MONETARY POLICY AND THE YIELD CURVE AT ZERO INTEREST*

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Abstract

To extract market expectations about monetary policy from the yield curve when the short-term interest rate is zero, we use Black’s (1995) model of interest rates as options. We employ the extended Kalman filter to estimate the model with Japan’s data. This enables us to compute the probability density function of duration of zero interest under the actual probability measure. We find that the mode of duration has substantially varied and reached to around 3 years. We also find that the mode of duration has closely tracked survey measures of inflation expectations. These results suggest that inflation expectations are the primary determinant of monetary policy expectations and the yield curves under a zero interest rate environment.

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1. Introduction

In response to the global financial crisis, central banks in major developed economies lowered their targets for the short-term interest rates to virtually zero. In the United States, for instance, the Federal Reserve (Fed) brought its target for the federal funds rate down, in steps, from 4.25 percent to a range of 0 to 0.25 percent in 2008. Although this target is not exactly zero percent, this can be regarded as the lowest possible or virtually zero, since Chairman Bernanke (2010) claims that a further reduction of the target range could disrupt some key financial markets and institutions, as near-zero returns might induce many participants and market-makers to exit. In fact, the Fed has kept this target range for more than three years. Nevertheless, the history of zero interest rate is still limited to study monetary policy in such a situation.

Research on the case of Japan would help us to understand a low interest rate environment, since Japan has experienced extended periods of low interest rates since the mid-1990s. In 1995, the Bank of Japan (BOJ) lowered its official discount rate from 1.75 percent to 0.5 percent, which was the historical low at the time. Afterwards, as shown in Figure 1, the overnight (O/N) call rate has not surpassed around 0.5 percent. The BOJ also adopted several policies to further lower the call rate, combined with commitments conditional on inflation. From February 1999, the BOJ adopted the zero interest rate policy, under which the call rate was lowered to the lowest possible. In April 1999, Governor Hayami of the BOJ announced that the zero interest rate policy would be maintained until deflationary concerns subside. Although the BOJ terminated the zero interest rate policy and raised its target call rate to 0.25 percent in August 2000, it started the quantitative monetary easing policy (QMEP) in March 2001. In the QMEP, the BOJ
used the current account balances at the BOJ as the operating target and led the call rate to effectively stay at zero percent. This policy was combined with another commitment to maintaining the QMEP at least until the inflation rate of core CPI, i.e. CPI excluding perishables, stayed sustainably positive. Although the BOJ raised its call rate target in 2006-07 to 0.5 percent, it began to lower the target again in response to the increased severity of global financial crisis in October 2008. At the time of writing this paper, the BOJ commits to keep the virtually zero interest rate policy, in which the call rate is encouraged to remain at around 0 to 0.1 percent, until it judges that the 1 percent goal of CPI inflation is in sight.1

Even with long historical data, it is still not easy to study monetary policy if the short-term interest rate has been at the zero lower bound. Since the short-term interest rate is the conventional monetary policy instrument, it has been used in the greatest part of empirical analyses of monetary policy. However, under a low interest rate environment, a central bank attempts to stimulate the economy without changing the target for the short-term interest rate by, for instance, making commitments. Thus we cannot observe monetary policy stance from the short-term interest rate in such a situation, once it is stuck at the zero lower bound. Yield curve data have a potential to solve this problem, since longer-term interest rates reflect market expectations about the future pass of monetary policy even when the short-term interest rate is zero. Figure 2 displays a typical curvature measure of the yield curve, the 5-year zero-coupon Japanese Government Bond

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1 More specifically, the BOJ commits to continue pursuing the powerful easing including the virtually zero interest rate policy until it judges that the 1 percent goal of CPI inflation is in sight on the condition that the BOJ does not identify any significant risk, including the accumulation of financial imbalances.
(JGB) yield minus the average of the 1-year and 10-year yields. This figure shows that in general the curvature measure was negative, or the yield curve was concave, when the BOJ conducted the QMEP as well as after the BOJ began to lower its call rate target in response to the global financial crisis. The curvature measure sharply increased in the second half of 2005, when many market participants started to expect the end of the QMEP in the near future, and turned positive in February 2006, just before the BOJ terminated the QMEP. These observations suggest that a negative curvature measure reflects the market expectations that the BOJ would not raise its target rate for a considerable period, and that intermediate-term interest rates such as the 5-year yield are particularly sensitive to monetary policy expectations. This supports our view that the yield curve data may be useful to extract market expectations about monetary policy, particularly in the short-to-intermediate future, even when the short-term interest rate is zero.

To study the yield curve data, affine term structure models (Duffie and Kan (1996)) are prevailingly used in the literature. But, many of affine models do not take into account of the zero lower bound, and allow the short-term interest rate to be negative. In the Cox, Ingersoll, and Ross (1985) (CIR) model, a popular affine model, the short-term interest rate is non-negative. However, since the zero lower bound is a reflecting boundary rather than a sticky boundary in the CIR model, the short-term interest rate is expected to rise from the zero lower bound immediately if it reaches zero, or to “bounce off” zero in the words of Black (1995). This property of model clearly contradicts with the experiences in major developed economies: The short-term interest

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2 The zero-coupon yield data are obtained from Bloomberg and are identical to those used for estimation later.
rate has been stuck at virtually zero for a prolonged period. Hamilton and Wu (2012) modify their affine term structure model by adding a constant risk-neutral probability that the economy will remain at the zero lower bound next period. But the probability under the risk-neutral measure, an artificial probability measure, is hard to interpret. Perhaps more importantly, even if we can interpret this as an approximation for that under the actual probability measure, the constant probability appears to be inconsistent with our experiences. For instance, market perceptions about the probability of the termination of the QMEP conducted by the BOJ significantly varied as will be shown in this paper. In sum, affine models are unsuitable to use for extracting market expectations about monetary policy when the short-term interest rate is zero.

To overcome the limitations of affine term structure models, we use Black’s (1995) model of interest rates as options, which has a sticky boundary and allows the probability of remaining at the zero lower bound to be time-varying. Black’s (1995) model employs a shadow rate that can take on negative values. The nominal spot rate equals the shadow rate when the shadow rate is positive and zero otherwise. As the shadow rate is more negative, the probability of remaining at the zero lower bound is higher, the expected duration of zero interest is longer, and the yield curve is more concave. While Black (1995) only suggests the idea of the model, Gorovoi and Linetsky (2004) formulate this idea. In a version of their model, they assume that the shadow rate follows an Ornstein-Uhlenbeck process, as the spot rate in Vasicek’s (1977) model. They calibrate this model by fitting to the cross-section data of JGB yields at a selected date. Ueno, Baba, and Sakurai (2006) calibrate this Vasicek-type Black’s model at each point of time independently with their five-year sample.
We generalize Gorovoi and Linetsky’s (2004) model by allowing the market price of risk to be time-varying, as often found to be essential to capture the features of U.S. yield data. We employ the extended Kalman filter to estimate this non-linear model using time-series data from 1995 to 2011. As already seen, Gorovoi and Linetsky (2004) and Ueno, Baba, and Sakurai (2006) only calibrate their models by fitting to the cross-section yield data, and allow time-variation of the parameter values. Since in term structure models, the yield curve reflects the probability density function of underlying factors computed based on the assumption of time-invariant parameters of factor processes, time-varying parameters result in time-inconsistency and overfitting. To avoid these problems, we estimate the time-invariant deep parameters of model. With such time-series estimation, we can also identify the market price of risk. This enables us to break down yields into expectations components, averages of current and expected short-term interest rates, and term premia. We can also compute the probability density function of duration time of zero interest rate under the actual probability measure, whereas Ueno, Baba, and Sakurai (2006) only compute that under the risk-neutral measure. We calculate the mode from the probability density function and use it as the measure of representative market expectation about the duration of zero interest.

Our main findings are summarized as follows. First, the shadow rate fell into negative territory even when the call rate was around 0.5 percent and before the BOJ started the zero interest rate policy in 1999. This result implies that 0.5 percent has been

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3 After the working paper version of this paper and another related paper, Ichiue and Ueno (2006, 2007), show time-series estimation results of shadow rate models, a few papers conduct some extended exercises. Kim and Singleton (2011) compare several models including two versions of Black’s model, and confirm that Black’s model captures many features of JGB yield data. In contrast to our paper, however, they do not derive any policy implications. Krippner (2012) estimates a shadow rate model using U.S. data.
regarded as indistinguishable from zero percent by market participants, plausibly because the BOJ has been considered to care about the functions of short-term money markets and to have no room to lower the call rate further.

Second, the mode of duration time of zero interest rate has substantially varied and reached around 3 years under the QEMP. This result casts doubt on the assumption of a constant probability of remaining at the zero lower bound as Hamilton and Wu (2012) made under the risk-neutral measure. In addition, the mode shows a clearly negative correlation with the shadow rate. This relationship suggests a convenient way of interpreting the shadow rate: the shadow rate varies reflecting the market expectations about duration of zero interest rate.

Finally, the shadow rate closely tracks survey measures of inflation expectations rather than the actual core CPI inflation rate. Since the shadow rate varies reflecting the expected duration of zero interest, we can regard that the expected duration is closely linked with expected inflation. This suggests that market participants expect the future path of monetary policy based on their prediction of inflation. The BOJ’s series of commitments conditional on inflation may have helped strengthen the link between market expectations about the shadow rate and those about inflation.

The rest of this paper is organized as follows. Section 2 describes our model. Section 3 discusses data and estimation method. Then, we show the parameter estimates and evaluate how much the model captures the data. Section 4 examines the relationship among the shadow rate, the expected duration of zero interest rate, and inflation expectations. Section 5 concludes the paper.
2. The Model

This section describes our model, a generalization of Gorovoi and Linetsky’s (2004) Vasicek-type Black’s model. The nominal spot rate \( i_t \), which is defined as an interest rate per annum, is nonnegative. The spot rate is represented as a function of a shadow rate \( x_t \):

\[
i_t = \max(\lambda_t, 0).
\]  

(1)

That is, the nominal spot rate is equal to the shadow rate if the shadow rate is positive, and equal to zero otherwise. The shadow rate is the only factor in this model and is assumed to follow an Ornstein-Uhlenbeck process represented as

\[
dx_t = \kappa(\theta - x_t)dt + \sigma dW_t,
\]  

(2)

where \( \theta > 0 \) is the long-run level of the shadow rate or spot rate, \( \kappa > 0 \) is the rate of mean reversion toward the long-run level, \( \sigma > 0 \) is the volatility parameter, and \( W_t \) is a standard Brownian motion. The market price of risk \( \lambda_t \) is a linear function of the shadow rate:

\[
\lambda_t = \lambda_0 + \lambda_1 x_t
\]  

(3)

with two parameters \( \lambda_0 \) and \( \lambda_1 \). Our model nests Gorovoi and Linetsky’s (2004), as the latter is obtained by setting \( \lambda_1 = 0 \). We generalize their model by allowing the market price of risk to be time-varying, as often found to be essential to capture the features of U.S. yield data.4

Under the no-arbitrage assumption, the \( T \)-year zero-coupon yield \( i^{(T)}(x_t) \) can

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4 Duffee (2002) proposes essentially affine models, in which the market prices of risk can vary independently of interest rate volatility. Following his idea, we generalize the Vasicek-type Black’s model by allowing the market price of risk to be conditional on the shadow rate even though the shadow rate volatility is invariant in the model.
be expressed as

\[
i^{(T)}(x_t) = \frac{-1}{T} \log \left( \tilde{E}_t \left[ \exp \left( -\int_0^T x_s ds \right) \right] \right),
\]

(4)

where \( \tilde{E}_t[\cdot] \) is the conditional expectation operator under the risk-neutral measure. The Ornstein-Uhlenbeck process (2) can be rewritten as

\[
dx_t = \kappa (\bar{\theta} - x_t) dt + \sigma d\tilde{W}_t,
\]

(5)

where \( \kappa \bar{\theta} = \kappa \theta - \sigma \lambda, \quad \kappa = \kappa + \sigma \lambda, \) and \( d\tilde{W}_t = dW_t + \lambda_t dt \). \( \tilde{W}_t \) is a standard Brownian motion under the risk-neutral measure.

Although we generalize Gorovoi and Linetsky’s (2004) model, their solution method is still applicable. That is, to solve (4) for obtaining model-implied bond yields \( i^{(T)}(x_t) \) given \( x_t \) and parameter values, we use Gorovoi and Linetsky’s (2004) closed form solution as

\[
i^{(T)}(x_t) = \frac{-1}{T} \log \left( \sum_{n=0}^{\infty} c_n e^{-\omega_n T} \phi_n(x_t) \right)
\]

(6)

where \( c_n \) and \( \omega_n \) are some scalars, \( \phi_n(\cdot) \) is a first-order differentiable function, and these depend on \( \kappa \), \( \bar{\theta} \), and \( \sigma \).\(^5\) While the closed form solution (6) is represented with an infinite sum of terms, we in practice use a partial sum up to the 5th term, which is enough to attain considerable accuracy according to our preliminary investigations.\(^6\)

3. Estimation

This section considers data and estimation. Subsection 3.1 describes the data and the

\(^5\) In addition to Gorovoi and Linetsky (2004), see Ueno, Baba, and Sakurai (2006), who compactly review Gorovoi and Linetsky’s (2004) closed form solution for the Vasicek-type Black’s model.

\(^6\) We confirmed that we obtained similar results with a lattice method.
estimation method. Subsection 3.2 discusses the parameter estimation results. Subsection 3.3 evaluates the fitting performance.

3.1 Data and the Estimation Method

We use data of the uncollateralized O/N call rate, the policy interest rate of the BOJ, as the counterpart of the model-implied spot rate. As for the JGB zero-coupon yield data, the maturities included are 1-, 2-, 5-, and 10-years. We use end-of-month data over the period from January 1995 to December 2011. The estimation results of Black’s model tend to be influenced by intermediate-term interest rates such as the 5-year yield, since these interest rates are relatively sensitive to the shadow rate when the short-term interest rate is stuck at the zero lower bound. The Japanese government started to increase markedly the share of the issuance of the intermediate-term bonds around 1995, which has contributed to higher liquidity and more reliable prices of such bonds. This is an important reason for our choice of starting date of sample.

To estimate our continuous-time model with the discrete-time data, the model should be discretized. The discretized shadow rate process is represented as the following AR(1) process:

\[ x_t = \mu + \phi x_{t-1} + \nu \eta_t, \]

where \( \mu = \theta(1-\phi) \), \( \phi = e^{-\kappa/12} \), \( \nu = \sigma \sqrt{1 - e^{-\kappa/6} + \kappa} \), and \( \eta_t \sim N(0,1) \) i.i.d.

To estimate the unobservable shadow rate process, we employ the extended Kalman filter. Conditionally linearized observation equations are obtained by linearizing equations (1) and (6) around the one-month-ahead linear least squares forecast of shadow rate in the previous month, which is defined as \( x_{t|t-1} = \mu + \phi x_{t-1} \) according to the
discretized shadow rate process (7). Then equation (1) can be linearized as
\[ i_t = 1\{x_{gl-1} \geq 0\} \cdot x_t , \]
where \( 1\{x_{gl-1} \geq 0\} \) equals one if the linear least squares forecast \( x_{gl-1} \) is nonnegative, and zero otherwise. The observed O/N call rate \( i_t^{ON} \) is assumed to equal the linearized model-implied spot rate plus a measurement error \( \varepsilon_t^{ON} \):
\[ i_t^{ON} = 1\{x_{gl-1} \geq 0\} \cdot x_t + \varepsilon_t^{ON} . \]  
(8)

We also linearize the model-implied \( T \)-year yields \( i_t^{(T)}(x_t) \) around the linear least squares forecast of shadow rate as \( i_t^{(T)}(x_t) = i_t^{(T)}(x_{gl-1}) + i_t^{(T)}(x_{gl-1}) \cdot (x_t - x_{gl-1}) \). We assume that the observed yields \( i_t^{(T)} \) equal the corresponding linearized model-implied yields plus measurement errors \( \varepsilon_t^{(T)} \):
\[ i_t^{(T)} = i_t^{(T)}(x_{gl-1}) + i_t^{(T)}(x_{gl-1}) \cdot (x_t - x_{gl-1}) + \varepsilon_t^{(T)} , \]  
(9)

for \( T = 1, 2, 5, \) and \( 10 \). The model-implied \( T \)-year yields and their first derivatives at the linear least squares forecast, \( i_t^{(T)}(x_{gl-1}) \) and \( i_t^{(T)}(x_{gl-1}) \), are computed with the Gorovoi and Linetsky’s (2004) closed form solution (6). The measurement errors, \( \varepsilon_t^{ON} \), \( \varepsilon_t^{(1)} \), \( \varepsilon_t^{(2)} \), \( \varepsilon_t^{(5)} \), and \( \varepsilon_t^{(10)} \), are assumed to follow serially and mutually uncorrelated normal distributions.

Now we have a state space representation with a state equation of (7) and five observation equations expressed with (8) and (9). We use the Kalman-filter-based maximum likelihood estimation to estimate five parameters in the model: \( \kappa \), \( \theta \), \( \sigma \), \( \lambda_0 \), and \( \lambda_1 \). In the Kalman filter, we set the initial value of shadow rate to 2.25 percent, the call rate in the starting date of our sample, January 1995.
3.2. Parameter Estimates

Table 1 reports the estimates of the five parameters with their asymptotic standard errors. This table shows that all parameters are significantly different from zero at the 1 percent level. The long-run level of spot rate $\theta$ is estimated to be 1.62 percent, even though the call rate has never surpassed around 0.5 percent since September 1995, a very early part of our sample. This result is obtained by virtue of the steep shape of Japan’s yield curve.

The significant estimate of the slope parameter of the market prices of risk $\lambda_1$ implies that term premia vary depending on the shadow rate. To evaluate how much the shadow rate has driven term premia, we decompose the model-implied yields into two components. The first one is an average of current and expected future short-term interest rates, or the expectations component, as suggested by the pure expectations hypothesis. The other one is the term premium, and is computed as the model-implied yield minus the expectations component. Figure 3 shows the decomposition of the 5-year yield, and shows that the time-variation of the expectations component is much larger than that of the term premium. Thus the shadow rate has played a little role in driving the term premium. Although not reported, we obtain similar results with the decompositions of the 1-, 2-, and 10-year yields.

Instead of further discussing the parameter estimates for the continuous-time shadow rate process, we examine those for the discretized dynamics, $\mu$, $\phi$, and $\sigma_\eta$, since these make it easier to get intuition. Table 2 shows that these parameter values are close to those of AR(1) model for the call rate estimated using the sample from January 1980 to December 1994, which does not overlap with the sample we used for estimating the Black’s model. This suggests that the estimated parameters of the Black’s model are
reasonable, because the call rate is a proxy for the shadow rate and thus their time-series properties should be similar when the call rate is not stuck at the zero lower bound.

3.3. Fitting Performance

Figure 4 displays the observed and model-implied yield curves averaged over the period from March 2001 to February 2006, when the BOJ conducted the QMEP and the yield curve was notably concave. Our model shows good performance in replicating the convex-shaped yield curve, while the model has some difficulty matching the observed 10-year yield. Table 3 reports the root mean squared errors (RMSEs) for the call rate and the JGB yields, and confirms what are suggested in Figure 4. Our model fits the observed 1- and 2-year yields well, while the model fit to the 10-year yield is less accurate. These results appear to show the limitations of our one-factor structure of model, and we may need at least two factors to match both the shorter and longer term yields. However, for our aim to extract market expectations about monetary policy in the short-to-intermediate future and to derive policy implications when the short-term interest rate is zero, the model fit to the shorter-maturity yields is essential. We thus regard our one-factor model as an attractive parsimony framework.

In Table 3, the RMSE of the 5-year yield is the second largest, although that is just around a half of the RMSE of the 10-year yield. Here we discuss the model fit of the 5-year yield, since such intermediate-term interest rates are relatively sensitive to market expectations about monetary policy and thus are important to extract policy expectations, when the short-term interest rate is zero. Figure 5 displays the observed and model-implied 5-year yields together with the residual. This figure displays that the
model-implied 5-year yield generally tracks the observed one, although there are some exceptions, which result in a somewhat large RMSE. The model underestimates the observed yield for the early part of sample, which may be partly because the shadow rate is tied to the call rate in the start of sample. On the other hand, the model tends to overestimate the observed yield in the recent sample. This may reflect, for instance, strong demand from rapid-growing and high-saving emerging countries for bonds issued in developed economies, as discussed by Bernanke (2005).

4. Results

This section uses the estimated model to examine several issues. Subsection 4.1 observes the estimated process of shadow rate, which is found to have often foreseen future monetary policy. Subsection 4.2 uses Linetsky’s (2004) method to compute the expected duration time of zero interest rate. Subsection 4.3 shows that the shadow rate is tightly linked to survey measures of inflation expectations. This suggests that market participants anticipate monetary policy through predicting inflation, based on the understanding that the monetary policy authority cares much about inflation.

4.1. The Shadow Rate

Figure 6 displays the shadow rate, the only factor of this model, together with the call rate. This figure shows that the shadow rate tended to be negative in the early part of our sample, even when the call rate was maintained around 0.5 percent and before the BOJ commenced the zero interest rate policy in February 1999. This suggests that 0.5 percent has been regarded as indistinguishable from zero percent in effect by market participants,
plausibly because the BOJ was considered to care about the functions of short-term money markets and to have no room to lower the call rate further. The shadow rate remained negative even after the BOJ terminated the zero interest rate policy and raised the call rate target to 0.25 percent in August 2000. This may suggest that market participants regarded that the BOJ terminated just an abnormal policy, which may have done serious damage to the functions of short-term money markets, and expected that near-zero interest rates such as 0.25 percent would be maintained for some time.

The shadow rate sharply declined around the start of the QMEP in March 2001, and reached its historical low, -3.3 percent, in May 2003. This is less negative than that reported in Ueno, Baba, and Sakurai (2006), who obtain a shadow rate that is even lower than -15 percent during 2002. Kim and Singleton (2011) claim that a -15 percent shadow rate is hard to be rationalized, and that the problem appears to be specific to the one-factor structure of Ueno, Baba, and Sakurai’s (2006) model. We disagree with this view since our one-factor model generates a reasonable size of shadow rate. Instead, our result suggests that such a large negative value obtained by Ueno, Baba, and Sakurai (2006) is attributable to calibrations using only cross-section yield data.

The shadow rate started to soar in the second half of 2005, before the BOJ terminated the QMEP in March 2006. This reflects the sharp rises in the intermediate-term yields when market participants began to predict a quicker end of the QMEP than previously expected. Although the shadow rate turned positive after the termination of the QMEP, it again fell into negative territory in October 2008, when the BOJ began to lower its call rate target in response to the increased severity of global financial crisis. Since then, the shadow rate has steadily declined to -2 percent.
4.2. The Expected Duration of Zero Interest

The last subsection illustrates that the shadow rate has varied reflecting market participants’ expectations about monetary policy, even when the call rate was at the zero lower bound. This subsection quantitatively shows the representative market expectation about the duration time of zero interest by applying Linetsky’s (2004) method. We define the first hitting time as the duration time until the shadow rate hits 0.25 percent:

\[
\tau_t = \begin{cases} 
\min(\tau \geq 0; x_{t+\tau} = 0.25\%) & \text{if } x_t < 0.25\% \\
0 & \text{otherwise} 
\end{cases}
\]  

(10)

For central banks in major developed economies, 0.25 percent is a typical unit of a target change in the policy interest rate. In fact, the BOJ raised its call rate target to 0.25% in August 2000 when it terminated the zero interest rate policy, and also in July 2006 slightly after it ended the QMEP in March 2006. Thus, this definition of first hitting time can be considered the duration of zero interest.

Ueno, Baba and Sakurai (2006) compute such first hitting time too. In comparison with theirs, our definition and method have several advantages. First, Ueno, Baba and Sakurai (2006) compute the first hitting time to zero percent rather than to 0.25 percent. Since market expectations about when central banks will raise their target interest rates influence the yield curves, our definition is better to examine. Second, Ueno, Baba and Sakurai (2006) compute the first hitting time only under the risk-neutral measure, just an artificial probability measure. On the other hand, our time-series estimation of Black’s model enables us to identify the market price of risk and thus compute the first hitting time under the actual probability measure. Finally, as mentioned before, their estimated shadow rate is unreliable due to their method, calibrations using
only cross-section yield data. Thus so is their result of the first hitting time.

Figure 7 displays the probability density functions of the first hitting-time as of the end of May 2003, when the shadow rate reached its historical low, and December 2005, around a half year before the BOJ raised its call rate target in July 2006. The probability density function has shifted leftward, which means the duration became shorter. The mode, which corresponds to the peak of the probability density function and can be interpreted as the main scenario of the duration, has shifted from 2.8 to 0.6 years. Since the mode is more convenient than the mean to compare with survey forecasts, which can be interpreted as the main scenarios of survey respondents, we use the mode as the measure of market expectations about duration time of zero interest rate.

Figure 8 exhibits the expected duration time, together with the shadow rate. The expected duration shows a clearly negative correlation with the shadow rate. This relationship suggests a convenient way of interpreting the shadow rate: the shadow rate varies reflecting the expected duration of zero interest rate.

Figures 7 and 8 suggest that the expected duration has substantially varied. This result casts doubt on the assumption of a constant probability of remaining at the zero lower bound as Hamilton and Wu (2012) made under the risk-neutral measure.

4.3. The Shadow Rate and Inflation Expectations

The previous subsections suggest that market expectations about the duration time of zero interest rate drive the yield curve when the short-term interest rate is at the zero lower bound. Then, how do market participants expect future monetary policies? It is plausible that inflation expectations substantially influence monetary policy expectations
and thus are the primary determinant of the shadow rate. Central banks in major
developed economies including the BOJ are obliged to attain price stability. The
literature of monetary policy including Taylor (1993) empirically shows that central
banks have responded to inflation, and theoretically shows that such responses are
desirable to stabilize economic activities. Even if the short-term interest rates are at the
zero lower bound, market participants expect that central banks will not raise their policy
rates until inflation rates are high enough. Central banks can back up such expectations
by making commitments conditional on inflation. In practice, the BOJ adopted various
types of commitments. The zero interest rate policy was committed to be maintained
until deflationary concerns subside. The QMEP was promised to be kept at least until the
core CPI inflation rate stays sustainably positive. Currently, the BOJ commits to keep the
virtually zero interest rate policy until it judges that the 1 percent goal of CPI inflation is
in sight.

Figure 9 shows the 2- and 5-years ahead survey forecasts of CPI inflation
reported in Consensus Forecasts, together with the shadow rate. Consensus Forecasts
reports survey forecasts for 2-years ahead or more only around the 10th of April and
October every year. Since it is natural to consider that the surveys are conducted a few
weeks before the publication, we plot these forecasts with the shadow rate in March and
September. The figure shows that the survey forecasts closely track the shadow rate. This
supports our view that inflation expectations are the primary determinant of the shadow
rate. This suggests, even when the nominal short-term rate is zero, central banks can
influence the yield curve by strengthening their commitments to future policy rates
conditional on inflation rates. Closely looking at the figure, the shadow rate appears to be
somewhat lower than expected by the survey forecasts for 2004-05. This may suggest that the effect of BOJ’ commitment, according to which the QMEP would be maintained at least until the core CPI inflation rate stays sustainably positive, was stronger as the inflation rate was higher.

Figure 10 displays the actual core CPI inflation rate together with the shadow rate. This figure shows that the shadow rate is much less related to the actual CPI inflation than with the survey measures of inflation expectations. Actual CPI inflation rates are subject to various short-term fluctuations. For instance, the leap and plunge in 2008 reflect the developments in the international commodity market: the WTI index of crude oil price reached its historical high of 145 dollars per gallon in July 3rd, 2008, and then plunged below 35 dollars per gallon in December of the same year. Figures 6 and 7 suggest that market participants have not been bothered by such short-term fluctuations of actual CPI inflation rate, and have attempted to find out the underlying trend of inflation to predict future monetary policy.

5. Conclusion
Recently, central banks in major developed economies lowered their targets for the short-term interest rates to virtually zero. The zero lower bound as well as the limited historical data makes it difficult to extract market expectations about monetary policy. To overcome this challenge, we use a generalized version of Vasicek-type Black’s model. We employ the extended Kalman filter to estimate the model with the JGB yield data. The estimated model enables us to compute the probability density function of the duration time of zero interest rate under the actual probability measure. We calculate the
mode from the probability density function and use it as the measure of representative market expectation about the duration of zero interest.

Our main findings are summarized as follows. First, the shadow rate fell into negative territory even when the call rate was around 0.5 percent and before the BOJ started the zero interest rate policy in 1999. This result implies that 0.5 percent has been regarded as indistinguishable from zero percent by market participants, plausibly because the BOJ has been considered to care about the functions of short-term money markets and to have no room to lower the call rate further.

Second, the mode of duration time of zero interest rate has substantially varied and reached around 3 years under the QEMP. This result casts doubt on the assumption of a constant probability of remaining at the zero lower bound as Hamilton and Wu (2012) made under the risk-neutral measure. In addition, the mode shows a clearly negative correlation with the shadow rate. This relationship suggests a convenient way of interpreting the shadow rate: the shadow rate varies reflecting the expected duration of zero interest rate.

Finally, the shadow rate closely tracks survey measures of inflation expectations rather than the actual core CPI inflation rate. Since the shadow rate varies reflecting the expected duration of zero interest, we can regard that the expected duration is closely linked with expected inflation. This suggests that market participants expect the future path of monetary policy based on their prediction of inflation. The BOJ’s series of commitments conditional on inflation may have helped strengthen the link between market expectations about the shadow rate and those about inflation.

Currently, the short-term interest rates in major developed economies are
virtually zero and this situation is likely to continue for some time. For instance, in January 2012, the Fed started to announce that it anticipates that economic conditions are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. In addition, even after the end of virtually zero interest rate, we need to use the time-series data of such a situation to secure sufficient samples for reliable empirical exercises. Therefore, we must keep handling the zero lower bound of interest rate perhaps at least for next few decades. The methods proposed in this paper may contribute to overcoming this challenge in future research studies.
References


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<td>$\sigma$</td>
<td>$\lambda_0$</td>
<td>$\lambda_1$</td>
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<td>0.152</td>
<td>0.0162</td>
<td>0.0129</td>
<td>-0.304</td>
<td>5.44</td>
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<tr>
<td>(0.031)</td>
<td>(0.0009)</td>
<td>(0.0008)</td>
<td>(0.018)</td>
<td>(1.13)</td>
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Note: Standard errors are in parentheses.
Table 2: Time Series Properties of Shadow Rate and Call Rate

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\phi$</th>
<th>$\nu$</th>
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<tbody>
<tr>
<td>Shadow rate</td>
<td>0.0002</td>
<td>0.9874</td>
<td>0.0037</td>
</tr>
<tr>
<td>1995-2011</td>
<td>(0.0000)</td>
<td>(0.0026)</td>
<td>(0.0002)</td>
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<tr>
<td>Call rate</td>
<td>0.0005</td>
<td>0.9853</td>
<td>0.0043</td>
</tr>
<tr>
<td>1980-1994</td>
<td>(0.0009)</td>
<td>(0.0144)</td>
<td>(0.0003)</td>
</tr>
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Note: Standard errors are in parentheses. The upper estimates are those for the discretized process of the shadow rate, and are computed using the maximum likelihood estimates of the continuous-time process of the shadow rate. Their standard errors are calculated with the delta method. The lower estimates are obtained by an OLS regression of AR(1) model for the call rate over the sample from January 1980 to December 1994.
Table 3: Root Mean Squared Errors

<table>
<thead>
<tr>
<th></th>
<th>O/N</th>
<th>1Y</th>
<th>2Y</th>
<th>5Y</th>
<th>10Y</th>
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<tbody>
<tr>
<td>O/N</td>
<td>0.21</td>
<td>0.09</td>
<td>0.04</td>
<td>0.28</td>
<td>0.51</td>
</tr>
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</table>

Note: This table reports the root mean squared errors for the overnight call rate, and 1-, 2-, 5-, and 10-year JGB yields (percent per annum).
Figure 1: Overnight Call Rate and JGB Yields

Note: This figure shows the uncollateralized overnight call rate, and 1-, 2-, 5-, and 10-year JGB yields.
Figure 2: Curvature Measure of JGB Yield

Note: This figure reports a curvature measure, which is calculated as the 5-year yield minus the average of the 1- and 10-year yields.
Figure 3: Expectations Component and Term Premium of 5-year Yield

Note: This figure shows the expectations component and model-implied term premium of the 5-year yield.
Note: This figure reports the actual and model-implied yield curves averaged over the period from March 2001 to February 2006.
Figure 5: Fitting Performance of 5-year Yield

Note: This figure shows the observed and model-implied 5-year yields together with the observation error.
Note: This figure shows the estimated shadow rate and the uncollateralized overnight call rate.
Figure 7: Probability Density Functions of Duration of Zero Interest

Note: This figure shows the probability density functions of zero interest rate in May 2003 and December 2005.
Figure 8: Expectations about Duration of Zero Interest

Note: This figure shows market expectations about the duration time of zero interest rate, which are computed as the mode of duration time (years), and the shadow rate.
Figure 9: Inflation Forecasts and Shadow Rate

(a) The 2-years ahead inflation forecast and the shadow rate

(b) The 5-years ahead inflation forecast and the shadow rate

Note: Panels (a) and (b) show the 2- and 5-years ahead survey forecasts about CPI inflation taken from Consensus Forecasts, respectively, together with the shadow rate.
Figure 10: CPI Inflation and Shadow Rate

Note: This figure shows the year-on-year change of CPI (excluding perishables) and the shadow rate.