\[
\bar{R}_Z = \alpha_Z - \gamma_Z Z. \quad (5)
\]

These assumptions imply that as the supply of long-term assets to the bank (or the investors’ demand for long-term loans) decreases the long-term rate will increase and as the demand for equity increases its expected return will decrease. Once again, these assumptions are used such that the main channel between portfolio choice and spreads can be highlighted; these supply conditions will be endogenous in the general equilibrium model presented in Section 4.

**Case 1 : \( \sigma_B > 1 \)**

If banks are risk averse, the equilibrium condition that determines the long-term rate is simply given by

\[
\mathbb{E} \left[ \left( \Pi^B_1 \right)^{-\sigma_B} (R_D - 1 + \rho) \right] = \beta \mathbb{E} \left[ \left( \Pi^B_2 \right)^{-\sigma_B} (R_L - R_D) \right]. \quad (6)
\]

It is easy to see that when expected profitability in period 1 increases relative to the profitability in period 2, due to higher expected return on equity, then the left-hand side term of (6) decreases, and in equilibrium, long-term rates will also decrease and the bank’s demand for long-term assets will increase. Note that the covariance between profitability (\( \Pi^B_1 \)) and future liquidity shortages (\( \rho \)) will also be important to determine the extent to which an increase in profitability affects the decrease in the LHS of (6). We will discuss the role of this covariance in more detail in the general equilibrium model. Also note that in this simple model we assume the deposit rate remains constant in period 1. In the main model of the paper this will also be uncertain, affecting the term premia.

**Case 2 : \( \sigma_B = 0 \)**

In both Case 1 and in the general equilibrium model (Section 4) we assume banks are risk averse. That delivers a straightforward link between expected profits and term premia; banks require lower premia in periods in which profits are expected to be high. However, we only require

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\(^7\)The VaR constraint binds since expected profit increases and VaR decreases as \( X_L \) and \( Z \) increase.

\(^8\)The bank takes \( \bar{R}_Z \) as given while selecting \( Z \).
that banks care about some measure of risk in order to establish a link between profitability and risk premium. As such, in this case we assume that banks are risk neutral but are subject to a value-at-risk constraint. Our main interest is to verify how term spreads move as the overall profitability (expected returns) of the bank’s portfolio varies. In doing so we aim at establishing a link between bank’s appetite to accumulate long-term assets, incurring the risk of maturity transformation, the equilibrium long-term rates and the expected performance of bank’s investments. The details of the solution of the model in this case is given in the appendix with the discussion provided here.

As $\alpha_Z$ (constant term that controls expected return on equity, see (5)) increases, the return on equity, holding demand for $Z$ constant, rises. That implies that if banks were to hold the same portfolio, their VaR (expected minimum return) would increase above the constraint. That would allow banks to increase the demand for both equity and long-term assets, increasing expected profit, until the constraint becomes binding again (akin to an income effect). Additionally, banks could increase demand for equity and decrease the demand for long-term assets, as equity became the relatively better asset (substitution effect)$^9$. As long as there is a gain in asset diversification in the bank’s portfolio (or $\alpha_Z\rho < 0$), then the income effect dominates$^{10}$ and the demand for long-term assets will increase as expected return on equity, or profitability, increases. Consequently, as portfolio returns increase, long-term rates decrease, leading to a flatter yield curve or narrower term spreads. As the expected profitability of banks increase they are willing to charge a lower premium to bear the risk of maturity transformation, increasing their balance sheet position until reaching the constraint on expected minimum return (or maximum expected losses). The additional expected profits can then be used to cover potential liquidity injections needed to maintain long-term assets in the balance sheet.

Therefore, in both Case 1 and Case 2, during periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, charging a lower risk premium. Higher profits on investments allow banks to cover for potential liquidity shortages. These fluctuations of expected bank profitability, therefore, lead to endogenous movements in term spreads. We now look at some macro empirical evidence on the relevance of financial business profitability in explaining real output growth and the linkages between spreads and expected profitability.

3. **Empirical Evidence**

As argued in the introduction there is strong evidence that US term spreads help predicting US real output growth (see, for instance, Rudebusch and Williams (2009)). Furthermore, Adrian, Shin, and Moench (2010) highlight the importance of financial sector variables, particularly the growth in financial intermediary asset holdings, in predicting several asset prices and risk measures. Finally, Adrian, Estrella, and Shin (2010) look at the link between term spreads, future banking asset growth and economic activity. We complement this macro empirical evidence by

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$^9$Shin (2009) provides a similar mechanism of the effect of VaR on bank’s portfolio decision, although he focuses on the the overall effect of reducing default probability on asset growth while we look into the cross-asset or portfolio balancing effect.

$^{10}$The strength of the substitution effect is directly related to the covariance between the asset returns. As the covariance decreases so does the substitution effect and more likely it is that the demand for long-term assets will increase with $\alpha_Z$. 

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looking particularly at the linkages between financial sector profitability, term spreads and output growth.

We use quarterly US data covering the period 1980Q1-2008Q2. Our data consists of seasonally adjusted US real GDP expressed in billions of chained 2005 Dollars as reported by the Bureau of Economic Analysis and financial business undistributed corporate profits as reported by the Flow of Funds Statistics of the US Federal Reserve. This measure is for all financial businesses, including not only commercial banks but also for instance mutual funds and security brokers and dealers (see the Appendix B for further details on the data used). Term spreads are computed using the US Treasury Bill rate in percent per annum and 10 year government bond rate in percent per annum as reported by the IMF/IFS.

We first investigate the additional marginal predictive content in movements of financial business profits to explain real output movements next to autoregressive components of real output, past term spreads and the T-Bill rate (which controls for the level of interest rates in the economy). We focus explicitly on correlation structures as the preliminary test of statistical connectedness between variables. The reduced form/information value approach is immune to questions of causality and exogeneity issues as first advocated by Sims (1972, 1980) and Friedman and Kuttner (1992) among others. Secondly, we study linkages between term spreads and the changes in expected financial sector profitability. Term spreads are inherently forward looking variables reflecting the future path of short-term rates plus the risk premia from holding long-term positions, and hence if the channel explained above exists, variation in spreads should contain information on the expected financial business profitability.

3.1. Information Content of Financial Business Profitability

Our primary interest is to investigate whether variations in financial sector profits contain exploitable information that will help predict variations in real output beyond those already predictable by using past variations in real output, term spreads and the short-term interest rates, similar in nature to the analysis conducted by Benati and Goodhart (2008).

Our specification for real output changes is given by

\[ \Delta y_t = \alpha + \sum_{i=1}^{8} \beta_i \Delta y_{t-i} + \sum_{i=1}^{8} \gamma_i (r_{L, t-i}^L - r_{S, t-i}^S) + \sum_{i=1}^{8} \delta_i \Delta \pi_{t-i} + \sum_{i=1}^{8} \zeta_i \Delta r_{S, t-i}^S + \epsilon_t \]  

(7)

The terms \( \Delta y_t \), \((r^L - r^S)\), \(\Delta \pi\) and \(\epsilon\) represent the annualized changes in output, the spread between 10 year government bond \((r^L)\) and 3-months Treasury Bill \((r^S)\), the annualized changes in financial business profits and an error term, respectively. We include 8 lags for independent variables as an inverted yield curve is found to contain predictive power for recessions within a 12 to 18 months period (see for instance, Estrella and Hardouvelis (1991) and Rudebusch and Williams (2009)). We test whether the lagged coefficients of each variable are jointly significant (Wald test). In the first estimation (A) we include all four explanatory variables with lags (as in equation (7)) and in a second estimation (B) we include variations in lagged output, lagged short-term interest rates

\[ \text{We have also included inflation in the benchmark estimation but all lags were shown to be jointly insignificant. Furthermore, we estimate a similar equation for financial profits and spreads and find that term spreads significantly explain changes in profits and vice versa. Results are available upon request.} \]
Table 1: Measuring the Marginal Predictive Content of Financial Business Profits for Output Growth

<table>
<thead>
<tr>
<th>Estimation A</th>
<th>Estimation B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980Q1-2008Q2</td>
<td>1980Q1-2008Q2</td>
</tr>
<tr>
<td>$\chi - Square$</td>
<td>$\chi - Square$</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>45.82</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>15.20</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>20.50</td>
</tr>
<tr>
<td>$\zeta_t$</td>
<td>19.93</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.58</td>
</tr>
</tbody>
</table>

and lagged spreads only, excluding financial sector profits (setting $\delta_t = 0$ in equation (7)). Table 1 reports the $\chi - Square$ statistics and the $p-values$ of the estimated coefficients for each given lag structured$^{12}$.

Results for Estimation A reveal that the marginal predictive content of both spreads and variations in financial business profits are statistically significant$^{13}$. That is variations in financial business profits add information in predicting real output movements next to variations in past real output, term spreads and short-term interest rates (see Estimation B).

3.2. Spreads and Expected Financial Business Profitability

In the previous section we argue that both spreads and financial business profitability are closely linked and are important to predict future output growth. Furthermore, term spreads are inherently forward looking variables reflecting the future path of short-term rates plus the risk premia from holding long-term positions, and hence variation in spreads should contain information on the expected profitability. For this purpose we follow Wright (2006) closely and analyze whether spreads today are associated with future decreases in profitability.

Let the yearly financial business profits to be equal to $\pi_Y = \sum_{i=1}^{4} \pi_i$. The change in profits at period $t$ is given by $\Delta \pi_Y = \frac{\sum_{i=1}^{4} \pi_i}{4}$. We then construct a binary variable ($D\pi_Y$) that takes the value 1 when financial business profitability is decreasing and zero otherwise. Formally,

$$D\pi_Y = \begin{cases} 1 & \text{if } \Delta \pi_Y < \Delta \pi_Y_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

We employ the following probit model to assess whether spreads are linked to future profitability, estimating$^{14}$

12 We use White heteroskedasticity consistent standard errors.

13 In order to gauge an idea of the stability of the information content of changes in financial business profits to explain output, we have recursively estimated Equation 7 in a rolling fashion; first estimation being for the period 1970Q1-2000Q1 and rolling estimation sample one period forward at a time. Wald tests indicate the association between financial business profits changes and real output is reasonably stable. Results are available upon request.

14 We estimate $D\pi_{Yt+6}$ on term spreads determined at time $t$, which reflect the slope of yield curve from quarter $t + 1$.
Table 2: Probit Results - Spreads and Future Profitability

<table>
<thead>
<tr>
<th>Term Spread ($\gamma_1$)</th>
<th>Coefficient</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term Spread ($\gamma_1$)</td>
<td>0.222</td>
<td>0.0187</td>
</tr>
</tbody>
</table>

Mc Fadden $R^2$ | 0.037

$P(D\pi^Y_{t+6} = 1) = \Phi(\gamma_0 + \gamma_1 (r^L_t - r^S_t))$,

where $\Phi(\_\_)$ denotes the standard normal cumulative distribution function. Wright (2006) performs a similar estimation linking term spreads with a dummy variable that takes the value of 1 when a recession occurs, concluding that low spreads are linked with a high probability of observing a recession. If $\gamma_1$ is significant then spreads are relevant in predicting the future (negative) movements in profitability. Table 2 shows the results. The estimated coefficient for the term spread is significant and positive. This suggests that high term spreads are associated with a higher probability of observing decreasing financial business profitability in the future. We also run an augmented probit model with spreads and the 3 month T-Bill. Results remain qualitatively the same.

This evidence presented here indicates that expected profitability may influence term spreads, providing support to the channel explained in our simple partial equilibrium model.

4. General Equilibrium Model

In Section 2 we presented a partial equilibrium model of bank’s portfolio choice, establishing a link between bank’s appetite to bear maturity transformation risk, its balance sheet holdings and its expected profitability from portfolio investments. During periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, increasing long-term asset holdings and charging lower risk premia. We also showed that at a macro level, expected financial business profitability and term spreads are found to be linked as our theoretical channel suggests. Finally, the portfolio choice discussed also provides a rationale for the link between balance sheet asset growth and risk premia pointed out by Adrian, Shin, and Moench (2010). In the remainder of the paper we focus on extending the simple portfolio choice model and embedding it into a DSGE framework to study endogenous fluctuations of spreads and its effects on economic activity and monetary policy.

The model economy is populated by a continuum $i \in [0, 1]$ of intermediate good firms, a final good producer, a continuum of households, banks, entrepreneurs and the central bank. Entrepreneurs borrow funds from a bank and transform consumption goods into capital. There are two types of entrepreneurs, one with access to a short-run investment project and one with a long-run investment opportunity available. This introduces a segmentation of short and long-term funding requirements, similar to the one stressed by Vayanos and Vila (2009). Firm $i$ hires labour to quarter $t + 40$ (3 month to 10 year points), since that binary variable depends on financial business profits from $t + 1$ until $t + 6$. 

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from households, produces a differentiated input using labour and the current capital stock, and at the end of the period sells the inputs to the final good firm and buys new capital from entrepreneurs. The final good firm combines all inputs to produce consumption goods that are then sold to households and entrepreneurs. We assume households (workers) receive the profits from banks and entrepreneurs, which are all of unit mass. Thus, only households consume. An equivalent alternative would be to follow a similar model structure of Gertler and Kiyotaki (2009), where a family is split into banks and consumers but consumption is done at the family level.

The bank receives deposits from households, provides loans to both entrepreneur types and buys equity from the intermediate firms. Note that long-term loans are issued at every period but do last for two periods, thus banks’ balance sheets will contain three loan agreements. These are: a short-term loan, a long-term loan and another long-term loan issued in the previous period. Finally, we assume that during the current period the bank might need to provide a liquidity injection to long-term entrepreneurs who borrowed in the previous period. Figure 1 shows the production and financial flows of the model.

### 4.1. Households

The household maximizes its expected discounted lifetime utility given by

$$\max_{C_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0, 1) \quad \sigma, \eta, \chi > 0$$

where $C_t$ denotes the household’s total consumption and $H_t$ denotes the composite labour index. The curvature parameters $\sigma, \eta$ are strictly positive. $\beta$ is the discount factor. The household faces the following budget constraint

$$C_t + \frac{D_t}{P_t} \leq \frac{W_t H_t}{P_t} + \frac{R_{t-1,CB} D_{t-1}}{P_t} + \frac{\tilde{\Pi}_t}{P_t}$$

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**Figure 1: Model Structure**

(a) Production Flows

(b) Financial Flows
where $W_t$ is the wage index and $R_{t,CB}$ is the rate of return on deposits $D_t$. The central bank sets $R_{t,CB}$ directly according to a monetary policy rule to be specified. We assume the only asset available to the “worker” is a deposit made directly to the financial intermediary$^{15}$, thus only banks invest in equities issued by the intermediate good firms and lend to entrepreneurs. Although not modeled here, one reason for that would be the existence of higher household-firm agency costs relative to bank-firm agency costs$^{16}$.

Finally, $\tilde{\Pi}_t = \tilde{\Pi}^{ES}_t + \tilde{\Pi}^{EL}_t + \Pi^B_t$ is the sum of the nominal profits, realized at period $t$, for entrepreneurs with short-term projects, with long-term projects and the bank, respectively, which is passed on to the household.

4.1.1. Optimal Wage Setting

Households supply a continuum of labour types $j \in [0, 1]$. The composite labour index $H_t$ is then given by

$$H_t = \left[ \int_0^1 H_{j,t}^{j+1} \right]^{\frac{1}{1-\varepsilon_w}}.$$

From the subsequent firms minimization problem we have that the demand for each labour type and the wage index are given by

$$H_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{-\varepsilon_w} H_t,$$

$$W_t = \left[ \int_0^1 W_{j,t}^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}}.$$

Households, when allowed (Calvo scheme with parameter $\omega_w$), set wages $W_{j,t}$ to maximize expected utility subject to the budget constraint and the labour demand equation. The main reason to include both price and wage rigidity is to ensure firm’s real profits are pro-cyclical after a productivity shock (see Carlstrom and Fuerst (2007)).

4.2. Entrepreneurs

Entrepreneurs are responsible for capital formation. We assume a set of mass unit of entrepreneurs has a short-term investment opportunity available, at each period. Another set of mass unit of entrepreneurs has a long-term (two periods) investment opportunity available, at each period. Thus, there are always three mass units of active entrepreneurs in the economy.

Short-term entrepreneurs borrow funds from the bank ($X_{S,t}$), buy consumption goods and transform it into capital next period with the following production function

$$y^{S}_{t+1} = \gamma S \ln(1 + X_{S,t}).$$

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$^{15}$One may assume workers make deposits on banks owned by other households.

$^{16}$Household-bank agency costs are not modelled. That would be an interesting extension, given that potential changes to the premium households require to fund banks could also affect the bank’s appetite to bear maturity transformation risk.
The capital produced is then sold to the intermediate good firms. The profits of these entrepreneurs are given by

$$\tilde{\Pi}^S_{t+1} = p_{t+1}q^S_t\gamma_S\ln(1 + X_{S,t}) - R_{t,S}P_tX_{S,t},$$

where $R_{t,S}$ is the gross interest rate on short-term borrowing and $q^S_t$ is the price of short-term capital denominated in consumption goods. Short-term entrepreneurs select $X_{S,t}$ to maximize expected profits.

Long-term entrepreneurs also borrow from the bank ($X_{L,t}$), buy consumption goods and transform it into capital after two periods with the following production function

$$y^L_{t+2} = \gamma_L \ln(1 + X_{L,t}),$$

where $\gamma_L > \gamma_S$.

The production function for short and long-term capital output ($yk^m$) take the form $\gamma_m\ln(1 + X_{m,t})$ for $m = \{S, L\}$ since we need (i) capital production to have decreasing returns (concave function) such that movements in borrowing rates influence the marginal propensity to invest\(^{17}\); (ii) long-term capital investment to be more productive than short-term capital due to the liquidity shock explained below (thus, $\gamma_L > \gamma_S$); and (iii) one unit of consumption invested to return more than one unit of capital goods, thus we normalize the production function to be the log of one plus the amount invested. That way, each unit of consumption good invested ($X_{m,t}$) is turned into one unit of capital plus an increment, which decreases as the amount invested increases, and whose overall size depends on the parameter $\gamma_m$ (this interpretation holds as long as $X_{m,t}$ is small and $\gamma_m$ close to one, which will be the case in our calibration).

Following the contributions of Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005) we assume long-term projects suffer from a potential liquidity shortage during the first period, thus they only get completed if an injection of $\rho_{t+1}X_{L,t}$ is done\(^{18}\). We assume the liquidity shock is given by $\rho_t = (1 - \rho_L)\bar{\rho} + \rho_L\rho_{t-1} + e^L_t,$ where $e^L_t$ is i.i.d.. As entrepreneurs have no income at period 1, this injection is provided by the bank. The capital produced is then sold to the intermediate good firms. Long-term entrepreneurs profits are then given by

$$\tilde{\Pi}^L_{t+2} = p_{t+2}q^L_{t+2}\gamma_L\ln(1 + X_{L,t}) - R_{t,L}P_tX_{L,t} - \rho_{t+1}X_{L,t}p_{t+1},$$

where $R_{t,L}$ is the gross interest rate on long-term borrowing and $q^L_t$ is the price of long-term capital denominated in consumption goods. Long-term entrepreneurs select $X_{L,t}$ to maximize expected profits.

Note that although it is natural to think of $\rho_t > 0$, or potential liquidity shortages, we could also have liquidity gains with $\rho_t < 0$, for instance due to an increase in the prospective gains from securitization of long-term assets in the banks’ balance sheets. In this case, instead of liquidity shortages, the banking sector is characterized by excess liquidity, which would give rise to an inverted yield curve. That might have been the case in the UK during the few years preceding

\(^{17}\)This assumption allows us to have the original Keynesian investment equation dependent on the interest rate.

\(^{18}\)As in Holmstrom and Tirole (1998), the injection is not added as an input to the production function, being just an additional cost (“money”) needed to complete the project.
the 2007 crisis. An interesting extension to the model left for future research is to make $\rho$ endogenous based on potential for securitization versus expected share of non-performing assets and consequent need for liquid funds/provisions.

4.3. Banks

At every period $t$ a bank, representing all financial business in the economy, acquires three types of nominal assets, a short-term debt ($P_t X_{S,t}$), a long-term debt ($P_t X_{L,t}$) and equity ($Z_t$). Furthermore, it has a long-term asset it carries over from last period ($P_{t-1} X_{L,t-1}$). Banks fund these investments with deposits ($D_t$) from households. Equities are acquired from the intermediate good producers. The investment in equity made at time $t$, $Z_t$, pays off a gross dividend at period $t+1$, denoted by $DIV_{t+1}$ (see detail below). Short-term entrepreneurs pay back the loan made at time $t$ in period $t+1$, providing a return to the bank of $(R_{S,t} - 1)P_t X_{S,t}$. Long-term entrepreneurs pay back the loan made at time $t-1$ in period $t+1$, providing a return to the bank of $(R_{L,t-1} - 1)P_{t-1} X_{L,t-1}$ where $R_{L,t-1}$ is the nominal long-term rate set at time $t-1$. Finally, long-term entrepreneurs that borrowed at time $t$ may require a liquidity injection at time $t+1$ of $\rho_{t+1}P_{t+1}$ per unit invested. Hence, bank’s real profits at period $t+1$ are given by

$$\Pi_{t+1}^B = \frac{1}{P_{t+1}} (DIV_{t+1} + (R_{L_{t-1}} - 1)P_{t-1} X_{L_{t-1}} + (R_{S,t} - 1)P_t X_{S,t} - D_t (R_{t, CB} - 1) - \rho_{t+1} X_{L,t} P_{t+1}).$$

We assume banks are risk averse. The only risk involved in the banking business in our model is the maturity transformation risk, since banks must commit to lend money to long-term investment opportunities having to acquire funds next period to re-finance this balance sheet commitment plus any additional liquidity injection needed. Risk aversion here implies that banks do not only care about the return on short and long-term assets, requiring them simply to pay the same expected return on average. Banks will weigh these returns according to the expected profitability of the entire portfolio, requiring higher premium to bear risk when overall profitability is low but accepting lower risk compensation when overall returns are high. Effectively, the bank will care about the covariance between the return of each asset and the return of the overall portfolio.

Note that even if banks were risk neutral, the limits on Value-at-Risk (VaR) banks normally abide to, would effectively imply that overall profitability of assets would influence banks’ required premium to bear maturity risk through the VaR constraint (as discussed in Section 2). Hence, the assumption that banks are risk averse reflects that some measure of overall riskiness and expected profitability affect their long-term rate setting decision or the premium they require for bearing maturity transformation risk. The bank maximization profit problem is given by

$$\max_{\{X_{S,t}, X_{L,t}\}_{t=0}} \mathbb{E}_0 \sum_{t=0}^{\infty} B^t \frac{\Pi_{t+1}^{B, \sigma_B}}{1 - \sigma_B}$$

s.t. $D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1}$,

---

$^{19}$In this paper we do not consider banking capital requirements.

$^{20}$We exclude $Z_t$ from the set of choice variables in the maximization since, as equity is the best asset in the portfolio, paying an expected return higher than the short-term rate, banks always demand the total amount of equity supplied by intermediate firms.
where $\sigma_B$ controls the degree of risk aversion.

4.4. Firms

The final good representative firm combines a continuum of intermediate inputs $i \in [0, 1]$ with the following production function

$$Y_t = \left[ \int_0^1 \frac{e^{y_{ijt}}}{y_{ijt}} \right]^{\frac{1}{\alpha}}.$$  

As standard this implies a demand function given by

$$y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t,$$

where the aggregate price level is

$$P_t = \left[ \int_0^1 P_{it}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

The intermediate sector is constituted of a continuum of firms $i \in [0, 1]$ producing differentiated inputs with the following constant returns to scale production function

$$y_{ijt} = A_t K_{ijt}^S \alpha \zeta K_{ijt}^L \alpha (1-\zeta) H_{ijt}^{1-\alpha},$$

where $A_t$ denotes the productivity level at time $t$ (which follows the standard AR1 process $\log(A_t) = \rho_t \log(A_{t-1}) + e_t$, with $e_t$ i.i.d.), $K_{ijt}^S$ is the capital stock originated from short-term projects, $K_{ijt}^L$ is the capital stock originated from long-term projects, and $H_{ijt}$ is the household composite labour used in production. Each firm hires labour and invests in both stocks of capital. Implicit here is the assumption that short-term and long-term capital are not perfect substitutes, which reflects the fact that long-term projects might have a distinct technological enhancement compared to capital based on short-run investments.

To characterize the problem of intermediate firms, we split their decision into a pricing decision given their real marginal cost and the production decision to minimize costs. Following the standard Calvo pricing scheme ($\omega$), firm $i$, when allowed, sets prices $P_{it}$ according to

$$\max_{P_{it}} E_t \left\{ \sum_{s=0}^{\infty} P_{i+s} Q_{i+s} (W_{ijt} + P_t q_{ijt}^S + P_t q_{ijt}^L) \right\},$$

subject to the demand function (8), where $Q_{i+s}$ is the economy’s stochastic discount factor, defined in the next section and $\Lambda_{i+s}$ is the firm’s $i$ real marginal cost at time $t + s$. To obtain the real marginal cost, we need to solve the firm’s intertemporal cost minimization problem. That is

$$\min_{K_{ijt}^S, K_{ijt}^L, H_{ijt}} E_t \left\{ \sum_{i=0}^{\infty} Q_{0,i} \left( W_{ijt} + P_t q_{ijt}^S + P_t q_{ijt}^L \right) \right\},$$
subject to the production function (9) and investment equation 
\[ I_m^t = K_{i,t+1}^m - (1 - \delta)K_{i,t}^m \] for \( m = \{S, L\} \)\(^{21}\), where \( \delta \) is the depreciation rate.

Finally, dividends\(^{22}\), which are paid one period after production takes place, are given by
\[
DIV_{i,t+1} = P_t Y_{i,t} - W_t H_{i,t} - P_t (q^S_{i,t} Y_{i,t}^S + q^L_{i,t} Y_{i,t}^L) + P_t (q^S_{i,t+1} K_{i,t+1}^S + q^L_{i,t+1} K_{i,t+1}^L) - Z_t,
\]
where \( Z_t = P_{t-1} (q_{S,t-1} K_{S,t-1}^S + q_{L,t-1} K_{L,t-1}^L) \). The first three terms comprise the profits (flow) and the last two the capital gains (due to changes in amount and price of capital held in the firm). We assume equities are bought (or evaluated) by banks at the beginning of time \( t \). Thus, the value of the firm at time \( t \), denoted \( Z_t \), is equal to the value of its capital holdings at the beginning of period \( t \) before production and investment in (new) capital takes place. The two types of capital used in production at time \( t \) are given by \( K_t^S \) and \( K_t^L \).

### 4.5. Market Clearing Conditions

The capital market clearing conditions are given by
\[
I_S^t = y k_t^S = \gamma_S \ln(1 + X_{S,t-1}) \quad \text{and} \quad I_L^t = y k_t^L = \gamma_L \ln(1 + X_{L,t-2}).
\]

The good market clearing condition, or the aggregate demand, is given by
\[
Y_t = C_t + X_{S,t} + X_{L,t} + p_t X_{L,t-1}.
\]

Furthermore, capital and labour markets across firms are aggregated such that
\[
K_t^m = \int_0^1 K_t^m d i \quad \text{for} \quad m = S, L \quad \text{and} \quad H_t = \int_0^1 H_{i,t} d j.
\]

The credit market clearing condition is
\[
\frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{Z_t}{P_t} + \frac{X_{L,t-1}}{\pi_t},
\]
where \( Z_t = \frac{(q_{S,t} K_S^t + q_{L,t} K_L^t)}{\pi_t} \) and \( \pi_t = \frac{p_t}{P_t} \).

Finally, we define the term spread (annual rate in percentage points) between long and short-term rate as
\[
t p_t = \frac{1}{2} ((R_{L,t} - 1) - (R_{t, CB} - 1) - (R_{t+1, CB} - 1)) 400.
\]

\(^{21}\)Note that the demand for each type of labour stated in the household wage setting problem can be obtained by minimizing the total cost of labour \( \int W_{i,j} H_{i,j} d j \) subject to the labour composite index.

\(^{22}\)Dividends here are in fact profits plus capital gains and represent the gross return on equity.
4.6. Equilibrium and Calibration

The equilibrium of the economy is defined as the Lagrange multiplier \( f \Lambda_t g \), the allocation set \( f C_t ; H_t ; K_{S+1} ; K_{L+1} ; X_{S,t} ; X_{L,t} ; Y_t ; D_t ; I_t^S ; I_t^L ; DIV_t ; \Pi^B_t \), and the vector of prices \( P_{jt} , \pi_t , w_t , w_{jt} , R_{L,t} , R_{t,CB} , q^S_t , q^L_t , R_s ; t_p \) such that the household, the final good firm, intermediate firms, entrepreneurs and the bank maximization problems are solved, and the market clearing conditions hold.

Details of the equations that determine the recursive equilibrium and the steady state of the economy are shown in Appendix C. Before discussing the results we quickly present the main parameter values used for the benchmark version of our model. As standard we set the goods market mark-up to 20\%, thus \( \varepsilon = 6 \). The labour market mark-up\(^{23}\) is set to 7.5\% or \( \varepsilon_w = 14 \). We set the discount factor \( \beta = 0.99 \); the intertemporal elasticity of substitution in consumption \( \sigma = 1 \); and the Frisch elasticity of labour supply \( \eta = 1 \). The Calvo price and wage parameters\(^{24}\) are \( \omega = 0.5 \) and \( \omega_w = 0.6 \). The depreciation rate is set to \( \delta = 0.05 \), the share of capital in production to \( \alpha = 0.36 \), and the share of short run capital to \( \zeta = 0.4 \). That ensures that at the steady state the share of long-term loans in total loans are 60\%. Fan, Titman, and Twite (2010) report that the debt maturity ratio, (that is, long-term interest bearing debt over total debt) is about 80\% in the US, 60\% in the UK, 55\% in Germany and 40\% in Japan during the period 1991-2006. They found that the median long-term debt ratio across 39 different countries is estimated to be around 60\%. We set the degree of risk aversion of banks to \( \sigma^B = 1 \), which is the same as the one for the household.

The steady state long-term rate is given by \( R_L = \frac{1}{\beta} + \frac{\rho}{\beta} \) and thus depends on the liquidity shortage at steady state (\( \bar{\rho} \)). We set \( \bar{\rho} = 0.0025 \), such that the 10 year term premium is roughly 100 basis points matching the US data (Rudebusch and Swanson (2008b)). We initially assume that the central bank follows a simple Taylor Rule with inflation parameter \( \varepsilon_\pi = 2.5 \) and output gap parameter \( \varepsilon_Y = 0.125 \). Note that higher values of \( \varepsilon_Y \) and lower values of \( \varepsilon_\pi \) easily lead to indeterminacy issues in models with cost channels (see Aksoy, Basso, and Coto-Martinez (2009)). Finally, we set the persistence of the productivity process \( \rho_A = 0.9 \) and the persistence of liquidity shocks \( \rho_L = 0.9 \), while setting the standard deviation of their respective \( i.i.d \) disturbances to \( \sigma_A = 0.01 \) and \( \sigma_L = 0.0001 \). The model is solved to a third order approximation using Dynare++ (without centralization).

5. Term Spreads and Economic Activity

In this section we analyze the mechanism that drives term spread fluctuations in our model, the effect of these on economic activity and the link between spread movements and future output growth.

\(^{23}\)While some contributions to the DSGE literature set \( \varepsilon_w = 21 \) others set \( \varepsilon_w = 2 \). Our results are unchanged when we vary \( \varepsilon_w \) within this range.

\(^{24}\)These are a bit smaller than the ones obtained in DSGE-based Bayesian estimations. However, all these studies have assumed wage and price indexation decreasing the effect of nominal rigidity on economic activity, while here for simplicity we do not. Our results are unchanged when higher degrees of price and wage rigidity are assumed.
5.1. Endogenous Term Spreads

We start by focusing on the benchmark model and the fluctuation of term spreads after a positive productivity shock. Figure 2 shows impulse responses of the main variables of interest. For all variables the percentage deviation from steady state is shown except for term spreads movements where the change in the percentage rate is reported (thus a 0.5 deviation implies a 50 basis point change in term spreads).

![Figure 2: Benchmark - Productivity Shocks](image)

As expected, a one standard deviation positive productivity shock leads to an increase in output, a decrease in inflation, and higher consumption and long-term investment. While both long-term and the base rate decline, the drop in the long rate is about 3 times larger than the base rate’s deviation leading to a fall in term spreads.

The movements in spreads is mainly due to the response of bank profits after the shock. Banks set a term premium according to the potential costs of liquidity shortage they may face in the future. In periods of higher profits, banks are less likely to suffer balance sheet problems in the event that liquidity injections in long-term projects are needed, since cash flows from profits can be used to cover for these injections. Therefore, bearing maturity risk in these states becomes relatively cheaper such that long-term rates fall, resulting in lower term spreads. In states where profits are expected to decrease the opposite occurs. Note that even though the bank is still subject to liquidity shortages on long-term investment funding in the future, these shortages may occur when banks have higher profits, and hence, we observe an expansion of long-term credit supply today. Consequently, after observing a positive productivity shock in period 1, banks set term
spreads low since they expect profits to be high in period 2, allowing them to potentially use these high profit flows to offset the liquidity injections needed.

Our empirical results highlight exactly that relationship between spreads and financial sector profitability. Higher term spreads indicate higher probability of observing lower future profits. The model also generates a negative link between bank’s asset holdings and term premia. Adrian, Shin, and Moench (2010) present empirical evidence of this relationship across various asset classes. Augmenting our model to include different assets in the bank’s portfolio would allow us to further explore the links between financial intermediation and the macroeconomy, providing additional theoretical support for their findings. In the benchmark specification an expected increase in profits of around 6% leads to a term spread adjustment of around 60 basis points. The standard deviation (quarterly) of Financial Business Profits used in our empirical analysis is around 8% and the standard deviation of term spreads, depending on the maturity, ranges between 50 to 75 basis points. Thus, the magnitudes of the movements in the model are roughly in line to those in the data. In Section 6 we compare the volatility of term spreads in the data and in our model economy in detail.

In order to further investigate the drivers of the endogenous movements in term spreads after productivity shocks we run three variants of our model and compare them to the benchmark case. In the first variant we set a higher steady state liquidity shortage ($\bar{\rho} = 0.0175$), and denote it $\text{high}_\rho$. In the second we set a high variance of liquidity shock ($v_l = 0.04$), denoting it $\text{high}_{v_l}$ and finally, in the third, we assume banks are more risk-averse setting ($\sigma_B = 3$), denoting it $\text{high}_{\sigma_B}$. Figure 3 shows the results.

Firstly, note that, in all three cases bank profits move in a very similar way after a positive productivity shock. In order to distinguish these three cases we need to uncover how the same response in profits leads to different dynamics in term spreads. In the first variant, we observe that spread movements are amplified under higher steady state liquidity shortages. The main intuition for this result is as follows. At steady state, banks set long-term rates higher than short-term rates to offset potential liquidity shortages and hence, the higher $\bar{\rho}$ or the higher the average need for liquidity injection, the higher will the steady state long-term rates be. Given that long-term rates are relatively high, an equivalent increase in bank profits (when compared to the benchmark case) induces a stronger adjustment in long-term rates, which in turn implies long-term rates falling by a greater amount than under the benchmark case.

The opposite occurs when the variance of the liquidity shock is high. Banks are willing to bear more maturity risk in periods of high profits since they know that high profits can be used to offset liquidity shortages. However, the more volatile are these shortages, the less certain the bank will be that high profits will be enough to offset them. Therefore, an equivalent movement in bank profits leads to smoother movements in long-term rates and term spreads after a positive productivity shock.

Finally, the third variant illustrates that the higher the degree of bank risk aversion, the more responsive to productivity shocks term spreads are. $\sigma_B$ effectively determines how fluctuations of bank profits influence the bank’s long-term rates decision. When $\sigma_B \to 0$ banks will set long-term rates to be a discounted sum of short-term rates and term spreads will be constant. This mechanism is the same as the one explored in the macro-finance literature where Epstein-Zin preferences are used to increase risk aversion in order to match volatility of risk/term premia (see Rudebusch and Swanson (2008a)).
The key equilibrium condition (see Appendix C for details) that determines the long-term rate, and consequently the term spread, comes from the bank’s portfolio decision. The bank will set long-term assets holdings ($X_L$) such that

$$E_t \left[ \Pi_B + 1 - \sigma_B \left( \frac{\pi_t}{\pi_{t+1}} + \rho_t \right) + \rho \Gamma \left( \Pi_B + 1 \right) \right] = \beta E_t \left[ 1 + \sigma_B \left( \frac{\Pi_B + 1}{\pi_{t+1}} \right) \right].$$

One can approximate this condition (the derivation is shown in Appendix D) to illustrate, formally, the effect of changes in bank profits on term spread fluctuations.

$$E_t \left[ \hat{p} + 0.5(\hat{p})^2 \right] = E_t \left[ \sigma^2 \left( \Pi_{t+1} - \Pi_{t+1} \right) + 0.5 \sigma^2 \left( \left( \Pi_{t+1} \right)^2 - \left( \Pi_{t+1} \right)^2 \right) + \pi_{t+2} - 0.5 \left( \pi_{t+2} \right)^2 + \rho \left( \pi_{t+1} + 0.5 \left( \pi_{t+1} \right)^2 \right) + CovTerms \right].$$

As easily verified, setting $\sigma_B = 0$, or assuming bank’s utility is linear on profits, eliminates all the effects of movements of profits on term spread decisions. Looking at the covariance terms we observe that the higher the covariance between profits and the liquidity shortage (first term), the lower term spreads will be, with the strength of the effect being positively associated with $\sigma_B$ and
\( \hat{\rho} \). Hence, as we increase \( \hat{\rho} \) movements in spreads, given expected changes in profits, are amplified. Finally, the increase in the variance of the liquidity shock \( v_l \) has two opposing effects. Firstly, it tends to raise spreads since \( E(\hat{\rho}_{t+1})^2 \) has a positive impact on spreads. Secondly, it becomes a stronger driver of the expected covariance between profits and \( \hat{\rho}_{t+1} \). The latter implies that the expected positive movement in profits due to the productivity shock will have little effect on the covariance term and as such one of the key drivers of the endogenous movements of spreads loses its significance. As a result of this effect on the covariance, higher volatility of liquidity shocks dampens the impact of productivity shocks on term spreads.

5.2. Yield Spreads and Output Growth

As reported by Rudebusch, Sack, and Swanson (2007), the Congressional Budget Office output gap and the 10Y term premium during the period 1960 and 2005 seem to be negatively related (see Figure 6 in their paper). Hence, the counter-cyclical movement of spreads obtained in our model after both a productivity and a monetary shock (not reported here) matches the overall characteristic of the US data. As stressed by Gürkaynak and Wright (2010), term structure models should generate a high slope of the yield curve at the beginning of recoveries from recessions and a flat yield curve during booms, feature which is related to the predictive power of yield spreads. Hamilton and Kim (2002) conclude that lower term premiums predict slower GDP growth, although this effect appears to be strong only in the short-run, while Wright (2006) shows that lower term premium raises the odds of a recession.

In order to verify if the dynamics of term spreads in our model is consistent with this feature we study the impact of a three period\(^{25}\) anticipated technology shock (Figure 4). That way, based on the information at time \( t \) banks form an expectation of future growth and profits which will affect long-term rates and thus term spreads. These then feed back to the economy influencing long-term investment and output.

We observe that output and long-term investment increase from \( t \) until \( t + 3 \) (time of the realization of the productivity shock). Therefore, if one is regressing output gains \( \hat{y}_{t+3} - \hat{y}_t \) on \( \hat{\rho}_t \) (a variant of Hamilton and Kim (2002) estimation), getting a positive parameter estimate must imply that \( \hat{\rho}_t > 0 \), which is what we obtain. The main driver of this result is the future path of bank profits. Bank profits will initially decrease, making it more costly to bear maturity risk, and hence long-term rates and spreads increase in period \( t \). Spreads are at their highest when output is at its lowest and expected to increase in the future. As we approach \( t + 3 \), bank profits will be increasing and spreads decreasing given that the premium banks charge to bear maturity risk is lower when their portfolio are expected to have higher returns. That leads to increasing long-term capital investment and consumption. At \( t + 3 \), when productivity is at its peak, output is at its highest and spreads at their lowest point; bank profits are then expected to decrease so that spreads start to increase thereafter. Therefore, as observed in the data, high sloped yield curve indicates future output is increasing while a flat yield curve indicates that output is at its peak.

Rudebusch, Sack, and Swanson (2007) refer to a potential contradiction while discussing the intuition behind the results of regressions of output growth on the level of term spreads (as the one

\(^{25}\)At time 1 (t) agents learn there will be a productivity shock at time 4 (t+3).
They point out that under a standard Keynesian view, low term spreads should result in higher investment and thus higher output in the future, not lower as the level regressions suggest. They in fact confirm this view in the data by estimating output differences $\gamma_{t+1} - \gamma_t$ on spread differences $\tilde{r}_{t}-\tilde{r}_{t-1}$, obtaining the expected negative parameter estimate. As opposed to the macro-finance literature where the yield curve is built based on the stochastic discount factor, the term premium here has a direct effect on long-term investment and output and thus this Keynesian mechanism is in place. As a result of that, our model also confirms the prediction that decreasing spreads leads to higher output. However, at the time the anticipated shock is known, period 1, bank profits are expected to remain low for the next two periods forcing banks to initially charge more for long-term commitments. Thus, term spreads are high but decreasing.

Note that Adrian, Estrella, and Shin (2010) assess empirically the link between banks, spreads and output growth. They propose that lower term spreads, holding riskiness constant, leads to lower net interest margins, which in turn leads to lower banking asset growth and hence, lower output. Effectively, this mechanism would occur for movements of spreads that are exogenous to the bank’s balance sheet decision proposed here, which is based on riskiness. We can obtain a similar link using our model if we alter $\gamma_L$ (the parameter that controls entrepreneurs demand for loans given the long-term interest rate - see equation (29)). As $\gamma_L$ decreases (exogenously), spreads and bank’s long-term asset holdings decrease, leading to lower output. Nonetheless, in the framework presented here, endogenous fluctuations of spreads are intrinsically linked to the riskiness of bank’s portfolio or asset holdings, preventing us from fully analyzing changes in
spreads holding riskiness constant, as their mechanism suggests.

5.3. Term Spreads and Macroeconomic Dynamics

As we mentioned earlier, in the DSGE asset-pricing models, which are now standard in the structural model of the yield curve literature (e.g. Rudebusch and Swanson (2008b)), current output is determined by the expected path of short-term rates, and hence, term premia or long-term rates have no effect on economic activity. In our model long-term rates are determined by banks, being a function of expected future short-term rates and potential liquidity shortages in banks’ balance sheets, and most importantly, they affect firm’s long-term capital investment decisions. That way, we purposely open a channel through which term premia fluctuations affect economic activity. The effect of these endogenous fluctuations of term spreads on output in our model can be highlighted by looking at the impulse responses after a productivity shock with $\sigma_B = 0$ (constant spreads), plotted against the benchmark case. Figure 5 shows the results.

Figure 5: Endogenous vs Constant Term Spreads

Countercyclical term spread movements lead to an amplification of output responses after a productivity shock, although the change in output seems fairly small comparing to the movement observed in spreads and long-term rates. The main reason for that is the shift in the composition of output. While, after a sharp decrease in long-term rates, long-term investment increases significantly relative to the case with constant spreads, the response of consumption follows the opposite pattern.\textsuperscript{26} On the one hand, given that long-term rates have decreased significantly, the base rate

\textsuperscript{26}Note that including variable capital utilization might dampen these movements in consumption.
set by the central bank does not need to fall as much. As a result, the demand channel (Euler equation) is dampened and consumption and inflation do not move as much as in the constant spread case. On the other hand, when spreads are constant, the central bank moves the base rate more aggressively, pushing consumption up and leading to higher inflation deviations.

If part of the consumption is financed by long-term borrowing (durable consumption) then the endogenous movements in term spreads would have a much stronger effect on output given that both consumption and investment would expand further relative to the constant spreads case. Moreover, monetary policy would be forced to be considerably less aggressive to control output and inflation volatility.

Finally, in all the analysis so far we kept \( \rho_t \) constant, discussing the impact of potential liquidity shortages in the future on the bank’s portfolio decision. In Section 7.2, we will use the model to replicate a feature of the recent crisis when banks faced significant liquidity shortages, shocking \( \rho_t \) positively, to study the impact of different policies interventions. However, there could be periods of increased liquidity in the banking sector when \( \rho_t \) actually decreases, leading to lower long-term rates and narrowing term spreads and consequently expanding economic activity. That could be a potential description of the US and UK economies during the period 2003-2007 where long-term rates fell significantly (the yield curve in the UK actually inverted). De Graeve, Emiris, and Wouters (2009) decompose this fall in long-term rates in the US and show that its main drivers were declining term spreads. During the same period we observed a boom in securitization or development of structured finance activities. These activities actually meant that long-term commitment/assets could be re-packaged and sold; increasing profits through fees or advantageous balance sheet operations. Effectively, banks found themselves operating in a market in which bearing maturity transformation was relatively cheap, or \( \rho \) was significantly smaller or even negative, bringing term spreads down.

Benati and Goodhart (2008) show that during the post-war period the marginal predictive content of spreads increased during periods where current (and future) monetary policy regimes were uncertain. They conjecture that the additional predictive power of the yield curve was due to higher long-term rates, which reflected this uncertainty, depressing output. They also observe that during the early 2000’s the marginal predictive content of spreads also increased while monetary regimes had been successfully established. They then conjecture that external forces (to monetary policy) were holding down long-term yields, relating the predictive content of spreads to the real yield curve which reflects structural conditions of the economy. The fluctuations of parameter \( \rho \) given the conditions in the banking sector can be one potential avenue to explain some of these external factors.

### 6. Yield Curve and Volatility

In our basic model, long-term funding is made for two periods while short-term funding is for one period. In order to compare the derived dynamics of term spreads movements in the model to the data, we ought to consider longer maturities. However, one of the main challenges is that, due to the liquidity premium, the expectations hypothesis does not hold. (See Dewachter, Iania, and Lyrio (2011) for new evidence on the rejection of the expectation hypothesis in term structure models.)
In order to obtain the zero coupon rate for a three period bond we need the zero coupon rate for two periods $R_{L,t}$ and the forward rate between period $t+2$ and $t+3$, denominated $f_{t+2,t+3}$. There are two possible ways, one is to use $R_{S,t+2}$, or the short-term rate of period $t+2$, such that it is equivalent to invest in a three period zero bond or first investing in a two period bond and then in a one period asset. However, that implies that the liquidity risk will only affect the investment from period 1 to 2. The alternative is to obtain what would be the premium from investing in long-term maturity assets from $t+2$ till $t+3$. That is setting $t+2f_{t+3} = \frac{R_{L,t+2}}{R_{S,t+1}}$ (note that the rate $R_{L,t+2}$ is for holding a long-term asset from period $t+1$ until $t+3$). We will set forward rates following the latter and hence $t+nf_{t+n+1} = \frac{R_{L,t+n+1}}{R_{S,t+n+1}}$, such that the future liquidity risk impacts the entire yield curve. We will therefore build the yield curve obtaining the forward rates using the forward looking long-term rates against the short-term rates at each period they are set. Let the price of a $n$ quarters zero coupon bond be $E_t[p_{t}^{(n)}]$. Let the risk neutral and liquidity risk free zero coupon bond be $E_t[\hat{p}_{t}^{(n)}]$. The pricing of each of these assets is given by

$$E_t[p_{t}^{(n)}] = \exp \left( -(R_{L,t} - 1) - \sum_{i=2}^{n-1} (f_{t+i,t+i+1} - 1) \right),$$

and

$$E_t[\hat{p}_{t}^{(n)}] = \exp \left( -\sum_{i=0}^{n-1} (R_{S,i+1} - 1) \right).$$

The term premium will therefore be

$$tp_{t}^{(n)} = \frac{1}{n} \left( \ln(p_{t}^{(n)}) - \ln(\hat{p}_{t}^{(n)}) \right) = (R_{L,t} - 1) + \sum_{i=2}^{n-1} (f_{t+i,t+i+1} - 1) - \sum_{i=0}^{n-1} (R_{S,i+1} - 1).$$

Note that the standard measure used in the macro-finance literature (see Rudebusch and Swanson (2008b)) is effectively given by

$$tp_{t}^{(n)} = \sum_{i=0}^{n-1} \left( \frac{1}{m_{t+i}} - 1 \right) - \sum_{i=0}^{n-1} (R_{S,i+1} - 1),$$

where $m_t$ is the stochastic discount factor obtained from the household’s Euler equation. As Rudebusch and Swanson (2008b) stress this measure invariably generates term spreads movements that do not match the data unless one assumes shocks with high standard deviations, which in turn worsens the model’s ability to match standard business cycle facts.

We start by comparing the volatility of the 1 and 2 years term spreads of this standard measure and the one in our model. We then compare the 5 years term spreads obtained in our model against the data (collected from Kim and Wright (2005)) and the volatility obtained by the baseline model\(^{27}\) of Rudebusch and Swanson (2008b) (denoted $RS_{Baseline}$). We simulate our model without liquidity shocks, thus all variability in spreads are due to endogenous movements after productivity and monetary policy shocks. In Table 3 we provide the results of two simulations, one with the benchmark parameters and one with a higher bank risk aversion parameter ($\sigma_B = 3$).

\(^{27}\)We use their code to calculate the 5Y term spread volatility since they only report the 10Y point.
Table 3: Volatility of Term Premium*

<table>
<thead>
<tr>
<th>Term Premium - ( t_p )</th>
<th>MF Premium - ( \tilde{t}_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Y</td>
<td>0.3868</td>
</tr>
<tr>
<td>2Y</td>
<td>0.1884</td>
</tr>
</tbody>
</table>

Banking Channel against the Data

<table>
<thead>
<tr>
<th>Data (1990 - 2011)</th>
<th>Benchmark</th>
<th>High ( \sigma_B )</th>
<th>( RS_{Baseline} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5Y</td>
<td>0.657</td>
<td>0.0745</td>
<td>0.1938</td>
</tr>
</tbody>
</table>

*0.5 denotes 50 basis points.

Firstly, as clearly seen the banking balance sheet/profitability channel explored in our model is able to generate considerably more volatile term spreads than the standard macro-finance measure. This occurs despite the fact that we kept the standard deviation of shocks to be in the order of 1% and the risk aversion parameter (benchmark) to be equal to 1, lower than the one used by Rudebusch and Swanson (2008b). Bank profits are considerably more volatile than consumption, delivering greater volatility of term spreads for the same degree of risk aversion and variance of exogenous shocks. Finally, when we compare our benchmark case with the data we come closer than the standard macro-finance model.

One important feature of the data, which is not replicated in our benchmark model, is the fact that both the mean and the standard deviation of term spreads tend to increase with maturity. In our model, while the mean increases, the standard deviation decreases. This is because we introduce only two securities in the banks’ balance sheet, a one period and a two periods security, hence a movement in spreads today affect the short end of the curve more heavily than the long end. Financial businesses would have a variety of securities with different maturities and risk profiles, including assets bearing long-term inflation risks. Extending the model to consider a richer bank portfolio and including long-term inflation risks may be, therefore, fruitful areas for further research.

Rudebusch and Swanson (2008a) have extended the standard macro-finance model to include long-term inflation shocks and adopt Epstein-Zin preferences to break the link between intertemporal elasticity of substitution and coefficient of risk aversion. They are able to deliver volatile term spreads and match the dynamics of the main macroeconomic variables. Although acknowledging the importance of long-term inflation risk as an important driver of term premium - as discussed by Gürkaynak, Levin, and Swanson (2010) while covering the regime changes in the UK in the last 20 years - there is also evidence that the dynamics of short-run rates and inflation expectations do not explain all the variability of long-term rates (De Graeve, Emiris, and Wouters (2009)) or its output predictive power (Benati and Goodhart (2008)). More importantly, as stressed by Gürkaynak and Wright (2010), the US treasury inflation protected securities (TIPS) forward rate dynamics have not been that different than their nominal counterparts (see Figure 5 in their paper), indicating the term premia are also influenced by real factors. Hence, we see the bank balance sheet channel explored here as a potential complement to variations in term spreads due to inflation risk premia. In fact, given that banks would be exposed to inflation risk while bearing maturity risk, a potential long-term inflation shock would also generate increased volatility of term...
spreads in our model without the need of increasing the degree of risk aversion.

7. Monetary Policy

In this section we firstly look at conventional monetary policies in the presence of endogenous term spreads, focusing on different short-term rate policy rules. We then look at unconventional policies during periods of large shocks to liquidity shortages.

7.1. Conventional

Our interest here is to provide an answer to the following question: Should the central bank directly change short-term rates given the fluctuations in term spreads? This direct adjustment of short-term rates could be relevant in our model since spreads feed back to the macroeconomic variables of the model through their effects on long-term investment. A similar question is tackled by Aksoy, Basso, and Coto-Martinez (2010) in the case of banking spreads movements. Their finding is that welfare increases when central banks explicitly take spreads into account while setting base rates.

In order to calculate welfare under different policy rules we first obtain a third order approximation solution of the main variables of the model and then approximate the unconditional mean of the expected utility function of the household using a standard Monte Carlo Method, thus

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{1-\sigma}}{1-\sigma} - \chi H_{i}^{1+\eta} \right) = \int_{-\infty}^{\infty} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{1-\sigma}}{1-\sigma} - \chi H_{i}^{1+\eta} \right) \right] f(z') dz'
\]

\[
\approx \frac{1}{M} \sum_{i=1}^{M} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{1-\sigma}}{1-\sigma} - \chi H_{i}^{1+\eta} \right) \right],
\]

where \( z' \) is a series of shocks, \( f(z') \) the probability distribution of these shocks and \( x'_i \) the realization of the endogenous variables under a specific series of shocks \( z'_i \) for \( i = 1 \) to \( M \).

We set the monetary rule to be

\[
\frac{R_{i,CB}}{R} = \left[ \left( \frac{\pi}{\bar{\pi}} \right)^{\epsilon_{\pi}} \left( \frac{Y}{\bar{Y}} \right)^{\epsilon_Y} \left( \frac{t_{p_i}}{t_{\bar{p}}} \right)^{\epsilon_{t_p}} \right].
\]

We observe that setting \( \epsilon_{t_p} \), the degree to which the central bank moves the base rate given fluctuations of term spreads, to be greater than 0.002, causes model indeterminacy (no matter how strongly the central bank responds to inflation). Nonetheless, for our benchmark case we observe that setting \( \epsilon_{t_p} = 0.001 \) leads to higher welfare. This is due to the fact that as monetary policy responds to spread fluctuations it is able to compensate for the distortion created by the liquidity shortages which is assumed to be positive at steady state. We confirm this conclusion by modifying the model assuming \( \bar{\rho} = 0 \), in which case banks no longer face liquidity shortages in long-term investments on average or at steady state. Setting \( \epsilon_{t_p} = 0.001 \) under this specification does not lead to higher welfare relative to the standard monetary rule where the base rate depends only on output and inflation deviations.
Hence, we conclude that although fluctuation of term spreads are important for economic activity, and monetary policy should take these fluctuations into account, the central bank should not respond explicitly to term spread movements, unless it wants to correct for steady state distortions. However, note that fluctuations in term spreads do affect output and inflation, and thus, even when relying on a standard monetary policy rule, the central bank is taking spread movements implicitly into account (see Figure 5).

7.2. Unconventional

The Federal Reserve Bank (FED) conducted two purchase programmes of long-term Treasuries and other long-term bonds, known as QE1 in 2008-2009 and QE2 in 2010-2011. These quantitative easing policies comprised of purchase of mortgage backed securities, Treasuries and “Agencies” from the private sector. Gagnon, Raskin, Remache, and Sack (2010) analyze the effectiveness of the Large-Scale Assets Purchases conducted by the FED. They find that the purchase programme lead to reductions in long-term interest rates on a range of securities, including some securities that were not included in the purchase programme, indicating that portfolio balancing effects were in play. They argue that the reductions in interest rate primarily reflect lower risk/liquidity premiums rather than lower expectations of future short-term rates. Krishnamurthy and Vissing-Jorgensen (2011) find that these QE policies in the U.S. lead to a significant decline in nominal rates on long-term safe assets (Treasuries and “Agencies”, assets which were more heavily traded by the FED) and only a small effect on less safe assets such as corporate rates and mortgage rates (assets which were less heavily influenced by FED market activity). Their results suggest that the effects of asset purchases on the duration of risk premium are small, while effects on liquidity-safety premium are substantial.

Beirne, Dalitz, Ejsing, Grothe, Manganelli, Monar, Sahel, Suec, Tapking, and Vong (2011) report on the effectiveness of the Covered Bond Purchase Programme (CBPP), which started in July 2009 for a period of 12 months in the Eurozone, and show that covered bond yields decreased by 12 basis points; the programme increased the liquidity of secondary market and that it managed to encourage lending. Joyce, Lasaosa, Stevens, and Tong (2010) report that the QE interventions in the UK led to a 100 basis point decrease in Gilt yields. Given the effects on other asset classes, although the purchase programme has been overwhelmingly of government securities, they also stress the importance of portfolio balancing effects. Finally, Borio and Disyatat (2010) provide a survey of different forms of possible unconventional monetary policies and argue that the main balance sheet channel operates though the central bank’s ability to reduce yields and ease financing constraints by altering the risk profile of private portfolios. Overall, a constant theme in these studies is the effect on long-term rates through lower term spreads being crucial for the effectiveness of the interventions and for allowing the financial market to continue funding economic activity.

Two main features of our model are particularly important in formalizing this type of interventions. First, fluctuations in term spreads are a relevant factor in determining output as long-term rates influence investment decisions, and hence, an intervention aimed at lowering long-term rates affects economic activity. Second, given that term spreads or long-term rate decisions are directly determined by fluctuations in future bank profits and changes in their balance sheet holdings, our model provides a new channel through which the effects arise. However, an important caveat,
which underlines a promising path for future research, is the fact that our bank portfolios are fairly simple, with only three assets. They do not include, for instance, housing debt/mortgages, thereby restricting the analysis of some of the portfolio balancing effects mentioned.

In order to study the main effects of QE policies we first introduce two types of unconventional monetary policies and then analyze their impact after a liquidity shortage shock. The first is a simple liquidity injection (QE) to banks financed by a lump-sum tax collected from households. Liquidity injection, which is costless to the receiving bank, is set such that $\xi^t = \xi X_{L,t-1} P_{t-1} \rho_1$, where $\xi_p = \frac{\xi}{Y_{t-1} - \frac{1}{X_t}} \frac{X_t}{L_t}$ and $\frac{X_t}{L_t}$ is the ratio of long to short run funding that would be in place without QE intervention. The liquidity injection is a proportion $\xi_p$ of the current bank’s long-term asset exposure, and its intensity depends on how skewed current investment funding is towards short-term relative to long-term funding. Note that this relative difference will be a direct function of future liquidity conditions.

The second unconventional policy is the existence of favorable conditions for banks to borrow funds from the central bank using their long-term asset positions as collateral. Favorable conditions in our context imply a lower rate of borrowing relative to the short-term funding currently available. Banks now decide the fraction of long-term assets ($\Theta_t$) they want to pledge as collateral to get funds from the central bank. Effectively, at time $t$, banks make a two period investment. At period $t+1$ they sell a portion ($\Theta_t$) of these assets to the central bank to get additional funds, promising to buy them back at $t+2$ before they mature. The total cost of central bank funding is $\Theta_t X_{L,t-1} P_{t-1} (R_{t,QE} - 1) + \frac{\rho_2}{2} \Theta_t^2$, where $R_{t,QE}$ is the borrowing rate. The term $\frac{\rho_2}{2} \Theta_t^2$ is included such that the marginal cost of this type of funding is increasing as usage increases. The bank problem now becomes

$$\max_{\{X_{S,t}, X_{L,t}, \Theta_t\}_t} E_0 \sum_{t=0}^\infty \beta_t \frac{\Pi_t^{1-\sigma_t}}{1-\sigma_t}$$

s.t. $D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1} - \Theta_t X_{L,t-1} P_{t-1}$

where

$$\Pi_{t+1}^{B_t} = \frac{1}{P_{t+1}} (DIV_{t+1} + (R_{L,t-1} - 1) P_{t-1} X_{L,t-1} + (R_{S,t} - 1) P_t X_{S,t} - D_t (R_{S,t} - 1) - \rho_{t+1} X_{L,t} P_{t+1} - \Theta_t X_{L,t-1} P_{t-1} (R_{Q,E,t} - 1) - \frac{\phi_{Q,E}}{2} \Theta_t^2).$$

The first order conditions in this case are

$$E_t \left[ \Pi_{t+1}^{B_t-\sigma_t} \left( \frac{(R_{S,t} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] = E_t \left[ \frac{\beta \Pi_{t+2}^{B_t-\sigma_t} (R_{L,t} - R_{t+1, CB} + \Theta_{t+1} (R_{S,t+1} - R_{Q,E,t+1}))}{\pi_{t+1} \pi_{t+2}} \left( R_{S,t} - R_{Q,E,t} \frac{X_{L,t-1}}{\pi_{t+1}} - \phi_{Q,E} \Theta_t \right) \right]$$

$$0 = \Pi_{t+1}^{B_t-\sigma_t} \left( (R_{S,t} - R_{Q,E,t}) \frac{X_{L,t-1}}{\pi_t \pi_{t+1}} - \phi_{Q,E} \Theta_t \right).$$

29
Finally, we assume the central bank sets \( R_{QE,t} = R_{S,t} \left( 1 - \phi_{re} \left( \frac{\delta_{L}}{\delta_{S}} - \frac{\delta_{S}}{\delta_{S}} \right) \frac{\bar{X}_L}{\bar{X}_S} \right) \). Thus, lower the long-term funding relative to short-term without intervention, more favorable central bank funding will be.

Figure 6: QE Policies

Figure 6 illustrates the results for a liquidity shock of 0.04 with \( \phi_{QE} = 0.35 \). That translates in the central bank covering roughly 90% of the liquidity shortage under the first intervention after the shock or, in the second case, buying roughly 10% of long-term assets from the bank’s balance sheets (see the bottom left graph in Figure 6). We observe that both QE policies do have a significant impact on long-term rates and term spreads. That leads to dampened responses of long-term investment and output relative to the case where only conventional monetary policy is used. The main difference between these two interventions is that asset purchases have a stronger initial impact since they immediately free up the balance sheet of banks which are then able to maintain long-term funding despite the liquidity shock. Liquidity injections do affect term spreads and eventually help sustain higher output; however, these require a longer period to work through the bank’s long-term rate setting as it protects them from future liquidity shortages. Note that we allow base rates to move freely in both cases, hence, short-term rates do not need to decrease as much while asset purchases are conducted. An important result is that inflation turns out to be significantly higher after QE interventions, thus even if nominal rates are close to 1, QE interventions will lead to lower real rates.

One of the important debates at central banks in the UK, the US and the Eurozone is at which point to unwind the large-scale purchases. Although not completely suited to give a definite answer
to such a question, we can use our model to verify the effectiveness of short-term asset purchase agreements, which sell securities back to banks after one period, and the interventions that allow banks to move long-term assets away from balance sheets for longer periods. In order to do that we modify the model such that long-term investments now require one year (4 quarters) commitments from firms and hence from banks. The appendix shows the details of the model and of each of the two QE interventions: one period asset purchases and three periods asset purchases. Figure 7 shows the results. We set $\phi_{QE}$ for each of these two interventions such that the portion of long-term assets bought by the central bank are matched (roughly 20%, see graph at the bottom left corner).

Figure 7: Short versus Long-term Asset Purchases

We observe that when the central bank holds assets for longer periods the same intervention in terms of assets purchases leads to lower levels of term spreads/long-term yields and to a lower decrease in long-term investment after a liquidity shock. There is a gain for the central bank to hold the securities bought in such interventions for longer periods of time since they are more effective in freeing up banks’ balance sheet, fomenting long-term funding. Obviously, these securities remain in the central bank balance sheet for longer and thus the monetary authority is taking significantly more risks than when it keeps securities for only one period. Finally, the long holding period interventions allow central banks to bring short-term rates back to their steady state levels sooner.

28Impulse responses for output and long-term capital stock are shown after the fourth period since that is the point changes to long-term investment done at time $t = 1$ start having effects.
8. Conclusions

Term spread fluctuations have relevant implications for macroeconomic outcomes and may predict output growth. Undoubtedly, inflation expectations or more generally long-term inflation risks are important determinants of these fluctuations. However, the observation that nominal and real yield curves move together in many instances suggests that other factors are in play. We propose a model that delivers endogenous variations in term spreads driven primarily by changes in banks’ portfolio decision and their appetite to bear the risk of maturity transformation. We show that fluctuations of banks’ portfolio future profitability affect their ability to cover for any liquidity shortage and hence influence the premium they require to carry maturity risk. Another feature that distinguishes our framework from macro-affine DSGE models is that fluctuations in term spread impact economic activity since they alter investment financing costs. Additionally, we also present empirical evidence on the relevance of financial business profits in explaining output dynamics and on the link between yield spreads and expected profitability.

While we present a model in which bank portfolios are fairly simple, we are able to match important features of the data. Our model suggests that factors external to monetary policy may contribute not only to the marginal predictive power of spreads but also to the understanding of the linkages between banks, spread movements and the macroeconomy.

Embedding this simple banking sector framework into a DSGE model allows us to analyze the interaction between these spread movements and conventional and unconventional policies. Spread movements effectively imply tighter or looser monetary conditions forcing the central bank to adjust short-term rates accordingly. Once again, spreads between different interest rates in the economy are shown to be crucial and should be explicitly included in models that analyze optimal policies. Unconventional policies are shown to have a strong impact on spread movements fomenting long-term investment and helping reduce output losses after negative liquidity shocks, matching the general view on the effects of recent asset purchases programmes. Finally, we show that asset purchases programmes that keep the assets in the central bank balance sheet for longer are more effective in offsetting a liquidity shock and allow the central bank to re-store short-term rates to steady state levels more quickly. This result indicates that the initial decision of the ECB to hold asset purchased under the CBPP programme until maturity gives more strength to this type of intervention.

The present work highlights three areas in which further research, exploring the role of bank’s portfolio decisions, may be fruitful. First, increasing the complexity of banks’ portfolios will provide a better understanding of this important channel, most notably, including (workers) housing investment funded by financial intermediation. That would mean term spread fluctuations would not only influence investment but also consumption, potentially amplifying the effects of spread movements, since as we observe, consumption and investment move in opposite directions compensating each other. Moreover, including other long-term asset classes may potentially allow us to study portfolio balancing effects after QE interventions. Second, making liquidity shortages endogenous based on the potential for securitization of long-term assets may be crucial to fully understand those factors behind spread movements and their marginal predictive power. Finally, final investors (after securitization) and bank sentiment or risk assessment could also be time varying affecting the linkage between long-term funding risks and economic activity.
References


Appendix A  Simple Model - Case 2

Given the assumption for the two stochastic processes \( (R_Z, \rho) \) and setting \( \sigma_B = 0 \), total profits \( \Pi^B \) are also normally distributed with mean \( \mu_{\Pi} = (\bar{R}_Z - R_D)Z + (\beta(R_L - R_D) - R_D - \bar{\rho})X_L \) and variance \( \sigma_{\Pi}^2 = Z^2\sigma_Z^2 + \beta^2X_L^2\sigma_p^2 + 2\sigma_{Z, p}X_LZ\sigma_Z\sigma_p \).

Using an approximation for the first percentile of the profit probability density function we then obtain

\[
0.01 = \frac{1}{2} \left[ 1 + erf \left( \frac{VaR - \mu_{\Pi}}{\sigma_{\Pi}\sqrt{2}} \right) \right]
\]  

(16)

where \( erf \) is the error function. We can differentiate the equation above with respect to \( X_L \) and \( Z \) to obtain

\[
\frac{\partial VaR}{\partial X_L} = (\beta(R_L - R_D) - R_D - \bar{\rho})\sigma_{\Pi}\sqrt{2} + \left[ VaR - \mu_{\Pi} \right] (\sigma_{\Pi})^{-1}\sqrt{2}(\beta^2X_L\sigma_p^2 + 2\sigma_{Z, p}Z\sigma_Z\sigma_p)
\]

\[
\frac{\partial VaR}{\partial Z} = (\bar{R}_Z - R_D)\sigma_{\Pi}\sqrt{2} + \left[ VaR - \mu_{\Pi} \right] (\sigma_{\Pi})^{-1}\sqrt{2}(Z\sigma_Z^2 + 2\sigma_{Z, p}X_L\sigma_Z\sigma_p)
\]

We then substitute these conditions into (1) - (5) and (16) to determine the equilibrium. Our main interest is to verify how term spreads (measure in basis points), defined as \( tp = \frac{1}{2} ((R_L - 1) - (R_D - 1) - (R_D - 1)) \times 10000 \), move as the overall profitability (expected returns) of the bank’s portfolio varies. In doing so we aim at establishing a link between banks’ appetite to accumulate long-term assets in the balance sheet, incurring the risk of maturity transformation, the equilibrium long-term rates and the expected performance of bank investments. As such we look at the equilibrium level of term premium as \( \sigma_Z \) (which controls expected returns on equity) varies. Figure 8 shows the results for the following parameter values \( R_D = 1.01 \) (base rate equal to a 4% annual), \( \bar{\rho} = 0.005 \) (annual spread of roughly 100 basis points), \( \sigma_Z = 0.03 \), \( \sigma_p = 0.01 \), \( \gamma_L = 6 \), \( \gamma_Z = 0.0006 \), \( \Lambda = -0.3 \) (the VaR limit implies a loss of roughly 2.5 standard deviations). The qualitative implications are unchanged when these are altered. Finally, the important parameter to determine the results is the correlation between the asset returns. We set it to -0.1 (allowing for gains of diversification). The impact on term spreads reverse when this correlation is positive, since in this instance the substitution effect will be greater than the income effect (see the discussion in the text). Note that the increase in banks’ balance sheet occur since both \( Z \) and \( X_L \) holding increase, although equity holding increase more sharply.

Appendix B  Data

This provides a description of the data used in the empirical study.
Figure 8: Assets and Term spreads as expected bank profits increase

- Treasury Bill Rate (Units: Percent per Annum), (Series ID: 60C..ZF) Source: International Financial Statistics/IMF
- Government Bond Yield: 10 year (Units: Percent per Annum), (Series ID: 61..ZF) Source: International Financial Statistics/IMF
- CPI All Items City Average (Units: Index Number), (Series ID: 64..ZF), Source: International Financial Statistics/IMF
- Real Gross Domestic Product, Seasonally Adjusted Annual Rate, (Series ID: GDPC96) Source: U.S. Department of Commerce: Bureau of Economic Analysis
- Financial Business; undistributed corporate profits excluding CCAdj, (FOF Code: FA796006403.Q), Source: Flow of Funds Accounts, Board of Governors of the Federal Reserve

Appendix C  Equilibrium Conditions and Steady State

The household maximization routines yield the following equilibrium conditions

$$ \beta E_t \left( \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = \frac{C_t^{-\sigma}}{R_t, CB} $$

and

$$ w_{j,t} = \frac{\varepsilon_w}{\varepsilon_w - 1} E_t \left\{ \sum_{x=0}^{\infty} \frac{C_t^{-\sigma}}{C_t^{-\sigma}} (\omega_{x, \beta}^x)^y H_t^{1+y} \left( \prod_{k=1}^{t} \pi_{t+k} \right)^{-1} \right\}. $$
This equation can be conveniently expressed in recursive form as such

\[
0 = f^w_{1,t} \frac{\varepsilon_w}{\varepsilon_w - 1} - f^w_{2,t} w_{j,t},
\]

\[
f^w_{1,t} = H_t \frac{\phi H}{C_t^{\sigma}} + E_t \left[ \beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} f^w_{1,t+1} \right],
\]

\[
f^w_{2,t} = H_t + E_t \left[ \beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{-1} f^w_{2,t+1} \right],
\]

where, \( w_{j,t} = W_{j,t}/P_t \) and \( w_t = W_t/P_t \), is given by

\[
w_t^{1-\varepsilon_w} = (1 - \omega_w)w_{j,t}^{1-\varepsilon_w} + \omega_w \pi_{t-1}^{-1}. \tag{19}\]

We assume firms discount future payoffs using the household’s stochastic discount factor given by

\[
Q_{i,t+1} = \beta E_t \left( \frac{C_{t+1}^{-\sigma}}{\pi_t + C_t^{-\sigma}} \right) = \frac{1}{R_{CB,t}}.
\]

Given that the purpose of our analysis is not to look at the effects of firm-specific capital we assume that there exists a capital market within firms. That way all firms will have the same labour-capital ratio and \( \Lambda_{t,i} = \Lambda_t \) for all \( i \), as in the case where a capital rental market is available. The net aggregate investment in (new) capital is then acquired from entrepreneurs. Note that, as shown by Sveen and Weinke (2007), the relevant difference of considering firm-specific capital is that the parameter on the marginal cost in the Phillips curve would be lower, increasing effective price stickiness. Our results are not qualitatively affected by this change.

Based on that, \( p_{i,t} \) is determined by solving the price setting maximization, substituting for the stochastic discount factor and using \( \Lambda_{t+s,j} = \Lambda_{t+s} \). That gives

\[
p_{i,t} = \varepsilon E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{\sigma}}{C_t^{\sigma}} (\omega \bar{\beta})^s \Lambda_{t+s} \frac{Y_{t+s}}{\prod_{k=1}^{t+s} \pi_{t+s-k}} \right\}.
\tag{20}\]

The recursive formulation is given by

\[
0 = f^w_{1,t} \frac{\varepsilon}{\varepsilon - 1} - f^w_{2,t} p_{i,t},
\]

\[
f^w_{1,t} = Y_t \Lambda_t + E_t \left[ \beta \omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{-1} f^w_{1,t+1} \right],
\]

\[
f^w_{2,t} = Y_t + E_t \left[ \beta \omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{-1} f^w_{2,t+1} \right],
\]

where, \( p_{i,t} = P_{i,t}/P_t \) and \( \pi_t = P_t/P_{t-1} \), is given by

\[
1 = (1 - \omega) p_{i,t}^{1-\varepsilon} + \omega \pi_{t-1}^{\varepsilon-1}. \tag{21}\]
From the firm cost minimization problem we obtain the demand for capital and labour. After rearranging the first order conditions and substituting for the stochastic discount factor $Q_{t,t+1}$, we obtain the following equilibrium conditions:

\begin{align*}
Y_t &= A_t K_t^{\frac{\alpha}{1-\alpha}} L_t^{(1-\alpha)/\alpha} H_t^{1-\alpha}, \\
\Lambda_t &= \frac{w_t H_t}{Y_t(1-\alpha)}, \\
q_t^S &= \beta E_t \left\{ \frac{C_t^{\sigma}}{\pi_{t+1}} \left[ \Lambda_{t+1} \frac{\alpha \bar{\zeta} Y_{t+1}^{1-\alpha}}{K_t^{\sigma}} + (1 - \delta) q_{t+1}^S \right] \right\}, \text{ and} \\
q_t^L &= \beta E_t \left\{ \frac{C_t^{\sigma}}{\pi_{t+1}} \left[ \Lambda_{t+1} \frac{(1 - \bar{\zeta}) Y_{t+1}^{1-\alpha}}{K_t^{\sigma}} + (1 - \delta) q_{t+1}^L \right] \right\}.
\end{align*}

Using the capital aggregation conditions, investment evolves according to

\begin{align*}
I_t^S &= K_{t+1}^S - (1 - \delta) K_t^S, \quad \text{and} \\
I_t^L &= K_{t+1}^L - (1 - \delta) K_t^L.
\end{align*}

From entrepreneurs maximization problems we obtain

\begin{align*}
X_{S,t} &= \frac{\gamma_t E_t [q_{t+1}^S \pi_{t+1}]}{R_{S,t}} - 1, \quad \text{and} \\
X_{L,t} &= \frac{\gamma_t E_t [q_{t+1}^L \pi_{t+1}]}{R_{L,t} + E_t [\rho_{t+1} \pi_{t+1}]} - 1.
\end{align*}

Finally, from the bank maximization problem we have that

\begin{align*}
E_t \left[ \Pi_{t+1}^{B} - \sigma_\rho \left( \frac{(R_{t,\text{CB}} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] &= R_{t,\text{CB}}, \quad \text{and} \\
\beta E_t \left[ \frac{1}{\pi_{t+1} \pi_{t+2}} \Pi_{t+2}^{B} - \sigma_\rho (R_{L,t} - R_{t,\text{CB}}) \right],
\end{align*}

where

\begin{align*}
\Pi_t^{B} &= \frac{DIV_t}{\pi_t} + \left( \frac{R_{t-2,\text{LB}} - 1}{\pi_t \pi_{t-1}} \right) X_{t-2} + \left( \frac{R_{t-1,\text{CB}} - 1}{\pi_t \pi_{t-1}} \right) \left( X_{t-1} - \frac{D_t}{\pi_t} \rho_t X_{L,t-1} \right), \\
DIV_t/\pi_t &= Y_{t-1} - w_{t-1} T_{t-1} - q_{t-1}^S T_{t-1} - q_{t-1}^L T_{t-1} \\
&+ q_{t-1}^S K_t^S + q_{t-1}^L K_t^L - \frac{q_{t-2}^S K_{t-1}^S + q_{t-2}^L K_{t-1}^L}{\pi_{t-1}}, \quad \text{and} \\
\frac{D_t}{\pi_t} &= X_{S,t} + X_{L,t} + \left( \frac{q_{t-1}^S K_t^S + q_{t-1}^L K_t^L}{\pi_t} \right) + \frac{X_{L,t-1}}{\pi_t}.
\end{align*}

\footnote{Once again we have used the fact that marginal costs are the same across firms.}
We define the term spread (annual rate in percentage points) between long and short-term rate as
\[
\tau_p = \frac{1}{2} ((R_{L,J} - 1) - (R_{t, CB} - 1) - (R_{t+1, CB} - 1)) 400. \tag{35}
\]

Finally, the central bank sets monetary policy according to
\[
R_{t, CB} = \left[ \left( \frac{\pi_t}{\bar{\pi}} \right)^{\frac{\epsilon}{\beta}} \left( \frac{Y_t}{\bar{Y}} \right)^{\frac{\epsilon_Y}{\beta}} \right]. \tag{36}
\]
where \( \bar{X} \) is the steady state value of \( X_t \). This is the standard monetary rule whereby the base rate responds to deviations in inflation and output. The recursive equilibrium is determined as the solution to equations (10) - (36).

Steady State From pricing equation (normalizing prices at steady state to 1) we have that
\[
\Lambda = \frac{\epsilon - 1}{\epsilon}.
\]

From wage pricing equation we have that
\[
\bar{w} = \frac{\epsilon_w \chi \bar{H}_w}{\epsilon_w - 1 \frac{C}{\sigma}}.
\]

From the firm problem we have that
\[
\begin{align*}
\Lambda &= \frac{\bar{w} H}{\bar{Y}(1 - \alpha)} \\
\bar{q}_s &= \frac{\beta \Lambda \alpha \bar{Y}}{\bar{K} S(1 - \beta(1 - \delta))} \\
\bar{q}_L &= \frac{\beta \Lambda \alpha (1 - \bar{Y})}{\bar{K} L(1 - \beta(1 - \delta))}.
\end{align*}
\]

From entrepreneurs problems we have that
\[
\begin{align*}
\bar{X}_S &= \gamma S \bar{q}_S - 1 \\
\bar{X}_L &= \frac{\gamma L \bar{q}_L}{R_L + \rho} - 1.
\end{align*}
\]

From the bank problem we have that
\[
\left( \frac{1}{\beta} - 1 \right) + \rho = \beta \left( R_L - \frac{1}{\beta} \right) \text{ or } R_L = \frac{1}{\beta^2} + \frac{\rho}{\beta}.
\]

The term spread at the steady state is given by
\[
\tau \bar{p} = \frac{1}{2} ((R_L - 1) - (1/\beta - 1) - (1/\beta - 1)) 400.
\]

Clearing conditions and investment flow equation determine that
Appendix D  Second Order Approximation of Long-term Rate Decision

Bank equilibrium condition is given by

\[ E_t \frac{1}{\pi_{t+1}} \left[ (\Pi_{t+1}^B)^{-\sigma^g} \left( (R_{CB,t} - 1) + \rho_{t+1} \right) \right] = \beta E_t \frac{1}{\pi_{t+1} \pi_{t+2}} \left[ (\Pi_{t+2}^B)^{-\sigma^g} (R_{L,t} - R_{CB,t+1}) \right], \]

and at the steady state

\[ \left( \frac{1}{\hat{\pi}} + \hat{\rho} \right) \frac{1}{\hat{\pi}} = R_L \text{ and } R_{CB} = \frac{1}{\hat{\beta}}. \]

Let \( W_1 = (R_{CB,t} - 1 + \rho_{t+1}) \) and \( W_2 = (R_{L,t} - R_{CB,t+1}) \).

**Approximation of Left-Hand Side (LHS)**

\[ E_t \frac{1}{\pi_{t+1}} \left[ (\Pi_{t+1}^B)^{-\sigma^g} \left( (R_{CB,t} - 1) + \rho_{t+1} \right) \right] = \]

\[ E_t \frac{1}{\pi_{t+1}} \left[ (\Pi_{t+1}^B)^{-\sigma^g} W_1 \right] \approx \]

\[ E_t \left[ -\sigma^g \Pi_{t+1}^B + 0.5 \sigma^g (\Pi_{t+1}^B)^2 + \hat{W}_1 + 0.5 (\hat{W}_1)^2 - \hat{\pi}_{t+1} + 0.5 (\hat{\pi}_{t+1})^2 - \sigma^g \Pi_{t+1}^B \hat{W}_1 + \sigma^g \Pi_{t+1}^B \hat{\pi}_{t+1} - \hat{W}_1 \hat{\pi}_{t+1} \right]. \]

From the definition of \( W_1 \)

\[ \hat{W}_1 = \frac{1}{\hat{\beta} \Gamma \hat{R}_{CB,t} + \hat{\rho} \Gamma \hat{\rho}_{t+1}}, \text{ where } \Gamma = \left( \frac{1}{\hat{\beta}} - 1 + \hat{\rho} \right), \text{ and} \]

\[ \hat{W}_1 + 0.5 (\hat{W}_1)^2 = \frac{1}{\hat{\beta} \Gamma} \left( \hat{R}_{CB,t} + 0.5 (\hat{R}_{CB,t})^2 \right) + \frac{\hat{\rho} \Gamma}{\hat{\beta} \Gamma} \left( \hat{\rho}_{t+1} + 0.5 (\hat{\rho}_{t+1})^2 \right). \]

Hence, LHS becomes

\[ E_t \left[ -\sigma^g \Pi_{t+1}^B + 0.5 \sigma^g (\Pi_{t+1}^B)^2 + \frac{1}{\hat{\beta} \Gamma} \left( \hat{R}_{CB,t} + 0.5 (\hat{R}_{CB,t})^2 \right) + \frac{\hat{\rho} \Gamma}{\hat{\beta} \Gamma} \left( \hat{\rho}_{t+1} + 0.5 (\hat{\rho}_{t+1})^2 \right) - \hat{\pi}_{t+1} + 0.5 (\hat{\pi}_{t+1})^2 - \sigma^g \Pi_{t+1}^B \hat{W}_1 + \sigma^g \Pi_{t+1}^B \hat{\pi}_{t+1} - \hat{W}_1 \hat{\pi}_{t+1} \right]. \]
Approximation of Right-Hand Side (RHS)

\[
\beta E \left[ \frac{1}{\pi_{t+1}\pi_{t+2}} \left( (\Pi^B_{t+2})^{-\sigma_B} (R_{L,t} - R_{CB,J+1}) \right) \right] =
\]

\[
\beta E \left[ \frac{1}{\pi_{t+1}\pi_{t+2}} \left( (\Pi^B_{t+2})^{-\sigma_B} W_2 \right) \right] \approx
\]

\[
E \left[ -\sigma^\prime \Pi^B_{t+2} \bar{\sigma} + 0.5\sigma^\prime \left( \Pi^B_{t+2} \right)^2 - \bar{\sigma} R_{L,t} + 0.5 \left( \bar{\sigma} R_{L,t} \right)^2 - \bar{\sigma} R_{CB,J+1} + 0.5 \left( \bar{\sigma} R_{CB,J+1} \right)^2 \right]
\]

From the definition of \( W_2 \)

\[
\bar{W}_2 = \left( \frac{1}{\beta} + \frac{\rho}{\Gamma} R_{L,t} - \frac{1}{\beta T} R_{CBJ+1} \right) \text{ where } \Gamma = \left( \frac{1}{\beta} - 1 + \bar{\rho} \right) \text{ and }
\]

\[
\bar{W}_2 + 0.5 \left( \bar{W}_2 \right)^2 = \frac{1}{\beta} + \frac{\bar{\rho}}{\Gamma} \left( R_{L,t} + 0.5 \left( R_{L,t} \right)^2 \right) - \frac{1}{\beta T} \left( R_{CBJ+1} + 0.5 \left( R_{CBJ+1} \right)^2 \right)
\]

Hence, RHS becomes

\[
E \left[ -\sigma^\prime \Pi^B_{t+2} \bar{\sigma} + 0.5\sigma^\prime \left( \Pi^B_{t+2} \right)^2 + \left( \frac{1}{\beta} + \frac{\rho}{\Gamma} R_{L,t} - \frac{1}{\beta T} R_{CBJ+1} \right) \bar{\sigma} R_{L,t} + 0.5 \left( \bar{\sigma} R_{L,t} \right)^2 - \bar{\sigma} R_{CB,J+1} + 0.5 \left( \bar{\sigma} R_{CB,J+1} \right)^2 - \bar{\sigma} R_{CBJ+1} + 0.5 \left( \bar{\sigma} R_{CBJ+1} \right)^2 \right]
\]

From the definition of term premium we have that \( \bar{\rho} = 0.5(R_{L,t} - R_{CB,J+1} - R_{CB,J+1}) \), hence\(^{30}\)

\[
\bar{\rho} + 0.5 \left( \bar{\rho} \right)^2 \approx \frac{1}{\beta} + \frac{\rho}{\Gamma} \left( R_{L,t} + 0.5 \left( R_{L,t} \right)^2 \right) - \frac{1}{\beta T} \left( R_{CBJ+1} + 0.5 \left( R_{CBJ+1} \right)^2 \right) - \frac{1}{\beta T} \left( R_{CB,J} + 0.5 \left( R_{CB,J} \right)^2 \right)
\]

We can now combine the LHS and RHS to get

\[
E \left[ \bar{\rho} + 0.5 \left( \bar{\rho} \right)^2 \right] = E \left[ \sigma^\prime \left( \Pi^B_{t+2} - \Pi^B_{t+1} \right) + 0.5\sigma^\prime \left( \Pi^B_{t+2} \right)^2 - \bar{\sigma} R_{L,t} + 0.5 \left( \bar{\sigma} R_{L,t} \right)^2 + \bar{\rho} \left( R_{L,t} + 0.5 \left( R_{L,t} \right)^2 \right) + \text{CovTerms} \right]
\]

where,

\[
\text{CovTerms} = -\sigma^\prime \Pi^B_{t+1} \left( \frac{1}{\beta} R_{CBJ+1} + \frac{\rho}{\Gamma} \bar{\sigma} R_{L,t} - \frac{1}{\beta T} R_{CBJ+1} \right) + \sigma^\prime \Pi^B_{t+2} \left( \frac{1}{\beta} R_{L,t} - \frac{1}{\beta T} R_{CBJ+1} \right) + \bar{\rho} \left( R_{L,t} + 0.5 \left( R_{L,t} \right)^2 \right) - \bar{\sigma} R_{L,t} + 0.5 \left( \bar{\sigma} R_{L,t} \right)^2 - \bar{\sigma} R_{L,t} + 0.5 \left( \bar{\sigma} R_{L,t} \right)^2.
\]

\(^{30}\)Note that the approximated signed is also used here since the denominator should be \( \left( \frac{1}{\beta} - 1 + \rho \right) \) and not \( \Gamma = \left( \frac{1}{\beta} - 1 + \rho \right) \).
Appendix E  Long-term Investment with 1Y maturity

If we assume long-term investments are done at period $t$ but mature at $t + 4$ then $X_{L,t}$ becomes

$$X_{L,t} = \frac{\gamma L E_t \left[ L_{t-4} \pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4} \right]}{R_{L,t} + E_t \left[ \pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4} \right]} - 1.$$  

And the long-term rate is set such that

$$\beta^3 E_t \left[ \frac{1}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \Pi_{t+4}^B -\sigma_B (R_{L,t} - R_{t+3,CB}) \right] = E_t \left[ \Pi_{t+1}^B -\sigma_B \left( \frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] +$$

$$\beta E_t \left[ \Pi_{t+2}^B -\sigma_B \left( \frac{(R_{t+1,CB} - 1)}{\pi_{t+1} \pi_{t+2}} + \rho_{t+2} \right) \right] +$$

$$\beta^2 E_t \left[ \Pi_{t+3}^B -\sigma_B \left( \frac{(R_{t+2,CB} - 1)}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} + \rho_{t+3} \right) \right].$$

Where

$$\Pi_t^B = div_t + \frac{(R_{t-4,L} - 1)}{\pi_{t-1} \pi_{t-2} \pi_{t-3}} \pi_{t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}), \text{ and}$$

$$d_t = X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_{t-1}} + \frac{X_{L,t-2}}{\pi_{t-1} \pi_{t-2}} + \frac{X_{L,t-3}}{\pi_{t-1} \pi_{t-2} \pi_{t-3}} + z_t.$$

We define the term premium (annual rate in percentage points) between long and short-term rate as

$$tp = \frac{1}{4} (R_{L,t} - R_{t,CB} - R_{t+1,CB} - R_{t+2,CB} - R_{t+3,CB} + 3) 400.$$  

Finally, the good market clearing condition is

$$Y_t = C_t + X_{S,t} + X_{L,t} + \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}).$$

Short-term asset purchase agreements

We assume banks can only repo the long-term asset that is about to mature. Profits and deposits are given by

$$\Pi_t^B = div_t + \frac{(R_{t-4,L} - 1)}{\pi_{t-1} \pi_{t-2} \pi_{t-3}} \pi_{t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-1} X_{L,t-4}}{\pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{Q,E,t} - 1) - \frac{\Theta_{Q,E}^2}{2} \Theta_{t-1}^2, \text{ and}$$

$$d_t = X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_t} + \frac{X_{L,t-2}}{\pi_{t-1}} + \frac{X_{L,t-3}}{\pi_{t-1} \pi_{t-2}} + z_t - \Theta_t \frac{X_{L,t-3}}{\pi_{t-1} \pi_{t-2}}.$$
Which implies
\[
\beta^t E \left[ \Pi^B_{t+4} - \sigma^b \left( \Pi_{t+1} \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}} + \rho_{t+1} \right) \right] = E_t \left[ \Pi^B_{t+1} - \sigma^b \left( \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}} + \rho_{t+1} \right) \right] \
+ \beta E_t \left[ \Pi^B_{t+2} - \sigma^b \left( \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}\pi_{t+2}} + \rho_{t+2} \right) \right] \
+ \beta^2 E_t \left[ \Pi^B_{t+3} - \sigma^b \left( \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}\pi_{t+2}\pi_{t+3}} + \rho_{t+3} \right) \right] \
0 = \Pi^B_{t+1} - \sigma^b \left( R_{CB,t} - R_{QE,t} \right) \frac{X_{L,t-3}}{\pi_{t+1}\pi_{t+2}\pi_{t+3} - \phi_{QE} \Theta_t}.
\]

Long-term asset purchase agreements

We assume banks can sell the long-term asset with the longest maturity and buy back before maturity. 

Profits and deposits are given by
\[
\Pi^B_t = div_t + \frac{(R_{t-4, L} - 1)}{\pi_{t-1}\pi_{t-2}\pi_{t-3}} X_{L,t-4} + (R_{t-1, CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_{t-1}} \
- \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-3} X_{L,t-4}}{\pi_{t-1}\pi_{t-2}\pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE} \Theta_t^2}{2}, \text{ and} \\
D_t = \rho_t X_{S,t} + \rho_t X_{L,t} + \rho_{t-1} X_{L,t-1} + \rho_{t-2} X_{L,t-2} + \rho_{t-3} X_{L,t-3} + \Theta_{t-1} \rho_{t-1} X_{L,t-1} + \Theta_{t-2} \rho_{t-2} X_{L,t-2} - \Theta_{t-3} \rho_{t-3} X_{L,t-3}.
\]

Which implies
\[
\beta^t E \left[ \Pi^B_{t+4} - \sigma^b \left( \Pi_{t+1} \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}} + \rho_{t+1} \right) \right] = E_t \left[ \Pi^B_{t+1} - \sigma^b \left( \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}} + \rho_{t+1} \right) \right] \
+ \beta E_t \left[ \Pi^B_{t+2} - \sigma^b \left( 1 - \Theta_{t+1} \right) \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}\pi_{t+2}} + \rho_{t+2} \right] \
+ \beta^2 E_t \left[ \Pi^B_{t+3} - \sigma^b \left( 1 - \Theta_{t+1} \right) \frac{\left( R_{t+1, CB} - 1 \right)}{\pi_{t+1}\pi_{t+2}\pi_{t+3}} + \rho_{t+3} \right] \\
\Pi^B_{t+1} - \sigma^b \left( R_{CB,t} - R_{QE,t} \right) X_{L,t-1} + \Pi^B_{t+2} - \sigma^b \left( R_{CB,t} - R_{QE,t} \right) X_{L,t-1} + \Pi^B_{t+3} - \sigma^b \left( R_{CB,t} - R_{QE,t} \right) X_{L,t-1} - \phi_{QE} \Theta_t = 0.
\]